HANDBOOK
FOR
MACHINE DESIGNERS AND DRAFTSMEN

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AND
DRAFTSMEN

BY

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PREFACE

As an editor, the author's heart has often ached at the manner in which contributions to technical journals of permanent value and usefulness form a procession to the limbo of forgotten things and benefit none but those under whose eyes they happen to fall at the date of publication. This volume is primarily an effort to rescue from the oblivion of the out of print such contributions as are of direct use in the design of machinery. The search for material has not, of course, been limited to periodicals but has extended to the transactions of many engineering societies, wherein information is nearly as effectively buried as in the back numbers of periodicals. In filling the gaps that remained after the search was completed, willing friends have come to the author's assistance.

Not only is this the way in which this volume has been prepared, but the author is convinced that it is the only way, and, more than this, that there should be deliberate co-operation between contributors, editors and collectors, with efforts focused on books of this character as the ultimate outcome.

To be more specific, the author is under no delusions regarding the many things that should be between these covers but that are not, nor of those others of which the data presented are inadequate but, now that a place has been provided for the preservation of information of the sort here gathered together, he hopes that increased activity in the preparation and the publication of such information will follow. He will certainly be glad to do his part toward the incorporation of such information in future editions. Assistance may be rendered in other ways than by preparing contributions. Wide as the search has been, it is not possible that all of the articles and papers that contain desirable data have been discovered. Those who know of such sources of information are invited to forward memoranda of the places where they may be found.

Due credit to those who have supplied material will be found scattered through the volume. From the many who have given willing help it is almost invidious to make selections, but the author feels that it would be an injustice not to make special mention of Mr. J. A. Brown, Mr. Axel Pedersen and Prof. J. B. Peddle.

F. A. H.

NEW YORK,
November, 1913.

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HANDBOOK FOR MACHINE DESIGNERS AND DRAFTSMEN

MECHANICAL PRINCIPLES OF DESIGN

"When a thing is wholly right it is pretty sure to look right, though it may be pretty bad and appear to be fairly good or be absolutely bad and not appear so to the casual observer. . . . When a thing is known to be bad and it looks right to an observer, it is time for him to cultivate his taste. . . . When a thing is

right long and never gets fixed until it is too bad to use" (*Professor Sweet*).

Next to extent of wearing surface, the chief essential of durability is fit. Whatever destroys fit limits durability. The chief enemy of fit is local wear, because local wear means change of shape and hence loss of fit. Conditions that favor local wear favor short life.

The *stationary* cross rail of a planer wears hollow at the center because it is most used there. The *moving* platen of a milling machine wears in the same way and for the same reason. In both cases the conditions favor local wear. Were the stationary and moving pieces of the same length, neither would wear hollow, and truth in both cases would be indefinitely prolonged. With this construction, local wear being impossible, the form and hence the fit,

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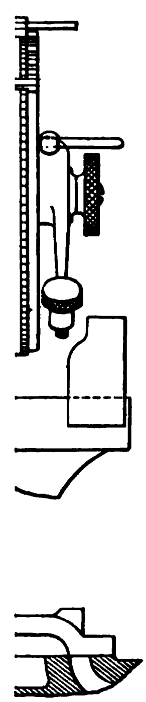


5.

FIGS. 1 to 5.—Examples of equal length wearing surfaces.

would destroy the accuracy of the machine, is prevented. Figs. 3, 4 and 5 are by PROFESSOR SWEET (*Amer. Mach.*, *Nov.* 17, 1904), who originated the principle. Fig. 3 shows the usual and bad construction of steam-engine valve-rod guides and Fig. 4 the correct construction in which the sliding surfaces are of equal length. Forty or fifty engines made in this way showed no wear after twelve or fifteen years of use while, should they wear, the slack can be taken up without refitting the wearing surfaces. Fig. 5 shows an application to the slide valve of a common steam engine. As commonly made, these

WANTED

CORRECTIONS—SUGGESTIONS—CONTRIBUTIONS

PLANS have been made for keeping this book continuously up to date. The general make-up, with each section beginning on a fresh page, while the illustration and table numbers of each section begin with No. 1, as well as the fact that the type forms have been preserved, will make it easy to drop old and introduce new material anywhere.

In this plan for continuous development the co-opera ion of all users of the book is solicited. Suitable contributions will be published first in the AMERICAN MACHINIST and will be paid for as contributions to that paper.

Contributions should be sent to the editor of the AMERICAN MACHINIST, 505 Pearl Street, New York. Other communications should be sent to the author at the same address.

Since future additions to this book will be published first in the AMERICAN MACHINIST, it is only necessary for each user of the book to follow the AMERICAN MACHINIST in order to keep his copy continuously up to date.

valves have the seat so long that the valve overruns but a short distance, the construction being due to the impression that increased surface gives increased life. This is bad practice, as the seat always wears concave. If it is designed to have the valve cut off at three-quarter stroke, the lap of the valve will be one-quarter the travel. If the ports and bridges are also one-quarter the travel, then by cutting away the valve face until it is only as long as the valve, as shown in Fig. 5, there will be the same wearing surface on the seat as on the valve and the two will remain straight and keep tight much longer than if made the common way.

There are cases to which the principle does not apply, an example being the beds and tables of planing machines. Here the chief load on the V's is the weight of the table which is not very stiff vertically. Were the bed and table of equal length, the flexibility of the table would lead, as it overruns, to a concentration of pressure at the ends of the bed and near the center of the table, leading to wear of the bed into a convex and of the table into a concave shape. In this and similar cases the bed should be the longer, F. A. Pratt's rule for planers under average conditions being that the length of the surfaces should be in the ratio of 17 to 12.

Equalized Wearing Surfaces

There are other cases in which the principle cannot be applied for obvious reasons, examples being the cross rails and saddles of planing and planer-type milling machines. In many such cases, including these examples, an alternative construction (also due to Professor Sweet) is available. As he has put it, "when conditions are known, flat guides may be made to stay flat and when conditions are not known, common practice may be improved." The principle here is to make the wearing surfaces proportional to their use. Fig. 6 shows this principle applied to the cross rail of a traversing machine, which is primarily a vertical spindle milling machine of the planer type. In the case of a planer the head on the cross rail does not move under the cut, whereas, in this case the head travels across under the cut. Hence, it was found that the cross rail wore out in the middle, and the cross rail was recessed as shown by the shaded sections, thus reducing the wearing surface where the head is used the least and *equalizing the wear*. This principle can be applied to great advantage wherever the wear is unequal. The exact extent to which it can be carried is undeterminable, because it is impossible to know how much the machine will be used with the head in the center or at the ends, but, as the surface is cut away, the result will be progressive improvement until too much is cut away and that is not apt to be done; while, if cut away too much, the result will be no worse than if cut away too little by the same amount, that is, it will still be better than if not cut away at all. Like the factor of safety, the amount to be cut away is a matter of judgment at first and of experience later.

In all cases the wearing surface of the guide should be cut away so that the slide shall overrun at the ends.

Assuming that the use of the head at different locations is proportional to its distance from the center of the rail, which is not far from average conditions, the correct method of laying out the widths of the parts to be recessed and of those to be left as lands is shown in the diagram below the cross rail. Draw the diagonal line across the guide surface. Locate the edge of the first recessed portion at *a*, when the distance from *a* to *b* gives the width *c* of the space to be recessed and the distance from *d* to *e* gives the width *f* of the land. Similarly, *gh* and *ij* give the width of the next space and land and so on to the end of the rail. The recesses are, of course, shallow—the principle is to get rid of the wearing surface where it does harm.

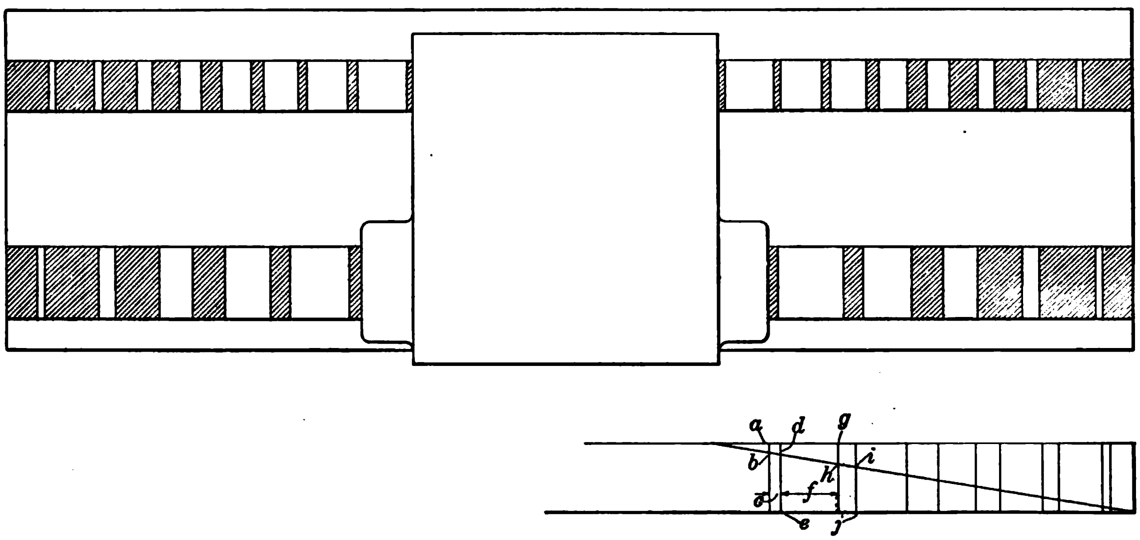


FIG. 6.—Bearing surfaces in proportion to use.

In Professor Sweet's traversing machine, as first made without the cut-away feature, the rail required refitting after two years' use, while, after it was cut away as above, it ran six years before refitting was necessary. Similarly a shaper slide, as first made without the cuts, required refitting after two or three years' use, while after being cut away it ran fifteen years.

Another illustration of the principle that things that do not tend to wear out of truth do not wear much, is found in the Schiele curve bearing, which see. The principle of this construction is uniformity of wear and it has remarkable durability. There is no question that its merits are not adequately appreciated.

The Narrow Guide

The narrow guide was first used by Professor Sweet on his traversing machine in 1886. The cross rail of that machine is shown in Fig. 7. Its merits, as contrasted with those of the usual construction, may best be realized by imagining the usual construction exaggerated in height, when its weakness against side tilting and its tendency toward local wear at the ends of the short guiding surfaces will be realized. Just as the usual construction is better than the exaggerated illustration, so the narrow guide is better than the usual form. The construction is such that there is vertical clearance at the top of the cross rail, the weight of the head being carried by the gib at the bottom.

Fig. 8 shows the narrow guide as applied to a lathe bed by John Lang and Sons, while Fig. 9 shows the natural development of the principle as adapted to the American V guide, the illustration being an end view of the Brown and Sharpe grinding machine. Professor Sweet advocated and practised the single V guide for lathe construction as early as 1876. Another application is found in the cross rail of the Bullard boring mill, Fig. 10, and still another in the arm of the Cincinnati-Bickford radial drill arm, Fig. 13.

Tubular Torsion Members

"The box section is the best form metal can be put into to resist the various strains machine frames are subjected to" (*Professor Sweet*).

The readiest way for the designer to learn the value of the tubular section as a torsion member is to compare, by twisting in his hands, two pieces of common pasteboard mailing tube, one complete and

The weakness of the slit tube is due to the absence of any provision for the longitudinal shearing stress. If, to meet structural or operative conditions, it becomes necessary to cut holes through the tube, it may be done without serious harm provided ample metal is left for the shearing stress.

The torsional stress on that member would make the bed of a lathe an ideal place for the tubular section, but for the necessity for getting rid of the chips, which makes the application of the complete tube impracticable. The Tangye lathe bed, Fig. 14, illustrates how the practical necessities can be met and most of the rigidity of the tubular section be preserved. Note that, in a partial tube of rectangular section, wide flanges *aa* are highly important.

In planer beds, unlike lathe beds, there is nothing to prevent the use of the tubular section with such openings as are required for the gears. The continued use of the ladder bed for planers is due to nothing more creditable than custom and precedent.

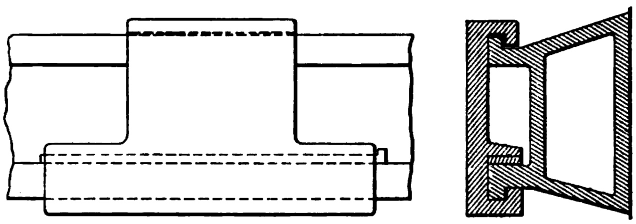


FIG. 7.

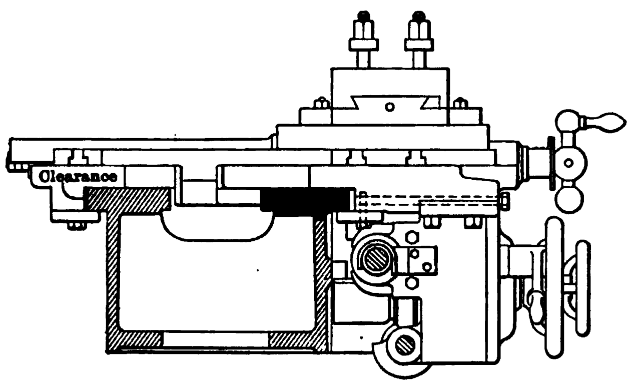


FIG. 8.

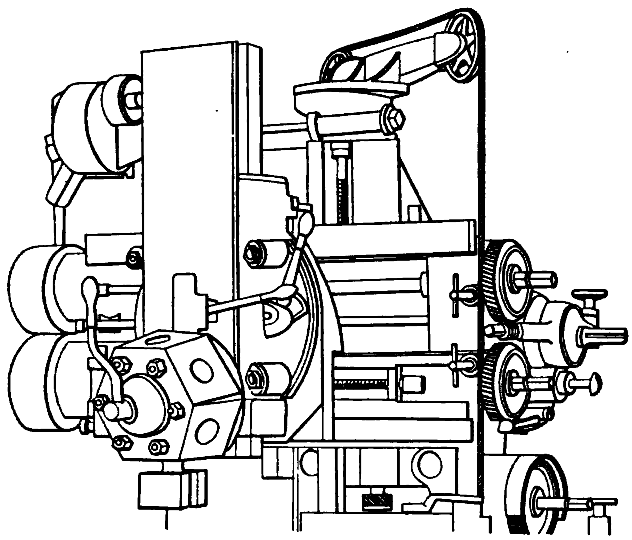


FIG. 10.

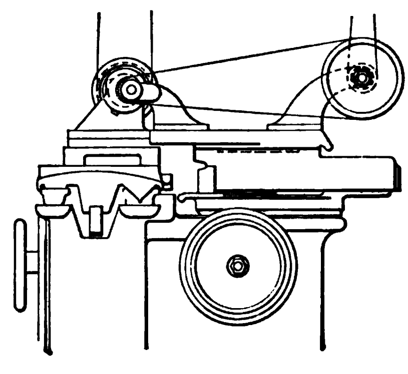


FIG. 9.

FIGS. 7 to 10.—Examples of the narrow guide.

the other slit down its length as shown in Fig. 11. The difference, which is simply startling and can scarcely be expressed in figures, is not a matter of the material but of the construction. Relatively speaking, the same difference exists between a cut and an uncut tube of iron. No possible addition of material can make up the loss due to slitting a tube. Next to a lath, the very common I-beam section, while ideal to resist bending, is about the worst possible distribution of metal to resist torsion.

An excellent example of the use of tubular sections in appropriate places is seen in Fig. 12, by the Beaman and Smith Co., in which both bed and upright are tubular. Another example is seen in Fig. 13, which is a section of the arm of a radial drilling machine by the Cincinnati-Bickford Tool Co. In the latter case the tubular section is combined with another correct construction—the narrow guide.

The Division of Functions

Many cases of improved design, when analyzed, are cases of the division or separation of the functions. The principle is of considerable application and deserves to be recognized.

The most common application of the principle is the well-known Pratt and Whitney pattern of turret lathe index ring and latch bolt, Fig. 15. Were the notches and bolt made of truncated V form, both sides would be equally concerned in the functions of locating and moving the plate, both must be made with equal accuracy and both would be subject to wear. In the construction shown, the functions are divided, the radial side doing the locating and the inclined side the moving to position. The result is that the radial side only need be of refined accuracy, while the wear is chiefly on the inclined side where it does no harm.

Another case is found in the loose center piece snap gage, Fig. 16. In the usual form of snap gage, one piece of metal determines the size of the gage and of the piece of work measured. In the form shown, the functions are divided, the center piece determining the size of the gage, while the jaws determine the size of the work. Wear is confined to the jaws and, after it occurs, the gage may be brought back to its original size by removing the jaws and lapping them flat. Note that, if a limit gage is to be made, further division of function must be made if the full advantage of the construction is to be realized. One jaw must be divided as in Fig. 17, the limits being made on the center piece, by which plan both limits once made, are permanent the lapping of the jaws flat being all that is necessary to remove the effects of wear.

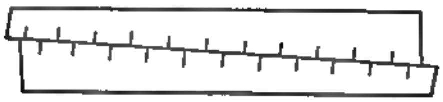


FIG. 11.

FIGS. 11 to 13.—Examples of the tubular torsion member.

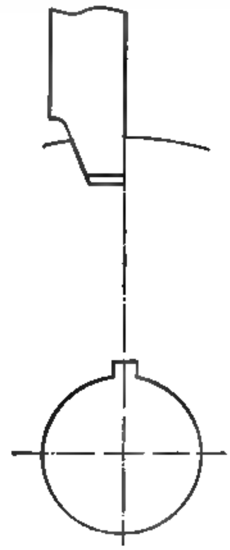


FIG. 15.

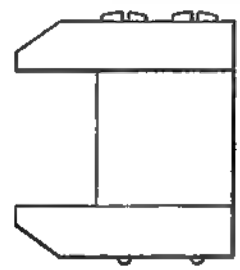


FIG. 16.

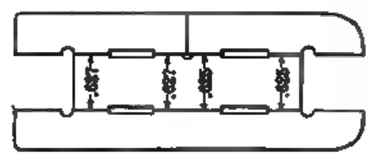


FIG. 17.

FIGS. 15 to 17.—Examples of the division of functions.

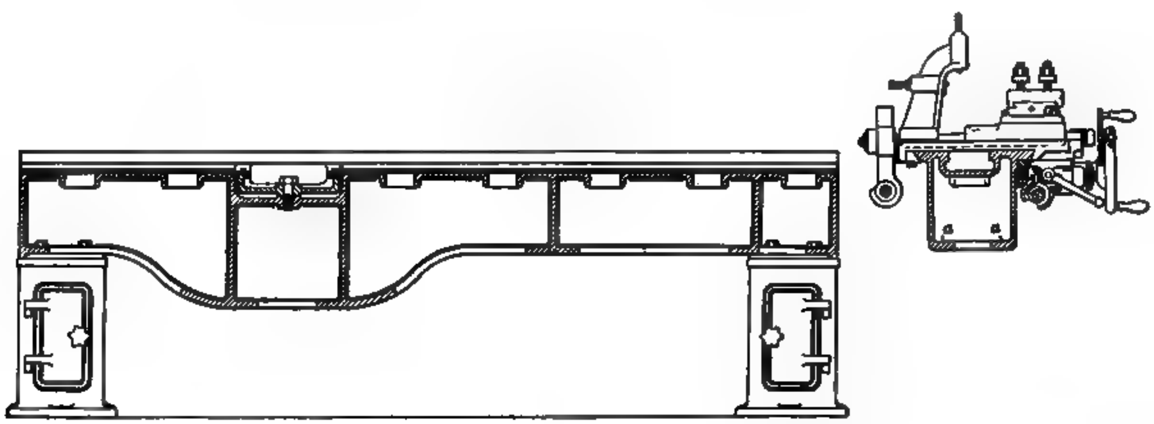


FIG. 14.—Example of a compromise tubular torsion member.

Another case is found in the Newall measuring machine head, Fig. 2. The functions of traversing and of carrying the weight of the parts are here divided, the screw *a* doing nothing but the traversing, bearings *bc* being provided to carry the weight. The chief cause of wear of the most essential piece—the screw—is thus removed.

All these are cases of obvious improvement and they are all applications of the principle of the division of functions.

Reducing the Overhang of Cranks

The common method of making overhung cranks with the hub on the rear side, Fig. 18, leads to an amount of overhang which may easily be reduced by reversing the position of the hub as in Fig. 19, by PROFESSOR SWEET (*Amer. Mach., Dec.* 8, 1904). The improvement is obvious and it costs nothing.

The Center of Pressure should be at the Center of a Bearing

The neglect of this principle is almost universal and leads to the bell-mouthed wear of the bearings—that is to local wear which always leads to short life. The correct construction is not always possible, although it is possible in many cases in which it is not used. A common case of bad construction is that of the rock shaft introduced to provide for the offset of the eccentric and valve rods of slide-valve steam engines. In the common construction, Fig. 20, the tendency is to oscillate back and forth around a vertical center line, wear the hole bell-mouthed at both ends and wear off the shaft in like manner. Fixing the rocker to the shaft as in Fig. 21 is better, as it not only throws the bearings farther apart but they are better lubricated as the oil can be introduced on the slack side. Such rocker arms are best when cast of hollow box section, as that form is best to resist torsion.

Where a form such as shown in Fig. 22 can be used, it is a great improvement if a line drawn from the center of one wrist to the center of the other passes centrally through the main bearing. The form shown in Fig. 23 is better still, for the reason that Fig. 21 is better than 20.

The same principle is embodied in the form shown in Fig. 24, which, however, requires ball connections for the eccentric rod, although it requires much less of a projection for the supporting bracket (*Professor Sweet, Amer. Mach., Dec.* 8, 1904).

Frames and Supports.

"Whenever inconsistent or useless things are stuck on to improve the appearance, they always fail. To be good, a design must be consistent" (*Professor Sweet*).

Any machine frame standing on three legs is free from twisting stress and from the resulting distortion. When machines have considerable height, as in the case of a lathe, the omission of one of the

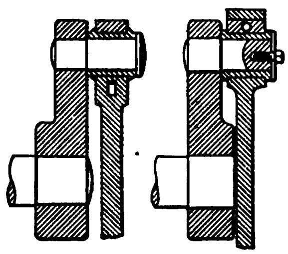


FIG. 18. FIG. 19.

FIGS. 18 and 19.—Reducing the overhang of cranks.

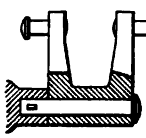


FIG. 20.

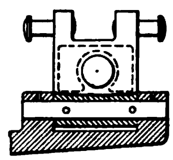


FIG. 21.

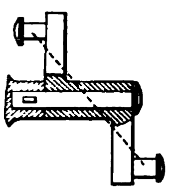


FIG. 22.

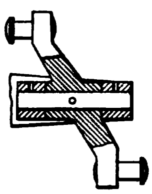


FIG. 23.

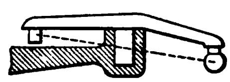


FIG. 24.

FIGS. 20 to 24.—Correct and incorrect constructions of rocker arms.

customary four legs would lead to a lack of stability, but this condition can be met by swivelling the leg to the frame as in Fig. 25, which shows the construction used at this point in Professor Sweet's Artisan speed lathe, which also has a tubular bed.

The customary location of supports under the extreme ends of machine frames leads to an unnecessary increase of span and of spring, while the placing of a third pair of legs under the center should be a last resort. Machines should be complete in themselves, whenever possible, and not depend on foundations for maintenance of form. In the planer bed, Fig. 26, the distance between the supports is no greater than in Fig. 27, and as the center of the load in planing would, in no case, overhang the supports more than a slight distance, the construction shown in Fig. 26 is quite as well supported as the other, and if the iron in the legs and the work to fit them were put into the casting, the bed could be brought down to the floor as in Fig. 28, greatly improving the structure.

Were the bed made of tubular section, with one leg under the back of each housing and one under the middle toward the other end, the results would be still better. Such a planer could be set anywhere on anything solid, and that is all that need or can be done (*Professor Sweet, Amer. Mach., Mar.* 9, 1910).

An example of correct frame design and support on these principles is seen in Fig. 29, which shows a Norton grinding machine. The underneath view shows the arrangement of the three points of support and of the connecting ribs.

One of the main points in designing frames is not to expose thin sections, as in the case of holes through plates and webs. In a standard or column 12 ins. on the sides, or in diameter, the exposed sections should not be less than $1\frac{1}{2}$ ins.; or, to make a rule, the exposed sections should be equal to an eighth of the extreme faces, as in Figs. 30 and 31. External beads should not be employed, because they convey an idea of thin sections unless their width corresponds to the flange *e*, or to the base flanges of the frame.

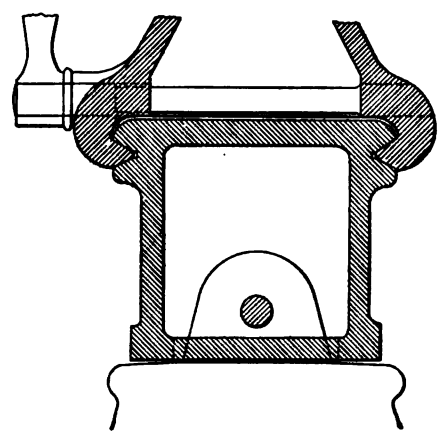


FIG. 25.—Example of a pivotted tailstock lathe leg and tubular bed.

Another feature that has a good deal to do with the symmetry of frames is the thickness of base flanges. These should follow the rule of exposed sections and equal an eighth of the faces or the diameter of the trunk above when the latter is either round or rectangular. This is required not only to produce harmony of dimensions, but to insure against accident by fracture.

The base flanges for frames or pedestals larger than 10 ins. in diameter should be cored out beneath, as shown in Fig. 32. The top corners of base flanges, when of the proportions named, should be

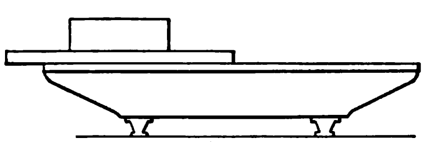


FIG. 26.

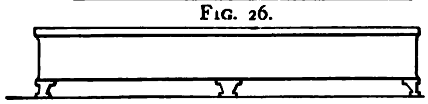


FIG. 27.

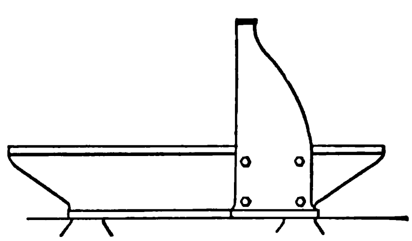


FIG. 28.

FIGS. 26 to 28.—Correct and incorrect supports for planers.

rounded to a radius of about one-fourth the thickness so as to avoid the contour indicated in Fig. 33, which is a monstrosity of the "ogee" order of architecture.

Struts are difficult things to bring into harmony with machine framing, especially when connected to cylindrical or rectangular sections as in Figs. 34, 35 and 36. When the frame is cored as in Fig. 35, the best way is to use a solid section for the strut as at *a*,

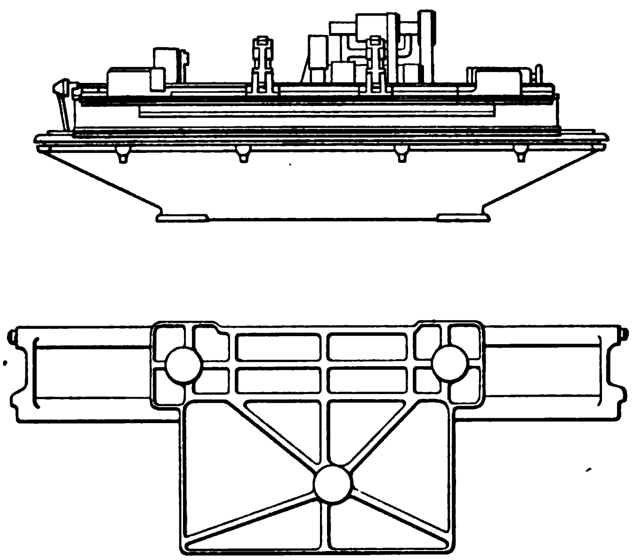


FIG. 29.—Example of correct frame design and support.

Figs. 34 and 36, the corners being slightly rounded so as to harmonize with the other members but never made semicircular. This is always wrong and looks so.

Struts, ties and braces should be in straight lines, unless set in curved intersections for reinforcement, as at *e*, in Fig. 37. If the corner at *a* is of short radius, as in pipe flanges, the brace *e* should be straight.

In rib sections, of which Fig. 38 shows examples, there is the common mistake of considering the web *a* as a principal member and the flange *e* a reinforcement. This leads to a thicker section for the plate, and is a waste of material. The web *a* is no more than a brace or tie, and should always be made as thin as it can be cast without cooling strains, usually not to exceed one-half as thick as the members *e*.

The curved form for bracing ribs as in Fig. 39 is still adhered to in most cases by habit, and because we reluctantly abandon the old idea of curves and ornament, but we are fast reaching the point when the shape shown in Fig. 40 will be substituted for the curves.

Fig. 41 shows a cored section which is especially suitable for large frames, and conveys an idea of an indented surface rather than of ribs, and is a means of relieving broad, flat surfaces that always look "skinny" unless perfectly flat and smooth. It also forms a reinforcement of corners, which are the weakest part, and for any machine of fine character the corners can be finished (*John Richards, Amer. Mach., June* 8, 1899).

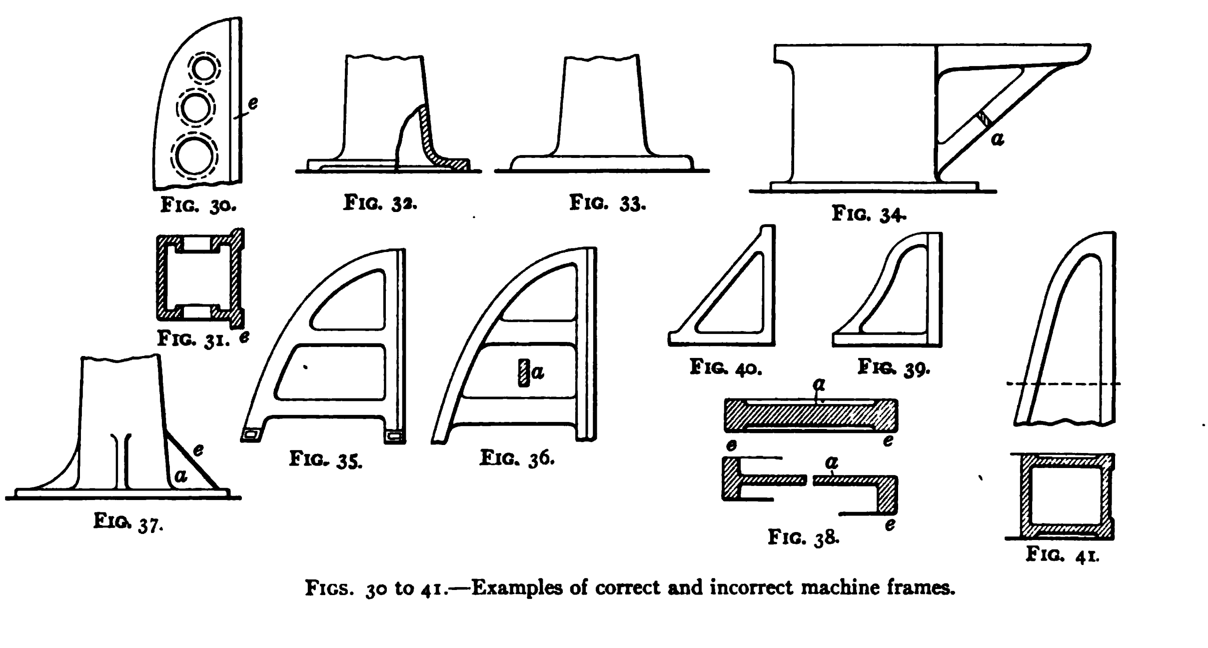
The outline sketch, Fig, 42, shows an appropriate base form from which to derive suitable frames for a great variety of purposes. Appropriate modifications to provide a base and attachments for bearings are shown in Fig. 43.

It will be a matter of astonishment to those who have not previously considered the matter, to discover the extent to which this form of the projecting beam or bracket enters into machine-tool framing. In that type called vertical machines, such as those for drilling, slotting, and planing, nearly all have this feature, and it has beside a wide place in horizontal supports that project from the main standards, such as tables for drilling, or other purposes. It is, therefore, well worth considering as a distinctive feature in design.

This form of standard is often spoiled by inharmonious bolted-on parts, such as have a ribbed section when the main member is hollow or cored. This is an incongruous thing, too common in practice. There is nothing saved and generally something lost by attaching ribbed parts to box framing. Good practice demands that all bearings requiring positive alignment be cast integral with the main frame and in harmony therewith (*John Richards, Amer. Mach., May* 25, 1899).

Charts in Systematic Design

The use of charts in systematic machine design is illustrated by a very simple case in Figs. 44–47 by H. S Britt (*Amer. Mach., Mar.* 22, 1906).



FIGS. 30 to 41.—Examples of correct and incorrect machine frames.

Assuming a line of sizes of any part of a type in which judgment is the chief element in the design to be in contemplation, two sizes near the extremes are first designed, after which the intermediate sizes are taken direct from a diagram, Fig. 46, which is first laid down to the sizes already designed.

For example, suppose it is desired to get up a line of boxes, such as are shown, for shaft sizes from 15/16 in. to 2 15/16 ins. inclusive. First the 15/16-in. size, Fig. 44, and the 2 15/16-in. size, Fig. 45, would each be laid out, the design and proportions being determined by the judgment of the designer. The chart, Fig. 46, is then constructed by plotting the values of each dimension for the large and small sizes and connecting the plotted points by straight lines, when the ordinates corresponding to the intermediate sizes determine that particular dimension for those sizes. The letters showing to what dimension each line refers correspond to those in Fig. 47. Part of

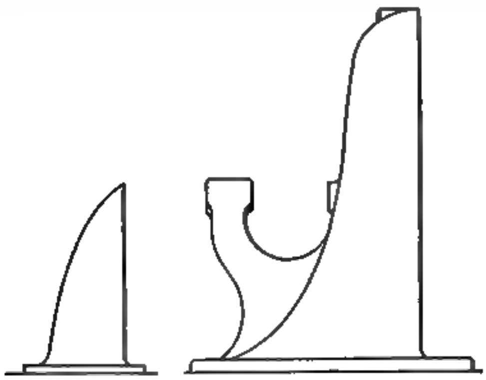


FIG. 42 FIG. 43.

FIGS. 42 and 43.—Correct frame construction.

the lines are laid off to the scale on the right side of the chart. These are distinguished from the remaining lines, which are to the scale on the left, by being dotted. From the chart the table, Fig. 47, is filled out.

For instance, suppose it is desired to find the width D for the 1 7/16-in. size. On the chart the intersection of the vertical line marked 1 7/16 and the inclined line D is found to be close to the horizontal line corresponding to 2 3/8 in. on the right-hand scale. The dimension thus obtained is entered in the table.

It will be noticed that there are no lines on the chart for dimensions F and G. A line is plotted for the distance from the center of the bolt holes to the outside of the metal around the bearing, or $\frac{1}{2}(F-B)$, and from this F is determined, the B column having previously been filled out. A line is also plotted for the distance from the center of the bolt holes to the ends of the bases or $\frac{1}{2}(G-F)$, the line in this case coinciding with the line for E. $\frac{1}{2}(G-F)$ having been obtained from the chart, G is found from F by addition in the same way as F was previously found from B.

The reason for determining these two dimensions in this indirect manner is that these dimensions depend partly upon B and partly upon the size of the bolts, and for that reason will not increase in a regular manner, the increase being greater whenever the size of bolt changes. In this particular, as in many others, judgment and discretion are necessary in the use of such a method.

In general, the lines thus found will not pass through the origin but above it. After some experience with the method, judgment will enable one to use it with only one originally designed size if the new sizes are not too much larger than it. The dimensions of one size being plotted, the lines are drawn through the plotted points and at such distances above the origin as experience indicates.

FIG. 46.

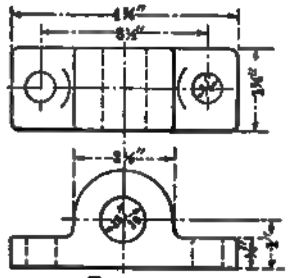


FIG. 44

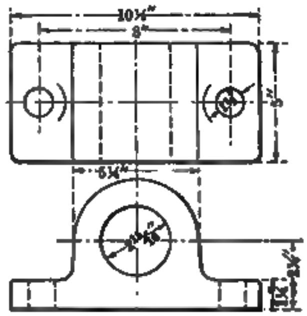


FIG. 45.

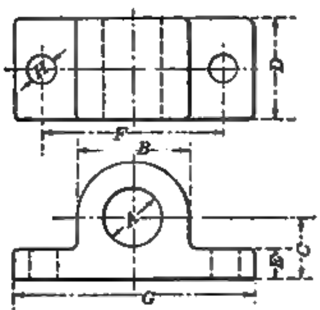


FIG. 47.

FIGS. 44 to 47.—Chart method for the systematic design of machine parts.

This method is safer if the new sizes are smaller than the original. When larger, caution should be used by comparing the resulting parts with the chart and correcting the latter if found desirable.

PLAIN OR SLIDING BEARINGS

For additional information on steam-engine bearings see Bearings for Steam Engines.

For additional information on gas-engine bearings see Bearings for Gas Engines.

For the fit allowances of bearings see Press and Running Fits.

TABLE 1.—PERMISSIBLE PRESSURES ON BEARINGS FOR STEAM ENGINES AND OTHER MACHINES

Kind of bearing and condition of operation	Allowable bearing pressure in pounds per square inch of projected area
Bearings for very low speeds and intermittent service as in turntables and bridges.	7000 to 9000
American Railroad Practice	
Locomotive cross-head pin bearings	3000 to 4000
Locomotive crank pin bearings	1500 to 1700
Locomotive driving wheel journal bearings	.... to 550
Car axle bearings	300 to 325
Tender axle bearings	.... to 425
British Railway Practice	
Locomotive crank pin bearings	.... to 1400
Locomotive cross-head pin bearings	.. . to 2000
Locomotive driving axle bearings	250 to 300
Car axle bearings	.... to 330
United States Naval Practice	
Main engine bearings	275 to 400
Main engine crank pin bearings	400 to 500
Steam turbine bearings (for weight alone)	.... to 85
Thrust bearings for torpedo boats	.... to 50
Merchant Marine Practice	
Main engine bearings	400 to 500
Main engine crank pin bearings	400 to 500
Thrust bearings	70
High-speed Stationary Engine Practice	
Main bearings (for dead load)	60 to 120
Main bearings (for steam load)	150 to 250
Crank pin bearings, overhung crank	900 to 1500
Crank pin bearings, center crank	400 to 600
Cross-head pin bearings	1000 to 1800
Slow-speed Stationary Engine Practice	
Main bearings (for dead load)	80 to 140
Main bearings (for steam load)	200 to 400
Crank pin bearings	800 to 1300
Cross-head pin bearings	1000 to 1500
Air Compressor Practice[1]	
Straight line, steam-driven, 100 lb. steam and air	
Main bearings	160 to 237
Crank pin bearings	565 to 700
Cross-head pin bearings	628 to 820
Straight line, belt-driven, center crank, 100 lb. steam and air	
Main bearings	122 to 220
Crank pin bearings	244 to 402
Cross-head pin bearings	400 to 785

[1] Canadian Ingersoll-Rand Company

TABLE 1.—PERMISSIBLE PRESSURES ON BEARINGS FOR STEAM ENGINES AND OTHER MACHINES.—(*Continued*)

Kind of bearing and condition of operation	Allowable bearing pressure in pounds per square inch of projected area
Straight line, belt-driven, side crank, 100 lb. steam and air	
Main bearings	178 to 227
Crank pin bearings	628 to 825
Cross-head pin bearings	628 to 825
Straight line, steam-driven, side crank, 100 lb. steam and air	
Main bearings	198 to 227
Crank pin bearings	462 to 825
Cross-head pin bearings	462 to 825
Duplex, Meyer cut-off, steam-driven, 100 lb. steam and air	
Main bearings	157 to 200
Crank pin bearings	644 to 855
Cross-head pin bearings	850 to 1370
Duplex Corliss valve gear, steam-driven, 100 lb. steam and air	
Main bearings	115 to 141
Crank pin bearings	513 to 708
Cross-head pin bearings	732 to 1150
Direct-connected, motor-driven main bearings	.... to 70
Gas Engine Practice	
Main bearings	500 to 700
Crank pin bearings	1500 to 1800
Cross-head pin bearings	1500 to 2000
Electrical Machinery Practice	
Generator and motor bearings	30 to 80
Main engine bearings, driving generators	40 to 80
Horizontal steam turbine bearings	40 to 60
Vertical steam turbine steps	200 to 1000

Rolling Mill Practice[1]

	Rubbing velocity, Ft. per Min.	
Pinion housing bearings	350 to 600	30 to 50[2]
Roll housing bearings	350 to 600	100 to 2000[2]
Table roller bearings	150	30 to 50
Table line-shaft bearings	150	30 to 50
Main bearings of shears	50 to 65	1800 to 2500

Miscellaneous Practice

Bearings for slow-speed and intermittent load as in punch presses, shears, and the like.	3000 to 4000
Main bearings of slow-speed pumping engines	.... to 600
Heavy line-shaft bearings, bronze or babbitt lined	100 to 150
Light line-shaft bearings, cast-iron	15 to 25
Heavy slow-speed step bearings	.... to 2000
Drill press thrust collars	.... to 325
Angular-thrust bearing for boring mill tables	.... to 75[3]

[1] Mesta Machine Company, Pittsburg, Pa.

[2] These factors are of value as showing good practice, not for purposes of design. The diameters and lengths of the bearings are determined by the requirement of strength in the pinion and roll necks and their housings.

[3] Practice of Bullard Machine Tool Company.

"Whenever it is possible to give journals end play, it will be found that they polish rather than cut and endure rather than wear out. The wearing surfaces should be of equal length in the box and shaft and be so controlled that the overrun will be equal at each end" (*Professor Sweet*).

Much of what follows is taken from the exhaustive treatise, Bearings and Their Lubrication, by L. P. Alford, to which the reader is referred for much additional information not to be found elsewhere.

The lack of a complete theory connecting the pressures, velocities and temperatures of bearings, until it was supplied by AXEL K. PEDERSEN (*Amer. Mach., Oct.* 10, 1912) and given below, has made it impossible to determine pressure factors of general application. Nevertheless, the factors in common use, within the fields from which they were obtained and to which they are intended to apply, are useful and adequate.

Tables 1 to 8 give such pressure factors. Tables 2 to 8 are by G. W. LEWIS and A. G. KESSLER (*Amer. Mach., Aug.* 31, *Sept.* 14, *Nov.* 9, 1911). They are the result of an extended investigation and correctly represent modern practice.

TABLE 2.—MAIN BEARING PRESSURES FOR STATIONARY GAS ENGINES

Horizontal						
D	4	8	12	16	20	Assumed
Dmb......	1.3	3.1	4.85	6.6	8.4	
Lmb......	2.6	6.75	10.8	14.9	19.1	
Amb......	3.4	20.9	52.5	98.5	160.5	*Dmb*×*Lmb*
Pm			250			Assumed
Kmb......	462	300	270	255	244	
Pm			300			Assumed
Kmb......	553	360	324	307	293	
Pm			350			Assumed
Kmb......	647	420	377	358	342	
Pm			400			Assumed
Kmb......	738	503	430	408	391	

Vertical						
D	4	8	12	16	20	Assumed
Dmb......	$1\frac{1}{2}$	$3\frac{1}{2}$	$5\frac{1}{2}$	$7\frac{1}{2}$	$9\frac{1}{2}$	
Lmb......	$3\frac{1}{2}$	$6\frac{3}{4}$	10	13	16	
Amb......	5.25	23.6	55	97.5	152	*Dmb*×*Lmb*
Pm			250			Assumed
Kmb......	300	267	258	258	258	
Pm			300			Assumed
Kmb......	359	320	310	310	310	
Pm			350			Assumed
Kmb......	415	373	360	360	360	
Pm			400			Assumed
Kmb......	481	414	414	414	414	

D = cylinder diameter, ins.
Dmb = main bearing diameter, ins.
Lmb = main bearing length, ins.
Amb = projected area main bearing (one) = *Dmb*×*Lmb*.
Pm = maximum explosion pressure, lbs. per sq. in. of piston face.
Kmb = maximum unit bearing pressure, lbs. per sq. in. considering explosion to occur on dead center.

TABLE 3.—CRANK-PIN BEARING PRESSURES FOR STATIONARY GAS ENGINES

Horizontal						
D	4	8	12	16	20	Assumed
Dcp......	$1\frac{1}{2}$	$3\frac{1}{8}$	$4\frac{3}{4}$	$6\frac{3}{8}$	$8\frac{1}{8}$	
Lcp......	$1\frac{5}{8}$	$3\frac{1}{4}$	$4\frac{7}{8}$	$6\frac{9}{16}$	$8\frac{3}{8}$	
Acp......	2.44	10.15	23.2	41.75	68	*Dcp*×*Lcp*=*Acp*
Pm			250			Assumed
Kcp.....	1290	1240	1220	1210	1150	From equation A
Pm			300			Assumed
Kcp.....	1550	1485	1450	1450	1390	From equation A
Pm			350			Assumed
Kcp.....	1800	1730	1710	1690	1620	From equation A
Pm			400			Assumed
Kcp.....	2060	1980	1950	1930	1850	From equation A

Vertical						
D	4	8	12	16	20	Assumed
Dcp......	$1\frac{5}{8}$	$3\frac{1}{4}$	$4\frac{7}{8}$	$6\frac{1}{2}$	$8\frac{3}{16}$	
Lcp......	$1\frac{5}{8}$	$3\frac{5}{8}$	$5\frac{5}{8}$	$7\frac{5}{8}$	$9\frac{5}{8}$	
Acp......	2.64	11.8	27.8	49.75	78.75	*Dcp*×*Lcp*
Pm			250			Assumed
Kcp.....	1190	1065	1035	1015	995	
Pm			300			Assumed
Kcp.....	1430	1280	1240	1215	1200	
Pm			350			Assumed
Kcp.....	1660	1490	1440	1420	1400	
Pm			400			Assumed
Kcp.....	1920	1720	1660	1620	1600	

D = cylinder diameter, ins.
Pm = maximum explosion pressure, lbs. per sq. in. of piston face.
Dcp = bearing diameter of crank pin, ins.
Lcp = bearing length of crank pin, ins.
Kcp = maximum unit bearing pressure, lbs. per sq. in.
Acp = *Dcp*×*Lcp*, sq. ins.

$$\text{Max. } Kcp = \pi\frac{D^2}{4}Pm \div (Dcp \times Lcp). \quad \text{(A)}$$

TABLE 4.—WRIST OR PISTON-PIN BEARING PRESSURES FOR STATIONARY GAS ENGINES

Horizontal						
D	4	8	12	16	20	
Dwp.....	0.93	1.62	2.76	4.36	6.42	From equation (A)
Lwp.....	1.6	2.8	4.77	7.52	11.15	From equation (B)
Awp.....	1.49	4.54	13.2	32.8	71.5	$Dwp\times Lwp$
Pm			250			Assumed
Kwp.....	2100	2760	2145	1530	1100	
Pm			300			Assumed
Kwp.....	2530	3320	2570	1840	1320	
Pm			350			Assumed
Kwp.....	2950	3880	3000	2150	1540	
Pm			400			Assumed
Kwp.....	3370	4425	3430	2455	1760	

$Dwp = .0143\ D^2 + .7$ in. (A)

$Lwp = 1.75\ Dwp$ (B)

D = cylinder diameter, ins.

Dwp = bearing diameter of piston pin, ins.

Lwp = bearing length of piston pin, ins.

Vertical						
D	6	8	12	16	20	
Dwp.....	$1\frac{5}{8}$	$1\frac{7}{8}$	$2\frac{1}{2}$	$3\frac{3}{8}$	$4\frac{1}{2}$	Equation (C)
Lwp.....	$2\frac{7}{8}$	$3\frac{3}{8}$	$4\frac{1}{2}$	$6\frac{1}{8}$	$8\frac{1}{8}$	Equation (D)
Awp.....	4.67	6.33	11.25	20.65	36.6	$Dwp\times Lwp$
Pm			250			Assumed
Kwp.....	1510	1990	2520	2430	2145	
Pm			300			Assumed
Kwp.....	1810	2380	3020	2920	2580	
Pm			350			Assumed
Kwp.....	2120	2780	3520	3410	3010	
Pm			400			Assumed
Kwp.....	2420	3190	4030	3900	3440	

$Dwp = .00795\ D^2 + 1\frac{3}{8}$ in. (C)

$Lwp = 1.82\ Dwp$ (D)

Awp = projected area piston pin, sq. ins.

Pm = maximum unit explosion pressure.

Kwp = maximum unit bearing pressure, lbs. per sq. in.

TABLE 5.—MAIN BEARING PRESSURES FOR AUTOMOBILE ENGINES

Center bearings					
D	4	$4\frac{1}{2}$	5	$5\frac{1}{2}$	
Dcb	$1\frac{9}{16}$	$1\frac{3}{4}$	$1\frac{7}{8}$	$2\frac{1}{16}$	From equation (5)
Lcb	$2\frac{3}{16}$	$2\frac{1}{2}$	$3\frac{1}{16}$	$3\frac{5}{8}$	From equation (6)
Acb	3.42	4.3	5.75	7.5	$Dcb\times Lcb$
Pm		250			Assumed
Kcb	690	615	620	575	
Pm		300			Assumed
Kcb	830	730	750	690	
Pm		350			Assumed
Kcb	970	855	870	810	
Pm		400			Assumed
Kcb	1100	980	1000	920	

D = cylinder diameter, ins.

Dcb = diameter of center bearing, ins.

Lcb = length of center bearing, ins.

Kcb = maximum unit bearing pressure, lbs. per sq. in.

Pm = maximum explosion pressure, lbs. per sq. in. of piston face.

$Dcb = .32\ D + .3$ in. (5)

$Lcb = 2.8\ Dcb - 2.2$ in. (6)

Front bearings					
D	4	$4\frac{1}{2}$	5	$5\frac{1}{2}$	
Dfb	$1\frac{9}{16}$	$1\frac{3}{4}$	$1\frac{7}{8}$	$2\frac{1}{16}$	From equation (7)
Lfb	$2\frac{13}{16}$	$2\frac{7}{8}$	3	$3\frac{5}{16}$	From equation (8)
Afb	4.2	5.02	5.63	6.6	$Dfb\times Lfb$
Pm		250			Assumed
Kfb	560	595	640	670	
Pm		300			Assumed
Kfb	665	710	775	800	
Pm		350			Assumed
Kfb	780	830	900	930	
Pm		400			Assumed
Kfb	890	945	1030	1075	

D = cylinder diameter, ins.

Dfb = diameter of front bearing, ins.

Lfb = length of front bearing, ins.

Kfb = maximum unit bearing pressure, lbs. per sq. in.

Pm = maximum explosion pressure, lbs. per sq. in. of piston face.

$Dfb = .32D + .3$ in. (7)

$Lfb = Dfb + 1\frac{1}{4}$ in. (8)

Rear bearings					
D	4	$4\frac{1}{2}$	5	$5\frac{1}{2}$	
Drb	$1\frac{9}{16}$	$1\frac{3}{4}$	$1\frac{7}{8}$	$2\frac{1}{16}$	From equation (9)
Lrb	3	4	$4\frac{5}{8}$	$5\frac{5}{8}$	From equation (10)
Arb	4.7	7	8.67	11.6	$Drb\times Lrb$
Pm		250			Assumed
Krb	565	495	495	465	
Pm		300			Assumed
Krb	660	575	575	535	
Pm		350			Assumed
Krb	760	660	655	605	
Pm		400			Assumed
Krb	855	735	735	675	

D = cylinder diameter, ins.

Drb = diameter of rear bearing, ins.

Lrb = length of rear bearing, ins.

Arb = projected bearing area, sq. ins.

Krb = maximum bearing pressure, lbs. per sq. in.

Pm = maximum explosion pressure, lbs. per sq. in. of piston face.

$Drb = .32D + .3$ in. (9). $Lrb = 5.3\ Drb - 5.3$ in. (10)

Relation of Speed and Pressure

The velocity of rubbing being, equally with the load, a factor in determining the work of friction which must be dissipated as heat, it follows that the velocity of rubbing should appear in a rational formula for the size of bearings. Such a formula has been given the form

$$pv = C,$$

in which p = pressure on projected area, lbs. per sq. in.

v = velocity of rubbing, ft. per min.

C = a constant determined from observation.

Values of this constant are much less numerous than those for simple pressure. Table 9 gives such authentic values as the author has been able to find.

The sources of these constants are as follows: (1) A. M. BENNETT (*Amer. Mach., June* 17, 1909), who bases his conclusions on a large number of bearings which operated under a rise of temperature not exceeding 72 deg. Fahr.; (2) H. P. BEAN (*Trans. A. S. M. E., Vol.* 27); (3), (4) and (7) JAS. CHRISTIE (*Proc. Engrs. Club of Phila.*, 1898);

TABLE 6.—WRIST-PIN BEARING PRESSURES FOR AUTOMOBILE ENGINES

D	4	$4\frac{1}{2}$	5	$5\frac{1}{2}$	
Dwp	$\frac{7}{8}$	1	$1\frac{3}{16}$	$1\frac{3}{8}$	From equation (1)
Lwp	2	2.25	$2\frac{5}{8}$	$3\frac{1}{16}$	From equation (2)
Awp	1.75	2.25	3.12	4.2	$Dwp \times Lwp$
Pm			250		Assumed
Kwp	1800	1780	1570	1420	Equation (a)
Pm			300		Assumed
Kwp	2150	2130	1890	1700	Equation (a)
Pm			350		Assumed
Kwp	2510	2480	2200	1980	Equation (a)
Pm			400		Assumed
Kwp	2870	2840	2510	2270	Equation (a)

D = cylinder diameter, ins.
Dwp = bearing diameter, ins.
Lwp = bearing length, ins.
Awp = projected bearing area, sq. ins.
Pm = maximum unit explosion pressure, lbs. per sq. in. of piston face.

$$Dwp = .34\,D - .53 \text{ in.} \quad (1)$$
$$Lwp = 2.25\,Dwp \quad (2)$$

Kwp = maximum unit bearing pressure, lbs. per sq. in.

$$Kwp = \frac{Pm\ D^2\ .7854}{Awp} \quad \text{(a)}$$

TABLE 7.—CRANK-PIN BEARING PRESSURES FOR AUTOMOBILE ENGINES

D	4	$4\frac{1}{2}$	5	$5\frac{1}{2}$	
Dcp	$1\frac{9}{16}$	$1\frac{3}{4}$	$1\frac{7}{8}$	$2\frac{1}{16}$	From equation (3)
Lcp	$2\frac{1}{16}$	$2\frac{3}{8}$	$2\frac{1}{2}$	$2\frac{3}{4}$	From equation (4)
Acp	3.32	4.17	4.68	5.68	$Dcp \times Lcp$
Pm			250		Assumed
Kcp	945	960	1050	1050	From equation (b)
Pm			300		Assumed
Kcp	1130	1150	1260	1260	From equation (b)
Pm			350		Assumed
Kcp	1320	1350	1470	1470	From equation (b)
Pm			400		Assumed
Kcp	1510	1540	1675	1675	From equation (b)

D = cylinder diameter, ins.
Dcp = bearing diameter, ins.
Lcp = bearing length, ins.
Acp = projected bearing area, sq. ins.
Kcp = maximum unit bearing pressure, lbs. per sq. in.
Pm = maximum unit explosion pressure, lbs. per sq. in. of piston face.

$$Dcp = .32D + .3 \text{ in.} \quad (3)$$
$$Lcp = 1.35\,Dcp \quad (4)$$
$$Kcp = \frac{Pm\ D^2\ .7854}{Acp} \quad \text{(b)}$$

TABLE 8.—AVERAGE RUBBING SPEED AND WORK OF FRICTION FOR AUTOMOBILE ENGINES

Computed for a piston speed of 1000 ft. per min. and a mean pressure of 20 lbs. per sq. in. of piston face.

D		4	$4\frac{1}{2}$	5	$5\frac{1}{2}$	Average
L		$4\frac{3}{8}$	5	$5\frac{1}{2}$	6	
RPM		1370	1200	1090	1000	
Rubbing speed in ft. per min. on bearings listed below.		560	550	535	540	546
V		9.35	9.15	8.9	9	9.1
$P_{(mean)}$		252	320	395	475	
Crank-pin bearing.	Km	76	77	84	84	80
	W	710	705	745	755	729
Center bearing.	Km	55	49.2	49.8	46	50
	W	515	450	445	415	456
Front bearing.	Km	44.7	47.5	52	53.4	49.4
	W	418	435	462	480	449
Rear bearing.	Km	56	45.5	43.5	38	45.7
	W	523	415	388	342	417

D = cylinder diameter, ins.
L = stroke, ins.
$P_{(mean)}$ = mean total pressure on piston for entire cycle, lbs. (assumed).
RPM = r.p.m. at 1,000 ft. per min. piston speed.
Km = mean unit bearing pressure, lbs. per sq. in.
V = rubbing speed, ft. per sec.
W = work of friction = $(Km \times fV)$ ft. lbs. per sec.
L = $1.1\,D$.

(5) and (6) FRED. W. TAYLOR (*Trans. A. S. M. E., Vol.* 27), whose figures are based on observations on eleven bearings in an overloaded mill; (8) and (9) G. W. DICKIE (*Trans. A. S. M. E., Vol.* 27); (10), (11), (12), (13) and (14) *The Mesta Machine Co.*

In interpreting these constants regard must be had for the influence of reciprocating and momentary loads. The former is seen in (2) and (3) and the latter in (11) and (14).

It is probable that the diversity of the constants is largely due to the inaccuracy of form of the equation, the probability being that the pressure should not be reduced in the same proportion that the speed is increased. A recognition of this is the basis of Edwin Reynold's rule for the main bearings of steam engines, which see.

TABLE 9.—PRODUCT OF PRESSURE, LBS. PER SQ. IN. OF PROJECTED AREA, AND VELOCITY, FT. PER MIN. OF BEARINGS

Kind of Bearings and Condition of Operation	Values of C
(1) Self-aligning ring-oiled bearings with continuous load in one direction	36,000–40,000
(2) Main bearings of Corliss engines (steam load only)	60,000–78,000
(3) Steam engine crank pins	200,000
(4) Steam engine cross head slide (figured on pressure at mid-stroke)	50,000
(5) Mill shafting with self-aligning ring-oiled babbitted bearings, highest admissible value	24,000
(6) Mill shafting with self-aligning cast-iron bearings, sight or wick oil feed or grease cups, should be less than	12,000
(7) 110,000 lbs. freight or axle journals at 10 miles per hour	60,000
(8) Water-cooled thrust bearings of steamships, customary value	37,500
(9) Water-cooled thrust bearings of steamships with extra care in water cooling	61,000
(10) Rolling-mill pinion-housing bearings	18,000
(11) Rolling-mill roll-housing bearings	60,000–70,000
(12) Rolling-mill table roller bearings	4,500–7,500
(13) Rolling-mill table line-shaft bearings	4,500–7,500
(14) Main bearings of rolling-mill shears	120,000

Relation of Speed, Pressure and Temperature

The following methods of bearing design are from the practice of the General Electric Co. and are the results of extended experimental investigations: It is very desirable in laying out bearings to keep the diameter as small as possible, consistent with sufficient strength of shaft and suitable deflection of the journals both inside and outside the bearings as the work of friction is thereby reduced. It is also very

Rubbing Speed, Ft. per Min.

FIG. 1.—Relation between rubbing speed and safe maximum pressure on bearings without artificial cooling for perfect film lubrication.

desirable to so dimension bearings that they are fairly well loaded, in order to avoid bulky machines and also because the coefficient of friction rises quite rapidly when the load is less than 50 lbs. per sq. in. of projected area.

When calculating the projected area of any bearing, especially if it is to be heavily loaded, the amount of space lost through the drain

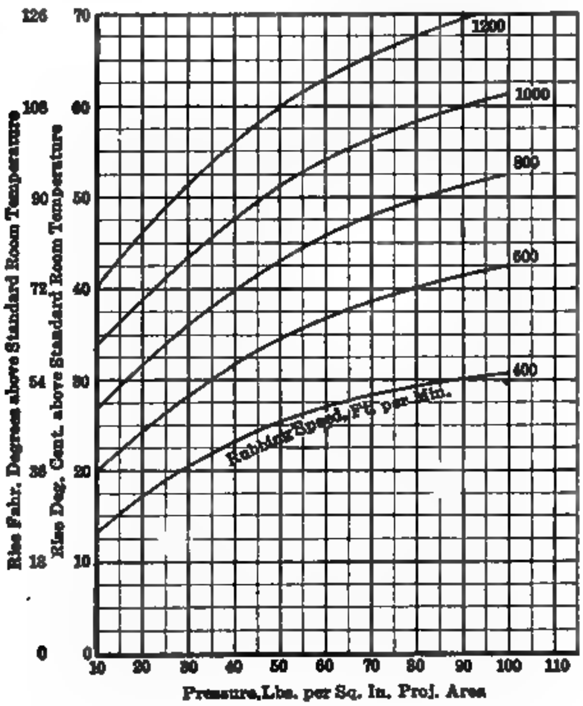


FIG. 2.—Temperature rise of oil-ring bearings in still air, room temperature 75 deg. Cent.—77 deg. Fahr.

grooves at both ends must be deducted. This is particularly important when the length of the bearing is small in proportion to the diameter.

It is also necessary—that this applies to all forms of lubrication—that there be no sharp corners on the edges of the oil distributing grooves or channels, but that these be gradually eased off so that the oil can be drawn in between the journal and the bearing. Sharp corners are invariably oil wipers and often absolutely prevent proper lubrication.

The heat generated in any bearing may be dissipated:

1. By radiation from the housings and conduction by the shaft.
2. By forcing cooled oil through the bearing.
3. By surrounding the bearing by some form of water jacket.

Bearings without artificial cooling are usually lubricated by oil rings or similar devices or by gravity feed. It is essential that an abundant supply of oil be delivered to all parts of the bearing by suitably arranging the channels so that a perfect film will be maintained at all times between the journal and bearing, and that there is no opportunity for the oil forming this film to escape through openings or grooves at the points of greatest pressure and thus allow the metals to come in contact.

The heat generated in bearings having no artificial cooling is conducted away and radiated by the housings. The great variation in the design of bearing housings and the different conditions of ventilation, etc., make it extremely difficult to predetermine the ultimate

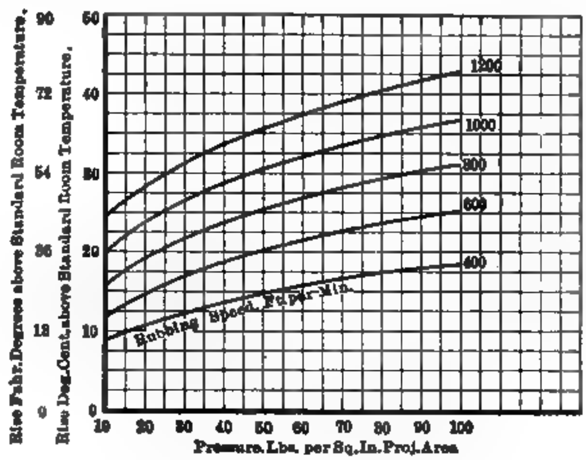


FIG. 3.—Temperature rise of oil-ring bearings for well ventilated condition but without artificial cooling, room temperature 25 deg. Cent.—77 deg. Fahr.

temperature of such bearings with any great accuracy, and it is always necessary to allow a considerable margin of safety.

Fig. 1 covers the range of pressure and speed ordinarily permissible in this type of bearing, while Figs. 2 and 3 show the ultimate temperatures for different speeds and loads. These curves were made up from the readings obtained from special bearings and afterward checked by the test records of a large number of machines—both of the pillow-block and shield types—which have gone through the testing department. Fig. 2 shows the temperatures to be expected under the most unfavorable conditions, that is, of a bearing so situated that no current of air can circulate about it, and therefore cooled by radiation only. There is, however, a considerable circulation of air about most machines, due to the fanning action of the revolving parts, and the ultimate temperatures to be expected in such cases are shown by the curves of Fig. 3. These curves apply to the great majority of open generators and motors, both of the pillow-block and end-shield types. When the machine is enclosed, or the free circulation of air in any way interrupted, higher temperatures will result, until finally the conditions of Fig. 2 are reached.

A part of the heat of the bearings of motors and generators is usually conducted away by the shaft and radiated by the spider and other revolving parts. When machines are totally enclosed or are connected to other machinery whose temperature is high, heat may be transmitted through the shaft to the bearing, thus raising the latter's temperature, and due allowance must be made for this.

When bearing pressures and speeds are unusually high it is often necessary to force oil under pressure into the bearings and advantage is often taken of this to keep the heating down by artificially cooling the oil. This method, although used to a very considerable extent, is usually not as efficient as a water jacket.

For all practical purposes, it may be considered that the entire heat generated is taken away by the oil, and it is therefore possible to predetermine the bearing temperature with considerable accuracy. Fig. 4 shows the ultimate temperature of bearings using the quantities of oil most commonly pumped through, and with the assistance of these curves the necessary amount can be determined. Intermediate speeds and pressures can be easily interpolated.

For pressures and speeds beyond the limits of this table, it is advisable to resort to water-jacket cooling.

In arranging bearings for this form of lubrication, care must be taken to force the oil to the point where the work is being done, as otherwise the oil coming from the bearing may be comparatively cool, while the bearing itself is much too warm. In addition to this, it means that an excessive amount of oil must be pumped through the bearing requiring unnecessarily large pumps, piping, etc. Experiments with such bearings show that when oil begins to run out of the ends quite freely, nothing is gained by forcing through a larger quantity.

A properly designed water jacket will carry away a very much larger amount of heat than will any form of forced oil lubrication, as the specific heat of water is very much higher. As is the case of forced oil lubrication the ultimate temperature of a well-designed water-jacket bearing can be very accurately determined. It is essential that the pipes or channels be located as close as possible to the surface of the lining where the work is being done, in order that the heat generated may be absorbed without danger of damage to the lining. Water circulated at some distance away from the lining surface is of comparatively little assistance, as heat may be generated so rapidly that the lining will be destroyed before the heat reaches the jacket. The water passages must also be so arranged that an even and continuous circulation is kept up in all parts.

With properly constructed passages, it is safe to assume that heat may be removed at the rate of from 3 to 5 h.p. for each gallon of water circulated per minute, and, where the conditions are unusually good and the jacketing carefully arranged, from 10 to 12 h.p. per gallon can be dissipated.

With water jackets any suitable method of lubrication may be used which will insure at all times a good film of oil between journal and bearing.

For designs of water-jacketed bearings, see below.

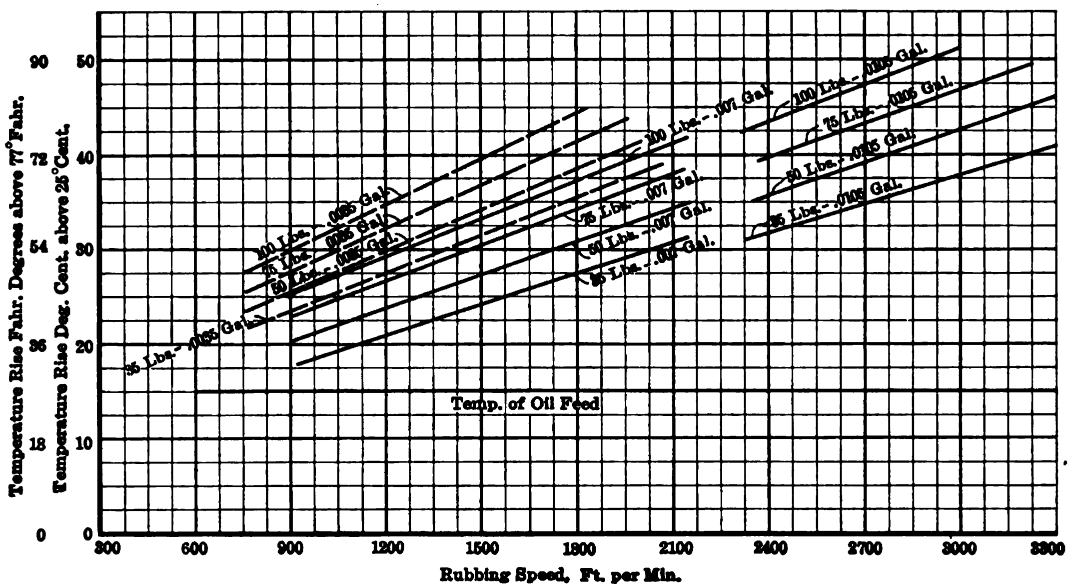
Conditions of Film Lubrication

The experiments of Beuchamp Tower (*Proc. I. M. E.*, 1885) demonstrated that, given flooded lubrication and suitable relations of speed and pressure, the condition of affairs between a journal and bearing becomes that illustrated in exaggerated form in Fig. 5. The rotating journal assumes an eccentric position in its bearing, and is separated from it by a circular wedge-shaped film of oil. The journal brings up more oil than can be carried around the space between journal and bearing, and some oil is therefore forced out sidewise and, the film of oil resisting this action by virtue of its viscosity, there is set up a wedging action which will support the bearing away from the journal against considerable pressure. By drilling holes in the bearing and inserting pressure gages, Mr. Tower found curves somewhat like a' c'' b' to represent the pressure at various (projected) points of the circumference of the journal. The film is thinnest, not at the point of application of external load, but at a point somewhat farther along in the direction of rotation.

The summation of these pressures was found to equal the total load on the bearing with a surprising degree of accuracy.

An immediate practical result of these experiments is the demonstration that the oil should be introduced at the point of no pressure.

Mr. Tower's experiments show that the action of high-speed bearings is entirely different from that of low-speed bearings. In the latter we have oily surfaces in actual rubbing contact. An accidental increase of temperature reduces the viscosity of the lubricant, which in turn increases the intimacy of contact, thereby bringing about additional cumulative increase of temperature. Such a bearing may be said to be, as regards temperature, in unstable equilibrium. A high-speed bearing, on the other hand, is in stable equilibrium. If the



Loads in lbs. per sq. in. of projected area. Oil feeds in gals. per min.

FIG. 4.—Relation between rubbing speed and rise in temperature for forced lubrication and three rates of oil feed, room temperature 25 deg. Cent.—77 deg. Fahr.

speed is sufficiently above the critical speed at which the film action is established to prevent the reduced viscosity from bringing the surfaces into actual contact, there is no reason why the heating action should be cumulative, and such bearings may safely be run at temperatures that would be unsafe below the critical speed.

Similar difference exists in the tendency to wear. H. M. MARTIN (*The Design and Construction of Steam Turbines*) says that steam turbine bearings, after years of use, show no signs of wear.

For these reasons the complete film system of lubrication should be aimed at whenever possible. DR. HERBERT F. MOORE (*Amer. Mach., Sept.* 10, 1903) determined experimentally the relation of pressure and rubbing speed at which the film breaks down and the lubrication becomes of the ordinary kind between oily surfaces. Dr. Moore's results are represented graphically by the full line of Fig. 6, the dotted line being an approximation represented by the equation:

$$P_{max} = 7.47\sqrt{v},$$

in which P_{max} = limiting pressure on projected area of bearing at which the oil film breaks down, lbs. per sq. in.

v = velocity of rubbing, ft. per min.

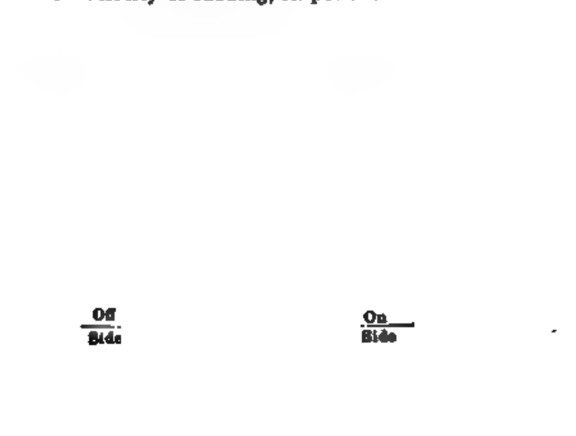


FIG. 5.—Journal and Bearing with film lubrication.

This equation is fundamental and is generally accepted. It forms the starting-point of the first complete theory connecting the pressures, velocities and temperatures of bearings, by AXEL. K. PEDERSEN, analytical expert the General Electric Co. (*Amer. Mach., Oct.* 10, 1912).

Mr. Pedersen's remarkable deductions are based on a large number of widely scattered experiments, including those of Beauchamp Tower, and are given below. It must be remembered that they apply to complete film lubrication only, the bearing proportions being determined from the conditions for preserving a perfect film at a permissible final bearing temperature.

Introducing a proper factor of safety, Mr. Pedersen obtains the equation:

$$d^3 = .068453\left(\frac{s}{x}W\right)\frac{st}{n}, \qquad (a)$$

in which d = diameter of bearing, ins.,

s = factor of safety,

$= \dfrac{P_{max}}{\text{actual pressure on projected area, lbs. per sq. in.}}$,

W = total load on bearing, lbs.,

l = length of bearing, ins.,

$x = \dfrac{l}{d}$,

n = r.p.m. of journal.

For each class of machinery, the ratio $x = \frac{l}{d}$ is a well-defined quantity. Following are customary values of this ratio:

Type of bearing	Values $l \div d$
Marine engine main bearings	1 to 1.5
Stationary engine main bearings	1.5 to 2.5
Ordinary heavy shafting with fixed bearings	2 to 3
Ordinary shafting with self-adjusting bearings	3 to 4
Generator and motor bearings	2 to 3
Machine-tool bearings	2 to 4

Equation (*a*) can readily be used for determining the diameter of the bearing. The factor of safety is selected by considering the importance of safe running. A factor of safety of 1 would indicate that the journal is running under limiting conditions, that is, that the oil film is on the point of breaking down. For ordinary light machinery, the factor of safety may be taken as low as 2 and for heavy (especially high-speed) machinery as high as 8 or even 10. As a good average 4 to 5 may be taken at the first trial.

FIG. 6.—Breaking-down point of perfect oil film.

The alignment chart, Fig. 7, was designed for the prompt solution of equation (*a*). The use of the chart is explained below it.

The diameter of the bearing being thus determined, the length is fixed by the selected ratio x or

$$l = xd \qquad (b)$$

The pressure on the projected area is

$$p = \frac{W}{l \times d} \qquad (c)$$

The fundamental consideration, in connection with the final bearing temperature and the specific losses, deals with the laws of friction and the heat-radiating capacity of a bearing. From the great number of test data available in regard to the coefficient of friction the following important fundamental principles may be stated: For a journal revolving above 500 ft. per min. (8.5 ft. per sec. approximately) we use the formula given by Lasche:

$$fp(t - 32) = 51.2 \qquad (d)$$

This formula is a very close practical approximation; actually, the coefficient of friction is not independent of the rubbing speed of the journal, but increases slightly with the speed up to a speed of about 2000 ft. per min.; this increase, however, may be neglected.

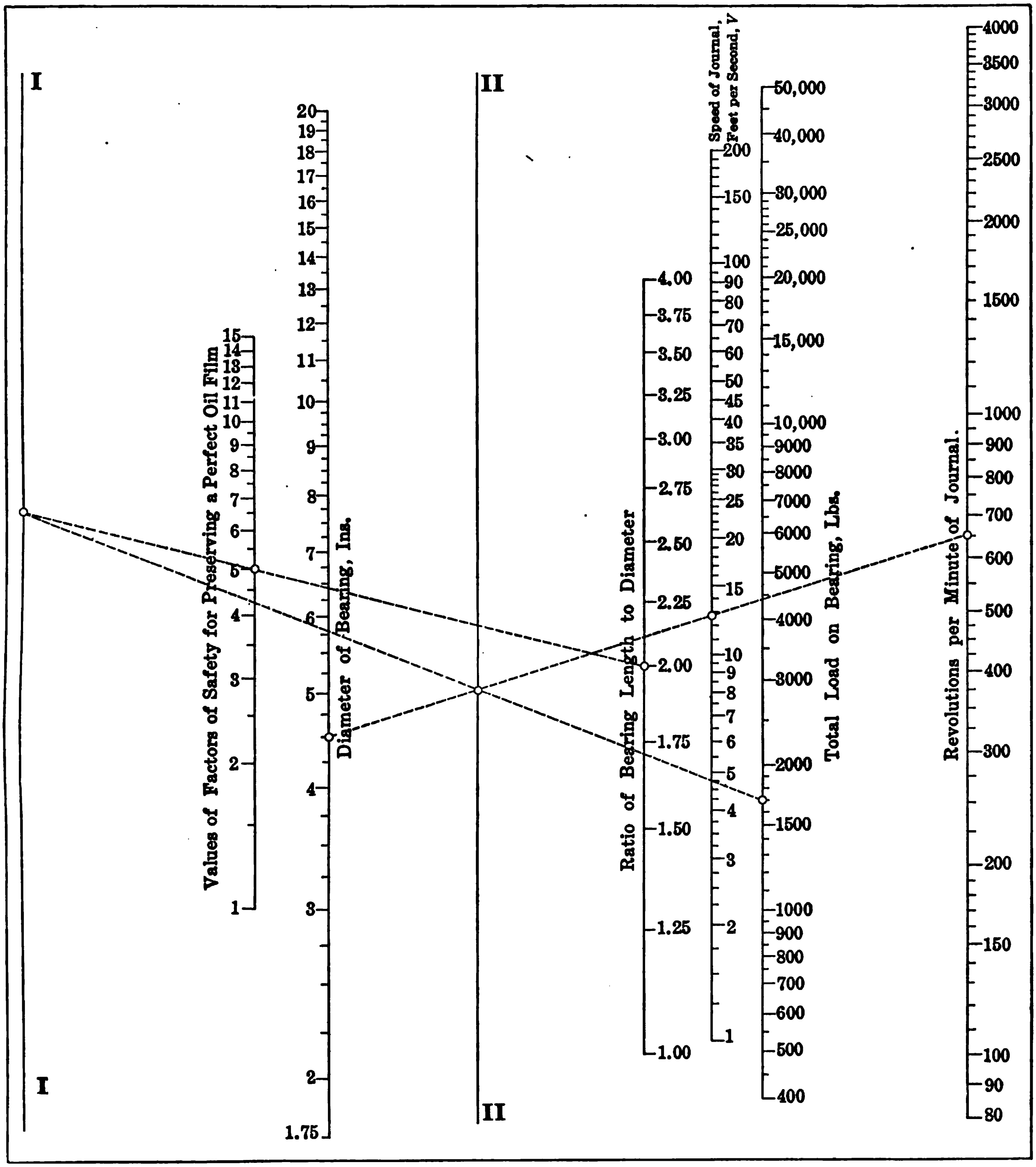


FIG. 7.—Dimensions of bearings for film lubrication.

Connect the ratio of length divided by diameter and the selected factor of safety and note the intersection with axis I; connect the intersection and the load and note the intersection with axis II; connect this intersection with the revolution per minute and read the diameter and rubbing speed from the approprfate scales. The chart may be read in the opposite direction if desired.

For a journal revolving *below* 500 ft. per min. (8.5 ft. per sec. approximately) the coefficient of friction is dependent on the velocity of rubbing. From many test data it has been concluded that for these speeds

$$fp(t-32)=2.3\sqrt{60v} \tag{e}$$

In (*d*) and (*e*) f = coefficient of friction,

p = pressure on projected area, lbs. per sq. in.

t = final bearing temperature, Fahr.

v = rubbing speed of journal, ft. per sec.

Formula (*e*) has been fully corroborated by comparison with the experiments of Beauchamp Tower (*Proc. I. M. E.*, 1885), the agreement being quite remarkable (*Amer. Mach., Oct.* 10, 1912).

The heat-radiating capacity of a bearing depends mainly upon the iron masses contained in it and upon the surrounding air. If the bearing is located in a place where the surrounding air is easily moved (ventilated bearing), and if the bearing contains large masses of iron, we have the best conditions possible. On the other hand, if the bearing and its housing are of comparatively small dimensions and the air is still, the heat-radiating capacity is at a minimum.

In the chart, Fig. 8, the first condition is represented by the point M for ventilated bearings, the second by the point N for still-air bearings. We may have a condition where the bearing contains large masses of iron, but is surrounded by still air; evidently, then, a point located approximately midway between the points M and N should be used.

The following formulas are very close approximations to the experiments by Lasche on the heat-radiating capacity of bearings. These experiments are given in chart form in "Bearings and Their Lubrication," by L. P. Alford.

For *ventilated* bearings we have the heat-radiating capacity in ft.-lbs. per sec. per sq. in. of the projected area of bearing expressed by

$$\frac{(t-t_o+33)^2}{1860} \tag{f}$$

and for still-air bearings

$$\frac{(t-t_o+33)^2}{3300} \tag{g}$$

in which t_o = temperature of room, Fahr.

t = final bearing temperature, Fahr.

The maximum friction loss must not be greater than the heat-radiating capacity of the bearing, otherwise artificial cooling must be resorted to. The friction loss in ft.-lbs. per sec. per sq. in. of projected bearing area is pfv, hence for ventilated bearings

$$pfv=\frac{(t-t_o+33)^2}{1860} \tag{h}$$

and for still-air bearings

$$pfv=\frac{(t-t_o+33)^2}{3300} \tag{i}$$

As the coefficient of friction follows different laws whether the rubbing speed is above or below 500 ft. per min., we must consider this in formulas (*h*) and (*i*)

For speeds about 500 ft. per min., we combine (*h*) and (*i*) with (*d*), and solving for v, we get for ventilated bearings

$$v=\frac{(t-32)(t-t_o+33)^2}{95232} \tag{j}$$

for still-air bearings

$$v=\frac{(t-32)(t-t_o+33)^2}{168960} \tag{k}$$

For speeds *below* 500 ft. per min., (*h*) and (*i*) are combined with (*e*); then for ventilated bearings

$$v=\sqrt[3]{\left\{\frac{(t-32)}{2.3\sqrt{60}}\times\frac{(t-t_o+33)^2}{1860}\right\}^2} \tag{l}$$

for still-air bearings

$$v=\sqrt[3]{\left\{\frac{(t-32)}{2.3\sqrt{60}}\times\frac{(t-t_o+33)^2}{3300}\right\}^2} \tag{m}$$

Equations (*j*), (*k*), (*l*), and (*m*) are the fundamental formulas plotting the chart, Fig. 8, as far as the determination of the final bearing temperature is concerned.

The chart also gives the specific losses y, namely

$$y=pf \tag{n}$$

Hence from (*d*) for speeds *above* 500 ft. per min., or 8.5 ft. per sec. approximately,

$$y=\frac{51.2}{t-32} \tag{o}$$

and from (*e*) for speeds *below* 500 ft. per min. or 8.5 ft. per sec., approximately,

$$y=\frac{2.3\sqrt{6cv}}{t-32} \tag{p}$$

The total friction loss in the bearing is obtained from

$$Y=yvld \text{ ft.-lbs. per sec.} \tag{q}$$

or

$$Y=\frac{yvld}{550}\text{ h.p.} \tag{r}$$

In equations (*n*)–(*r*)

y = specific losses; that is, the losses due to friction in ft.-lbs. per sec. per sq. in. of projected bearing area for each foot of rubbing speed of the journal,

Y = total friction losses in the bearing.

The use of the chart, Fig. 8, is as follows: (*a*) To determine the final bearing temperature: Locate the proper value of the rubbing speed on the AA scale, connect this point with points N or M or some intermediate point on the line NM, according to the condition of the surrounding air and the design of the bearing. The connecting line locates a point on the BB axis. Trace from here horizontally to the curve giving the proper temperature of the room, thence vertically down to the temperature scale and read the final bearing temperature.

(*b*) To determine the specific losses y: Here different methods must be employed, one for speeds above 8.5 ft. per sec., and another for speeds below 8.5 ft. per sec.

Rubbing speeds *above* 8.5 ft. per sce.:

From the final bearing temperature trace parallel to BB axis to the dotted curve CC, thence horizontally to the right to the axis DD and read the value of y, the specific loss.

Rubbing speeds *below* 8.5 ft. per sec.:

From the final bearing temperature trace parallel to the BB axis to the dotted curve CC, thence horizontally to the left to the BB axis, thus locating a point on this axis. Now connect this point with the proper value of the rubbing speed on the speed scale EE; the connecting line intersects a point on the specific-loss scale FF, where the specific loss y is read. The general procedure is shown by the connecting lines on the chart.

Example 1.—Design a motor bearing for the following data:

Ventilated bearing and large masses of iron;

Ratio of length to diameter = 2;

Factor of safety for preserving perfect oil film = 5;

Total load on bearing = 1700 lbs.;

Revolutions per minute = 650;

Temperature of room = 75 deg. Fahr.

From the chart, Fig. 7, we get the diameter d = 4.5 ins., approximately, hence from equation (*b*)

$$l=x\times d=2\times 4.5=9 \text{ ins., the length,}$$

then from equation (*c*)

$$p=\frac{W}{l\times d}=\frac{1700}{9\times 4.5}=42 \text{ lbs. per sq. in.}$$

The chart, Fig. 7, also determines the rubbing speed

$$v=12.7 \text{ ft. per sec., nearly.}$$

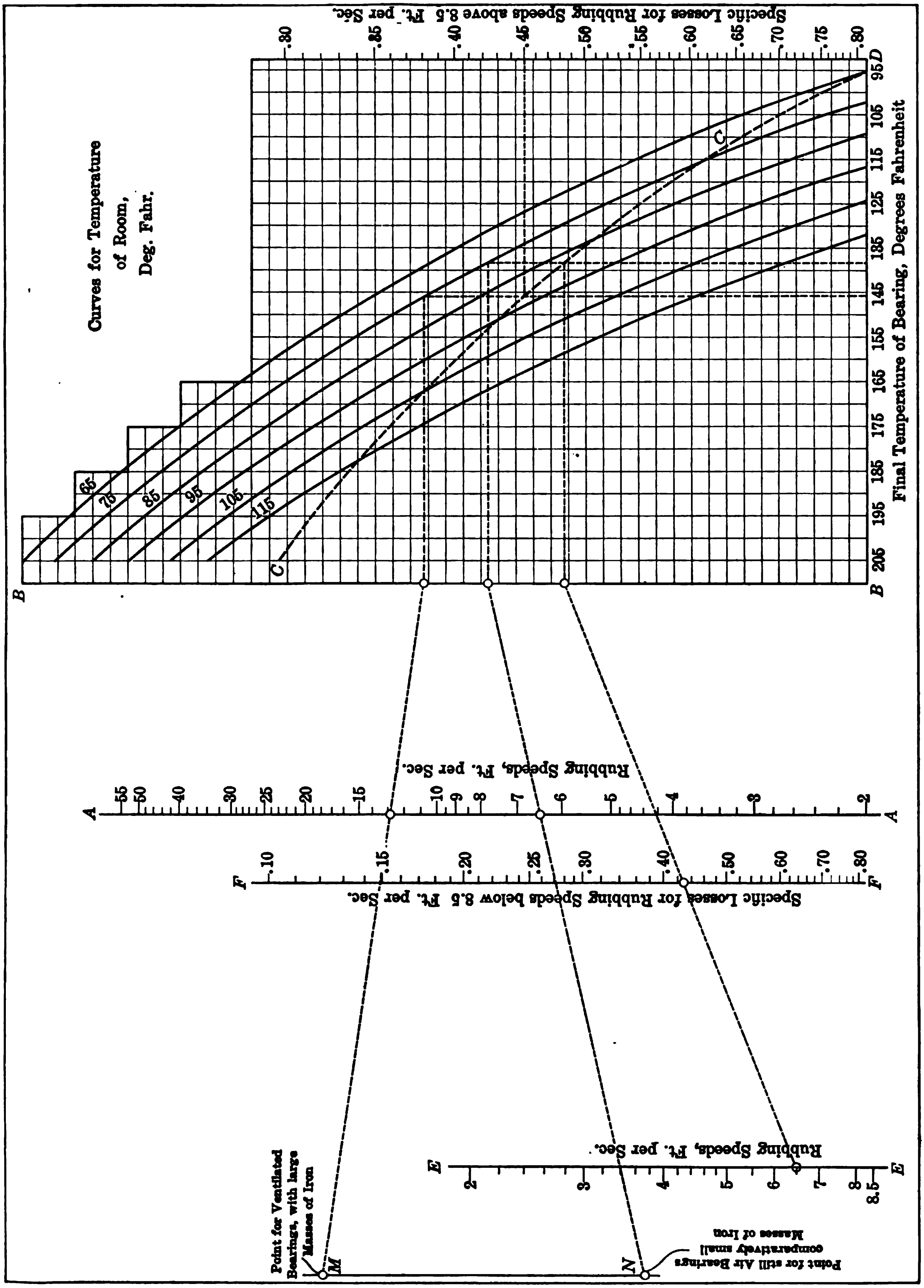


FIG. 8.—Final temperatures and specific losses of bearings.

Locating the rubbing speed 12.7 on the AA scale of Fig. 8, and proceeding as previously explained, noting that the rubbing speed is above 8.5 ft. per sec., we get the final bearing temperature

$$t=146 \text{ deg. Fahr., nearly.}$$

This is by no means an excessive temperature. The tendency of machinery builders, however, is to limit the final bearing temperatures to approximately 150 deg. Fahr.; this is very conservative. According to Bearings and Their Lubrication, by L. P. Alford: "In practice it is necessary to design bearings to run at a much lower temperature than will cause damage, because of the requirements of the average customer. Such a maximum temperature is from 140 to 160 deg. Fahr. It is probably true that the average bearing could just as well run at a temperature of 200 deg. Fahr. . . . .

The actual temperatures of large bearings was the subject of observations by A. M. Mattice (*Trans. A. S. M. E., Vol.* 27), who states that examination of the temperatures of a large number of main bearings of engines of various makes showed more large engines running with bearings at temperatures over than under 135 deg. Fahr. Many bearings were running at over 150 deg., some considerably higher, and in one case a continuous temperature of 180 deg. was found, and in all of these cases the bearings were giving no trouble.

H. M. Martin (*The Design and Construction of Steam Turbines*) says "turbines in which the bearing temperature is constantly about 195 deg. Fahr. have given no trouble in practice, but a more usual limit of temperature is 165 deg. Fahr."

The bearing oil loses its lubricating qualities at a temperature about 250 deg. Fahr., approximately. Returning to our example, we get the specific loss y, which in this case is read on the DD scale

$$y=.451.$$

Hence from (q) $Y=yvld=.451\times 12.7\times 9\times 4.5=232$ ft.-lbs. per sec.

Example 2.—Given a bearing running at a rubbing speed of 6.5 ft. per sec. and the conditions of a still-air bearing with small iron masses; Fig. 8 must now be used according to the rules for speeds below 8.5 ft. per sec. We get the final bearing temperature = 138 deg. Fahr., and the specific loss on scale FF, $y=.43$.

The maximum allowable final bearing temperature at a given room temperature determines the maximum speed at which the journal can be run without artificial cooling; thus, in the first example, if 146 deg. Fahr. is considered as the maximum allowable bearing temperature, we cannot run this bearing at a higher speed than 12.7 ft. per sec. without artificially cooling the bearing.

In the following, we shall only consider cooling by means of water.

If D = the temperature difference, deg. Fahr., of the water before and after cooling (a practical, average value of D is 20 to 25 deg. Fahr.),

V_1 = actual rubbing speed of journal, ft. per sec.,

V_2 = maximum speed in ft. per sec., at which bearing can be run at the maximum allowable temperature without water cooling,

y = specific loss in bearing, corresponding to the rubbing speed V_2, and determined by the chart,

then to *keep* the bearing at a temperature corresponding to V_2, we must use

$$Q=\frac{y(V_1-V_2)ld}{108\ D} \qquad (s)$$

gallons of water per min.

Example 3.—Suppose, for instance, that we wish to run the bearing in Example 1 at a speed of 20 ft. per sec., but that the maximum allowable temperature must be kept at 146 deg. Fahr., then we have

$V_1=20$ ft. per sec.,

$V_2=12.7$ ft. per sec.,

corresponding to a temperature of 146 deg. Fahr. at 75 deg. Fahr. room temperature, $y=.451$, as previously determined (see Example No. 1), then, using the value $D=20$ deg. Fahr., we get from (s)

$$Q=\frac{.451(20-12.7)}{108\times 20}\times 9\times 4.5,$$
$$=.062 \text{ gal. of water per min.}$$

A very important use of Fig. 8 thus consists in the possibility of determining the limiting speed at which a bearing can be run without artificial cooling at a given maximum bearing temperature. If high final bearing temperatures are allowed, very high rubbing speeds may be used; in fact, the allowable speeds increase much faster than the corresponding temperatures; thus, at a final bearing temperature of, say, 195 deg. Fahr., and at a room temperature of 75 deg. Fahr., Fig. 8, determines a limiting rubbing speed of 40 ft. per sec. (against 12.7 ft. per sec. at 146 deg. Fahr.), for a ventilated bearing and a limiting rubbing speed of 23 ft. per sec. for a still-air bearing.

In determining these speeds, the chart is read in the opposite direction, by starting at the final bearing temperature, then tracing parallel to the BB axis until the room-temperature curve is reached, thence horizontally to the left to the BB axis. The point reached on this axis is then connected with M or N (as the case may be), and the connecting line intersects the speed axis AA at points, which give the maximum allowable rubbing speeds at the given maximum bearing temperatures.

Materials for Bearings

Materials for bearings form an endless subject of discussion. The author is convinced that cast-iron is entitled to a far wider use than it has received. It has the well-known property of taking on a glazed surface which is practically proof against wear. As John Richards has put it, "there is no doubt that prejudice or mistrust prevents the use of iron bearings in many cases where they are best." Failures of cast-iron bearings are charged to the material, while failures of other bearings are charged to fate or luck.

Those who oppose the use of cast-iron fail to recognize the numerous cases for which its use is so habitual that nothing else is thought of. Of these, the most striking are the unbalanced slide valves of common steam engines, which work under heavy loads and very indifferent lubrication. Eccentrics and eccentric straps, especially of locomotives, work under scarcely less favorable conditions of load and lubrication, with the additional condition of high speed. The tables of planers and boring mills and all manner of sliding bearings in machine tools form additional illustrations. For steam engine cross-head slides and cross heads nothing equals it. Finally, the line-shaft hangers made by Wm. Sellers & Co. since prior to 1850 have been made of this material. These bearings have run for thirty years without appreciable wear.

A test of cast-iron and other materials for live spindle lathe bearings was made by the R. K. Le Blond Machine Tool Co. (*Amer. Mach., Mar.* 23, 1911). Four experimental 18-in. lathes were fitted with different combinations of bearing materials, as follows:

1. Hardened steel spindle with cast-iron boxes.
2. Soft steel spindle with babbitt boxes.
3. Hardened steel spindle with bronze boxes.
4. Soft steel spindle with bronze boxes.

The soft steel spindles were of 60-carbon crucible steel; the bronze was made to the specifications of the Pennsylvania Railroad Company. After the end of some 8 years' service and treatment as far as possible identical for all four lathes, it was found that their rating as regards absence of wear and general satisfaction was in the order as given above; that is, the hardened-steel spindle with cast-iron boxes was the best combination. Both spindle and boxes were in as good condition as when placed in the lathe, and from all appearances and tests showed absolutely no wear.

Mr. Le Blond adds the following general observations: The question of bearing metals is a question of affinity. One metal has

an affinity for a certain other metal, and as very often illustrated in life, a soft spindle may be married to a bronze box when its affinity is babbitt. In other words, a successful bearing must be composed of two metals of entirely different degrees of hardness and disposition. The only exception to this rule is cast-iron and cast-iron.

It is a matter of general knowledge that a soft-steel spindle and a soft-steel bearing will immediately cut and run together; in fact, it is practically impossibile to lubricate this combination so it will not cut.

A soft-steel spindle and bronze bearing is probably the next worst combination, as the metals are very similar in hardness and under the very best conditions will scratch and cut.

A soft-steel spindle and cast-iron bearing will give splendid wear if properly lubricated, but will not stand for the slightest neglect.

A soft-steel spindle and babbitt will give excellent service, stand for a great deal of abuse; in fact is as near a fool proof proposition as any.

The hardened-steel spindle and cast-iron will stand as much neglect as any combination of metals, has a much longer life, will retain its accuracy for an indefinite period, withstand intermittent pressure or a series of blows which would peen out and loosen babbitt, and, from our experience, the most fool-proof bearing in the world to-day is the cast-iron and hard spindle. It has indefinite life, requires absolutely no adjustment and will stand the maximum of abuse.

The original patent of Isaac Babbitt (issued in 1839) was not for the alloy known by his name, but for the method of its application. The exact formula used by the inventor is not known. Tin, copper and antimony were the ingredients, and from the best sources of information the original proportions in per cent. were as follows:

Tin = 89.3 or 83.3 or 89.1.
Copper = 3.6 or 8.3 or 3.7.
Antimony = 7.1 or 8.3 or 7.4.

This metal, when carefully prepared, is one of the best metals in use for lining boxes that are subjected to heavy weight and wear.

A concise summary of modern practice with composition bearing alloys is given by JOHN F. BUCHANAN (*The Foundry*, 1906) thus:

To make the best grade of babbitt or anti-friction metal, proceed as follows: Select the purest metals that can be had, and the most suitable recipe for the duty of the alloy; make a preliminary mix of the refractories in a plumbago crucible, and pour it out for "hardening." Melt the metal which forms the basis of the alloy (it may be tin, lead, or zinc), and dissolve the hardening therein, at a gentle heat, using sawdust, tallow, or powdered sal-ammoniac for a flux. For making a large quantity in the ordinary brass furnace, make a cast-iron crucible 2 ins. smaller than the diameter of the furnace; lower it into the furnace and lute round. One word of caution is needed here. Zinc should not be melted in an iron pot, but if melted in a plumbago crucible it may be poured and mixed with the other components of the alloy already melted in the pot.

The utility of babbitt metal is not to be gaged by its cost per pound. A cheap babbitt (lead or zinc base), well made, may give better service than a costly mixture which has been carelessly blended. Generally speaking, the commercial grade numbers of bearing metals are for: 1, light loads and high speeds; 2, medium loads and moderate speeds; 3, heavy loads and slow or moderate speeds; and 4, heavy loads and high speeds. Such grading is reasonable, for the hardness of the alloys increases with the numbers, and price does not count.

Babbitt metal, correctly speaking, is a tin alloy, but modern engineering practice and commercial usage favor the application of the name to all metals capable of the same duty as babbitt. Hence we get three series of babbitt or anti-friction metals: 1st, the tin series; 2d, the lead series; 3d, the zinc series. Tin is the most polishable of the soft metals, and it alloys readily with any of the useful metals employed for minimizing the friction of machinery; it has been made the basis of the best anti-friction alloys. Lead is undoubtedly the best anti-friction medium among metals, but it lacks stiffness to stand up to the work. Copper is the ideal bond for zinc alloys, and zinc is the most expansible and durable of metals. Zinc babbitts cast well, wear well, and fit snugly to the bearing. Owing to its highly crystalline structure, antimony, the principal hardening element, should not exceed 20 per cent., as it is apt to separate and rub out of the alloy—17 per cent. has been fixed as the limit by an eminent authority.

The mutual relations of the metals determine the mechanical properties of the alloys. Zinc and antimony are too much alike to be used simultaneously, and tin alloys, without copper, are apt to spread under heavy loads. Due to its poor affinity for lead and tin and its low atomic volume, aluminum is not a suitable metal for anti-friction alloys. Bismuth, on the contrary, is a decided advantage up to about 1.5 per cent. This metal has been freely used in the production of some modern alloys, notably those with low fusibility, low contraction and high atomic volume. In Table 10 are given some special mixtures which have given complete satisfaction for the duty stated, and in Table 11 are given four grades of mixtures.

In each case the metals represented by the figures 7, 17, and 6 constitute the "hardening." These are copper-hardened alloys—the copper content being over 5 per cent.—and provide a series of cheap, serviceable, anti-friction metals.

TABLE 10.—MISCELLANEOUS BEARING METALS

For lining	Tin	Lead	Zinc	Antimony	Copper	Bismuth
Dynamos: high-speed.	88	......	......	8	3.5	.5
Marine engines........	77	17	......	3	3	......
Eccentrics............	5	78	......	15	2	.25
Submerged bearings...	40	48	......	10	2	......
Main bearings........	34	44	......	16	6	......
Slides, thrusts.........	63	......	30	2.5	2.5	......
Railway trucks........	42	......	56	......	2	......
Axle-boxes (by analysis)	74.22	13.50	1.80	6.55	3.60	......
Anti-acid metal (by analysis).	78.84	14.75	......	trace	3.70	......
Plastic metal..........	80	10	......	1	8	1
Genuine babbitt (hard)	80	......	......	10	10	......
Genuine babbitt (No. 2)	88	......	......	9	8	......
Universal bearing metal	6	78	......	16	......	.25
Anti-friction castings...	24	......	80	......	4	......

TABLE 11.—A SERIES OF COPPER HARDENED ALLOYS

Grades	1	2	3	4
Tin..............	77	77	17	....
Zinc..............	....	17	77	77
Lead..............	17	7	7	17
Antimony..........	7	....	....	6
Copper............	6	6	6	7

The composition of many common bearing metals, as determined in the laboratories of the Pennsylvania Railroad and published by DR. DUDLEY (*Journal of the Franklin Institute, Feb.*, 1892), is given in Tables 12 and 13.

The bearing metal known as the standard of the Bureau of Steam Engineering of the United States Navy, also called anti-friction or anti-attrition metal, has this composition:

Best refined copper....................	3.7 per cent.
Banca tin..........................	88.8 per cent.
Regulus of antimony................	7.5 per cent.

The percentages are by weight. The mixture must be well fluxed with borax and rosin in mixing.

TABLE 12.—COMPOSITION OF BEARING METALS (PER CENT.)

Name of metal	Copper	Tin	Lead	Anti-mony	Zinc	Iron
Camelia metal.	70.2	4.25	14.75	...	10.2	.55
Anti-friction metal.	1.6	98.13	...	...	...	trace
White metal....	...	...	87.92	12.08	...	...
Car brass lining.	...	trace	84.87	15.1	...	...
Salgee metal...	4.01	9.91	1.15	...	85.57	...
Graphite bearing metal.	...	14.38	67.73	16.73	...	not determined
		Graphite—none				
Antimonial lead	...	...	80.69	18.83	...	...
Carbon bronze.	75.47	9.72	14.57	...	...	...
		Carbon—possible trace				
Cornish bronze.	77.83	9.6	12.4	...	trace	trace
		Phosphorus—trace				
Delta metal....	92.39	2.37	5.1	...	...	.007
Magnolia metal	...	...	83.55	16.45	...	...
		Traces of iron, copper, zinc and possibly bismuth				
American anti-friction metal.	...	...	78.44	19.6	.98	.65
Tobin bronze..	59	2.16	.31	...	38.4	.11
Graney bronze.	75.8	9.2	15.06	...	...	...
Damascus bronze.	76.41	10.6	12.52	...	...	...
Manganese bronze.	90.52	9.58	...	...	...	...
		Manganese—none				
Ajax metal....	81.24	10.98	7.27	...	...	...
		Phosphorus or arsenic		.37		
American anti-friction metal.	...	...	88.32	11.93	...	...
Harrington bronze.	55.73	.97	...	...	42.67	.68
Carbox metal...	...	...	84.33	14.38	trace	.61
Hard lead.....	...	...	94.4	6.03	...	...
Phosphor bronze.	79.17	10.22	9.61	...	...	...
		Phosphorus	.94			
Ex. B. metal...	76.8	8	15.0	...	...	...
		Phosphorus	.2			

TABLE 13.—COMPOSITION OF BEARING METALS (PER CENT)

	Copper	Lead	Tin	Antimony	Nickel	Phosphorus	Sulphur	Zinc
Plastic bronze........	64	30	5	...	1	...	...	...
Phosphor bronze......	79.7	95	10	...	...	.8	...	...
Cyprus bronze........	64.75	30	5	...	...	...	.25	...
Plumbic bronze.......	50	50	...	...	...	...	...	...
Parsons white brass...	2.25	.15	64.9	...	...	...	...	32.93
Demo bronze.........	60.67	32.97	4.6	...	2.1	...	...	...
Standard babbitt.....	3.7	...	88.89	7.41	...	...	...	...
Shonberg M. M. metal	2.5	.25	58.38	...	...	...	...	38.93
Souther babbitt......	7	...	84	9	...	...	...	...
German babbitt......	5.55	...	88.33	11.11	...	...	...	...

The mixing of this anti-friction metal is a trick which must be learned. The best practice is to melt the copper, tin and antimony [illegible] adding the tin to the copper and the antimony to this mixture, fluxing it with borax with the proportion of about 1½ lbs. to 175 lbs. of the mixture; but satisfactory results are obtained by melting the copper first, dropping the cold tin into the melted copper and adding the antimony, which has been separately melted. This metal is carefully skimmed before pouring, and is poured into pigs and carried into stock as it stands.

The journal bronze used on battleships of the United States Navy has this composition: Copper 82 to 84, tin 12.5 to 14.5, zinc 2.5 to 4.5, iron (max.) 0.06, lead (max.) 1.00, all in per cent., with a normal of 83–13½–3½. It is used for bearings, bushings, sleeves, slides, guide gibs, wedges on watertight doors and all parts subject to considerable wear.

Albert E. Guy gives the composition of the high-speed babbitt used in De Laval steam turbines as: copper 10, tin 80, and antimony 10 per cent. For low speeds the metal used is: lead 77, tin 6 and antimony 17 per cent.

The Mesta Machine Company, on rolling mill work, uses two grades of babbitt and a bronze. For the general run of work, a lead babbitt is satisfactory having this composition in per cent.: lead 75, tin 12.5, antimony 12.5. For high rubbing speeds a mixture is made of 1 part of the above and 2 parts of genuine babbitt. This genuine babbitt, alone, is used on rolling mill engines and in bearings subjected to shock and pound. Its composition in per cent. is: tin 82, copper 5.4, antimony 12.6. The bronze is a tough copper-tin-lead alloy very similar to Pennsylvania Railroad metal.

The alloys division of the Standards Committee of the Society of Automobile Engineers in their report for June, 1911, specifies four bearing metals as follows:

BABBIT METAL, SPECIFICATION NO. 24

Tin..................................	84 per cent.
Antimony............................	9 per cent.
Copper..............................	7 per cent.

A variation of 1 per cent. either way will be permissible in the tin, and 0.5 per cent. either way will be permissible in the antimony and copper. The use of other than virgin metals is prohibited. No impurity will be permitted other than lead, and that not in excess of 0.25 per cent.

NOTE: This grade of babbitt is special, owing to the large amount of copper contained therein. It is used for the connecting-rod bearings of gasoline motor bearings, locomotive work, or for any service where machinery designers are confronted with severe operating conditions.

WHITE BRASS, SPECIFICATION NO. 25

Copper.......................	3.00 to 6.00 per cent.
Tin, not less than..............	65.00 per cent.
Zinc..........................	28.00 to 30.00 per cent.

Metal containing more than 0.25 per cent. impurities may be rejected.

NOTE: This alloy gives good results in automobile engines, but provision should be made to have it generously lubricated.

PHOSPHOR BRONZE BEARING METAL, SPECIFICATION NO. 26

Copper......................	80.00 per cent.
Tin..........................	10.00 per cent.
Lead........................	10.00 per cent.
Phosphorus.................	0.05 to 0.25 per cent.

Impurities in excess of 0.25 per cent. will not be permitted.

NOTE: This is a metal similar to that specified by many railroads for various purposes. It is an excellent composition where good anti-frictional qualities are desired, standing up exceedingly well under heavy loads and severe usage. It should be used only against hardened steel in automobile construction.

RED BRASS, SPECIFICATION NO. 27

Copper	85.00 per cent.
Tin	5.00 per cent.
Lead	5.00 per cent.
Zinc	5.00 per cent.

A tolerance of 1 per cent. plus or minus will be allowed in the above percentages. Impurities in excess of .25 per cent. will not be permitted.

NOTE: A high grade of composition metal, and an excellent bearing where speed and pressure are not excessive. Largely used for light castings, and possesses good machining qualities.

The following particulars regarding *Westinghouse practice with babbitted bearings* are by JESSE L. JONES, metallurgist Westinghouse Electric & Mfg. Co. (*Amer. Mach., Apr.* 18, 1912). The company has adopted two principal babbitts—a tin-base babbitt that is very easy flowing and suited to pouring extremely thin linings. This babbitt is much tougher and but slightly softer than the original genuine babbitt formula which is often referred to as the U. S. Government Standard.

The second is a lead-base babbitt that contains considerable tin, flows well and is much tougher and but slightly softer than the usual babbitts of the Magnolia class.

Some use is also made of the lead-antimony, a hard genuine babbitt, and other special formulas that customers may specify.

In order to insure the best results in bearings, only the very best grades of copper, lead, tin and antimony are used. The use of drossy lead, off grades of tin and antimonial lead results in inferior babbitt and unsatisfactory bearings, and is therefore most carefully guarded against.

While the amount of copper in most babbitts is small, the use of the electrolytic grades is to be preferred, as some of the Lake brands are high in arsenic and this may cause poor adherence of the babbitt lining to bronze shells.

Most of the brands of lead on the market are almost chemically pure but they contain varying amounts of dross and oxide and the only practical way of testing them is to run down 100 lbs. or more in a graphite crucible, boil up with green hickory wood, skim off the dross and weigh the clean lead. The same brand of lead may be very clean at one time and drossy at another, and the melting loss in making babbitt from it will vary accordingly, as will also the anti-frictional qualities.

There is no real economy in using an off grade of tin running from 93 to 98 per cent. of tin, instead of Straits, as it is necessary to pay for the tin content at the market price of tin, and the lead content at the market price of lead, so that all that is obtained gratis is a little iron, antimony, dross, etc., that will increase the melting loss and add nothing to the quality of the babbitt.

The grade of antimony to be used has been the subject of very extensive practical tests. It has been found that in some cases the better brands, having almost identical chemical analysis, give quite different results in the finished babbitt in regard to hardness. As antimony is used as a hardening agent, and as the total amount used in any babbitt is relatively small, the brand which has given the best practical results, although it is the highest priced antimony on the market, has been adopted.

No adequate explanation has as yet been found to show why this particular brand gives better results than other brands of practically identical composition, but this fact has been checked so often that it is now accepted without question.

Having secured the best materials obtainable they are melted together in the proper proportions to produce the grade of babbitt desired. It is customary in making a genuine babbitt to combine the copper and antimony, or the copper, antimony and part of the tin to form a preliminary alloy or hardener. This is mixed with the rest of the tin, thus giving a more uniform product.

A temperature of about 900 deg. Fahr. should be used in mixing a babbitt to secure satisfactory alloying, and the surface of the metal should be protected from oxidization by a layer of powdered charcoal. Dross is removed by boiling up with green hickory wood, and the babbitt may be deoxidized by means of vanadium, manganese, aluminum, magnesium, sodium, etc. When all new metals are used, deoxidization is, as a rule, unnecessary.

Before pouring into ingots, the temperature of the babbitt should be lowered considerably, especially if water-cooled molds are not used, as a finer grain is thus secured.

For pouring the ingots a bucket-shaped ladle with a bail and handle and a long, square-nosed pouring spout should be used. It gives a good surface as the metal is less agitated in the pouring than when the ordinary ladle is used. A few ounces of the babbitt should first be poured into the mold, the stream interrupted for a second and then the pouring of the ingot completed. A cushion for the stream is thus formed and the surface is smoother as a result. Small air bubbles are removed by touching with a wooden pick before the metal solidifies.

Taking so much pains to obtain ingots of good appearance may seem unnecessary when the babbitt is for one's own use, but it has been found that the nicer the appearance of the ingots, the better the bearings turned out, as the workman babbitting the bearings will take more pains with his work than when rough-looking ingots are given him.

The Brinell hardness test has been found satisfactory as a shop test for securing uniformity in the babbitt. Tests are taken from the top, middle, and bottom of each kettle of the ingot metal and similar control tests are made daily on each of the various babbitt pots throughout the works where the bearings are filled.

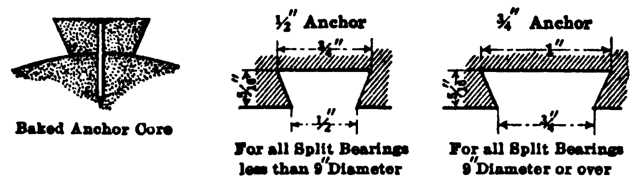


FIG. 9. FIG. 10.

FIGS. 9 and 10.—Anchor core and standard anchors of Westinghouse babbitt bearings.

Bending, fluidity and peening tests are made daily on strips 12 × ¼ × ¼ in. Analysis, tensile, compression and specific-gravity tests are also made occasionally, while a babbitt inspector, who is a thoroughly practical man, has general supervision of all babbitt pots and the pouring of all bearings.

Bearing shells for stationary apparatus are usually made from cast-iron, because of its rigidity and cheapness. Where mechanical strength, a certain amount of toughness and cheapness are desired, malleable iron is used.

Shells of cast steel are made for some customers but they are not recommended as they do not retain their shape.

For street cars, etc., standard phosphor-bronze shells are used, because with such a bearing the return of a car to the barn is assured even if the babbitt melts and runs from the bearing.

To prevent the babbitt lining from flowing, due to the revolution of the axle, all iron bearings are provided with cast anchor holes. These are made by adding to the green-sand core of the casting, baked anchor cores, secured with brads as shown in Fig. 9.

There are two sizes of anchors used, ½ and ¾ in. as shown in Fig. 10. Where bearings are bored before babbitting the cores are made of such length that the holes will be standard after boring. To help the molder in setting the cores, the pattern maker spots the pattern so that it will leave small center marks on the green-sand core. Along the straight lips of each half bearing, the anchor holes should be very numerous and as close to the edge as is possible in casting.

With bronze shells, undercut grooves or anchor holes, drilled in diagonally, may be added to prevent the lining loosening in case the bearing has been poorly tinned, but if properly tinned and babbitted, these are unnecessary. The greater the amount of babbitt in the anchor holes of a bronze bearing the greater will be the shrinkage and the more likely the lining will be to be loose and spongy.

A bearing with large anchor holes seldom gives a clear, bell-like sound when struck with a hammer. But if the anchor holes are few and small, the bearing properly tinned and poured with a thin lining, the babbitt becomes an integral part of the bearing, can only be

Rough boring all bearings before babbitting is desirable, as it gives a lining of the babbitt of uniform thickness, a uniform grain and hence a uniform rate of wear.

All iron shells are heated before babbitting to a temperature that will just admit handling them, say 350 deg. Fahr. This heating is done preferably in an oven, but it may be done over a coke or gas fire. In the latter case, especially with bronze bearings, the inner surface that is to be babbitted must be turned upward, otherwise a greasy deposit will form on the bearing that will prevent a good job of tinning, and hence the proper adherence of the babbitt.

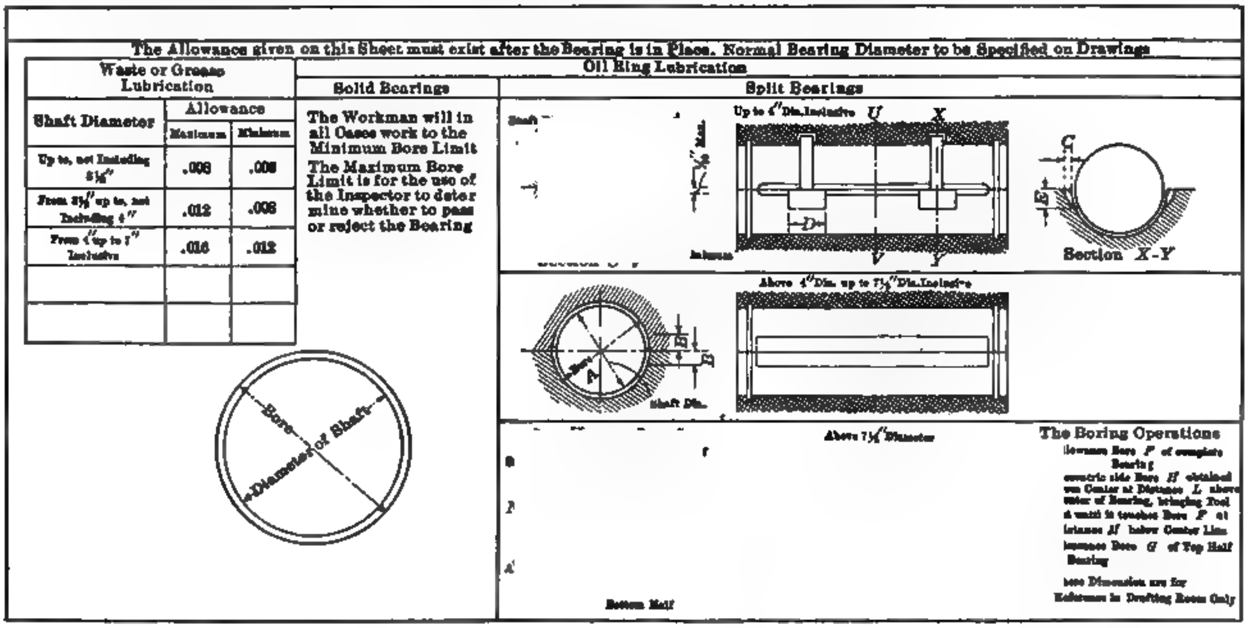


Fig. 11.—Standard bore finishes of Westinghouse babbit bearings.

Nominal shaft diameter	Horizontal			Vertical		limits + on bore Max.	Nominal shaft diameter From	Not inc.	Bore 4 allowance	B	C	D	E	Nominal shaft diameter From	Not inc.	Lip	Allowance Bore F	Bore G	Bore H (X)	K (X)	L	M	N	P
Up to ¾ inclusive	.0015	.001	.002	.002	.001	.0015	1¼	1½	+.002		3/32	¼	⅛	Not inc 7½	8	¼	+.004	+.007	+.068	.032	3/32	2	¼	3/16
Above ¾ up to 1 inclu.	.003	.002	.004	.003	.0015	.002	1½	2	+.002		3/32	¼	⅛	8	9	¼	+.004	+.008	+.068	.032	3/32	2	¼	3/16
Above 1 up to 1½ inclu.	.004	.003	.006	.005	.002	.003	2	2¼	+.002		3/32	¼	⅛	9	10	¼	+.005	+.009	+.068	.032	3/32	2¼	¼	3/16
Above 1½ up to 2½ inclu.	.006	.004	.008	.006	.002	.004	2¼	2½	+.002		⅛	¼	⅛	10	11	¼	+.005	+.010	+.100	.047	3/16	2½	¼	3/16
Above 2½ up to 4 inclu.	.008	.005	.011	.008	.003	.005	2½	2¾	+.002		⅛	¼	⅛	11	1′-0″	¼	+.006	+.011	+.100	.047	3/16	2¾	¼	3/16
Above 4 up to 4½ inclu.	.009	.006	.012	.009	.004	.006	2¾	3	+.002		⅛	¼	⅛	1′-0″	1′-1″	¼	+.006	+.012	+.100	.047	3/16	3	¼	3/16
Above 4½ up to 5 inclu.	.010	.007	.013	.009	.004	.006	3	3½	+.003		⅛	¼	⅛	1′-1″	1′-2″	¼	+.006	+.013	+.100	.047	3/16	3¼	¼	3/16
Above 5 up to 5½ inclu.	.011	.008	.014	.009	.005	.006	3½	4	+.003		⅛	¼	⅛	1′-2″	1′-3″	½	+.007	+.014	+.102	.047	3/16	3½	¼	3/16
Above 5½ up to 6 inclu.	.012	.009	.015	.009	.005	.006	4	4½	+.004		⅛	¼	⅛	1′-3″	1′-4″	½	+.007	+.015	+.102	.047	3/16	3¾	¼	3/16
Above 6 up to 7 inclu.	.014	.011	.017	.009	.005	.006	4½	5	+.004	½				1′-4″	1′-6″	½	+.008	+.016	+.102	.047	3/16	4	¼	3/16
Above 7 up to 8 inclu.	.015	.012	.018	.009	.005	.006	5	5½	+.005	¾				1′-6″	1′-8″	½	+.009	+.018	+.136	.064	¼	4½	1	¼
Above 8 up to 9 inclu.	.016	.013	.019	.009	.005	.006	5½	6	+.005	1				1′-8″	1′-10″	½	+.010	+.020	+.136	.064	¼	5	1	¼
Above 9 up to 10 inclu.	.017	.014	.020	.015	.008	.010	6	6½	+.006	1¼				1′-10″	2′-0″									
Above 10 up to 15 inclu.	.018	.015	.021	.015	.010	.012	6½	7½	+.006	1¼				2′-0″										
							7½		+.007	1½														

stripped off with great difficulty and leaves a white frost on the bronze.

Iron bearings are cleaned in the tumbling barrel, or by the sand blast at the foundry. It is usually necessary to clean out the anchor holes by hand before babbitting, or even to pickle in hydrofluoric acid (especially on bearings provided with oil-ring lubrication), because any adherent sand will be loosened by the hammering necessary in adjusting the mandrel, and this sand mingling with the babbitt when poured will ruin the bearing.

The tinning of bronze shells is best done by immersing them in a pot of molten solder of half and half composition, using a saturated solution of zinc chloride as a flux, applied with a mop of clean woolen waste. Immediately after tinning, the bearing is placed on the mandrel and babbitted. Unless there is a clean film of molten solder over the entire surface to be babbitted there will not be a perfect adherence of the layer of babbitt.

This will also be true if babbitt has been used for the tinning, as the babbitt has a much higher melting-point than the solder, and

maintaining a clear molten film with it is difficult. The presence of arsenic in the babbitt, due to the use of cheap antimony, or antimonial lead, will result in loose linings also.

In order to avoid blow-holes and imperfections in the babbitt lining, it is very necessary to coat all mandrels with a very thin coating of clay wash. Put a pound or two of Jersey red clay in a pail of water and stir until suspended, then plunge the heated mandrel into it. The mandrel will soon dry and the molten babbitt will lie on it, giving a smooth surface, free from bubbles.

This makes it possible to line a bearing with as little as $\frac{1}{16}$ in. of babbitt and the surface will be so smooth that only .008 to .010 in. need be machined out for the finish. Brass shells from $1\frac{1}{2}$ to $4\frac{1}{2}$ ins. in diameter are usually lined with $\frac{3}{32}$ in. of babbitt and .014 to .016 in. machined out. Iron shells are lined with $\frac{1}{8}$ in. of babbitt and $\frac{1}{16}$ in. machined out.

The use of the clay is especially necessary where oil gets on the mandrels. The oil causes the babbitt to blister. Half an hour's babbitting will not suffice to burn off the oil, but if the clay wash is used the oil is covered up and smooth bearings result.

Cast-iron shells are rarely if ever tinned, as such tinning cannot be depended upon to hold the lining in place. If the shells are made hot enough for the solder to alloy with the iron, the solder will oxidize and will not adhere. If kept cool enough not to burn the solder, the solder will fail to alloy with the iron, and hence will peel off when cool.

The babbitt is melted in cast-iron kettles holding about 500 lbs., and fired by gas. On first melting the new ingots, or in remelting the babbitt which has solidified after standing in the kettle, it will be found that the tin in the babbitt will commence to liquate at about 450 deg. Fahr.; hence it is necessary for satisfactory work to heat the babbitt to about 850 deg. Fahr. on starting up, and stir very thoroughly before pouring into the bearings, as otherwise the babbitt will not be of uniform composition.

After once thoroughly alloyed in this manner, there is comparatively little tendency for the tin to liquate, so long as the temperatures given as satisfactory pouring temperatures are maintained, although stirring of the babbitt during the pouring process is desirable.

The importance of the pouring operation may seem to be exaggerated in this statement, but if it leads the manufacturer to employ a skilled workman for pouring bearings, instead of a laborer, the slight exaggeration will be justified, for the skilled workman will not only pour the bearing properly, but he will also see to it that the quality of the babbitt, its temperature and the tinning are what they should be.

The temperature at which the babbitt is poured is important. If much above 900 deg. Fahr. the shrinkage is very pronounced, and porous areas result, while the babbitt will be dirty and oxidized and its antifrictional qualities injured. Uniformity of temperature is desirable, and this is maintained by the use of a delicate thermostat.

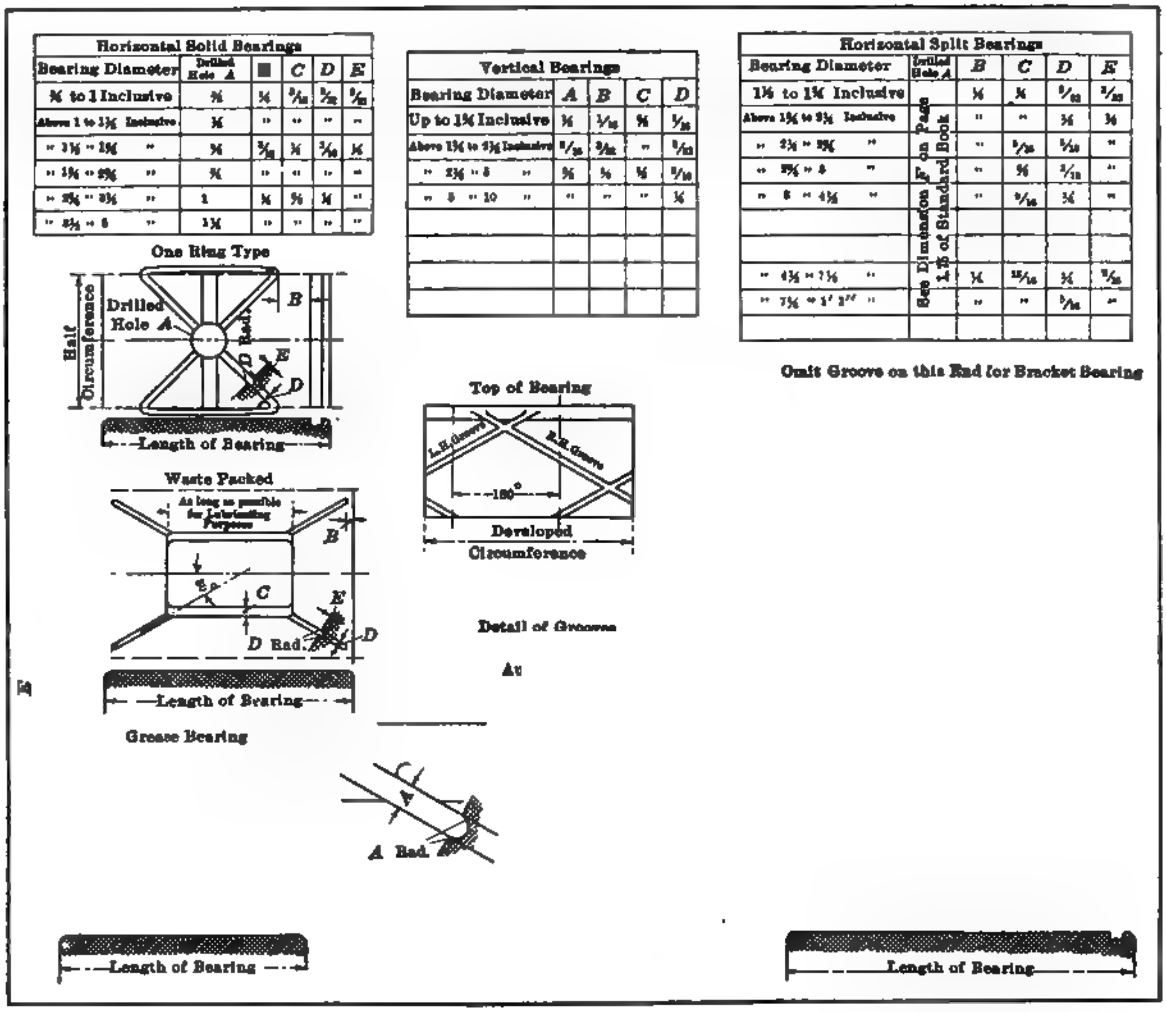


FIG. 12.—Westinghouse practice for oil grooves in bearings.

The thermostat is set for 860 deg. Fahr. and the gas is shut off when 880 deg. Fahr. is reached, or if the temperature falls below 840 deg. Fahr. more gas is turned on.

The shape of the lips of the ladle used for pouring bearings is very important. The lips should not be sharp but rounded, so that the stream will not strike either mandrel or shell, otherwise a burnt streak will result. A broad stream or an intermittent stream will produce porous areas or masses of blow-holes. A good pourer will keep both elbows close to his body, use a hand leather, so that he can grasp the handle of the ladle near the bowl, and hold his body almost rigid while pouring, thus avoiding any surging of the metal in the ladle, or splashing.

If the pourer is not very skillful, a sheet-iron bridge may be riveted to the lip of the ladle so that it will extend some distance below the surface of the metal. It can be adjusted so that it will give a stream of the diameter found best for the bearing being poured. This will not only regulate the stream out but keep the dross out of the bearing.

Bearings are preferably poured in a vertical position. Some half bearings are poured with the convex side upward through holes cast in the shells for the purpose. Very large bearings are usually poured with the concave side upward.

All solid bearings are broached on a broaching machine, which is also used for pushing out the mandrel. This operation heats the bearing, making it necessary to allow it to reach the room temperature before making the finishing cut.

D = Diameter of Shaft $F = 2.47D + \frac{1}{4}''$
$A = D + 2t = 1.1D + \frac{1}{4}''$ $t = .05D + \frac{1}{8}''$
$B = 1.8D + \frac{1}{4}''$ $d = .25D$
$C = 1.45D + \frac{1}{4}''$ $s = .35D$
$E = 1.9D + \frac{1}{4}''$ $e = .08D + \frac{1}{8}''$

FIG. 13.—A plain bearing with formulas for dimensions.

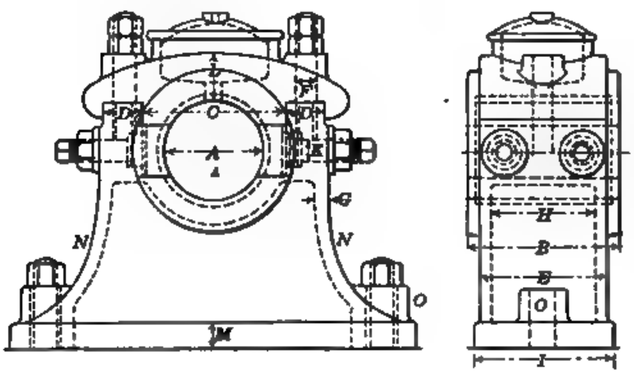


FIG. 14.—Heavy pedestal bearings with table of dimensions.

DIMENSIONS IN INCHES

A	B	C	D	E	F	G	H	I	J	K	L	M
4	6½	5½	2	5	1	⅝	3½	5½	1⅛	1⅛	2¼	1¼
4½	7	6	2¼	5½	1⅛	¾	4	6¼	1¼	1¼	2⅜	1⅜
5	8	6¾	2½	6	1¼	¾	4½	6¾	2	1¼	2½	1½
6	10	8	2¾	8	1⅜	⅞	6	9	1¾	1¼	3	1⅝
7	12	9½	3	10	1½	⅞	8	11	1¾	1½	3¼	1¾
8	13	10½	3½	10¾	1⅝	⅞	8½	12	2	1½	3½	1⅞
9	15	11½	4	12½	1⅝	⅞	10	14	2	1½	4	2
10	16	12½	4½	13½	1¾	1	10½	15	2¼	1¾	4½	2¼
12	20	15	5	17	2	1⅛	14	19	2½	2	5	2½

REMARKS.—In the column F there are two bolts to shafts 7 ins. diameter; above that, four bolts in each bearing. The side brasses or cheeks are set up with screws K, but in a manner free from the common objection to this method. The screws are inserted from the inside, and have enlarged ends to give bearing enough to meet all requirements. The recesses to receive these enlarged ends can be cored in the main casting, and the cost of construction is no more than in the case of common set-screws, which should never be employed unless of very large size. The curves at N are developed to suit the height and area of base required. The bosses at O should be at least the depth of the plinth or base flange. Two are preferable for shafts larger than 5 ins. in diameter. When the caps become heavy the oil box can be made rectangular to remove useless metal, and is preferable in that form for bearings exceeding 5 ins. in diameter.

When pedestal bearings of this kind are made without side brasses, or with a half shell on top, the transverse dimensions can be reduced, and should always be as small as possible. For mounting on masonry a sole plate should be used. This is generally required for the lateral adjustment of shafts.

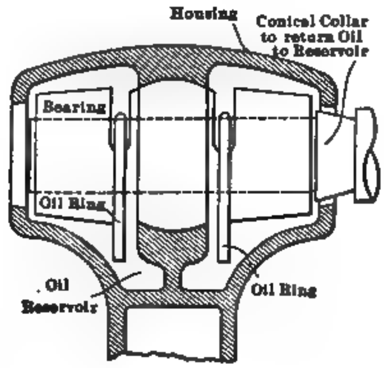


FIG. 15.—A ring-oiled bearing.

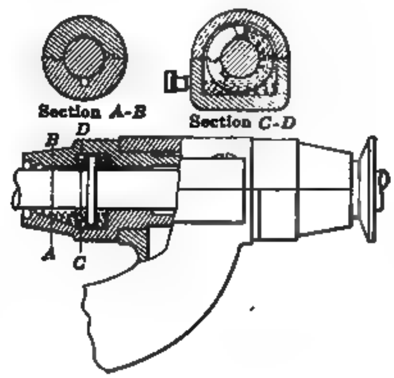


FIG. 17.—An improvement on the ring-oiled bearing.

The necessary allowances for the bore finishes of bearings are shown in Fig. 11, which gives the results of many years of experience. For additional information on this subject, see Press and Running Fits.

Oil grooves are cut in the finished bearings by hand because, as a rule, the babbitt lining is too thin to permit their being cast. Standard forms of grooving are shown in Fig. 12.

The most important element in the production of a satisfactory bearing is the pouring. The quality of the babbitt is important, the use of a thermostat is important, the tinning is important, but more depends on the actual pouring of the lining than on any other one element.

Regarding oil grooves, there is great diversity of opinion and practice. With film lubrication their presence on the pressure side of a bearing would seem more likely to do harm than good, while with ordinary lubrication the reverse is true. Some advocate blind-ended grooves to avoid escape of oil, while others object to blind grooves because of their liability to become clogged and useless.

With film lubrications, open-ended grooves are obviously inadmissible. One point is settled—the edges should be well rounded to facilitate the entrance of oil to the bearing and the same is true of the meeting edges of split boxes. Sharp edges of grooves act as oil scrapers, not oil distributors.

Bearing Design

A drawing of a simple split bearing with formulas for leading dimensions is given in Fig. 13, by C. F. BLAKE (*Amer. Mach., Nov.* 28, 1901), while Fig. 14 and the accompanying table give dimensions of heavy four-part bearings by JOHN RICHARDS (*A Manual of Machine Construction*).

The self-aligning (ball and socket) construction was introduced in 1849 by Wm. Sellers & Co., as a feature of line shaft hanger bearings. It has now come into extended use for large bearings of high-class machinery. In connection with the oil ring, first published by PROFESSOR SWEET (*Engineering, Jan.*, 1868), it is shown in Figs. 15 and 16, Fig. 15 being a typical section of a bearing fitted with both devices. Fig. 16, with the accompanying table of dimensions, gives the practice of the General Electric Co. Bearings up to and including 9 ins. diameter have two, and above that size four rings.

Fig. 17 shows an improvement on the oil ring for high speeds, by the Builders' Iron Foundry (*Amer. Mach., Feb.* 10, 1898), and applied by them to grinding and polishing stands. The loose ring is replaced by a collar, which is forced on the shaft and revolves with it. The collar dips into a capacious oil cellar below as usual, and a wide circumferential channel is cast in the box for the ring to revolve in,

FIG. 16.—Dimensions of ball and socket, ring-oiled bearings.

Nominal dia. of bearings	Dimensions in Inches														
A	B	C	D	E	F	H	J	K	L	*N	P	R	S	T	*U
7	21	½	22	14½	5¾	1	4¾	12½	10¾	1¾	5¾	6⅝	2¼		
8	24	½	25	16½	6⅜	1¼	4¾	14	12½	1¾	6½	7¾	2⅜		
9	27	½	28	18½	7½	1¼	5¼	15¾	14	2½	7	8⅝	3		
10	30	¾	31½	20½	8½	1½	5¾	17¾	16	2¾	6½	8¾	2½	3¾	5⅝
11	33	¾	34½	22½	8¾	1½	6	19½	17½	3⅜	7½	9½	2¾	4¾	6
12	36	¾	37½	24½	9½	1½	6½	21½	19	4	8½	10⅝	3	4¾	6
14	42	¾	43½	28½	9½	1¾	6¾	25	21½	5	10	12½	3¾	6⅜	7
15	45	¾	46½	30½	9¾	1¾	7	26½	23½	7	11¾	13½	4¾	7	7

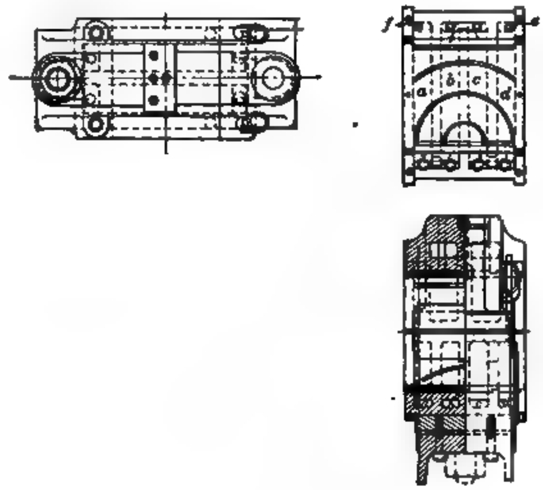
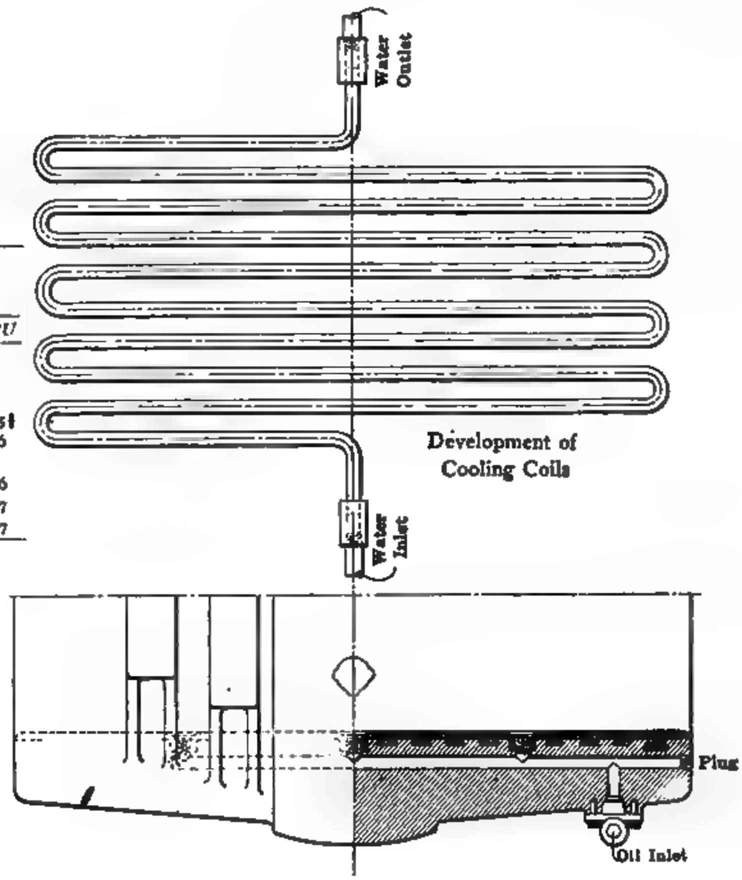


FIG. 18.—A water-jacketed bearing.



Section on Center Line Half Elevation Half Section on Center Line

FIG. 19.—Water-cooled bearing with forced lubrication.

except that at the top this channel is obstructed by projections *a*, which provide only sufficient room for the collar to revolve freely. Their office is to scrape the oil from the ring. This not only deposits it on the top of the box, but the force with which the oil strikes the projections causes it to shoot down the channels provided for it endwise of the bearing. Collecting grooves are provided at the end of the bearing, as well as free return channels to the oil cellar. The obvious result is a positive flooded circulation of oil throughout the bearing.

Fig. 18 shows a water-cooled bearing, without self-alignment, for a large vertical engine, by the Union Iron Works (*Amer. Mach., Oct.* 12, 1905). Four passages *a b c d* are cast in the lower half of the bearing, the ribs which separate the passages having openings at alternate ends to provide a continuous flow, as indicated by arrows in the plan. The ends of the outer passages have tapped holes at *e f* for the water-pipe connections. The caps of the bearings have no water connections.

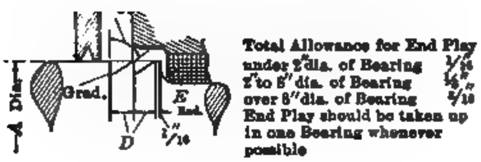


FIG. 20.—Standard oil-retaining grooves.

Size of bearing	A	B	C	D	E	I	
1¼	2	2 1/16	3/16	¼	⅛	5/16	3/16
1½	2¼	2 5/16	5/16	¼	⅛	5/16	3/16
2	2¾	2 13/16	5/16	¼	⅛	5/16	3/16
2¼	[illegible]	[illegible]	5/16	¼	⅛	5/16	3/16
2½	3¼	3 5/16	5/16	5/16	⅛	5/16	5/16
2¾	3½	3 9/16	5/16	5/16	⅛	5/16	3/16
3	3¾	3 13/16	5/16	⅜	3/16	5/16	5/16
3½	4¼	4 5/16	5/16	3/16	3/16	5/16	5/16
4	4¾	4 13/16	⅜	3/16	3/16	5/16	5/16
4½	5¼	5 5/16	⅜	3/16	¼	5/16	⅜
5	6¼	6 5/16	⅜	5/16	¼	5/16	⅜
5½	6½	6 11/16	⅜	5/16	½	5/16	⅜
6	7½	7 9/16	⅜	5/16	½	[illegible]	⅜
7	8½	8 11/16	⅜	5/16	½	5/16	⅜
8	9½	9 11/16	7/16	⅝	½	7/16	7/16
9	10½	10 11/16	7/16	⅝	½	7/16	7/16
10	12	12 1/16	7/16	[illegible]	½	[illegible]	½
11	13	13 1/16	7/16	[illegible]	½	7/16	½
12	14½	14 9/16	7/16	⅞	½	7/16	½
13	15½	15 9/16	7/16	1	½	7/16	½
14	16½	16 9/16	7/16	1	½	7/16	½
15	17½	17 11/16	7/16	1⅛	½	7/16	½

The General Electric Co. find water cooling to be increasingly effective as the cooling surfaces are brought nearer to the actual bearing surfaces. Their preferred construction of water-cooled bearing, having also forced lubrication, is shown in Fig. 19. A grid of cooling pipe is laid in recesses in the bearing sheet in such manner as to be imbedded in the babbitt. Both pipe and babbitt anchors are exaggerated in size in the illustration.

Standard oil-retaining grooves, as applied by the General Electric Co. to split bearings, are shown in Fig. 20 and the accompanying table.

End Play of Shafts

The well-known freedom of large shafts, when in motion, to move endwise under small forces is thus explained by LUCIAN E. PICOLET (*Amer. Mach., Dec.* 15, 1910).

Fig. 21 represents a loosely fitted bearing resting upon a journal and carrying a load *P*. When the journal rotates, the bearing is maintained in position by a force $f\,P$, acting opposite to the direction of rotation and equivalent to the tangential effort due to the friction, f being the friction coefficient.

For the present purpose the journal and its bearing may be represented as in Fig. 22, by a block supporting the load P and resting upon a flat surface. If the block slide uniformly under the force X, obviously $X = f\,P$. Suppose now, another force F be applied perpendicular to X, as in Fig. 23. When the block is on the point of sliding, the resultant R must be $f\,P$, since the resistance to sliding is the same in all directions. It at once follows that the application of the force F, however small, changes the direction of sliding from the direction of X to the direction of R and a gradual creeping takes place in the direction of F. It is clear that this end motion will occur, no matter how small F is, or how great the friction, because R will always be the diagonal of the rectangle formed by the forces X and F, and R will change its angle, though not its value, to suit the value of F.

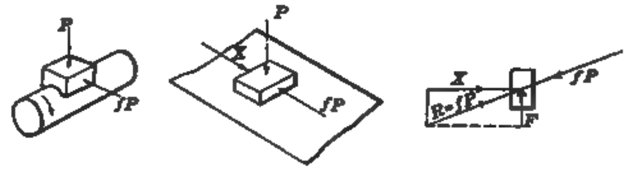


FIG. 21. FIG. 22. FIG. 23.
FIGS. 21 to 23.—End motion of a rotating journal.

FIG. 24.—Revolving element of the Glocker-White turbine governor.

Advantage may frequently be taken of the freedom of revolving shafts to move endwise in the construction of machines of which delicate adjustment is an essential feature.

An example is the well-known dead load tester for pressure gages, in which turning the plunger completely frees it of endwise friction. Another example is found in the Glocker-White governor, applied by the I. P. Morris Co. to four 13,000-h.p. turbines at Niagara Falls (*Amer. Mach., Aug.* 6, 1908), and shown in Fig. 24.

The fly balls, *a*, are of special construction to suit the peculiar requirements of turbine governing. Through links, *b*, they act on the sleeve, *c*, through which connection is made with the other mechanism of the governor. The driving shaft, *d*, is cut off at *e*, the spindle, *f*, being supported stationary at its upper end, sleeve, *c*, revolving upon it. The result is absolute freedom of sleeve, *c*, to respond to the forces exerted by the fly balls.

Bearings in which end motion takes place are free from the tendency to streak, which characterizes bearings with closely fitted end flanges.

Thrust Bearings

Thrust bearings are much less favorably situated as regards lubrication than journal bearings, as, except in the Kingsbury bearing, which see below, the film-forming tendency is absent. The best that can be expected of such bearings is oily surface lubrication. The discovery of the film-forming tendency of journal bearings by Beauchamp Tower explained the well-known fact that thrust bearings must be subjected to smaller unit pressures than journal bearings.

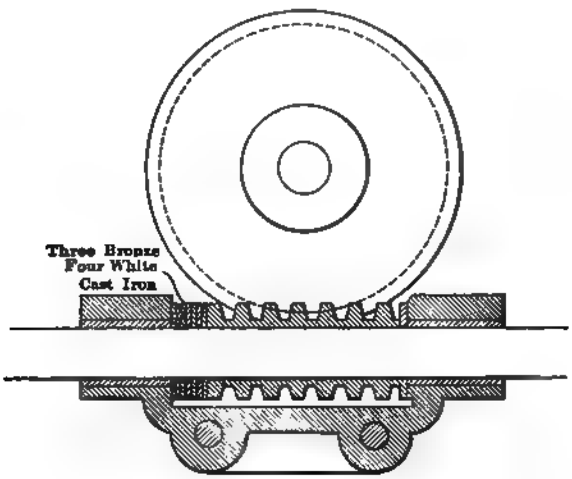


FIG. 25.—Multiple washer thrust bearing.

Much less information is available on thrust than on journal bearings. A few figures for unit loads will be found in Table 1 of Permissible Loads on Bearings and in Table 9 of Products of Pressure and Velocity of Bearings.

A construction of multiple washer thrust bearing having peculiar merit, applied by the Newton Machine Tool Works to worm drives, is shown in Fig. 25 (*Amer. Mach., Jan.* 20, 1898).

When several loose washers are interposed between the shaft collar and the face of the shaft bearing, it is obvious that slipping may occur between any pair of faces, and that this slipping will take place between those surfaces which at the moment offer the least friction. Should these surfaces from any cause increase their resistance, the slipping will be at once transferred to another joint, the various surfaces acting as mutual safety valves to one another, any surface which gets into the condition of incipient heating or cutting being at once relieved by another taking up the work. The holes in the washers are larger than the shaft on which they are placed. This construction introduces an irregular compound motion of the surfaces upon one another, the advantages of which are well understood.

It would seem to the author that these holes might well be ⅛ in. larger than the shaft. The washers should be covered to avoid criticism by the unthinking.

Washers of vulcanized fiber have been used with conspicuous success in thrust bearings. They are used by the G. A. Gray Co. in the thrust bearings of their spiral geared planers, each bearing consisting of two fiber and one hardened steel disk. Fiber washers are also common in drilling machine thrusts. S. P. Yeo reports a test of the material under severe conditions (*Amer. Mach., Oct.* 24, 1907), as follows:

Our experiment was on two disks 9 ins. diameter with a 4-in. hole ⅝ in. thick. We used regular commercial red fiber, bored and turned carefully with cut oil grooves in it. The conditions these washers worked under were as follows: number of hours per day running, 9; revolutions per minute, 15; pressure per square inch, 350 lbs.; disks running in oil.

We found upon examining these disks after one month's service that the fiber had worn to a glazy surface and showed very little wear. The life of such a pair of disks was 1½ years; the same size in bronze lasted about three months.

Fig. 26 shows the step bearing of large Curtis vertical steam turbines. The bearing plate, or lower block, is of cast-iron rigidly

FIG. 26.—Step bearing of the vertical Curtis steam turbine.

held by the frame. The block is guided at the sides and carried on a large screw, passing through a steel nut and coming in contact with a steel block set in the bearing plate. It is essential that this plate should be rigidly held, that its upper face should be a true plane set at right angle to the shaft axis, and that the clearance should be so small that the relative alinement of blocks and shaft cannot vary appreciably. The step plate is likewise of cast-iron and keyed to the lower end of the shaft. Both plates are recessed so that the surfaces of contact are collars. Directly above the step plate is a cylindrical guide bearing. The contact faces of the blocks must be truly parallel, and the contact surfaces of the end of the screw beneath the lower block and its mate must be likewise true and free from convexity. Oil is introduced through the center of the screw, passes upward, enters the recess in the center of the plate, passes out between the contact surfaces, and ascends upward through the guide bearing, as indicated by the arrows in the illustration. In service the plates are actually separated by a lubricating film; some four or five times as much lubricant as is necessary is usually pumped as a safeguard. The greater the quantity the greater the separation between the plates and the less the danger of cutting out. The actual separation of the plate is, of course, only a few thousandths of an

inch. From this fact it can be seen that this type of bearing calls for the best of workmanship.

True film lubrication takes place in the Kingsbury thrust bearing, Fig. 27 (*Amer. Mach., Mar.* 13, 1913), which shows a bearing installed as part of a 17,500-h.p. hydraulic turbine generator of the Pennsylvania Water & Power Co. at McCalls Ferry, Penn. The diameter of the bearing is 48 ins. and it carries a load of 410,000 lbs. at a speed of 94 r.p.m. The illustration shows the stationary element, the upper babbitted surfaces of the shoes *D* being the surfaces on which the revolving step bears. The shoes are segmental, as shown by the detached one at the left. A recess in the lower side of each shoe receives a block *E* one end of which is spherical and rests on the similar spherical end of a second block *H*. When in action, the segmental blocks tilt slightly and admit a film of oil between themselves and the revolving step, on which film the load is carried. Wedges *F* are for adjustment. A continuous circulation of oil is maintained, an inner retaining ring *A* and an outer ring *B* retaining the oil, of which holes, *C*, establish the level.

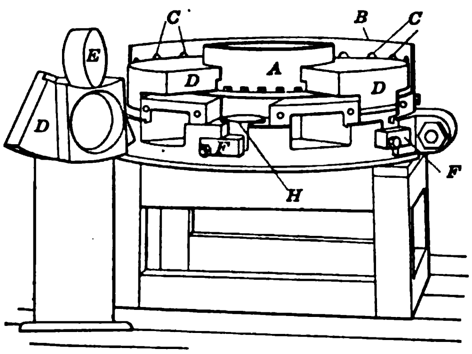


FIG. 27.—The Kingsbury thrust bearing.

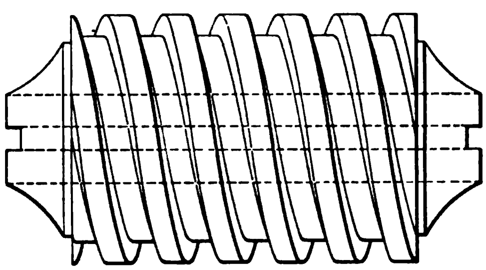


FIG. 28.—The Schiele bearing as a worm step.

No general data connecting the pressure and speed at which the oil film is established have been published, to the author's knowledge. Experiments at the works of the Westinghouse Machine Co. developed the fact that at a mean rubbing speed of 3240 ft. per min. the enormous pressure of 7000 lbs. per sq. in. was sustained without destroying the film, although, under this pressure, the babbitt began to flow. In the bearing shown, the mean rubbing speed is 900 ft. per min. and the pressure 350 lbs. per sq. in. Tested under these conditions, with a circulation of 15 gals. of light mineral oil per min., the mean temperature rise of the oil was 4½ deg. Fahr., while the coefficient of friction reached the remarkably low figure of .0008.

The reason for the unsatisfactory wear of step bearings lies in the fact that the wear of any bearing is proportional, other things being equal, to the product of the pressure on and the velocity of the rubbing surfaces; and in a new flat step bearing, while the pressure is uniformly distributed over the surface, the velocity is greater as the distance from the center increases. Consequently the bearing wears much faster at the outside edges than in the center, and its effective area is practically reduced by wear, throwing increased pressure on the remaining portion and further increasing the tendency of the outer portion of the remaining effective surface to wear. The whole bearing is thus rapidly worn away in detail, as it were.

This action is reduced if the bearing surface is a ring instead of an entire disk, and it is for this reason that collar bearings, such as the thrust bearings of steamships, do much better than step bearings.

Theory indicates and practice confirms that the Schiele curve bearing, shown in Fig. 28 as the bearing of a worm, overcomes this

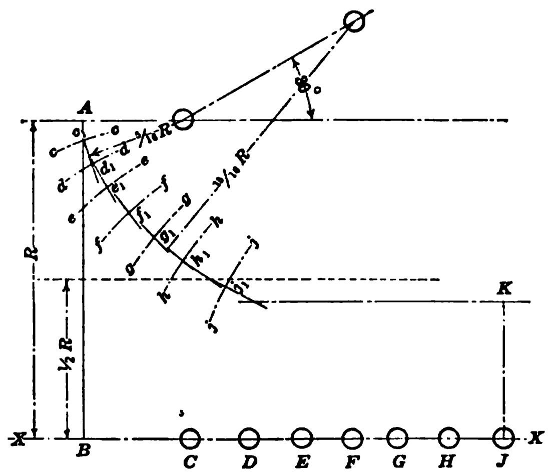


FIG. 29.—Exact and approximate methods of laying out the Schiele curve.

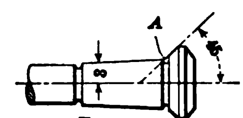


FIG. 30.

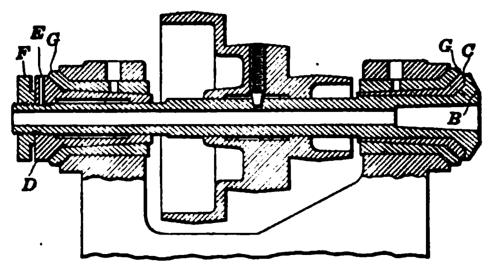


FIG. 31.

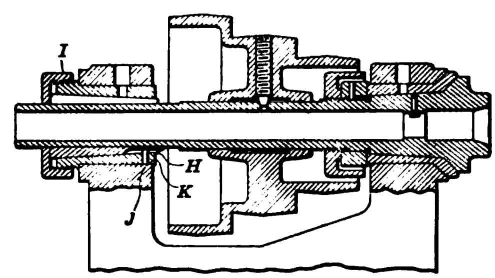


FIG. 32.

FIGS. 30 to 32.—The double cone spindle bearing.

action. This construction is one which the author feels has been unduly neglected. It conforms to Professor Sweet's dictum that "things that do not tend to wear out of truth do not wear much." Its basic principle is uniform wear, because its form is such that the pressure at different diameters increases as the velocity decreases. The cost of its construction is doubtless the chief cause of its neglect, but in heavy, rough machinery, in which unbored babbitted

bearings are admissible, the extra cost is confined to the journal where it is not serious.

The construction of the Schiele curve, together with an approximation to it which is practically sufficient, is thus explained by J. E. JOHNSON, JR. (*Amer. Mach., Apr.* 21, 1904), who has had ample experience with it in rugged work, and who unqualifiedly endorses it, one of his bearings having run for seven years without appreciable wear.

X X, Fig. 29, is the center line of the shaft, *A B* the maximum radius of the thrust bearing, and *A* the point of beginning of the curve. With *A B* as a radius from any point *C* a short distance from *B* on the axis, strike a short arc *c c*, intersecting *A B* at *c*. Draw the line *c C* or a small part of it next *c*, and from point *D* with the same radius proceed similarly, obtaining the line *d D* and successively with points *E F G*, etc., until the outer portion of last line drawn comes down to the minimum radius desired for the bearing or to the radius of the shaft passing through the thrust bearing. In practice there is little to be gained by making *J K* less than half of *A B*. A smooth curve tangent to all the short lines outside their intersecting arcs is readily drawn and is the curve desired. The whole operation can be done in less time than it takes to describe it.

For all practical purposes the curve may be drawn as two arcs of circles, as shown in the sketch, first an arc struck from the outside line of the bearing, prolonged, with a radius of 5–16 *R*, then tangent to this a second arc of radius 15–16 *R*, with its center on a line diverging from the outer line of the bearing at 30 deg. passing through the center of the first arc.

The double cone bearing has approximately the properties of the Schiele bearing and has found wide use for the spindles of precision machine tools. It originated during the early days of American watch making and was developed from the Schiele construction in order to take advantage of the grinding machine, since Schiele curves cannot be made on those machines.

Figs. 30, 31 and 32, by JAS. DANGERFIELD (*Amer. Mach., Mar.* 27, 1913) show approved constructions. The angles commonly used are 3 and 45 deg., though angles of 4 and 5 deg. have been used in place of 3 deg.

Fig. 30 shows a typical journal with angles of 3 and 45 deg., the groove *A* being for clearance in grinding. Fig. 31 shows one form of construction. In this case the spindle *B* is soft, the front bearing *C* is hardened and shrunk on; the rear bearing is the sliding sleeve *D*, with a pin *E* to key it to the spindle, and adjusted by the split binding nut *F*. The bearings *C* and *D* are ground in place on their spindle to fit the bushes *G*, which are usually of hardened steel, but bronze, cast-iron and babbitt metal have been used.

Fig. 32 shows a construction by Mr. Dangerfield in which both end thrusts are on the front bearing. The rear bearing is straight. *H* is a split taper sleeve closed by the nut *I* and keyed to its bush *K* by the pin *J*. In case of wear of the side bearing causing shake, the thrust bearing is ground off enough to bring the side bearing to a fit.

BALL AND ROLLER BEARINGS

The following information on this subject is taken largely from a paper read before the A. S. M. E. (*Trans. Vol.*, 29) and articles in periodicals by Henry Hess, the data sheets published by the Hess-Bright Mfg. Co. and the exhaustive treatise, Bearings and Their Lubrication, by L. P. Alford.

Successful performance of ball bearings depends upon the following factors: (*a*) A high degree of accuracy as regards sphericity and uniformity of diameter of the balls. The tolerance in first-class bearings between the diameters of the balls in any one bearing is .0001 in. (*b*) A high (very high) degree of surface finish. (*c*) High elastic limit of the materials. (*d*) Hardness, and especially uniform hardness, throughout each and all balls—case-hardened balls or races are inadmissible. (*e*) True rolling contact.

Requirement (*e*) has caused the failure of more *designs* of ball bearings than any other. Many designs which, theoretically, provide true rolling, require a perfection of workmanship to realize that condition which is impracticable and which has led to the failure of those designs. This is illustrated in Figs. 1–6. Fig. 1 shows a step bearing which, correctly made, fulfills the requirement. The points of contact *a*, *b*, *c*, *d* lie in the elements of a cone having its apex at the center of the shaft. Slight errors of workmanship throw the points of contact out of their correct positions, as in Fig. 2, introduce sliding and bring about failure. Similarly, Fig. 3, which is correct in theory, is subject to maladjustment which destroys correct action, and the same remark applies to Fig. 4. Fig. 5 is fatally defective because of sliding, while Fig. 6 is initially correct provided the contact points *L M* are diametrically opposed, but the correctness is subject to destruction by maladjustment. The danger of maladjustment, when adjustment is provided, is so great as to almost justify adding the absence of adjustment to the list of essential features.

Fig. 5 is a type inherited from the bicycle. Its faulty design explains its rapid failure. Fig. 6, also a bicycle bearing, is permissible in that application because of the light load which it carries. It is not suitable for heavy duty.

Successful operation of ball bearings requires attention to the following points:

Bearings must be lubricated. The oft-repeated statement that ball bearings can be run without lubricant is pernicious.

Bearings must be kept free of grit, moisture and acid. This prohibits the use of lubricants that contain or develop free acids.

The inner race must be firmly secured to the shaft. It is best to do this by a light drive fit, reinforced by binding between a substantial shoulder and a nut.

The outer race must be a slip fit in its seat.

When thrust is taken in both directions it should be by the same bearing. This avoids all strains due to flexure of the shaft or of the housing or due to temperature variation and, while doing away with the considerable shop costs inseparable from correct lengthwise dimensioning, avoids the danger of excessive end loads from forcible assembly consequent on an inaccurate lengthwise location of parts.

More than one bearing should never be dismembered at a time, in order to avoid the danger of mixing balls from different bearings; such balls from different bearings are apt to vary more than is permissible for the individual bearing.

Lubricants may range from the lightest of spindle oils at high speeds to fairly heavy greases at low speeds. The less frequent the attention given, the heavier should be the lubricant. An excess of lubricant, enough to force out at the closures, should be employed whenever the entrance of grit or moisture is to be feared. Lubricants containing or developing acid or containing free alkali must be avoided, as must those that become rancid.

A ball bearing, like a plain bearing, must have a running clearance, but in the well-made ball bearing this clearance is much smaller than in a plain bearing; in all new bearings this freedom (radial freedom) is less than .001 in. The radial freedom is accompanied by an endwise (axial) freedom of one race with reference to the other; this will vary with the ball diameter and ranges from .0006 in. to .00[illegible] in. for new bearings.

A properly made and not overloaded ball bearing will not show wear in the ordinary sense of plain bearings, *i.e.*, a reduction of the diameter and increase in bore. That and reduction in ball diameter can occur only when abrasive grit is admitted to the bearing. Grit will quickly grind down a bearing at a rate depending only upon the sharpness of the grit and the amount of time the bearing is exposed to it.

An overloaded bearing will not be worn, but the surfaces of the balls and races will be destroyed; that will show first by minute pin holes and later by flaking. A large ball will take more than its share of the load and may therefore bring about all the appearances of an overloaded bearing; to avoid this, no ball must vary by more than .0001 in. from its fellows in the same bearing. It is on this account that bearings must never be dismembered, as otherwise balls are likely to be mixed; neither must repairs be made by adding balls to a set. Such repairs should be undertaken only by the maker, who has full sets of even-size balls available. The balls of commerce are never sold within the necessary limits.

Fig. 4. Fig. 5. Fig. 6.

Figs. 1 to 6.—Correct but impracticable designs of ball bearings.

Rust is absolutely destructive to a ball bearing. It is very readily recognized; even in a bearing which has been cleaned so that no red rust is to be seen, the presence of more or less pronounced pits and excoriations, not only on the race surfaces, but also on the other parts of the bearings, is clear evidence. These pits are very distinct in appearance from those due to overload; aside from that, overload pits are necessarily confined to the balls and the ball tracks.

Although the presence of acid or alkali in many lubricants is well known, its destructive effect is not generally conceded, but attributed to rust and overload. Nevertheless, it is a very serious menace with some lubricants; its ravages are clearly enough distinguished from overload, because they are found elsewhere than on the balls and ball tracks; the marks are also quite distinct from rust marks;

the acid marks often are pits, but always show also clearly-defined irregular etchings, similar to, though less pronounced than, those produced by acid etching of damascened gun-barrels.

A very considerable rocking freedom of the one race with reference to the other as the result of grit cutting will do no harm. An amount of rocking that at first seems alarming in the individual bearing may be due to a radial clearance of but few thousandths of an inch. It is the radial clearance which determines the further usefulness of the bearing; as two bearings are always used at some distance apart in the support of a shaft or wheel, the rock is governed by the radial freedom of the bearings and the distance between them, not by the angular freedom of the bearings individually. To determine its true radial freedom, the bearing must be so held that the races are moved only crosswise without any lengthwise or tilting motion.

Bearings in which the balls or ball tracks are pitted or roughed up from rust, acid or overload are usually beyond repair.

Bearings that are ground down by grit so as to be loose, can be put in good order by refilling with a new set of larger balls. The same amount of care must be exercised to have all these balls within .0001 in. as with a new bearing; it will not do to put in a few new balls only, nor will it do to accept a dealer's belief that the balls in his

between the balls, have become the accepted design by the Hess-Bright Mfg. Co. Some other manufacturers prefer the cut race filled with balls and without separators.

Running tests and accumulated experience have proven that this type of bearing with separators will also carry a thrust load far beyond what would be expected or what calculations based on the wedging action would indicate. The safe thrust-carrying capacity of such bearings is $\frac{1}{15}$ of the radial-load capacity, though under special conditions more may be imposed, depending on the relation of ball diameter, race curvature and number of balls.

Typical correct mountings of bearings for radial loads—combined in some cases with moderate thrust—are shown in Figs. 9–12.

Fig. 9 shows a mounting for radial load without thrust.

The inner race *A* should be a light drive fit on the shaft, and should further be securely clamped between a shoulder on the shaft and a nut, or their equivalents.

The shoulder *C* should not be too small; about half the thickness of the inner race for small bearings, and about $\frac{3}{4}$ the thickness of the inner race for large bearings, is good design.

The nut, when well set up, should be firmly secured against jarring loose. An effective device consists of a split spring-wire ring with one end bent inward to pass through a hole drilled through the

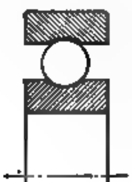


Fig. 7.

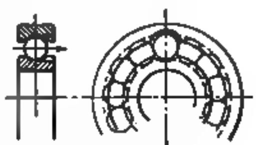
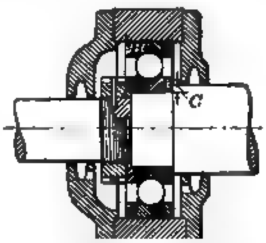


Fig. 8.



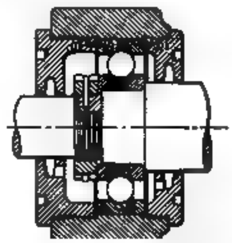


Fig. 11.

Fig. 12.

Figs. 7 to 12.—Radial bearings for radial and thrust loads.

bin are within the necessary limit. Unless this grinding action has lasted too long, it will be practicable to restore the bearing to somewhere near its original condition; even though such refilled bearing should be somewhat looser than a new one, that does not justify the expense of a replacement.

Radial Bearing Mountings

Fig. 7 shows the principle of the modern form of radial bearing. The curvature of the race cross-section is an important factor in the carrying capacity of the bearing. The local groove, Fig. 8, to permit assembling the balls, if used, must be confined to the stationary race and be placed at the unloaded side of the bearing. This requires two designs, one with the cut in the outer race, if the shaft revolves, and one with it in the inner race, if the housing revolves, or else the load must be limited to that permissible with straight races. Moreover, at high speeds, the rotation of the balls at the filling opening injures the balls and races. For these reasons uncut races with such a number of balls as may be then assembled, and separators

nut into the shaft, the body of the ring lying in a circumferentia groove turned in the nut.

The outer race *B* should be a slip fit in the box; it should not be bound endwise.

Failure to securely clamp the inner race on the shaft may produce trouble. It has been found that the hard race occasionally cuts into the relatively soft shaft, particularly when loads are heavy and of a pounding or vibratory character. A reliance on a drive fit alone is not safe. Such fit may be poorly made, it may be destroyed by occasional dismantling, or it may be destroyed by the load peening down the shaft surface.

Failure to mount the outer race with a slip fit may preven ring-from taking up an unrestrained position with reference to th he felt race and the balls and so produce an end thrust uncontemp trip with the initial selection of the bearing that might soon prove dest need not

Fig. 10 shows a mounting for combined radial and thrust . This arrangement differs from that of Fig. 9 only in having the d box. race *B* of the bearing secured endwise in the case with slight cle con-ance each side. Any thrust parallel to the axis of the shaft or a

Fig. 15.

Fig. 19.

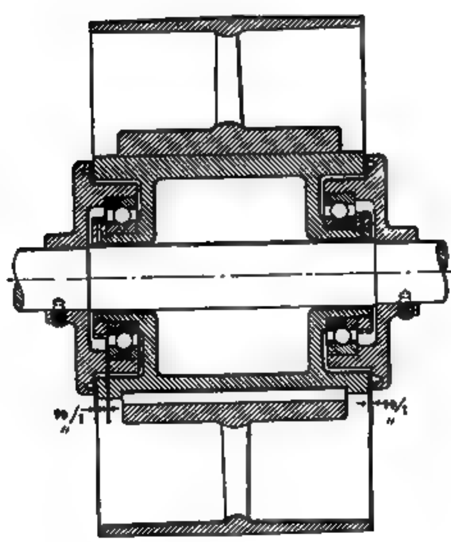


Fig. 14.

Fig. 18.

Fig. 17.

Fig. 13.

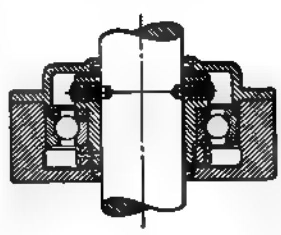


Fig. 16.

Figs. 13 to 19.—Various applications of ball bearings.

tendency of the shaft endwise will be taken up by the bearing acting as a thrust bearing.

Fig. 11 is a combination of the elements shown in Figs. 9 and 10; the remarks already given apply here also.

Whenever there are two or more bearings on a shaft the parts must be so arranged that whatever end thrusts there may be will be taken in both directions on the same bearing. Frequently designers make the mistake of taking thrust in one direction on one bearing and the opposite thrust on the other bearing. In that case any inaccuracy in the machining of shoulders on the shaft or in the case will cause a destructive thrust to be set up through the balls and bearings as soon as the end nut on the shaft is set up. Similarly, deflections of shaft or housing, or temperature variations will set up such end thrusts.

Aside from the avoidance of these possible sources of trouble, the construction recommended has the advantage of less shop cost, because avoiding otherwise necessarily very accurate work.

The illustration also shows the use of a distance sleeve to permit of the endwise binding of the inner races of both bearings by one nut only.

Fig. 12 shows a construction for taking radial and thrust loads on separate radial bearings. A radial bearing is mounted as near the load as may be and in the usual way to take only radial load. Beyond it a similar radial bearing is similarly mounted, but with its outer race clamped endwise to take thrust. To prevent the radial load being imposed on this bearing the seat is counterbored to free the bearing diameter

The use of the radial type to take thrust is preferable for speeds above 1500 r.p.m. as its carrying capacity is but little affected by speed. If the thrust direction alternates there will be a slight endwise play of the shaft, since the inner race has an axial or endwise freedom of .0006 in. for the small bearings to .006 in. for the larger ones; this will be increased by too heavy loading.

Fig. 13 shows a mounting for loose pulleys, conveyor rolls and the like, on horizontal shafts consisting of a standard inner hub on which a loose pulley of any desired diameter, and of not more than the specified width, may be secured by means of a key. The inner races are a light drive fit on a sleeve which is held by set screws or keys on the shaft. The outer races are a slip fit in their housings and one is confined with a slight end clearance, while the other race is free endwise.

Fig. 14 shows a mounting for a mule pulley on a vertical shaft. The shaft is stationary and the pulley rotates. Two objects are gained by the peculiar form of mounting shown. One is retention of oil, which would tend to throw out, owing to centrifugal force, if the outer races rotated

in the usual manner. The other object is distribution of wear around both races. With a stationary shaft it is evident that clamping the inner race causes all the wear to take place at a single point of that race. Where the outer race is fixed and the shaft rotates, concentration of wear is prevented by the standard mounting, which allows the outer race to creep slowly around. In the mounting illustrated herewith the outer race is attached to the shaft and the inner race rotates with the pulley. Thus the wear on the inner race is distributed by rotation, and the outer race distributes its own wear by creeping. The objection to allowing the inner race to creep when mounted directly on the shaft is that the contact surface between the inner race and the shaft is not large enough to sustain the load without peening or wear of the shaft, or both.

It is evident that oil will be retained without splashing around both upper and lower races, even if the shaft be reversed end for end. A certain amount of tilting of the shaft is also permissible.

Fig. 15 shows a typical mounting for a high-speed spindle and pulley. In machine-tool work, especially in grinding, it is highly important that the spindle be subjected to no unnecessary forces producing sidewise wear of the bearing. The pulley, therefore, should be supported by bearings independent of those in which the spindle runs. The spindle itself is mounted in ball bearings in the usual way. The back end of the spindle extends without contact through a stationary tubular support on which is mounted a second pair of ball bearings, whose sole function is to support the pulley. Driving connection between the pulley and the spindle is provided by a pair of splines or feathers riveted into the hub of the pulley and fitting loosely in keyways in the spindle. These splines permit the pulley bearings to be out of line with the spindle bearings or to wear faster than the latter without impairing the true running of the spindle.

The rear ball bearing of the spindle (*i.e.*, the one in the center) has a special outer race without a groove. This construction is provided against the possibility of the spindle expanding from the heat developed at the wheel when grinding. With the form of race shown, the inner race can move indefinitely lengthwise without the outer race being forced to follow it, which it might fail to do if it also expanded slightly.

Figs. 16, 17 and 18 show mountings on shafts without shoulders, with provision for securely clamping the inner race of the ball bearing, while distributing the peening effect of the load on a sufficient length of shaft. Fig. 16 shows a radial bearing mounted in the usual manner on a sleeve, the inner race clamped by means of a nut against a substantial shoulder on a sleeve. The sleeve is locked to the shaft by means of two set screws with fitted ends sunk into the shaft. A spring ring is then snapped into a circumferential groove in the sleeve and into the slots in the set screws, securely locking them against jarring loose.

Fig. 17 includes a collar thrust bearing. Lighter thrust in the reverse direction is taken by the radial bearing. The set screws must be large enough to let their pilot ends take the end thrust without danger of shearing. The set-screw heads must, of course, be accessible from the end or through suitable holes.

Fig. 18 shows an adapter bearing used chiefly on line shafts. The adapter consists of two steel sleeves or bushes; the outer bush is driven home to its shoulder with a light drive fit. This bush is bored out taper to suit the inner split bush. The inner bush should be driven home on the shaft when the bearing and outer sleeve are in place; that will firmly and truly bind the whole on the shaft. The taper is sufficient for adaptation to the ordinary variation in shaft sizes. After the inner bush is driven home good and hard on the shaft, the split collar is brought against the bearing and clamped on the projecting bush end, thus safeguarding against the tendency of vibration to back out the taper bush.

For shafts requiring great steadiness of movement, as in precision grinding machines, ball bearings alone have been found inadequate and Fig. 19 shows the application of a floating bush to a thickness-grinding machine at the Underwood Typewriter Works, (W. M. Byorkman, *Amer. Mach.*, *Feb.* 9, 1911).

The bushes are located close to the ball bearings, and have inside and outside clearances of about $\frac{3}{1000}$ in. They have also end clearances sufficient to permit appreciable movement. The bushes carry no load, but the clearances fill with oil which acts like a dashpot to suppress the minute vibrations which would otherwise arise in the ball bearings. Not being loaded, the bushes do not wear, and their only resistance is that due to the viscosity of the oil. To prevent endwise movement of the spindle, the two thrust ball bearings near the right-hand end of the spindle are provided with an adjustable take-up, which allows metal to metal contact to be made without crushing.

In Figs. 9–12 the groove and lip end closures shown are very effective in excluding dust and grit and in retaining oil. They should be bored out no more than $\frac{1}{64}$ in. larger than the diameter of the shaft. Figs. 20–23 show still more effective arrangements.

Fig. 20 is a modification useful where much very destructive fine grit is afloat or liquid is encountered under slight pressure. The second outer groove is filled with a semi-solid grease that makes a definite, frictionless packing. Its wearing away is compensated from a hand- or spring-operated grease cup; the latter type is preferable, but demands a proper balance between grease consistency under various temperatures and the spring pressure. The hand-operated cup is more definite for all conditions, provided it is occasionally set up.

Fig. 21 is used where liquid under considerable pressure must be kept out. For occasional submersion the outer groove is simply drained outward with a free drain hole. For continuous submersion this drain hole must be connected to a pipe whose end is clear and low enough below the liquid to drain.

Many constructors prefer to pin their faith to some positive felt packing for keeping lubricant in and foreign matter out. Felt washers, as usually arranged, soon lose their contact with the shaft as they grow hard and are then worn away.

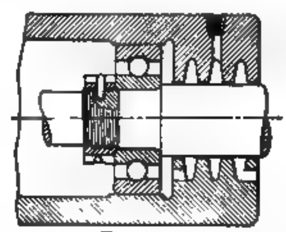


Fig. 20. Fig. 21

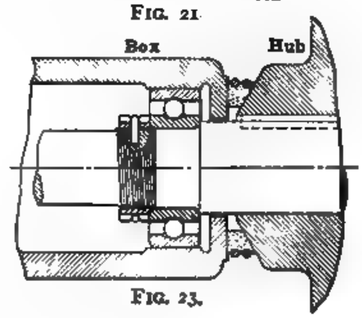


Fig. 22. Fig. 23.

Figs. 20 to 23.—Oil-retaining and dust-excluding devices.

Fig. 22 shows an arrangement for sealing at the shaft. A spring-wire ring encircles the felt washer; its pressure will force the felt into sealing contact. If the washer is laid up from a strip with scarfed ends instead of stamped from a sheet the spring ring need not be so stiff.

Fig. 23 shows a seal applied between a rotating hub and fixed box. The hub face and ring, being both beveled will permit of very considerable wear.

The felt should be thoroughly soaked in a good cylinder oil. Mutton tallow and similar acid producers must not be employed.

Collar Thrust-bearing Mountings

In collar thrust bearings, aligning washers are necessary to secure

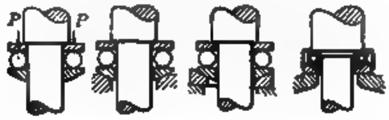


FIG. 24. FIG. 25. FIG. 26. FIG. 27.

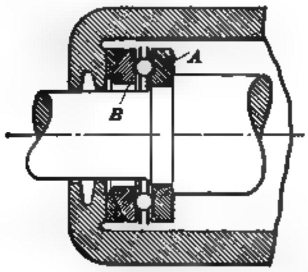


FIG. 28.

FIG. 29.

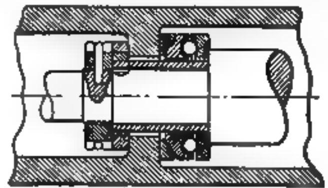


FIG. 30.

FIG. 31.

FIG. 32.

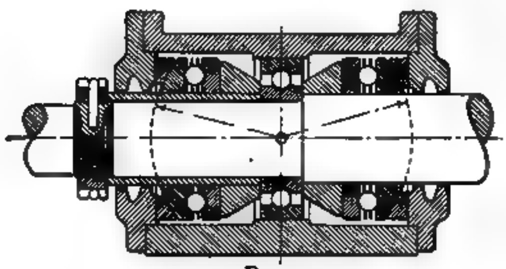


FIG. 33.

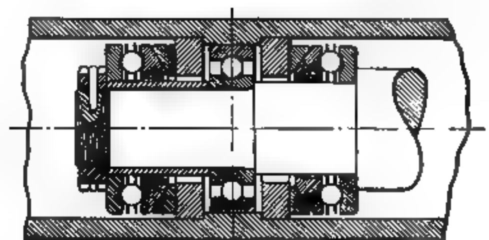


FIG. 34.

FIGS. 24 to 34.—Principles and arrangements of collar thrust bearings.

uniform loading of the balls. These washers frequently take the incorrect form shown in Fig. 24. One plate is convexed to rest in a concaved aligning seat. This form is wrong in that both plates fit the shaft and so cannot move relatively. Fig. 25 shows this corrected by freeing the lower plate from the shaft. This allows for certain errors, but not for all. Fig. 26 shows the full solution for every possible error of machining or of deflection. An aligning washer is added to receive the lower plate; this washer also is free of the shaft and also free of the seat, so that it may move crosswise. In Fig. 27 a complete unit-handling ball bearing is shown, which consists of two plates, an interposed set of balls, the lower plate convexed to seat in a concaved universal aligning washer, and a cage to hold all together.

These constructions are safeguards to provide for small unavoidable errors. It is clear that large errors will be accompanied by large eccentric action of the aligning washers, which will be accompanied by friction.

Typical correct mountings of collar thrust bearings are shown in Figs. 28–38.

The collar type of bearing, at speeds below 1500 r.p.m., will take higher thrust loads than the radial type.

The bore of the rotating plate *A*, Fig. 28, is ground to a definite size and is usually seated on the shaft. The bore of the fixed plate *B* is rather larger.

In order to insure an even distribution of the load over the entire series of balls, the seating surface of the stationary plate is spherical. If the thrust load is apt to be relieved sufficiently to permit a separation of the plates this must be prevented by a suitable arrangement, since otherwise the cage with the balls would drop slightly and the balls would be pinched as the pressure was again put on.

Wherever shaft deflections are liable to occur, the thrust bearing must have an underlying washer, or a special aligning washer must be inserted under the ball-seated race. In either case the aligning washer must be free to shift laterally to accommodate itself to the shaft deflection. A ball-seated thrust bearing without the aligning washer

FIGS. 35 to 38.—Foot step bearings.

is good only for angular misalignment, not for deflections involving lateral shift of the shaft axis.

By arranging two bearings to face in opposite directions, Fig. 29, thrust in opposite directions is taken up on ball bearings. Care must

be exercised that no undue amount of thrust is set up by the initial adjustment of the nut behind the bearing. Setting up must not be carried to the point of feeling the bearing resistance; ball-bearing friction is so low that it is perceptible to the touch only under loads that mean serious overload.

When the thrust in one direction is light, a plain disk step may answer to replace the second ball thrust, as shown in Fig. 30.

Combinations of radial and collar bearings—the former taking radial and the latter thrust loads—are shown in Figs. 31–34.

In certain combinations of sizes the rotating plate of the collar bearing might be large enough to come into contact with the stationary outer race of the radial bearing. The insertion of a washer will prevent that (Fig. 31). The inner race at A should be a light press fit.

Where the radial load causes heavy hammering and is large as compared with the thrust, it is well not to rely merely on the press fit of the radial-bearing inner race nor yet on the end-clamping due to the thrust load. By interposing, as in Fig. 32, a locked nut between the collar and the radial bearings, the latter will be securely clamped endwise. This is the preferred construction: it should be used wherever possible.

The action of the spherical seat in compensating for deflections of shaft and housing and inaccuracies of alignment will be best when the center of the spherical seat lies as nearly under the center of the radial bearing as is possible.

The arrangements shown in Figs. 33 and 34 have been advantageously employed in motor boats to take the thrust of the propeller shaft as well as its weight. Similar combinations are in use in steam yachts of 300 h.p. and in worm-driven elevators.

The arrangement of Fig. 33 is preferable. If the center of the radial bearing be also the exact center of the ball seats of both thrust bearings, the aligning washers may be dispensed with. Generally, however, it is more convenient to use them and shorten the length of the block, incidentally providing for small errors and deflections.

Figs. 35 and 36 show suitable arrangements for the lower end journals of vertical shafts.

Figs. 37 and 38 show mountings for end thrust on upper or intermediate journals, which differ from the mountings for lower end journals in the provision made for oiling. A cup is carried upward along the shaft to a height that will ensure the balls being about half immersed.

Fig. 37 takes thrust on collar bearing only. The oil cup A may be tightly spun or threaded in, or otherwise arranged to prevent the escape of oil in any way other than by overflow at its top.

Fig. 38 takes thrust and radial load on collar and radial bearings. The thrust arrangement is practically that of Fig. 37. In order to provide an oil level for the radial bearing this is not mounted directly on the shaft, but on a collar which has an annular space next to the shaft into which the oil cup A projects upward to a sufficient height.

The two oil spaces may be separately filled, or the lower one by overflow from the upper.

Figs. 35 and 37 take thrust only. Figs. 36 and 38 take radial and thrust loads on separate bearings. The sleeve or bush between the two bearings will prevent any contact between the rotating plate of the collar bearing and the stationary outer race of the radial bearing.

The action of the spherical seat in compensating for deflections of shaft and housing and inaccuracies of alignment will be best when the center of the spherical seat lies as nearly in the center of the radial bearing as is possible.

The Load Capacity of Ball Bearings

The load-carrying capacity of radial ball bearings, according to the practice of the Hess-Bright Mfg. Co. (based on the researches made for it by Professor Stribech), may be determined from the formula:

$$P = knd^2$$

in which P = load on bearing,
n = number of balls required to fill the races,
d = diameter of balls,
k = a coefficient depending on the type of bearing, the material and the speed.

For Hess-Bright bearings, in which the radius of curvature of the outer race groove $= \frac{9}{13}d$, and that of the inner race groove $= \frac{27}{52}d$, separated balls being used and uniformly distributed load and uniform speed below 3000 r.p.m. being assumed, $k = 9$ (for load in lbs. and d in units of $\frac{1}{8}$ in.). For full type bearings with the filling opening in one race at the unloaded side, otherwise as above, $k = 5$. For both ball tracks interrupted by filling openings, inelastic cage separators for the balls, or for full ball type; and speeds not over 2000 r.p.m. with a uniform distributed load, $k = 2.5$. For thrust load on a radial bearing of the first type in this tabulation $k = 0.9$.

In general, the larger the number of balls, the smaller the value of k. The radial load bearing is, within the limits stated, practically unaffected by the speed as to its carrying capacity.

Collar thrust bearings are made of three general types. In the first both races are flat; in the second one race is flat the other grooved; in the third both races are grooved. The load-carrying equation given by Mr. Hess is:

$$P = \frac{k_1 nd^2}{\sqrt[3]{S}}$$

in which P = load on bearing,
n = number of balls,
d = diameter of balls,
S = r.p.m., not exceeding 3000,
k_1 = a coefficient depending on the material and the shape of the ball races.

For the materials used by the Hess-Bright Mfg. Co. and for races having grooves with a cross-sectional radius equal approximately to $.82d$, $k_1 = 25$ to 40 (for loads in lbs. and d in units of $\frac{1}{8}$ in.). For unhardened steel, such as is occasionally used for very large races and where there is no hammering or sharp blows, $k_1 = .5$. When one or both races are flat k_1 should be reduced to one-fourth the above value.

The Standard Roller Bearing Company give the following load-capacity formulas for ball thrust bearings having a groove in each washer, stating that the ratings obtained from their use are very conservative and give a condition of loading under which a bearing can be guaranteed if properly installed and lubricated.

The formula for the light-type bearing with 17 balls is:

$$P = 32{,}000 \frac{nd^2}{a\sqrt{S}}$$

in which P = load on bearing, lbs.,
d = ball diameter, ins.,
n = number of balls,
a = pitch diameter of the ball grooves, ins.,
S = speed of the shaft, r.p.m.

The formula for the medium and heavy bearings having 11 and 9 balls respectively is:

$$P = 19{,}200 \frac{nd^2}{a\sqrt{S}}$$

The notation is the same as given above.

Ball thrust bearings are commonly of the two-point type to which according to the S. R. B. Co., the preceding formulas apply.

Variations in speed cut down the carrying capacity; sharp variations of small amplitude, particularly at high speed, have the more marked effect. Their reducing action is similar to the battering effect of sharp load variations.

Load variations reduce carrying capacity, the effect increasing with the amount of the load change and the rapidity of such change.

Accumulated experience with various classes of mechanisms is so far the only available guide for estimating the reductions in the constants k that must be made to take these influences into account.

The frictional resistances of ball bearings have, by actual measurement, been found to vary from .0011 to .0095. These are the coefficients of friction referred to the shaft diameter, thus permitting direct comparison with those of sliding friction. The higher values are due to conditions that cause a preponderance of sliding as compared with rolling friction. It must be remembered that there is no such thing as a bearing having only rolling friction; that might be possible were balls and races made originally with absolute truth of surfaces and were such truth then maintained by the absence of deformation under load. Ball bearings having a coefficient of friction materially above .0015 under the greatest allowable load are inadmissible because too short-lived. The high resistance indicates the presence of too large an element of sliding.

A good ball bearing will have a coefficient of friction, independent of the speed within wide limits, and approximating .0015. This coefficient will rise to approximately .0030 under a reduction of the load to about one-tenth of the maximum.

Dimensions of Ball Bearings

The dimensions of ball bearings are well standardized. Oddly enough, ball diameters are universally expressed in inches and fractions thereof, while the dimensions of the races are given in either millimeters or English units. German builders use millimeters with a single exception where a firm has developed a series in English units, adapting them for the British trade, though that firm also uses chiefly ball bearings in millimeters. Even in England most ball bearings are made to millimeters as is also the more general practice of American manufacturers who have followed the German example. This general adoption of the millimeter dimensions is due to the fact that early German makers rehabilitated the ball bearing by the development of the principles and construction data of the modern type and secured a wide vogue for their products that made the sizes standard. As a rule each manufacturer makes a wide and narrow type of radial bearing, with three series for each; namely, light, medium, and heavy. In some cases a fourth series has been standardized, known as extra heavy.

TABLE 1.—DIMENSIONS OF RADIAL BALL BEARINGS—LIGHT SERIES

No. of bearing	Bore		Diameter		Width		Corner at bore of inner race		Radial load, lbs.
	Mm.	Ins.	Mm.	Ins.	Mm.	Ins.	Mm.	Ins	
200	10	0.39370	30	1.18110	9	0.35433	1	0.04	120
201	12	0.47244	32	1.25984	10	0.39370	1	0.04	140
202	15	0.59055	35	1.37795	11	0.43307	1	0.04	160
203	17	0.66929	40	1.57481	12	0.47244	1	0.04	250
204	20	0.78740	47	1.85040	14	0.55118	1	0.04	320
205	25	0.98425	52	2.04725	15	0.59055	1	0.04	350
206	30	1.18110	62	2.44095	16	0.62992	1	0.04	550
207	35	1.37795	72	2.83465	17	0.66929	1	0.04	600
208	40	1.57481	80	3.14962	18	0.70866	2	0.08	860
209	45	1.77166	85	3.34647	19	0.74803	2	0.08	950
210	50	1.96851	90	3.54332	20	0.78740	2	0.08	1000
211	55	2.16536	100	3.93702	21	0.82677	2	0.08	1160
212	60	2.36221	110	4.33072	22	0.86614	2	0.08	1550
213	65	2.55906	120	4.72443	23	0.90551	2	0.08	1670
214	70	2.75591	125	4.92128	24	0.94488	2	0.08	1820
215	75	2.95277	130	5.11813	25	0.98425	2	0.08	2130
216	80	3.14962	140	5.51183	26	1.02362	3	0.12	2650
217	85	3.34647	150	5.90554	28	1.10236	3	0.12	2850
218	90	3.54332	160	6.29924	30	1.18110	3	0.12	3400
219	95	3.74017	170	6.69294	32	1.25984	3	0.12	3750
220	100	3.93702	180	7.08664	34	1.33858	3	0.12	3950
221	105	4.13387	190	7.48035	36	1.41732	3	0.12	4600
222	110	4.33072	200	7.87405	38	1.49607	3	0.12	5000

TABLE 2.—DIMENSIONS OF RADIAL BALL BEARINGS—MEDIUM SERIES

No. of bearing	Bore		Diameter		Width		Corner at bore of inner race		Radial load, lbs.
	Mm.	Ins.	Mm.	Ins.	Mm	Ins.	Mm.	Ins.	
300	10	0.39370	35	1.37795	11	0.43307	1	0.04	200
301	12	0.47244	37	1.45669	12	0.47244	1	0.04	240
302	15	0.59055	42	1.65355	13	0.51181	1	0.04	280
303	17	0.66929	47	1.85040	14	0.55118	1	0.04	370
304	20	0.78740	52	2.04725	15	0.59055	1	0.04	440
305	25	0.98425	62	2.44095	17	0.66929	1	0.04	620
306	30	1.18110	72	2.83465	19	0.74803	2	0.08	860
307	35	1.37795	80	3.14962	21	0.82677	2	0.08	1100
308	40	1.57481	90	3.54332	23	0.90551	2	0.08	1450
309	45	1.77166	100	3.93702	25	0.98425	2	0.08	1750
310	50	1.96851	110	4.33072	27	1.06299	2	0.08	2100
311	55	2.16536	120	4.72443	29	1.14173	2	0.08	2400
312	60	2.36221	130	5.11813	31	1.22047	2	0.08	2800
313	65	2.55906	140	5.51183	33	1.29921	3	0.12	3300
314	70	2.75591	150	5.90554	35	1.37795	3	0.12	4000
315	75	2.95277	160	6.29924	37	1.45669	3	0.12	4400
316	80	3.14962	170	6.69294	39	1.53544	3	0.12	5000
317	85	3.34647	180	7.08664	41	1.61418	3	0.12	5700
318	90	3.54332	190	7.48035	43	1.69292	3	0.12	6400
319	95	3.74017	200	7.87405	45	1.77166	3	0.12	7000
320	100	3.93702	215	8.46460	47	1.85040	3	0.12	7700
321	105	4.13387	225	8.85830	49	1.92914	3	0.12	8400
322	110	4.33072	240	9.44886	50	1.96851	3	0.12	10000

TABLE 3.—DIMENSIONS OF RADIAL BALL BEARINGS—HEAVY SERIES

No. of bearing	Bore		Diameter		Width		Corner at bore of inner race		Radial load, lbs.
	Mm.	Ins.	Mm.	Ins.	Mm.	Ins.	Mm.	Ins.	
403	17	0.66929	62	2.44095	17	0.66929	1	0.04	850
404	20	0.78740	72	2.83465	19	0.74803	2	0.08	1050
405	25	0.98425	80	3.14962	21	0.82677	2	0.08	1320
406	30	1.18110	90	3.54332	23	0.90551	2	0.08	1600
407	35	1.37799	100	3.93702	25	0.98425	2	0.08	1900
408	40	1.57481	110	4.33072	27	1.06299	2	0.08	2200
409	45	1.77166	120	4.72443	29	1.14173	2	0.08	2500
410	50	1.96851	130	5.11813	3f	1.22047	2	0.08	3400
411	55	2.16536	140	5.51183	33	1.29921	3	0.12	3900
412	60	2.36221	150	5.90554	35	1.37795	3	0.12	4400
413	65	2.55906	160	6.29924	37	1.45669	3	0.12	4900
414	70	2.75591	180	7.08664	42	1.65355	3	0.12	6200
415	75	2.95277	190	7.48035	45	1.77166	3	0.12	6600
416	80	3.14962	200	7.87405	48	1.88977	3	0.12	7300
417	85	3.34647	210	8.26775	52	2.04725	3	0.12	8580
418	90	3.54332	225	8.85830	54	2.12599	3	0.12	10000
419	95	3.74017	250	9.84256	55	2.16536	3	0.12	11880
420	100	3.93702	265	10.43311	60	2.36221	3	0.12	14000

Thrust bearings are usually made in three series, light, medium, and heavy. The heavier the bearing the larger the balls for a given strength.

At the 1911 spring meeting of the Society of Automobile Engineers, the standards committee rendered a report containing Tables 1, 2, and 3, giving recommended standard dimensions for the light, medium, and heavy series, respectively, for radial ball bearings. These standards are referred to as Ball Bearings Standards A. The last column gives a radial load rating in pounds for each bearing. In this connection the committee reported:

TABLE 4.—DIMENSIONS OF HESS-BRIGHT LIGHT-WEIGHT SERIES COLLAR THRUST BEARINGS
With Ball-separating Cage (Fig. 39)

Brg. No.	A		B		D		E		F	G	R		r		X		Balls		Load in lbs. and r.p.m.									Crane-hook load	Wgt. of brg. lbs.
	mm.	ins.	mm.	ins.	mm.	ins.	mm.	ins.	ins.	ins.	mm.	ins.	mm.	ins.	mm.	ins.	No.	Dia., ins.	1500	1000	500	300	150	100	50	25	10		
1009	45	1.7717	75	2.95	19	0.75	47	1.85	$1\frac{25}{32}$	$3\frac{13}{32}$	70	2.76	1.5	.059	45.5	1.79	21	$\frac{5}{16}$	640	770	900	1155	1410	1630	2200	2970	4,180	5,280	0.66
1010	50	1.9685	80	3.15	20	0.79	52	2.05	$1\frac{31}{32}$	$3\frac{9}{16}$	75	2.95	1.5	.059	48.9	1.92	23	$\frac{5}{16}$	705	860	990	1265	1540	1760	2420	3255	4,620	5,940	0.84
1011	55	2.1654	90	3.54	22	0.87	57	2.24	$2\frac{3}{16}$	4	90	3.54	1.5	.059	62.1	2.44	21	$\frac{3}{8}$	860	990	1210	1595	1980	2245	3035	4005	5,500	9,020	1.23
1012	60	2.3622	95	3.74	22	0.87	62	2.44	$2\frac{3}{8}$	$4\frac{3}{16}$	95	3.74	1.5	.059	66.5	2.62	22	$\frac{3}{8}$	905	1045	1265	1675	2090	2355	3170	4180	5,940	9,460	1.32
1013	65	2.5591	100	3.94	23	0.91	67	2.64	$2\frac{9}{16}$	$4\frac{13}{32}$	100	3.94	1.5	.059	69.9	2.75	21	$\frac{7}{16}$	1100	1320	1705	2090	2530	2970	4025	5280	7,480	12,320	1.50
1014	70	2.7559	105	4.13	24	0.94	72	2.83	$2\frac{3}{4}$	$4\frac{5}{8}$	100	3.94	1.5	.059	67.8	2.67	22	$\frac{7}{16}$	1155	1385	1785	2200	2775	3125	4225	5500	7,920	13,200	1.54
1015	75	2.9528	110	4.33	24	0.94	77	3.03	$2\frac{31}{32}$	$4\frac{25}{32}$	110	4.33	1.5	.059	77.6	3.06	23	$\frac{7}{16}$	1210	1455	1870	2310	2860	3255	4400	5785	8,140	13,640	1.65
1016	80	3.1496	120	4.72	25	0.98	83	3.27	$3\frac{5}{32}$	$5\frac{3}{16}$	115	4.53	1.5	.059	80.6	3.17	25	$\frac{7}{16}$	1320	1585	2035	2530	3080	3520	4775	6270	9,020	14,740	1.98
1017	85	3.3465	125	4.92	25	0.98	88	3.46	$3\frac{11}{32}$	$5\frac{13}{32}$	125	4.92	1.5	.059	90.3	3.56	27	$\frac{7}{16}$	1430	1715	2200	2750	3300	3830	5170	6765	9,680	16,060	2.09
1018	90	3.5433	130	5.12	25	0.98	93	3.66	$3\frac{1}{2}$	$5\frac{19}{32}$	130	5.12	1.5	.059	94.7	3.73	28	$\frac{7}{16}$	1485	1785	2310	2860	3410	3960	5370	7040	10,120	16,500	2.20
1019	95	3.7402	135	5.31	26	1.02	98	3.86	$3\frac{3}{4}$	$5\frac{3}{4}$	130	5.12	1.5	.059	92.6	3.65	29	$\frac{7}{16}$	1540	1850	2420	2970	3520	4115	5545	7260	10,340	17,160	2.33
1020	100	3.9370	140	5.51	26	1.02	103	4.06	$3\frac{15}{16}$	6	140	5.51	1.5	.059	106.0	4.17	30	$\frac{7}{16}$	1595	1915	2530	3080	3630	4270	5720	7525	10,780	17,820	2.49

|–D–|

FIG. 39.

With Ball Separator, Aligning Washer and Enclosing Cage (Fig. 40)

Bearing No.	H		I		J		K		M		O		Wgt. of brg. and encl., lbs.
	mm.	ins.	mm.	ins.	mm.	ins.	mm.	ins.	mm.	ins.	mm.	ins.	
1009KU...	53	2.09	78	3.07	70	2.76	79.5	3.13	21	0.83	3.0	0.12	1.10
1010KU...	58	2.28	83	3.27	75	2.95	84.5	3.33	22	0.87	3.0	0.12	1.21
1011KU...	63	2.48	94	3.70	85	3.35	95.5	3.76	24	0.94	3.0	0.12	1.76
1012KU..	68	2.68	99	3.90	90	3.54	100.5	3.96	24	0.94	3.5	0.14	1.87
1013KU...	73	2.87	105	4.13	95	3.74	107.0	4.21	25	0.98	3.5	0.14	2.09
1014KU...	78	3.07	109	4.29	100	3.94	111.0	4.37	26	1.02	3.5	0.14	2.31
1015KU..	83	3.27	114	4.49	105	4.13	115.5	4.55	26	1.02	3.5	0.14	2.53
1016KU.	90	3.54	124	4.88	115	4.53	126.0	4.96	27	1.06	3.5	0.14	2.86
1017KU	95	3.74	129	5.08	120	4.72	130.5	5.14	27	1.06	3.5	0.14	3.08
1018KU	100	3.94	134	5.28	125	4.92	135.5	5.33	27	1.06	3.5	0.14	3.30
1019KU..	105	4.13	138	5.43	130	5.12	140.0	5.51	28	1.10	3.5	0.14	3.41
1020KU.	110	4.33	144	5.67	135	5.31	145.5	5.73	28	1.10	3.5	0.14	3.63

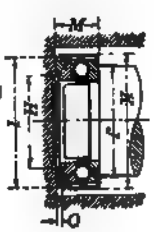


FIG. 40.

TABLE 5.—DIMENSIONS OF HESS-BRIGHT MEDIUM-WEIGHT SERIES COLLAR THRUST BEARINGS
With Ball Separator, (Figs. 41 and 42)

Brg. No.	A mm.	A ins.	B mm.	B ins.	C mm.	C ins.	D mm.	D ins.	E mm.	E ins.	F ins.	G ins.	R mm.	R ins.	r mm.	r ins.	X mm.	X ins.	Balls No.	Balls ins.	Crane hook ld.	Load 1500	1000	500	300	150	100	50	25	10	Weight of brg. lbs.
1102	10	0.3937	30	1.18	32	1.26	14	0.55	12	0.47	9/16	1 5/16	25	0.98	1	.04	9.9	0.39	8	1/4	1100	145	190	245	285	330	395	540	660	1100	.09
1103	15	0.5906	35	1.38	35	1.38	15	0.59	17	0.67	3/4	1 15/32	30	1.18	1	.04	13.4	0.53	10	1/4	1320	185	245	310	365	430	505	660	825	1320	.13
1104	20	0.7874	42	1.65	42	1.65	16	0.63	22	0.87	15/16	1 21/32	35	1.38	1	.04	16.6	0.65	13	1/4	1760	240	320	395	485	550	650	870	1080	1760	.20
1105	25	0.9843	47	1.85	47	1.85	17	0.67	27	1.06	35/32	2 1/16	35	1.38	1	.04	14.4	0.57	16	1/4	2420	295	395	495	585	680	770	1045	1355	2200	.24
1106	30	1.1811	53	2.09	55	2.17	18	0.71	32	1.26	1 3/8	2 3/16	40	1.57	1	.04	17.8	0.70	18	3/8	3520	330	440	550	660	770	880	1175	1520	2420	.33
1107	35	1.3780	62	2.44	64	2.52	21	0.83	37	1.46	1 1/2	2 19/32	50	1.97	1.5	.06	23.8	0.94	17	7/16	4400	440	550	660	880	990	1285	1725	2245	3300	.51
1108	40	1.5748	64	2.52	66	2.60	21	0.83	42	1.65	1 3/4	2 21/32	50	1.97	1.5	.06	22.4	0.88	18	7/16	5500	550	660	770	990	1210	1395	1905	2400	3520	.53
1109	45	1.7717	73	2.87	75	2.95	25	0.98	47	1.85	1 15/16	3 1/8	60	2.36	1.5	.06	28.5	1.12	17	1/2	7700	660	770	880	1210	1540	1890	2585	3255	4620	.81
1110	50	1.9685	78	3.07	80	3.15	25	0.98	52	2.05	1 7/8	3 5/16	65	2.56	1.5	.06	33.	1.30	19	1/2	8800	770	880	1100	1430	1760	2110	2880	3650	5060	.88
1111	55	2.1654	88	3.46	90	3.54	28	1.10	57	2.24	2 5/16	4	70	2.76	1.5	.06	33.8	1.33	18	7/16	9900	880	1100	1320	1650	2200	2465	3255	4355	6380	1.25
1112	60	2.3622	90	3.54	92	3.62	28	1.10	62	2.44	2 1/2	4 1/8	75	2.95	1.5	.06	38.1	1.50	19	7/16	11000	990	1210	1540	1870	2420	2605	3430	4620	6820	1.28
1113	65	2.5591	100	3.94	102	4.02	32	1.26	67	2.64	2 7/16	4 3/8	80	3.15	2	.08	38.5	1.52	18	1/2	13200	1210	1430	1760	2200	2640	3235	4235	5720	8360	1.78
1114	70	2.7559	103	4.06	105	4.13	32	1.26	72	2.83	2 7/8	4 1/2	85	3.35	2	.08	42.8	1.69	19	1/2	15400	1320	1540	1980	2420	3080	3300	4335	6070	8800	1.80
1115	75	2.9528	110	4.33	110	4.33	32	1.26	77	3.03	2 15/16	4 11/16	90	3.54	2	.08	47.1	1.85	20	1/2	16280	1430	1650	2090	2530	3300	3485	4555	6380	9240	2.09
1116	80	3.1496	115	4.53	118	4.65	35	1.38	82	3.23	3 1/32	5 1/8	95	3.74	2	.08	48.5	1.91	19	9/16	17600	1540	1760	2420	2640	3740	4180	5455	8790	11000	2.35
1117	85	3.3465	125	4.92	132	5.20	38	1.50	88	3.46	3 15/32	5 1/4	105	4.13	2	.08	54.8	2.16	19	5/8	22000	1870	2090	2860	3300	4400	4950	6600	9295	13200	3.19
1118	90	3.5433	135	5.32	135	5.32	38	1.50	93	3.66	3 5/8	5 23/32	110	4.33	2.5	.10	59.2	2.33	20	5/8	23100	1980	2200	2970	3520	4620	5225	6950	9790	13860	3.74
1119	95	3.7402	140	5.51	145	5.71	41	1.61	98	3.86	3 7/8	6 1/8	115	4.53	2.5	.10	60.2	2.37	19	11/16	26400	2200	2530	3520	4180	5280	5995	7965	11255	15400	4.25
1120	100	3.9370	150	5.91	150	5.91	41	1.61	103	4.06	3 15/16	6 3/8	125	4.92	2.5	.10	70.2	2.76	20	11/16	29040	2420	2640	3740	4400	5500	6510	8745	11440	16280	4.95
1121	105	4.1339	155	6.10	157	6.18	46	1.81	108	4.25	4	6 1/2	130	5.12	2.5	.10	69.2	2.72	19	3/4	33000	2640	3080	3960	4840	5940	7370	9845	12640	17600	5.50
1123	115	4.5276	165	6.50	167	6.57	49	1.93	118	4.65	4 5/8	7 1/4	140	5.51	3	.12	74.8	2.94	19	13/16	39600	2860	3520	4840	5500	7040	8635	11605	14830	22000	6.42
1125	125	4.9213	175	6.89	180	7.09	52	2.05	128	5.04	4 7/8	7 3/4	150	5.91	3	.12	80.2	3.16	19	7/8	46200	3080	4180	5280	6380	8140	10340	13970	17600	24200	7.63
1128	140	5.5118	200	7.87	206	8.11	58	2.28	143	5.63	5 5/8	8 1/4	170	6.69	3	.12	92.5	3.64	19	1	61600	3740	4840	6600	8140	10560	13510	18215	23010	28600	11.46

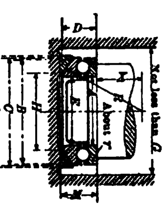


FIG. 41.

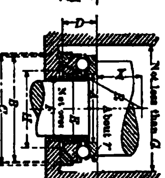


FIG. 42.

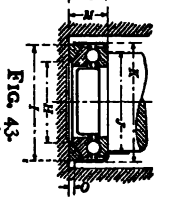


FIG. 43.

With Ball Separator, Aligning Washer and Enclosing Case (Fig. 43)

Bearing No.	H mm.	H ins.	I mm.	I ins.	J mm.	J ins.	K mm.	K ins.	M mm.	M ins.	O mm.	O ins.	Wgt. of brg. and encl., lbs.
1102 KU	14	0.55	35	1.38	25	0.98	36.	1.42	15.	0.59	3.	0.12	0.15
1103 KU	19	0.75	38	1.50	30	1.18	39.	1.53	17.	0.67	3.	0.12	0.22
1104 KU	25	0.98	45	1.77	37	1.46	46.	1.81	18.	0.71	3.	0.12	0.33
1105 KU	30	1.18	50	1.97	42	1.65	51.	2.01	19.	0.75	3.5	0.14	0.40
1106 KU	35	1.38	59	2.32	47	1.85	60.5	2.38	20.	0.79	4.	0.16	0.55
1107 KU	43	1.69	67	2.64	56	2.20	68.5	2.70	23.	0.91	4.	0.16	0.84
1108 KU	48	1.89	69	2.72	57	2.24	70.5	2.78	23.	0.91	4.	0.16	0.88
1109 KU	53	2.09	78	3.07	66	2.60	79.5	3.13	27.	1.06	4.	0.16	1.32
1110 KU	58	2.28	83	3.27	71	2.79	84.5	3.33	27.	1.06	3.5	0.14	1.43
1111 KU	64	2.52	94	3.70	81	3.19	95.5	3.76	30.	1.18	4.5	0.18	1.87
1112 KU	69	2.72	96	3.78	83	3.27	97.5	3.84	30.	1.18	4.5	0.18	1.98
1113 KU	74	2.91	105	4.13	92	3.62	107.	4.21	34.	1.34	4.5	0.18	2.75
1114 KU	79	3.11	109	4.29	95	3.74	111.	4.37	34.	1.34	4.5	0.18	2.77
1115 KU	84	3.31	114	4.49	102	4.02	115.5	4.55	34.	1.34	4.5	0.18	3.19
1116 KU	89	3.50	124	4.88	107	4.21	126.	4.96	37.	1.46	4.5	0.18	3.52
1117 KU	96	3.78	138	5.43	117	4.61	140.	5.51	40.	1.57	5.	0.20	4.62
1118 KU	101	3.98	141	5.55	127	5.00	143.	5.63	40.	1.57	5.	0.20	5.50
1119 KU	107	4.21	151	5.94	132	5.20	153.	6.02	43.5	1.71	6.	0.24	6.16
1120 KU	112	4.41	156	6.14	142	5.59	158.	6.22	44.	1.73	6.	0.24	7.15
1121 KU	118	4.65	163	6.42	147	5.79	165.	6.50	49.	1.93	6.	0.24	7.92
1123 KU	128	5.04	173	6.81	156	6.14	175.	6.89	52.	2.05	5.5	0.22	9.24
1125 KU	139	5.47	186	7.32	166	6.53	188.	7.40	56.	2.20	6.	0.24	11.00
1128 KU	155	6.10	212	8.35	191	7.52	214.	8.42	62.	2.44	7.	0.28	16.06

"Attention is called to the fact that the capacities given in the tables are based upon ball bearings manufactured of suitable workmanship and of suitable material and running at uniform speed and uniform radial load, the speed not exceeding 500 r.p.m.

"It is further suggested in explanation of the load standards that it cannot be expected that all conditions will be covered by the loads given. For conditions of shock, end thrust, and a combination of the two, greater factors of safety will have to be used."

The dimensions and capacities of Hess-Bright thrust-collar bearings, light and medium weight series, are given in Tables 4, 5 and 6.

The center of R is on the center line; its location is not quite definite, but will be approximated by drawing the ball seat tangent to a corner fillet r at the base.

The inch dimensions are the nearest equivalents to the even mm. in which the bearings are made. The loads cited are safe for steady speeds and constant loads. Consult the manufacturer regarding loads for speeds above 1500 r.p.m.

The bearings shown in Fig. 40 are the same as those shown in Fig. 39 but with balled seat washer and enclosure. For loads and speeds, also for dimensions of bearings themselves, consult the upper table.

It is advisable to give the aligning washer some freedom of sidewise movement as shown, to permit it to assume its best position in case of shaft oscillation. With such freedom allowed, the shaft may be a snug fit in the upper race.

If side freedom cannot be given the aligning washer, the shaft must be a loose fit in the upper race.

Roller Bearings

The well-known roller bearings with hollow, cylindrical, helical, flexible rollers made by the Hyatt Roller Bearing Co. are used in machinery in general, on line shafting and in automobiles. The rollers are wound from flat strip steel into a closed helical coil. Thus they are flexible and can adapt themselves to slight irregularities in either journal or box without causing excessive pressure. The cylindrical hollows in the rollers serve as storage spaces for lubricant and the helical interstices distribute the oil the entire length of the box. One half of the rollers in a box have a right-hand helix, the other half left hand.

In the form of bushings, two types are made, known as the standard type and the high-duty type. In the standard type the rollers are of carbon steel with an outer shell or lining of special analysis sheet steel. The rollers run in contact with the shaft or journal. Where the bearing surface is generous it is satisfactory to operate the rollers direct on soft-steel surfaces.

In the second type the rollers are of nickel steel, 3½ per cent. nickel, and heat treated. The lining is tubular and a tubular sleeve is provided to slip over the shaft or journal. Both of these parts are also heat treated. Several devices are used to cage the rollers, which are squared on the ends to thrust against the ends of the box. As the allowable unit-bearing pressures of the high-duty rollers are higher

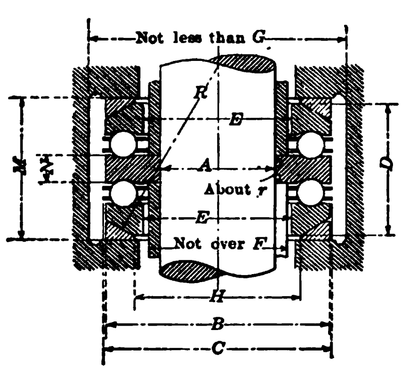


FIG. 44.

TABLE 6.—DIMENSIONS OF HESS-BRIGHT MEDIUM-WEIGHT SERIES TWO-DIRECTION COLLAR THRUST BEARINGS WITH BALL-SEPARATING CAGES (Fig. 44)

Brg. No	A		B		C		D		E		F	G	H		M		N		R		r		Balls		Load in lbs., and r.p.m.									Wt. of brg.
	mm.	ins.	mm.	ins.	mm.	ins.	mm.	ins.	mm.	ins.	ins.	ins.	mm.	ins.	mm.	ins.	mm.	ins.	mm.	ins.	mm.	ins.	No.	Dia.	1500	1000	500	300	150	100	50	25	10	alone lbs.
2102	5	0.1968	30	1.18	32	1.26	27.1	1.07	12	0.47	11/32	1 5/16	14	0.55	29.1	1.15	6.5	0.26	25	0.98	0.5	.02	8	1/4	145	190	245	285	330	395	540	660	1100	.20
2103	10	0.3937	35	1.38	35	1.38	28.1	1.11	17	0.67	7/16	1 13/32	19	0.75	32.1	1.26	6.5	0.26	30	1.18	0.5	.02	10	1/4	185	245	310	365	430	505	660	825	1320	.26
2104	13	0.5118	42	1.65	42	1.65	30.1	1.19	22	0.87	3/4	1 23/32	25	0.98	34.1	1.34	7.5	0.30	35	1.38	0.5	.02	13	1/4	240	320	395	485	550	650	870	1080	1760	.37
2105	15	0.5906	47	1.85	47	1.85	32.	1.26	27	1.06	15/16	2 1/16	30	1.18	36.	1.42	8.	0.32	35	1.38	0.5	.02	16	1/4	295	395	495	585	680	770	1045	1355	2200	.48
2106	20	0.7874	53	2.09	55	2.17	32.4	1.28	32	1.26	1 1/8	2 5/16	35	1.38	36.4	1.43	7.	0.28	40	1.57	0.5	.02	18	1/4	330	440	550	660	770	880	1175	1520	2420	.59
2107	25	0.9843	62	2.44	64	2.52	37.	1.46	37	1.46	1 1/4	2 21/32	43	1.69	41.	1.61	7.	0.28	50	1.97	0.5	.02	17	5/16	440	550	660	880	990	1285	1725	2245	3300	.90
2108	30	1.1811	64	2.52	66	2.60	37.	1.46	42	1.65	1 1/2	2 23/32	48	1.89	41.	1.61	7.	0.28	50	1.97	0.5	.02	18	5/16	550	660	770	990	1210	1395	1905	2400	3520	1.12
2109	35	1.3780	73	2.87	75	2.95	45.4	1.79	47	1.85	1 11/16	3 1/16	53	2.09	49.4	1.94	9.	0.35	60	2.36	0.5	.02	17	3/8	660	770	880	1210	1540	1890	2585	3255	4620	1.54
2110	40	1.5748	78	3.07	80	3.15	45.4	1.79	52	2.05	1 7/8	3 5/16	58	2.28	49.4	1.94	9.	0.35	65	2.56	0.5	.02	19	3/8	770	880	1100	1430	1760	2110	2880	3650	5060	1.65
2111	45	1.7717	88	3.46	90	3.54	52.4	2.06	57	2.24	2 1/16	3 11/16	64	2.52	56.6	2.23	12.	0.47	70	2.76	1.	.04	18	7/16	880	1100	1320	1650	2200	2465	3255	4355	6380	2.46
2112	50	1.9685	90	3.54	92	3.62	52.4	2.06	62	2.44	2 1/4	4 1/32	69	2.72	56.6	2.23	12.	0.47	75	2.95	1.	.04	19	7/16	990	1210	1540	1870	2420	2605	3430	4620	6820	2.64
2113	55	2.1654	100	3.94	102	4.02	56.6	2.23	67	2.64	2 7/16	4 11/32	74	2.91	60.6	2.39	11.	0.43	80	3.15	1.	.04	18	1/2	1210	1430	1760	2200	2640	3235	4235	5720	8360	3.10
2114	60	2.3622	103	4.06	105	4.13	56.6	2.23	72	2.83	2 5/8	4 5/16	79	3.11	60.6	2.39	11.	0.43	85	3.35	1.	.04	19	1/2	1320	1540	1980	2420	3080	3300	4335	6070	8800	3.19
2115	65	2.5591	110	4.33	110	4.33	60.	2.36	77	3.03	2 13/16	4 11/16	84	3.31	64.	2.52	15.	0.59	90	3.54	1.	.04	20	1/2	1430	1650	2090	2530	3300	3485	4555	6380	9240	4.51
2116	70	2.7559	115	4.53	118	4.65	66.	2.60	82	3.23	3	5 5/16	89	3.50	70.	2.76	15.	0.59	95	3.74	1.	.04	19	9/16	1540	1760	2420	2640	3740	4180	5455	8790	11000	4.62
2117	75	2.9528	125	4.92	132	5.20	70.	2.76	88	3.46	3 1/4	5 1/2	96	3.78	74.	2.91	16.	0.63	105	4.13	1.	.04	19	5/8	1870	2090	2860	3300	4400	4950	6600	9295	13200	6.05
2118	80	3.1496	135	5.32	135	5.32	70.	2.76	93	3.66	3 3/8	5 23/32	101	3.98	74.	2.91	16.	0.63	110	4.33	1.	.04	20	5/8	1980	2200	2970	3520	4620	5225	6950	9790	13860	7.70
2119	85	3.3465	140	5.51	145	5.71	75.4	2.97	98	3.86	3 5/8	6 1/16	107	4.21	80.4	2.17	16.	0.63	115	4.53	1.	.04	19	11/16	2200	2530	3520	4180	5280	5995	7965	11255	15400	7.92
2120	90	3.5433	150	5.91	150	5.91	75.4	2.97	103	4.06	3 13/16	6 1/4	112	4.41	81.4	3.20	16.	0.63	125	4.92	1.	.04	20	11/16	2420	2640	3740	4400	5500	6510	8745	11440	16280	9.24
2121	95	3.7402	155	6.10	157	6.18	85.	3.35	108	4.25	4	6 13/16	118	4.65	91.	3.58	18.	0.71	130	5.12	1.	.04	19	3/4	2640	3080	3960	4840	5940	7370	9845	12640	17600	12.10
2123	105	4.1339	165	6.50	167	6.57	90.	3.54	118	4.65	4 3/8	7 5/16	128	5.04	96.	3.78	19.	0.75	140	5.51	1.	.04	19	13/16	2860	3520	4840	5500	7040	8635	11605	14830	22000	14.30
2125	110	4.3307	175	6.89	180	7.09	93.2	3.67	128	5.04	4 3/4	7 11/16	139	5.47	101.2	3.98	18.	0.71	150	5.91	1.	.04	19	7/8	3080	4180	5280	6380	8140	10340	13970	17600	24200	16.50
2128	120	4.7244	200	7.87	206	8.11	102.4	4.03	143	5.63	5 3/8	8 1/2	155	6.10	110.4	4.35	18.	0.71	170	6.69	1.	.04	19	1	3740	4840	6600	8140	10560	13510	18215	23010	28600	22.00

than for the standard rollers the high-duty bearings have the practical advantage of being shorter for the same load than the standard bearings.

The shafting bearing is of the standard type with a horizontally

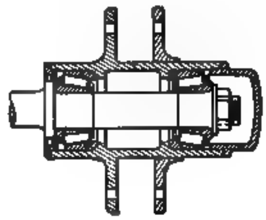


Fig. 46.

Figs. 45 and 46.—Conical roller bearings.

split box so that it can be put on a shaft anywhere and does not have to be slipped on over the end.

The allowable pressures for the standard or commercial type are fixed by the quality of the shaft or journal against which the rollers bear. For low speeds up to 50 r.p.m. the maximum limit lies between 400 to 500 lbs. per sq. in. of projected journal area. The method of housing, particularly in its relation to the distribution of load, and quality of lubrication have an influence in determining these limits.

With an increase of speed the allowable pressure decreases. For line-shaft bearings up to $3\frac{15}{16}$ ins. dia. running up to 600 r.p.m., 30 lbs. per sq. in. is considered good practice. For larger shafts the same factor is allowable up to speeds of 400 r.p.m.

The high-duty bearings carry much greater unit loads. A rating of 750 lbs. per sq. in. at 1000 r.p.m. is conservative.

A limiting maximum speed for the standard or commercial bearings is about 1500 r.p.m.; for the high-duty bearings, 3000 r.p.m.

Solid roller journal bearings divide into two classes, the first using cylindrical rollers, and the second conical rollers. Both have their extensive applications. In general, the cylindrical roller is long and small in diameter compared with its length, while the conical roller is usually short and with less difference between its length and diameter. The cylindrical roller is used on machinery in general, while the conical roller has had its special extensive application in automobile practice.

The formula for the capacity of solid roller journal bearings used by the Standard Roller Bearing Co. is as follows:

$$P = 130{,}000\,\frac{d^2nl}{3s}$$

in which P = load on bearing, lbs.,
d = diameter of rollers, ins.,
n = number of rollers,
l = length of each roller, ins.,
s = circumferential speed of each roller, ft. per min.

Conical roller journal bearings have had their most extensive use in automobile practice. Figs. 45 and 46 are from designs of the Standard Roller Bearing Company. Fig. 45 shows two constructions for automobile hubs having two single rows of rollers, and Fig. 46 shows in section a double taper roller bearing.

The load-carrying capacity is given by the following formula, from the practice of the Standard Roller Bearing Co.:

$$P = 130{,}000\,\frac{d^2nl}{3s}$$

in which P = load on bearing, lbs.,
d = mid-diameter of the rollers, ins.,
n = number of rollers,
l = contact length of a roller with its bearing washer, ins.,
s = mean circumferential speed of each roller, ft. per min.

Roller bearings have been used with conspicuous success as thrust bearings under enormous loads, a bearing of this type for a speed of 100 r.p.m. and a load of 2,000,000 lbs. (subsequently increased to 2,250,000 lbs.) by the Standard Roller Bearing Co., being shown in Fig. 47 (*Amer. Mach., July* 14, 1910).

The rollers are cylindrical and in short sections—a feature which reduces the differential sliding to an amount where it does no harm. The bearings consist of three elements:

Treads, consisting of two heat-treated, tempered and accurately ground steel washers or plates.

Roll cage, usually of bronze, complete with rolls of steel, heat-treated, tempered and accurately ground; retaining band, ball thrust, etc.

Leveling device, consisting of two washers or plates, one face of each being convexed and concaved respectively, thus providing a

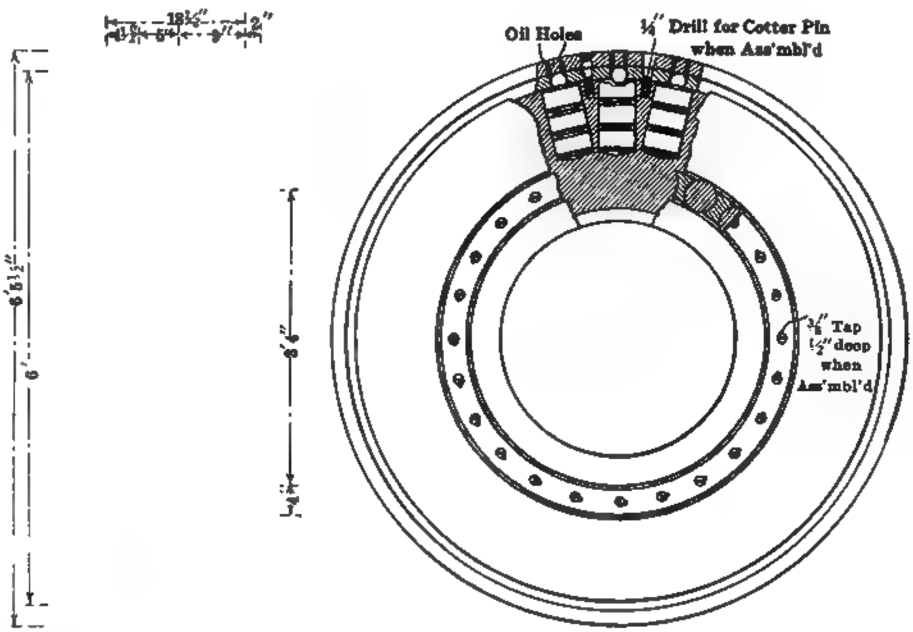


Fig. 47.—Roller thrust bearing carrying a load of 2,225,000 lbs. at 100 r.p.m.

ball-and-socket base for the bearing and insuring an equal distribution of the load, even though all parts adjacent be not in exact alignment.

The general dimensions of the cage are 2 ft. 7 ins. inside diameter by 6 ft. 5½ ins. outside diameter. Details of construction are given in Fig. 47. The bearings shown are inclosed in the casing of the machine, all voids around bearings being packed with grease, and large compression grease cups provided to supply lubricant as consumed.

The after, or right-hand tread washer, was required to be extra thick, to avoid deflection, as it is supported at its outer edge only. The forward, or left-hand tread washer, is amply supported over its entire face. The plates are made of high-grade, special alloy steel, forged into form as washers. They are bored, turned and faced to approximate dimensions, allowances being made for grinding. The washers are then subjected to heat treatment, are hardened, drawn, and then carefully ground to very close limits, the two faces being as nearly parallel as human ingenuity and highest-grade grinding machinery can produce, each face being in itself perfectly level and parallel.

The thrust cages are of phosphor bronze, carefully machined, special care being exercised in the location of the slots which carry the rollers. The rollers are manufactured from a high-grade, special alloy steel, carefully heat treated, and all rollers are ground true cylinders, the error in parallelity for a given roll and diameter of all rolls in the bearing being held within .0002 in., plus or minus, of the nominal diameter. Heavy steel retaining bands are provided which encircle the bronze cage and retain the rollers in their respective slots, a steel ball being provided at the end of each roll slot to care for the end thrust of the rolls in the slot, due to centrifugal force and the force generated by reason of the rolls being guided by the cage in a circular path, instead of their natural tangential path.

Large numbers of such bearings are in use, other notable examples by the same constructors being at the Niagara Falls power house, where they sustain a normal load ot 156,000 lbs. (extreme load 190,000 lbs.) at a speed of 250 r.p.m.

SHAFTS AND KEYS

For shaft couplings, including the Hooke universal coupling, see Index.

The strength of shafts subject to simple torsion may be determined from the formula:

$$M = .1963\ d^3S$$

in which M = twisting moment, lb.-ins.,

d = diameter of shaft, ins.,

S = fiber stress at outer fiber, lbs. per sq. in.

Suitably transformed this becomes:

$$d = \sqrt[3]{\frac{321000}{S} \times \frac{\text{h.p.}}{\text{r.p.m.}}}.$$

Experience shows that for simple torsion and for short countershafts a suitable value of $\frac{321000}{S}$ is 75, giving for this condition:

$$d = \sqrt[3]{\frac{75 \times \text{h.p.}}{\text{r.p.m.}}},$$

More exact calculations of cases involving combined bending and torsion, when justified by the known data, may be made by the formula:

$$M_e = M_b + \sqrt{M_b^2 + M_t^2}$$

in which M_e = equivalent torsion moment,

M_b = applied bending moment,

M_t = applied twisting moment.

The results given by all the above formulas may be obtained graphically by the use of Fig. 1, by PROF. J. H. BARR (*Amer. Mach, June* 11, 1908) which can be used without numerical computations for determining the following factors: (1) The diameter of a shaft for given moments and intensity of fiber stress. (2) The intensity of the stress in a given shaft under known moments. (3) The moment in a given shaft corresponding to any intensity of stress.

As plotted it covers all possible moments and shaft diameters, and all intensities of stress up to and including 15,000 lbs. per sq. in.

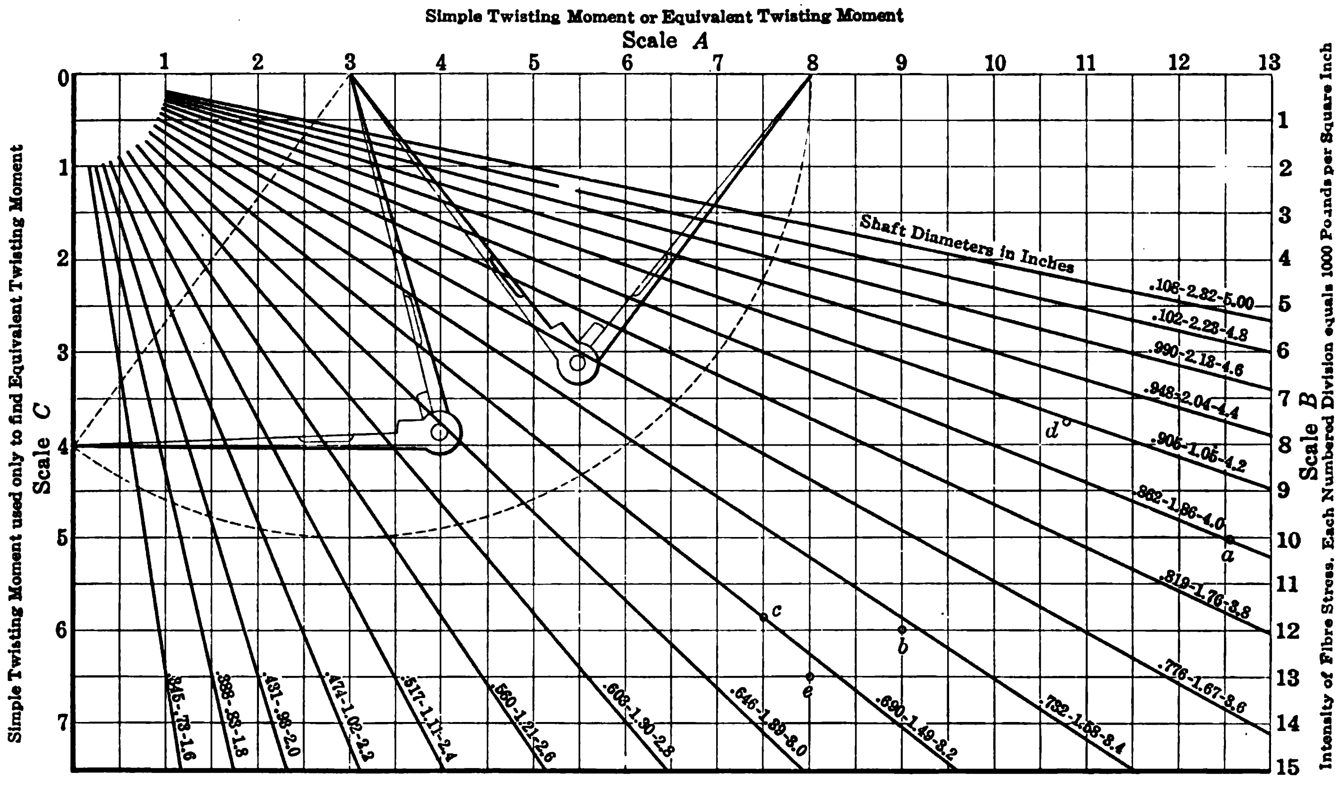


FIG. 1.—Diameters of shafts under bending and twisting moments.

which corresponds to a value of 4280 lbs. per sq. in. for S.

For the combined bending and torsion of line shafts with hangers about 8 ft. apart, a rough and ready allowance for the bending action is made by making the formula read:

$$d = \sqrt[3]{\frac{100 \times \text{h.p.}}{\text{r.p.m.}}},$$

Similarly, for the more concentrated bending action of first movers or jack shafts, the formula becomes:

$$d = \sqrt[3]{\frac{125 \times \text{h.p.}}{\text{r.p.m.}}}.$$

(*a*) To find the required diameter of a shaft for a given twisting moment and a given intensity of stress: Scale A represents the twisting moment in pound inches. Each numbered division may represent 100, 1000 or 10,000 lb.-ins., as the nature of the problem demands. Scale B represents the intensity of the fiber stress, each numbered division representing 1000 lbs. per sq. in. Locate points on the scales A and B corresponding with the quantities given in the problem. From these points erect perpendiculars to the scales and find their point of intersection. If this point falls on one of the diagonals, the shaft diameter required will be the smallest quantity given on the

diagonal if each numbered division on the A-scale has been taken as 100 lb.-ins. of twisting moment. If each division has been taken to represent 1000 lb.-ins. of moment, the shaft diameter will be the intermediate number on the diagonal. Similarly, if each division has been taken to represent 10,000 lb-ins. of moment, the shaft diameter will be the largest quantity given on the diagonal. If the point of intersection of the perpendiculars does not fall on a diagonal the values for a diagonal through that point can be obtained by interpolation.

(*b*) To find the intensity of stress for a shaft of given diameter and acted upon by a given twisting moment: Find the diagonal corresponding to the shaft diameter or interpolate for its position. Determine the point on the A-scale representing the twisting moment, letting each numbered division represent 100 lb.-ins. of moment if the smallest quantity on the diagonal represents the diameter given, or 1000 lb.-ins. if the diameter is represented by the intermediate quantity on the diagonal, or 10,000 lb.-ins. if the diameter is represented by the largest number on the diagonal. Trace a horizontal from this point on the A-scale until it intersects the diagonal representing the diameter. From this point of intersection trace a vertical until it intersects the B-scale. This last point of intersection will represent the intensity of stress required.

(*c*) To find the twisting moment which will produce a given intensity of stress in a shaft of a given diameter: Locate on scale B the point representing the intensity of stress which is given and drop a perpendicular until it intersects the diagonal representing the diameter of the shaft. If necessary interpolate for the position of this diagonal. From this point of intersection trace a horizontal until it intersects the A-scale. The point thus found represents the twisting moment required. Each numbered division equals 100 lb.-ins. if the smallest quantity on the diagonal represents the shaft diameter, or 1000 lb.-ins. if the intermediate value on the diagonal represents the shaft diameter, or 10,000 lb.-ins. if the largest value on the diagonal represents the shaft diameter.

(*d*) To find the required diameter of a shaft for a given bending moment and a given intensity of fiber stress: Multiply the given bending moment by 2 and proceed as under (*a*).

(*e*) To find the intensity of stress in a shaft of given diameter acted upon by a given bending moment: Multiply the given bending moment by 2 and proceed as under (*b*).

(*f*) To find the bending moment in a shaft of given diameter and under a given intensity of stress: Solve as under (*c*) and divide the moment thus found by 2; the quotient will be the bending moment required.

(*g*) To find the diameter of a shaft required for a given combined twisting and bending moment and with a given intensity of fiber stress: Lay out the value of the bending moment on scale A as outlined for the twisting moment under (*a*). Similarly, lay out the twisting moment on scale C, using the same value for each scale division as was used for each division of scale A. Set a pair of dividers as shown on the chart to the length between these two points. Add this length, to which the dividers have been set, to the length previously plotted on the A-scale. The point thus found will represent the value of the combined twisting and bending moment or the equivalent twisting moment. Use this point in the same way as the point representing the twisting moment was used under (*a*) and solve for the shaft diameter.

(*h*) To find the intensity of stress for a shaft of given diameter, acted upon by given combined twisting and bending moments: Find the equivalent twisting moment as outlined under (*g*), then solve for the intensity of stress as indicated under (*b*).

(*i*) To find the equivalent twisting moment for a shaft acted upon by a combined twisting and bending moment, having given the diameter of the shaft and the intensity of the fiber stress: Use the method outlined under (*c*) for a simple twisting moment; the result will be the equivalent twisting moment. For a given equivalent twisting moment there will be an indefinite number of sets of values for the simple twisting and bending moments. If either bending or twisting moment is known, the solution can be made directly. If the ratio of these moments is known, their values can be found by trial with the dividers by using the chart.

The range of the chart can be still further increased by giving larger values to each numbered scale division of scale A. If each scale division of scale A represents 100,000 lb.-ins. of moment, the corresponding shaft diameter is 10 times the smallest quantity on the corresponding diagonal. If each scale division of scale A represents 1,000,000 lb.-ins. of moment, the corresponding shaft diameter is 10 times the intermediate quantity on the corresponding diagonal, and so on.

Hollow Shafts

The diameters of hollow shafts may be obtained from the formula:

$$d^3 = \frac{d_1^4 - d_2^4}{d_1}$$

in which d = diameter of solid shaft,
d_1 = outside diameter of hollow shaft,
d_2 = inside diameter of hollow shaft.

the two shafts having the same strength.

The diameters of hollow shafts having given weight relations with solid shafts of the same strength may be determined from Fig. 2 by HENRY HESS (*Amer. Mach, Sept.* 3, 1896). The use of the chart is best shown by an example:

Required, hollow shafts of the same strength as a solid shaft 9 ins. diameter. Follow the curve, starting at 9 ins. at the left, to its intersection with, say, the 10-in. dia. line; then trace vertically downward to the internal diameter boundary line, which starts from zero of the diameter scale, and find that it is intersected at $7\frac{3}{8}$ ins. diameter; *i.e.*, a solid shaft of 9 ins. dia. is equivalent in strength to a hollow one of 10 ins. with a $7\frac{3}{8}$-in. hole. Similarly, a 12-in. with $10\frac{1}{2}$-in. hole, and a 16-in. shaft with a $15\frac{3}{16}$-in hole, are found to be equivalents.

Should it be desired to find the equivalent hollow shaft weighing, say, 50 per cent. of the 9-in. shaft, then the weight-percentage curve at the right, starting from 9 ins., is traced to the 50 per cent. vertical, which it is found to intersect at a diameter of $10\frac{3}{4}+$. Tracing the intersection of this diameter with the 9-in. curve at the left, downward to the internal diameter boundary, gives a hole of $8\frac{5}{8}$ ins.; *i.e.*, a hollow shaft of $10\frac{3}{4}$ ins. dia., with an $8\frac{5}{8}$-in. hole, is the equivalent in strength of a solid shaft of 9 ins. dia., but has only half its weight.

The results given by the chart, while not always agreeing with those given by calculations, are in sufficiently close accord for all practical purposes.

As a matter of fact, the hollow shaft will be stronger than the solid shaft, to which it is nominally equivalent, because the centers of solid shafts of any considerable size are defective and of little value, and it is to get rid of these defects that hollow forging process is resorted to.

The Torsion of Shafts

The torsion of shafts under a given stress may be determined from the formula

$$\theta = \frac{2\,lM}{.196\,Gd^4},$$

in which θ = angle of torsion in circular measure,
l = length of shaft, ins.,
M = torsion moment, lb.-ins.,
G = modulus of transverse elasticity,
= 11,000,000 for machinery steel,
d = diameter of shaft, ins.

FIG. 2.—Solid and hollow shafts of equal strength and their weight relations.

For a torsion moment, M, of 1000 lb.-ins. and a length, l, of 1 in., this becomes, when converted from circular measure to degrees,

$$\theta = \frac{.05315}{d^4},$$

in which θ = angle of torsion, deg.

The results given by this formula may be obtained graphically from Fig. 3 by W. H. RAEBURN (*Amer. Mach, June* 22, 1905), in which the quantity $\frac{.05315}{d^4}$ has been calculated for diameters from 1 to 5 ins. and plotted in the chart, the use of which is best shown by an example:

Required, the angle of torsion for the shaft shown in connection with the chart. The ordinate to the curve at $2\frac{1}{4}$ ins. diameter is .00207, which, multiplied by 12—the turning moment in thousands of lb.-ins.—and by 60—the length of the shaft in ins.—gives 1.49 deg., the angle through which the shaft will twist.

The maller chart for shafts between 1 and 2 ins. diameter is to a smaller scale than the one for larger shafts, but the larger chart can be used for shafts below 2 ins. diameter, if it be remembered that a shaft half the diameter of another will twist through 16 times as great as angle, 16 being the fourth power of 2. Thus these charts can be used for shafts either smaller or larger than those directly given.

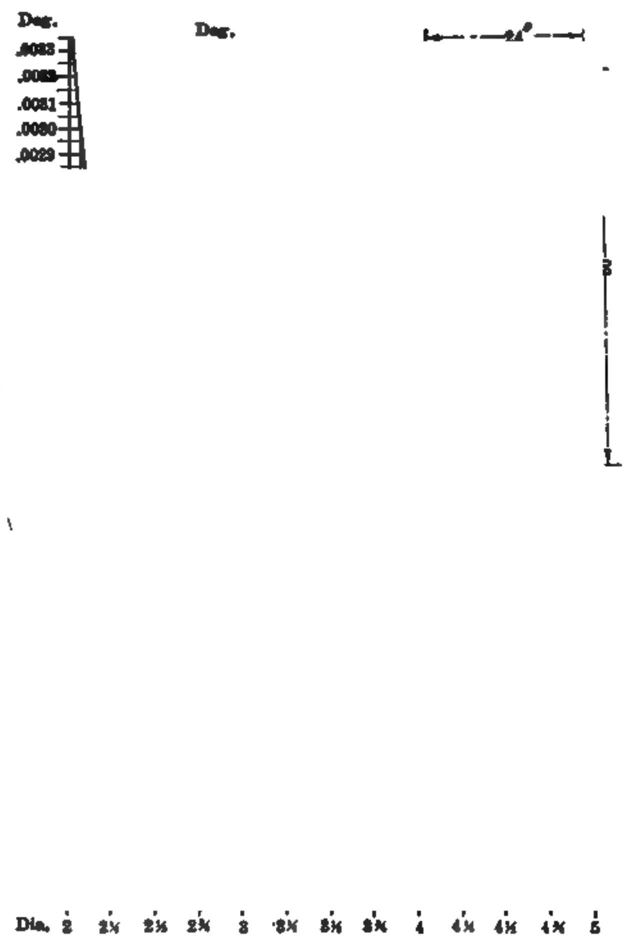


FIG. 3.—The torsion of machinery steel shafts.

The Critical Speed of Shafting

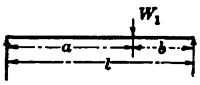
The critical speed of shafting has not received adequate experimental investigation. A comprehensive mathematical treatment of the subject has been supplied by S. H. Weaver, supervisor of mechanical calculation, the General Electric Co. (*Journal A. S. M. E., June*, 1910) and the reliability of the resulting formulas has been established by extended experience. A margin of 15 per cent. between the calculated critical speeds and the actual speeds of steam turbines has been found to give complete security from vibration in hundreds of cases. The following general observations and formulas are from Mr. Weaver's paper:

All critical-speed calculations assume an unbalanced load. It is practically impossible to balance a rotating mass so that its center of gravity exactly coincides with the mechanical axis of rotation. As the mass starts to rotate, the center of gravity will rotate in a very small radius around the shaft center. The rotation of the center of gravity at this small radius produces a centrifugal force which acts radially outward from the shaft center through the center of gravity, and rotates around the shaft with the center of gravity, causing the shaft to rotate in a bowed condition. The bowed shape will in itself

TABLE I.—CRITICAL SPEEDS OF SHAFTS

Single Concentrated Load. General Formulæ

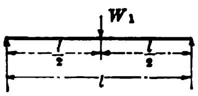
$$N_1 = \frac{187.7}{\sqrt{\Delta_1}} \qquad (1)$$



$$N_1 = 387{,}000\frac{d^2}{ab}\sqrt{\frac{l}{W_1}} \qquad (2)$$

$$N_1 = 187.7\sqrt{\frac{1}{\Delta_1}} \qquad (1)$$

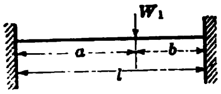
$$\Delta_1 = \frac{W_1a^2b^2}{3EIl} \qquad (3)$$



$$N_1 = 1{,}550{,}500d^2\sqrt{\frac{1}{W_1l^3}} \qquad (4)$$

$$N_1 = 187.7\sqrt{\frac{1}{\Delta_1}} \qquad (1)$$

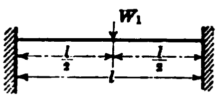
$$\Delta_1 = \frac{W_1l^3}{48EI} \qquad (5)$$



$$N_1 = 387{,}000d^2\sqrt{\frac{l^3}{W_1a^3b^3}} \qquad (6)$$

$$N_1 = 187.7\sqrt{\frac{1}{\Delta_1}} \qquad (1)$$

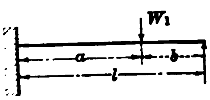
$$\Delta_1 = \frac{W_1a^3b^3}{3EIl^3} \qquad (7)$$



$$N_1 = 3{,}100{,}850d^2\sqrt{\frac{1}{W_1l^3}} \qquad (8)$$

$$N_1 = 187.7\sqrt{\frac{1}{\Delta_1}} \qquad (1)$$

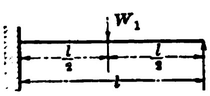
$$\Delta_1 = \frac{W_1l^3}{192EI} \qquad (9)$$



$$N_1 = 775{,}200\frac{d^2l}{ab}\sqrt{\frac{l}{W_1a(3l+b)}} \qquad (10)$$

$$N_1 = 187.7\sqrt{\frac{1}{\Delta_1}} \qquad (1)$$

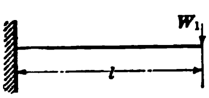
$$\Delta_1 = \frac{W_1a^3b^2}{12EIl^3}(3l+b) \qquad (11)$$



$$N_1 = 2{,}337{,}000\frac{d^2}{l}\sqrt{\frac{1}{W_1l}} \qquad (12)$$

$$N_1 = 187.7\sqrt{\frac{1}{\Delta_1}} \qquad (1)$$

$$\Delta_1 = \frac{7}{768}\,\frac{W_1l^3}{EI} \qquad (13)$$



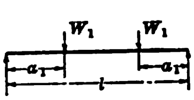
$$N_1 = 387{,}000d^2\sqrt{\frac{1}{W_1l^3}} \qquad (14)$$

$$N_1 = 187.7\sqrt{\frac{1}{\Delta_1}} \qquad (1)$$

$$\Delta_1 = \frac{W_1l^3}{3EI} \qquad (15)$$

Two Concentrated Loads. General Formulæ

$$N_1 \text{ and } N_2 = 132.2\sqrt{\left(\frac{K_1}{W_1}+\frac{K_2}{W_2}\right) \pm \sqrt{\left(\frac{K_1}{W_1}-\frac{K_2}{W_2}\right)^2 + \frac{4K_3^2}{W_1W_2}}} \qquad (16)$$



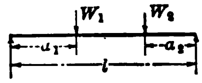
$$C = \frac{6EI}{(l-a_1-a_2)^2[l(3l-2a_1-2a_2)-(a_2-a_1)^2]} \qquad (17)$$

$$K_1 = C\frac{2}{a_1^2}\,l(l-a_2)^2 \qquad (18)$$

$$K_2 = C\frac{2}{a_2^2}\,l(l-a_1)^2 \qquad (19)$$

$$K_3 = C\frac{l(l^2-a_2^2-a_1^2)-a_1a_2(a_1-a_2)}{a_1a_2} \qquad (20)$$

N_1 and N_2 = substitute in equation (16)



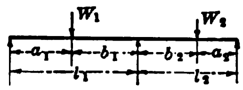
$$N_1 \text{ and } N_2 = 187.7\sqrt{\frac{K_1 \pm K_3}{W_1}} \qquad (21)$$

$$N_1 = 548{,}400\frac{d^2}{a_1(l-2a_1)}\sqrt{\frac{l}{W_1}} \qquad (22)$$

$$N_2 = 548{,}400\frac{d^2}{a_1}\sqrt{\frac{1}{W_1(3l-4a_1)}} \qquad (23)$$

$$N_2 = 187.7\sqrt{\frac{1}{\Delta_1}} \qquad (24)$$

$$\Delta_1 = \frac{W_1a_1^2}{6EI}(3l-4a_1) \qquad (25)$$



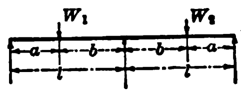
$$C = \frac{3EI}{4l_1l_2(l_1+l_2)-l_1(l_2+a_2)^2-l_2(l_1+a_1)^2} \qquad (26)$$

$$K_1 = \left(\frac{l_1}{a_1b_1}\right)^2 C[4l_2(l_1+l_2)-(l_2+a_2)^2] \qquad (27)$$

$$K_2 = \left(\frac{l_2}{a_2b_2}\right)^2 C[4l_1(l_1+l_2)-(l_1+a_1)^2] \qquad (28)$$

$$K_3 = \frac{l_1l_2}{a_1b_1a_2b_2}C(l_1+a_1)(l_2+a_2) \qquad (29)$$

N_1 and N_2 = substitute in equation (16)

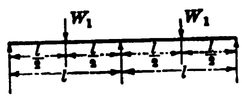


$$C = \frac{3EIl}{2a^2b^2[4l^2-(l+a)^2]} \qquad (30)$$

$$K_1 = K_2 = C[8l^2-(l+a)^2] \qquad (31)$$

$$K_3 = C(l+a)^2 \qquad (32)$$

N_1 and N_2 = substitute in equation (16)



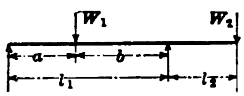
$$N_1 = 1{,}547{,}000\,d^2\sqrt{\frac{1}{W_1l^3}} \qquad (33)$$

$$N_1 = 124.3\sqrt{\frac{1}{\Delta_1}} \qquad (34)$$

$$N_2 = 2{,}337{,}000d^4\sqrt{\frac{1}{W_1l^3}} \qquad (35)$$

$$N_2 = 187.7\sqrt{\frac{1}{\Delta_1}} \qquad (36)$$

$$\Delta_1 = \frac{7}{768}\,\frac{Wl^3}{EI} \qquad (37)$$



$$C = \frac{12EI}{a^2l_2^2[4b^2l_1(l_1+l_2)-(l_1^2-a^2)^2]} \qquad (38)$$

$$K_1 = Cl_1^2l_2^2(l_1+l_2) \qquad (39)$$

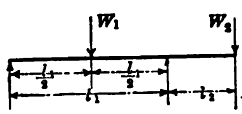
$$K_2 = Ca^2b^2l_1 \qquad (40)$$

$$K_3 = \frac{C}{2}al_1l_2(l_1^2-a^2) \qquad (41)$$

N_1 and N_2 = substitute in equation (16)

$$\Delta_1 = \frac{W_1a^2b^2}{3EIl_1} - W_2\frac{al_2}{6EIl_1}(l_1^2-a^2) \qquad (42)$$

$$\Delta_2 = \frac{W_2l_2^2}{3EI}(l_1+l_2) - W_1\frac{al_2}{6EIl_1}(l_1^2-a^2) \qquad (43)$$



$$C = \frac{3EI}{l_1^3l_2^2(l_1+l_2)-\frac{9}{16}l_1^4l_2^2} \qquad (44)$$

$$K_1 = 16Cl_2^2(l_1+l_2) \qquad (45)$$

$$K_2 = Cl_1^3 \qquad (46)$$

$$K_3 = 3Cl_1^2l_2 \qquad (47)$$

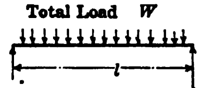
N_1 and N_2 = substitute in equation (16)

$$\Delta_1 = \frac{W_1l_1^3}{48EI} - \frac{W_2l_2l_1^2}{16EI} \qquad (48)$$

$$\Delta_2 = \frac{W_2l_2^2(l_1+l_2)}{3EI} - \frac{W_1l_2l_1^2}{16EI} \qquad (49)$$

Distributed Loads. Δ = Maximum Static Deflection

Total Load W



$$N_1 = 2{,}210{,}410d^2\sqrt{\frac{1}{Wl^3}} \qquad (50)$$

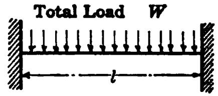
$$N_1 = 211.4\sqrt{\frac{1}{\Delta}} \qquad (51)$$

$$N_1 = 4{,}705{,}000\frac{d}{l^2} \quad \text{(shaft alone)} \qquad (52)$$

$$N = [1, 4, 9, 16, \text{etc.}]N_1 \qquad (53)$$

$$\Delta = \frac{5}{384}\,\frac{Wl^3}{EI} \qquad (54)$$

Total Load W



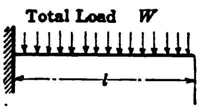
$$N_1 = 4{,}973{,}400d^2\sqrt{\frac{1}{Wl^3}} \qquad (55)$$

$$N_1 = 212.7\sqrt{\frac{1}{\Delta}} \qquad (56)$$

$$N_1 = 10{,}586{,}740\frac{d}{l^2} \quad \text{(shaft alone)} \qquad (57)$$

$$N = [1, 2.78, 5.45, 9, \text{etc.}]N_1 \qquad (58)$$

$$\Delta = \frac{Wl^3}{384EI} \qquad (59)$$



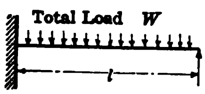
$$N_1 = 787,450 d^2 \sqrt{\frac{I}{Wl^3}} \quad (60)$$

$$N_1 = 233.3 \sqrt{\frac{I}{\Delta}} \quad (61)$$

$$N_1 = 1,676,214 \frac{d}{l^2} \quad \text{(shaft alone)} \quad (62)$$

$$N = [1,\ 6.34,\ 17.6,\ 43.6,\ \text{etc.}]N_1 \quad (63)$$

$$\Delta = \frac{Wl^3}{8EI} \quad (64)$$



$$N_1 = 3,453,715 d^2 \sqrt{\frac{I}{Wl^3}} \quad (65)$$

$$N_1 = 212.9 \sqrt{\frac{I}{\Delta}} \quad (66)$$

$$N_1 = 7,351,600 \frac{d}{l^2} \quad \text{(shaft alone)} \quad (67)$$

$$N = [1,\ 3.24,\ 6.8,\ 11.6,\ \text{etc.}]N_1 \quad (68)$$

$$\Delta = \frac{Wl^3}{185EI} \quad (69)$$

Weights in pounds and dimensions in inches.
N = critical speed in r.p.m.
N_1 = first critical speed in r.p.m.
N_2 = second critical speed in r.p.m.
Δ_1 = static deflection in ins. at W_1 if shaft is horizontal.
Δ_2 = static deflection in ins. at W_2 if shaft is horizontal.
d = diameter of shaft in ins.; I = moment of inertia.
E = 29,000,000; consider vertical shafts as horizontal.

increase the circle in which the center of gravity rotates; this increases the centrifugal force, and in turn the shaft deflection. This action continues until finally a state of equilibrium is reached where the force of the shaft deflection is equal and opposite to the centrifugal force of the mass. This condition of equilibrium for any speed can be mathematically calculated from the formulas of centrifugal force and shaft deflection. As the angular speed increases from zero the deflection increases, until the latter becomes theoretically infinite.

This is the condition of maximum vibration produced by the shaft, and the critical number of revolutions can be definitely calculated.

Beyond the critical-speed value, the vibration amplitude becomes negative, and as the speed is increased, approaches the limit of zero; in other words, above the critical speed the center of gravity revolves inside the bow of the shaft, or in a smaller circle than the shaft center; and the tendency of the rotating mass is to rotate about its own center of gravity, and not about the mechanical center. It approaches its center of gravity as a limit for infinite speed. It is also determined from the calculations that for a single concentrated load the critical-speed phenomena occur when the revolutions synchronize with the natural period of vibration of the shaft. No satisfactory explanation has been given of the detail action at the critical speed, or of the manner in which the center of gravity passes from the outside to the inside of the bow of the shaft. Theoretically the deflection or bow of the shaft becomes infinite at the critical speed. Practically it does not, because of the resistance of the air and probably the need of the factor of time to accumulate energy.

In machines where the normal running speed is higher than the critical speed, the shaft is made just strong enough to withstand the deflection in passing through the critical speed, and as weak or flexible as possible for the smooth running above the critical speed. The weaker the shaft, the lower the critical speed, the nearer approach to rotation about the center of gravity, and the less bow or deflection in the shaft.

This solution is satisfactory so far as the critical value and deflection of the rotating mass is concerned, and it affords a simple explanation of the actions of a rotating body. But in the design of a machine the vibrations of the frame or supporting structure are of equal or greater importance. The shaft rotating in its bowed condition has a reaction on the bearing points, the reaction rotating with the shaft. This force is the impressed vibration that causes the frame to vibrate. If we determine the shaft deflection during rotation, or the location of the shaft axis at any instant, we can find the amount of the force of the shaft, or the impressed vibration on the frame.

These considerations and calculations may be extended to any two concentrated loads with either two or three bearing supports. In these cases the equation gives two values of critical speed for two concentrated loads regardless of the method of support, for either two or three bearings. It can be further shown that with a uniformly loaded shaft there is an infinite number of critical speeds and that the shaft becomes bowed differently for each one.

All formulas developed for critical speed, for both concentrated and distributed loads, apply to vertical shafts as well as horizontal. Although some formulas use the static deflection Δ, this is an equivalent deflection and can be used for vertical shafts by considering them horizontal.

As previously pointed out, theoretically the vibrations become infinite at the critical speed; actually they do not, but the vibrations are at a maximum point. The vibrations will begin at a certain speed, increase as the speed increases, and with still increasing speed will after a while die away. The vibrations may be felt over a considerable range, and the exact point of maximum value is difficult to detect. It is, therefore, advisable to keep the running speed at least 20 per cent. away from the critical value; and if the normal speed is between two critical values, careful calculations should be made for the point of minimum vibration.

Under ordinary circumstances the speed should be considerably below the critical value, as then the balance need not be particularly good. When the speed is considerably above the critical value, the vibration is almost proportional to the unbalance, or distance of the center of gravity from the center of the shaft, and the flexibility of the shaft; and the balance should be good, to prevent injury to the shaft and excessive vibration when passing through the critical speed.

A machine may be run very close to or at the critical speed, but the alignment and play of bearings, all mechanical details and the balance will require extra care, so that a troublesome and more expensive machine results before it is in good operating condition. The machine will run smoothly for a considerable time, until some mechanical fit or play causes a slight unbalance and immediately sets up excessive vibrations.

The equations for vibration further show that during the critical-speed period the vibrations increase theoretically with the time, so that in machines running above the critical speed there is less vibration at the critical-speed point when it is rapidly passed over. The equations also show a transfer of energy; the kinetic energy from the unbalance being transformed into the potential energy of the shaft deflection, so that a machine with nearly perfect balance may run smoothly for considerable time at the critical speed before vibrations appear.

With excessive vibration in passing through the critical speed there is a considerable tendency to spring the shaft by giving it a permanent set. This is most dangerous when the machine is first started, before it has a running balance.

The data required for the solution of critical-speed problems are the same as those for shaft deflection at loads. As the shaft is usually of variable diameter, and its stiffness is increased by a long hub, an ideal shaft of uniform diameter and equal stiffness, or for the same deflection, must be assumed. The loads are usually concentrated with an ideal point of application. The weights and distances between bearings and loads are the same in the ideal as in the actual case. Experience has shown that when the largest shaft diameter and uniform load cover about one-third of the span, approximately the same deflection is given for the load concentrated with a uniform shaft of the largest diameter. The weight of the shaft can be divided among the concentrated loads. As formulas have not been developed for more than two loads, when more than two loads are given they must be transformed into two resultant loads that would give the same deflection. For this case, two critical speeds will be found, one of which is usually far from the working speed.

Friction of Line Shafting

The friction of line shafting fitted with plain bearings formed the subject of an extended series of observations comprising 188 separate

tests by C. A. Graves, Chf. Engr. Edison Electric Illuminating Co. of Brooklyn (*Amer. Mach, June* 5, 1913).

The shafts were driven by electric motors and the power absorbed was obtained from the current readings. The hangers and loose pulleys on the line and counter-shafts were counted; the machines were stopped and the power input of the motor was measured. Next the belts were removed one at a time and readings were taken after each belt was removed until the belt connecting the motor and line shaft only remained. Finally, this was removed and the readings were taken with the motor running free. In many instances the belts were replaced in reverse order in order to check the results.

The difference between the power conserved with all belts on and with the motor running free gives the power absorbed by the bearings, subject to a slight correction due to the varying efficiency of the motor at various loads. The friction due to the increased load on the bearings when the machines are at work was not measured. It is believed to amount to about 20 per cent., but, in any case, it should be charged to the work done.

The hangers and loose pulleys were treated in the same manner as equivalents, because the tests showed that the average loose pulley, either on a line or counter-shaft, absorbed nearly as much, and in some cases more, power than a hanger.

Table 2 gives the results of the tests classified by industries, and in Table 3 the same data have been reclassified in accordance with diameters of the shafts.

Mr. Graves also tested the shafts (eight in number) of a factory with a complete equipment of Hyatt roller bearings. The results of these tests are given in Table 4. The average of the table is .0286 h.p. per bearing.

TABLE 2.—FRICTION CONSUMED BY LINE SHAFTING FITTED WITH PLAIN BEARINGS. CLASSIFIED BY INDUSTRIES

No. of installations tested	Class of industry	Horse-power per bearing		
		Max.	Min.	Mean
6	Stone working.......	.245	.124	.191
7	Wood working.......	.318	.015	.117
25	Clothing mfg.........	.037	.010	.027
50	Machine shops.......	.237	.025	.066
100	Various..............	.321	.015	.119

TABLE 3.—FRICTION CONSUMED BY LINE SHAFTING FITTED WITH PLAIN BEARINGS. CLASSIFIED BY SIZE OF SHAFT

No. of bearings	Size of shafts, ins.	Average r.p.m.	Horse-power per bearing		
			Max.	Min.	Mean
66	$\frac{15}{16}$–1	428	.052	.010	.036
706	$1\frac{1}{8}$	382	.079	.016	.033
37	$1\frac{1}{4}$	425	.119	.040	.062
492	$1\frac{1}{2}$	392	.193	.035	.089
155	$1\frac{3}{4}$	218	.113	.029	.c78
409	2	242	.300	.028	.133
21	$2\frac{1}{4}$	264	.321	.124	.257
83	$2\frac{1}{2}$	243	.300	.085	.255

The diameter 2 ins. is to be understood as including everything between $1\frac{15}{16}$ and $2\frac{1}{8}$ ins. and so for the other sizes.

TABLE 4.—HORSE-POWER CONSUMED BY LINE SHAFTING FITTED WITH HYATT ROLLER BEARINGS

Section of shaft..............	1	2	3	4	5	6	7	8
Number of hangers............	22	7	11	7	7	7	7	8
Number of loose pulleys........	40	24	20	19	5	8	2	22
Total number of bearings.......	62	31	31	26	12	15	9	30
H.p. to drive shafting	1.716	.858	.724	.804	.288	.549	.281	.938
H.p. per bearing	.027	.028	.026	.031	.024	.036	.031	.031
R.p.m. of shaft................	275	300	300	200	275	200	240	180
Diameter of shafting, ins.......	2	2	2	$1\frac{15}{16}$	$1\frac{15}{16}$	$1\frac{15}{16}$	$1\frac{15}{16}$	$1\frac{15}{16}$

The power lost in friction of shafting formed the subject of an investigation by PROF. C. H. BENJAMIN (*Trans. A. S. M. E., Vol.* 18). Sixteen factories were investigated, the results appearing in Table 5.

Indicator cards were taken from the engine during the day, at intervals of about 1 hr., while the factory was in operation. During the noon hour, or after working hours at night, cards were taken from the same engine, when it was driving the line and countershafts only, no machines being in operation. Averages of these two sets of cards were assumed to show respectively the total horse-power and the friction horse-power.

The figures in the column headed Horse-power to Drive Shafting include the power required to overcome the friction of the engine itself and that of all the shafting and counters. If a deduction of 10 be made from the percentages in the last column, they would show approximately the power required to drive shafting and counters alone.

The friction of line shafting fitted with plain and ball bearings formed the subject of experiments by Dodge and Day, which were reported by HENRY HESS (*Trans. A. S. M. E., Vol.* 31). As a result of the experiments, Mr. Hess concludes that:

When the belts from line shaft to countershaft pull all in one direction and nearly horizontally the saving due to the substitution of ball bearings for plain bearings on the line shaft may be safely taken as 35 per cent. of the bearing friction.

When ball bearings are used also on the countershafts the savings will be correspondingly greater and may amount to 70 per cent. or more of the bearing friction.

These percentages of savings are percentages of the friction work lost in the plain bearings; they are not percentages of the total power transmitted. The latter percentage will depend upon the ratio of the total power transmitted to that absorbed in the line and countershafts.

The power consumed in the plain line and countershafts varies, as is well known, from 10 to 60 per cent. in different industries and shops. The substitution of ball bearings for plain bearings on the line shaft only will thus result in savings of total power of $35 \times .10 = 3.5$ per cent. to $35 \times .60 = 21$ per cent. By using ball bearings on the countershafts also, the saving of total power will be from $70 \times .10 = 7$ per cent. to $70 \times .60 = 42$ per cent.

For additional information on the friction of line shafting see Index.

Keys and Keyways.

The common driven key for securing a crank or gear to a shaft is commonly made with a width of one-fourth the diameter of the shaft up to about a 4-in. shaft, and above that somewhat narrower, say $1\frac{3}{8}$ ins. for a 6-in., $1\frac{3}{4}$ ins. for an 8-in., and $2\frac{1}{4}$ ins. for a 10-in. shaft. The depth should be from five-eighths to three-fourths the width. If the work is at all severe, the length should be not less than $1\frac{1}{2}$ times the diameter of the shaft. The taper is commonly $\frac{1}{8}$ in. per ft.

This type of key is, however, a poorly designed thing at best; and under heavy duty, especially when the stresses alternate in direction, such keys are the source of much trouble. They seldom fail by shearing, but frequently fail from deformation due to the turning-over tendency of the forces to which they are subjected. Calculations of dimensions based on the shearing stress are, therefore, largely futile. There is no doubt that the success of the Woodruff key (which see below) is largely due to its better resistance to the forces which tend to turn it over.

For the ends of shafts the Nordberg Mfg. Co. uses round keys, Figs. 4 and 5, which are much better than the customary form. The tendency toward deformation is absent; they are in true shear and are a driven fit in the direction of the shear, no one of these statements being true of the common construction. Moreover, with the taper reamer once provided, they are much cheaper than the square key. To overcome the tendency of the drill to crowd over into the cast-iron

TABLE 5.—POWER REQUIRED TO DRIVE ENGINE AND LINE AND COUNTERSHAFTS FITTED WITH PLAIN BEARINGS

Number	Nature of work	Total length of line shaft, ft.	Diam. of line shafts, ins.	Revolutions per minute	Number of bearings	Number of belts	Average width of belts in inches	Number of counters	Number of machines	Number of men	Total horse-power	H. P. to drive machines	H. P. to drive shafting	P. C. to drive shafting	Plant running at what capacity
1	Wire drawing and polishing.....	1130	2½, 3¾; 4, 6	170	115	89	4	69	....	....	400.0	243.0	157.0	39.2	One-half
2	Steel stamping and polishing....	580	3, 3½	200	68	28	6	27	18	78	74.0	17.0	57.0	77.0	One-third
3	Boiler and machine work........	1530	2½, 3	150	46	53	5½	47	43	152	38.6	13.3	25.3	65.6	Two-thirds
4	Bridge machinery.............	1460	2½, 3; 4	110	142	92	4½	79	69	80	59.2	11.3	47.9	80.7	Nearly full
5	Heavy machine work...........	1130	3	190	110	141	4	96	68	300	112.0	48.0	64.0	57.0	Full
6	Heavy machine work...........	1065	2, 3; 4	180, 150; 150	114	192	4	152	123	225	168.0	77.0	91.0	54.2	Full
7	Light machine work............	748	1½, 1¾; 2, 3	135, 135; 135, 150	101; 101	217	3	133	250	200	40.4	19.7	20.7	51.2	Full
8	Manufacturers' small tools......	500	2, 3	114	58	335	3	314	313	226	74.3	34.3	40.0	53.8	Full
9	Manufacturers' small tools......	900	1¾, 2½	175, 136	102	217	3	202	258	100	47.2	22.7	24.5	51.8	Full
10	Sewing machines and bicycles...	2490	2, 6	150	274	521	3	403	454	400	190.0	82.0	108.0	56.9	Full
11	Sewing machines...............	1470	2, 3; 4	160, 160; 125	184	484	3	435	179	350	107.0	32.5	74.5	69.7	Full
12	Screw machines and screws......	1800	2, 2¼; 2½, 3	180	180	486	3	392	428	320	241.0	127.0	114.0	47.3	Full
13	Steel wood screws..............	674	1½, 2; 3	175, 160; 175	96	131	4	89	392	140	117.0	100.0	17.0	14.5	One-fourth
14	Manufacturers' steel nails.......	988	2½	200	74	187	3	175	184	58	91.6	45.9	45.7	49.9	Full
15	Planing mill....................	165	3	267	19	45	6	40	53	8	39.2	10.6	28.6	73.0	Full
16	Light machine work............	275	2	175	37	48	4	27	30	....	8.3	4.3	4.0	48.6	One-half

hub, a small pilot hole *a*, Fig. 5, is first drilled in the joint, after which a second hole *b*, as large as the proposed keyway will admit, is drilled in the shaft. Standard dimensions are given in Table 6.

The Kennedy key, Fig. 6, has found large use in the Pittsburg district in the most rugged rolling-mill work, for which it does better than any other. For such work the keys are made approximately one-quarter of the shaft diameter, and located in the gear so that diagonals through two corners of the keys intersect at the center of the bore. The taper of ⅛ in. per ft. should be on the top for a driving fit, the sides being a neat fit. The shaft is first bored for a press fit, then rebored about $\frac{1}{64}$ of the shaft diameter off the center, the keyways being cut in the eccentric side. That portion of the bore opposite

TABLE 6.—NORDBERG STANDARD ROUND KEYS

Dia. of shaft, ins.	Dia. of reamer, ins.	Cutting length of reamer, ins.	Dia. of shaft, ins.	Dia. of reamer, ins.	Cutting length of reamer, ins.
2 $\frac{15}{16}$–3	¾	4¼	13		
3 $\frac{7}{16}$–3½	⅞	4½	14	2 $\frac{9}{16}$	12
3⅞–4	1	4⅞	15		
4⅜–4½	1⅛	5	16		
5	1¼	4⅝	17	3⅛	13
5½	1⅜	4⅞	18		
6	1½	6⅛	19		
7			20	3 $\frac{11}{16}$	
8	1⅝	6⅞ and 8	21		
9			22		
10			23	4¼	14¼
11	2	10¼	24		
12					

Reamer diameters are at the small end. Taper $\frac{1}{16}$ in. per ft., measured on the diameter.

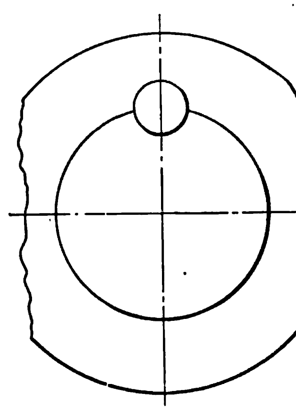


FIG. 4.

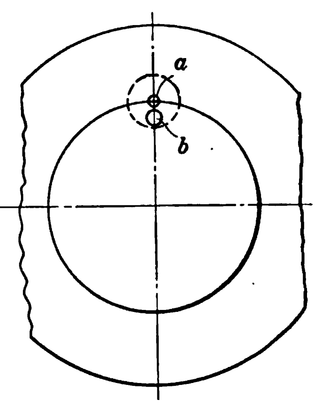


FIG. 5.

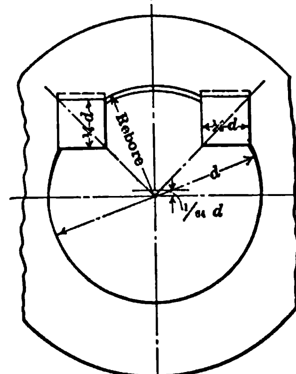


FIG. 6.

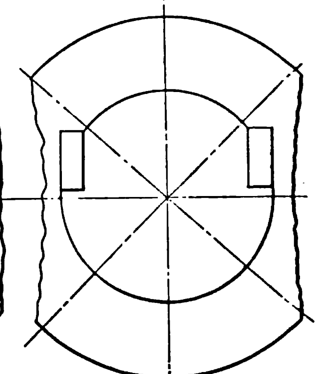


FIG. 7.

FIGS. 4 to 7.—Improved forms of keys.

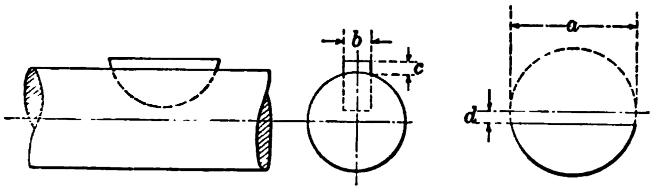


TABLE 7.—WOODRUFF STANDARD KEYS

No. of key	Diameter of key	Thickness of key	Depth of keyway	Center of stock, from which key is made, to top of key	No. of key	Diameter of key	Thickness of key	Depth of keyway	Center of stock, from which key is made, to top of key
	a	b	c	d		a	b	c	d
1	1/2	1/16	3/64	3/64	B	1	5/16	5/32	1/16
2	1/2	3/32	3/64	3/64	16	1 1/8	3/16	3/32	5/64
3	1/2	1/8	1/16	3/64	17	1 1/8	7/32	7/64	5/64
4	5/8	3/32	3/64	1/16	18	1 1/8	1/4	1/8	5/64
5	5/8	1/8	1/16	1/16	C	1 1/8	5/16	5/32	5/64
6	5/8	5/32	5/64	1/16	19	1 1/4	3/16	3/32	5/64
7	3/4	1/8	1/16	1/16	20	1 1/4	7/32	7/64	5/64
8	3/4	5/32	5/64	1/16	21	1 1/4	1/4	1/8	5/64
9	3/4	3/16	3/32	1/16	D	1 1/4	5/16	5/32	5/64
10	7/8	5/32	5/64	1/16	E	1 1/4	3/8	3/16	5/64
11	7/8	3/16	3/32	1/16	22	1 3/8	1/4	1/8	3/32
12	7/8	7/32	7/64	1/16	23	1 3/8	5/16	5/32	3/32
A	7/8	1/4	1/8	1/16	F	1 3/8	3/8	3/16	3/32
13	1	3/16	3/32	1/16	24	1 1/2	1/4	1/8	7/64
14	1	7/32	7/64	1/16	25	1 1/2	5/16	5/32	7/64
15	1	1/4	1/8	1/16	G	1 1/2	3/8	3/16	7/64

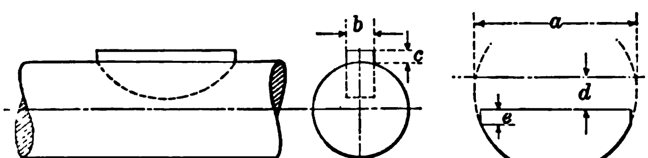


TABLE 8.—WOODRUFF SPECIAL KEYS

Number of key	Diameter of key	Thickness of key	Depth of keyway	Center of stock, from which key is made, to top of key	Width of flat	Number of key	Diameter of key	Thickness of key	Depth of keyway	Center of stock, from which key is made, to top of key	Width of flat
	a	b	c	d	e		a	b	c	d	e
26	2 1/8	3/16	3/32	17/32	3/32	31	3 1/2	7/16	7/32	13/16	1/16
27	2 1/8	1/4	1/8	17/32	3/32	32	3 1/2	1/2	1/4	13/16	1/16
28	2 1/8	5/16	5/32	17/32	3/32	33	3 1/2	9/16	9/32	13/16	1/16
29	2 1/8	3/8	3/16	17/32	3/32	34	3 1/2	5/8	5/16	13/16	1/16
30	3 1/2	3/8	3/16	13/16	1/16						

TABLE 9.—DIAMETERS OF SHAFTS AND SUITABLE WOODRUFF KEYS

Diameter of shaft	Number of key	Diameter of shaft	Number of key	Diameter of shaft	Number of key
5/16–3/8	1	7/8–15/16	6,8,10	1 3/8–1 7/16	14,17,20
7/16–1/2	2,4	1	9,11,13	1 1/2–1 5/8	15,18,21,24
9/16–5/8	3,5	1 1/16–1 1/8	9,11,13,16	1 11/16–1 3/4	18,21,24
11/16–3/4	3,5,7	1 3/16	11,13,16	1 13/16–2	23,25
13/16	6,8	1 1/4–1 5/16	12,14,17,20	2 1/16–2 1/2	25

the keys remains as originally bored. This feature is not essential but is of obvious convenience in assembling and disassembling.

For less severe duty this type of key may be made narrower, as in

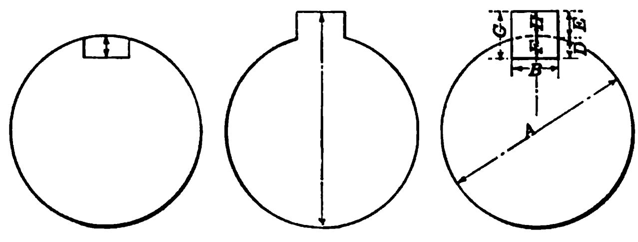


FIG. 8. FIG. 9. FIG. 10.

TABLE 10.—DIMENSIONS OF KEYS AND KEYWAYS

Size of shaft	Width of key	Taper key (1/8 in. per foot): Thickness, large end	Depth in shaft measured at edge	Depth in hole at edge	Depth in shaft measured at center	Depth in hole at center	Straight key: Thickness	Depth in shaft at edge	Depth in hole at edge	Depth in shaft at center	Depth in hole at center
A	B	C	D	E	F	H	C	D	E	F	H
1/4	1/16	3/64	1/64	1/32	.0195	.0272	1/16	1/32	1/32	.0352	.0273
5/16	3/32	1/16	1/32	1/32	.0384	.0240	3/32	3/64	3/64	.0540	.0397
3/8	3/32	1/16	1/32	1/32	.0374	.0252	3/32	3/64	3/64	.0528	.0409
1/2	1/8	3/32	3/64	3/64	.0548	.0389	1/8	1/16	1/16	.0704	.0546
5/8	5/32	1/8	1/16	1/16	.0724	.0525	5/32	5/64	5/64	.0880	.0682
3/4	3/16	5/32	5/64	5/64	.0900	.0662	3/16	3/32	3/32	.1056	.0819
7/8	7/32	3/16	3/32	3/32	.1076	.0798	7/32	7/64	7/64	.1232	.0955
1	1/4	3/16	3/32	3/32	.1096	.0778	1/4	1/8	1/8	.1408	.1092
1 1/4	5/16	1/4	1/8	1/8	.1448	.1051	5/16	5/32	5/32	.1761	.1364
1 1/2	3/8	5/16	5/32	5/32	.1800	.1324	3/8	3/16	3/16	.2113	.1637
1 3/4	7/16	3/8	3/16	3/16	.2153	.1596	7/16	7/32	7/32	.2465	.1910
2	1/2	3/8	3/16	3/16	.2192	.1557	1/2	1/4	1/4	.2817	.2183
2 1/4	9/16	1/2	1/4	1/4	.2857	.2142	9/16	9/32	9/32	.3169	.2456
2 1/2	5/8	1/2	1/4	1/4	.2897	.2112	5/8	5/16	5/16	.3522	.2728
2 3/4	11/16	5/8	5/16	5/16	.3561	.2688	11/16	11/32	11/32	.3874	.3000
3	3/4	5/8	5/16	5/16	.3601	.2648	3/4	3/8	3/8	.4226	.3274
3 1/2	7/8	11/16	3/8	3/8	.4306	.3193	7/8	7/16	7/16	.4931	.3819
4	1	3/4	3/8	3/8	.4385	.3114	1	1/2	1/2	.5635	.4365

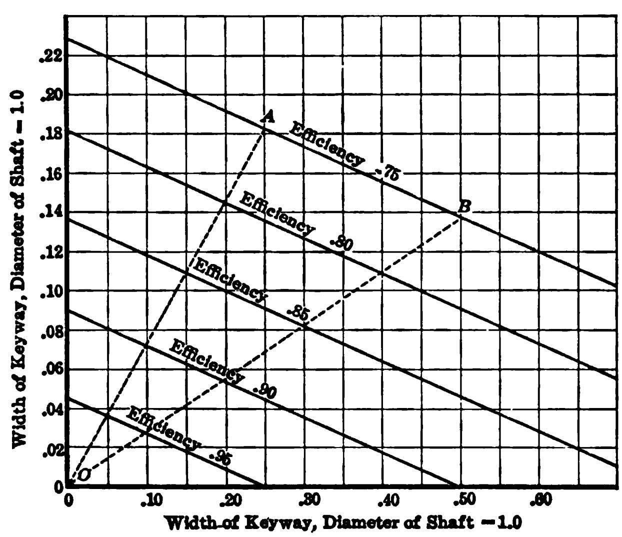


FIG. 11.—The weakening effect of keyways on shafts.

Fig. 7, and may be accepted as a sovereign cure for key troubles. In all cases the taper should be between the top and bottom faces.

The Woodruff system of keys, because of its increased depth, obviates

the tendency of common square keys to turn over in ... seats. It has come into large use, especially in machine-too. construction. Tables 7, 8 and 9 give the dimensions.

The best method of dimensioning drawings of keyways in shafts which are to be cut on the milling machine is that shown in Fig. 8. By adjusting the cutter until it just marks the shaft and then sinking it for depth by reading the index dial, the correct depth is quickly obtained. The best method of dimensioning keyways in hubs is that shown in Fig. 9, the convenience of which, to the workman, is obvious. The figures for these methods of dimensioning are easily obtained from Table 10, the reference letters of which correspond with those of Fig. 10.

The weakening effect of keyways on shafts formed the subject of experiments by Dr. H. F. Moore (*Bulletin University of Illinois Engineering Experiment Station, No.* 42). The results of the experiments are given graphically in Fig. 11. Dr. Moore defines the efficiency of a shaft with keyway as the ratio of strength at the elastic limit of a shaft with keyway to the strength at the elastic limit of a similar shaft without keyway, and it is this efficiency which may be obtained from the chart. To use the chart locate a point defining the size of the keyway which will, in general, fall between two lines representing values of efficiency, and the efficiency of the shaft in question may then be estimated with sufficient accuracy. The space within the triangle *OAB* represents the range covered by the tests actually performed, and covers the proportions of keyways commonly used in practice.

BELTS AND PULLEYS

The driving power of leather belts is summed up by PROF. W. W. BIRD (*Journal Worcester Polytechnic Institute, Jan.*, 1910) as follows:

The h.p. that a belt will transmit depends upon the effective tension and the belt speed. The effective tensions depend upon the difference in the tensions of the two sides of the belt and on the surface friction, which depends upon the ratio of the tensions and the angle of wrap.

Experiments and practice have shown that a belt of single thickness will stand a stress of 60 lbs. per in. of width and give good results, that is, it will only require an occasional taking up and will have a fairly long life. The corresponding values for double and triple belts are 105 and 150 lbs. per in. of width provided the pulleys are not too small.

Experiments have shown that on small pulleys the ratio of the tensions should not exceed 2, on medium pulleys 2.5, and on large pulleys 3. The larger the pulley, the better the contact, the thinner the belt, and the better the contact for the same size of pulley. When the pulley diameter in ft. is three times the thickness of the belt in ins. or in this proportion, we get equivalent results for different thicknesses of belts. This gives a method of classifying pulleys. The belt has to adjust itself in passing over a pulley due to its own thickness. Some adjustment is also necessary on account of the crowning of the pulley. These adjustments account for the different ratios for the various pulley diameters. The effects of the crown and pulley diameters are not usually considered in belt rules, which is a grave mistake. The ratios are for 180 deg. wrap and decrease with less contact.

The creep of the belt depends upon its elasticity and the load, and experiments have shown that this should not exceed 1 per cent. in good practice. In order to keep this creep below 1 per cent., it is necessary to limit the difference of tension per in. of width of single belt to 40 lbs. The corresponding values for double and triple belts are 70 and 100 lbs. per in. of width. These figures are based on an average value of 20,000 for the running modulus of elasticity of leather belting.

Table 1 has been prepared on the basis of these limitations and gives a value for F in the equation

$$\text{h.p.} = \frac{V \times W}{F} \text{ or } W = \frac{\text{h.p.} \times F}{V}$$

in which h.p. is the horse-power, V the belt velocity in ft. per min., and W the width in ins.

Table 2 gives corrected values for F when the arc of contact or wrap is greater or less than 180 deg. On large pulleys the creep may exceed 1 per cent. if the wrap is over 180 deg., as the increased friction gives a greater difference of tensions.

TABLE 1.—CONSTANTS FOR THE DRIVING POWER OF LEATHER BELTS

Diameter of pulley	Under 8 ins.	8–36 ins.	Over 3 ft.	Under 14 ins.	14–60 ins.	Over 5 ft.	Under 21 ins.	21–84 ins.	Over 7 ft.
Thickness of belt	Single	Single	Single	Double	Double	Double	Triple	Triple	Triple
Factor........................	1100.0	920.0	830.0	630.0	520.0	470.0	440.0	370.0	330.0
Difference of tensions.........	30.0	36.0	40.0	52.5	63.0	70.0	75.0	90.0	100.0
Per cent. of creep............	0.74	0.89	0.99	0.74	0.89	0.99	0.74	0.89	0.99
Ratio of tensions.............	2.0	2.50	3.0	2.0	2.50	3.0	2.0	2.50	3.0
Tension on tight side.........	60.0	60.0	60.0	105.0	105.0	105.0	150.0	150.0	150.0

The use of the tables is shown by the following examples:

How much horse-power will a 4-in. single belt transmit at a speed of 4600 ft. per min., passing over a 12-in. pulley? The factor is 920, therefore,

$$\frac{4600 \times 4}{920} = 20 \text{ h.p.}$$

How wide should a belt be in order to transmit 50 h.p. at 2000 ft. per min. on a 36-in. pulley?

$$W = \frac{50 \times 830}{2000} = 20.7\text{-in. single belt.}$$

This gives a width of single belt which is beyond the usual limit, 8 ins. being considered good practice for the maximum width of a single belt.

$$W = \frac{50 \times 520}{2000} = 13\text{-in. double belt.}$$

How wide should a single belt be in order to transmit 2 h.p. at 600 ft. per min. over a 4-in. pulley with 140 deg. wrap?

TABLE 2.—CONSTANTS FOR THE DRIVING POWER OF LEATHER BELTS

220°	210°	200°	190°	180°	170°	160°	150°	140°	130°	120°
980	1010	1040	1070	1100	1140	1180	1220	1270	1330	1400
810	830	860	890	920	950	990	1040	1100	1170	1240
730	750	770	800	830	860	890	930	980	1030	1100
560	570	590	610	630	650	670	700	730	760	800
460	470	840	500	520	540	570	600	630	660	700
420	430	440	450	470	490	510	530	560	590	630
390	400	410	420	440	460	480	500	520	540	560
320	330	340	350	370	390	410	430	450	470	490
290	300	310	320	330	340	360	380	400	420	440

In this case take the factor 1100 from Table 1 and in Table 2 find a corrected value for 1100 under 140 deg. of 1270.

$$W = \frac{2 \times 1270}{600} = 4.23\text{-in. single belt.}$$

How wide a belt is required for 300 h.p. at 2000 ft. per min. over a 10-ft. pulley?

$$W = \frac{300 \times 470}{2000} = 70.5\text{-in. double belt.}$$

This is too wide. Good practice calls for a change to triple at 48 ins. unless for some special reason a narrower belt is necessary.

$$W = \frac{300 \times 330}{2000} = 49.5\text{-in. triple belt.}$$

The belt speed is limited by centrifugal force, but below 5000 ft. per min. the loss on this account is largely compensated for by the

increase of friction due to the decrease in the time element of the contact, caused by the increased velocities.

The results given by these factors are well within working values and the belts will probably transmit 50 per cent. more power than these factors, but at the expense of the life of the belt. A liberal allowance at the beginning means less annoyance, fewer delays in taking up the belts, longer life and less cost for renewals and repairs.

The dimensions of belts in relation to the power transmitted and the effective pull may be obtained from Figs. 1 and 2. Fig. 1 conforms to the usage of Wm. Sellers & Co. as deduced from the experiments made for them by WILFRED LEWIS (*Trans. A. S. M. E., Vols.* 7 and 20). Fig. 2 conforms to the recommendations of CARL G. BARTH (*Trans. A. S. M. E., Vol.* 31) based on a re-analysis of the same experiments combined with a study of the extended observations of belts in service by F. W. TAYLOR (*Trans. A. S. M. E., Vol.* 15). Mr. Barth's recommendations are intended to secure maximum economy of belts and of upkeep. He considers the proper working loads of belts when proportioned from this viewpoint to be so well within the capacity of any good joint that the kind of joint may be ignored.

The author suggests that the Sellers chart be followed for main driving belts and that Mr. Barth's chart be used for machine belts.

For arcs of contact other than 180 deg., the power transmitted and the effective pull, as given by the charts, are to be multiplied by the factors of the table below the charts.

The charts should not be understood as giving, or as intended to give, the ultimate capacity of belts. As with every other machine member, the question regarding a belt is not how much it can be made to do, but how much it should be made to do. As a matter of fact, and as shown by the figures given below, belts will carry loads materially in excess of those imposed by either chart.

An examination by the author of belt fly-wheels of 12 Corliss engines, ranging between 50 and 380 rated h.p. by three high-class builders (*Amer. Mach., Oct.* 28, 1897), gave the following values for the number of ft. per min. travel of 1-in. belt for each h.p. of the rated capacity of the engines:

Average of 12	480
Maximum	750
Minimum	375

For additional information on main driving belts for steam engines, see Steam-engine Belts.

As an example of heavy duty, SAMUEL WEBBER (*Amer. Mach., Feb.* 22, 1894) cites a main driving belt 30 ins. wide, $\frac{1}{4}$ in. thick, running 3900 ft. per min. on a 5-ft pulley and transmitting 556 h.p. for a period of 6 yrs., giving 210 ft. per min. travel of 1-in. belt per h.p.

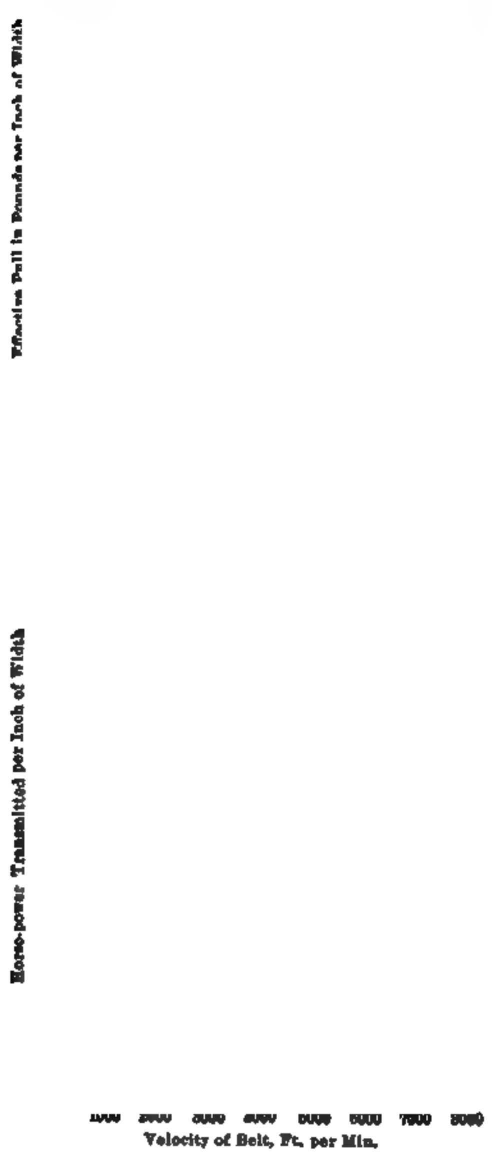


FIG. 1.—Sellers belt formula.

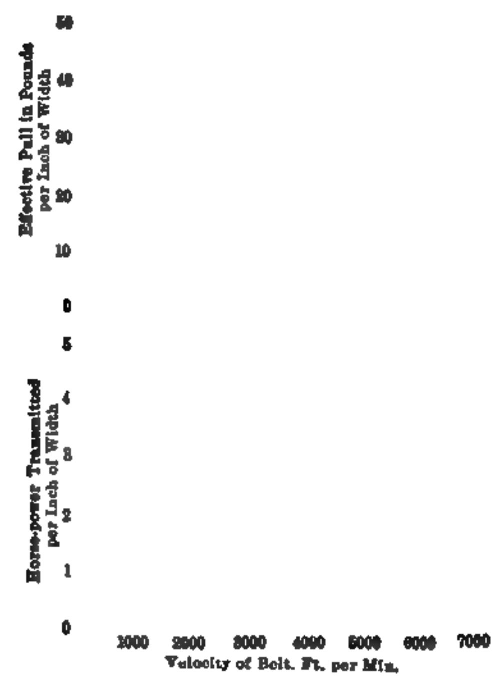


FIG. 2.—Barth belt formula.

Factors to be used for different arcs of contact.

Arc of contact, deg	90	100	110	120	130	140	150	160	170	180	190	200	210
Factor	.65	.70	.75	.79	.83	.87	.91	.94	.97	1.00	1.03	1.05	1.07

The arc of contact of a belt on the smaller of two pulleys may be found by the following rule and Table 3, by Wm. Cox (*Amer. Mach., July* 20, 1905):

Divide the difference between the diameters of the two pulleys in inches by the distance between their centers, also in inches. Let the quotient equal x. Now from the accompanying table find, in line with such ascertained values of x, the corresponding angle of the arc of contact of the belt on the smaller pulley.

Example: Two pulleys of 80 and 30 ins. diameter are spaced 120 ins. apart, center to center; what is the arc of contact on the smaller pulley?

$$\frac{80-30}{120}=.416=x$$

Opposite .416 in the x column we find the arc of contact to be 156 deg.

Table 3.—Arcs of Contact of Belts on Pulleys

x	Angle, degrees	x	Angle, degrees	x	Angle, degrees
.000	180	.347	160	.684	140
.017	179	.364	159	.700	139
.035	178	.382	158	.717	138
.052	177	.399	157	.733	137
.070	176	.416	156	.749	136
.087	175	.433	155	.765	135
.105	174	.450	154	.781	134
.122	173	.467	153	.797	133
.139	172	.484	152	.813	132
.157	171	.501	151	.829	131
.174	170	.518	150	.845	130
.192	169	.534	149	.861	129
.209	168	.551	148	.877	128
.226	167	.568	147	.892	127
.244	166	.584	146	.908	126
.261	165	.601	145	.923	125
.278	164	.618	144	.939	124
.296	163	.635	143	.954	123
.313	162	.651	142	.970	122
.330	161	.668	141	.985	121
.347	160	.684	140	1.000	120

The comparative transmitting capacities of pulleys made of different materials formed the subject of tests by Prof. W. M. Sawdon (*Proc. Nat. Asso. of Cotton Mfrs.*, 1911). The results of these tests reduced to an arc of contact of 180 deg. and 250 lbs. per sq. in. tension on the tight side are given in Table 4. The tests were made at a belt speed of 2200 ft. per min.

Idler Pulleys and Quarter Twist Belts

The idler pulley may be made the source of great benefit, when properly laid out, although commonly looked upon as an unmixed evil. Used as a simple tightener, it is not to be recommended, but when so laid out to increase the arc of contact it may be made to reduce the tensions.

Fig. 3 (*Amer. Mach., May* 26, 1910) shows the correct location of the idler pulley. Its obvious effect is to increase the arc of contact, especially on the smaller pulley where most needed. This, in turn, reduces the necessary tension on the slack side, increases the difference of tensions, that is, the effective tension, and reduces the tension on the tight side for a given effective tension, these reduced tensions leading, in turn, to a corresponding decrease of pressure on the bearings. The idler should be on the slack side of the belt and near the smaller pulley. Either pulley may drive. Additional benefit

Table 4.—Comparative Power-Transmitting Capacities of Pulleys of Various Materials

Kind of pulley	Comparative transmitting capacity at 2 per cent. slip
Cast iron	100.0
Cast iron with corks proj. .04 in.	107.0
Cast iron with corks proj. .015 in	112.1
Wood	105.6
Wood with corks proj. .075 in.	104.8
Wood with corks proj. .03 in.	104.8
Paper	137.5
Paper with corks proj. .087 in.	122.0
Paper with corks proj. (about) .015 in	133.2

may be obtained by mounting the idler on a weighted arm arranged to swivel about the center of the smaller pulley, as shown in Fig. 4.

With this construction the tensions are independent of the elasticity of the belt and the objection to short belt transmissions disappears. Similarly, the weight of the belt in vertical transmissions no longer reduces the tensions on the lower pulley, and such transmissions become entirely practicable. Again, the effect of centrifugal force in causing the belt to leave the smaller pulley, reducing the arc of contact and carrying air between pulley and belt is overcome.

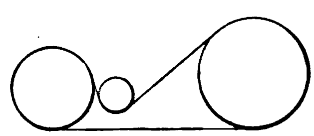


Fig. 3.

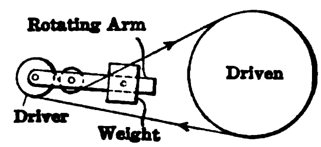


Fig. 4.

Figs. 3 and 4.—Correct arrangement of idler pulleys.

The layout of quarter twist belts is shown in Fig. 5, the rule being that the central plane of each pulley must pass through the point of delivery of the other pulley. This construction should be used with narrow belts running on pulleys at a good distance apart only. Quarter twist belts will drive in one direction only.

Guide pulleys should be substituted for quarter twist belts whenever possible, and Figs. 6–14 show various arrangements of such pulleys. The rule is that the intersection of the central planes of consecutive pulleys shall be tangent to both pulleys.

Thus in Fig. 7, in which pulleys A and B are of the same size, and either of which may drive in either direction and the shafts are at right angles, the intersection of the central planes of B and C' is obviously tangent to both and so for the other pulleys. In Fig. 8, A is larger than B and the same condition holds, as it does also in the increasingly complex arrangements of Figs. 9 and 10.

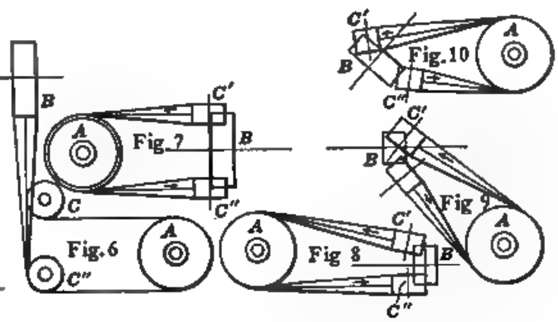
In Figs. 11 and 12 A or B may drive. In Fig. 11 C or D is loose on the shaft, while in Fig. 12 both C and D are loose. The loose pulleys should, if possible, be placed on the slack side of the belt. In Fig. 13 the guide pulleys revolve, nominally, at the same speed, but nevertheless one of them should be loose in order to provide for the differential action due to any slight difference in their diameters. Fig. 14 shows a power distribution system through a 16-story building by means of vertical shafts, a single guide pulley only being used for each belt. Similar constructions distribute the power from the vertical shafts to line shafts on each floor.

In all of the constructions shown, except that of Fig. 5, the belt will drive in either direction, the arrows being for assistance in tracing the motion.

Holes through floors for quarter-twist vertical belts may be laid out by

the method shown in Figs. 15–18 by M. H. BALL (*Mchy., Sept.*, 1912). The basic rule is that the center of the face of one of the pulleys at a point level with the center of its shaft must be in the same vertical line as the similar point on the other pulley, as indicated in the illustrations. The direction in which the pulleys are to turn determines which of their sides must be in line, as it is always the sides

FIG. 5.—Quarter twist belt.



FIGS. 6 to 10.—Substitutes for quarter twist belts.

from which the belt leaves the pulleys which should line up. Fig. 15 shows how the pulleys should be set when the lower pulley turns to the left, as indicated by the arrow. Fig. 16 shows the setting when the lower pulley is driven in the opposite direction. The rules given apply to the aligning of pulleys at other angles as well, an example of which is shown in Fig. 17.

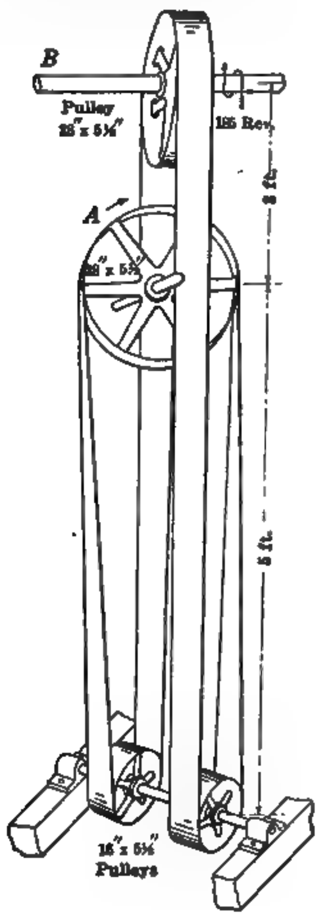


FIG. 13.

Fig. 18 shows a method of laying out the floor holes for the drive indicated in Fig. 16. First draw an outline (plan view) of the two pulleys on the floor in full size, directly below and above the respective pulleys to be connected by the belt. A starting-point for this

FIG. 11.

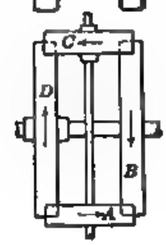


FIG. 12.

FIGS. 11 and 12.—Substitutes for quarter twist belts.

layout can readily be found with a plumb bob. Then draw the center lines *AB* and *CD* through the faces of the pulleys, and divide the diameter of each pulley into eight parts, as shown, numbering the divisions 1, 2, 3, etc. The numbers of the divisions must start from

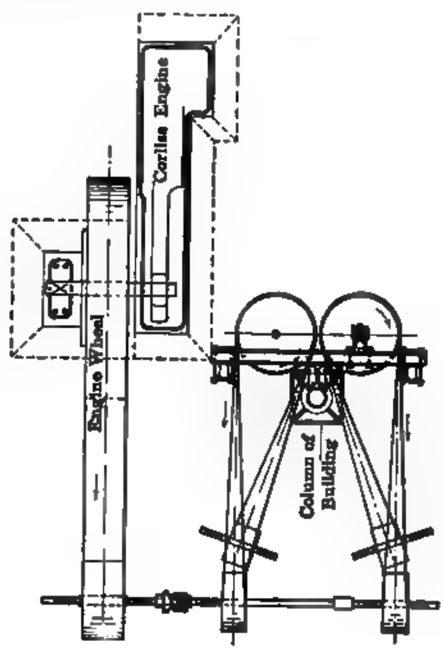


FIG. 14.

FIGS. 13 and 14.—Substitutes for quarter twist belts.

the sides of the pulleys which are opposite the arrow points shown in the plan view indicating the direction of rotation. Next, measure the distances from center to center of the shafts and from the center of the upper shaft to the floor. In the example shown the distance from center to center of the shafts is 96 ins., and the distance from the center of the upper shaft to the floor is 42 ins. As $96 \div 8 = 12$, each division on the diameter of the pulleys is equivalent to 12 ins. Further, $42 \div 12 = 3\frac{1}{2}$, which represents the number of spaces that the center of the belt will be from the center points of the sides of the upper pulley, as indicated at *J* and *K* in the engraving. Draw the line *EJ* through the point thus located in the rectangle representing the upper pulley. Then strike an arc with *J* as center and *EJ* as radius, as indicated, and divide it into eight equal parts. As we are working on the top side of the floor, and as our measurements from the floor to the upper shaft determine how many spaces we are to set off, start numbering these divisions from the point *E*, the line *EJ* being parallel to the face of the upper pulley; then set off $3\frac{1}{2}$ spaces from *E*, thus determining point *L*, and draw line *LO* through *J*, making *JO* equal to *LJ*. This line indicates the position of the center of the belt at the floor line and a line of the same length parallel to it through *K* indicates the other center line of the belt at the floor line.

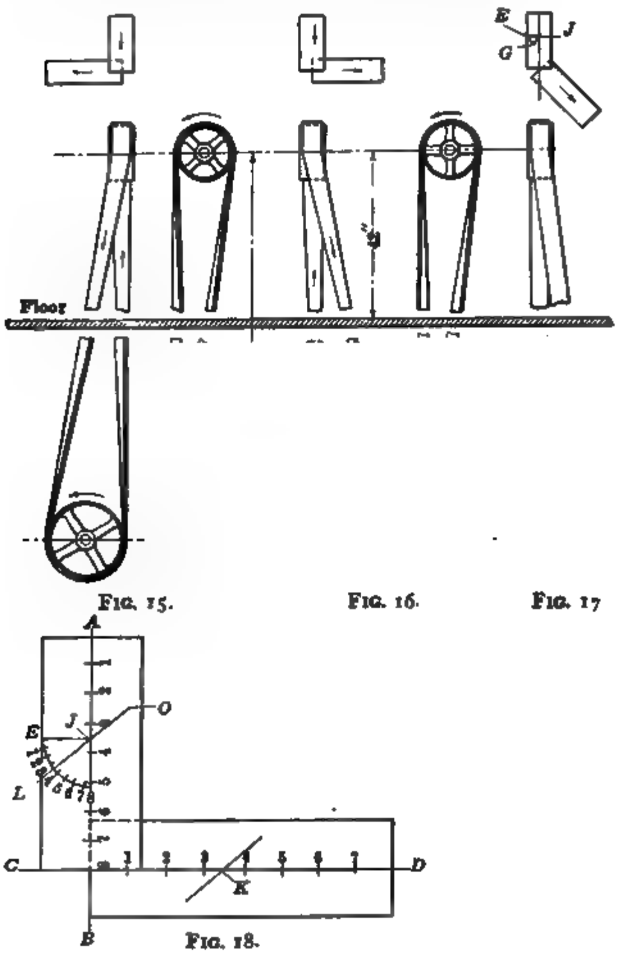


FIG. 15. FIG. 16. FIG. 17

FIG. 18.

FIGS. 15 to 18.—Laying out holes in floors for quarter twist belts.

The layout for an angle of other than 90 deg., as indicated in Fig. 17, differs only in that the arc on the pulley outline extends only from the line *EJ* to the line *GJ*, Fig. 17, this latter line being parallel with the face of the lower pulley. Any number of divisions more or less than eight may be used if preferred.

Mule-pulley stands may be laid out by the method shown in Figs. 19 and 20 by FRED HOWE (*Woodcraft, June*, 1912).

Before the problem can be laid out as in Fig. 19 the diameters of the pulleys and the distances between the shafts must be known. Assume that shafts *A* and *C* are each horizontally 4 ft. 6 ins. from the mule pulleys, which are to be centered at *E*, and that the shafts are vertically 18 ins. apart. Draw two horizontal lines, 18 ins. apart, through the centers of *A* and *C*, and locate these pulleys upon their lines as shown.

Pulley *A* is represented 4 ft. 6 ins. from pulley *C*, and the line *H* at the center of pulley *A* indicates the point where the turn in the belt is to be located.

Having drawn a vertical line *H* 4 ft. 6 ins. from *C*, measure off another 4 ft. 6 ins. from the last vertical line, and draw a third line at *B*. This line represents the location of shaft *AB*, were the belt stretched out in a straight line, without passing around the mule stand *E*. The inclined lines from pulley *C* to pulley *B* show the exact path or slope of the belt at every portion of its length from one pulley to the other.

Let it now be assumed that the mule pulleys *E* are 10 ins. in diameter. Measure back 5 ins. from the middle vertical line *H*, and draw another vertical *E*, which will be the center of the mule stand; measure from the horizontal lines through *C* and *A* to the lines *G* and *F*, and, scaling the drawing, shows about 2 ins. and $3\frac{1}{4}$ ins. or by calculation $2\frac{1}{4}$ and $3\frac{3}{8}$ ins., respectively.

If desired, the belt may be made to run with the mule pulleys level with one of the pulleys, either the upper or the lower, or they may be located anywhere between the two extremes, but the principle involved is the same, no matter where the mule pulleys may be located. The method here shown places the mules directly in line with pulleys *C* and *A*, so that the belt makes an even drop from one pulley to the other.

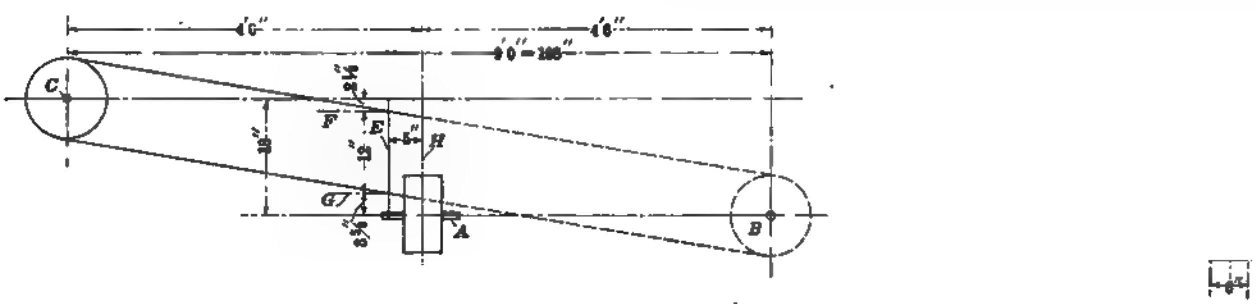


FIG. 19. FIG. 20.

FIGS. 19 and 20.—Laying out mule pulley stands.

Should it be proposed to locate the mule pulleys at any other point between *C* and *A*, the sum of the distances between the mules and the two pulleys is taken as above, and pulley *B*, laid down as described, no matter where between them line *E* may come. Should line *E* be moved toward or from shaft *C*, and the belt length

remain the same, there will be no change in the angle at which the mule shaft must be placed. And this angle is the same from the vertical as the angle of the belt is from the horizontal, viz., one in six, or the mule shaft must be suspended 2 ins. to the ft. of its length out of plumb.

Fig. 20 shows the arrangement of the mules as found from Fig. 19. The mule pulleys are 10 ins. in diameter, and it will be assumed that all the pulleys are 6-in. face. Pulleys *C* and *B* being equal in diameter, the mules must be a distance apart equal to the diameter of the pulleys *C* and *B*, or 12 ins. As the center of the face of each mule pulley must be placed with the middle of its face upon vertical line *H*, and as the shafts of the mules pitch 1 in 6, it is evident that the centers of the mule shafts will be 2 ins. apart on centers, and scaling the drawing, Fig. 20, shows this to be the case.

The mule pulleys located as shown will run perfectly, keeping the belt square upon both driver and driven pulleys. In fact, in any belt transmission of similar character it is only necessary to look to two points. The first of these is that the upper mule pulley be so arranged that it receives the belt fair and square from drive pulley *C*. In fact, the upper mule pulley may be placed at any angle or at any distance from *C* and the belt will track perfectly as long as the face of the mule pulley is fair to receive the belt squarely from pulley *C*. It makes no difference at what angle the belt leaves the mule pulley, except that this pulley must be so located that the belt leading away therefrom shall lead or track directly and fairly toward pulley *B*. That is all that is necessary for the upper mule pulley.

The lower mule pulley may be so placed that it shall receive the lower fold of the belt fair and square from pulley *A*. Nothing else is so necessary as that the mule-pulley is located so that the belt guides fair toward the receiving side or face of pulley *C*. This means that the lower mule shall be moved bodily so as to guide the belt toward *C* and turned at the angle which may be necessary to receive the belt from pulley *A*. It makes no difference at what angle—within limits, of course—a belt leaves a pulley so long as the belt guides toward that pulley squarely at an angle of 90 deg. and on the center line of the face.

In practice, it is usually necessary to locate a pulley so that it delivers the belt square to the next pulley, and then turn the pulley in or out, up or down, without moving it from its location, until it will receive the belt fairly from the last pulley over which the belt has passed. This applies alike to open belts, crossed belts, quarter-turn belts and mule belts, as in the present instance.

Taking advantage of this fact, it is possible to move the mule pulleys a little so that both may be placed upon a single shaft. Referring again to Fig. 20, it will be noted that the mule shafts are parallel and only 2 ins. out of line. But it should not be forgotten that the shafts are 2 ins. out of line in another direction, for, if the eye be placed to the right, so as to look along the direction of shaft *AB*, then the upper mule pulley will be found 2 ins. out of alignment with the lower pulley, and to bring both the mule shafts into alignment, the lower one must be moved 2 ins. to the left, while the upper one must be moved 2 ins. directly from the observer, toward the pulley on shaft *AB*.

The reason why pulleys can be moved thus, and still allow the belts upon them to run properly, is due to the characteristic explained above. Take the case of the lower mule pulley: The belt, running in the direction of the arrow, leaves pulley *B*, Fig. 20, guided toward the lower mule pulley. Note what would happen were this pulley to be moved 2 ins. to the left. The belt as it left pulley *B* would be twisted slightly to the left but would still hit the mule pulley fairly. But as it is immaterial how a belt leaves a pulley, no harm is done in moving the lower mule pulley 2 ins. to the left. Its angle is not changed, therefore it receives the belt properly, and that is all that is necessary.

Next, push the upper mule pulley horizontally backward 2 ins. This brings the two mule shafts in line as viewed from the right side, and causes the upper fold of the belt to twist a little as it leaves pulley *C*, but the angle of the upper mule pulley not having been disturbed, it still receives the belt fairly from *C*, and still delivers the belt squarely toward pulley *B*. Therefore, both mule pulleys may be placed upon a single shaft by making the slight changes described.

When the pulleys upon shafts *B* and *C* are of unequal diameters, it will be necessary to use separate mule shafts and to adjust the shaft of each mule pulley square to the line drawn from one pulley to the other, as in Fig. 19. Otherwise, there is no change in the method of locating the mule pulleys and obtaining the angles of their shafts.

Should it be found necessary to run the belt at two different angles, instead of using the same angle from pulley *C* to pulley *B*, lay down both angles in Fig. 19, adjust the mule shaft to right and left to be square with the line over face of pulleys from *C* to *E* in Fig. 19, and then adjust the mule stand to and from the beholder, to be perpendicular to the belt line from *E* to *B*, Fig. 19. The lower mule shaft is to be adjusted in like manner, but to agree with the lower belt line in Fig. 19. Thus, when the pulleys are of unequal size, and the belts leave and reach pulleys *C* and *B* at unlike angles, there will be correspondingly different angles given to the mule shafts, to the right and left, and forward or backward.

Length of Belts

The calculation of the length of a belt is occasionally necessary to meet cases where endless belts are to be carried over pulleys at considerable distances apart. Carl G. Barth (*Amer. Mach., Mar.* 12, 1903) gives the following formulas ofincreasing accuracy in the order given:

$$L=\frac{(D+d)\pi}{2}+Cx+2C \qquad (a)$$

$$L=\frac{(D+d)\pi}{2}+Cx\frac{12}{12-x}+2C \qquad (b)$$

$$L=\frac{(D+d)\pi}{2}+Cx\frac{60-13x}{6c-18x}+2C \qquad (c)$$

in which L = length of belt (open)
D = diameter of larger pulley
d = diameter of smaller pulley
C = center distance
$x=\left(\frac{D-d}{2C}\right)^2$

All dimentions are to be taken in the same units—feet or inches as preferred.

Mr. Barth has tested these formulas by applying them to the limiting case (beyond what is possible in practice) in which $d=o$ and $C=\frac{D}{2}$, the correct value of L being $D\pi$. Under this test, formula (*a*), which is identical with Rankine's well-known formula, gives a result which is a little over 2 per cent. short, formula (*b*) a result which is about 1 per cent. short, and formula (*c*) a result which is less than four-tenths of 1 per cent. short.

The length of a crossed belt is given by the exact formula

$$L=2\left\{\sqrt{C^2-D'^2}+D\left(\pi-\cos.^{-1}\frac{D'}{C}\right)\right\}$$

in which the notation is as before with the addition that

$$D'=\frac{D+d}{2}$$

Steel Belts

Steel belts have been used to a considerable extent in Germany and with apparent success. The joint construction, shown in Fig. 21 (*Amer. Mach., Dec.* 24, 1908), consists of two steel plates, an under and an upper, between which the ends of the belts are joined. These plates taper from a thickened section at the center to comparatively thin edges. The ends of the outer locking pieces are prolonged. It was discovered that when these extensions were not provided the

belt would break near the inner pieces just after leaving the pulley, probably owing to the rapid straightening of the belt after its rapid motion over the pulley. In the size illustrated, the upper plate is made with a series of holes in order to lighten it. Both of these plates are shaped to a circular arc, whose radius is equal to the radius of the smallest pulley on which the joint is to be used. Thus, for a given joint there is a minimum limiting diameter of pulley on which it can run, but no similar maximum limiting diameter; for a given joint can be used on pulleys of any diameter larger than the one to which the plates are particularly fitted.

The belt itself is made of a uniform quality of steel of an even thickness and is tempered. The ends are carefully brought together, fitted and soldered with a special solder that flows at a comparatively low temperature to avoid drawing the temper of the belt. This joining is then placed between the two plates already described, and these plates are fastened together by means of screws, as shown in the illustration.

A number of interesting claims are made for these belts. Three of the most striking are: The small amount of slipping of the belt on the pulley, given in figures as less than $\frac{1}{10}$ of 1 per cent., the narrow width of the belt compared with leather belts, the proportion being about as 1 to 5, and the great speed at which these belts can be run, given as 100 m. per sec., or say 19,000 ft. per min. This latter figure is striking when compared with the limiting factor, usually given for leather belting as 4000 ft. per min. It is very common to run these steel belts at a speed of 50 m. per sec., or say, roughly, 10,000 ft. per min. They have been used for driving belts in machine

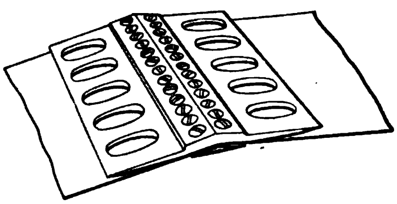


FIG. 21.—German steel belt joint.

shops and other manufacturing establishments, installations of 250 h.p. having been made. Table 5 gives some comparative data between a rope drive, a leather-belt drive, and steel-belt drive for 100 h.p., transmitted by pulleys 10 m. apart at a speed of 200 r.p.m. and a diameter of 1 m. The metric measurements are not translated into English measurements because the table is of comparative interest only.

TABLE 5.—COMPARISON OF ROPE, LEATHER-BELT AND STEEL-BELT DRIVES

Item	Rope drive	Leather-belt drive	Steel-belt drive
Breadth of belt space....	6 ropes 45 mm. in diameter	500 mm.	100 mm.
Breadth of pulley.......	380 mm.	500 mm.	110 mm.
Weight of pulley........	1000 kg.	520 kg.	270 kg.
Weight of rope or belt...	240 kg.	140 kg.	13 kg.
Total weight of drive....	1240 kg.	660 kg.	283 kg.
Cost of pulleys..........	720 marks	400 marks	250 marks
Cost of ropes or belts....	600 marks	1300 marks	750 marks
Total cost.............	1340 marks	1700 marks	1000 marks
Power lost in per cent...	13%	6%	.5%
Power lost in horse-power	13 h.p.	6 h.p.	.5 h.p.

More recent information regarding several successful German installations of steel belts is given by R. K. CRONKHITE (*Amer. Mach.*, *Nov.* 21, 1912), the final conclusions being that:

Steel belts from half to one-third, and in some cases one-quarter, the width of leather belts will do the same work as the leather belt without trouble.

Steel belts do not stretch or slip after being placed on the pulleys, and are not affected by variations in temperature to any perceptible extent, which makes them very reliable for use in damp places, such as laundries. They are especially adapted for use in paint and varnish works, as the accumulations of paint and other sticky substances can be washed off with gasoline and the belt kept in good condition.

Being narrower than leather or other types of belting, they require pulleys of narrower face, which is an item in the equipment of a new plant or the installation of new drives in any factory.

From the investigations made, it can be said that their first cost is considerably below that of leather or rubber belting.

The experiments have demonstrated that steel belts are more sensitive than other types and that the shafts and pulleys on which they run must be in line and level or the belt will invariably run to the *low* side of the pulley, out of line, and will run off if the pulley is too much out of line. The use of canvas on the face of the pulleys is of decided

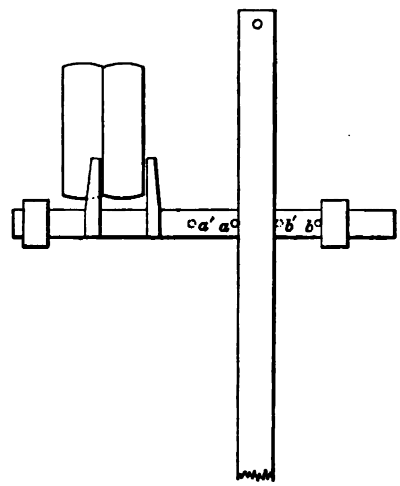


FIG. 22.—Improved belt shipper.

advantage in connection with steel belting, as it forms a bed or cushion for the belt to run on, and at the same time greatly increases the pulling power of the belt. A special rubber covering, suggested by experiments, has proved satisfactory.

In replacing a leather belt with a steel belt where the pulleys are crown faced, it is necessary to build the crown up to a flat face, as steel belts will not run on crown-faced pulleys. They will, of course, run on plain uncovered iron or steel, as well as on wood pulleys, but the use of the canvas or rubber covering is so beneficial that it seems almost necessary to good service.

Belt Shippers

An improved belt shipper, which completely overcomes the common nuisance of the belt refusing to remain where put, is shown in Fig. 22. The difficulty is due to the weight of the shipper pole, which tends to bring the pole to the vertical position, with the belt half on each pulley. In this position the machine will not stand still if it has no work to do, and it will not drive if it has work to do. The arrangement shown gets rid of this effect of the gravity of the pole, with the result that the belt stays on either tight or loose pulley as desired.

The sketch represents a shifter made of wood, the improvement consisting in having the pole play between pegs *ab* on the fork bar. The fork is shown in position to guide the belt to the loose pulley. The pole hangs in a vertical position against peg *a*. If the belt is to be shifted, the pole is pushed to the left as usual, and with the usual result, except that after the pole is dropped by the hand, it swings back by gravity to the vertical position again. The previous movement of the fork bar will, however, have moved the pegs to the posi-

tions $a'b'$, so that the pole will then be in contact with peg b in its new position b', ready to push the bar in the opposite direction whenever wanted, after which the pole will again return to the vertical position. The belt fork always stays where left, and, the pulleys being crowned, the belt also stays where put. The same result may be obtained by increasing the space between the forks—making this space span both pulleys instead of one, as usual.

Pulleys

The dimensions of cast-iron pulleys may be obtained from the following formulas and tables by J. W. See (*Amer. Mach., July* 23, 1881).

$$B=A\times.0625+.5.$$
$$C=A\times.04+.3125.$$
$$D=A\times.025+.2.$$
$$E=A\times.016+.125.$$

in which A = diameter of pulley,
B = width of arms at center of pulley,
C = width of arms at circumference of pulley,
D = thickness of arms at center of pulley,
E = thickness of arms at circumference of pulley.

All dimensions in inches. Change decimal results to the nearest sixteenths.

Mr. See also supplies Table 6 of pulley dimensions, and the following instructions:

The pattern spiders should be of iron, parted, dowelled, the ends of the arms turned to size in the lathe, and the shallow recess H turned in the hub seat. The rims may be iron or wood, as policy suggests. The drawing shows how to shape straight and curved arms. The table gives dimension of arms where they would cross the rim and cross the center. The hub seat H is of such size as to receive quite a range of standard hub patterns, and make a nice, smooth job without sharp corners. The ends of the arms may be drilled to receive screws put through edge of rim to hold strings together, if parted rim pattern is used. Some will prefer a single narrow rim to be drawn for any width. Some shops follow the vicious plan of casting all pulleys the full width of, say, a 9-in. pattern, and then cutting to width in the lathe, using a special or drawing pattern for wider rims.

The Rims.—Columns FG show the thickness at center and edge in the rough. The crown will be right for all widths. Pattern should be large enough to let casting finish to exact size—a matter very often neglected. All pulleys for general work should be ½ in. wider than belt. A good pulley trade calls for iron rim patterns of sundry widths to change on loose spider.

The Spider.—The table gives size of arms at rim and center crossing, the diameter of the center web, radius of the fillets, and diameter of the hub seat H, which is ⅛ in. deep in all cases. The table makes the hub seat large enough to receive good-sized hubs, and still look right with small ones.

To Draw Curved Arms.—Draw full size the diameter A; step off six points, a b c d e f; at each of these points strike circles C, of size given in table column C; strike circles at pulley center, sizes from columns B and I in table; with c for a center strike arcs h and i, the radius being to each side of circle B; midway between these arcs and the points j and k locate points l and m; with k for center and cl for radius strike arc n; with j as center and mc as radius strike arc o; with kn for radius sweep inside of arm touching circle B, center being somewhere on arc n; with jo for radius sweep outside of arm, touching circle B, center being somewhere on arc o.

For Straight Arms.—Draw lines touching circles B and C. Draw fillets p, touching edges of arms, and circle I. With one-half of I, minus F for radius, cut off the arms. Radius of q equals one-half of C.

The edge view, or section of arms, as in Fig. 2, is made by circles E E and D from table, and side lines touching these circles. Radius of y equals E. Make these fillets nice, and thus avoid all sharp internal angles.

Section of Arm, Fig. 26.—Draw circles r and s, representing width and thickness of arm; make t u equal to v w; with u v for radius and u as center, draw sides of arm; put in circle x, touching the sides and the circle r. The fillet p should have half circle section and present pure blended surface.

The Hub Pattern, Fig. 27.—The intention is to have the hub patterns fit all pulley patterns within reason. Table gives diameter and lengths. The flanges should fit easily in the hub seats in the spider patterns. Radius of y is ⅜ in. in all cases. Fillet is quarter circle. Hub patterns should be of wood. Core prints should be turned on the pattern solid. The prints are one size on all hubs. Make full set of straight core boxes 1 ft. long, and have in each two sliding ends to give shape of prints. By this means but few core boxes are needed, and the hubs and cores will interchange nicely for all common work. Taper both prints if desired.

The above formulas and tables make no distinction between pulleys for single and double belts. For double-belt pulleys the author suggests the formulas:

$$C=A\times.05+.75,$$
$$E=\tfrac{1}{2}C,$$
F and G ½ more than for single-belt pulleys.

The only suitable number of arms in a pulley, wheel or gear which is to be chucked by the arms is a multiple of 3, as such numbers permit strapping at three points without distortion.

The above formulas and table are suitable for all ordinary cases of stock pulleys. For special cases and extra large pulleys, Fig. 29 by S. E. Freeman (*Amer. Mach., Dec.* 3, 1896), which gives the practice of the Todd and Stanley Mill Furnishing Co. may be used. As will be seen, it is adapted for use in laying out rope sheaves as well as belt pulleys.

To use the chart, substitute the given dimensions in the proper formula; find the value of the quantity under the cube root sign. Find this same quantity on the base line and trace upward to the various lines where read the required dimensions. Examples will be found below the chart.

For the design of hollow pulley arms from their solid equivalents. see Arms of Spur Gears.

The static strength of belt pulleys formed the subject of experiments by Prof. C. H. Benjamin (*Amer. Mach., Sept.* 22, 1898). The general conclusions arrived at are as follows:

1. That the bending moments on pulley arms are not evenly distributed by the rim, but are greatest on the arm near the tight side of belt.

2. That there are bending moments at both ends of the arm, that at the hub being much the greater, the ratio depending on the relative stiffness of rim and arms. An increase of the width of rim will undoubtedly help the arms.

The rules deduced from the experiments for the rational design of cast-iron pulleys are as follows:

1. Multiply the net pull of belt by a suitable factor of safety and by the length of arm in inches. Divide this product by one-half the number of arms and use the quotient for a bending moment. Design the hub end of arm by the usual rules to resist this moment.

2. Make the rim ends of arms one-half as strong as the hub ends.

Parting split pulleys half-way between the arms, Fig. 30, is a source of danger at high speed, as has been demonstrated by the experiments of Professor Benjamin (see Bursting Strength of Fly-wheels). This location of the joint is even worse in pulleys than in fly-wheels because the thinness of the rim provides less strength to resist the centrifugal bending stress. The construction shown is particularly bad because

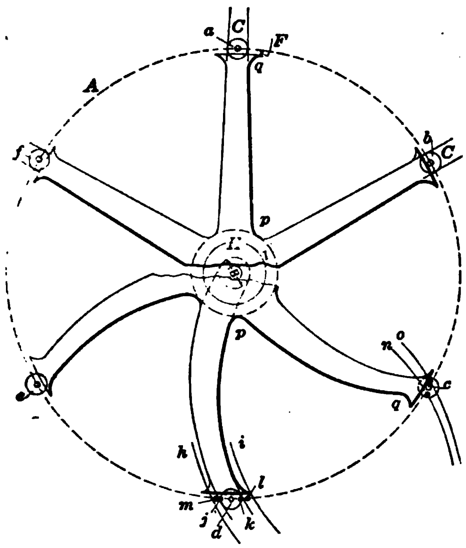


FIG. 23.

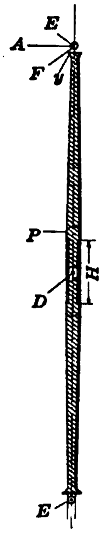


FIG. 24.

the absence of a joint at the inner ends of the lugs aggravates the other bad conditions. Fig. 31, by PROFESSOR SWEET (*Amer. Mach., Jan.* 12, 1905), is a well considered design in which the weakness due to the parting is practically eliminated.

Overhanging pulleys should be avoided, but when that is impossible the usual construction, Fig. 32, may be greatly improved by adopting the plan shown in Fig. 33.

In Fig. 32 the pulley A is secured by the set screw C to the driving shaft B, which runs in the bushing E carried in the bracket D. The

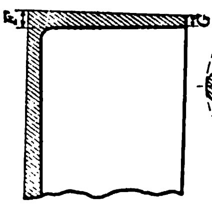


FIG. 25.

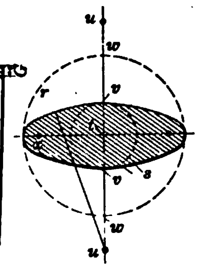


FIG. 26.

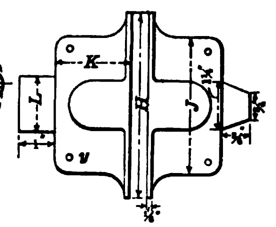


FIG. 27.

TABLE 6.—DIMENSIONS OF CAST-IRON PULLEYS

Diam. of pulley	Width of arms at center	Width of arms at circum.	Thickness of arms at center	Thickness of arms at circum.	Thickness of rim at center	Thickness of rim at edge	Diameter of hub seat	Distance across web
A	B	C	D	E	F	G	H	I
6	$\frac{7}{8}$	$\frac{9}{16}$	$\frac{5}{16}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{16}$	$2\frac{3}{4}$	3
7	$\frac{15}{16}$	$\frac{9}{16}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{16}$	$2\frac{3}{4}$	3
8	1	$\frac{5}{8}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{16}$	$2\frac{3}{4}$	3
9	$1\frac{1}{16}$	$\frac{5}{8}$	$\frac{7}{16}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{16}$	$3\frac{1}{2}$	4
0	$1\frac{1}{8}$	$\frac{11}{16}$	$\frac{7}{16}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{16}$	$3\frac{1}{2}$	4
1	$1\frac{3}{16}$	$\frac{3}{4}$	$\frac{7}{16}$	$\frac{5}{16}$	$\frac{5}{16}$	$\frac{3}{16}$	$3\frac{1}{2}$	4
2	$1\frac{1}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	$\frac{5}{16}$	$\frac{5}{16}$	$\frac{3}{16}$	$4\frac{1}{4}$	5
3	$1\frac{5}{16}$	$\frac{13}{16}$	$\frac{1}{2}$	$\frac{5}{16}$	$\frac{5}{16}$	$\frac{3}{16}$	$4\frac{1}{4}$	5
4	$1\frac{3}{8}$	$\frac{7}{8}$	$\frac{1}{2}$	$\frac{5}{16}$	$\frac{5}{16}$	$\frac{3}{16}$	$4\frac{1}{4}$	5
5	$1\frac{7}{16}$	$\frac{7}{8}$	$\frac{9}{16}$	$\frac{3}{8}$	$\frac{5}{16}$	$\frac{3}{16}$	$4\frac{1}{4}$	5
6	$1\frac{1}{2}$	$\frac{15}{16}$	$\frac{9}{16}$	$\frac{3}{8}$	$\frac{5}{16}$	$\frac{3}{16}$	$4\frac{1}{4}$	5
7	$1\frac{9}{16}$	1	$\frac{5}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{4}$	$4\frac{1}{4}$	5
8	$1\frac{5}{8}$	1	$\frac{5}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{4}$	$4\frac{1}{4}$	5
9	$1\frac{11}{16}$	$1\frac{1}{16}$	$\frac{11}{16}$	$\frac{7}{16}$	$\frac{3}{8}$	$\frac{1}{4}$	$4\frac{1}{4}$	5
0	$1\frac{3}{4}$	$1\frac{1}{8}$	$\frac{11}{16}$	$\frac{7}{16}$	$\frac{3}{8}$	$\frac{1}{4}$	$4\frac{1}{4}$	5
2	$1\frac{7}{8}$	$1\frac{3}{16}$	$\frac{3}{4}$	$\frac{7}{16}$	$\frac{3}{8}$	$\frac{1}{4}$	5	6
4	2	$1\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{4}$	5	6
6	$2\frac{1}{8}$	$1\frac{3}{8}$	$\frac{13}{16}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{4}$	5	6
8	$2\frac{1}{4}$	$1\frac{7}{16}$	$\frac{7}{8}$	$\frac{9}{16}$	$\frac{3}{8}$	$\frac{1}{4}$	5	6
0	$2\frac{3}{8}$	$1\frac{1}{2}$	$\frac{15}{16}$	$\frac{9}{16}$	$\frac{3}{8}$	$\frac{1}{4}$	5	6
2	$2\frac{1}{2}$	$1\frac{9}{16}$	1	$\frac{5}{8}$	$\frac{7}{16}$	$\frac{5}{16}$	5	7
4	$2\frac{5}{8}$	$1\frac{5}{8}$	1	$\frac{5}{8}$	$\frac{7}{16}$	$\frac{5}{16}$	5	7
6	$2\frac{3}{4}$	$1\frac{3}{4}$	$1\frac{1}{8}$	$\frac{11}{16}$	$\frac{7}{16}$	$\frac{5}{16}$	5	7
8	$2\frac{7}{8}$	$1\frac{13}{16}$	$1\frac{1}{8}$	$\frac{11}{16}$	$\frac{7}{16}$	$\frac{5}{16}$	5	7
0	3	$1\frac{7}{8}$	$1\frac{3}{16}$	$\frac{3}{4}$	$\frac{7}{16}$	$\frac{5}{16}$	7	8
2	$3\frac{1}{8}$	2	$1\frac{1}{4}$	$\frac{3}{4}$	$\frac{7}{16}$	$\frac{5}{16}$	7	8
4	$3\frac{1}{4}$	2	$1\frac{5}{16}$	$\frac{13}{16}$	$\frac{7}{16}$	$\frac{5}{16}$	7	8
6	$3\frac{3}{8}$	$2\frac{1}{8}$	$1\frac{3}{8}$	$\frac{13}{16}$	$\frac{7}{16}$	$\frac{5}{16}$	7	8
8	$3\frac{1}{2}$	$2\frac{1}{4}$	$1\frac{7}{16}$	$\frac{7}{8}$	$\frac{1}{2}$	$\frac{3}{8}$	7	9

Diam. of pulley	Width of arms at center	Width of arms at circum.	Thickness of arms at center	Thickness of arms at circum.	Thickness of rim at center	Thickness of rim at edge	Diameter of hub seat	Distance across web
A	B	C	D	E	F	G	H	I
50	$3\frac{5}{8}$	$2\frac{5}{16}$	$1\frac{7}{16}$	$\frac{7}{8}$	$\frac{1}{2}$	$\frac{3}{8}$	7	9
52	$3\frac{3}{4}$	$2\frac{3}{8}$	$1\frac{1}{2}$	$\frac{15}{16}$	$\frac{1}{2}$	$\frac{3}{8}$	7	9
54	$3\frac{7}{8}$	$2\frac{7}{16}$	$1\frac{1}{2}$	$\frac{15}{16}$	$\frac{9}{16}$	$\frac{7}{16}$	7	10
56	4	$2\frac{1}{2}$	$1\frac{9}{16}$	1	$\frac{9}{16}$	$\frac{7}{16}$	7	10
58	$4\frac{1}{8}$	$2\frac{5}{8}$	$1\frac{5}{8}$	1	$\frac{9}{16}$	$\frac{7}{16}$	9	11
60	$4\frac{1}{4}$	$2\frac{11}{16}$	$1\frac{11}{16}$	$1\frac{1}{16}$	$\frac{9}{16}$	$\frac{7}{16}$	9	11
62	$4\frac{3}{8}$	$2\frac{3}{4}$	$1\frac{3}{4}$	$1\frac{1}{16}$	$\frac{5}{8}$	$\frac{1}{2}$	9	11
64	$4\frac{1}{2}$	$2\frac{7}{8}$	$1\frac{13}{16}$	$1\frac{1}{8}$	$\frac{5}{8}$	$\frac{1}{2}$	9	12
66	$4\frac{5}{8}$	$2\frac{15}{16}$	$1\frac{13}{16}$	$1\frac{1}{8}$	$\frac{5}{8}$	$\frac{1}{2}$	9	12
68	$4\frac{3}{4}$	3	$1\frac{7}{8}$	$1\frac{3}{16}$	$\frac{5}{8}$	$\frac{1}{2}$	9	12
70	$4\frac{7}{8}$	$3\frac{1}{16}$	$1\frac{15}{16}$	$1\frac{3}{16}$	$\frac{5}{8}$	$\frac{1}{2}$	9	12
72	5	$3\frac{1}{16}$	2	$1\frac{1}{4}$	$\frac{5}{8}$	$\frac{1}{2}$	9	12

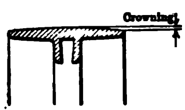
Hubs

Diameter of shaft	Diameter of hub	Length of half	Diameter of print
	J	K	L
1 to $1\frac{1}{4}$	$2\frac{1}{4}$	$1\frac{1}{4}$	$\frac{3}{4}$
$1\frac{5}{16}$ to $1\frac{1}{2}$	$2\frac{5}{8}$	$1\frac{1}{2}$	$1\frac{1}{8}$
$1\frac{9}{16}$ to $1\frac{3}{4}$	$3\frac{1}{8}$	$1\frac{3}{4}$	$1\frac{1}{4}$
$1\frac{13}{16}$ to 2	$3\frac{5}{8}$	2	$1\frac{1}{2}$
$2\frac{1}{16}$ to $2\frac{1}{4}$	4	$2\frac{1}{4}$	$1\frac{3}{4}$
$2\frac{5}{16}$ to $2\frac{1}{2}$	$4\frac{1}{2}$	$2\frac{1}{2}$	2
$2\frac{9}{16}$ to $2\frac{3}{4}$	5	$2\frac{3}{4}$	$2\frac{1}{4}$
$2\frac{13}{16}$ to 3	$5\frac{1}{4}$	3	$2\frac{1}{2}$
$3\frac{1}{8}$ to $3\frac{1}{2}$	$6\frac{3}{8}$	$3\frac{1}{2}$	$2\frac{3}{4}$
$3\frac{5}{8}$ to 4	$7\frac{1}{4}$	4	$3\frac{1}{4}$
$4\frac{1}{8}$ to $4\frac{1}{2}$	$8\frac{1}{8}$	$4\frac{1}{2}$	$3\frac{3}{4}$
$4\frac{5}{8}$ to 5	9	5	$4\frac{1}{4}$

collar *F*, which is taper-pinned to the shaft prevents end play. This design is bad, as the bush *E* wears bell-mouthed at the pulley end and the bending effect on the shaft due to the pull of the belt on the pulley increases as the wear on the bush increases. This gives combined bending and torsion on the shaft in transmitting the drive.

In the improved design, Fig. 33, these difficulties are overcome. The bush is prolonged and the pulley runs upon its periphery. The drive is transmitted through the collar *G*, which is secured to the pulley, and also taper-pinned to the shaft. The collar *F* is the same as in Fig. 32. Thus the shaft is subject to torsion alone, or practically so.

The correct arrangement of tight and loose pulleys is shown in Fig. 34, by PROFESSOR SWEET (*Amer. Mach.*, *Jan.* 12, 1905), the hub of the tight pulley being shortened and that of the loose pulley lengthened *at both ends* to make it central with the pulley face. Fig. 35 sacrifices length of bearing where it is most needed, and Fig. 36 is cer-



Crowning	Width of Face
1/16	Under 6″
3/32	6 to 12
1/8	12 " 18
5/32	18 " 24
3/16	24 " 30
7/32	30 " 36
1/4	36 " 48
5/16	48 " 60
3/8	60 & Over

FIG. 28.—Crowning for belt pulleys.

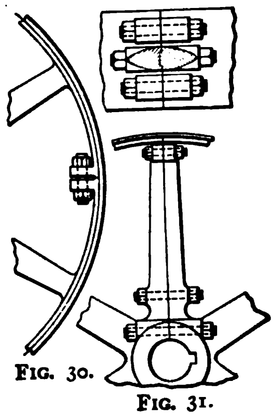


FIG. 30. FIG. 31.

FIGS. 30 and 31.—Correct and incorrect parting of split pulleys.

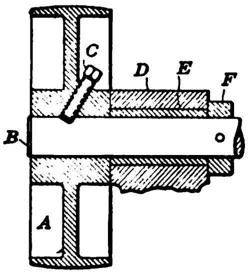


FIG. 32.

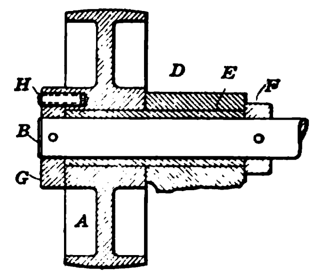
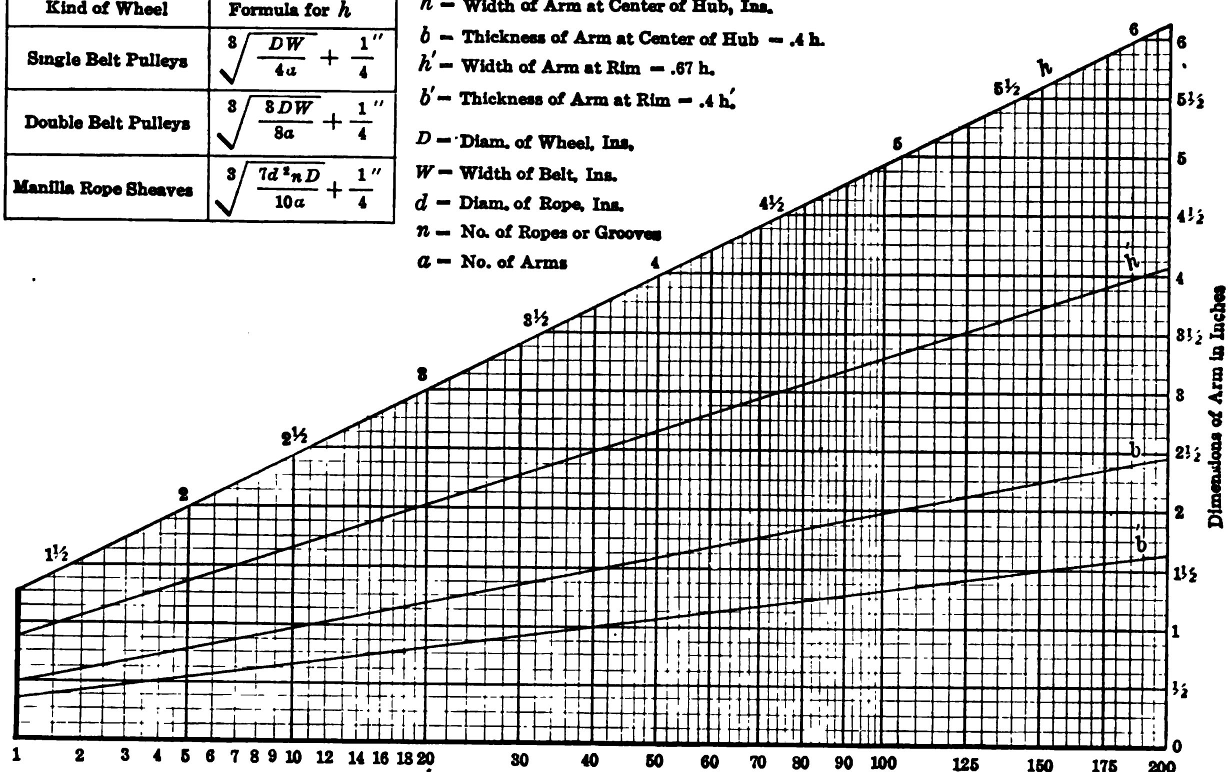


FIG. 33.

FIGS. 32 and 33.—Correct and incorrect design of overhung pulleys.

Kind of Wheel	Formula for h
Single Belt Pulleys	$\sqrt[3]{\frac{DW}{4a}}+\frac{1''}{4}$
Double Belt Pulleys	$\sqrt[3]{\frac{8DW}{8a}}+\frac{1''}{4}$
Manilla Rope Sheaves	$\sqrt[3]{\frac{7d^2nD}{10a}}+\frac{1''}{4}$

h — Width of Arm at Center of Hub, Ins.
b — Thickness of Arm at Center of Hub = .4 h.
h' — Width of Arm at Rim = .67 h.
b' — Thickness of Arm at Rim = .4 h′.
D — Diam. of Wheel, Ins.
W — Width of Belt, Ins.
d — Diam. of Rope, Ins.
n — No. of Ropes or Grooves
a — No. of Arms



To find the dimensions of the arms of a 48 in. single belt pulley having 6 arms and for a 12 in. belt: substituting these factors in the quantity under the cube root sign for single belt pulleys gives $\frac{48 \times 12}{4 \times 6} = 24$. Locate 24 on the bare line, trace upward and read $h = 3\frac{1}{8}$, $b = 1\frac{1}{4}$, $h' = 2\frac{1}{8}$, and $b' = \frac{7}{8}$, all in ids.

Again for a rope sheave 8 ft. diameter having 6 arms and for 8, $1\frac{1}{2}$ in. ropes: substitute as before in the proper formula and obtain $\frac{7 \times 125^2 \times 8 \times 96}{10 \times 6} = 140$. Locate 140 on the bare line, trace upward and read $h = 5\frac{1}{2}$, $h' = 3\frac{5}{8}$, $b = 2\frac{3}{16}$ and $b' = 1\frac{1}{2}$ ins.

FIG. 29.—Dimensions of arms of belt pullers and rope sheaves.

tain to wear bell-mouthed. The chambered construction, Fig. 37 is appropriate on tight pulleys only.

A radical cure for the loose pulley nuisance is shown in Fig. 38 by E. J. ARMSTRONG (*Amer. Mach., Apr.* 24, 1900). The reason for the trouble with loose pulleys is that the shaft wears smaller and the hole larger, resulting in a destruction of the fit.

In the illustration, the pulley is shown bored out and a bush inserted which is kept from revolving by a rod *a* secured in any convenient way. With this construction the wear is confined to one side of the bush which continues to fit the curve of the hole. The bush should be of cast-iron, with which the wear is almost negligible.

About $\frac{3}{16}$ in. o each 1 in. of shaft diameter is a good proportion for the thickness of the bush, giving proper wearing thickness without unduly enlarging the bore of the pulley. Several oil holes should be drilled through both top and bottom, and if an oil groove is cut from the oil cup cast on the bush, so as to lead the oil inside along the shaft, no other means of oiling is ordinarily needed.

When arranged as usual, loose pulleys are much more effectively lubricated with grease than with oil, the former remaining in place much better than the latter. For small pulleys the grease cup may be tapped into the end of the shaft—a suitable hole lengthwise the shaft and another crosswise within the pulley hub carrying the grease to the bearing.

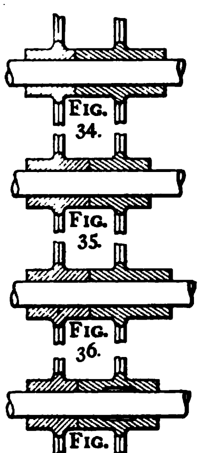
A self-oiling loose pulley is shown in Fig. 39 by H. J. WHITE (*Amer. Mach., June* 22, 1905). The bushing is of hard composition and the oil holes are plugged with hard felt or rattan. Mr. White says that with $\frac{3}{4}$ pt. of oil in the oil space these pulleys will run three months without attention.

*The bursting strength of pulleys*of various materials and constructions formed the subject of experimental tests by PROF. C. H. BENJAMIN (*Journal A. S. M. E., June,* 1910) similar to those on fly-wheels (see Bursting Strength of Fly-wheels). The results are given in Table 7. The cast-iron pulleys Nos. 11 and 12 were not fractured.

TABLE 7.—RESULTS OF BURSTING TESTS OF BELT PULLEYS

No. of test	Kind of material in pulleys	Rim: Style	Rim: Diameter inches	Rim: Breadth inches	Rim: Depth inches	Weight pounds	Bursting speed: r.p.m.	Bursting speed: Peripheral speed ft. per sec.
1	wood	solid	24	6.25	1.62	29.37	2720	284.7
2	wood	solid	24	6.25	1.62	29.37	2550	266.9
3	wood	2 sections	24	6.5	1.78	29.67	2210	231.8
4	wood	2 sections	24	6.5	1.78	29.67	2110	220.8
5	wood	2 sections	24	6.5	1.78	28.81	2390	251.0
6	wood	2 sections	24	6.5	1.78	28.81	2430	254.3
7	wood	2 sections	24	6.5	1.78	28.81	2360	247.0
8	wood	2 sections	24	6.5	1.78	28.81	2420	253.3
9	wood	2 sections	24	6.5	1.78	28.81	2570	258.5
10	wood	2 sections	24	6.5	1.78	28.81	2535	244.4
11	cast-iron	solid	24	6.0	0.406	70.44	3720	389.4
12	cast-iron	solid	24	6.0	0.406	70.44	3380	353.8
13	paper	solid	24	6.0	1.75	77.37	2820	295.2
14	paper	solid	24	6.0	1.75	77.37	2930	306.7
15	steel	2 sections	24	6.75	0.0625	41.75	2240	234.5
16	steel	2 sections	24	6.75	0.0625	41.75	2240	234.5

F. P. READ (*Power, Apr.* 22, 1913) reports the repeated failure, at a rim speed of 5937 ft. per min., of 84×12 ins. cast-iron split pulleys of the usual type with lugs and bolts half-way between the arms.



FIGS. 34 to 37.—Correct and incorrect arrangements of tight and loose pulleys.

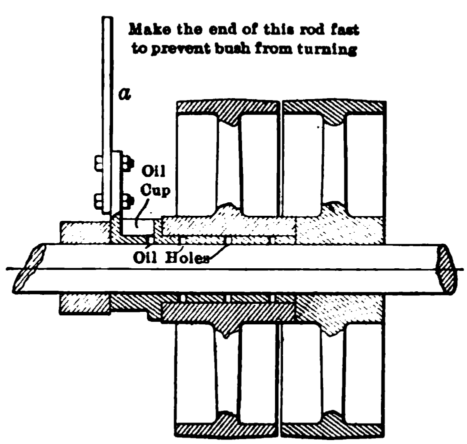


FIG. 38.—A cure for the loose pulley nuisance.

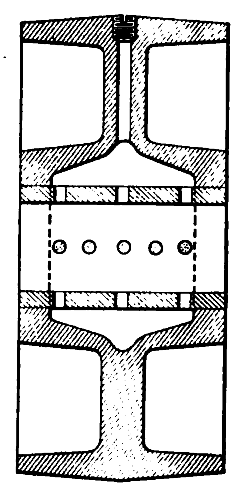
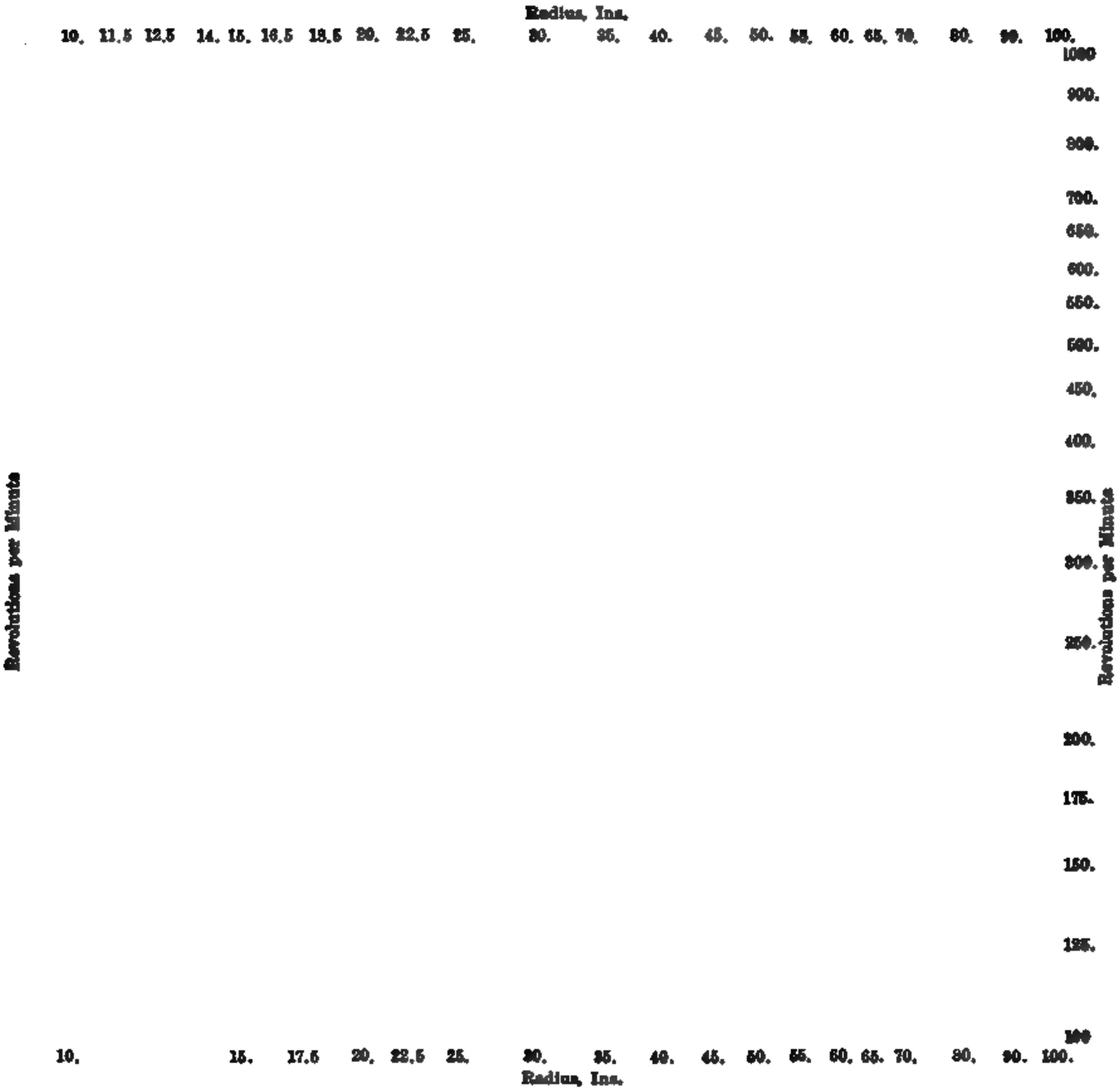


FIG. 39.—Self-oiling loose pulley.

FLY-WHEELS



For cast iron.

To find the rim tension in a cast iron wheel 40 ins. radius running 350 r.p.m.: Find 40 on the base line and 350 on the vertical scale; multiply the stress by 100, and so for 4 ins. radius read as for 40 (that is, 4 × 10) and divide the stress read by 100.

FIG. 1.—Centrifugal

Stresses in Fly-wheels

The stress in a ring revolving about an axis passing through its center due to centrifugal force is similar to that in a boiler shell due to internal pressure and is given, for any material, by the formula:

$$S = \frac{wv^2}{2.68} \qquad (a)$$

in which S = stress on section, lbs. per sq. in.
w = weight of material, lbs. per cu. in.
v = velocity of center of gravity of rim, ft. per sec.

For cast-iron having $w = .26$ this becomes:

$$S = .097v^2 \qquad (b$$

For steel having $w = .28$ it becomes:

$$S = .1045v^2 \qquad (c$$

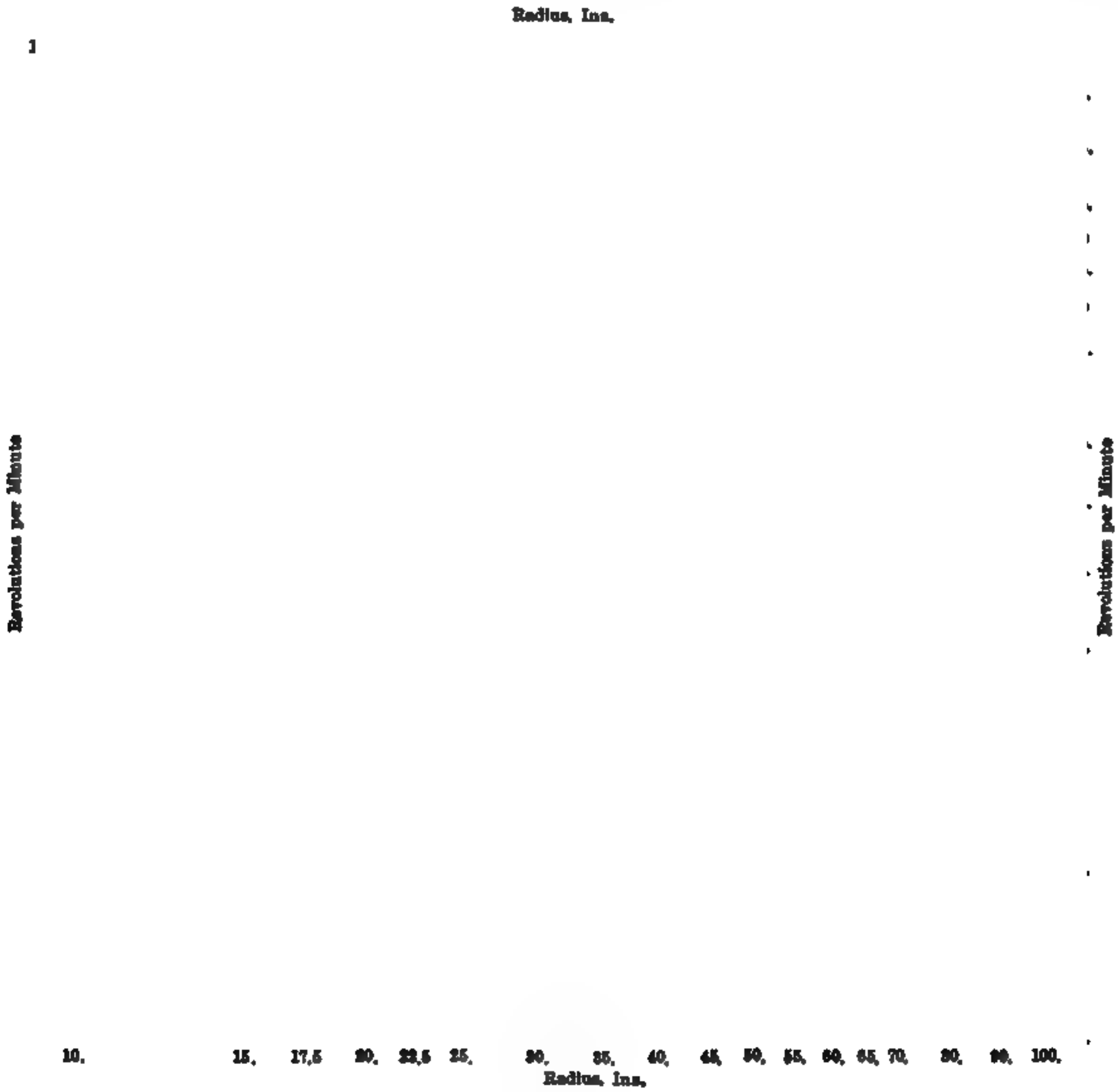
For both iron and steel it becomes, with sufficient accuracy for fly-wheel calculations:

$$S = \frac{v^2}{10} \qquad c$$

To find the total stress on the section for the calculation of the

dimensions of link and other joints, multiply the stress per sq. in. by the area of the section in sq. ins.

The rim tension may be obtained without calculation from Fig. 1 by P. MULLER (*Amer. Mach., Nov.* 28, 1901). The diagram for steel will also serve for wrought iron, which has practically the same specific gravity. The use of the charts is shown by an example below them.



For steel.

trace to their intersection and read 1450 lbs. per sq. in., rim tension. For a wheel of 400 ins. radius read as for 40 (that is, $\frac{400}{10}$) and

tension in fly-wheels.

The velocity of the rim at which bursting may be expected is given, for any material, by the formula:

$$V = 1.64\sqrt{\frac{te}{w}} \qquad (e)$$

in which V = bursting velocity of rim, ft. per sec.
t = tensile strength of material, lbs. per sq. in.
w = weight of material, lbs. per cu. in.
e = efficiency of construction, for values of which see Table 1.

For cast-iron, taking 19,000 lbs. per sq. in. as the tensile strength and .26 lb. per cu. in. as the weight, this becomes:

$$V = 443\sqrt{e} \qquad (f)$$

Before the experiments of PROF. C. H. BENJAMIN (summarized below) were published, the value of unity would have been substituted in formula (*f*) for the efficiency of construction of a wheel cast in one piece. Those experiments show this procedure to be incorrect, such wheels giving way at velocities materially below those to be expected from the tensile strength of the material—the efficiency of the construction being .85. Had deeper rim sections been used in the experiments, a larger value would probably have been found. The efficiencies to be substituted for other constructions are given in Table 1.

For steel, taking 60,000 lbs. per sq. in. as the tensile strength and .28 lbs. per cu. in. as the weight, the formula becomes:

$$V = 757\sqrt{e} \qquad (g)$$

No experiments have been made on steel wheels to determine their actual efficiencies of construction.

The most essential fact disclosed by these formulas is that the stress increases with the square of the speed, doubling the speed multiplying the stress by four and neutralizing a factor of safety of four based on the stress. Much greater increases of stress are therefore possible with fly-wheels than with steam boilers, and fly-wheels are correspondingly more dangerous than boilers. The Fidelity and Casualty Company, which insures both boilers and fly-wheels, finds the hazard on fly-wheels materially to exceed that on boilers.

These formulas should be used with caution when designing fly-wheels, as they are now known to have much less direct application than was formerly supposed. The condition of simple tension assumed, while true for an ideal revolving ring without arms, is seriously modified by the action of the arms of actual wheels in restraining the free expansion of the rim. Because of this restraint, each rim section between adjacent arms is in the condition of a beam under a uniformly distributed load, the load being the centrifugal force of the material of the section and the stress due to this beam action is added to the simple tension stress, the resulting stress being always greater than that given by the above formulas.

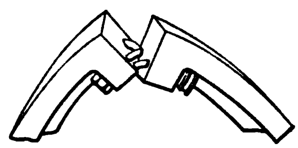


FIG. 2.—Method of failure of fly-wheels with flanged joints.

The beam action is especially serious in the case of built-up wheels with joints located, as usual, half-way between the arms. A joint in this position is equivalent to a joint in the middle of a beam and not to a simple splice in a tension member.

The beam action may be reduced by increasing the number of arms. Such increase reduces both the weight and length of the segments and the fiber stress due to the beam action—not the total fiber stress—is, hence, other things being equal, inversely as the square of the number of arms.

Attention was first called to this beam action by J. B. STANWOOD (*Trans. A. S. M. E., Vol.* 14) and the truth of his analysis has been experimentally proven by PROFESSOR BENJAMIN (*Trans. A. S. M. E., Vols.* 20 and 23), who tested model fly-wheels to destruction by revolving them in a bomb-proof casing at increasing speeds until they gave way. Fig. 2, from a photograph of an actual case, shows the manner of failure of the common flanged and bolted joint located midway between the arms, and demonstrates not only the reality of this beam action but its preponderating importance in wheels of this construction.

It is to be especially noted that, in repeated instances, wheels with this type of joint gave way through the solid rim and without failure of the bolts, as shown in Fig. 2, although the strength of the bolts was less than one-third that of the rim section, showing that the strength of this joint, as calculated in the usual way from the strength of the bolts, has nothing to do with the effective strength of the wheel.

The results of these experiments have been corroborated by the experience of the fly-wheel insurance department of the Fidelity and Casualty Company which has had several cases of failure of band fly-wheels with joints of the type shown in Fig. 2, in which the joint section went bodily out of the wheel and, in two cases, without affecting the remainder of the wheel or even bringing it to a stop.

The beam action becomes an increasing factor as the radial dimension of the rim decreases and is at its maximum in thin-rim belt pulleys.

Wheels having joints at the points of contrary flexure, that is, at one-fourth the distance from one arm to the next, have been repeatedly proposed as better adapted to meet the conditions of the beam action than those placed midway between the arms. Such wheels were tested by Professor Benjamin and found not to be appreciably stronger than those of the midway joint construction.

Professor Benjamin's experiments are summarized in Table 1. The figures of the table are the averages of the experimental results, the number of wheels of each type tested ranging from two to four, except in the case of column 5 of which construction but one was tested.

Regarding the wheel in column 3, Professor Benjamin considers that "if the tie rods had been more carefully designed and constructed, a greater speed could have been attained."

For similar tests of belt pulleys see Bursting Strength of Belt Pulleys.

W. H. Boehm, superintendent of the boiler and fly-wheel insurance departments of the Fidelity and Casualty Company, has calculated the very useful Table 2 of safe speeds of cast-iron wheels of various types. The table is figured for a margin of safety, based on speed, of approximately three or a factor of safety, based on the stress, of nine. The table assumes the solid wheel to have an efficiency of construction of unity, which is not borne out by Professor Benjamin's tests and the table doubtless slightly overestimates the strength of the wheels. The Fidelity and Casualty Company accepts for insurance wheels having a factor of safety on stress of five, equivalent to a margin of safety on speed of 2.24. The company frequently insists on the addition of tie rods, Table 1, column 3, to wheels with bolted flange joints.

The fly-wheel cast in one piece is subject to uncertain initial strains due to shrinkage, but it is, nevertheless, by far the best of all common constructions. This is shown by Professor Benjamin's experiments and is, moreover, shown by common experience in which the failure of such wheels is the rarest of accidents.

Construction of Fly-wheels

In the design of wheels cast in one piece, the uncertainty of the shrinkage strains makes calculations regarding the strength of the arms of more than doubtful value. The author's empirical formulas for the arms of such wheels (*Amer. Mach., April* 23, 1896) have been used in the design of wheels from 33 ins. to 8 ft. diameter, and have been compared with wheels up to 20 ft. diameter with very satisfactory results. The formulas contain a factor for the diameter and another for the cross-section of the rim together with the usual constant. The author prefers a rectangular section having its greatest dimension radial, as it best resists the beam action, but the formulas provide for other sections by considering all sections of the same area as equivalents and taking the side of a square equal in area to the section as the base of the factor for the section.

Referring to Fig. 3 for the notation, the formulas for the arm section at the outer end are:

$$x = \tfrac{7}{8}\text{ in.} + .04d + .153c$$
$$y = \tfrac{1}{2}x$$

all dimensions being in inches. The author's preference regarding the dimensions a and b is to make $b = \tfrac{3}{8}a$.

The taper of the arms each side the center line should be from $\tfrac{1}{4}$ to $\tfrac{3}{8}$ in. per ft. in the side view and $\tfrac{1}{8}$ to $\tfrac{3}{16}$ in. per ft. in the edge view, depending on the size of the hub. The arm section is preferably that made by two circular arcs rounded over at the edges, as shown in Fig. 2, such section having a much more pleasing appearance than the more usual ellipse. The arms are usually six in number, but the same formulas may be used for a greater number of arms.

For many cases in which a fly-wheel is desired but without definite requirements to permit calculations of the section for weight, satisfactory wheels will be obtained by making

$$c = 1\text{ in.} + .08d$$

A superior fly-wheel by the Mesta Machine Company is shown in Fig. 4 (*Amer. Mach., July* 20, 1911). This wheel, which is of 17 ft. diameter, was designed for a rim speed of 10,000 ft. per min. The material is air-furnace iron having a tensile strenght of 30,000 lbs. per sq. in. The wheel was divided as shown in order to reduce the

spongy center of large sections, and the rim section is deep to reduce the beam action. The arms were cast with the rims but free at the hub ends, the hub being a separate casting of steel. Long sweeping curves connect the arms with the rim and hub ends. Complete calculations were made for the stresses at various sections, the extreme values being, for the arm section 3000, and for the rim section 2410 lbs. per sq. in.

Another superior fly-wheel (patented by G. M. Hinkley) is shown in Fig. 5 (*Amer. Mach., May* 17, 1900). It is used by the Allis Chalmers Company in their band-saw mills in which the rim speeds are regularly 10,000 ft. per min.—a figure that has, in some instances, been run up to 12,000 ft. At the date of publication about 300 of these wheels had been made, none of which had failed. The aim of the construction is to enable the wheel to relieve itself of shrinkage strains in cooling. The arms are arranged diagonally and pass from one side of the wheel rim to the opposite end of the hub, alternate arms being staggered with one another. As first cast, the hub is in two pieces, the central portion marked *a* being vacant. After the wheel has become entirely cold, this space is filled by pouring in molten iron. As poured, the ends of the hub are separated by a core ⅜ in.

TABLE 1.—SUMMARY OF PROFESSOR BENJAMIN'S EXPERIMENTS ON THE STRENGTH OF FLY-WHEELS

	1 Solid wheel, 6 arms	2 Wheel in halves, flange joint, 6 arms	3 Wheel in halves, reinforced joint, 6 arms	4 Wheel in halves, link joint, 6 arms	5 Segmental wheel, link joint, 8 segments	6 Rim in halves, pad joint, 6 arms	7 Solid rim with separate spider, 6 arms	8 Solid rim with 24 tangent spokes
								
Rim speed at failure, feet per second	395	194	225	305	256	223	393	424
Rim speed at failure, feet per minute	23,700	11,640	13,500	18,300	15,360	13,380	23,580	25,440
Apparent rim tension at failure., lbs. per sq. in. by formula (*d*)	15,625	3,764	5,062	9,302	6,502	4,973	15,445	17,978
Comparative rim speeds at failure	100	49	57	77	65	56½	100	107
Comparative rim tensions at failure	100	24	32½	60	42	32	99	115
Efficiency of construction, *e* in formulas (*e*) and (*f*), assuming 19,000 lbs. per sq. in. tensile strength of cast-iron	.85	.19	.26½	49	34	.26	.84	.94

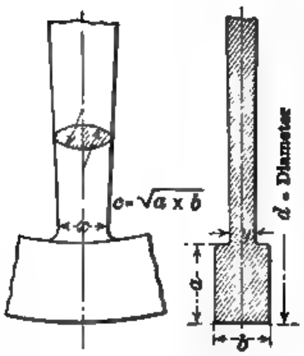


FIG. 3.—Arms of fly-wheels cast in one piece.

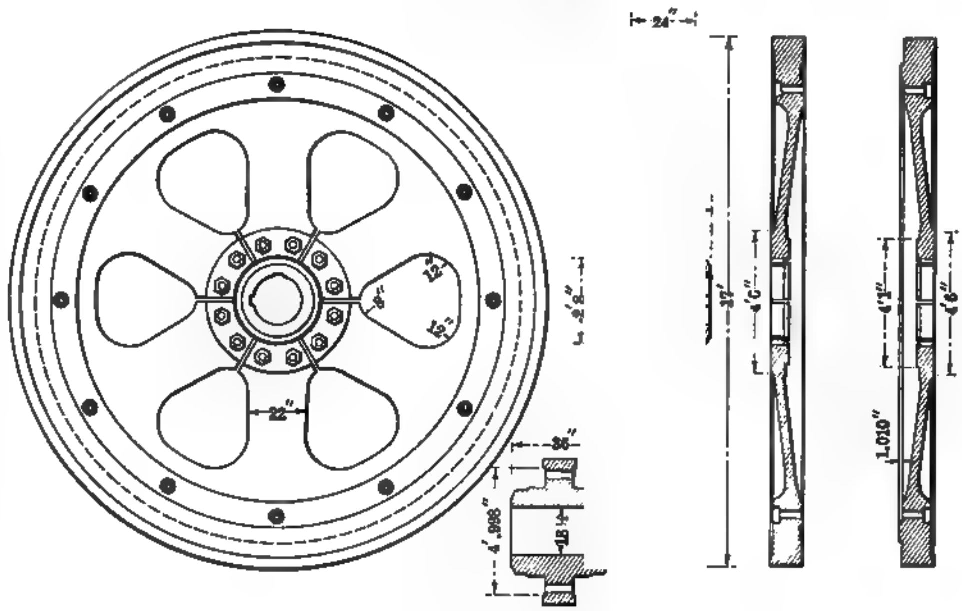


FIG. 4.—The Mesta fly-wheel.

Table 2.—Safe Speeds of Cast-iron Fly-wheels. Margin of Safety on Speed Approximately Three. Figures for Pad Joint do not Seem to be Justified by Professor Benjamin's Experiments

Type of wheels and maximum obtainable efficiency of rim-joint

Diam. in feet	No joint 1.00 Rev. per min.	Flange joint .25 Rev. per min.	Pad joint .50 Rev. per min.	Link joint .60 Rev. per min.
1	1910	955	1350	1480
2	955	478	675	740
3	637	318	450	493
4	478	239	338	370
5	382	191	270	296
6	318	159	225	247
7	273	136	193	212
8	239	119	169	185
9	212	106	150	164
10	191	96	135	148
11	174	87	123	135
12	159	80	113	124
13	147	73	104	114
14	136	68	96	106
15	128	64	90	99
16	120	60	84	92
17	112	56	79	87
18	106	53	75	82
19	100	50	71	78
20	95	48	68	74
21	91	46	65	70
22	87	44	62	67
23	84	42	59	64
24	80	40	56	62
25	76	38	54	59
26	74	37	52	57
27	71	35	50	55
28	68	34	48	53
29	66	33	47	51
30	64	32	45	49

If the revolutions given in the table be increased 20 per cent. the margin of safety on speed will be reduced to *two and one-half;* if the revolutions be increased 50 per cent. the margin of safety will be reduced to *two*.

thick, but the shrinkage of the rim compresses the arms and increases this space to about 1½ ins. After pouring the central portion, the ends are fastened together with bolts having the ends riveted over.

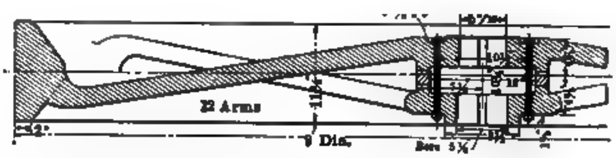
These wheels are made of 8, 9 and 10 ft. diameter, the 8-ft. wheels having weights ranging between 5000 and 8000 lbs., the 9-ft. wheels between 6000 and 10,000 lbs., and the 10-ft. wheels between 10,000 and 12,000 lbs.

Another superior high-speed wheel by E. S. Newton (*Amer. Mach., June* 21, 1900) used without failure in band saw mills at speeds of 10,000 ft. per min., is shown in Fig. 6. Wheels of 8 ft. diameter weigh about 6000 lbs. They have cast-iron rims and hubs with 16 wrought-iron, not steel, arms 1¾ ins. square. These arms are upset at each end and carefully tinned as far as they enter the cast-iron. They are also staggered. The rim is poured one day and the hub the next. The figure shows a wheel of 5 ft. diameter.

Unusually large high-speed wheels have been called for in the construction of electric power-houses. Fig. 7 shows such a wheel by the Allis Chalmers Company, located in one of the power-houses of New York City (*Amer. Mach., May* 24, 1900).

Except in its hub, the wheel is of steel throughout. The arms are hollow. The most striking feature lies in the reinforcing plates which are riveted to the sides of the rim casting. There are eight of these on each side, and the arrangement of the rivets will be seen to be such that the plates break joints with one another in such manner that there are fourteen effective plates in the weakest sections. The estimated weight of this wheel is 310,000 lbs.

While this wheel has the joints half way between the arms the number of arms is such as to greatly reduce the beam action.



Rim tension at 10,000 ft. per min., rim velocity by formula (*d*) 2777 lbs. per sq. in., no failures.

Fig. 5.—The Allis-Chalmers band saw mill fly-wheel.

Rim tension at 10,000 ft. per min., rim velocity by formula (*d*) 2777 lbs. per sq. in., no failures.

Fig. 6.—The Newton band saw mill fly-wheel.

A wheel for a rim speed of 15,000 *ft. per min.* is shown in Fig. 8 (*Amer. Mach., Jan.* 27, 1913). The wheel is in use at the mills of the Illinois Steel Company at South Chicago and was made by the Westinghouse Electric and Manufacturing Company. Its diameter is 13 ft. 2 ins., its weight 100,000 lbs., and its normal speed 375 r.p.m.

The assembled wheel shown is made with a cast-steel spider *A*, which has 12 arms made of a double 7½×4-ins. square section, the corners being well rounded with a 1½-in. radius. The arms have a liberal fillet at the hub and flange ends. The hub has a bearing of 26 ins. on the shaft, which has a double-stepped fit and driving through a feather key. The rim is machined with 12 notches *B* 2⅝ ins. deep, 2$\frac{11}{16}$ ins. at the outer periphery, having taper sides of 27 deg.

The laminated sheets *C*, which occupy a width of 1 ft. 9¼ ins., are made from .0281 in. bessemer (not annealed) sheet steel, 12 being used for a circumference. Each sheet is made with two dovetails fitting into the notches machined in the spider rim.

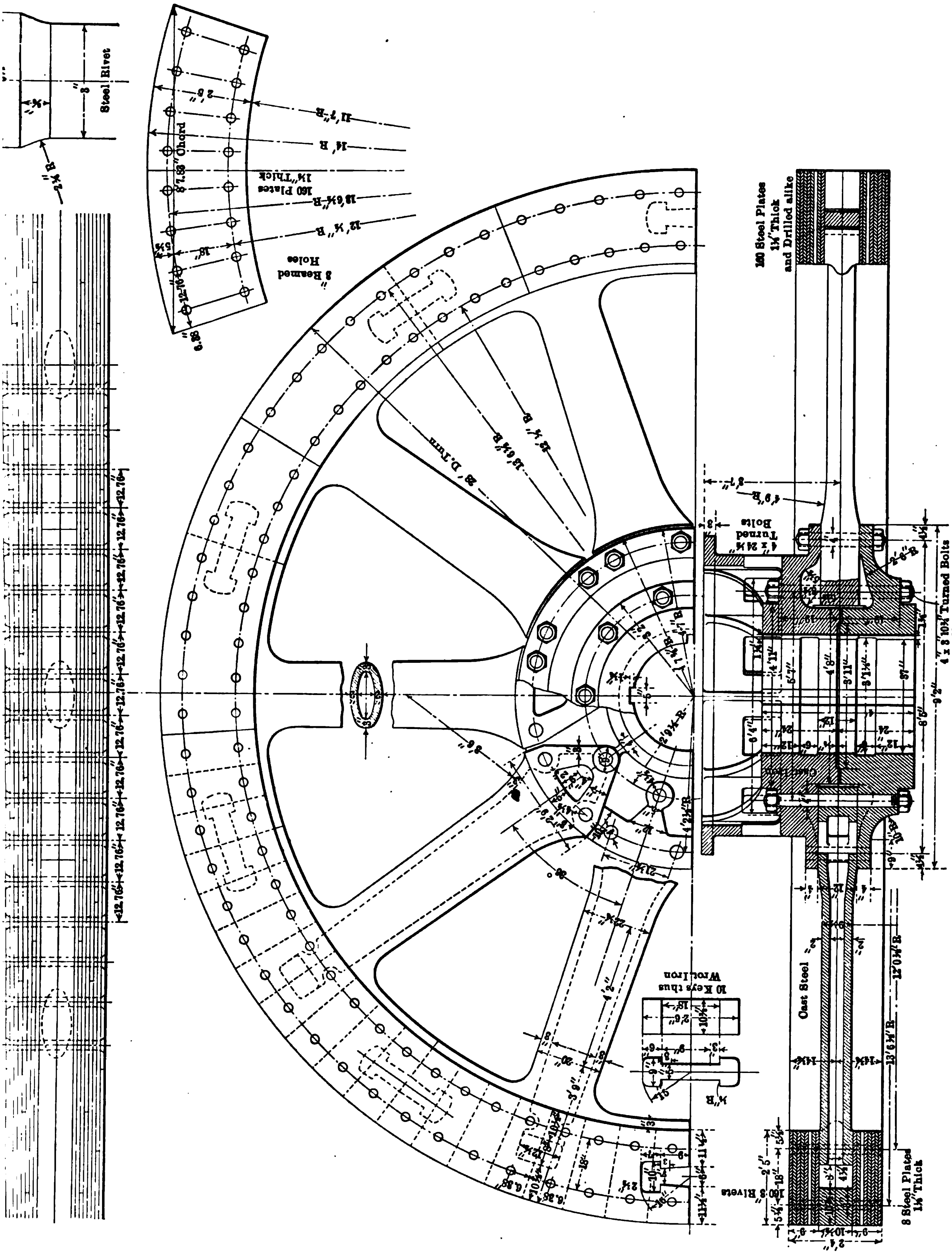


FIG. 7.—The Allis-Chalmers power house fly-wheel.

On each outer side of the laminated sheets is an end plate made of cast steel. Each end plate is accurately drilled, reamed and fitted with ten $1\frac{1}{4}$ cold-rolled steel bolts D, the ends of which are fitted with hexagon nuts which set into counterbores in the plate.

The laminated sheets are assembled with overlapping joints and when clamped together very little strain comes upon the bolts, as the thin sheet construction gives a very high slipping resistance with a comparatively light pressure. The bolts pass through the end plates and sheets.

In the center of the group of laminated sheets is inserted a punching of the same dimensions as the sheets, but fitted with six notches in each sheet, 3 ins. in width and $1\frac{1}{2}$ ins. deep. These notches are used for barring the engine, a special barring engine being attached to the equalizer set.

Fly-wheel joints having an efficiency of 100 *per cent.* or more have been made by JOHN FRITZ (*Trans. A. S. M. E.*, 1899) and by H. V.

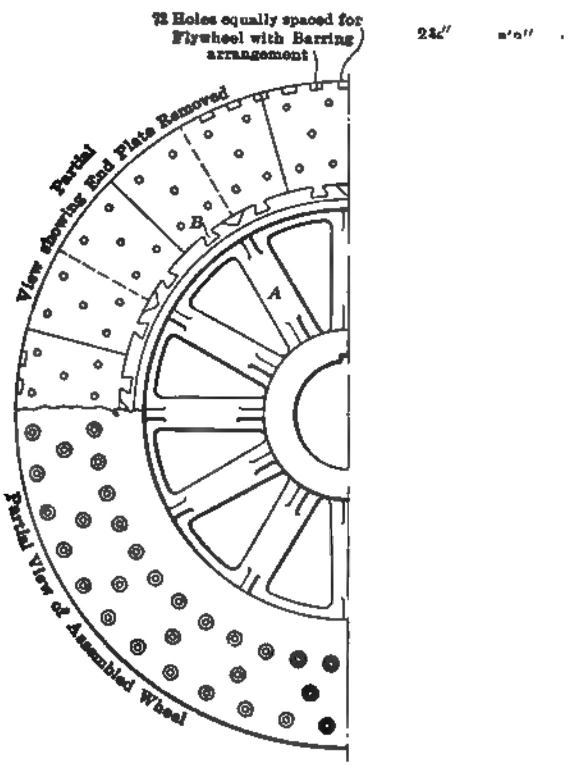


FIG. 8.—The Westinghouse fly-wheel for a rim speed of 15,000 ft. per min.

HAIGHT (*Amer. Mach., Feb.* 28, 1907). The two constructions are based on the same principle, Mr. Haight's wheel being shown in Fig. 9.

The reason why ordinary joints are weaker than the parts which are joined is that those parts are cut away to provide room for the joining pieces. Mr. Haight's plan is to cut away the rim section throughout its circumference, carrying the section which is imposed by the joining pieces all the way around the rim, the result being that the rim is not weakened by making provision for the joining pieces. There is, in fact, no difficulty in making the joining pieces stronger than the rim, and hence this joint may have an efficiency exceeding 100 per cent.

Moreover, the usual form of wheel-rim section involves a spongy center, which adds its due quota to the weight of the rim and to the strains to be carried by the rim section, while it adds very little to the strength of the section. Mr. Haight's construction involves a ribbed form of rim, by which this spongy center is largely eliminated, and hence the section of his castings should be stronger than that of the usual form; and, inasmuch as the link can then be made of the same strength as the casting, the conclusion would seem to be inevitable that the wheel as a whole should be stronger than a solid wheel having the usual section of rim.

Dividing the wheel with the joints at the arms neutralizes the beam action of the customary construction which Professor Benjamin's experiments have shown to be so injurious.

The removal of the metal which would ordinarily occupy the channels *aa* and its addition to the other parts of the section, where it acts to strengthen the section at the link as well as the remainder of the wheel, accomplishes the seemingly impossible.

The bolt through the arm was placed there to provide for machining the rim. It is not needed to reinforce the link.

In Mr. Fritz's design the same result is obtained by a cored section, Fig. 10, the action of the core being the same as that of the channels

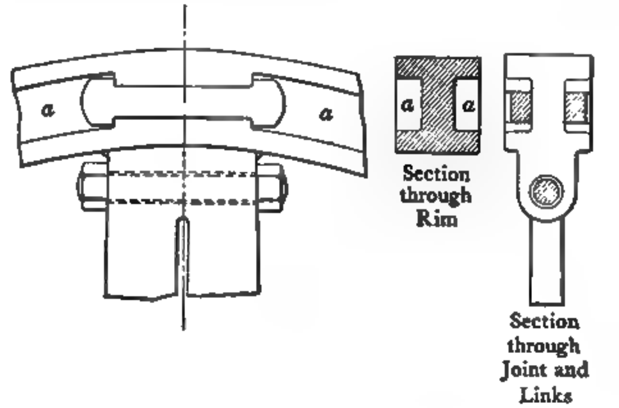


FIG. 9.—The Haight 100 per cent. efficiency fly-wheel joint.

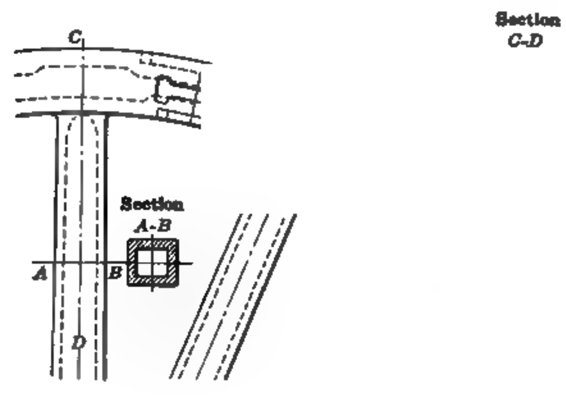


FIG. 10.—The Fritz 100 per cent. efficiency fly-wheel joint.

aa, Fig. 9, of Mr. Haight's construction. The joints are midway between the arms but the great number of arms (16) reduces the beam action to a probably negligible amount. The arms are hollow and join the rim segments by curves which avoid abrupt change of section. Four I links of unequal length are used at each joint, the object of the inequality being to distribute the stresses due to the links. Many of these wheels of 20 to 30 ft. diameter have been applied to the most severe rolling-mill duty and they have never failed.

The design of band fly-wheels is, as a rule, worse than that of plain fly-wheels. The thinness of the rim increases the stress due to the beam action and, with joints midway between the arms, such wheels are unsafe.

In wheels of a size suitable for casting in halves, which includes the great majority, double arms should be placed at the joint, as in Fig. 11 by J. B. STANWOOD (*Amer. Mach., Apr.* 4, 1907). This is a marked improvement over the midway joint, but it may be still further

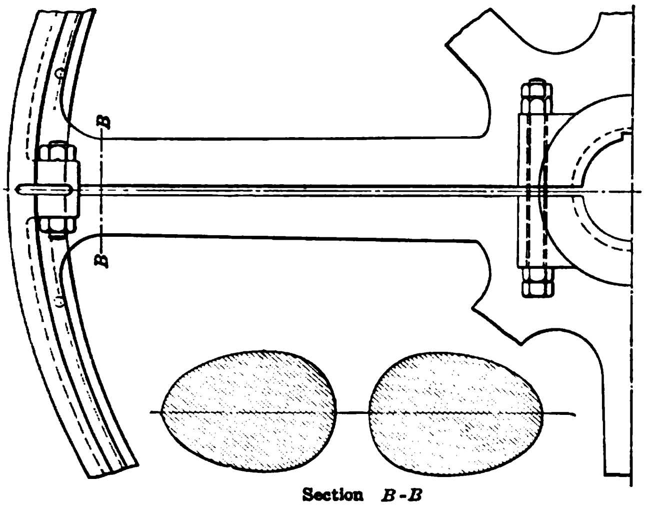
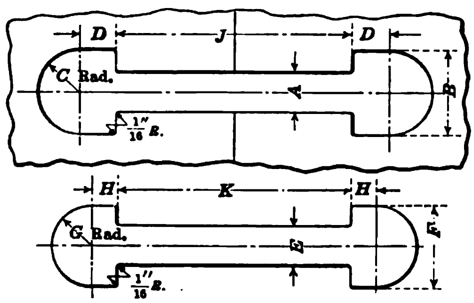


FIG. 11.—The Stanwood split band fly-wheel.

improved by adapting the Haight principle, as suggested by PROFESSOR BENJAMIN (*Amer. Mach., April* 11, 1907) and shown in Fig. 12.

The ribs should be deep, both to resist the beam action and to bring the links more nearly to the neutral axis of the section. For ordinary cases the arms may be proportioned in accordance with the author's formulas for wheels in one piece.

In segmental wheels the joints should be placed at the arms. Such a wheel of 22 ft. diameter, 96 ins. face and for three belts, by the Providence Steam Engine Co. (*Amer. Mach., Nov.* 11, 1895), is shown in Fig. 13. The space required for the pad for the arm joint makes the application of the Haight principle more difficult in these wheels, but the more nearly it is adhered to the better the wheel will be.



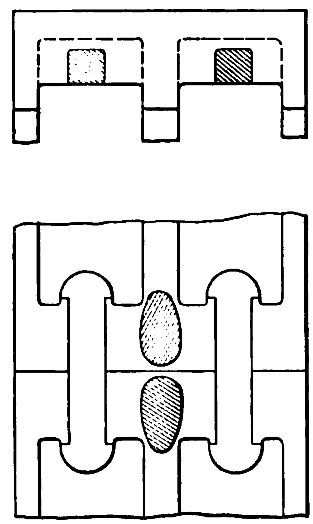


FIG. 12.—The Haight principle applied to split band fly-wheels.

TABLE 3.—DIMENSIONS OF SHRINK LINKS AND SHRINKAGE ALLOWANCE. PRACTICE OF THE GENERAL ELECTRIC COMPANY

Dimensions of standard keyways

A	J	A	J
3/4	4 1/2	1 1/2	9, 10 1/2, 12
1	5, 5 1/2, 6		
1 1/4	6, 7 1/2, 9		

Dimensions of keyway				Dimensions of key				Dimensions of keyway				Dimensions of key				Length of keyway and key							
A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*J*	*K*	*J*	*K*	*J*	*K*	*J*	*K*
3/4	1 1/2	3/4	5/8	23/32	1 3/8	11/16	7/16	4 3/4	8 3/4	4 3/8	3 11/16	4 11/16	8 5/8	4 5/16	3 1/2	4	3.995	12	11.988	20	19.980	28	27.972
1	2	1	7/8	31/32	1 7/8	15/16	11/16	5	9 1/8	4 9/16	3 13/16	4 15/16	9	4 1/2	3 5/8	4 1/2	4.495	12 1/2	12.487	20 1/2	20.479	28 1/2	28.471
1 1/4	2 3/8	1 3/16	1	1 7/32	2 1/4	1 1/8	13/16	5 1/4	9 5/8	4 13/16	4	5 3/16	9 1/2	4 3/4	3 13/16	5	4.995	13	12.987	21	20.979	29	28.971
1 1/2	2 7/8	1 7/16	1 1/4	1 15/32	2 3/4	1 3/8	1 1/16	5 1/2	10	5	4 3/16	5 7/16	9 7/8	4 15/16	4	5 1/2	5.494	13 1/2	13.486	21 1/2	21.478	29 1/2	29.470
1 3/4	3 1/4	1 5/8	1 3/8	1 23/32	3 1/8	1 9/16	1 3/16	5 3/4	10 1/2	5 1/4	4 3/8	5 11/16	10 3/8	5 3/16	4 3/16	6	5.994	14	13.986	22	21.978	30	29.970
2	3 3/4	1 7/8	1 9/16	1 31/32	3 5/8	1 13/16	1 3/8	6	11	5 1/2	4 5/8	5 15/16	10 7/8	5 7/16	4 7/16	6 1/2	6.493	14 1/2	14.485	22 1/2	22.477	30 1/2	30.469
2 1/4	4 1/4	2 1/8	1 13/16	2 3/16	4 1/8	2 1/16	1 5/8	6 1/4	11 3/8	5 11/16	4 3/4	6 3/16	11 1/4	5 5/8	4 9/16	7	6.993	15	14.985	23	22.977	31	30.969
2 1/2	4 5/8	2 5/16	1 15/16	2 7/16	4 1/2	2 1/4	1 3/4	6 1/2	11 7/8	5 15/16	5	6 7/16	11 3/4	5 7/8	4 13/16	7 1/2	7.492	15 1/2	15.484	23 1/2	23.476	31 1/2	31.468
2 3/4	5	2 1/2	2 1/8	2 11/16	4 7/8	2 7/16	1 15/16	6 3/4	12 1/4	6 1/8	5 1/8	6 11/16	12 1/8	6 1/16	4 15/16	8	7.992	16	15.984	24	23.976	32	31.968
3	5 1/2	2 3/4	2 5/16	2 15/16	5 3/8	2 11/16	2 1/8	7	12 3/4	6 3/8	5 5/16	6 15/16	12 5/8	6 5/16	5 1/8	8 1/2	8.491	16 1/2	16.483	24 1/2	24.475	32 1/2	32.467
3 1/4	6	3	2 1/2	3 3/16	5 7/8	2 15/16	2 5/16	7 1/4	13 1/4	6 5/8	5 9/16	7 3/16	13 1/8	6 9/16	5 3/8	9	8.991	17	16.983	25	24.975	33	32.967
3 1/2	6 1/2	3 1/4	2 3/4	3 7/16	6 3/8	3 3/16	2 9/16	7 1/2	13 5/8	6 13/16	5 11/16	7 7/16	13 1/2	6 3/4	5 1/2	9 1/2	9.490	17 1/2	17.482	25 1/2	25.474	33 1/2	33.466
3 3/4	6 7/8	3 7/16	2 7/8	3 11/16	6 3/4	3 3/8	2 11/16	7 3/4	14 1/8	7 1/16	5 15/16	7 11/16	14	7	5 3/4	10	9.990	18	17.982	26	25.974	34	33.966
4	7 3/8	3 11/16	3 1/8	3 15/16	7 1/4	3 5/8	2 15/16	8	14 5/8	7 5/16	6 1/8	7 15/16	14 1/2	7 1/4	5 15/16	10 1/2	10.489	18 1/2	18.481	26 1/2	26.473	34 1/2	34.465
4 1/4	7 3/4	3 7/8	3 1/4	4 3/16	7 5/8	3 13/16	3 1/16	....	....	....	....	....	....	....	....	11	10.989	19	18.981	27	26.973	35	34.965
4 1/2	8 1/4	4 1/8	3 1/2	4 7/16	8 1/8	4 1/16	3 5/16	....	....	....	....	....	....	....	....	11 1/2	11.488	19 1/2	19.480	27 1/2	27.472	35 1/2	35.464

The absence of shrinkage strains in segmental wheels makes feasible the application of the usual formulas for the strength of beams to their design. In large wheels the arms should be of I-beam or, better still, of oval hollow section. The total load should be taken as that due to the full pressure of the steam acting when the crank is at a right angle with the center line and a factor of safety of not less than 10 should be included.

For the design of hollow fly-wheel arms from their solid equivalents see Arms of Spur Gears.

The Regulating Power of Fly-wheels

Not much actual use is made of analytical methods in the determination of the weight of steam-engine fly-wheels—resort being usually made to comparison with existing wheels. As will be seen below, the formula for the weight of wheels designed for a given fineness of regulation includes two coefficients—one for the steam distribution and piston speed and the other for the degree of regulation desired. The determination of the former for any given engine is so laborious that it is seldom made. Because of this the two coefficients have been frequently combined into one. The resulting formulas, while rational in form and sufficiently correct for the types of engine and the services from which they have been derived, are of limited application and, worse yet, their limits are unknown.

The determination of the coefficient for the steam distribution and piston speed by analytical methods compels the resort to simplifying assumptions which vitiate, if they do not destroy, the value of the conclusions. The determination has, however, been made graphically with all necessary accuracy and for a wide range of conditions by Karl Mayer (*Zeitschrift des Vereines Deutscher Ingenieure*, 1893) and translated by Emil Theiss (*Amer. Mach., Sept.* 7 and 14, 1893). Herr Mayer, with infinite care and patience, constructed a series of rotative effort diagrams from which a series of values of this coefficient was determined.

In the operation of a fly-wheel under a varying impulse and a constant resistance, the velocity fluctuates between two limits which are expressed by the equation:

$$\frac{\text{mean velocity}}{\text{greatest velocity}-\text{least velocity}}=\text{a quantity called the } \textit{coefficient of steadiness},$$

which is the reciprocal of the coefficient of fluctuation used by some writers.

The value of the coefficient of steadiness having been selected to suit the character of the load, the weight of the wheel is then determined to suit. Since an early cut-off and low piston speed will deliver more irregular impulses than a late cut-off and high piston speed, it is obvious that these factors also affect the weight of the wheel.

The formula for the weight of the wheel is as follows:

$$W=id\frac{i.h.p.}{v^2\times r.p.m.}$$

in which W = weight of wheel rim, lbs.,
i = coefficient for steam distribution and piston speed,
d = coefficient of steadiness,
$i.h.p.$ = indicated horse-power,
v = mean velocity of wheel rim, ft. per sec.,
$r.p.m.$ = revolutions per minute.

Herr Mayer's determinations of the value of i are given in Table 4. Two assumptions run through the table: The length of the connecting rod is uniformly taken as five times the crank and the weight of the reciprocating parts is taken at an average value as given by a formula. The captions p, $.7p$ and o refer to the compression, which, in column p, is to the initial pressure; in column $.7p$, to seven-tenths of that pressure, while, in column o, there is no compression. Herr Mayer's values of the permissible coefficient of steadiness, d, together with additional values, from Unwin's Elements of Machine Design, are given in Table 5.

With the values of i determined, it is a comparatively simple matter for any engine builder to determine the values of d for his own wheels and thereafter to design others in a strictly rational manner.

In doing this, and, indeed, in any application of this method, it should be noted that the value of i increases as the $i.h.p.$ decreases—that is, as the cut-off is shortened. Values of the $i.h.p.$ for the points of cut-off included in Table 3 should therefore be determined and the calculation of the weight of the wheel be made for the maximum value of the product of i and $i.h.p.$ in order that the regulation may be satisfactory under the worst condition.

While useful for purposes of comparison, the sections of Table 4 for two- and three-cylinder engines have, probably, little real application. Wheels dimensioned in accordance with them would, no doubt, be so light as to be structurally too weak for use

In all that has been said, the weight of the arms and hub has been ignored. Their weight is so considerable while their effect is so small that, when applying the formula to existing wheels, their weight should be subtracted from the gross weight of the wheel. Calculations of many large wheels have shown that the weight of arms and hub combined make up about 35 per cent. of the weight of the entire wheel. Their fly-wheel effect, on the other hand, adds but from 7½ to 10 per cent. to the value of the rim.

Fly-wheels for Intermittent Work

The design of fly-wheels for intermittent work, such as punching, shearing, etc., is based upon an entirely different procedure. The loss of energy being equal to the work done, the weight and velocity of the wheel must be such that the loss of energy does not involve an undue reduction of speed. The fundamental formulas are:

$$E=\frac{W}{2g}\left(v_1^2-v_2^2\right) \tag{a}$$

$$W=\frac{2gE}{v_1^2-v_2^2} \tag{b}$$

in which E = loss of energy of wheel = work to be done, ft.-lbs.,
W = weight of wheel, lbs.,
v_1 = normal or full velocity, ft. per sec.,
v_2 = reduced velocity after work is done, ft. per sec.,
g = acceleration of gravity = 32.2

In equation (*b*) the reduced velocity may be expressed as a fraction of the normal velocity, that is, $v_2=av_1$, giving

$$W=\frac{2gE}{v_1^2-a^2v_1^2}$$

$$=\frac{2gE}{(1-a^2)\sqrt{v_1^2}} \tag{c}$$

For belt-driven machines the limiting low velocity is that at which the belt runs off the pulley. According to Wilfred Lewis (*Trans. A. S. M. E., Vol.* 7) the experiments of Wm. Sellers & Co. showed that this would take place when the slip exceeded 20 per cent. of the belt speed, that is, a in equation (*c*) should not be less than .8. Introducing this value and the numerical value of g gives for the limiting condition for belt driving

$$W=180\frac{E}{v_1^2}.$$

Since in most cases the reduction of speed is momentary only, while in the experiments it was continued for some time, the limiting condition or one not far from it would seem to be admissible when other conditions do not prevent its use.

Strict accuracy in calculations involving the energy of fly-wheels requires that the weight used shall be the weight of the entire wheel and that the velocity be that at the center of gyration. The calculation of the radius of gyration of such bodies as fly-wheels is laborious and is seldom made. The usual method is to make the calculations for the weight of the rim only and for the velocity at the center of gravity of the rim section.

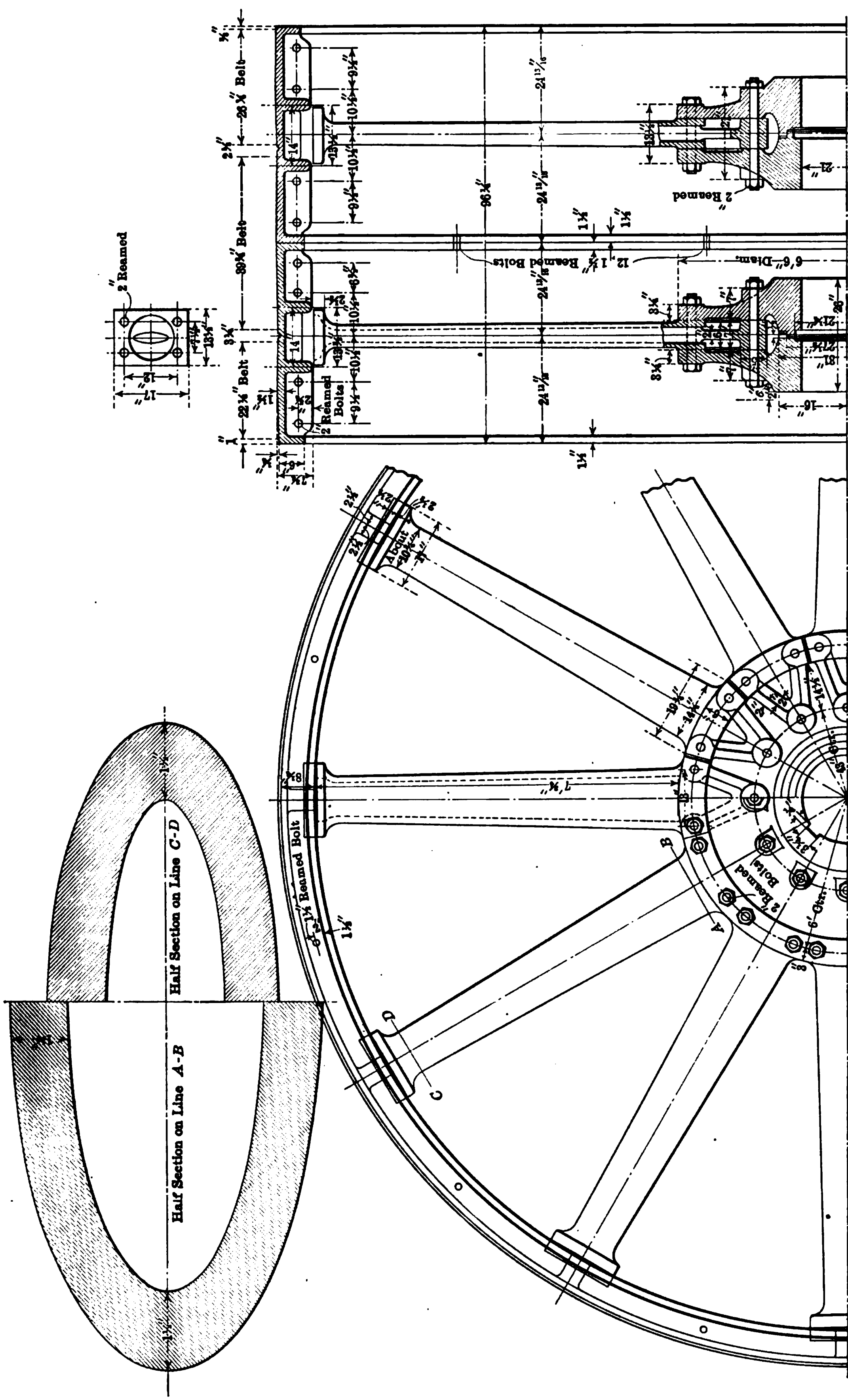


FIG. 13.—The Providence Steam Engine Co.'s band fly-wheel.

TABLE 4.—VALUES OF THE COEFFICIENT i FOR STEAM DISTRIBUTION AND PISTON SPEED FOR SUBSTITUTION IN FORMULA

Single Cylinder Non-condensing Engines

Piston speed	Cut-off [illegible]			[illegible]			[illegible]			[illegible]		
	Compression to											
	p	$.7p$	o	p	$.7p$	o	p	$.7p$	o	p	$.7p$	o
200	272,690	241,530	218,580	242,010	220,280	209,170	220,760	207,230	201,920	193,340	187,670	182,840
400	240,810	209,890	187,430	208,200	188,880	179,460	188,510	176,080	170,040	174,630	167,860	167,860
600	194,670	165,450	145,400	168,590	151,440	136,460	165,210	150,710	146,610	....	....	....
800	158,200	132,020	108,690	162,070	148,540	135,260	....	....	....	....	....	....

Two Cylinder Engines with Cranks at 90 Deg.

Piston speed	Cut-off [illegible]			[illegible]			[illegible]			[illegible]		
	Compression to											
	p		o	p		o	p		o	p		o
200	71,980	mean 70,560	60,140	59,420	mean 58,510	54,340	49,272	mean 49,210	50,000	37,920	mean 36,710	36,950
400	70,160			57,000			49,150			35,500		
600	70,040			57,480			49,220					
	70,040			60,140								

Single Cylinder Condensing Engines

Piston speed	Cut-off 1/10			[illegible]			[illegible]			[illegible]		
	Compression to											
	p	$.7p$	o	p	$.7p$	o	p	$.7p$	o	p	$.7p$	o
200	292,730	241,770	180,180	265,560	226,310	176,560	234,160	206,030	173,660	217,980	195,400	171,000
400	212,910	171,970	117,380	194,550	163,030	117,870	174,380	151,680	118,350	166,290	146,610	121,730
600	141,900	127,530	124,630	148,780	143,710	140,090	....	....	....	....	....	....

Single Cylinder Condensing Engines

Piston speed	[illegible]			[illegible]			[illegible]		
	Compression to								
	p	$.7p$	o	p	$.7p$	o	p	$.7p$	o
200	204,210	185,250	167,140	189,600	173,900	161,830	172,690	165,930	156,990
400	164,720	148,780	133,080	174,630	164,970	151,680	....	....	....

Three Cylinder Engines With Cranks at 120 Deg.

Piston speed	Cut-off [illegible]		[illegible]		[illegible]		[illegible]	
	Compression to							
	p	o	p	o	p	o	p	o
200	33,810	32,240	33,810	35,500	34,540	33,450	35,260	32,370
800	30,190	31,570	35,140	33,810	36,470	32,850	33,810	32,370

TABLE 5.—VALUES OF THE COEFFICIENT OF STEADINESS

	Values of d
For engines operating	
Hammering and crushing machinery	5
Pumping and shearing machinery	20 to 30
Weaving and paper-making machinery	40
Flour milling machinery	50
Spinning machinery	50 to 100
Ordinary driving engines with belt transmission	35
Gear-wheel transmission	50
Unwin's Elements of Machine design gives	
For engines operating	
Machine tools	35
Textile machinery	40
Spinning machinery	50 to 100
Electric machinery	150
Electric machinery direct driven	300

Determinations of fly-wheel effects using the entire weight of the wheel and the velocity of the center of gyration, have been much simplified by O. S. BEYER (*Amer. Mach., Oct.* 17, 24, 1912). The simplification grows out of the fact that examination of a large number of fly-wheels for use on punching and similar presses has shown the quite constant relation that the weight of the rim is equal to 68.6 and of the arms and hub 31.4 per cent. of the weight of the entire wheel. Using these percentages Mr. Beyer has calculated Table 6 for a variety of rim sections which embraces almost everything occurring in practice. With the aid of Figs. 14–17, this table will answer any question relating to the functions of fly-wheels used for intermittent work.

The use of this table is as follows: To find the velocity in ft. per sec. of the center of gyration of a fly-wheel, select from the rim sections shown in Table 6 and marked *a, b, c, d, e, f* and *g*, the one nearest, as to ratio of width to thickness, to that of the wheel, then locate in the first column the outer diameter of the wheel and trace over to the column headed by the same letter that identifies the selected rim section.

The number found in this column gives the velocity in ft. per sec. of the center of gyration of the wheel when running at the rate of 1 r.p.m. Multiplying this number by the r.p.m. the wheel actually makes, gives the required velocity of the center of gyration.

In the same manner the velocity in ft. per sec. of the outer circumference is found by tracing over from the outer diameter of the wheel in the first column, to the last column. The number there found is the velocity of the outer circumference of the wheel at 1 r. p. m., and multiplied by the actual r.p.m. of the wheel, gives the required velocity of the outer circumference.

For sizes between those given in the table interpolation is necessary.

The velocity at the center of gyration having been obtained from

Table 6, the energy for a wheel of given weight may be obtained from Figs. 14 and 15 (also by Mr. Beyer) which are identical, except that Fig. 14, for low velocities, is on a larger scale.

To find the energy of a body of a given weight and moving with a given velocity, find on the scale of velocity the point that corresponds to the velocity in ft. per sec. at which the body is moving, and trace upward to the curve; then, from the point thus located on the curve, trace over to the scale of energy; the number identified on that scale, if multiplied with the weight of the body in lbs., will produce the required energy in ft.-lbs.

The charts may be used with equal facility in the reverse direction to find the velocity at which a fly-wheel of known weight and diameter must be run in order to contain a given amount of energy. To do this divide the given energy for which the velocity is required by the weight of the body; the quotient being the amount of energy contained in each lb. of the body's weight. Locate this quotient on the scale of energy, in Fig. 14 or Fig. 15, trace over to the energy curve and down to the scale of velocity, where the required velocity in ft. per sec. may be read off. Then turn to the velocity table and find the velocity number corresponding to the outer diameter and type of rim section of the wheel, and divide by this number the velocity in ft. per sec. just found. The quotient is the r.p.m. the wheel must make in order to contain the given amount of energy.

A leading question in connection with the design of fly-wheels for intermittent work relates to the permissible reduction of velocity at each operation. The extreme value for belt-driven fly-wheels is 20 per cent., at which the belt is liable to run off the pulley. This represents an abstraction of 36 per cent. of the energy. According to Mr. Beyer, for press work, the extent to which the diminution of the velocity of a fly-wheel is practicable depends upon the frequency, as compared with the velocity, with which the fly-wheel is drawn upon for energy. Thus: If an ordinary fly-wheel press, running at 90 r.p.m., is tripped at regular intervals, say 15 times per min., the velocity may each time be diminished to the extent of 10 per cent. or even more. But if the press is run continuously, no greater diminution than from 5 to 6 per cent. should be reckoned with.

In the case of a heavily-geared drawing press, having an engine directly connected, or running under conditions otherwise favorable to readily restoring the velocity to normal, the fly-wheel may be brought almost to a standstill.

Obviously, other factors enter the problem, but they are as manifold as the kinds of work that may be done in the same press.

Problems relating to the reduction of velocity and energy of fly-wheels may be solved by the use of Figs. 16 and 17, also by Mr. Beyer, which are almost self-explanatory. Thus, in Fig. 16, locate the permissible reduction of velocity on the vertical scale, trace horizontally to the curve and then down to the horizontal scale, where read the fractional part of the energy given out with the given reduce tion of velocity. Obviously the chart may be used in the reverse direction with equal facility.

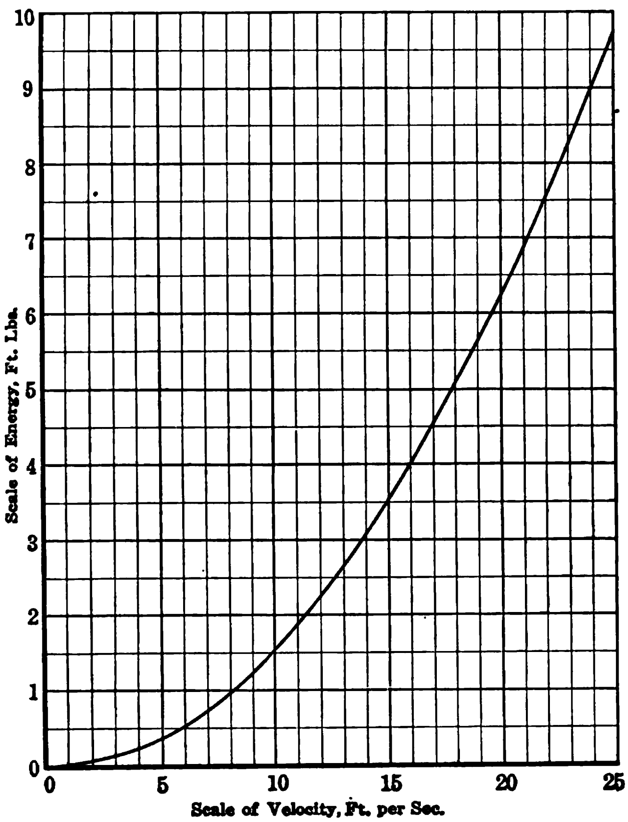


FIG. 14.

E = energy, ft. lbs.
W = weight of body, lbs.
v = velocity, ft. per sec.
$g = 32.2$

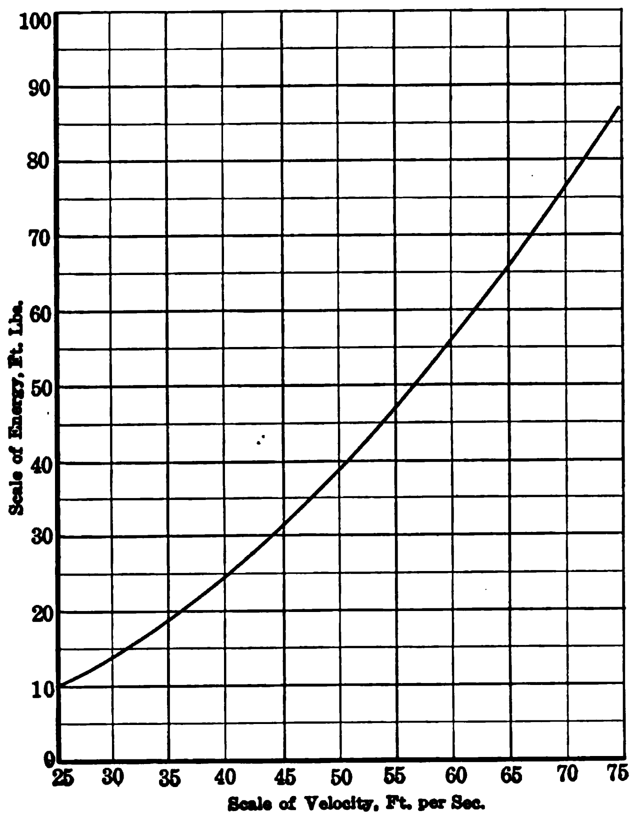


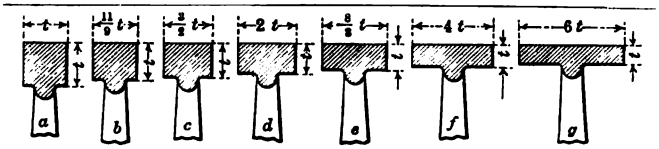
FIG. 15.

$$E = \frac{Wv^2}{2g}$$

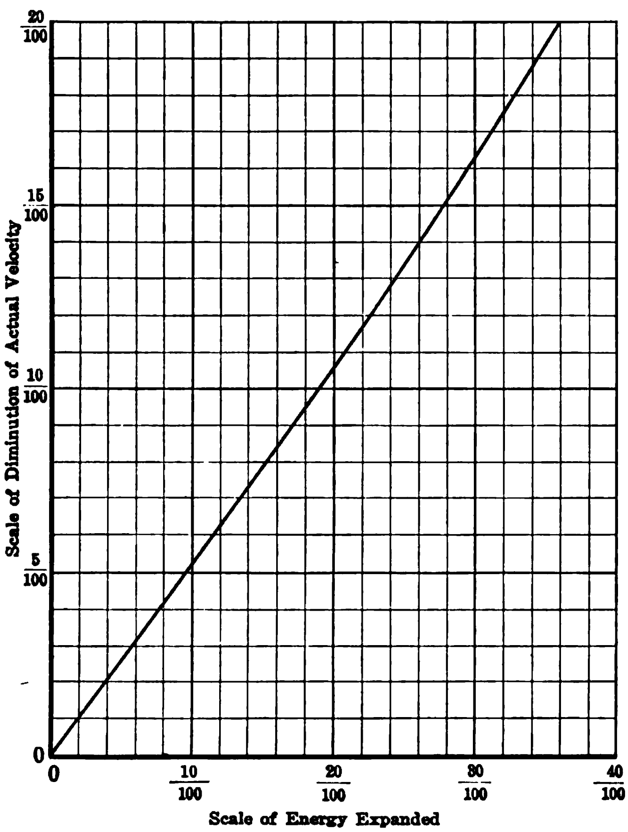
$$v = \frac{2gE}{W}$$

FIGS. 14 and 15. Energy of 1 lb. at various velocities.

TABLE 6.—VELOCITIES IN FEET PER SECOND OF CENTER OF GYRATION AND OF OUTER CIRCUMFERENCE FOR DIFFERENT CROSS-SECTIONS OF RIM, AND DIFFERENT DIAMETERS OF FLY-WHEELS RUNNING AT 1 R.P.M.



Outer diameter of wheel ins. — Cross-section of rim	Velocity of center of gyration, in ft. per sec. for sections: a	b	c	d	e	f	g	Velocity of outer circumference, ft. per sec.
12	.0406	.0411	.0417	.0423	.0428	.0434	.0440	.0524
18	.0609	.0617	.0625	.0634	.0642	.0651	.0660	.0785
24	.0812	.0822	.0834	.0845	.0857	.0868	.0880	.1047
30	.1014	.1028	.1042	.1056	.1071	.1085	.1100	.1309
36	.1217	.1233	.1250	.1268	.1285	.1302	.1319	.1571
42	.1420	.1439	.1459	.1479	.1499	.1519	.1539	.1833
48	.1623	.1644	.1667	.1690	.1713	.1736	.1759	.2094
54	.1826	.1850	.1876	.1901	.1927	.1953	.1979	.2356
60	.2029	.2055	.2084	.2113	.2141	.2170	.2199	.2618
66	.2232	.2261	.2292	.2334	.2356	.2387	.2419	.2880
72	.2435	.2466	.2501	.2535	.2570	.2604	.2639	.3142
78	.2638	.2672	.2709	.2747	.2784	.2821	.2859	.3403
84	.2841	.2877	.2917	.2958	.2998	.3038	.3079	.3665
90	.3043	.3083	.3126	.3169	.3212	.3255	.3299	.3927
96	.3246	.3288	.3334	.3380	.3426	.3473	.3519	.4189
102	.3449	.3494	.3543	.3592	.3641	.3690	.3739	.4451
108	.3652	.3699	.3751	.3803	.3855	.3907	.3958	.4712
114	.3855	.3905	.3959	.4014	.4069	.4124	.4178	.4974
120	.4058	.4110	.4168	.4225	.4283	.4341	.4398	.5236



v_1 = normal velocity, ft. per sec. E_1 = normal energy, ft. lbs.
v_2 = loss of velocity, ft. per sec. E_2 = loss of energy, ft. lbs.

$$\frac{E_1}{E_2}=\frac{1}{1-\left(1-\frac{v_2}{v_1}\right)^2}$$

FIG. 16.—Relation of energy expended and loss of velocity.

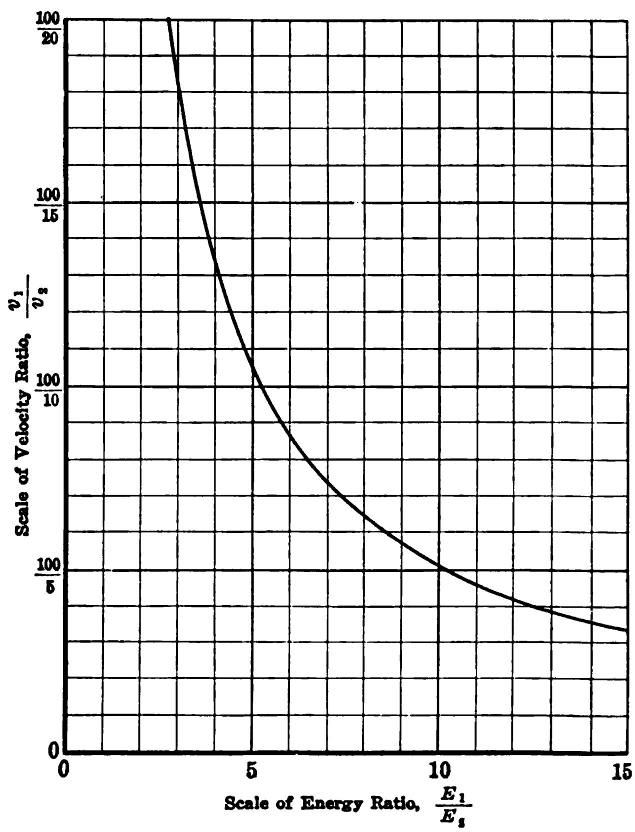
Similarly Fig. 17 gives the relation between loss of energy and of velocity. If, for example, a fly-wheel is to furnish 200 ft.-lbs. of energy during each cycle, but the working conditions of the press are such as to require the diminution of the velocity to be kept within the limit of 7.5 per cent. we turn to Fig. 17.

Locating on the scale of velocity ratio $\frac{v_1}{v_2}$ the one given; namely, $\frac{100}{7.5}$, and tracing over to the curve, and down to the scale of energy ratio $\frac{E_1}{E_2}$, the number 6.957 will be found, and is the ratio of the total energy the wheel must have to the energy to be expended at a diminution of the velocity not exceeding 7.5 per cent.

In other words, the energy to be expended is to be multiplied with that number, to produce the total energy, or $200\times6.957=1391.4$ ft.-lbs. From this total energy, the diameter being generally derived from surrounding conditions, the weight, velocity in ft. per sec. and the r.p.m. are readily settled with the aid of Figs. 14 and 15, and the velocity table.

Another problem occurs when the amount of the expended energy is limited to a certain ratio to the total energy and this also may be solved by the aid of Fig. 17.

Supposing, the total energy a fly-wheel requires to be 5 times the energy it may expend, the resulting velocity ratio is then found by locating the ratio 5 on the scale of energy ratio $\frac{E_1}{E_2}$, and tracing up to the curve, and over to the scale of velocity ratio $\frac{v_1}{v_2}$, where the number $\frac{100}{10.56}$ will be found, which indicates that, if the total energy of the wheel to the energy to be expended is to be as 5 is to 1, then the corresponding velocities must be as 100 is to 10.56.



$$\frac{v_1}{v_2}=\frac{1}{1-\sqrt{1-\frac{E_2}{E_1}}}$$

FIG. 17.—Relation of total and expanded energy to loss of velocity.

CONE PULLEYS AND BACK GEARS

Graphical Solution

The geometrical progression of speeds for driving machines by the cone pulley and back gear, or other means, is now generally accepted as correct. By this is meant that each speed should equal the one next below it, multiplied by a constant ratio. According to CARL G. BARTH (*Amer. Mach., Jan.* 11, 1912), the ideal value for this constant ratio in machine-tool practice is the fourth root of two or 1.189. The smallest ratio which Mr. Barth has found in the best speeded lathes of to-day is somewhat greater than this, being about 1.25. The ratios found in machine tools having the usual pattern of wide range cone pulley, range between 1.5 and 1.75, while ratios as high as 2 are occasionally found. Such ratios are too large to permit the selection of economical speeds for the work.

Find the resulting ratio in the base line as at *a*. Trace upward to the curve for the desired number of speeds as at *b*. Trace to the left and read the required ratio of successive speeds as at *c*.

To find the desired speeds construct a diagram as in Fig. 2, by PROFESSOR SWEET (*Amer. Mach., Oct.* 13, 1898). Lay off *ab* to any scale and call it unity. Lay off *ac* to the same scale and equal to the ratio found in Fig. 1. The most convenient method of doing this is to take *de* and *fc*, Fig. 1, for *ab* and *bc*, Fig. 2, respectively. Draw the verticals through *b* and *c*, Fig. 2, and lay off *bd* to any scale to represent the lowest number of revolutions. Draw *ad* and extend it to *e*, through which draw the horizontal *ef*, when *bf* will represent the second speed to the same scale that *bd* represents the first speed. Proceed in this way as indicated in the diagram, finding points *g*, *h*, *i*, etc., for the various speeds. Should the diagram extend

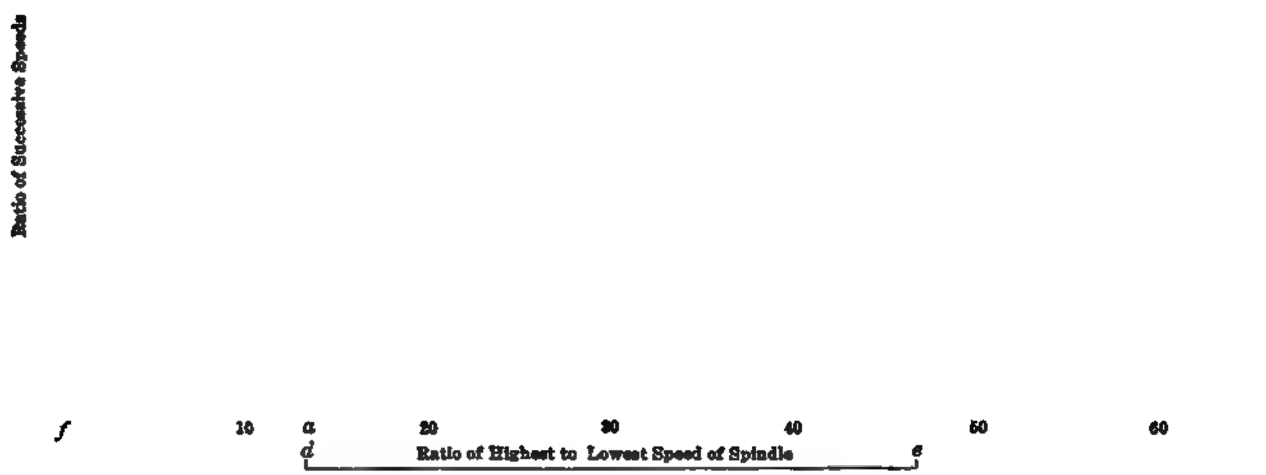


FIG. 1.—The geometrical speed ratio of cone pulleys.

For feeds of machine tools the geometrical progression is also in general, though not universal, use. About the only type of machine for which this arrangement of feeds is still a matter of controversy is the drilling machine.

The data given or assumed are usually the highest and lowest revolutions per minute of the machine spindle, the number of speeds and the diameter of the largest pulley, this last being determined by the available room. The assumed number of speeds should be regarded as a trial number only and subject to correction, should the ratio, due to that number, be found too high or too low.

There are three steps in the process: (1) finding the ratio; (2) finding the speeds; (3) finding the diameters.

To find the constant ratios consult the chart, Fig. 1, by PROF. H. F. MOORE (*Amer. Mach., May* 21, 1912) and proceed as follows: Divide the largest by the smallest r.p.m. of the machine spindle.

beyond the limits of the paper, lay down the last value found on the paper to a reduced scale, and proceed as before.

When finding the diameters of the steps three cases exist.

Case I. Crossed belts.

Case II. Open belts with pulleys at sufficient distance apart to make it unnecessary to compensate the tendency of the changing belt angle to alter the length of the belt. Machine tools driven from overhead countershafts are illustrations of this case.

Case III. Open belts with pulleys so near together that the tendency of the changing belt angle to alter the length of the belt must be compensated. Foot-lathe drives and many speed cones are examples of this case.

Since in Case I the belt length is constant, while in Case II it is so nearly constant that it may be regarded as such, the two cases may be treated as one. The distinguishing feature of these cases is that

the sum of the diameters of mating steps is constant, while in Case III this sum is not constant.

To find the diameters of the steps for cases I and II, the cones being alike, as is usual, and having an odd number of steps, proceed as in Fig. 3: Draw a horizontal through a and from a lay down the speeds to the same scale as in Fig. 2 or to any convenient scale, giving ab, ac, ad, etc. Draw a vertical aO and make aO equal to the middle speed, ad. Draw bO, cO, dO, etc.; lay down R_1 equal to the radius of the largest step and draw gh. Through h, the intersection of the highest speed line Of with gh, draw hi at an angle of 45 deg. Now R_1r_1, R_2r_2, R_3r_3, etc., are the radii of mating steps.

The speed of the countershaft is ad.

If the cones have an even number of steps there is no middle speed, point d is initially unknown and O cannot be located at the start.

In the case of unequal cones, two mating steps are naturally known and are used to locate O as in Fig. 5: Make jk equal to the radius of the largest driven step and bj equal to the radius of the smallest driving step (or, if those dimensions are given, make fl equal to the radius of the largest driving, and lm equal to the radius of the smallest driven, step). Draw and extend bk (or draw and extend fm) given O. Locate i by laying down the largest driven step r_5 (or locate h by laying down the largest driving step R_1), draw hi at an angle of 45 deg. and proceed as before.

The speed of the countershaft an is found by drawing On at right angles with ih. If the cones are very unlike On may fall without the field of the other constructions.

The ratio of the back gear in all cases is the ratio of the highest (or lowest) direct to the highest (or lowest) back-gear speed.

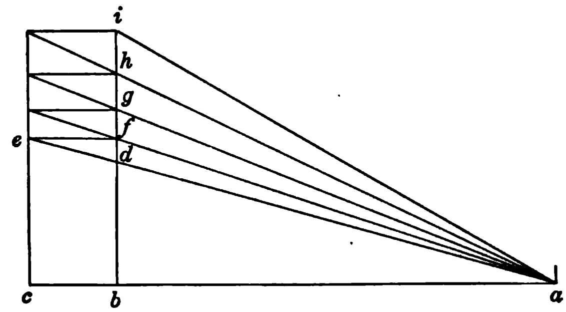


FIG. 2.—Finding the speeds.

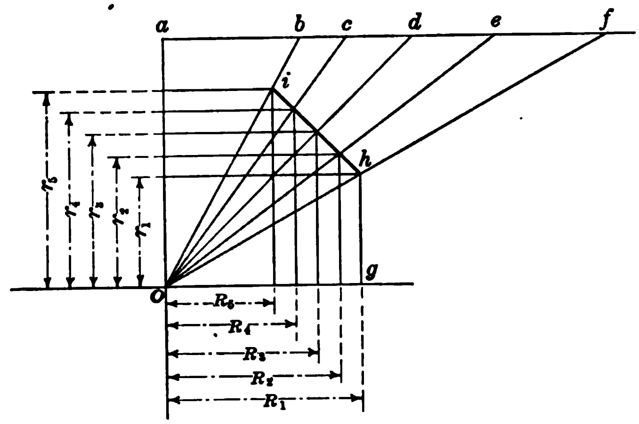


FIG. 3.—Finding the Diameters for Cases I and II. Equal cones having an odd number of steps.

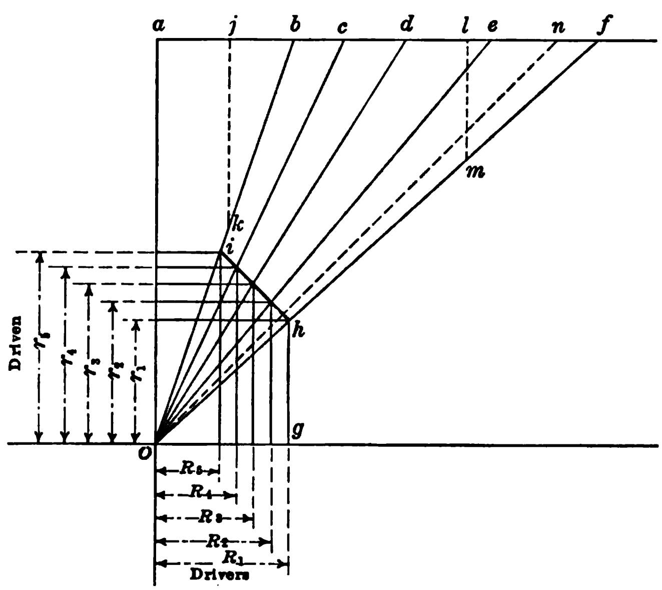


FIG. 5.—Finding the diameters for Cases I and II. Unequal cones having either an odd or an even number of steps.

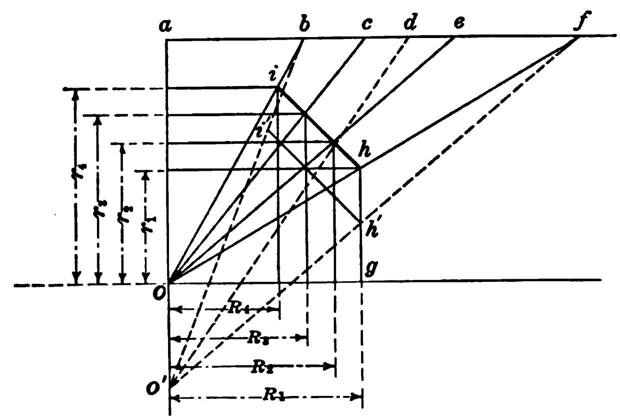


FIG. 4.—Finding the diameters for Cases I and II. Equal cones having an even number of steps.

FIGS. 2 to 5.—Graphical method of laying out cone pulleys.

Proceed as in Fig. 4: Lay down the speeds as before and locate O' at any convenient point on aO'. Draw $O'b$ and $O'f$; lay down R_1, find h' and draw $h'i'$ at 45 deg. as before. The coordinates of $h'i'$ will give mating cones, but not equal cones. Find the center of $h'i'$ and through it draw $O'd$, thus locating d. Now make $aO = ad$ and find the mating steps as before. The speed of the countershaft is again ad.

This construction is approximate only, its accuracy increasing as OO' becomes more nearly equal to zero. To obtain a second and very close approximation, repeat the construction by drawing a line through O and the center of hi, thus finding a new point d, from which lay out a new point O as before.

To find the diameters for Case III proceed as follows, by PROFESSOR MOORE (*Amer. Mach., Feb.* 26, 1903): First draw Fig. 6 in which R and r represent two of the mating radii. Draw the tangent ab and extend it by the distances ac, bd equal to the length of the arcs ae, bf.

To do this use Rankine's approximate method (*Machinery and Millwork*) thus: Bisect the arc bf at g (because Rankine's method should not be used for arcs greater than 90 deg.). Draw the chord bg and extend it, making $bh = \frac{bg}{2}$. From h as a center strike the arc gi giving $bi =$ arc bg. Repeat bi giving $bd =$ arc bf. Similarly, find c giving $ac =$ arc ae when cd obviously equals one-half the length of the belt.

Lay off dj equal to 3.1416 to any scale and dk equal to unity to the same scale, dk making any angle with cd. Lay off $cl = OO'$ and draw jk and lm parallel to it giving dm, which is the radius of the middle step if the cones have an odd number of steps, or of a hypothetical middle step, which is not actually used in case the pulleys are to have an even number of steps.

In Fig. 7 lay down this radius Rm, as shown (Figs. 6 and 7 are drawn to different scales), thus finding the point e. Lay down also the radii of the steps R, r given at the beginning. Through points aeb draw the arc of a circle, and proceed as in Fig. 3, R_n and r_n being mating radii for speed cn. As in Fig. 3 the line cf gives the speed of the countershaft.

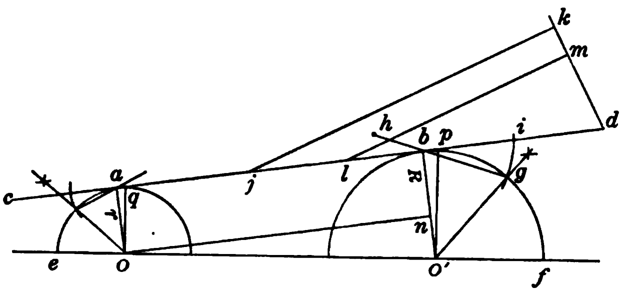


FIG. 6.—Finding the middle step for Case III.

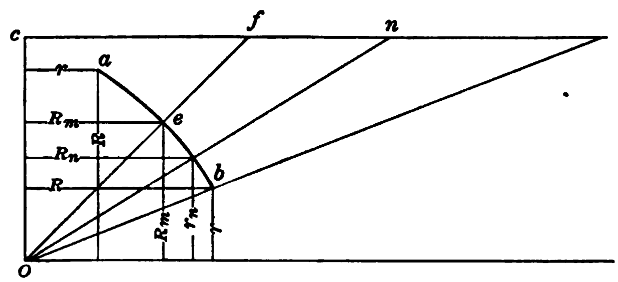


FIG. 7.—Finding the diameters for Case III.

FIGS. 6 and 7.—Graphical method of laying out cone pulleys.

The arc ab gives the required compensation for the angle of the belt and provides that a belt length which is correct for one pair of steps will be correct for all others. The circle is not mathematically correct but is a remarkably close approximation.

If the drive is too large to be laid down on the drawing board to a reasonable scale, the length of the belt may be obtained by calculating $ab = On$, Fig. 6 (OO' and $O'n$ of the right-angled triangle $OO'n$ being known) and also calculating the quadrants pf and qe, leaving the arcs bp, aq to be calculated or stepped off. These are such a small part of the whole that no material error will result from stepping them off on a small scale drawing.

The High-power Cone Pulley

The power transmitted by cone pulleys may be greatly increased by increasing the diameter of the small step, but without increasing the over-all dimensions, as explained in a paper read before the Cincinnati Metal Trades Association in 1893 by H. M. Norris.

Fig. 8 shows the standard and Fig. 9 the Norris design. The comparative powers transmitted by the two constructions may be best shown by actual figures. Calling the highest belt speed in Fig. 8—that obtained with the belt on the 4-in. step—100, the slowest—that on the 12-in. step—will be

$$100 \times \tfrac{4}{12} = 33\tfrac{1}{3}$$

To maintain the same r.p.m., the highest belt speed in Fig. 9 must be

$$100 \times \frac{11\frac{17}{32}}{4} = 288+$$

and the lowest will be

$$288 \times \frac{11\frac{17}{32}}{13} = 255+$$

The smallest step of Fig. 8 is too small for a double belt, while the opposite is true for Fig. 9. To obtain the ratio of power capacities we must multiply the belt-speed ratio by a suitable ratio for this, say $\frac{10}{7}$, and also by the ratio of the belt widths, $\frac{4}{2\frac{1}{2}}$. Doing this we obtain:

$$\frac{\text{Power capacity Fig. 9 small step}}{\text{Power capacity Fig. 8 small step}} = \frac{288}{100} \times \frac{10}{7} \times \frac{4}{2\frac{1}{2}} = 6.5+$$

$$\frac{\text{Power capacity Fig. 9 large step}}{\text{Power capacity Fig. 8 large step}} = \frac{255}{33\frac{1}{3}} \times \frac{10}{7} \times \frac{4}{2\frac{1}{2}} = 17.5-$$

That is, the capacity of Fig. 9 on the small step is $6\frac{1}{2}$ and on the large step, where the gain is most needed, $17\frac{1}{2}$ times that of Fig. 8.

In the cases shown there is a slight increase in the diameter of the large step but, without this increase, the gain would be nearly as large. So large an increase as the one shown is, of course, seldom needed. Many cone-pulley drives are, however, weak in capacity at the slow speeds and Mr. Norris's plan points out the remedy.

The cone pulley shown in Fig. 9 gives a smaller total range of speeds than the one shown in Fig. 8, and, if the range of Fig. 8 is required, additional back gears are necessary. If double gears be used a five-step cone will give 15 speeds against 10 in Fig. 8, while a three-step cone will give 9. Fifteen are unnecessary and 9 are, in most cases, enough. It is this reduction in the number of steps that gives the increase in belt width. The additional back gear will, however, increase the over-all length of the headstock slightly if the entire gain is to be realized.

The tendency of the belt to climb the side of the pulley against which it runs may be prevented by recessing the sides of the steps as shown in Fig. 10. The recess should be of ample depth to prevent the belt reaching its bottom.

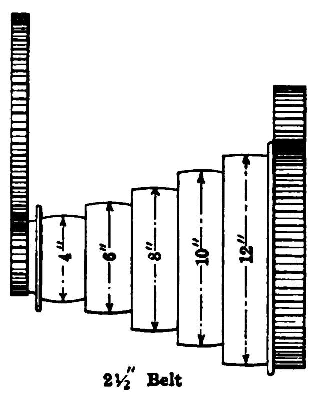


FIG. 8.—Conventional design of cone pulley.

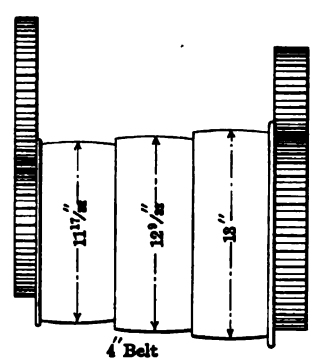


FIG. 9.—Norris design of cone pulley.

Slide-Rule Solution

The slide rule may be used for solving cone-pulley problems in cases which do not involve belt angles so large as to require compensation for belt length. The following explanation of this application of the instrument is by ROBERT A. BRUCE (*Amer. Mach., Aug.* 18, 1904).

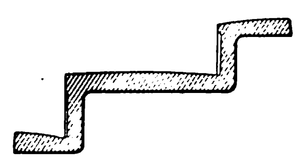


FIG. 10.—Preventing the climbing tendency of belts.

The method is best explained by taking an actual case: Fastest speed 280; slowest speed 20; number of speeds 12.

Lay off a straight line such that the length AB, Fig. 11, is equal to the distance between the points 20 and 280 on the B-scale of the slide rule and divide it into eleven equal parts—*i.e.*, one less than the total number of speeds. To do this draw BC of indefinite length and at any convenient angle. Space off eleven equal spaces of any convenient length. Join the last point D with A and, by a series of parallels through the remaining points, find the required divisions. The extreme and intermediate dividing marks will then form twelve graduations at equal intervals, and on applying the scale so that 20 comes opposite the first and 280 opposite the last, as in Fig. 11, the numbers found opposite the remaining ten divisions will be *the intermediate speeds required.* The accuracy thus attained is sufficiently close for the purpose in view. A record of the speeds thus obtained may be made by writing opposite each graduation of the divided line the corresponding scale reading of the slide rule.

The percentage rise or drop in changing from any speed to the one above or below may be at once obtained by inspection. For, applying the scales so that 100 of the slide-rule scale is opposite one graduation of the paper scale, it will be seen that the next graduation on the right falls opposite 127 on the scale, while the nearest graduation on the left lies against 78.5. The interpretation of these figures is that the percentage drop of speed is 100−78.5, or 21½ per cent., and the percentage rise of speed is 127−100, or 27 per cent.

To obtain the ideal speeds so found the scheme to be adopted must be settled by the peculiar circumstances of the case, rather than by hard and fast rules. We will therefore assume two different cases:

Let us first of all assume a single-speed countershaft and a cone with six steps and back gearing, the six quickest speeds being delivered direct by coupling the cone to the spindle, and the slower speeds being secured by the use of back gearing. Let us also suppose that by the conditions of the problem the diameter of the largest cone step is fixed at 20 ins. The speed of the countershaft is equal (whether the number of cone steps is odd or even) to the geometric mean of the fastest and slowest driven cone speeds. If we therefore bisect that portion of AB lying between 7 and B, that is, the lowest and highest direct-cone speeds, we obtain a line marked "Countershaft speed

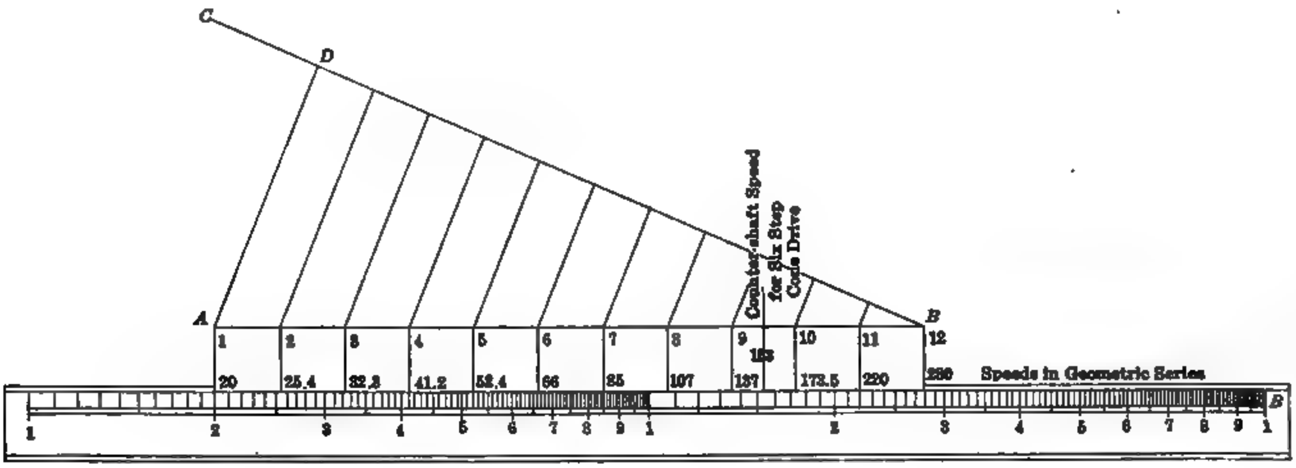


Fig. 11.

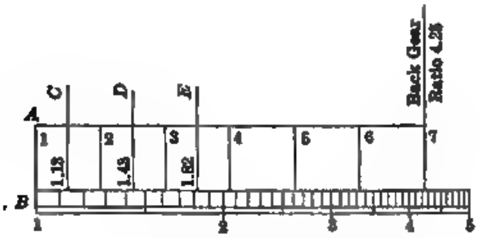


Fig. 12.

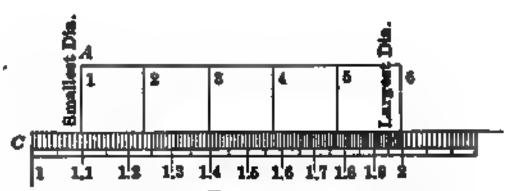


Fig. 13.

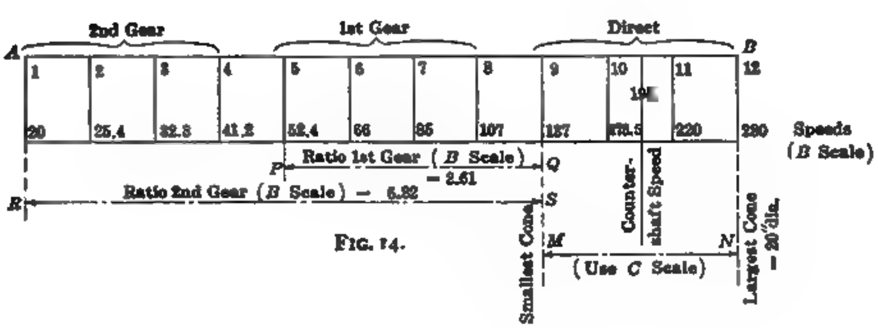


Fig. 14.

Fig. 15.

Figs. 11 to 15.—Slide rule method of laying out cone pulleys and back gears.

for six-step cone drive," which at once gives the scale reading 153 for the countershaft speed.

The ratio of back gear is found by shifting the slide so that the point A comes opposite the point 1 on the scale, as in Fig. 12, when we shall have 4.25 as the scale reading opposite 7, this giving the ratio of the back gear, because 1 represents the lowest back-gear speed and 7 the lowest direct-cone speed which, obviously, is the back-gear ratio.

The sizes of the cones now require fixing, and if lines C, D, E, midway between the main divisions 1 and 2, 2 and 3, 3 and 4, Fig. 12, are drawn with the slide in the position shown, we get scale readings of the ratios of the cones.

This result follows from the fact that if the counter-cones and driven cones are similar, the ratio of the diameter of steps equidistant from the middle is the square root of the ratio of their respective speeds. Thus in a three-speed cone where the diameter of the largest cone is twice the diameter of the smallest, the fastest speed is four times the slowest.

Thus C gives 1.13 for the ratio $\dfrac{\text{dia. cone 3}}{\text{dia. cone 4}}$

D gives 1.43 for the ratio $\dfrac{\text{dia. cone 2}}{\text{dia. cone 5}}$

E gives 1.82 for the ratio $\dfrac{\text{dia. cone 6}}{\text{dia. cone 1}}$

Now cone 6 being fixed on as 20 ins. in diameter, cone 1 is found by direct proportion to be as nearly as possible 11 ins. In practice the diameters of the intermediate cones would usually be taken in arithmetical progression, the results so obtained being sufficiently near the values sought. But if closer results are required, remembering that the sum of the diameters of a pair of cone steps is constant and equal to 31 ins. (20 ins.+11 ins.) if r is the ratio of any pair the diameter of one of them is

$$\frac{31\,r}{r+1}$$

and that of the other

$$\frac{31}{r+1}$$

If the cones are to have equal steps the lines C, D, E, of Fig. 12, may be entirely dispensed with. The diameter of the largest cone being settled by practical conditions, that of the smallest can be found direct as illustrated in Fig. 13. Opposite 6 in the line AB place 20 (the diameter of the largest cone) of the C-scale of the slide rule, and opposite the first graduation of AB find 10.97, or say 11 ins. on the C-scale.

It will be seen therefore that the above operations have involved: (*a*) the measurement of the distance between two points of the B-scale, (*b*) the transfer of this distance to the paper, (*c*) its division into eleven parts, and (*d*) the further bisection of one of these parts by a line. The results obtained by direct reading are: The appropriate speeds, the countershaft speed, the gearing ratio and the sizes of the cones, all of which are obtained without calculation. The method to be adopted for finding suitable gears to give the required ratio will be explained later.

The second variation is to employ a four-speed cone delivering its motion either direct or through two changes of gearing. The first step would be exactly the same as before, the distance AB being the scale distance between fastest and slowest speeds on the B-scale, and the intermediate speeds being read direct as before. The further scheme of operations is shown in Fig. 14.

The countershaft speed is obtained at the same time as the intermediate speeds by taking the scale reading of a line bisecting the fastest and slowest cone speed lines.

The gear ratios are obtained on the B-scale, from P to Q and from R to S, the unit of the slide-rule scale being placed respectively at P and R. The size of the smallest cone is given by the reading MN on the C scale, 20 (*i.e.*, diameter of the largest cone step) being placed at N and the diameter of smallest cone being given on the C scale at M as 14 ins.

When finding the teeth in the wheels the clue is the easily remembered fact that in a pair of wheels in which the ratio of the faster divided by the slower is r the number of teeth in the pinion is

$$\frac{\text{sum of number of teeth}}{r+1}$$

Now, the sum of the number of teeth is always fixed when the centers of the wheels and the pitch have been determined. In compound gears the simplest case is back gearing where both pairs are equal. The total ratio of the gearing has been determined in the foregoing cases by a simple reading on the B-scale, see Fig. 12, where the total ratio is 4.25. Using the C-scale, however, we should obtain the square root of this ratio and should thus have the ratio of each pair of wheels. Thus if 1 on the C-scale is put opposite A in Fig. 13 the line 7 would come opposite 2.06. And if the total number of teeth in each pair were 150, we should have as the number of teeth in pinion $\dfrac{150}{2.06+1}$ or 49 nearly, and of the wheel, $150-49=101$.

Returning to the case in Fig. 14, a convenient method of obtaining the changes of gear would be as in Fig. 15, where only three sizes of wheels are used, all of the same pitch but of decreasing breadths as we move from right to left. The fast or first gear is through the equal wheels AA and CB, the total ratio being 2.61. If the number of teeth as before is 150, then the number of teeth in C is obviously $\dfrac{150}{3.61}$ or 43, and for B we have number of teeth$=150-43=107$, while A has 75 teeth. The simplicity of the processes explained is obvious and the method can be modified by anyone understanding the principles of the logarithmic scale.

Arithmetical Solution

Arithmetical calculation may be used for solving cone pulley problems in cases which do not involve belt angles so large as to require compensation for belt length. The following systematic procedure is by P. V. VERNON (*Trans. Manchester Asso. of Engnrs.*, 1903).

In the preceeding solutions the slowest speed was selected as the starting-point from which the others were obtained by working upward. Mr. Vernon inverts this process and begins with the fastest speed from which the others are obtained by working downward. Under the former method the ratio between the speeds is more than one; under the latter it is less than one, the two values being reciprocals.

To calculate the ratio for the latter method, divide the slowest by the fastest r.p.m., and find the logarithm of the quotient. Divide this logarithm by the number of speeds less one and find the natural number corresponding to this logarithm, which number will be the required ratio. For the former method divide the fastest by the slowest r.p.m., and proceed as before

This calculation may be replaced by Table 1 of ideal speed ranges with sufficient accuracy for practical purposes, the greatest error introduced in the speed ratio being one-half of 1 per cent.

To find the ratio and the speeds for the example shown in Fig. 16, in which it is required to find the correct proportions of gears and cone pulley for an ordinary back-geared headstock[1] to produce twelve speeds varying from 280 down to 20 per min. The cone pulley is to have three steps, the largest 12 ins. diameter, and will, of course, be driven from a two-speed countershaft. This example is representative of a type of headstock largely used on medium-sized turret lathes.

Referring to the table of ideal speed ranges it will be seen that the first number in each column is 1000, so that a corresponding range of speeds in the table would have twelve speeds varying from 1000

[1] The construction called *back gear* in the United States is in England called *double gear.*

down to $\frac{1000 \times 20}{280}$, or from 1000 down to 71.4. Refer to table and look along horizontal line No. 12 for the figure nearest 71.4. This will be found to be 74.7 in the 21 per cent. column, which is probably near enough for the purpose, and fixes the common ratio required at .79, or 21 per cent. of drop from speed to speed.

The percentage of drop from speed to speed will therefore be 21 per cent. with 280 as a maximum. Plot out speeds as follows, either by calculation or slide rule: 280, 221, 174.7, 138, 109, 86.1, 68, 53.7, 42.4, 33.5, 26.46, 20.9.

To find these speeds by logarithms, note that the logarithm of the ratio is the common difference of the logarithms of the speeds, therefore, find the logarithm of the highest speed and from it subtract the logarithm of the ratio, the result being the logarithm of the second speed. From this logarithm subtract again the logarithm of the ratio and the result will be the logarithm of the third speed, and so on.

To find the correct proportions of gears and cone pulley to produce the above speeds:

The set of speeds as obtained above may now be arranged as in Fig. 17, which represents in a simple way the general arrangement or typical form for obtaining geometrical ranges for all combinations of gears, cone pulleys, and countershaft changes.

The arrangement consists of two main divisions of six speeds each, one division being entirely direct speed and the other entirely back geared, each division giving half the range and without any overlapping.

Each main division consists of two sub-divisions of three speeds

TABLE 1.—IDEAL SPEED RANGES

	10 per cent.	11 per cent.	12 per cent.	13 per cent.	14 per cent.	15 per cent.	16 per cent.	17 per cent.	18 per cent.	19 per cent.	20 per cent.	21 per cent.	22 per cent.	23 per cent.	24 per cent.	25 per cent.	26 per cent.	27 per cent.	28 per cent.	29 per cent.	30 per cent.
1	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
2	900	890	880	870	860	850	840	830	820	810	800	790	780	770	760	750	740	730	720	710	700
3	810	792	774	757	740	722	706	689	672	656	640	624	608	593	578	562	548	533	518	504	490
4	729	705	681	658	636	614	593	572	551	531	512	493	474	456	439	421	405	389	373	358	343
5	656	627	599	573	547	522	498	474	452	430	410	389	370	351	333	316	300	284	269	254	240
6	590	558	527	498	470	444	418	394	371	349	328	308	289	271	253	237	222	207	193	180	168
7	531	497	464	433	404	377	351	327	304	282	262	243	225	208	193	178	163	151	139	128	118
8	478	442	408	377	348	320	295	271	249	229	210	192	176	160	146	133	121	110	100	91	82.3
9	430	393	359	328	299	272	248	225	204	185	168	152	137	123	111	100	89.4	81	72.2	64.5	57.6
10	387	350	316	284	257	231	208	187	167	150	134	120	107	95	84.5	75	66.1	59	52.4	45.8	40.3
11	349	312	278	247	221	197	175	155	137	121	107	94.6	83.3	73.2	64.2	56.2	48.9	43	37.4	32.5	28.2
12	314	277	245	215	190	167	147	128	113	98.4	86	74.7	65	56.4	48.8	42.2	36.2	31.3	26.9	23	19.8
13	282	247	215	187	163	142	123	107	92.4	79.7	68.7	59.1	50.7	43.4	37.1	31.6	26.8	22.8	19.4	16.3	13.8
14	254	220	189	162	140	121	103	88.5	75.7	64.5	55	46.6	39.5	33.4	28.2	23.7	19.8	16.6	13.9	11.6	9.7
15	229	195	167	141	121	103	87	73.4	62.1	52.3	44	36.9	30.8	25.7	21.4	17.8	14.6	12.15	10	8.2	6.8
16	206	174	147	123	104	87.3	73	60.9	50.9	42.3	35.2	28.1	24	19.8	16.3	13.3	10.8	8.87	7.23	5.8	4.7
17	185	155	129	107	89.3	74.2	61.3	50.5	41.7	34.2	28.16	23	18.7	15.2	12.4	10	8	6.47	5.21	4.14	3.32
18	167	138	113	93	76.8	63	51.5	41.5	34.2	27.7	22.5	18.2	14.6	11.7	9.4	7.5	5.9	4.7	3.75	2.94	2.32
19	150	122	99.8	80.1	66	53.6	43.2	34.8	28	22.4	18	14.3	11.4	9	7.14	5.6	4.4	3.44	2.7	2.08	1.63
20	135	109	87.8	70.4	56.8	45.5	36.3	28.9	23	18.1	14.9	11.3	8.9	6.9	5.42	4.2	3.25	2.51	1.94	1.48	1.14
21	121	97	77.3	61.3	48.7	38.7	30.5	24	18.9	14.7	11.5	8.9	6.94	5.4	4.12	3.2	2.4	1.53	1.39	1.05	
22	109	86.3	68	53.3	41.9	32.9	25.6	19.9	15.5	11.9	9.2	7.1	5.4	4.1	3.13	2.4	1.78	1.33	1		
23	98.4	76.8	59.8	46.3	36	27.9	21.5	16.5	12.7	9.6	7.37	5.6	4.22	3.2	2.38	1.8	1.31	0.97			
24	88.5	68.3	52.6	40.3	30.9	23.7	18	13.7	10.4	7.8	5.9	4.4	3.29	2.46	1.81	1.3	0.97				
25	79.7	60.8	46.3	35	26.6	20.1	15.2	11.4	8.5	6.3	4.72	3.39	2.56	1.89	1.37	1					
26	71.7	54.1	40.7	30.5	22.9	17.1	12.7	9.4	7	5.1	3.78	2.67	2	1.45	1.04						
27	64.5	48.1	35.8	26.5	19.6	14.5	10.7	7.7	5.7	4.14	3.02	2.1	1.56	1.11							
28	58.1	42.8	31.5	23	16.9	12.4	9	6.5	4.7	3.35	2.42	1.57	1.22								
29	52.2	38.1	27.7	20	14.5	10.5	7.5	5.4	3.8	2.71	1.93	1.32	0.95								
30	47	33.9	24.4	17.4	12.5	8.9	6.3	4.5	3.16	2.2	1.55	0.82									
31	42.3	30.2	21.4	15.1	10.7	7.5	5.3	3.7	2.6	1.78	1.25										
32	38.1	26.8	18.9	13.2	9.2	6.4	4.5	3	2.12	1.44	0.99										
33	34.3	23.9	16.6	11.4	7.9	5.4	3.7	2.5	1.74	1.17											
34	30.8	21.3	14.6	9.9	6.8	4.6	3.1	2.1	1.43	0.95											
35	27.7	18.9	12.8	8.6	5.86	3.9	2.6	1.76	1.17												
36	25	16.7	11.3	7.5	5	3.3	2.2	1.46	0.96												
37	22.5	14.8	9.9	6.5	4.3	2.8	1.8	1.2													
38	20.25	13.2	8.7	5.7	3.7	2.4	1.5	1													
39	18.2	11.7	7.6	4.9	3.2	2	1.3														
40	16.4	10.4	6.7	4.3	2.75	1.74	1.1														

each, one for the fast and one for the slow countershaft speed, the sub-divisions being in the same order in each main division. Each sub-division consists of a group of three speeds, one for each step of the cone pulley, all in the same order and without overlapping.

Suppose, in the example, that the countershaft and machine cones are identical; then *the countershaft speeds* will be 221 and 109 r.p.m.

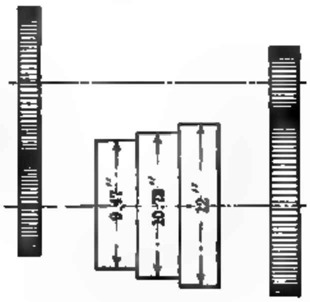


FIG. 16.—Lathe headstock with plain back gears, counter-shaft speeds 221 and 109 r.p.m., back gear ratio 4.11 to 1.

Direct Speed or Back Gear	Direct Speed						Back Gear					
Counter-shaft Fast or Slow	Fast			Slow			Fast			Slow		
Step of Cone	1	2	3	1	2	3	1	2	3	1	2	3
Speeds	280	221	174.7	138	109	86	68	53.7	42.4	33.6	26.46	20.9

FIG. 17.

Direct Speed or Back Gear	Direct Speed						Back Gear 4.11 to 1					
Counter-shaft Speed	221			109			221			109		
Step of Cone	1	2	3	1	2	3	1	2	3	1	2	3
Speeds	280	221	174.7	138	109	86	68	53.7	42.4	33.6	26.5	20.9

FIG. 18.

FIGS. 17 and 18.—Preliminary and final speed determinations for the case of Fig. 16.

Note.—If the cone pulleys have an odd number of steps, the countershaft speed equals the speed of the driven cone with the belt on the middle step. If the number of steps is even, the countershaft speed

$$= \text{quickest speed of cone} \times \sqrt{\frac{\text{slowest}}{\text{quickest}} \text{ speed of cone}} \qquad (a)$$

The largest diameter of the cone pulley is given as 12 in. The *smallest diameter* then is equal to

$$\frac{\text{largest diameter of cone} \times \text{countershaft speed}}{\text{quickest speed of cone}}$$

$$= \frac{12 \times 221}{280} = 9.47 \text{ ins.} \qquad (b)$$

The middle step = half the sum of the other two steps = 10.73 ins.

The ratio of the back gears may be obtained by inspection of the range of speeds of Fig. 17, from which it will be seen that the ratio required is in the proportion of the highest direct speed to the highest back-gear speed, or as 280 : 68 = 4.11 to 1. The gears should be proportioned to give this ratio to the nearest tooth.

The required data are now complete, and the actual speeds may be laid out as in Fig. 18.

If the gears which can be used will not give exactly 4.11 to 1, because of the necessity for using an integral number of teeth, the nearest approximation must be used.

6

For double back gears, as shown in Fig. 19, the conditions chosen are that the cone pulley shall have three steps, the largest 18 ins. diameter, and to be driven from a two-speed countershaft. Eighteen speeds are required from 4½ up to 300 r.p.m., the corresponding range in Table 1 of ideal speed ranges being 18 speeds, varying from 1000 down to $\frac{1000 \times 4.5}{300} = 15$

By looking along horizontal line No. 18, we find that 14.6 in the 22 per cent. column is the nearest figure to 15, and gives probably a near enough percentage for the purpose. If a greater degree of accuracy be required, the percentage of drop can be slightly changed to suit, but as the adjacent columns only vary from each other by a difference of 1 per cent., the table will be found to fulfill all practical requirements.

The required speed range will then have a drop from speed to speed of 22 per cent., with 300 as a maximum.

Plot out the speeds in a similar manner to the first example by calculation or slide rule, as shown in Fig. 20.

It should be noted that Fig. 20 has exactly the same general form as in the first example, an extra division, however, being required for the extra gear ratio. The calculation is just as simple as in the first example, although rather longer, and it will be shown later that the method is equally applicable to the most complicated arrangements of gearing.

The countershaft speeds will be 234 and 111 per min., as will be seen by inspection of Fig. 20, being equal to the second and fifth spindle speeds.

The largest diameter of cone pulley is given as 18 ins. The smallest diameter will therefore be $\frac{18 \times 234}{300} = 14.4$, say 14 ins. The cone pulley will therefore have diameters of 14, 16 and 18 ins.

Arrangement of speeds:

1st group: 6 speeds, direct.

2d group: 6 speeds, $\frac{A}{B} \times \frac{C}{D} = 4.44$ to 1.

3d. group: 6 speeds, $\frac{E}{F} \times \frac{C}{D} = 19.7$ to 1.

FIG. 19.—Lathe headstock with double back gears. Countershaft speeds 234 and 111 r.p.m.

The two gear ratios may be found by inspecting the table of speeds, from which it will be seen that the ratio required for the low gear is in the proportion of the first to the seventh speed.

$$= 300 : 67.7 = 4.44 \text{ to } 1$$

The ratio for the high gear is in the proportion of the first to the thirteenth speed.

$$= 300 : 15.2 = 19.7 \text{ to } 1$$

This gear ratio is exactly the square of the first gear ratio. The three divisions of speeds will thus have gears forming a geometrical progression with a common ratio equal to the first gear ratio thus:

	Single speed	First gear	Second gear
	1	1 × 4.44	1 × 4.44 × 4.44
or	1	4.44	19.7

The above holds good for all arrangements planned by this method, no matter how many gear changes may be used.

The required data are now complete, and the speeds may be laid out as in Fig. 21.

In the lowest line of Fig. 21 the percentage of drop from speed to speed is given, and it will be observed that this is very close to the 22 per cent. aimed at. If all the figures were worked out to sufficient places of decimals, exactly 22 per cent. would be obtained. It is not necessary, however, to overdo the calculations, or much

	1st Group						2nd Group						3rd Group					
Direct Speed or Low or High Gear	Direct Speed						Low Gear						High Gear					
Counter-Shaft Fast or Slow	Fast			Slow			Fast			Slow			Fast			Slow		
Step of Cone	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
Speeds	300	234	182.5	142.8	111	86.5	67.5	52.6	41.1	32	25	19.5	15.2	11.8	9.2	7.2	5.6	4.4

FIG. 20.

	1st Group						2nd Group						3rd Group					
Direct Speed or Low or High Gear	Direct Speed						Low Gear 4.44 to 1						High Gear 19.7 to 1					
Counter-Shaft Speed	234.			111.			234.			111.			234.			111.		
Step of Cone	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
Speeds	300	234	182	142.7	111	86.3	67.7	52.7	40.9	32.1	25	19.4	15.2	11.8	9.2	7.2	5.6	4.35
% Drop		22	22.2	21.6	22.2	22.2	21.5	22.1	22.3	21.5	22.1	22.4	21.6	22.3	22	21.7	22.2	22.3

FIG. 21.

FIGS. 20 and 21.—Preliminary and final speed determinations for the case of Fig. 19.

time can be wasted without any corresponding gain. In many cases the gear ratios obtainable will introduce a slight error, which would more than extinguish the extra accuracy so obtained. In all practical work approximations are permissible, providing that the errors are small and are known. The gear should be proportioned to give the above speeds as nearly as the pitches will allow.

For more complex arrangements of back gears, as shown in Fig. 22, it is assumed that it is desired to find the correct proportions of gears and cone pulley for a headstock arranged to run direct or through any of four separate ratios of gearing, the cone pulley to have four steps, the largest diameter being 24 ins., and driven from a two-speed countershaft giving 40 speeds varying from 1 up to 150 per min. Fig. 22 shows the arrangement in diagrammatic form. The total ratio of speed range required being 150 to 1, the corresponding range in the table will vary from 1000 down to $\frac{1000}{150}=6.66$.

By examining horizontal line 40 in the table we find that 6.7 in the 12 per cent. column is the nearest figure to 6.66, and is near enough for the purpose.

The required speed range should then have a percentage of drop from speed to speed of 12 per cent., with 150 as a maximum. Plot out speeds, following the same method as in the previous examples, as shown in Fig. 23.

The cone pulley in this example has four steps and the countershaft speeds must therefore be calculated, there being no middle step.

The fast countershaft speed, by formula (a)

$$=150\times\sqrt{\frac{102}{150}}=123.6$$

say, 124 r.p.m.

The slow countershaft speed may be found from the table of speeds given in Fig. 23, being equal to

$$\frac{\text{fast countershaft speed}}{\text{ratio of 1st and 5th speeds}}$$

$$=124\div\frac{150}{90}=74.4$$

The largest diameter of the cone pulley is given as 24 ins. The smallest diameter then by formula (b)

$$=\frac{\text{largest diameter of cone}\times\text{fast countershaft speed}}{\text{quickest speed of cone}}$$

$$=\frac{24\times124}{150}=19.84$$

The cone pulleys will therefore have steps 19.840, 21.227, 22.614, and 24 ins. diameter, the diameters being in arithmetical progression with a common difference of 1.387 ins.

If the drop in diameter between the steps of the cone is considerable, or if the drive is very short, it would be necessary to calculate the intermediate speeds separately, as equal differences in diameter do not give equal percentages of speed change. The calculation is made as follows:

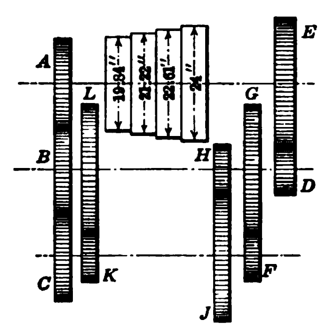
Let x = diameter of required step of driven cone.
y = diameter of required step of driving cone.
a = largest diameter of cone pulley,
b = smallest diameter of cone pulley,
c = intermediate speed required,
d = speed of driving cone.

Then

$$x=\frac{ad+bd}{c+d}\quad y=a+b-x$$

To determine the gear ratios in the last example it will be seen by inspection of Fig. 23 that the ratios required for the four sets of gears are in the proportions of the first speed to the ninth, seventeenth, twenty-fifth and thirty-third respectively. These ratios are in geometrical progression with a common ratio of 2.783 to 1, and work out at 2.78, 7.74, 21.55, and 60. As the ratios of the various sets of gears are in geometrical progression, the first gear ratio only need actually be calculated, the second, third and fourth ratios being the square cube and the fourth power respectively of the first.

Comparison of Figs. 23 and 24 shows that the desired range of speeds has been obtained within limits that are as accurate as the requirements.



A, B, C are equal gears.

$\frac{D}{E}$, $\frac{F}{G}$, $\frac{H}{J}$, $\frac{K}{L}$ are equal ratios = 2.78 to 1.

Arrangement of speeds:

1st group: 8 speeds, direct.

2d group: 8 speeds, $\frac{A}{B}\times\frac{D}{E}=2.78$ to 1.

3d group: 8 speeds, $\frac{A}{C}\times\frac{F}{G}\times\frac{D}{E}=7.74$ to 1.

4th group: 8 speeds, $\frac{A}{B}\times\frac{H}{J}\times\frac{F}{G}\times\frac{D}{E}=21.55$ to 1.

5th group: 8 speeds, $\frac{A}{C}\times\frac{R}{L}\times\frac{H}{J}\times\frac{F}{G}\times\frac{D}{E}=60$ to 1

FIG. 22.—Lathe headstock with multiple back gears. Countershaft speeds 124 and 74.4 r.p.m.

	1st Group								2nd Group								3rd Group								4th Group								5th Group							
Direct Speed or 1st, 2nd, 3rd or 4th Gear	Direct Speed								1st Gear								2nd Gear								3rd Gear								4th Gear							
Counter-shaft Fast or Slow	Fast				Slow				Fast				Slow				Fast				Slow				Fast				Slow				Fast				Slow			
Step of Cone	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Speeds	150	132	116	102.2	90	79.1	69.6	61.3	53.9	47.4	41.7	36.7	32.3	28.4	25	22	19.36	17	15	13.2	11.6	10.2	8.98	7.9	6.96	6.11	5.38	4.73	4.16	3.66	3.22	2.83	2.5	2.19	1.92	1.69	1.49	1.3	1.15	1.01

FIG. 23.

	1st Group								2nd Group								3rd Group								4th Group								5th Group							
Direct Speed or 1st, 2nd, 3rd or 4th Gear	Direct Speed								1st Gear 2.78 to 1								2nd Gear 7.74 to 1								3rd Gear 21.55 to 1								4th Gear 60 to 1							
Counter-shaft Speed	124				74.4				124				74.4				124				74.4				124				74.4				124				74.4			
Step of Cone	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Speeds	150	132	115	102	90	79.2	69.6	61.2	53.9	47.4	41.7	36.7	32.3	28.4	25	22	19.37	17	15	13.17	11.6	10.2	9	7.9	6.96	6.12	5.38	4.73	4.17	3.67	3.22	2.88	2.5	2.2	1.98	1.7	1.5	1.3	1.15	1.02
% Drop		22	22	22	21.7	22	22	22	21.8	22	22	22	21.7	22	22	22	21.8	22	22	22	21.8	22	22	22	21.8	22	22	22	21.7	22	22	22	21.8	22	22	22	21.7	22	22	22

FIG. 24.

FIGS. 23 and 24.—Preliminary and final speed determinations for the case of Fig. 22.

Planetary Back Gears

The proportions of planetary back gears have been worked out by E. J. LEES (*Amer. Mach., March* 1, 1906) as given in Table 2. Three idlers are used to give correct balance when locked up and driven direct. The general arrangement is shown in Fig. 25, which necessitates that the number of teeth in all gears shall be divisible by three.

TABLE 2.—PROPORTIONS OF PLANETARY BACK GEARS

Size No.	1	2	3	4	5	6	7	8	9	10	11	12
Diam. of pulley	8	8	10	10	12	12	15	15	15	15	18	18
Face of pulley	3	3	3½	3½	4½	4½	5½	5½	6½	6½	7½	7½
Width of belt	2½	2½	3	3	4	4	5	5	6	6	7	7
Approx. h.p. at 300 r.p.m.	2¼	2¼	4½	4½	7½	7½	11	11	13	13	18	18
Shaft diam. *D*	1½	1½	1½	1½	1⅝	1⅝	1¾	1¾	2	2	3	3
Pitch diam. pinion *A*	2¼	4½	2 4/7	5 1/7	2 4/7	5 1/7	2 4/7	6	3	9	5	13
No. teeth in *A*	18	36	18	36	18	36	18	42	18	54	15	39
Pitch diam. internal gear *B*.	9	9	10 2/7	10 2/7	10 2/7	10 2/7	12 6/7	12 6/7	18	18	25	25
No. teeth in *B*	72	72	72	72	72	72	90	90	108	108	75	75
Pitch diam. idler *C*	3⅜	2¼	3 6/7	2 4/7	3 6/7	2 4/7	5 1/7	3 3/7	7½	4½	10	6
No. teeth in *C*	27	18	27	18	27	18	36	24	45	27	30	18
No. of idlers	3	3	3	3	3	3	3	3	3	3	3	3
Diam. pitch of gears	8	8	7	7	7	7	7	7	6	6	3	3
Face of gears	1¼	1¼	1½	1½	1⅝	1⅝	1¾	1¾	2	2	3	3
Ratio	5 to 1	3 to 1	5 to 1	3 to 1	5 to 1	3 to 1	6 to 1	3.142 to 1	7 to 1	3 to 1	6 to 1	2.923 to 1

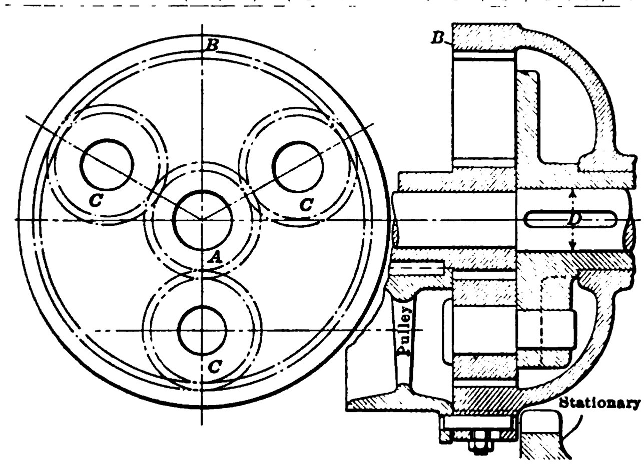


FIG. 25.—Planetary back gearing.

Gear Ratios for Motor Drives

Gear ratios for motor drives as pointed out by W. OWEN (*Amer. Mach., March* 28, 1907) are frequently arranged to advance in multiples of the total speed ratio of the motor—a method which results in the highest speed with each gear in, duplicating the lowest with that gear out and in reducing the total range. These results are shown in Fig. 26, in which the motor ratio is three to one. Were

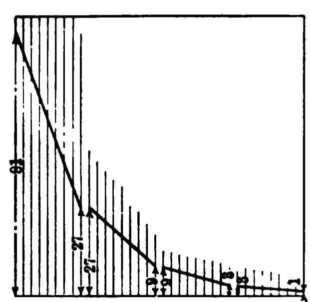


FIG. 26.

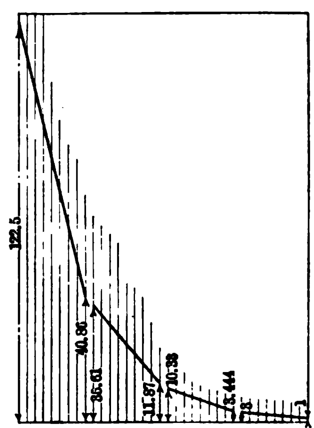


FIG. 27.

FIGS. 26 and 27.—Back gear ratios for motor drives.

the speeds made to advance as the gears are thrown out the result would be that shown in Fig. 27, which gives a total range of $\frac{122.5}{81} =$ 1.51 times that of Fig. 26.

The speeds of variable-speed motors are frequently arranged in arithmetical progression, as indicated in the illustrations which, while not as it should be, does not prevent the gear ratios being in geometrical progression, thus giving most of the advantages of that system.

Let S = highest motor speed,

s = lowest motor speed,

n = number of speeds on motor,

r = ratio of advance.

Were the speeds of the motor arranged in geometrical progression the ratio of advance would be

$$r = \sqrt[n-1]{\frac{S}{s}}$$

and, using this for the gear changes, the ratio of first gear change

$$= \left(\sqrt[n-1]{\frac{S}{s}}\right)^n$$

ratio of second gear change $= \left\{ \left(\sqrt[n-1]{\frac{S}{s}}\right)^n \right\}^2$

and so on for the other gear changes.

Table 3 of back gear ratios for motor drives gives the correct gear ratios for most cases arising in practice.

TABLE 3.—BACK GEAR CHANGES FOR MOTOR DRIVES

Total speed ratio	Total number of speeds	Ratio of advance	Motor variation	Number of speeds on motor	Number of changes by gears	Ratios of gear changes: First	Second	Third	Fourth	Fifth
20.07	40	1.080	2	10	4	2.16	4.667	10.08	.....	.....
20.7	36	1.090	2	9	4	2.18	4.753	10.36	.....	.....
21.53	32	1.104	2	8	4	2.208	4.873	10.76	.....	.....
22.53	28	1.122	2	7	4	2.244	5.035	11.30	.....	.....
24.21	24	1.149	2	6	4	2.298	5.279	12.14	.....	.....
26.84	20	1.190	2	5	4	2.380	5.665	13.48	.....	.....
31.45	27	1.148	3	9	3	3.444	11.87	.....	.....	.....
31.96	16	1.260	2	4	4	2.520	6.351	16.01	.....	.....
34.43	30	1.130	3	10	3	3.39	11.49	.....	.....	.....
36.84	24	1.169	3	8	3	3.507	12.3	.....	.....	.....
38.9	21	1.200	3	7	3	3.6	12.96	.....	.....	.....
41.86	18	1.247	3	6	3	3.741	14.0	.....	.....	.....
45.24	12	1.415	2	3	4	2.830	7.998	22.63	.....	.....
46.65	15	1.316	3	5	3	3.948	15.59	.....	.....	.....
50.52	35	1.122	2	7	5	2.244	5.035	11.30	25.35	.....
56.1	12	1.442	3	4	3	4.326	18.72	.....	.....	.....
56.98	30	1.149	2	6	5	2.298	5.279	12.14	27.87	.....
63.8	25	1.190	2	5	5	2.38	5.665	13.48	32.09	.....
80.48	20	1.260	2	4	5	2.52	6.351	16.01	40.33	.....
80.91	9	1.732	3	3	3	5.196	27.07	.....	.....	.....
116.7	40	1.130	3	10	4	3.39	11.49	38.95	.....	.....
121.9	36	1.148	3	9	4	3.444	11.87	40.86	.....	.....
127.9	15	1.415	2	3	5	2.830	7.998	22.63	64.15	.....
127.9	36	1.149	2	6	6	2.298	5.279	12.14	27.87	64.04
129.1	32	1.169	3	8	4	3.507	12.3	43.15	.....	.....
140.2	28	1.200	3	7	4	3.6	12.96	46.65	.....	.....

Gear-box Construction

The substitution of gear boxes for cone pulleys in feed gearing for machine tools has led to numerous constructions. The following analysis of some of the leading arrangements by A. M. SOSA (*Amer. Mach., Feb.* 15, 1906) will be of assistance to beginners in this field of work.

Fig. 28 shows the most usual way of applying the cone-gear mechanism, in which R indicates the driving and D the driven shaft. This arrangement is most economical for drives, where the power transmitted is constant. The largest gear gives the slowest speed and maximum torque, the smallest *vice versa*, and the pressure on all gear teeth, as well as lineal velocity, is constant.

Fig. 29 is the inverse of Fig. 28. The cone is on the driving and the sliding pinion on the driven shaft, which may be connected to the feed screw, or it may be the feed screw itself. The sliding pinion is of the same diameter as the largest cone-gear, and gives the 1 to 1 ratio, which is the fastest, and requires the maximum torque.

This construction gives low speeds, as one revolution of the screw gives a feed equal to its pitch, and very coarse pitches are most in use. The power is a maximum for the coarsest feed and is transmitted through two large gears instead of two small pinions. The large gear on the screw reduces the pressure on the gear teeth, permits the use of small pitches and gives a compact arrangement.

At the same time an increase of lineal velocity of the gear teeth is the result, but this is a rather desirable feature when low speeds are concerned. The ratio of speeds obtained by this method is as the ratio of the diameter of the largest and smallest cone-gears.

For a ratio 4 to 1, starting with a 14-tooth pinion, the largest would be a 56-tooth gear. This difference in diameter of gears is cumbersome and marks the limit as to the cone ratio. Next to the feed-gear box or in some other part of the machine is usually found a second box containing four gears and a clutch, called the speed clutch box, which is nothing more than a back gear.

It seems more economical, when possible, to place this back gear on the cone gear itself as shown in Fig. 30, in which the cone is in one piece and runs loose. The clutch slides, keyed on the shaft, and transmits the motion to the cone directly or through the back gear. With gears 4 to 1 in diameter, as shown, a ratio of feeds of 32 to 1 in round numbers is conveniently obtained.

Fig. 31 shows a combination for six feeds, speed ratio 12 to 1 and gear ratio 3 to 1. This is operated by one lever, the tumbler lever only. If the running speeds are low, it does not seem to be an objection to have all the gears running. This cone is in two parts; the three gears at the left are keyed to shaft R and the three at the right run loose, are in one piece on the same shaft and receive motion through the back gear as shown. Compounding two cones in this manner gives a very large ratio, with relatively small gears.

Fig. 32 illustrates the use of four gears with a diameter ratio of 2 to 1 only, giving a ratio for the cone of 8 to 1. The arrangement is the same as in Fig. 31.

In Fig. 33 the number of teeth are given, the smallest pinion having 14 and the largest gear 28 teeth. The first two gears at the left are keyed to shaft R, the other two running loose on a sleeve. The cone runs loose and is in one piece. Both clutches slide on keys on the cone shaft, and the other three gears run loose on the cone shaft. This arrangement seems very convenient for screw cutting. The table of threads per inch is shown in the figure. When the feeds per inch are arranged in successive groups, and each group is a multiple of the previous one, the gear ratios can be easily seen. The last group gives more directly the ratios of cone gears. The first number of first and second groups, gives the ratios for the clutch gears 1 to 1—2 to 1. The first number of the third group (4), gives the back-gear ratio 4 to 1. And the first number of the last group is the product of clutch gear (2) and back gear (4), and represents their combination.

Fig. 34 represents a type of drive with cone and sliding gears. The compounding of cones suggested for feed gears would not seem advisable for the reasons previously explained, but compounding the tumbler gear would not change the conditions materially, and double the number of speeds may be obtained. What is generally the idler is, in this case, made of two gears in one piece, running loose on the stud. The ratio of these gears is equal to the percentage of increase of the two next speeds, in this case about 1.2 to 1. The cone gears are spaced at a distance equal to the width of face plus clearance. The gears are drawn to scale, relative to each other, and represent a geometric progression of 16 speeds with a ratio of 20 to 1. The total number of gears used is only nine.

For heavy drives it is not possible to make combinations in the manner previously stated, and the back gears are more successfully grouped separately.

Fig. 35 shows a combination of four gears. The ratios are as 1, 2½, 6¼ and 15⅝, to 1. Different considerations enter into this problem, such as the distribution and the alignment of bearings, the elimination of sleeves and running fits under pressure or torsion and reducing the number of clutches, operating levers and interlocking devices.

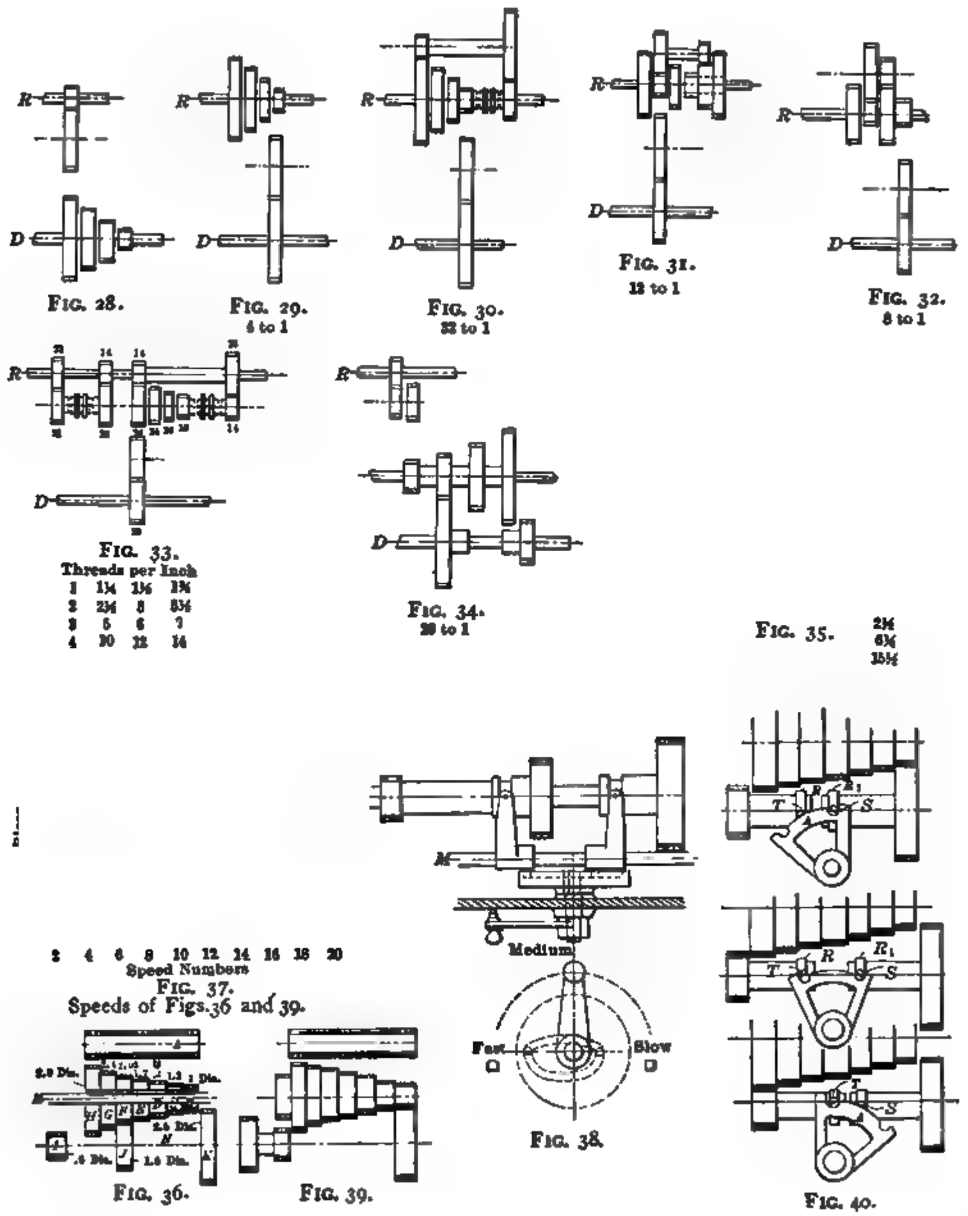
Other typical gear box arrangements are discussed by H. T. MILLAR (*Amer. Mach., Dec.* 14, 1905) as follows:

It is possible to obtain twenty-one changes in geometrical progression with twelve gears and four shafts. Fig. 36 is a development of the motion, the dimensions of which show the relative sizes.

It is always better to lay the gears out in this manner first, then to figure the absolute sizes in consideration of the actual requirements of the case, which may limit the size of the largest or smallest of the gears. The speed ratio is 34 to 1 and the rises are shown in the chart, Fig. 37. Wheels I, J and K, Fig. 36, gear with corresponding wheels H, F and B. I, J and K are mounted on a splined shaft, which is the final shaft of the motion. The connection between

Returning to Fig. 36, the dimensions of the gears BC—H rise in geometric ratio and need no comment.

With some rises it is impossible to obtain a correct ratio between F and J, having to keep a fixed center distance, and Fig. 39 shows the obvious remedy. In a similar manner four pairs of wheels could be used, giving four changes for every speed of the shaft above. The wheels on the driven shaft would need to be coupled



FIGS. 28 to 40.—Typical arrangements of geared feed boxes.

pinion A and the train BC—H is made by the ordinary sliding wheel and tumble shaft, not shown; the third shaft of the motion M has seven speeds of revolution, corresponding to the gears mounted on it. For each of these the final shaft N has three, obtained by putting either I, J or K into gear. Obviously only one of these must be in mesh at a time, and it is an advantage to have only one handle to move them. If two handles are used they must be interlocked. The details of a cam arrangement are shown in Fig. 38. The stirrups slide loosely on the rod M, and are moved by the pins running in the cam, which turns half a revolution.

together in pairs and moved in and out of mesh by a cam, or a pair of interlocking segments to be described later.

When there are only two gears on the driven shaft, as on the Brown & Sharpe gear cutter, the segment A, Fig. 40, provides a neat method of moving the slow and fast gears. As is evident from the sketch it moves one gear out of mesh before putting the other in, and prevents the breakage which might occur through leaving both gears half in mesh. It also enables the gears to be put in while in motion if necessary. The rings R, kept from revolving by a rod passing through eyes at the back of the shaft, have semicircular

pins set in them. Starting at the top sketch, the ring R_1 is moved endwise until the pin S comes out of the recess in the segment. At that time the end of the segment comes in contact with the other pin T, moving it along; the pin S being now clear of the segment.

The great advantage of all these arrangements is that no gear is in mesh except those actually doing the work and the number of these is kept at a minimum.

If the speeds shown in Fig. 41 are near enough to correct progression, the layout of Fig. 42 meets some cases. It is impossible to obtain absolutely correct rises, but those shown are near enough for the majority of purposes. As shown there are eighteen changes with a ratio of 34 to 1. The three wheels keyed on the left-hand end of shaft G are stationary endwise. The sliding wheels CBA and DEF are moved on their shafts by cams laid out on both sides of plate P. Lever R fulfills a double purpose; it acts as an index for the nine different positions of the cam plate and it also stops the gears before any change takes place. When the plate is turned, it lifts lever R, which is connected by a shaft to clutch S. The clutch K puts in either the slow or fast gears. If the motion had to be placed in an inaccessible position a bevel or other gear could be mounted in place of the handle, and connections made by shafts to some convenient position.

It is well to notice that the wheels in this motion should have varying widths. Each wheel has a different periphery speed and consequently a different load on the pitch line.

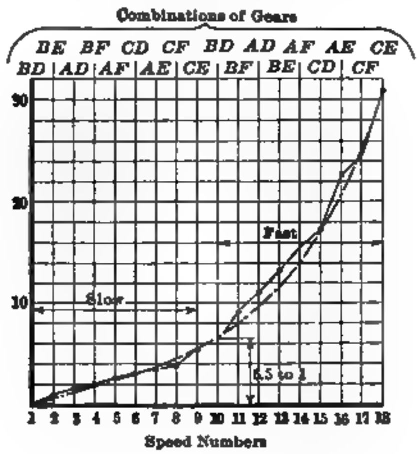


FIG. 41.—Speeds of Fig. 42.

FIG. 42.—Typical arrangement of geared feed boxes.

SPUR GEARS

The movement away from the epicycloidal and toward the involute system of gear-tooth profile has now reached the point where, for gears of small and moderate sizes, the involute system is practically universal. The details of this system are, however, still a subject of controversy. For heavy mill gearing, whether cut or cast, the epicycloidal system is still in large use.

Fig. 1 illustrates the generation of an involute. Two rollers, the circumferences of which are the *base circles*, are connected by a tangent cord which carries a tracing point as shown. When the parts are moved as shown by the arrows, the cord being kept taut, the tracing point traces an involute on the card *abcd* attached to the lower roller. This involute is a correct tooth profile for the lower one of a pair of gears having pitch circles as shown. The profile for the upper gear is traced in the same way by attaching the card to the upper roller.

In order to satisfy the geometrical conditions it is only necessary that the diameters of the rollers have the same *ratio* as the pitch circles. Since an indefinite number of rollers having a given ratio are possible, it follows that an indefinite number of involutes and tooth profiles are possible for every pair of pitch circles.

The line *ef* of the cord is the *line of action* and the angle *egh* between this line and the common tangent to the pitch circles is the *angle of obliquity* or the *pressure angle.* This angle is obviously determined by the diameters of the base circles selected and the value of the angle is the leading subject of controversy.

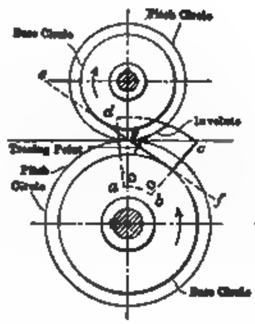


Fig. 1.—Generation of an involute.

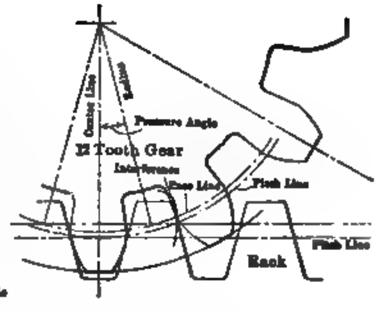


Fig. 2.—Interference of involute gear teeth.

A feature of the involute system is the *interference* between the profiles when high- and low-numbered gears are in mesh. This interference, for a pressure angle of 14½ deg. and between a twelve-tooth pinion and a rack is illustrated in Fig. 2, which shows how the outer end of the rack tooth cuts into the flank of the pinion tooth. The interference grows less as the number of teeth in the pinion is increased and in gears of the 14½-deg. system having the usual addendum, it disappears with a pinion of 30 teeth meshing with a rack.

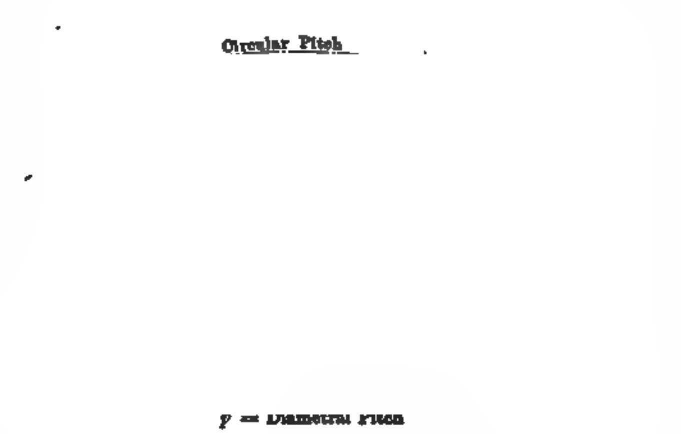


Fig. 2 shows the interference for true involute teeth of 14½ deg. obliquity which, as a matter of fact, are not made. The discussion of the angles has been obscured by its limitation to true involutes. As made by the Brown & Sharpe Mfg. Co. the tooth outlines, whatever the obliquity, are modified by rounding the points of the teeth in order to accommodate unavoidable imperfections of workmanship and bring about more quiet action. This rounding also permits filling in the undercut of 14½ deg. low-numbered pinions, thus restoring most of the loss of strength due to the undercut of unmodified involute profiles.

The advocates of unmodified involute profiles urge an increase of obliquity as a means of avoiding the undercut. As the angle

Table 1.—Details of Involute Gear-tooth Systems

	Brown and Sharpe	Sellers	Hunt stub tooth	Logue stub tooth	Fellows stub tooth	Adamson stub tooth
Pressure angle	14½ deg.	20 deg.	14½ deg.	20 deg.	20 deg.	15 deg.
Addendum	.3183P or $\frac{1}{p}$	3P	.25P or $\frac{.7854}{p}$	25P or $\frac{.7854}{p}$	See below for values of addendum.	.25P
Working depth	.6366P or $\frac{2}{p}$	......	50P or $\frac{1.5708}{p}$	.50P or $\frac{1.5708}{p}$		.50P
Whole depth	6866P or $\frac{2.1571}{p}$		.55P or $\frac{1.7279}{p}$	55P or $\frac{1.7279}{p}$	...	57P
Clearance	.05P or $\frac{.1571}{p}$	.05P	.05P or $\frac{.1571}{p}$	.05P or $\frac{.1571}{p}$	...	.07P

Values of Fellows Addendum

Diametral pitch	4	5	6	7	8	9	10	12
Addendum, in.	$\frac{1}{5}$	$\frac{1}{7}$	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{10}$	$\frac{1}{11}$	$\frac{1}{12}$	$\frac{1}{14}$

Involute Tooth Systems

The most common pressure angle is 14½ deg., being that of the Brown and Sharpe system.[1] The pressure angle of the Sellers system—due to Wilfred Lewis—is 20 deg., which also is the angle of the Fellows[2] and Logue stub-tooth systems. The Hunt stub-tooth system has an angle of 14½ deg.

[1] The Brown and Sharpe Mfg. Co. make cutters for other as well as for the standard angle.

[2] The Fellows Gear Shaper Co. make cutters for a pressure angle of 14½ deg. also.

is increased the number of pinion teeth requiring modification for interference is reduced until, with an angle of 22½ deg., the length of the tooth remaining unchanged, it disappears under the extreme condition of a 12-tooth pinion meshing with a rack.

Most constructors have hesitated to adopt so large an angle and a compromise suggestion has been made to increase the angle to 20 deg. and at the same time reduce the length of the teeth. The Fellows and Logue stub-tooth systems (the latter used by R. D. Nuttall Co.) embody these features.

The shorter tooth has also been strongly advocated for heavy mill gearing. It seems to have found greater adoption for this purpose in England than in the United States, the chief user in the latter country, so far as known to the author, being the C. W. Hunt Co. The object of this change in heavy gearing is to secure increased strength. Shortening the teeth without increasing the pressure angle will not avoid interference with pinions having as few as 12 teeth. The C. W. Hunt Co. prefer not to use pinions having less than 19 teeth.

Table 1 gives the principal details of the above-named systems and of the Adamson (British) system, the notation being given in the illustration above the table.[1] The Adamson (*Jos. Adamson & Co.*) system is based on the recommendations of Michael Longridge, the leading advocate of the stub tooth in Great Britain.

Dimensions of Gear Teeth

The dimensions of gears by diametral pitch and of the Brown and Sharpe standard may be determined from the well-known formulas of Table 2 by the Brown & Sharpe Mfg. Co. in which

p = diametral pitch or the number of teeth to 1 in. of diameter of pitch circle.

P = circular pitch or the distance from the center of one tooth to the center of the next on the pitch circle, ins.

D' = diameter of pitch circle, ins.	Larger wheel	These wheels run together
D = whole diameter, ins.		
N = number of teeth		
V = velocity		
d' = diameter of pitch circle, ins.	Smaller wheel	
d = whole diameter, ins.		
n = number of teeth		
v = velocity		

a = distance between centers of the two wheels, ins.

b = number of teeth in both wheels.

t = thickness of tooth or cutter on pitch circle, ins.

D'' = working depth of tooth, ins.

f = amount added to depth of tooth for rounding the corners and for clearance, ins.

$D''+f$ = whole depth of tooth, ins.

π = 3.1416.

The examples placed opposite the formulas are for a *single* wheel of 12 pitch 6.166 or $6\frac{2}{12}$ ins. diameter, etc., and in the case of the *two* wheels the larger has the *same* dimensions. The velocities are respectively 1 and 2.

A list of tooth parts according to the Brown & Sharpe system will be found in Tables 6 and 8 and a list of useful multipliers in Table 5. See also Table 19.

[1] In one respect the notation of the illustration is not in universal use, some writers defining dedendum as the entire depth below the pitch line = dedendum + clearance as here defined.

TABLE 2.—FORMULAS FOR DIMENSIONS OF INVOLUTE GEARS OF THE BROWN AND SHARPE STANDARD

FOR A SINGLE WHEEL

Formulas Examples

$$p=\frac{N+2}{D}=\frac{72+2}{6.166}, \text{ or } \frac{72+2}{6\frac{2}{12}}=12. \tag{1}$$

$$p=\frac{N}{D'}=\frac{72}{6}=12. \tag{2}$$

$$D'=\frac{D\times N}{N+2}=\frac{6.166\times 72}{72+2}=6. \tag{3}$$

$$D'=\frac{N}{p}=\frac{72}{12}=6. \tag{4}$$

$$N=pD'=12\times 6=72. \tag{5}$$

$$N=pD-2=12\times 6.166-2, \text{ or } 12\times 6\tfrac{2}{12}-2=72. \tag{6}$$

$$D=\frac{N+2}{p}=\frac{72+2}{12}=6.166, \text{ or } 6\tfrac{2}{12}. \tag{7}$$

$$D=D'+\frac{2}{p}=6+\frac{2}{12}, \text{ or } 6+.166=6.166. \tag{8}$$

$$t=\frac{1.57}{p}=\frac{1.57}{12}=.130. \tag{9}$$

$$D''=\frac{2}{p}=\frac{2}{12}=.166, \text{ or } \tfrac{2}{12}. \tag{10}$$

$$f=\frac{t}{10}=\frac{.130}{10}=.013. \tag{11}$$

$$D''+f=.166+.013=.179. \tag{12}$$

$$P=\frac{\pi}{p}=\frac{3.1416}{12}=.262. \tag{13}$$

$$p=\frac{\pi}{P}=\frac{3.1416}{.262}=12. \tag{14}$$

FOR A PAIR OF WHEELS

Formulas Examples

$$b=2ap=2\times 4.5\times 12=108 \tag{15}$$

$$n=\frac{bV}{v+V}=\frac{108\times 1}{3}=36. \tag{16}$$

$$N=\frac{nv}{V}=\frac{36\times 2}{1}=72. \tag{17}$$

$$n=\frac{NV}{v}=\frac{72\times 1}{2}=36. \tag{18}$$

$$N=\frac{bv}{v+V}=\frac{108\times 2}{3}=72. \tag{19}$$

$$n=\frac{pD'V}{v}=\frac{12\times 6\times 1}{2}=36. \tag{20}$$

$$V=\frac{nv}{N}=\frac{36\times 2}{72}=1. \tag{21}$$

$$v=\frac{NV}{n}=\frac{72\times 1}{36}=2. \tag{22}$$

$$v=\frac{pD'V}{n}=\frac{12\times 6\times 1}{36}=2. \tag{23}$$

$$D=\frac{2a(n+2)}{b}=\frac{2\times 4.5\times(72+2)}{108}=6.166. \tag{24}$$

$$b=\frac{2a(n+2)}{b}=\frac{2\times 4.5\times(36+2)}{108}=3.166. \tag{25}$$

$$a=\frac{b}{2p}=\frac{108}{2\times 12}=4.5. \tag{26}$$

$$D'=\frac{2av}{v+V}=\frac{2\times 4.5\times 2}{3}=6. \tag{27}$$

$$d'=\frac{2aV}{v+V}=\frac{2\times 4.5\times 1}{3}=3. \tag{28}$$

$$a=\frac{D'+d'}{2}=\frac{6+3}{2}=4.5. \tag{29}$$

Table 3 gives dimensions of gear teeth of the 14½-deg. system and Table 4 gives similar dimensions of 20-deg. stub teeth both according to the practice of the Fellows Gear Shaper Co.

TABLE 3.—DIMENSIONS OF GEAR TEETH OF 14½ DEGREES PRESSURE ANGLE WITH STANDARD ADDENDUM—Fellows System

Thickness of tooth..................	= 1.5708 ÷ diametral pitch.
Addendum..........................	= 1.0000 ÷ diametral pitch.
Clearance, gear shaper gear..........	= .2500 ÷ diametral pitch.
Clearance, milled gear...............	= .1571 ÷ diametral pitch.
Whole depth, gear shaper gear.......	= 2.2500 ÷ diametral pitch.
Whole depth, milled gear............	= 2.1571 ÷ diametral pitch.

Diametral pitch	Thickness of tooth	Addendum	Clearance		Whole depth	
			Gear shaper gear	Milled gear	Gear shaper gear	Milled gear
1	1.5708	1.0000	.2500	.1571	2.2500	2.1571
1½	1.0472	.6667	......	.1047	......	1.4381
2	.7854	.5000	......	.0785	......	1.0785
2½	.6283	.4000	......	.0628	......	.8628
3	.5236	.3333	......	.0524	......	.7190
4	.3927	.2500	.0625	.0393	.5625	.5393
5	.3142	.2000	.0500	.0314	.4500	.4314
6	.2618	.1667	.0417	.0262	.3750	.3595
7	.2244	.1429	.0357	.0224	.3214	.3081
8	.1963	.1250	.0312	.0196	.2812	.2696
9	.1745	.1111	.0278	.0175	.2500	.2397
10	.1571	.1000	.0250	.0157	.2250	.2157
12	.1309	.0833	.0208	.0131	.1875	.1798
14	.1122	.0714	.0179	.0112	.1607	.1541
16	.0982	.0625	.0156	.0098	.1406	.1348
18	.0873	.0555	.0139	.0087	.1250	.1198
20	.0785	.0500	.0125	.0078	.1125	.1079
22	.0714	.0455	.0114	.0071	.1023	.0980
24	.0654	.0417	.0104	.0065	.0938	.0899
26	.0604	.0385	.0096	.0060	.0865	.0829
28	.0561	.0357	.0089	.0056	.0804	.0770
30	.0524	.0333	.0083	.0052	.0750	.0719
32	.0491	.0312	.0078	.0049	.0703	.0674

Approximate Gear-tooth Outlines

Approximate gear-tooth outlines may be determined by the use of any of the various odontographs that have been proposed; of these probably the most accurate (in fact very accurate) and most generally available is that by GEO. B. GRANT (*Amer. Mach., May 22, July* 3, 1890, and A Treatise on Gear Wheels) which is repeated below in Tables 7 and 9.

To draw the involute tooth first draw the pitch, addendum, root and clearance lines and space the pitch line for the teeth as in Fig. 3. Draw the base line one-sixtieth of the pitch diameter inside the pitch line.

Take the face radius from Table 7, multiply or divide it as called for by the table, take the resulting radius in the dividers and draw in the faces from the pitch line to the addendum line from centers on the base line. Take the tabular flank radius from the table, multiplying or dividing it as before, and draw in the flanks from the pitch line to the base line. Draw straight radial flanks from the base line to the root line and round them into the clearance line.

TABLE 4.—DIMENSIONS OF GEAR TEETH OF 20 DEG. PRESSURE ANGLE WITH REDUCED ADDENDUM—Fellows Stub-tooth System

Thickness of tooth same as for 14½-deg. gear of same pitch as numerator of stub-tooth pitch fraction.

Addendum, clearance, depth of space and whole depth of tooth same as for 14½-deg. gear shaper, gear of same pitch as denominator of stub-tooth pitch fraction.

Diametral pitch	Thickness of tooth	Addendum	Clearance	Whole depth of tooth
4/5	.3927	.2000	.0500	.4500
5/7	.3142	.1429	.0357	.3214
6/8	.2618	.1250	.0312	.2812
7/9	.2244	.1111	.0278	.2500
8/10	.1963	.1000	.0250	.2250
9/11	.1745	.0909	.0227	.2045
10/12	.1571	.0833	.0208	.1875
12/14	.1309	.0714	.0179	.1607

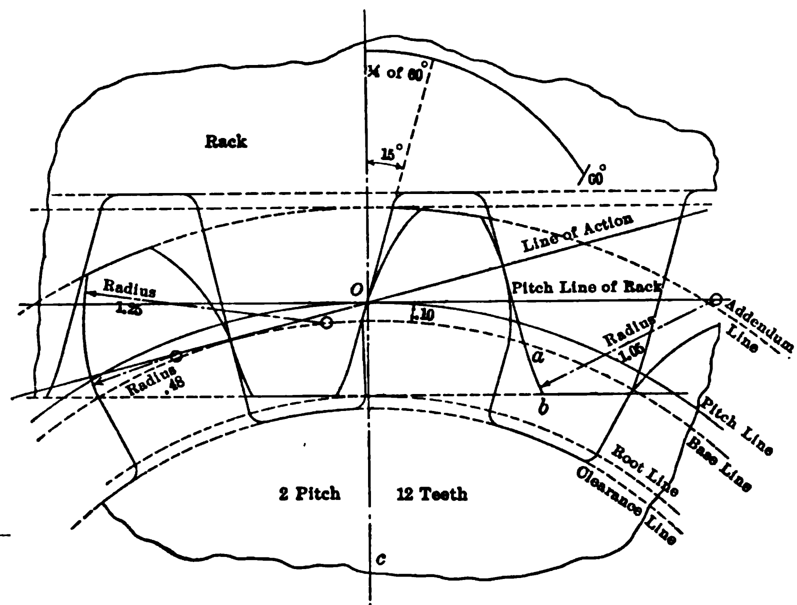


FIG. 3.—Grant's odontograph for involute teeth.

Fig. 3 shows the resulting radii and centers for a pinion of 2 diametral pitch with 12 teeth and for a rack meshing with it.

Special rule for the rack: Draw the sides of the rack tooth, Fig. 3, as straight lines inclined to the line of centers *co* at an angle of 15 deg. Draw the outer half *ab* of the face one-quarter of the whole length of the tooth from a center on the pitch line and with

$$\text{radius} = \frac{2.10 \text{ ins.}}{\text{diametral pitch}}$$
$$= .67 \times \text{circular pitch}$$

If the gear is to have more than 30 teeth the rounding of the ends of the rack teeth is unnecessary.

TABLE 5.—MULTIPLIERS AND THEIR LOGARITHMS FOR FINDING DIAMETERS OF SPUR GEARS FROM THE CIRCULAR PITCH

To find pitch diameter: Multiply the number of teeth by the multiplier for the pitch.
To find outside diameter for standard (B. & S.) addendum: Add two to the number of teeth and proceed as before.

P.	Mult'r.	Log.	P.	Mult'r.	Log.	P.	Mult'r	Log.	P.	Mult'r.	Log.
1/16"	.019894	$\bar{2}$.298722	2/5"	.127324	$\bar{1}$.104910	1 3/16"	.377993	$\bar{1}$.577484	2 1/8"	.676408	$\bar{1}$.830209
1/10"	.031831	$\bar{2}$.502850	7/16	.139261	$\bar{1}$.143829	1 1/4"	.397887	$\bar{1}$.599760	2 1/4"	.716197	$\bar{1}$.855033
1/9"	.035368	$\bar{2}$.548610	1/2"	.159155	$\bar{1}$.201820	1 5/16"	.417782	$\bar{1}$.620950	2 3/8"	.755986	$\bar{1}$.878514
1/8"	.039789	$\bar{2}$.599763	9/16"	.179049	$\bar{1}$.252972	1 3/8"	.437676	$\bar{1}$.641153	2 1/2"	.795775	$\bar{1}$.900789
1/7"	.045473	$\bar{2}$.657754	5/8"	.198944	$\bar{1}$.298731	1 7/16"	.457570	$\bar{1}$.660457	2 5/8"	.835563	$\bar{1}$.921979
1/6"	.053052	$\bar{2}$.724702	2/3"	.212207	$\bar{1}$.236760	1 1/2"	.477465	$\bar{1}$.678942	2 3/4"	.875352	$\bar{1}$.942183
3/16"	.059683	$\bar{2}$.775851	11/16"	.218838	$\bar{1}$.340132	1 9/16"	.497359	$\bar{1}$.696670	2 7/8"	.915141	$\bar{1}$.961488
1/5"	.063662	$\bar{2}$.803880	3/4"	.238732	$\bar{1}$.377911	1 5/8"	.517253	$\bar{1}$.713703	3"	.954930	$\bar{1}$.979971
2/9"	.070735	$\bar{2}$.849634	13/16"	.258627	$\bar{1}$.412674	1 11/16"	.537148	$\bar{1}$.730094	3 1/8"	.994718	$\bar{1}$.997700
1/4"	.079577	$\bar{2}$.900788	7/8"	.278521	$\bar{1}$.444858	1 3/4"	.557042	$\bar{1}$.745888	3 1/4"	1.034507	.014733
2/7"	.090945	$\bar{2}$.958779	15/16"	.298415	1.474821	1 13/16"	.576936	$\bar{1}$.761128	3 3/8"	1.074296	.031124
5/16"	.099472	$\bar{2}$.997701	1"	.318310	$\bar{1}$.502850	1 7/8"	.596831	$\bar{1}$.775851	3 1/2"	1.114085	.046916
1/3"	.106103	$\bar{1}$.025728	1 1/16"	.338204	$\bar{1}$.529179	1 15/16"	.616725	$\bar{1}$.790092			
3/8"	.119366	$\bar{1}$.076881	1 1/8"	.358099	$\bar{1}$.554003	2"	.636619	$\bar{1}$.803879			

TABLE 6.—TOOTH PARTS BY CIRCULAR PITCH, BROWN AND SHARPE SYSTEM. THE 2D, 9TH AND 10TH COLUMNS RELATE ALSO TO WORMS. From a Practical Treatise on Gearing by The Brown & Sharpe Mfg. Co.

To obtain the size of any part of a circular pitch not given in the table, multiply the corresponding part of 1" pitch by the pitch required.

Circular pitch	Threads or teeth per inch linear	Diametral pitch	Thickness of tooth on pitch line	Addendum	Working depth of tooth	Depth of space below pitch line	Whole depth of tooth	Width of thread-tool at end	Width of thread at top	Circular pitch	Threads or teeth per inch linear	Diametral pitch	Thickness of tooth on pitch line	Addendum	Working depth of tooth	Depth of space below pitch line	Whole depth of tooth	Width of thread-tool at end	Width of thread at top
P	$\frac{1''}{P}$	p	t	s	D''	$s+f$	$D''+f$	$P\times .3095$	$P\times .3354$	P	$\frac{1''}{P}$	p	t	s	D''	$s+f$	$D''+f$	$P\times .3095$	$P\times .3354$
2	1/2	1.5708	1.0000	.6366	1.2732	.7366	1.3732	.6190	.6707	1/2	2	6.2832	.2500	.1592	.3183	.1842	.3433	.1547	.1677
1 7/8	8/15	1.6755	.9375	.5968	1.1937	.6906	1.2874	.5803	.6288	4/9	2 1/4	7.0685	.2222	.1415	.2830	.1637	.3052	.1376	.1490
1 3/4	4/7	1.7952	.8750	.5570	1.1141	.6445	1.2016	.5416	.5869	7/16	2 2/7	7.1808	.2187	.1393	.2785	.1611	.3003	.1354	.1467
1 5/8	8/13	1.9333	.8125	.5173	1.0345	.5985	1.1158	.5029	.5450	3/7	2 1/3	7.3304	.2143	.1364	.2728	.1578	.2942	.1326	.1437
1 1/2	2/3	2.0944	.7500	.4775	.9549	.5525	1.0299	.4642	.5030	2/5	2 1/2	7.8540	.2000	.1273	.2546	.1473	.2746	.1238	.1341
1 7/16	16/23	2.1855	.7187	.4576	.9151	.5294	.9870	.4449	.4821	3/8	2 2/3	8.3776	.1875	.1194	.2387	.1381	.2575	.1161	.1258
1 3/8	8/11	2.2848	.6875	.4377	.8754	.5064	.9441	.4256	.4611	4/11	2 3/4	8.6394	.1818	.1158	.2316	.1340	.2498	.1125	.1219
1 1/3	3/4	2.3562	.6666	.4244	.8488	.4910	.9154	.4127	.4471	1/3	3	9.4248	.1666	.1061	.2122	.1228	.2289	.1032	.1118
1 5/16	16/21	2.3936	.6562	.4178	.8356	.4834	.9012	.4062	.4402	5/16	3 1/5	10.0531	.1562	.0995	.1989	.1151	.2146	.0967	.1048
1 1/4	4/5	2.5133	.6250	.3979	.7958	.4604	.8583	.3869	.4192	3/10	3 1/3	10.4719	.1500	.0955	.1910	.1105	.2060	.0928	.1006
1 3/16	16/19	2.6456	.5937	.3780	.7560	.4374	.8156	.3675	.3982	2/7	3 1/2	10.9956	.1429	.0909	.1819	.1052	.1962	.0884	.0958
1 1/8	8/9	2.7925	.5625	.3581	.7162	.4143	.7724	.3482	.3773	1/4	4	12.5664	.1250	.0796	.1591	.0921	.1716	.0774	.0838
1 1/16	16/17	2.9568	.5312	.3382	.6764	.3913	.7295	.3288	.3563	2/9	4 1/2	14.1372	.1111	.0707	.1415	.0818	.1526	.0688	.0745
1	1	3.1416	.5000	.3183	.6366	.3683	.6866	.3095	.3354	1/5	5	15.7080	.1000	.0637	.1273	.0737	.1373	.0619	.0671
15/16	1 1/15	3.3510	.4687	.2984	.5968	.3453	.6437	.2902	.3144	3/16	5 1/3	16.7552	.0937	.0597	.1194	.0690	.1287	.0580	.0629
7/8	1 1/7	3.5904	.4375	.2785	.5570	.3223	.6007	.2708	.2934	2/11	5 1/2	17.2788	.0909	.0579	.1158	.0670	.1249	.0563	.0610
13/16	1 3/13	3.8666	.4062	.2586	.5173	.2993	.5579	.2515	.2725	1/6	6	18.8496	.0833	.0531	.1061	.0614	.1144	.0516	.0559
4/5	1 1/4	3.9270	.4000	.2546	.5092	.2946	.5492	.2476	.2683	2/13	6 1/2	20.4203	.0769	.0489	.0978	.0566	.1055	.0476	.0516
3/4	1 1/3	4.1888	.3750	.2387	.4775	.2762	.5150	.2321	.2515	1/7	7	21.9911	.0714	.0455	.0910	.0526	.0981	.0442	.0479
11/16	1 5/11	4.5696	.3437	.2189	.4377	.2532	.4720	.2128	.2306	2/15	7 1/2	23.5619	.0666	.0425	.0850	.0492	.0917	.0413	.0447
2/3	1 1/2	4.7124	.3333	.2122	.4244	.2455	.4577	.2063	.2236	1/8	8	25.1327	.0625	.0398	.0796	.0460	.0858	.0387	.0419
5/8	1 3/5	5.0265	.3125	.1989	.3979	.2301	.4291	.1934	.2096	1/9	9	28.2743	.0555	.0354	.0707	.0409	.0763	.0344	.0373
3/5	1 2/3	5.2360	.3000	.1910	.3820	.2210	.4120	.1857	.2012	1/10	10	31.4159	.0500	.0318	.0637	.0368	.0687	.0309	.0335
4/7	1 3/4	5.4978	.2857	.1819	.3638	.2105	.3923	.1769	.1916	1/16	16	50.2655	.0312	.0199	.0398	.0230	.0429	.0193	.0210
9/16	1 7/9	5.5851	.2812	.1790	.3581	.2071	.3862	.1741	.1886	1/20	20	62.8318	.0250	.0159	.0318	.0184	.0343	.0155	.0168

TABLE 7.—GRANT'S ODONTOGRAPH FOR INVOLUTE TEETH

Pressure angle = 15 deg.

Addendum = .3183 × circular pitch $= \dfrac{1}{\text{diametral pitch}}$

Clearance $= \dfrac{\text{addendum}}{8}$

No. of teeth	Divide by the diametral pitch: Face radius	Divide by the diametral pitch: Flank radius	Multiply by the circular pitch: Face radius	Multiply by the circular pitch: Flank radius
10	2.28	0.69	0.73	0.22
11	2.40	0.83	0.76	0.27
12	2.51	0.96	0.80	0.31
13	2.62	1.09	0.83	0.34
14	2.72	1.22	0.87	0.39
15	2.82	1.34	0.90	0.43
16	2.92	1.46	0.93	0.47
17	3.02	1.58	0.96	0.50
18	3.12	1.69	0.99	0.54
19	3.22	1.79	1.03	0.57
20	3.32	1.89	1.06	0.60
21	3.41	1.98	1.09	0.63
22	3.49	2.06	1.11	0.66
23	3.57	2.15	1.13	0.69
24	3.64	2.24	1.16	0.71
25	3.71	2.33	1.18	0.74
26	3.78	2.42	1.20	0.77
27	3.85	2.50	1.23	0.80
28	3.92	2.59	1.25	0.82
29	3.99	2.67	1.27	0.85
30	4.06	2.76	1.29	0.88
31	4.13	2.85	1.31	0.91
32	4.20	2.93	1.34	0.93
33	4.27	3.01	1.36	0.96
34	4.33	3.09	1.38	0.99
35	4.39	3.16	1.39	1.01
36	4.45	3.23	1.41	1.03
37–40		4.20		1.34
41–45		4.63		1.48
46–51		5.06		1.61
52–60		5.74		1.83
61–70		6.52		2.07
71–90		7.72		2.46
91–120		9.78		3.11
121–180		13.38		4.26
181–360		21.62		6.88

TABLE 8.—TOOTH PARTS BY DIAMETRAL PITCH, BROWN AND SHARPE SYSTEM.
From a Practical Treatise on Gearing by The Brown & Sharpe Mfg. Co.

To obtain the size of any part of a diametral pitch not given in the table, divide the corresponding part of 1 diametral pitch by the pitch required.

Diametral pitch	Circular pitch	Thickness of tooth on pitch line	$\frac{1}{p}$ or the addendum	Working depth of tooth	Depth of space below pitch line	Whole depth of tooth	Diametral pitch	Circular pitch	Thickness of tooth on pitch line	$\frac{1}{p}$ or the addendum	Working depth of tooth	Depth of space below pitch line	Whole depth of tooth
p	P	t	s	D''	$s+f$	$D''+f$	p	P	t	s	D''	$s+f$	$D''+f$
½	6.2832	3.1416	2.0000	4.0000	2.3142	4.3142	15	.2094	.1047	.0666	.1333	.0771	.1438
¾	4.1888	2.0944	1.3333	2.6666	1.5428	2.8761	16	.1963	.0982	.0625	.1250	.0723	.1348
1	3.1416	1.5708	1.0000	2.0000	1.1571	2.1571	17	.1848	.0924	.0588	.1176	.0681	.1269
1¼	2.5133	1.2566	.8000	1.6000	.9257	1.7257	18	.1745	.0873	.0555	.1111	.0643	.1198
1½	2.0944	1.0472	.6666	1.3333	.7714	1.4381	19	.1653	.0827	.0526	.1053	.0609	.1135
1¾	1.7952	.8976	.5714	1.1429	.6612	1.2326	20	.1571	.0785	.0500	.1000	.0579	.1079
2	1.5708	.7854	.5000	1.0000	.5785	1.0785	22	.1428	.0714	.0455	.0909	.0526	.0980
2¼	1.3963	.6981	.4444	.8888	.5143	.9587	24	.1309	.0654	.0417	.0833	.0482	.0898
2½	1.2566	.6283	.4000	.8000	.4628	.8628	26	.1208	.0604	.0385	.0769	.0445	.0829
2¾	1.1424	.5712	.3636	.7273	.4208	.7844	28	.1122	.0561	.0357	.0714	.0413	.0770
3	1.0472	.5236	.3333	.6666	.3857	.7190	30	.1047	.0524	.0333	.0666	.0386	.0719
3½	.8976	.4488	.2857	.5714	.3306	.6163	32	.0982	.0491	.0312	.0625	.0362	.0674
4	.7854	.3927	.2500	.5000	.2893	.5393	34	.0924	.0462	.0294	.0588	.0340	.0634
5	.6283	.3142	.2000	.4000	.2314	.4314	36	.0873	.0436	.0278	.0555	.0321	.0599
6	.5236	.2618	.1666	.3333	.1928	.3595	38	.0827	.0413	.0263	.0526	.0304	.0568
7	.4488	.2244	.1429	.2857	.1653	.3081	40	.0785	.0393	.0250	.0500	.0289	.0539
8	.3927	.1963	.1250	.2500	.1446	.2696	42	.0748	.0374	.0238	.0476	.0275	.0514
9	.3491	.1745	.1111	.2222	.1286	.2397	44	.0714	.0357	.0227	.0455	.0263	.0490
10	.3142	.1571	.1000	.2000	.1157	.2157	46	.0683	.0341	.0217	.0435	.0252	.0469
11	.2856	.1428	.0909	.1818	.1052	.1961	48	.0654	.0327	.0208	.0417	.0241	.0449
12	.2618	.1309	.0833	.1666	.0964	.1798	50	.0628	.0314	.0200	.0400	.0231	.0431
13	.2417	.1208	.0769	.1538	.0890	.1659	56	.0561	.0280	.0178	.0357	.0207	.0385
14	.2244	.1122	.0714	.1429	.0826	.1541	60	.0524	.0262	.0166	.0333	.0193	.0360

TABLE 9.—GRANT'S ODONTOGRAPH FOR EPICYCLOIDAL TEETH

$$\text{Addendum} = .3183 \times \text{circular pitch}$$
$$= \frac{1}{\text{diametral pitch}}$$
$$\text{Clearance} = \frac{\text{addendum}}{8}$$

Number of teeth in the gear		For one diametral pitch. For any other pitch divide by that pitch				For one inch circular pitch. For any other pitch multiply by that pitch			
		Faces		Flanks		Faces		Flanks	
Exact	Intervals	Rad.	Dis.	Rad.	Dis.	Rad.	Dis.	Rad.	Dis.
10	10	1.99	0.02	− 8.00	4.00	0.62	0.01	−2.55	1.27
11	11	2.00	0.04	−11.05	6.50	0.63	0.01	−3.34	2.07
12	12	2.01	0.06	∞	∞	0.64	0.02	∞	∞
13½	13–14	2.04	0.07	15.10	9.43	0.65	0.02	4.80	3.00
15½	15–16	2.10	0.09	7.86	3.46	0.67	0.03	2.50	1.10
17½	17–18	2.14	0.11	6.13	2.20	0.68	0.04	1.95	0.70
20	19–21	2.20	0.13	5.12	1.57	0.70	0.04	1.63	0.50
23	22–24	2.26	0.15	4.50	1.13	0.72	0.05	1.43	0.36
27	25–29	2.33	0.16	4.10	0.96	0.74	0.05	1.30	0.29
33	30–36	2.40	0.19	3.80	0.72	0.76	0.06	1.20	0.23
42	37–48	2.48	0.22	3.52	0.63	0.79	0.07	1.12	0.20
58	49–72	2.60	0.25	3.33	0.54	0.83	0.08	1.06	0.17
97	73–144	2.83	0.28	3.14	0.44	0.90	0.09	1.00	0.14
290	145–300	2.92	0.31	3.00	0.38	0.93	0.10	0.95	0.12
∞	Rack	2.96	0.34	2.96	0.34	0.94	0.11	0.94	0.11

To draw the epicycloidal tooth, first draw the pitch, addendum, root and clearance lines and space the pitch line for the teeth as in Fig. 4.

Draw the line of flank centers outside the pitch line at the tabular distance from it ("dis." in table) obtained from Table 9 and the line of face centers at the tabular distance ("dis.") inside of the pitch circle. Take the face radius ("rad.") in the dividers and draw the face curves from centers on the line of face centers. Take the flank radius ("rad.") and draw the flank curves from centers on the line of flank centers.

Table 9 gives the distances and radii for 1 diametral and 1-in. circular pitch. For other pitches multiply or divide as directed in the table.

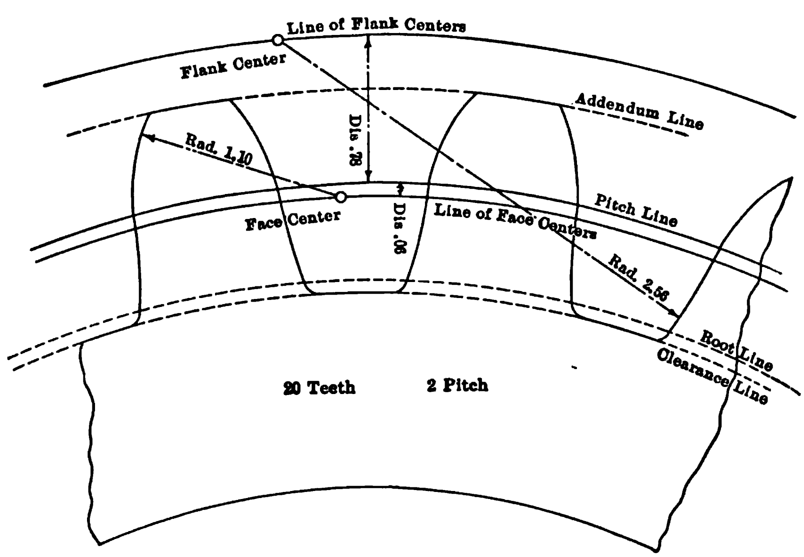


FIG. 4.—Grant's odontograph for epicycloidal teeth.

Fig. 4 shows the resulting distances and radii for a pinion of 2 diametral pitch with 20 teeth.

Strength of Spur Gears by Calculation

The working loads on spur gears are commonly determined from the formula proposed by WILFRED LEWIS (*Proc. Engrs. Club of Philadelphia*, 1892) as follows:

$$W = SPfy$$

in which W = pressure on teeth, lbs.,
S = fiber stress, lbs. per sq. in., this stress being dependent on the speed in accordance with Tables 10 and 11 below,
P = circular pitch, ins.,
f = face, ins.,
y = a factor for different numbers and forms of teeth in accordance with Table 12.

TABLE 10.—VALUES OF FACTOR S IN THE LEWIS FORMULA FOR STRENGTH OF GEARS

Pitch-line, speed ft. per min.	100 or less	200	300	600	900	1200	1800	2400
Cast-iron	8,000	6,000	4,800	4,000	3,000	2,400	2,000	1,700
Steel	20,000	15,000	12,000	10,000	7,500	6,000	5,000	4,300

The high-class, alloy-steel, heat-treated transmission gears of automobiles carry stresses materially in excess of those given in Table 10. F. M. Heldt, after analyzing the data of a large number of such gears, publishes the stresses of Table 11 as giving good results in intermittently meshed gears (*The Horseless Age, Apr.* 10, 1912). For constantly meshed gears the figures of the table should be reduced 15 per cent. A piston speed of 1000 ft. per min. was assumed when calculating the pitch-line speed.

TABLE 11.—VALUES OF FACTOR S IN THE LEWIS FORMULA DEDUCED FROM AUTOMOBILE PRACTICE

Pitch-line speed, ft. per min.	500	600	700	800	900	1,000	1,100	1,200
Alloy steel case hardened.	30,000	27,000	24,000	21,000	18,000	15,000		
Chrome nickel and chrome vanadium steel hardened all through.	60,000	53,000	47,000	42,000	38,000	34,000	30,000	27,000

The values of the factor y may also be obtained from Fig. 6 by ROBERT A. BRUCE (*Amer. Mach., Nov.* 21, 1901) which gives these values for not only standard proportions of teeth but for stub teeth which are now coming into use. The stub teeth for which the chart is drawn are somewhat shorter than those of the Hunt and Logue systems, which see, but reasonable allowances may be made for the difference.

The diagonal line shows a method of determining teeth of different systems but of equal strength. Thus, gears of 15 deg. obliquity, addendum .3183 P, 16 teeth; 20 deg. obliquity, addendum .3183 P, 20 teeth; 14½ deg. obliquity, addendum $\frac{P}{5}$, 29 teeth; and 20 deg. obliquity, addendum $\frac{P}{5}$, 35 teeth have the same strength, the diameter, face and speed being the same.

The Strength of Spur Gears by Graphics

The working loads on spur gears may be determined graphically from Fig. 7 which has been constructed

TABLE 12.—VALUES OF FACTOR y IN THE LEWIS FORMULA FOR STRENGTH OF GEARS

Number of teeth	Value of factor y			Number of teeth	Value of factor y			Number of teeth	Value of factor y		
	Involute 20°	Involute 15° cycloidal	Radial flanks		Involute 20°	Involute 15° cycloidal	Radial flanks		Involute 20°	Involute 15° cycloidal	Radial flanks
12	.078	.067	.052	20	.102	.090	.060	43	.126	.110	.068
13	.083	.070	.053	21	.104	.092	.061	50	.130	.112	.069
14	.088	.072	.054	23	.106	.094	.062	60	.134	.114	.070
15	.092	.075	.055	25	.108	.097	.063	75	.138	.116	.071
16	.094	.077	.056	27	.111	.100	.064	100	.142	.118	.072
17	.096	.080	.057	30	.114	.102	.065	150	.146	.120	.073
18	.098	.083	.058	34	.118	.104	.066	300	.150	.122	.074
19	.100	.087	.059	38	.122	.107	.067	rack	.154	.124	.075

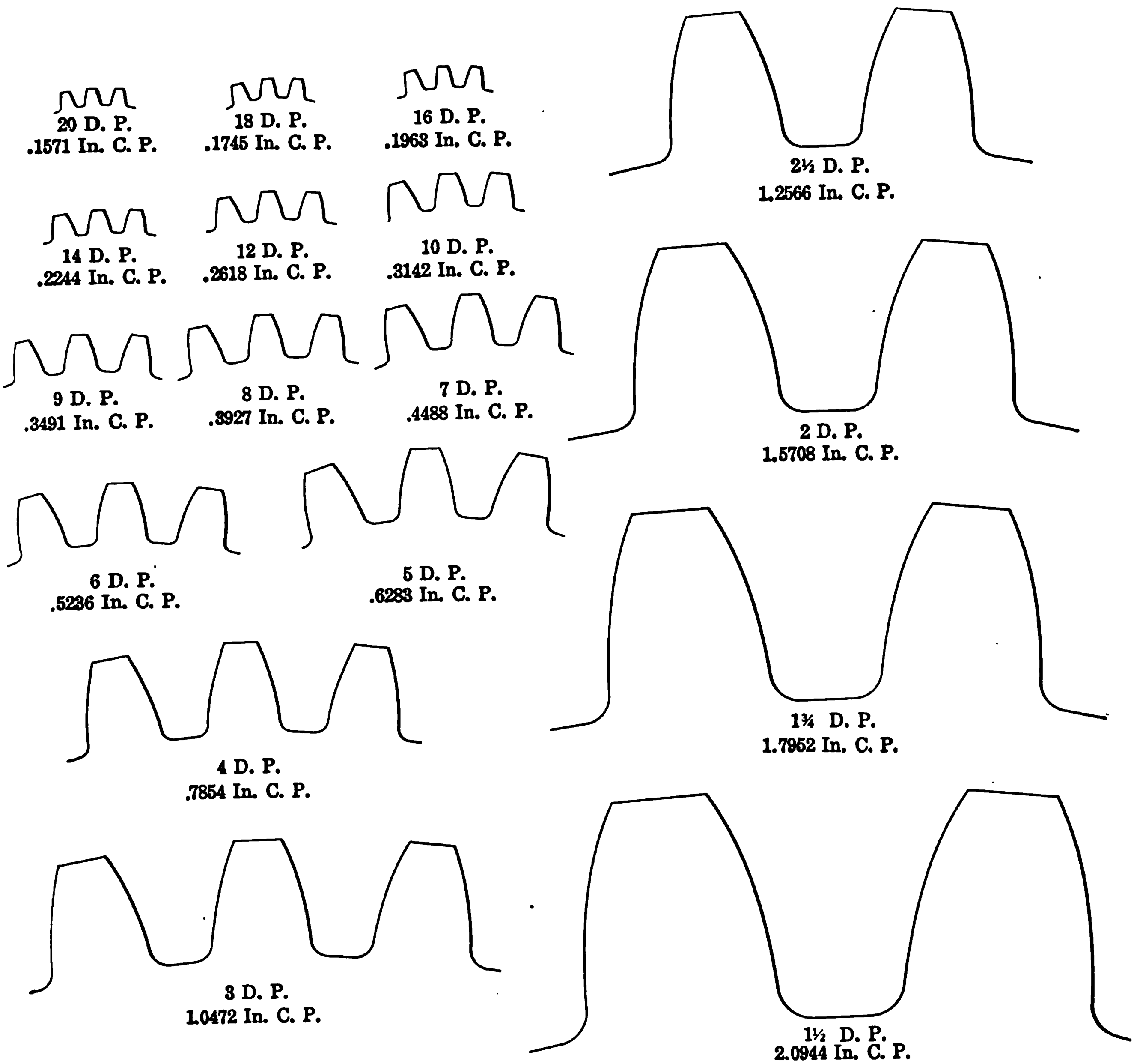


FIG. 5.—Gear teeth of full size, involute profile, 14½ degrees pressure angle.

to represent the Lewis formula by ROBERT A. BRUCE (*Amer. Mach.*, *May* 31, 1900).

The main chart, which applies to cast-iron and steel gears and to circular and diametral pitches, gives directly a preliminary false value for the load, which must be corrected by the use of the proper supplementary reduction scale below.

The use of the chart is best shown by an example: Required the working load on a cast-iron spur gear of 30 teeth, 2-in. pitch, 5-in.

face, at a pitch-line velocity of 1000 ft. per min., the teeth being of the 15-deg. involute form. Find 2-in. pitch on the left-hand vertical scale, trace to the right until the diagonal for 5-in. face is reached, then down for cast-iron or up for steel and read the preliminary false load of 10,000 lbs. for cast-iron or 25,000 lbs. for steel. Next apply the dividers to the reduction scale for 15-deg. involute and cycloidal teeth and take up the distance between 30 teeth and 1000 ft. pitch-line velocity. Step this off to the left from the preliminary false load and read the answers, 3000 lbs. for cast-iron and 7500 lbs. for steel.

Note that in the case of the reduction scale for 20-deg. involute teeth, the overlapping part indicates that with that tooth it is occasionally necessary to *add to* the preliminary false load. This is the case only when the selected point on the number-of-teeth scale is to the *right* of the selected point on the pitch-line-velocity scale.

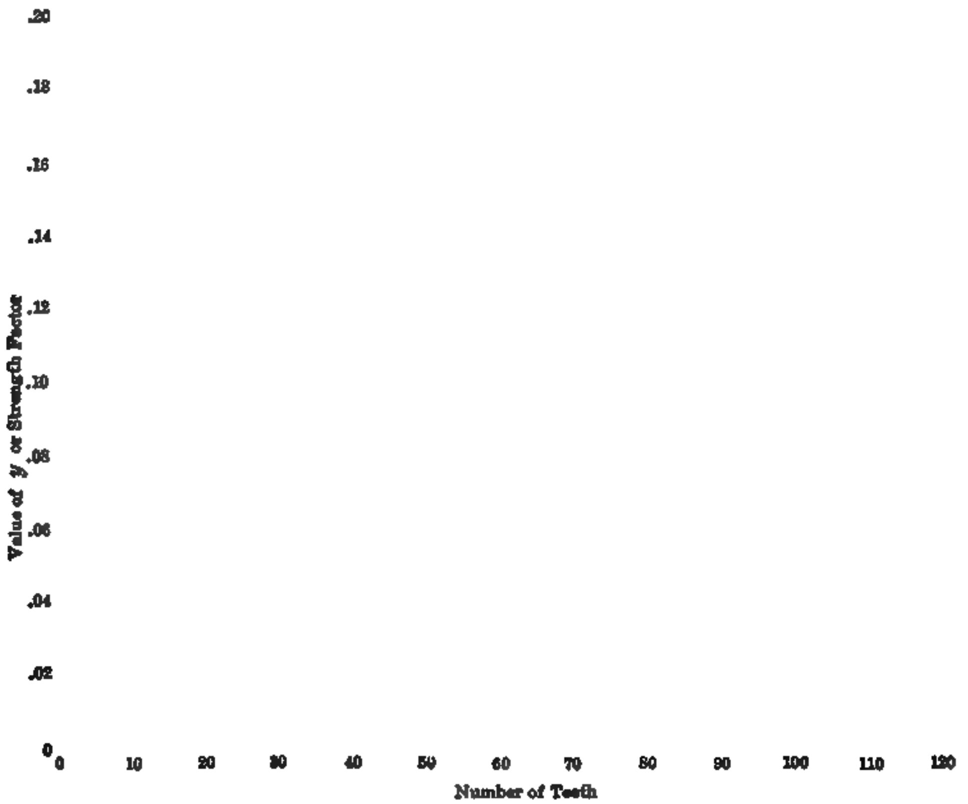


FIG. 6.—Values of y in the Lewis formula for the strength of gear teeth.

The working strength of shrouded gear teeth, according to WILFRED LEWIS (*Amer. Mach., Jan.* 30, 1902) is, for double shrouding to the full depth of the teeth, about 25 per cent. in excess of that of unshrouded teeth. For very narrow faces an increased load up to about 50 per cent. may be used. On the other hand, for single shrouding, Mr. Lewis reduces the increase to 10 per cent.

Many machine designers believe that gears are capable of carrying materially heavier loads than those determined by the Lewis formula —a belief that is supported by tests of gears to destruction by PROF. GUIDO H. MARX (*Trans. A. S. M. E., Vol.* 34). The gears tested were of 10 diametral pitch, which is too small a size to justify general deductions extending to heavy gears, although the indications of the experiments are clear.

Professor Marx concludes that the Lewis formula underestimates the static strength of gears and overestimates the effect of an increase of speed. The static breaking strength as determined from the Lewis formula is from 60 to 66⅔ per cent. of the actual strength developed in the tests, while, as the speed was increased, much larger discrepancies in the same direction were found between calculated and observed results.

Professor Marx concludes that a rational formula for the strength of gear teeth should include a factor for the arc of action and he embodies this conclusion and the other results of his experiments in the following formula:

$$W = \frac{spf}{k}\left(.154 - \frac{1.26}{n}\right)v\alpha$$

in which W = safe working load at pitch line, lbs.,

s = modulus of rupture = 39,000 in these tests but ordinarily to be taken = 36,000,

p = circular pitch, ins.,

f = width of face, ins.,

k = factor of safety,

n = number of teeth in gear,

v = velocity coefficient from Table 13,

α = arc of action coefficient from Table 14.

TABLE 13.—VELOCITY COEFFICIENTS

Velocity at pitch line, ft. per min.	0	100	150	200	300	400	500
Coefficient, v.	1	80	75	72	70	68	66

TABLE 14.—ARC OF ACTION COEFFICIENTS

Ratio: $\frac{\text{arc of action}}{\text{pitch arc}}$	1	1.4	1.6	1.7	1.8	1.9	1.95	2.00
Coefficient, α	1	1.05	1.1	1.15	1.24	1.38	1.47	1.00

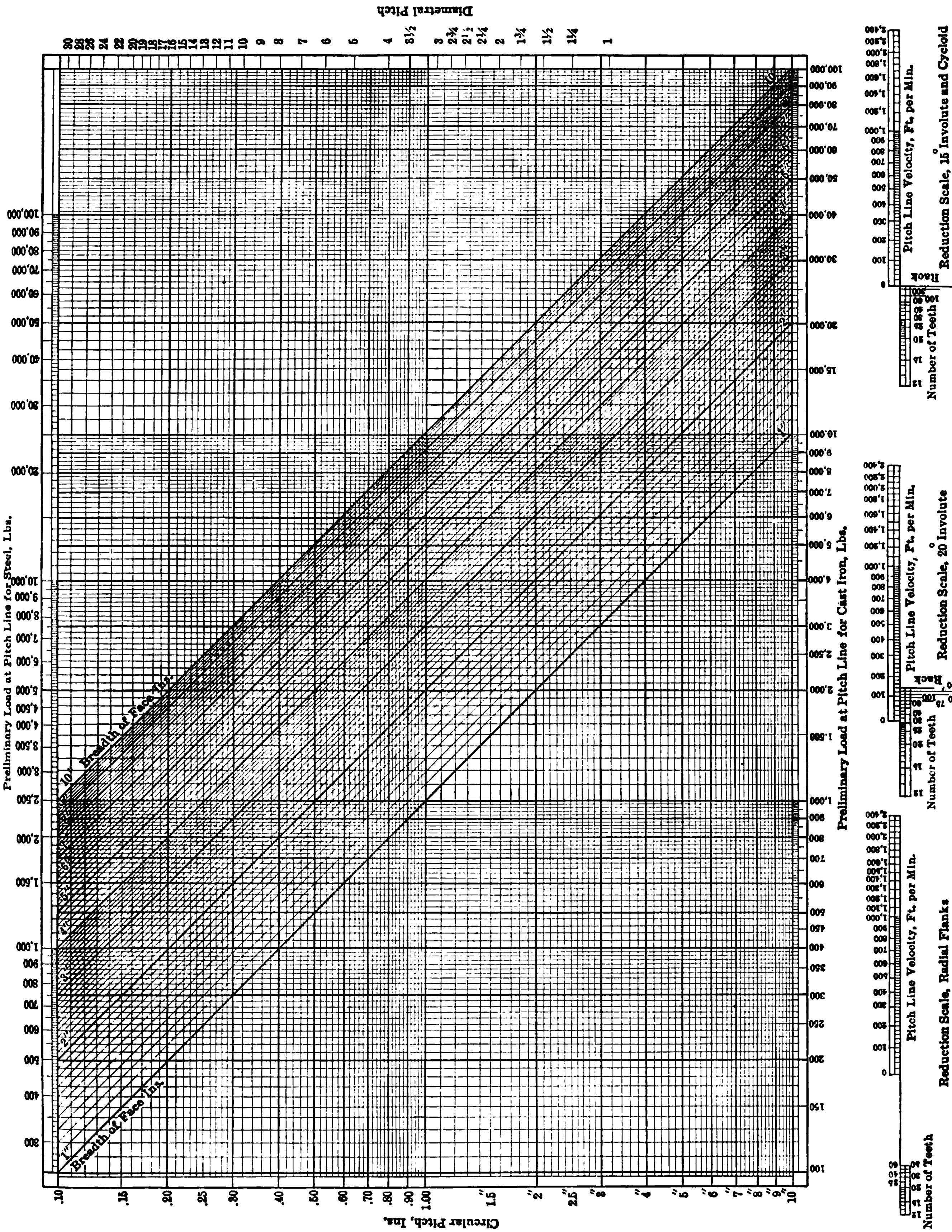


FIG. 7.—The strength of spur-gear teeth to the Lewis formula.

A list of spur gears that failed in service has been supplied by MICHAEL LONGRIDGE (*Proc. I. M. E.*, 1897). This list is repeated in Table 15 of which the last column has been added by the author.

TABLE 15.—SPUR GEARS THAT FAILED IN SERVICE

Power transmitted	Circumferential speed, ft. per min.	Mean pressure on teeth	Pitch of teeth	Breadth of teeth	Number of teeth in pinion	Velocity ratio, or ratio of revs. of pinion and wheel	Pressure per inch of pitch and face
I.H P.	Ft.	Lbs.	Ins.	Ins.	No.	Ratio	Lbs.
700	2,280	10,100	4.5	14	42	3.8	160
1,000	2,356	14,000	5.4	17	41	3.4	153
1,000	2,334	14,200	5.0	18	43	3.2	158
1,000	2,261	14,600	5.5	18	41	3.3	148
1,000	2,241	14,700	5.6	18	46	2.5	146
1,000	2,208	14,950	5.0	18	47	3.2	166
1,000	2,200	15,000	5.62	18	46	3.4	140
1,080	2,318	15,400	5.25	16¾	43	3.0	176
1,100	2,401	15,000	5.0	17	47	2.9	177
1,100	2,406	15,100	5.5	18	50	2.3	153
1,100	2,410	15,050	5.0	17	47	2.9	177
1,130	2,242	16,600	5.8	18	43	3.1	160
1,150	2,320	16,300	5.75	18	46	3.1	158
1,150	2,320	16,350	5.75	18	47	3.0	158
1,190	2,323	16,900	5.7	18	47	3.0	163
1,200	2,209	17,900	5.0	19	52	3.0	189
1,200	2,418	16,400	4.75	19	52	3.2	182
1,220	2,209	18,200	5.0	18¾	49	3.2	195
1,360	2,325	19,300	4.5	18	71	2.8	239

Strength of Bronze, Rawhide and Cloth Gears

The working loads of gears of bronze are less definitely known than those of iron or steel, but, for bronze of high quality, it is probably safe to impose loads one and one-half times those placed on cast-iron teeth of the same dimensions.

The working capacity of rawhide gears, according to W. H. DIEFENDORF, Chief Engineer, New Process Rawhide Co. (*Amer. Mach.*, *Apr.* 6, 1911) may be determined from the allowance of a pressure of 150 lbs. per in. of face for gears of 1 in. circular pitch. For other pitches the pressure allowance is to be taken in direct proportion, except that in no case should the pressure exceed 250 lbs. per in. of face.

These figures are to be applied to gears made of the highest grade of rawhide only. For lower grades the unit pressure should be reduced 15 per cent. or more. The figures are also intended for pinions having all rawhide working face. Pinions with bronze flanges having teeth cut through and forming part of the working face may be loaded with 10 to 25 per cent. greater pressures according to the grade of the bronze and the thickness of the flanges.

The unit pressure is not changed with the velocity or the number of teeth.

The practice of the General Electric Company with rawhide gears, according to A. SCHEIN (*General Electric Review*, *Apr.*, 1913), is to apply the Lewis formula using the stresses of Table 16 with the proviso that the dimensions must pass a further test because of the characteristics of these pinions due to heating. This test is embodied in the formula:

$$C = \frac{W \times V}{F \times N}$$

in which C = heating coefficient which must not exceed the values given in Table 17,

W = total load at pitch diameter, lbs.,
V = velocity at pitch diameter, ft. per min.,
N = number of teeth,
F = width of face, ins.

TABLE 16.—VALUES OF FACTOR S IN THE LEWIS FORMULA FOR RAWHIDE PINIONS

Speed at pitch dia. in ft. per min.	200	400	600	800	1000	1200	1400	1600	1800	2000
Stresses in lb. per sq. in.	3600	3300	3100	2800	2600	2400	2200	2000	1900	1800

TABLE 17.—MAXIMUM ALLOWABLE VALUES OF C FOR HEATING, RAWHIDE PINIONS

Diametral pitch........	1	1½	2	2½	3	3½	4
C....................	1600	1400	1200	1000	900	800	600

Rawhide gears should have some degree of lubrication. The best lubricant is a mixture of graphite and lard oil or tallow—never mineral oil.

The working capacity of cloth pinions, according to the General Electric Co., at whose works they originated, is, for most if not all services, equal to that of cast-iron gears of the same dimensions. The width of the cloth face should be equal to the face of the mating gear plus the aggregate end play of both shafts. The shrouds should never be permitted to run on the mating gear.

The limiting peripheral speed of metallic spur gears is about 2000 ft. per min. Ordinary cut gears begin to be objectionably noisy at peripheral speeds of about 1200 ft. per min.

Strength of Herringbone Gears

The working capacity of herringbone gears, in accordance with the practice of the Falk Co., American makers of the Wuest herringbone gears, is given by the formulas:

$$P = \frac{h.p. \times 33{,}000}{V}$$

$$P = \frac{p}{2.5} WK$$

in which $h.p.$ = horse-power.
P = tooth pressure, lbs.
K = admissible stress, lbs. per sq. in. in accordance with Fig. 8.
p = circular pitch, ins. For diametral pitch take nearest circular pitch.
W = total width of face, ins., including non-bearing width equal to 1 p.
V = velocity of pitch circle, ft. per min.

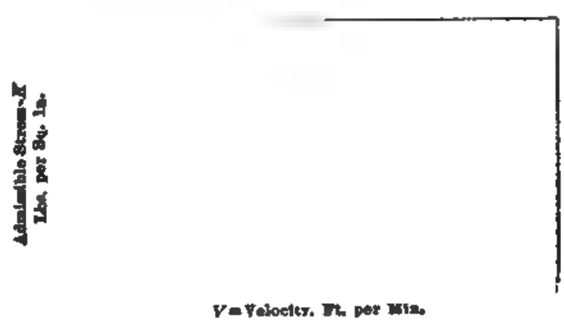


FIG. 8.—Values of admissible stress in herringbone gears of various materials.

Table 18 and Fig. 8 have been prepared from this formula by J. E. HOLVECK (*Mchy.*, *June*, 1913). The desirable speed limit, on account of noise, if the gears do not run in oil, is 1500 ft. per min.

TABLE 18.—HORSE-POWER OF HERRINGBONE GEARS

Pitch	Pitch dia., inches	Face, inches	Horse-power										
			Velocity	400	600	800	1000	1200	1400	1500	1600	1800	2000
6 D.P.	3½	4	Cast iron	7	10	12	14	15	17	18	19	20	21
			Cast steel	13	18	23	27	31	35	36	37	40	42
.5236" C.P.		5¼	Cast iron	9	13	16	18	20	·22	24	25	26	28
			Cast steel	17	24	30	35	41	46	47	49	53	55
5 D.P.	4.2	5	Cast iron	10	14	17	20	23	25	26	27	29	31
			Cast steel	19	27	34	40	46	51	53	55	59	63
.6283" C.P.		6¼	Cast iron	13	18	21	25	29	31	32	34	36	39
			Cast steel	24	34	43	50	58	64	66	69	74	79
4 D.P.	5¼	6¼	Cast iron	16	22	27	31	36	40	41	43	46	49
			Cast steel	29	41	53	62	71	80	83	86	93	99
.7854" C.P.		7¾	Cast iron	20	27	34	39	44	50	51	53	57	61
			Cast steel	36	51	66	77	88	99	103	107	116	122
3½ D.P.	6	7½	Cast iron	21	29	36	42	47	52	55	57	61	65
			Cast steel	39	55	70	83	95	105	110	114	123	131
.8976" C.P.		9	Cast iron	26	36	45	52	58	65	68	71	76	81
			Cast steel	48	68	87	103	118	130	136	141	154	162
3 D.P.	7	8¾	Cast iron	28	39	48	55	63	69	71	75	80	84
			Cast steel	51	72	92	110	126	139	146	151	165	173
1.0472" C.P.		10½	Cast iron	36	50	61	70	80	89	90	96	102	107
			Cast steel	65	92	117	140	160	177	184	192	210	220
2½ D.P.	8.4	10	Cast iron	41	57	70	80	91	101	106	109	116	122
			Cast steel	74	105	134	160	185	202	211	220	240	251
1.2566" C.P.		12½	Cast iron	51	71	87	100	114	126	132	137	145	152
			Cast steel	93	130	167	200	228	253	264	274	300	314
2 D.P.	10½	12½	Cast iron	64	89	109	125	143	158	166	171	182	195
			Cast steel	116	164	207	250	285	316	329	342	375	392
1.5708" C.P.		15½	Cast iron	79	111	136	154	178	196	206	213	226	242
			Cast steel	144	204	259	309	355	393	410	425	465	488
1¾ D.P.	12	14½	Cast iron	86	118	146	166	190	210	219	228	241	258
			Cast steel	155	218	279	331	378	420	438	455	496	522
1.7952" C.P.		18	Cast iron	106	146	180	206	234	261	272	282	300	320
			Cast steel	192	271	345	413	470	521	545	562	610	710
1½ D.P.	14	16	Cast iron	110	152	187	215	244	270	282	292	311	333
			Cast steel	199	280	358	426	488	540	564	·585	640	671
2.0944" C.P.		20	Cast iron	137	190	234	269	305	338	343	366	388	406
			Cast steel	249	350	447	534	610	675	707	732	800	838

Much higher speeds may be used satisfactorily but with correspondingly increased wear and noise. The pinion is commonly of tougher material than the gear, because of its greater wear, and the chart, Fig. 8, gives the admissible stress for various materials as developed by extended experience. The formulas and table are based on a tooth angle of 23 degrees. Note that, for the present purpose, the pitch is measured on the circumference—not on the normal.

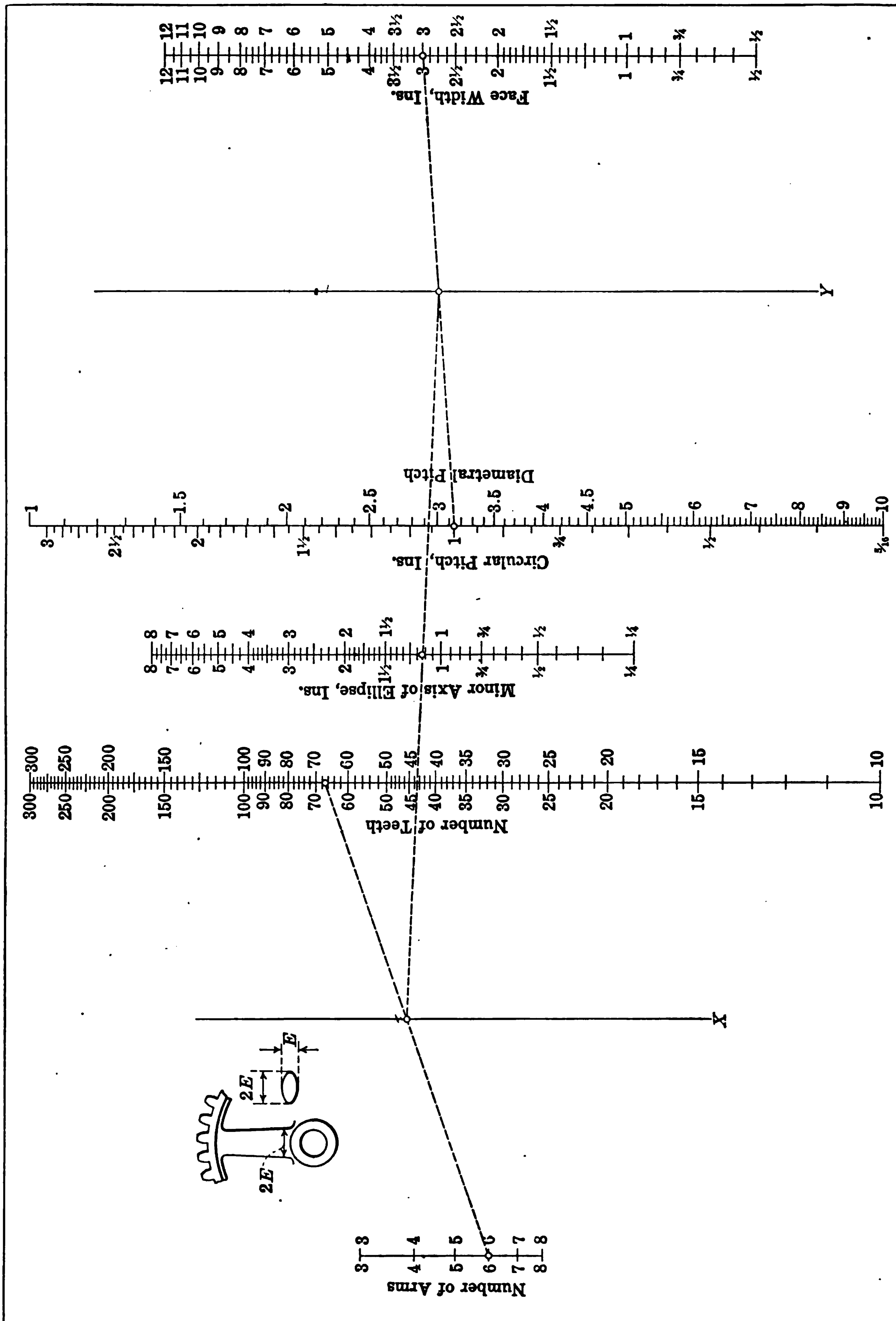
Dimensions of Spur Gear Parts

The dimensions of the arms of spur gears may be obtained from the following formulas and chart. The formulas are by HENRY HESS (*Amer. Mach., Apr.* 29, 1897) and represent the practice of the Niles Tool Works. They are based on an equal distribution of the pitch line load among the arms (which are assumed to act as cantilevers) and consider the sole load at the pitch line to be that given by the Lewis formula for the strength of gear teeth. For elliptical cross-sections:

$$E = \sqrt[3]{\frac{(N-7)P^3R}{20A}} \text{ for circular pitch}$$

$$E = \sqrt[3]{\frac{(N-7)\pi^3R}{20Ap^3}} \text{ for diametral pitch}$$

in which E = thickness of arm at hub, ins.
$2E$ = width of arm at hub, ins.
N = number of teeth.
P = circular pitch, ins.
p = diametral pitch.
R = ratio of face divided by circular pitch.
A = number of arms.



Join the number of arms and the number of teeth and note the intersection with axis X; join the pitch and width of face and note the intersection with axis Y; join the intersections and from the central scale and read the minor axis of the arm section at the hub. The solution given is for 6 arms, 67 teeth, 1 in. circular pitch, giving 1¼ ins.+for the thickness of the arm.

Fig. 9.—Dimensions of the arms of spur gears.

For other cross-sections:

$$Z = \frac{P^3R(N-7)}{50A} \text{ for circular pitch}$$

$$Z = \frac{\pi^3R(N-7)}{50p^3A} \text{ for diametral pitch}$$

in which Z = section modulus of arm at rim.

The same results for elliptical cross-sections may be obtained from Fig. 9 by PROF. J. B. PEDDLE (*Amer. Mach., Feb.* 13, 1913). The use of the chart is explained below it.

For large arms the designer will frequently prefer a cored section. A satisfactory one is that of Fig. 10, in which major and minor axes of both core and arm are relatively as 2 to 1. By equating the moduli of resistance for solid and hollow elliptical sections of these proportions, it is found that $E^3 = \frac{D^4 - d^4}{D}$, in which E is the thickness of the solid arm as obtained by chart or formula; d and D are dimensions of the cored arm. See Figs. 10 and 11.

In order to lessen the work of making the core box by substituting flat surfaces for curved ones, an approximation like Fig. 11 will add but slightly to the weight, as is shown by the ellipse dotted in for comparison.

The outlines are formed of circular arcs struck from four centers, which approximate very closely to the true ellipse and look better. The construction of the core sides is readily apparent from the sketch.

A suitable taper is 1 in 32 and 16, respectively, for the arm thickness and width, this gives a pleasing appearance for a moderately long arm, but it is not a hard-and-fast rule, as a greater or lesser taper may be employed to suit the designer's fancy without affecting the strength of the arm, unless the taper is made so excessive as to bring the dimensions at the rim down to one-half of those at the base.

As the tooth and arm are of the same material, the method is satisfactory for all cast gears, but this must not be interpreted to mean that this or any other formula will prevent shrinkage strain due to relatively large hubs or very heavy rims; where these occur, great care must be exercised in the foundry, and it will also not be amiss to add a generous amount of metal to the arms.

Other dimensions of spur gears may be obtained from the following formulas by C. H. LOGUE (*Amer. Mach., Sept.* 30, 1909) the notation being given in connection with Fig. 12.

$$M = \frac{3.927}{p}, \text{ or } 1.25P$$

$$M' = \frac{5.026}{p}, \text{ or } 1.60\,P$$

$$R' = \frac{2.868}{p}, \text{ or } .913P$$

$$W' = \frac{2.157}{p}, \text{ or } .6866P$$

$$R = \frac{1.769}{p}, \text{ or } .563P$$

Mean cross-section of arm = $1.3 \times t \times F$

$$A = \sqrt{\frac{\text{mean section} \times 1.27}{2,\ 2\tfrac{1}{2} \text{ or } 3}}$$

$$E = (2,\ 2\tfrac{1}{2} \text{ or } 3)A$$

(the same multiplier being used when finding both A and E)

$$D = d + \tfrac{3}{4}\sqrt[3]{NF} \text{ if reinforcements are used opposite keyways.}$$

Otherwise

$$D = 2d$$

The number of arms may be as follows:

Diameter	No. of arms
Up to 60 ins.	6
60 to 80 ins.	8
Over 80 ins.	10

For convenience of chucking without distortion the most desirable number of arms is either 6 or 9.

The width of face of spur gears according to traditional rules, should be from two and one-half to three times the circular pitch. The increased pressures put upon steel gears would, however, seem to call for greater widths since the increased *strength* of steel as compared with cast-iron is not accompanied by correspondingly increased *wearing properties*. Accordingly we find the gears of street railway cars with faces of five times the pitch.

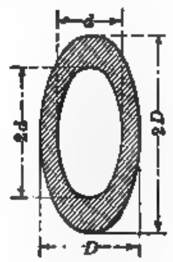
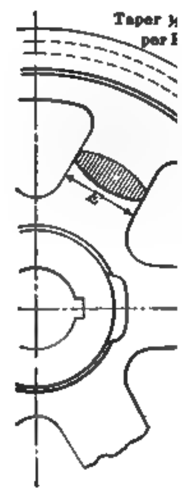


FIG. 10. FIG. 11.

FIGS. 10 and 11.—Proportions of hollow arms for large gears.



p = diametral pitch.
P = circular pitch, ins.
t = thickness of tooth at pitch line, ins.
N = number of teeth.

FIG. 12.—Notation of C. H. Logue's formulas for the dimensions of spur gears.

The factoring of numbers is frequently required in the calculation of trains of gearing. Table 20 by JOHN PARKER (*Amer. Mach., Dec.* 5, 1907) gives the smallest prime factors of all numbers between 1 and 9999. The top horizontal line gives the thousands and hundreds, the left vertical column the tens and units, and the body of the table the smallest prime factors. To find the other factors divide by the factor found and consult the table again. If no factor appears opposite a given number, the number is prime. Numbers divisible by 2 and 5 are omitted. Such numbers should be divided by 2 or 5 before consulting the table.

Example.—Required the smallest prime factor of 979. In the first table find 9 in the top horizontal line and 79 in the left vertical column. At the intersection is 11—the smallest prime factor. Similarly, consulting the table again, we find no entry for 971, showing that number to be prime.

TABLE 19.—PITCH DIAMETERS OF SPUR GEARS WHEN CIRCULAR PITCH IS GIVEN

By GEO. W. KLAGES (*Amer. Mach., Sept.* 22, 1910).

The Outside Diameter of a Gear Whose Addendum is Standard is the same as the Pitch Diameter of a Gear Having two more Teeth.

Circular pitch	1/7 in.	1/6 in.	1/5 in.	1/4 in.	5/16 in.	1/3 in.	3/8 in.	7/16 in.	1/2 in.	9/16 in.	5/8 in.	11/16 in.	3/4 in.	13/16 in.	7/8 in.	1 in.	1 1/8 ins.	1 1/4 ins.	1 1/2 ins.	1 3/4 ins.	2 ins.	Circular pitch
No. of teeth	Pitch diameter																					No. of teeth
11	.500	.584	.700	.875	1.094	1.167	1.313	1.532	1.751	1.970	2.188	2.407	2.626	2.845	3.064	3.501	3.939	4.377	5.252	6.127	7.003	11
12	.546	.637	.764	.955	1.194	1.273	1.432	1.671	1.910	2.149	2.387	2.626	2.865	3.103	3.342	3.820	4.297	4.775	5.730	6.684	7.639	12
13	.591	.690	.828	1.035	1.293	1.379	1.552	1.810	2.069	2.328	2.586	2.845	3.103	3.362	3.621	4.138	4.655	5.172	6.207	7.241	8.276	13
14	.637	.743	.891	1.114	1.393	1.485	1.671	1.950	2.223	2.507	2.785	3.064	3.342	3.621	3.899	4.456	5.013	5.570	6.684	7.798	8.913	14
15	.682	.796	.955	1.194	1.492	1.592	1.790	2.089	2.387	2.686	2.984	3.283	3.581	3.879	4.178	4.775	5.371	5.968	7.162	8.356	9.549	15
16	.728	.849	1.019	1.273	1.591	1.698	1.910	2.228	2.546	2.865	3.183	3.501	3.820	4.138	4.456	5.093	5.730	6.366	7.639	8.913	10.186	16
17	.773	.902	1.082	1.353	1.691	1.804	2.029	2.367	2.706	3.044	3.382	3.720	4.058	4.397	4.735	5.411	6.088	6.764	8.117	9.470	10.822	17
18	.819	.955	1.146	1.432	1.790	1.910	2.148	2.507	2.865	3.223	3.581	3.939	4.297	4.655	5.013	5.730	6.446	7.162	8.594	10.027	11.459	18
19	.864	1.008	1.210	1.512	1.890	2.016	2.268	2.646	3.024	3.402	3.780	4.158	4.536	4.914	5.292	6.048	6.804	7.560	9.072	10.584	12.096	19
20	.909	1.061	1.273	1.592	1.989	2.122	2.387	2.785	3.183	3.581	3.979	4.377	4.775	5.172	5.570	6.366	7.162	7.958	9.549	11.141	12.732	20
21	.955	1.114	1.337	1.671	2.089	2.228	2.507	2.924	3.342	3.760	4.178	4.596	5.013	5.431	5.849	6.684	7.520	8.356	10.027	11.698	13.369	21
22	1.000	1.167	1.401	1.751	2.188	2.334	2.626	3.064	3.501	3.939	4.377	4.814	5.252	5.690	6.127	7.003	7.878	8.753	10.504	12.255	14.006	22
23	1.046	1.220	1.464	1.830	2.288	2.440	2.745	3.203	3.660	4.118	4.576	5.033	5.491	5.948	6.406	7.321	8.236	9.151	10.982	12.812	14.642	23
24	1.091	1.273	1.528	1.910	2.387	2.546	2.865	3.342	3.820	4.297	4.775	5.252	5.730	6.207	6.684	7.639	8.594	9.549	11.459	13.369	15.279	24
25	1.137	1.326	1.592	1.989	2.487	2.653	2.984	3.481	3.979	4.476	4.974	5.471	5.968	6.466	6.963	7.958	8.952	9.947	11.937	13.926	15.915	25
26	1.182	1.379	1.655	2.069	2.586	2.759	3.103	3.621	4.138	4.655	5.172	5.690	6.207	6.724	7.241	8.276	9.311	10.345	12.414	14.483	16.552	26
27	1.228	1.432	1.719	2.149	2.686	2.865	3.223	3.760	4.297	4.834	5.371	5.909	6.446	6.983	7.520	8.594	9.669	10.743	12.891	15.040	17.189	27
28	1.273	1.485	1.783	2.228	2.785	2.971	3.342	3.899	4.456	5.013	5.570	6.127	6.684	7.241	7.798	8.913	10.027	11.141	13.369	15.597	17.825	28
29	1.319	1.538	1.846	2.308	2.885	3.077	3.462	4.038	4.615	5.192	5.769	6.346	6.923	7.500	8.077	9.231	10.385	11.539	13.846	16.154	18.462	29
30	1.364	1.592	1.910	2.387	2.984	3.183	3.581	4.178	4.775	5.371	5.968	6.565	7.162	7.759	8.356	9.549	10.743	11.937	14.324	16.711	19.099	30
31	1.410	1.645	1.973	2.467	3.084	3.289	3.700	4.317	4.934	5.551	6.167	6.784	7.401	8.017	8.634	9.868	11.101	12.334	14.801	17.268	19.735	31
32	1.455	1.698	2.037	2.546	3.183	3.395	3.820	4.456	5.093	5.730	6.366	7.003	7.639	8.276	8.913	10.186	11.459	12.732	15.279	17.825	20.372	32
33	1.501	1.751	2.101	2.626	3.283	3.501	3.939	4.595	5.252	5.909	6.565	7.222	7.878	8.535	9.191	10.504	11.817	13.130	15.756	18.382	21.008	33
34	1.546	1.804	2.164	2.706	3.382	3.607	4.058	4.735	5.411	6.088	6.764	7.440	8.117	8.793	9.470	10.822	12.175	13.528	16.234	18.939	21.645	34
35	1.592	1.857	2.228	2.785	3.482	3.714	4.178	4.874	5.570	6.267	6.963	7.659	8.356	9.052	9.748	11.141	12.533	13.926	16.711	19.496	22.282	35
36	1.637	1.910	2.292	2.865	3.581	3.820	4.297	5.013	5.729	6.446	7.162	7.878	8.594	9.311	10.027	11.459	12.892	14.324	17.189	20.053	22.918	36
37	1.683	1.963	2.355	2.944	3.681	3.926	4.417	5.152	5.889	6.625	7.361	8.097	8.833	9.569	10.305	11.777	13.250	14.722	17.666	20.611	23.555	37
38	1.728	2.016	2.419	3.024	3.780	4.032	4.536	5.292	6.048	6.804	7.560	8.316	9.072	9.828	10.584	12.096	13.608	15.120	18.144	21.168	24.192	38
39	1.773	2.069	2.483	3.103	3.880	4.138	4.655	5.431	6.207	6.983	7.759	8.535	9.310	10.086	10.862	12.414	13.966	15.518	18.621	21.725	24.828	39
40	1.819	2.122	2.546	3.183	3.979	4.244	4.775	5.570	6.366	7.162	7.958	8.753	9.549	10.345	11.141	12.732	14.324	15.915	19.099	22.282	25.465	40
41	1.864	2.175	2.610	3.263	4.078	4.350	4.894	5.710	6.525	7.341	8.157	8.972	9.788	10.604	11.419	13.051	14.682	16.313	19.576	22.839	26.101	41
42	1.910	2.228	2.674	3.342	4.178	4.456	5.013	5.849	6.684	7.520	8.356	9.191	10.027	10.862	11.698	13.369	15.040	16.711	20.053	23.396	26.738	42
43	1.955	2.281	2.737	3.422	4.277	4.562	5.133	5.988	6.844	7.699	8.555	9.410	10.265	11.121	11.976	13.687	15.398	17.109	20.531	23.953	27.375	43
44	2.001	2.334	2.801	3.501	4.377	4.668	5.252	6.127	7.003	7.878	8.754	9.629	10.504	11.380	12.255	14.006	15.756	17.507	21.008	24.510	28.011	44
45	2.046	2.387	2.865	3.581	4.476	4.775	5.372	6.267	7.162	8.057	8.952	9.848	10.743	11.638	12.533	14.324	16.114	17.905	21.486	25.067	28.648	45
46	2.092	2.440	2.928	3.661	4.576	4.881	5.491	6.406	7.321	8.236	9.151	10.067	10.982	11.897	12.812	14.642	16.472	18.303	21.963	25.624	29.284	46
47	2.137	2.493	2.992	3.740	4.675	4.987	5.610	6.545	7.480	8.415	9.350	10.285	11.220	12.155	13.090	14.961	16.831	18.701	22.441	26.181	29.921	47
48	2.183	2.546	3.056	3.820	4.775	5.093	5.730	6.684	7.639	8.594	9.549	10.504	11.459	12.414	13.369	15.279	17.189	19.099	22.918	26.738	30.558	48
49	2.228	2.599	3.119	3.899	4.874	5.199	5.849	6.824	7.799	8.773	9.748	10.723	11.698	12.673	13.647	15.597	17.547	19.496	23.396	27.295	31.194	49
50	2.274	2.653	3.183	3.979	4.974	5.305	5.968	6.963	7.958	8.952	9.947	10.942	11.937	12.931	13.926	15.915	17.905	19.894	23.873	27.852	31.831	50
51	2.319	2.706	3.247	4.058	5.073	5.411	6.088	7.102	8.117	9.132	10.146	11.161	12.175	13.190	14.204	16.234	18.263	20.292	24.351	28.409	32.468	51
52	2.365	2.759	3.310	4.138	5.173	5.517	6.207	7.241	8.276	9.311	10.345	11.380	12.414	13.449	14.483	16.552	18.621	20.690	24.828	28.966	33.104	52
53	2.410	2.812	3.374	4.218	5.272	5.623	6.327	7.381	8.435	9.490	10.544	11.598	12.653	13.707	14.762	16.870	18.979	21.088	25.306	29.523	33.741	53
54	2.456	2.865	3.438	4.297	5.371	5.729	6.446	7.520	8.594	9.669	10.743	11.817	12.891	13.966	15.040	17.189	19.337	21.486	25.783	30.080	34.377	54
55	2.501	2.918	3.501	4.377	5.471	5.836	6.565	7.659	8.753	9.848	10.942	12.036	13.130	14.224	15.319	17.507	19.695	21.884	26.260	30.637	35.014	55
56	2.546	2.971	3.565	4.456	5.570	5.942	6.685	7.798	8.913	10.027	11.141	12.255	13.369	14.483	15.597	17.825	20.053	22.282	26.738	31.194	35.651	56
57	2.592	3.024	3.629	4.536	5.670	6.048	6.804	7.938	9.072	10.206	11.340	12.474	13.608	14.742	15.876	18.144	20.412	22.680	27.215	31.751	36.287	57
58	2.637	3.077	3.692	4.615	5.769	6.154	6.923	8.077	9.231	10.385	11.539	12.693	13.846	15.000	16.154	18.462	20.770	23.077	27.693	32.318	36.924	58
59	2.683	3.130	3.756	4.695	5.869	6.260	7.043	8.216	9.390	10.564	11.738	12.911	14.085	15.259	16.433	18.780	21.128	23.475	28.170	32.865	37.561	59
60	2.728	3.183	3.820	4.775	5.968	6.366	7.162	8.355	9.549	10.743	11.937	13.130	14.324	15.518	16.711	19.099	21.486	23.873	28.648	33.422	38.197	60

TABLE 19.—PITCH DIAMETERS OF SPUR GEARS WHEN CIRCULAR PITCH IS GIVEN (*Continued*)

By GEO. W. KLAGES (*Am. Mach., Sept.* 22, 1910).

The Outside Diameter of a Gear Whose Addendum Is Standard Is the Same as the Pitch Diameter of a Gear Having Two More Teeth.

Circular pitch	1/7 in.	1/6 in.	1/5 in.	1/4 in.	5/16 in.	1/3 in.	3/8 in.	7/16 in.	1/2 in.	9/16 in.	5/8 in.	11/16 in.	3/4 in.	13/16 in.	7/8 in.	1 in.	1 1/8 ins.	1 1/4 ins.	1 1/2 ins.	1 3/4 ins.	2 ins.	Circular pitch
No. of teeth	Pitch diameter																					No. of teeth
61	2.774	3.236	3.883	4.854	6.068	6.472	7.281	8.495	9.708	10.922	12.136	13.349	14.563	15.776	16.990	19.417	21.844	24.271	29.125	33.980	38.834	61
62	2.819	3.289	3.947	4.934	6.167	6.578	7.401	8.634	9.868	11.101	12.334	13.568	14.801	16.035	17.268	19.735	22.202	24.669	29.603	34.537	39.470	62
63	2.865	3.342	4.011	5.013	6.267	6.684	7.520	8.773	10.027	11.280	12.533	13.787	15.040	16.293	17.547	20.053	22.560	25.067	30.080	35.094	40.107	63
64	2.910	3.395	4.074	5.093	6.366	6.790	7.639	8.913	10.186	11.459	12.732	14.006	15.279	16.552	17.825	20.372	22.918	25.465	30.558	35.651	40.744	64
65	2.956	3.448	4.138	5.173	6.466	6.897	7.759	9.052	10.345	11.638	12.931	14.224	15.518	16.811	18.104	20.690	23.276	25.863	31.035	36.208	41.380	65
66	3.001	3.501	4.202	5.252	6.565	7.003	7.878	9.191	10.504	11.817	13.130	14.443	15.756	17.069	18.382	21.008	23.634	26.261	31.513	36.765	42.017	66
67	3.047	3.554	4.265	5.332	6.665	7.109	7.998	9.330	10.663	11.996	13.329	14.662	15.995	17.328	18.661	21.327	23.993	26.658	31.990	37.322	42.653	67
68	3.092	3.607	4.329	5.411	6.764	7.215	8.117	9.470	10.822	12.175	13.528	14.881	16.234	17.586	18.939	21.645	24.351	27.056	32.468	37.879	43.290	68
69	3.138	3.660	4.393	5.491	6.864	7.321	8.236	9.609	10.982	12.354	13.727	15.100	16.472	17.845	19.218	21.963	24.709	27.454	32.945	38.436	43.927	69
70	3.183	3.714	4.456	5.570	6.963	7.427	8.356	9.748	11.141	12.533	13.926	15.319	16.711	18.104	19.496	22.282	25.067	27.852	33.422	38.993	44.563	70
71	3.229	3.767	4.520	5.650	7.063	7.533	8.475	9.887	11.300	12.713	14.125	15.537	16.950	18.362	19.775	22.600	25.425	28.250	33.900	39.550	45.200	71
72	3.274	3.820	4.584	5.730	7.162	7.639	8.594	10.027	11.459	12.892	14.324	15.756	17.189	18.621	20.053	22.918	25.783	28.648	34.377	40.107	45.837	72
73	3.320	3.873	4.647	5.809	7.261	7.745	8.714	10.166	11.618	13.071	14.523	15.975	17.427	18.880	20.332	23.237	26.141	29.046	34.855	40.664	46.473	73
74	3.365	3.926	4.711	5.889	7.361	7.851	8.833	10.305	11.777	13.250	14.722	16.194	17.666	19.138	20.611	23.555	26.499	29.444	35.332	41.221	47.110	74
75	3.410	3.979	4.775	5.968	7.460	7.958	8.952	10.444	11.936	13.429	14.921	16.413	17.905	19.397	20.889	23.873	26.857	29.842	35.810	41.778	47.746	75
76	3.456	4.032	4.838	6.048	7.560	8.064	9.072	10.584	12.096	13.608	15.120	16.632	18.144	19.655	21.168	24.192	27.215	30.239	36.287	42.335	48.383	76
77	3.501	4.085	4.902	6.127	7.659	8.170	9.191	10.723	12.255	13.787	15.319	16.850	18.382	19.914	21.446	24.510	27.574	30.637	36.765	42.892	49.020	77
78	3.547	4.138	4.966	6.207	7.759	8.276	9.311	10.862	12.414	13.966	15.518	17.069	18.621	20.173	21.725	24.828	27.932	31.035	37.242	43.449	49.656	78
79	3.592	4.191	5.029	6.287	7.858	8.382	9.430	11.001	12.573	14.145	15.717	17.288	18.860	20.431	22.003	25.146	28.290	31.433	37.720	44.006	50.293	79
80	3.638	4.244	5.093	6.366	7.958	8.488	9.549	11.141	12.732	14.324	15.915	17.507	19.099	20.690	22.282	25.465	28.648	31.831	38.197	44.563	50.930	80
81	3.683	4.297	5.156	6.446	8.057	8.594	9.669	11.280	12.891	14.503	16.114	17.726	19.337	20.949	22.560	25.783	29.006	32.229	38.675	45.120	51.566	81
82	3.729	4.350	5.220	6.525	8.157	8.700	9.788	11.419	13.051	14.682	16.313	17.945	19.576	21.207	22.839	26.101	29.364	32.627	39.152	45.677	52.203	82
83	3.774	4.403	5.284	6.605	8.256	8.806	9.907	11.558	13.210	14.861	16.512	18.163	19.815	21.466	23.117	26.420	29.722	33.025	39.629	46.234	52.839	83
84	3.820	4.456	5.347	6.684	8.356	8.912	10.027	11.698	13.369	15.040	16.711	18.382	20.053	21.725	23.396	26.738	30.080	33.423	40.107	46.972	53.476	84
85	3.865	4.509	5.411	6.764	8.455	9.019	10.146	11.837	13.528	15.229	16.910	18.601	20.292	21.983	23.674	27.056	30.438	33.820	40.584	47.349	54.113	85
86	3.911	4.562	5.475	6.844	8.555	9.125	10.265	11.976	13.687	15.398	17.109	18.820	20.531	22.242	23.953	27.375	30.796	34.218	41.062	47.906	54.749	86
87	3.956	4.615	5.538	6.923	8.654	9.231	10.385	12.116	13.846	15.577	17.308	19.039	20.770	22.500	24.231	27.693	31.155	34.616	41.539	48.463	55.386	87
88	4.002	4.668	5.602	7.003	8.753	9.337	10.504	12.255	14.005	15.756	17.507	19.258	21.008	22.759	24.510	28.011	31.513	35.014	42.017	49.020	56.023	88
89	4.047	4.721	5.666	7.082	8.853	9.443	10.624	12.394	14.165	15.935	17.706	19.477	21.247	23.018	24.788	28.330	31.871	35.412	42.494	49.577	56.659	89
90	4.093	4.775	5.729	7.162	8.952	9.549	10.743	12.533	14.324	16.114	17.905	19.695	21.486	23.276	25.067	28.648	32.229	35.810	42.972	50.134	57.296	90
91	4.138	4.828	5.793	7.242	9.052	9.655	10.862	12.673	14.483	16.293	18.104	19.914	21.725	23.535	25.345	28.966	32.587	36.208	43.449	50.691	57.932	91
92	4.184	4.881	5.857	7.321	9.151	9.761	10.982	12.812	14.642	16.473	18.303	20.133	21.963	23.794	25.624	29.284	32.945	36.606	43.927	51.248	58.569	92
93	4.229	4.934	5.920	7.401	9.251	9.867	11.101	12.951	14.801	16.652	18.502	20.352	22.202	24.052	25.902	29.603	33.303	37.004	44.404	51.805	59.206	93
94	4.274	4.987	5.984	7.480	9.350	9.973	11.220	13.090	14.960	16.831	18.701	20.571	22.441	24.311	26.181	29.921	33.661	37.401	44.882	52.362	59.842	94
95	4.320	5.040	6.048	7.560	9.450	10.080	11.340	13.230	15.120	17.010	18.900	20.790	22.679	24.569	26.459	30.239	34.019	37.799	45.359	52.919	60.479	95
96	4.365	5.093	6.111	7.639	9.549	10.186	11.459	13.369	15.279	17.189	19.099	21.008	22.918	24.828	26.738	30.558	34.377	38.197	45.837	53.476	61.115	96
97	4.411	5.146	6.175	7.719	9.649	10.292	11.578	13.508	15.438	17.368	19.298	21.227	23.157	25.087	27.016	30.876	34.736	38.595	46.314	54.033	61.752	97
98	4.456	5.199	6.239	7.799	9.748	10.398	11.698	13.648	15.597	17.547	19.496	21.446	23.396	25.345	27.295	31.194	35.094	38.993	46.791	54.590	62.389	98
99	4.502	5.252	6.302	7.878	9.848	10.504	11.817	13.787	15.756	17.726	19.695	21.665	23.634	25.604	27.573	31.513	35.452	39.391	47.269	55.147	63.025	99
100	4.547	5.305	6.366	7.958	9.947	10.610	11.937	13.926	15.915	17.905	19.894	21.884	23.873	25.863	27.852	31.831	35.810	39.789	47.746	55.704	63.662	100
	Standard addendum, depth of space below pitch line, and whole depth of tooth																					
Addendum..	.04547	.05305	.06366	.07958	.09947	.10610	.11937	.13926	.15915	.17905	.19894	.21884	.23873	.25863	.27852	.31831	.35810	.39789	.47746	.55704	.63662	..Addendum
D. of space bel. P.L.	.05262	.06139	.07366	.09208	.11510	.12277	.13812	.16114	.18415	.20717	.23019	.25321	.27623	.29925	.32227	.36831	.41435	.46039	.55246	.64454	.73662	D. of space bel. P.L.
W.D........	.09809	.11444	.13732	.17166	.21457	.22887	.25748	.30040	.34331	.38622	.42014	.47205	.51496	.55788	.60079	.68662	.77245	.85827	1.02093	1.20158	1.37324	........W.D.

TABLE 20.—PRIME FACTORS OF NUMBERS FROM 1 TO 9999

Nos. 1 to 2499

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
01–99	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Nos. 2501 to 4999

	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49
01–99	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Nos. 5001 to 7499

	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74
01–99	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Nos. 7501 to 9999

	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99
01–99	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

BEVEL GEARS

The dimensions and angles of bevel gears may be calculated from the formulas of Tables 1 and 2 which have been arranged by JOHN EDGAR (*Amer. Mach., Apr.* 13, 1905). The notation of the formulas is given in Figs. 1, 2 and 3.

TABLE 1.—FORMULAS FOR DIMENSIONS AND ANGLES OF BEVEL GEARS WITH SHAFTS AT RIGHT ANGLES

		Pinion		Gear
Diametral pitch	P	....	P	....
Number of teeth	N_1	$D_1 \times P$	N_2	$D_2 \times P$
Pitch diameter	D_1	$N_1 \div P$	D_2	$N_2 \div P$
Outside diameter	OD_1	$D_1 + d_1$	OD_2	$D_2 + d_2$
Diameter increase[1]	d_1	$\frac{2 \cos \phi}{P}$	d_2	$\frac{2 \sin \phi}{P}$
Center angle	ϕ	$\tan \phi = \frac{N_1}{N_2}$	θ	$90 - \phi$
Angle increment	δ	$\tan \delta = \frac{2 \sin \phi}{N_1}$	δ	....
Face angle	.....	$\theta + \delta$	.....	$\phi + \delta$
Cutting angle	.....	$\phi - \delta$	.....	$\theta - \delta$
Number of teeth for which to select cutter.	N'	$\frac{N_1}{\cos \phi}$	N''	$\frac{N_2}{\sin \phi}$
Backing increment[1]	B_1	$\frac{\sin \phi}{P}$	B_2	$\frac{\cos \phi}{P}$

[1] Note that backing increment is the same as one-half the diameter increase of the mating gear.
For Notation see Fig. 1.

The dimensions and angles of bevel gears having shafts at right angles may, in most cases, be obtained from Table 3 by CHAS. WATTS (*The Engr., Aug.* 13, 1909).

The proportion of the gears, that is, the ratio of the number of teeth in the large gear divided by the number of teeth in the pinion, being known, the center angles are read directly from columns *B*.

To find the outside diameters, add the diameter increment to the pitch diameters. The diameter increment is found by dividing the quantity in column *F*, for the large or small gear respectively, by the diametral pitch.

To find the face angles add the angle increment to the center angles. The angle increment is found by dividing the quantity in column *E* by the number of teeth in the large gear.

To find the cutting angles subtract the angle decrement from the center angles. The angle decrement is found by dividing the quantity in column *D* by the number of teeth in the large gear. The cutting angle thus obtained gives the standard Brown and Sharpe depth for involute teeth.

TABLE 2.—FORMULAS FOR DIMENSIONS AND ANGLES OF BEVEL GEARS WITH SHAFTS NOT AT RIGHT ANGLES

		Pinion		Gear
Diametral pitch	P	....	P	....
Number of teeth	N_1	$D_1 \times P$	N_2	$D_2 \times P$
Pitch diameter	D_1	$N_1 \div P$	D_2	$N_2 \div P$
Outside diameter	OD_1	$D_1 + d_1$	OD_2	$D_2 + d_2$
Diameter increase	d_1	$\frac{2 \cos \phi}{P}$	d_2	$\frac{2 \cos \theta}{P}$
Center angle—shafts at less than 90 deg.	ϕ	$\tan \phi = \frac{\sin O}{\frac{N_2}{N_1} + \cos O}$	θ	$O - \phi$
Center angle—shafts at greater than 90 deg.	ϕ	$\tan \phi = \frac{\cos V}{\frac{N_2}{N_1} - \sin V}$	θ	$O - \phi$
Angle of shafts	Ω	....	.....	....
	V	$O - 90°$	.....	....
Angle increment	δ	$\tan \delta = \frac{2 \sin \phi}{N_1}$	.....	....
Face angle	.....	$(90° - \phi) + \delta$	.....	$(90° - \theta) + \delta$
Cutting angle	.....	$\phi - \delta$	.....	$\theta - \delta$
Number of teeth for which to select cutter.	N'	$\frac{N_1}{\cos \phi}$	N''	$\frac{N_2}{\cos \theta}$
Backing increment	B_1	$\frac{\sin \phi}{P}$	B_2	$\frac{\sin \theta}{P}$

For Notation see Figs. 2 and 3.

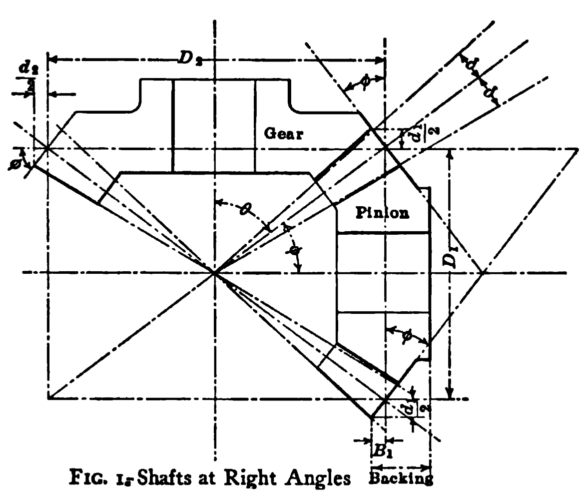


FIG. 1. Shafts at Right Angles

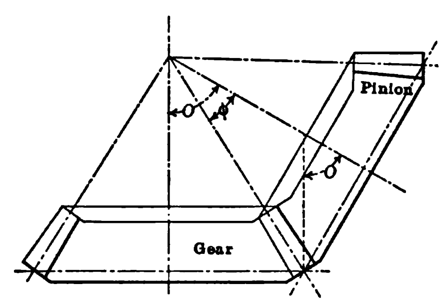


FIG. 2. Shafts at less than 90 Degrees

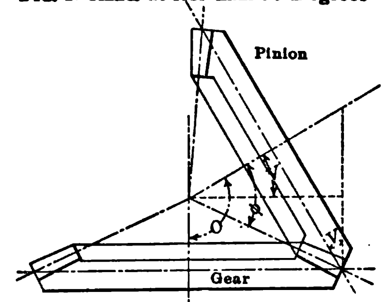


FIG. 3. Shafts at more than 90 Degrees

FIGS. 1 to 3.—Notation of formulas for dimensions and angles of bevel gears.

TABLE 3.—DIMENSIONS AND ANGLES OF BEVEL GEARS WITH SHAFTS AT RIGHT ANGLES

For notation see page 107.

Proportion of wheels	F		E	D	B	
	Large wheel	Small wheel			Large wheel	Small wheel
1 to 1	1.414 ÷ J	1.414 ÷ J	80°-36' ÷ K	93°-20' ÷ K	45°	45°
1 to 1.020	1.400 ÷ J	1.428 ÷ J	81°-18' ÷ K	94°-14' ÷ K	45°-34'	44°-26'
1 to 1.040	1.386 ÷ J	1.442 ÷ J	82°- 0' ÷ K	95°- 9' ÷ K	46°- 7'	43°-53'
1 to 1.050	1.379 ÷ J	1.448 ÷ J	83°-16' ÷ K	95°-35' ÷ K	46°-24'	43°-36'
1 to 1.100	1.345 ÷ J	1.479 ÷ J	85°- 4' ÷ K	97°-39' ÷ K	47°-43'	42°-17'
1 to 1.111	1.338 ÷ J	1.486 ÷ J	85°-27' ÷ K	98°- 5' ÷ K	48°-00'	42°-00'
1 to 1.125	1.328 ÷ J	1.494 ÷ J	85°-57' ÷ K	98°-39' ÷ K	48°-22'	41°-38'
1 to 1.143	1.317 ÷ J	1.505 ÷ J	86°-31' ÷ K	99°-19' ÷ K	48°-48'	41°-12'
1 to 1.150	1.312 ÷ J	1.509 ÷ J	86°-45' ÷ K	99°-35' ÷ K	48°-59'	41°- 1'
1 to 1.166	1.301 ÷ J	1.518 ÷ J	87°-18' ÷ K	100°-13' ÷ K	49°-24'	40°-36'
1 to 1.180	1.293 ÷ J	1.526 ÷ J	87°-43' ÷ K	100°-41' ÷ K	49°-43'	40°-17'
1 to 1.200	1.280 ÷ J	1.535 ÷ J	88°-21' ÷ K	101°-24' ÷ K	50°-12'	39°-48'
1 to 1.240	1.255 ÷ J	1.557 ÷ J	89°-32' ÷ K	102°-46' ÷ K	51°- 7'	38°-53'
1 to 1.250	1.249 ÷ J	1.561 ÷ J	89°-47' ÷ K	103°- 4' ÷ K	51°-20'	38°-40'
1 to 1.280	1.231 ÷ J	1.576 ÷ J	90°-37' ÷ K	104°- 1' ÷ K	52°-00'	38°-00'
1 to 1.285	1.227 ÷ J	1.579 ÷ J	90°-47' ÷ K	104°-12' ÷ K	52°- 8'	37°-52'
1 to 1.300	1.219 ÷ J	1.585 ÷ J	91°- 9' ÷ K	104°-38' ÷ K	52°-26'	37°-34'
1 to 1.320	1.208 ÷ J	1.594 ÷ J	91°-39' ÷ K	105°-12' ÷ K	52°-51'	37°- 9'
1 to 1.333	1.200 ÷ J	1.600 ÷ J	91°-59' ÷ K	105°-35' ÷ K	53°- 7'	36°-53'
1 to 1.350	1.190 ÷ J	1.607 ÷ J	92°-24' ÷ K	106°- 4' ÷ K	53°-28'	36°-32'
1 to 1.400	1.162 ÷ J	1.627 ÷ J	93°-41' ÷ K	107°-25' ÷ K	54°-28'	35°-32'
1 to 1.420	1.151 ÷ J	1.635 ÷ J	94°- 2' ÷ K	107°-56' ÷ K	54°-51'	35°- 9'
1 to 1.428	1.147 ÷ J	1.638 ÷ J	94°-12' ÷ K	108°- 7' ÷ K	55°-00'	35°-00'
1 to 1.440	1.141 ÷ J	1.642 ÷ J	94°-27' ÷ K	108°-25' ÷ K	55°-13'	34°-47'
1 to 1.450	1.135 ÷ J	1.646 ÷ J	94°-39' ÷ K	108°-39' ÷ K	55°-24'	34°-36'
1 to 1.480	1.120 ÷ J	1.657 ÷ J	95°-16' ÷ K	109°-22' ÷ K	55°-57'	34°- 3'
1 to 1.500	1.109 ÷ J	1.664 ÷ J	95°-41' ÷ K	109°-50' ÷ K	56°-19'	33°-41'
1 to 1.520	1.099 ÷ J	1.670 ÷ J	96°- 5' ÷ K	110°-16' ÷ K	56°-40'	33°-20'
1 to 1.550	1.084 ÷ J	1.680 ÷ J	96°-37' ÷ K	110°-54' ÷ K	57°-10'	32°-50'
1 to 1.560	1.079 ÷ J	1.684 ÷ J	96°-48' ÷ K	111°- 7' ÷ K	57°-20'	32°-40'
1 to 1.600	1.060 ÷ J	1.696 ÷ J	97°-31' ÷ K	111°-56' ÷ K	58°-00'	32°-00'
1 to 1.640	1.041 ÷ J	1.706 ÷ J	98°-11' ÷ K	112°-42' ÷ K	58°-38'	31°-22'
1 to 1.650	1.036 ÷ J	1.710 ÷ J	98°-21' ÷ K	112°-53' ÷ K	58°-47'	31°-13'
1 to 1.666	1.029 ÷ J	1.715 ÷ J	98°-36' ÷ K	113°-11' ÷ K	59°- 2'	30°-58'
1 to 1.680	1.023 ÷ J	1.718 ÷ J	98°-49' ÷ K	113°-25' ÷ K	59°-14'	30°-46'
1 to 1.700	1.014 ÷ J	1.724 ÷ J	99°- 7' ÷ K	113°-46' ÷ K	59°-32'	30°-28'
1 to 1.720	1.005 ÷ J	1.730 ÷ J	99°-25' ÷ K	114°- 7' ÷ K	59°-50'	30°-10'
1 to 1.750	.992 ÷ J	1.736 ÷ J	99°-50' ÷ K	114°-36' ÷ K	60°-15'	29°-45'
1 to 1.760	.988 ÷ J	1.739 ÷ J	100°- 0' ÷ K	114°-46' ÷ K	60°-24'	29°-36'
1 to 1.800	.971 ÷ J	1.748 ÷ J	100°-31' ÷ K	115°-23' ÷ K	60°-57'	29°- 3'
1 to 1.840	.955 ÷ J	1.757 ÷ J	101°- 3' ÷ K	116°- 0' ÷ K	61°-29'	28°-31'
1 to 1.850	.951 ÷ J	1.759 ÷ J	101°-10' ÷ K	116°- 8' ÷ K	61°-37'	28°-23'
1 to 1.880	.940 ÷ J	1.765 ÷ J	101°-30' ÷ K	116°-32' ÷ K	61°-59'	28°- 1'
1 to 1.900	.932 ÷ J	1.770 ÷ J	101°-45' ÷ K	116°-47' ÷ K	62°-14'	27°-46'
1 to 1.920	.924 ÷ J	1.774 ÷ J	102°- 0' ÷ K	117°- 3' ÷ K	62°-29'	27°-31'
1 to 1.950	.912 ÷ J	1.779 ÷ J	102°-19' ÷ K	117°-27' ÷ K	62°-51'	27°- 9'
1 to 1.960	.908 ÷ J	1.781 ÷ J	102°-26' ÷ K	117°-34' ÷ K	62°-58'	27°- 2'
1 to 2.000	.894 ÷ J	1.789 ÷ J	102°-51' ÷ K	118°- 3' ÷ K	63°-26'	26°-34'
1 to 2.040	.880 ÷ J	1.796 ÷ J	103°-15' ÷ K	118°-31' ÷ K	63°-53'	26°- 7'
1 to 2.080	.866 ÷ J	1.802 ÷ J	103°-39' ÷ K	118°-58' ÷ K	64°-20'	25°-40'
1 to 2.100	.859 ÷ J	1.805 ÷ J	103°-49' ÷ K	119°-10' ÷ K	64°-32'	25°-28'
1 to 2.120	.853 ÷ J	1.809 ÷ J	104°- 0' ÷ K	119°-23' ÷ K	64°-45'	25°-15'
1 to 2.160	.840 ÷ J	1.815 ÷ J	104°-21' ÷ K	119°-46' ÷ K	65°- 9'	24°-51'
1 to 2.200	.830 ÷ J	1.821 ÷ J	104°-40' ÷ K	120°- 9' ÷ K	65°-33'	24°-27'
1 to 2.240	.815 ÷ J	1.826 ÷ J	105°- 0' ÷ K	120°-32' ÷ K	65°-57'	24°- 3'
1 to 2.250	.812 ÷ J	1.827 ÷ J	105°- 5' ÷ K	120°-37' ÷ K	66°- 2'	23°-58'
1 to 2.280	.803 ÷ J	1.831 ÷ J	105°-19' ÷ K	120°-53' ÷ K	66°-19'	23°-41'
1 to 2.300	.797 ÷ J	1.834 ÷ J	105°-27' ÷ K	121°- 2' ÷ K	66°-30'	23°-30'
1 to 2.333	.788 ÷ J	1.838 ÷ J	105°-42' ÷ K	121°-19' ÷ K	66°-48'	23°-12'
1 to 2.360	.780 ÷ J	1.841 ÷ J	105°-52' ÷ K	121°-31' ÷ K	67°- 2'	22°-58'

TABLE 3.—DIMENSIONS AND ANGLES OF BEVEL GEARS WITH SHAFTS AT RIGHT ANGLES—(*Continued*)

Proportion of wheels	*F*		*E*	*D*	*B*	
	Large wheel	Small wheel			Large wheel	Small wheel
1 to 2.400	.769 ÷ *J*	1.846 ÷ *J*	106°— 9′ ÷ *K*	121°—50′ ÷ *K*	67°—23′	22°—37′
1 to 2.440	.758 ÷ *J*	1.850 ÷ *J*	106°—24′ ÷ *K*	122°— 8′ ÷ *K*	67°—43′	22°—17′
1 to 2.480	.747 ÷ *J*	1.855 ÷ *J*	106°—39′ ÷ *K*	122°—26′ ÷ *K*	68°— 3′	21°—57′
1 to 2.500	.743 ÷ *J*	1.857 ÷ *J*	106°—46′ ÷ *K*	122°—34′ ÷ *K*	68°—12′	21°—48′
1 to 2.520	.738 ÷ *J*	1.859 ÷ *J*	106°—53′ ÷ *K*	122°—40′ ÷ *K*	68°—21′	21°—39′
1 to 2.560	.727 ÷ *J*	1.863 ÷ *J*	107°— 7′ ÷ *K*	122°—57′ ÷ *K*	68°—40′	21°—21′
1 to 2.600	.718 ÷ *J*	1.866 ÷ *J*	107°—18′ ÷ *K*	123°—11′ ÷ *K*	68°—57′	21°— 3′
1 to 2.640	.708 ÷ *J*	1.870 ÷ *J*	107°—32′ ÷ *K*	123°—25′ ÷ *K*	69°—15′	20°—45′
1 to 2.666	.702 ÷ *J*	1.872 ÷ *J*	107°—39′ ÷ *K*	123°—34′ ÷ *K*	69°—26′	20°—34′
1 to 2.700	.694 ÷ *J*	1.875 ÷ *J*	107°—50′ ÷ *K*	123°—47′ ÷ *K*	69°—41′	20°—19′
1 to 2.720	.690 ÷ *J*	1.877 ÷ *J*	107°—57′ ÷ *K*	123°—53′ ÷ *K*	69°—49′	20°—11′
1 to 2.760	.681 ÷ *J*	1.880 ÷ *J*	108°— 7′ ÷ *K*	124°— 6′ ÷ *K*	70°— 5′	19°—55′
1 to 2.800	.672 ÷ *J*	1.883 ÷ *J*	108°—17′ ÷ *K*	124°—18′ ÷ *K*	70°—21′	19°—39′
1 to 2.840	.664 ÷ *J*	1.886 ÷ *J*	108°—28′ ÷ *K*	124°—30′ ÷ *K*	70°—36′	19°—24′
1 to 2.880	.656 ÷ *J*	1.889 ÷ *J*	108°—37′ ÷ *K*	124°—41′ ÷ *K*	70°—51′	19°— 9′
1 to 2.900	.651 ÷ *J*	1.891 ÷ *J*	108°—43′ ÷ *K*	124°—53′ ÷ *K*	70°—59′	19°— 1′
1 to 3.000	.632 ÷ *J*	1.897 ÷ *J*	109°— 6′ ÷ *K*	125°—14′ ÷ *K*	71°—34′	18°—26′
1 to 3.100	.614 ÷ *J*	1.903 ÷ *J*	109°—26′ ÷ *K*	125°—37′ ÷ *K*	72°— 7′	17°—53′
1 to 3.200	.596 ÷ *J*	1.909 ÷ *J*	109°—45′ ÷ *K*	126°— 0′ ÷ *K*	72°—39′	17°—21′
1 to 3.333	.575 ÷ *J*	1.915 ÷ *J*	110°— 8′ ÷ *K*	126°—25′ ÷ *K*	73°—18′	16°—42′
1 to 3.400	.564 ÷ *J*	1.919 ÷ *J*	110°—20′ ÷ *K*	126°—38′ ÷ *K*	73°—37′	16°—23′
1 to 3.450	.557 ÷ *J*	1.921 ÷ *J*	110°—26′ ÷ *K*	126°—46′ ÷ *K*	73°—50′	16°—10′
1 to 3.500	.549 ÷ *J*	1.923 ÷ *J*	110°—34′ ÷ *K*	126°—55′ ÷ *K*	74°— 3′	15°—57′
1 to 3.550	.542 ÷ *J*	1.925 ÷ *J*	110°—41′ ÷ *K*	127°— 3′ ÷ *K*	74°—16′	15°—44′
1 to 3.600	.535 ÷ *J*	1.927 ÷ *J*	110°—48′ ÷ *K*	127°—11′ ÷ *K*	74°—29′	15°—31′
1 to 3.631	.531 ÷ *J*	1.928 ÷ *J*	110°—52′ ÷ *K*	127°—15′ ÷ *K*	74°—36′	15°—24′
1 to 3.684	.524 ÷ *J*	1.930 ÷ *J*	110°—59′ ÷ *K*	127°—23′ ÷ *K*	74°—49′	15°—11′
1 to 3.736	.517 ÷ *J*	1.932 ÷ *J*	111°— 5′ ÷ *K*	127°—30′ ÷ *K*	75°— 1′	14°—59′
1 to 3.777	.512 ÷ *J*	1.934 ÷ *J*	111°—10′ ÷ *K*	127°—36′ ÷ *K*	75°—10′	14°—50′
1 to 3.789	.510 ÷ *J*	1.934 ÷ *J*	111°—11′ ÷ *K*	127°—37′ ÷ *K*	75°—13′	14°—47′
1 to 3.833	.505 ÷ *J*	1.935 ÷ *J*	111°—16′ ÷ *K*	127°—43′ ÷ *K*	75°—23′	14°—37′
1 to 3.888	.496 ÷ *J*	1.937 ÷ *J*	111°—22′ ÷ *K*	127°—48′ ÷ *K*	75°—35′	14°—25′
1 to 3.944	.492 ÷ *J*	1.938 ÷ *J*	111°—27′ ÷ *K*	127°—53′ ÷ *K*	75°—46′	14°—14′
1 to 4.000	.485 ÷ *J*	1.940 ÷ *J*	111°—33′ ÷ *K*	128°— 3′ ÷ *K*	75°—58′	14°— 2′
1 to 4.111	.472 ÷ *J*	1.943 ÷ *J*	111°—45′ ÷ *K*	128°—16′ ÷ *K*	76°—21′	13°—39′
1 to 4.176	.466 ÷ *J*	1.945 ÷ *J*	111°—50′ ÷ *K*	128°—22′ ÷ *K*	76°—32′	13°—28′
1 to 4.235	.459 ÷ *J*	1.946 ÷ *J*	111°—54′ ÷ *K*	128°—28′ ÷ *K*	76°—43′	13°—17′
1 to 4.312	.452 ÷ *J*	1.948 ÷ *J*	112°— 1′ ÷ *K*	128°—35′ ÷ *K*	76°—57′	13°— 3′
1 to 4.375	.446 ÷ *J*	1.949 ÷ *J*	112°— 6′ ÷ *K*	128°—40′ ÷ *K*	77°— 7′	12°—53′
1 to 4.428	.440 ÷ *J*	1.951 ÷ *J*	112°—10′ ÷ *K*	128°—45′ ÷ *K*	77°—17′	12°—43′
1 to 4.500	.434 ÷ *J*	1.952 ÷ *J*	112°—15′ ÷ *K*	128°—51′ ÷ *K*	77°—28′	12°—32′
1 to 4.571	.427 ÷ *J*	1.954 ÷ *J*	112°—20′ ÷ *K*	128°—57′ ÷ *K*	77°—40′	12°—20′
1 to 4.666	.419 ÷ *J*	1.955 ÷ *J*	112°—26′ ÷ *K*	129°— 3′ ÷ *K*	77°—54′	12°— 6′
1 to 4.800	.408 ÷ *J*	1.958 ÷ *J*	112°—35′ ÷ *K*	129°—13′ ÷ *K*	78°—14′	11°—46′
1 to 5.000	.392 ÷ *J*	1.961 ÷ *J*	112°—45′ ÷ *K*	129°—26′ ÷ *K*	78°—41′	11°—19′
1 to 5.142	.382 ÷ *J*	1.963 ÷ *J*	112°—53′ ÷ *K*	129°—34′ ÷ *K*	79°— 0′	11°— 0′
1 to 5.230	.375 ÷ *J*	1.964 ÷ *J*	112°—57′ ÷ *K*	129°—39′ ÷ *K*	79°—11′	10°—49′
1 to 5.385	.365 ÷ *J*	1.966 ÷ *J*	113°— 3′ ÷ *K*	129°—46′ ÷ *K*	79°—29′	10°—31′
1 to 5.461	.360 ÷ *J*	1.967 ÷ *J*	113°— 6′ ÷ *K*	129°—49′ ÷ *K*	79°—37′	10°—23′
1 to 5.538	.355 ÷ *J*	1.968 ÷ *J*	113°—10′ ÷ *K*	129°—53′ ÷ *K*	79°—46′	10°—14′
1 to 5.666	.348 ÷ *J*	1.969 ÷ *J*	113°—16′ ÷ *K*	129°—59′ ÷ *K*	79°—59′	10°— 1′
1 to 5.750	.342 ÷ *J*	1.970 ÷ *J*	113°—18′ ÷ *K*	129°— 2′ ÷ *K*	80°— 8′	9°—52′
1 to 5.833	.338 ÷ *J*	1.971 ÷ *J*	113°—20′ ÷ *K*	130°— 5′ ÷ *K*	80°—16′	9°—44′
1 to 5.916	.333 ÷ *J*	1.972 ÷ *J*	113°—23′ ÷ *K*	130°— 9′ ÷ *K*	80°—25′	9°—35′
1 to 6.000	.328 ÷ *J*	1.972 ÷ *J*	113°—26′ ÷ *K*	130°—12′ ÷ *K*	80°—33′	9°—27′

The use of the table is best shown by an example:

Gears of 60 and 30 teeth; 8 diametral pitch

Proportion 60 to 30 = 2 to 1

Outside dia. of large gear $= \text{pitch dia.} + F = \frac{60}{J} + \frac{.894}{J} = \frac{60}{8} + \frac{.894}{8}$

$= 7\frac{1}{2} + .112 = 7.612$ ins.

Center angle of large gear $= 63°\ 26'$

Face angle of large gear $= 63°\ 26' + E = 63°\ 26' + \frac{102°\ 51'}{K}$

$= 63°\ 26' + \frac{102°\ 51'}{60} = 63°\ 26' + 1°\ 43'$

$= 65°\ 9'$

Cutting angle of large gear $= 63°\ 26' - D = 63°\ 26' - \frac{118°\ 3'}{K}$

$= 63°\ 26' - \frac{118°\ 3'}{60} = 63°\ 26' - 1°\ 59'$

$= 61°\ 27'$

The dimensions and angles of miter gears, of diametral pitch may, in most cases, be taken from Table 4 by Wm. G. Thumm (*Amer. Mach., June* 13, 1907).

The profiles of the teeth of bevel gears are laid out on the developed backing cones as indicated in Fig. 5. The number of teeth contained in the circumference of the developed cone is to be calculated by dividing the actual number of teeth in the gear by the cosine of α. The number of teeth thus found is to be used when consulting Grant's odontograph, which see, for the various radii, the profile being drawn as for this number of teeth and as for a spur gear of pitch radius OA.

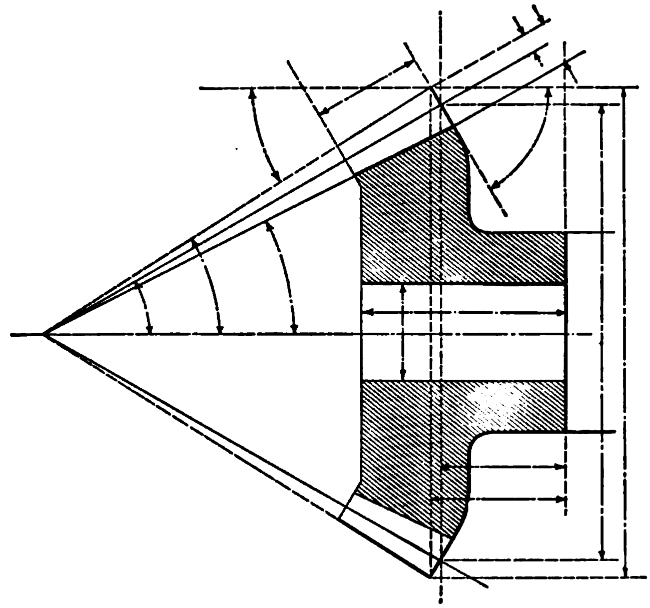


Fig. 4.—Needed shop dimensions of bevel gears.

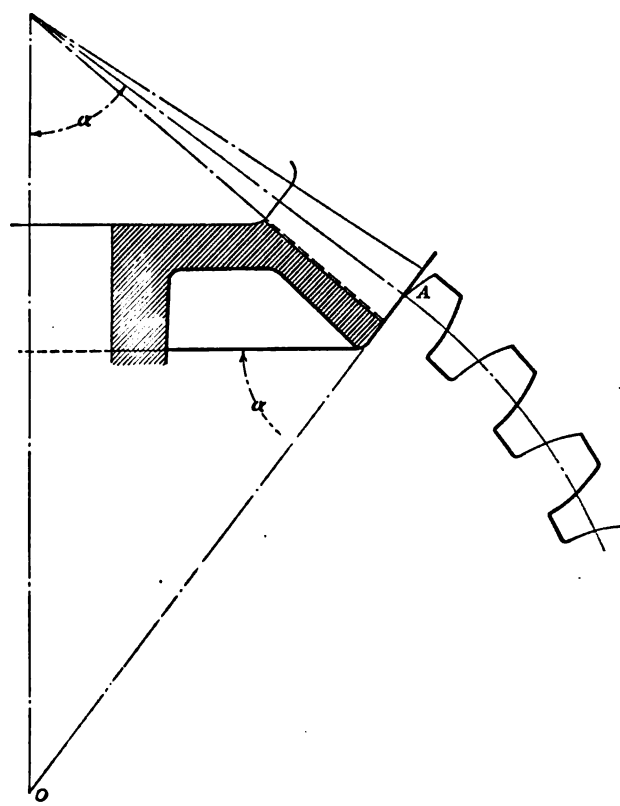


Fig. 5.—Profiles of bevel gear teeth.

Strength of Bevel Gears by Calculation

The working loads on bevel gears may be determined from the formula proposed by Wilfred Lewis (*Proc. Eng. Club of Philadelphia,* 1892) as follows:

$$W = SPfy\frac{d}{D}$$

in which W = pressure on teeth, lbs.,

S = fiber stress, lbs. per sq. in., this stress being dependent on the speed in accordance with Table 10 given in connection with the Lewis formula for spur gears, which see,

P = circular pitch, ins.,

f = face, ins.,

y = a factor for different numbers and forms of teeth in accordance with Table 12 given in connection with the Lewis formula for spur gears, which see. In selecting this factor for bevel gears the actual number of teeth is to be multiplied by the secant of one-half the pitch cone apex angle, the result being the equivalent number of teeth for which y is to be selected,

d = inside pitch diameter, ins..

D = outside pitch diameter, ins.,

The formula presupposes that d is not less than $\frac{2}{3}D$, which it should never be.

Strength of Bevel Gears by Graphics

The working loads on bevel gears may be determined by the following method, by Robert A. Bruce (*Amer. Mach., May* 31, 1900) which is based on and gives the same results as the Lewis formula:

First, find the face of a spur gear equivalent to the actual face on the bevel gear by the use of Fig. 6. Instructions for use are given below the chart.

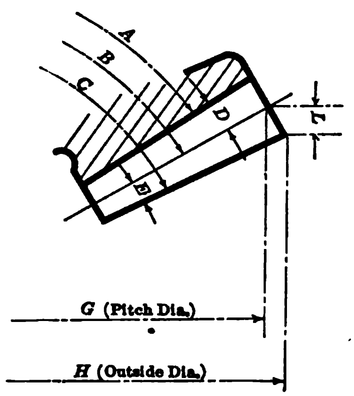
Second, find the number of teeth of a spur gear equivalent to the actual number of the bevel gear by the use of Fig. 7, for which instructions for use will be found below it.

Third, using the equivalent face width and number of teeth, follow the instructions for Mr. Bruce's chart for the strength of spur-gear teeth, Fig. 7 of the section on Spur Gears.

If desired, the Lewis factor y may be found by tracing downward to the second curve of Fig. 7 of this section and thence horizontally to the value of the factor at the right, as shown.

Selecting Bevel Gears from Stock Lists

Commercial or listed bevel gears for shafts at right angles may frequently be used for shafts at other angles, especially if the re-



A = Cutting angle $= B - D$
B = Center angle
C = Face angle $= B + E$
D = Angle decrement
E = Angle increment
F = Diameter increment
G = Pitch diameter
H = Outside diameter $= G + F$
J = Diametral pitch
K = Number of teeth in large wheel
L = From pitch line to outside angle $= \frac{1}{2}$ diameter increment of mating wheel
$G + F = H$
Angle B + angle E = angle C
Angle B − angle D = angle A
The values of F, E, D, B are shown in table

Notation of Table 3.

TABLE 4.—DIMENSIONS AND ANGLES OF MITER GEARS

No. of teeth	Pitch diameters								Outside diameters								Face angle	Cut angle
				5 P.	6 P.	7 P.	8 P.	10 P.	2 P.	3 P.	4 P.	5 P.	6 P.	7 P.	8 P.	10 P.		
12	6	4	3	2 2/5	2	1 5/7	1 1/2	1 2/10	6.71	4.48	3.35	2.68	2.24	1.92	1.68	1.34	51° 43′	38° 17′
13	6 1/2	4 1/3	3 1/4	2 3/5	2 1/6	1 6/7	1 5/8	1 3/10	7.21	4.80	3.60	2.88	2.40	2.06	1.81	1.44	51° 12′	38° 48′
14	7	4 2/3	3 1/2	2 4/5	2 1/3	2	1 3/4	1 4/10	7.71	5.14	3.85	3.08	2.57	2.20	1.93	1.54	50° 47′	39° 13′
15	7 1/2	5	3 3/4	3	2 1/2	2 1/7	1 7/8	1 5/10	8.21	5.46	4.10	3.28	2.73	2.35	2.06	1.64	50° 23′	39° 37′
16	8	5 1/3	4	3 1/5	2 2/3	2 2/7	2	1 6/10	8.71	5.80	4.35	3.48	2.90	2.49	2.18	1.74	50° 03′	39° 57′
17	8 1/2	5 2/3	4 1/4	3 2/5	2 5/6	2 3/7	2 1/8	1 7/10	9.21	6.14	4.60	3.68	3.07	2.63	2.31	1.84	49° 45′	40° 15′
18	9	6	4 1/2	3 3/5	3	2 4/7	2 1/4	1 8/10	9.71	6.48	4.85	3.88	3.24	2.77	2.43	1.94	49° 29′	40° 31′
19	9 1/2	6 1/3	4 3/4	3 4/5	3 1/6	2 5/7	2 3/8	1 9/10	10.21	6.80	5.10	4.08	3.40	2.92	2.56	2.04	49° 15′	40° 45′
20	10	6 2/3	5	4	3 1/3	2 6/7	2 1/2	2	10.71	7.14	5.35	4.28	3.57	3.06	2.68	2.14	49° 03′	40° 57′
21	10 1/2	7	5 1/4	4 1/5	3 1/2	3	2 5/8	2 1/10	11.21	7.46	5.60	4.48	3.73	3.20	2.81	2.24	48° 51′	41° 09′
22	11	7 1/3	5 1/2	4 2/5	3 2/3	3 1/7	2 3/4	2 2/10	11.71	7.80	5.85	4.68	3.90	3.35	2.93	2.34	48° 41′	41° 19′
23	11 1/2	7 2/3	5 3/4	4 3/5	3 5/6	3 2/7	2 7/8	2 3/10	12.21	8.14	6.10	4.88	4.07	3.49	3.06	2.44	48° 31′	41° 29′
24	12	8	6	4 4/5	4	3 3/7	3	2 4/10	12.71	8.48	6.35	5.08	4.24	3.63	3.18	2.54	48° 22′	41° 38′
25	12 1/2	8 1/3	6 1/4	5	4 1/6	3 4/7	3 1/8	2 5/10	13.21	8.80	6.60	5.28	4.40	3.77	3.31	2.64	48° 14′	41° 46′
26	13	8 2/3	6 1/2	5 1/5	4 1/3	3 5/7	3 1/4	2 6/10	13.71	9.14	6.85	5.48	4.57	3.92	3.43	2.74	48° 07′	41° 53′
27	13 1/2	9	6 3/4	5 2/5	4 1/2	3 6/7	3 3/8	2 7/10	14.21	9.46	7.10	5.68	4.73	4.06	3.56	2.84	48° 00′	42° 00′
28	14	9 1/3	7	5 3/5	4 2/3	4	3 1/2	2 8/10	14.71	9.80	7.35	5.88	4.90	4.20	3.68	2.94	47° 54′	42° 06′
29	14 1/2	9 2/3	7 1/4	5 4/5	4 5/6	4 1/7	3 5/8	2 9/10	15.21	10.14	7.60	6.08	5.07	4.35	3.81	3.04	47° 47′	42° 13′
30	15	10	7 1/2	6	5	4 2/7	3 3/4	3	15.71	10.48	7.85	6.28	5.24	4.49	3.93	3.14	47° 42′	42° 18′
31	15 1/2	10 1/3	7 3/4	6 1/5	5 1/6	4 3/7	3 7/8	3 1/10	16.21	10.80	8.10	6.48	5.40	4.63	4.06	3.24	47° 37′	42° 23′
32	16	10 2/3	8	6 2/5	5 1/3	4 4/7	4	3 2/10	16.71	11.14	8.35	6.68	5.57	4.77	4.18	3.34	47° 32′	42° 28′
33	16 1/2	11	8 1/4	6 3/5	5 1/2	4 5/7	4 1/8	3 3/10	17.21	11.46	8.60	6.88	5.73	4.92	4.31	3.44	47° 27′	42° 33′
34	17	11 1/3	8 1/2	6 4/5	5 2/3	4 6/7	4 1/4	3 4/10	17.71	11.80	8.85	7.08	5.90	5.06	4.43	3.54	47° 23′	42° 37′
35	17 1/2	11 2/3	8 3/4	7	5 5/6	5	4 3/8	3 5/10	18.21	12.14	9.10	7.28	6.07	5.20	4.56	3.64	47° 19′	42° 41′
36	18	12	9	7 1/5	6	5 1/7	4 1/2	3 6/10	18.71	12.48	9.35	7.48	6.24	5.35	4.68	3.74	47° 15′	42° 45′
37	18 1/2	12 1/3	9 1/4	7 2/5	6 1/6	5 2/7	4 5/8	3 7/10	19.21	12.80	9.60	7.68	6.40	5.49	4.81	3.84	47° 11′	42° 49′
38	19	12 2/3	9 1/2	7 3/5	6 1/3	5 3/7	4 3/4	3 8/10	19.71	13.14	9.85	7.88	6.57	5.63	4.93	3.94	47° 08′	42° 52′
40	20	13 1/3	10	8	6 2/3	5 5/7	5	4	20.71	13.80	10.35	8.28	6.90	5.92	5.18	4.14	47° 01′	42° 59′
42	21	14	10 1/2	8 2/5	7	6	5 1/4	4 2/10	21.71	14.48	10.85	8.68	7.24	6.20	5.43	4.34	46° 56′	43° 04′
44	22	14 2/3	11	8 4/5	7 1/3	6 2/7	5 1/2	4 4/10	22.71	15.14	11.35	9.08	7.57	6.49	5.68	4.54	46° 50′	43° 10′
46	23	15 1/3	11 1/2	9 1/5	7 2/3	6 4/7	5 3/4	4 6/10	23.71	15.80	11.85	9.48	7.90	6.77	5.93	4.74	46° 46′	43° 14′
48	24	16	12	9 3/5	8	6 6/7	6	4 8/10	24.71	16.48	12.35	9.88	8.24	7.06	6.18	4.94	46° 42′	43° 18′
50	25	16 2/3	12 1/2	10	8 1/3	7 1/7	6 1/4	5	25.71	17.14	12.85	10.28	8.57	7.35	6.43	5.14	46° 37′	43° 23′
54	27	18	13 1/2	10 4/5	9	7 5/7	6 3/4	5 4/10	27.71	18.48	13.85	11.08	9.24	7.92	6.93	5.54	46° 31′	43° 29′
58	29	19 1/3	14 1/2	11 3/5	9 2/3	8 2/7	7 1/4	5 8/10	29.71	19.80	14.85	11.88	9.90	8.49	7.43	5.94	46° 24′	43° 36′
60	30	20	15	12	10	8 4/7	7 1/2	6	30.71	20.48	15.35	12.28	10.24	8.77	7.68	6.14	46° 21′	43° 39′

quired gears have a ratio of unity, and the bevel-gear ratio may frequently be made unity when the speed ratio is not by adding a pair of spurs to the train. The following explanation of this fact and of the method of selecting the gears from gear-maker's lists is due to W. C. CONANT (*Amer. Mach., June* 13, 1901).

In Fig. 8, AB and CD are right-angle pairs having the same cone apex, pitch and tooth system. Inspection will show that B and C will mesh together properly. Given the shaft angle NOM and the ratio of B and C, the problem is to find the right-angle pairs AB and CD from which to select B and C. Going further, it is equally evident from Fig. 9 that gear A may mesh with an indefinite number of gears $BCDE$, provided that the pitch cones intersect at the common point O, and the gears $BCDE$ may all be members of right-angle pairs, each combination AB, AC, AD, AE giving different angles of shafts and different speed ratios $\frac{A}{B}$, $\frac{A}{C}$, $\frac{A}{D}$, $\frac{A}{E}$.

The conditions given in practice are, two shafts intersecting at any angle to run at any speed ratio, to find from standard lists bevel gears that will meet the requirements.

Referring to Fig. 8, let OM and ON be the center lines of two

shafts and let C and B be the pitch lines of any two bevel gears that will give the required speed ratio. Draw OP perpendicular to OM and dotted line from R perpendicular to OP; this latter line to represent the pitch line of a gear mating with B to form a right-angle pair. By a similar construction draw D to form a right-angle pair with C. It is evident from the figure that any diameter gear B having a ratio with its mate A of $\frac{B}{A}$ and any diameter gear C having a ratio with its mate D of $\frac{C}{D}$ will run correctly together provided ratio $\frac{B}{C}$ is a constant, the most favorable case being that when $\frac{B}{C}=1$, gears B and C being identical. To solve the problem, it is only necessary to find the ratios $\frac{B}{A}$ and $\frac{C}{D}$ and select the proper member from each pair. We have,

$$\text{Angle } MON = \text{angle } MOR + \text{angle } NOR \qquad (a)$$

Since $$OR=\frac{B}{\sin MOR} \text{ and } OR=\frac{C}{\sin NOR}$$

therefore $$\frac{B}{\sin MOR}=\frac{C}{\sin NOR}$$

or, $$\sin NOR=\frac{C}{B}\sin MOR$$

Let $$\frac{C}{B}=R$$

when $$\sin NOR=R\sin MOR \qquad (b)$$

From a table of natural sines select such values for the angles MOR and NOR that their sum is MON and that $\sin NOR$ is R times $\sin MOR$, thus satisfying equations (a) and (b). If the gears are to be equal, R becomes 1 and the angles MOR and NOR are equal.

We have next to find the values of $\frac{A}{B}$ and of $\frac{D}{C}$.

Since $$OR=\frac{B}{\sin MOR}$$

and $$OR=\frac{A}{\sin ROP}=\frac{A}{\cos MOR}$$

therefore $$\frac{B}{\sin MOR}=\frac{A}{\cos MOR}$$

or $$\frac{A}{B}=\frac{\cos MOR}{\sin MOR}$$

Let $$\frac{A}{B}=r$$

when $$r=\frac{\cos MOR}{\sin MOR} \qquad (c)$$

In the same manner it can be shown that

$$\frac{D}{C}=r'=\frac{\cos NOR}{\sin NOR} \qquad (d)$$

Example.—Required a pair of gears to connect two shafts at an angle of 60 deg., the speed ratio $\frac{C}{B}=R=1$, giving at once $MOR=NOR=30$ deg. Substituting the sine and cosine of 30 deg. in (c) or (d) we have:

$$r=\frac{.866}{.500}=1.73$$

and we have only to find a pair of right-angle gears of the required strength having this ratio; for example, gears having 24 and 42 teeth give a ratio of 1.75, which is sufficiently accurate for the class of work for which cast gears are used. Of these we may obviously use either the pinion or the gear.

If the required gears are to have a different ratio, or say $\frac{C}{B}=R=\frac{2}{1}$, we find by inspection of a table of natural sines angles to satisfy equations (a) and (b). Thus we find that

$\sin 41°(=.656)=2\sin 19°=(2\times.326)$ nearly, and that $41°+19°=60°$, which is to say that angle $NOR=41°$ and angle $MOR=19°$.

Substituting the values of the sines and cosines in (c) and (d) we get

$$r=\frac{.9455}{.3255}=2.9$$

and $$r'=\frac{.7547}{.6560}=1.15$$

Breath of Face of Equivalent Spur Wheel, Ins.

.10 .15 .20 .25 .30 .35 .40 .45 .50

Values of r = Breadth of Face of Bevel Wheel Divided by Length of Slant Side of Pitch Cone

Divide the width of face by the length of slant height of pitch cone, the result being r; find the value of r on the base line, trace vertically to the curve, horizontally to the diagonal for the actual face width and vertically to the top where read the equivalent face width. The example is for $r=.25$ and actual face width $=5$ ins., the equivalent face width being 3.85 ins.

FIG. 6.—The face of bevel gears reduced to the equivalent face of spur gears.

From a catalogue list we now select the pinion of a pair of right-angle gears having a ratio of 2.9 and we will suppose that a pair found having 17 and 50 teeth answer the requirements of strength.

Since the speed ratio of the shafts is to be 2, the gear to run with the 17-tooth pinion must have $17 \times 2 = 34$ teeth and it must come from

Equivalent Number of Spur Teeth

Values of Lewis Variable

Half Angle of Pitch Cone in Degrees

Find the half angle of the pitch cone on the base line; trace vertically to the curve of secants, horizontally to the diagonal for the actual number of teeth, and vertically to the equivalent number of spur teeth at the top. The example is for half cone angle = $31\frac{1}{2}$ deg. and actual number of teeth = 34, the equivalent number of teeth being 40.

Fig. 7.—The number of teeth of bevel gears reduced to the equivalent number of teeth of spur gears.

a pair in which it mates with a gear having $34 \times 1.15 = 39$ teeth. Therefore the 17-tooth pinion selected from the first pair and the 34-tooth pinion from the second are the gears required.

In order to determine whether the pinion or gear of a given pair is to be used, observe the following rule:

When the angle *MOR* is less than 45 deg., use the pinion of the pair *B-A*, and conversely when the angle *MOR* is greater than 45 deg., use the gear of the pair *B-A*. As a corollary it follows that the same rule applies to the pair *D-C*, using the angle *NOR* as the critical angle.

As Fig. 8 is drawn with angle *MON* less than 90 deg., Fig. 10 is drawn to show angle *MON* greater than a right angle. It will be ob-

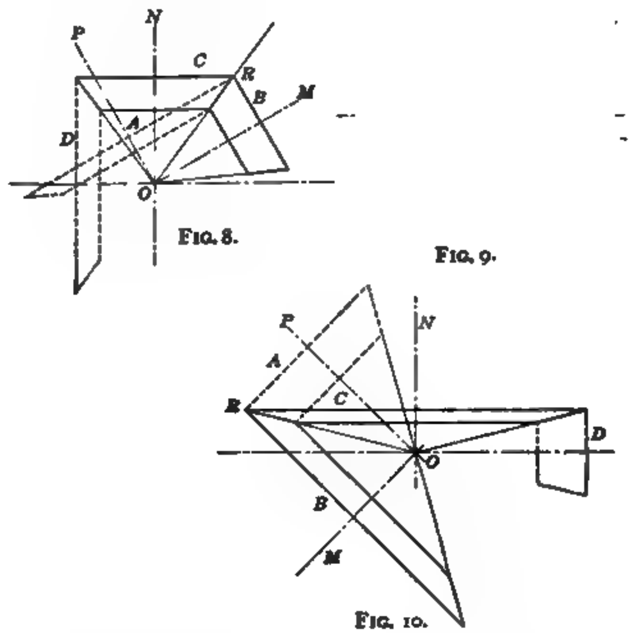


Fig. 8.

Fig. 9.

Fig. 10.

Figs. 8 to 10.—Stock bevel gears for shafts at any angle.

served that the obtuse angle uses larger gears than the acute angle, of which advantage may be taken in cases where strength is needed and consequently large gears required.

While it will generally be possible to find a pair containing one of the required gears, it will usually be found more difficult to find a pair from which the mating gear can be selected. That is to say, having found the 17-tooth pinion, it will be difficult to find a 34-tooth pinion belonging to a pair having a ratio of 1.15 with the same pitch and face. There is also the danger that the teeth of the two pairs from which the mating gears are selected may have been designed for different systems and that therefore the mating gears will not run well together. For these reasons it is better to use two gears of the same size and make the required speed change at some other point. At the worst, if only a single stock gear is used, that much pattern work will be saved.

FRICTION GEARS

The working loads on friction gearing formed the subject of a series of experiments by PROF. W. F. M. GOSS (*Trans. A. S. M. E., Vol.* 29). Various materials were tested for both the fibrous and the metal wheels. The materials of the fibrous wheels were straw fiber, straw fiber with belt dressing, leather fiber, leather, leather-faced iron, sulphite fiber, and tarred fiber.

The straw-fiber wheels were worked out of blocks built up of square sheets of straw board laid one upon another with a suitable cementing material between them and compacted under heavy hydraulic pressure. In the finished wheel the sheets appear as disks, the edges of which form the face of the wheel.

The wheel of straw fiber with belt dressing was similar to that of straw fiber, except that the individual sheets of straw board from which it was made had been treated, prior to their being converted into a block, with a belt dressing, the composition of which is unknown.

The leather-fiber wheel was made up of cemented layers of board, as were those already described; but in this case the board, instead of being of straw fiber, was composed of ground sole-leather cuttings, imported flax and a small percentage of wood pulp. The material is very dense and heavy.

The leather wheel was composed of layers or disks of sole leather.

The leather-faced iron wheel consisted of an iron wheel having a leather strip cemented to its face. After less than 300 revolutions the bond holding the leather face failed and the leather separated itself from the metal of the wheel. This wheel proved entirely incapable of transmitting power and no tests of it are recorded.

The wheel of sulphite fiber was made up of sheets of board composed of wood pulp. The sulphite board is said to have been made on a steam-drying continuous-process machine in the same way as is the straw board.

The tarred-fiber wheel was made up of board composed principally of tarred rope stock, imported French flax and a small percentage of ground sole-leather cuttings.

Each of the fibrous driving wheels was tested in combination with driven wheels of the following materials: Iron, aluminum, type metal.

Regarding the metallic wheels the conclusions are that those driving wheels which are the more dense work more efficiently with the iron follower than with either the aluminum or type-metal followers; but in the case of the softer and less dense driving wheels, and especially in the case of those in which an oily substance is incorporated, driven wheels of aluminum and type metal are superior to those of iron. Finely powdered metal which is given off from the surface of the softer metal wheels seems to account for this effect, and the character of the driving wheels is perhaps the only factor necessary to determine whether its presence will be beneficial or detrimental. Finally, with reference to the use of soft-metal driven wheels, it should be noted that no combination of such wheels with a fibrous driver appears to have given high frictional results. Except when used under very light pressures, the wear of the type metal was too rapid to make a wheel of its material serviceable in practice.

Regarding the fibrous wheels the conclusions are that the addition of belt dressing to the composition of a straw-fiber wheel is fatal to its frictional qualities. The highest frictional qualities are possessed by the sulphite-fiber wheel which, on the other hand, is the weakest of all wheels tested. The leather fiber and tarred fiber are exceptionally strong; and the former possesses frictional qualities of a superior order. The plain straw fiber, which in a commercial sense is the most available of all materials dealt with, when worked upon an iron follower possesses frictional qualities which are far superior to leather, and strength which is second only to the leather fiber and the tarred fiber.

A review of the data discloses the fact that several of the friction wheels tested developed a coefficient of friction which in some cases exceeded .5. That is, such wheels rolling in contact have transmitted from driver to driven wheels a tangential force equal to 50 per cent. of the force maintaining their contact. These wheels, also, were successfully worked under pressures of contact approaching 500 lbs. per in. in width. Employing these facts as a basis from which to calculate power, it can readily be shown that a friction wheel a foot in diameter, if run at 100 r.p.m., can be made to deliver in excess of 25 h.p. for each inch in width. It is certainly true that any of the wheels tested may be employed to transmit for a limited time an amount of power which, when gaged by ordinary measures, seems to be enormously high; but obviously, performance under limiting conditions should not be made the basis from which to determine the commercial capacity of such devices. In view of this fact, it is important that there be drawn from the data such general conclusions with reference to pressures of contact and frictional qualities as will constitute a safe guide to practice.

The recommended contact pressures, which are one-fifth of the ultimate resistance established by tests under destructive pressures, are given in Table 1.

TABLE 1.—WORKING CONTACT PRESSURES PER INCH OF FACE

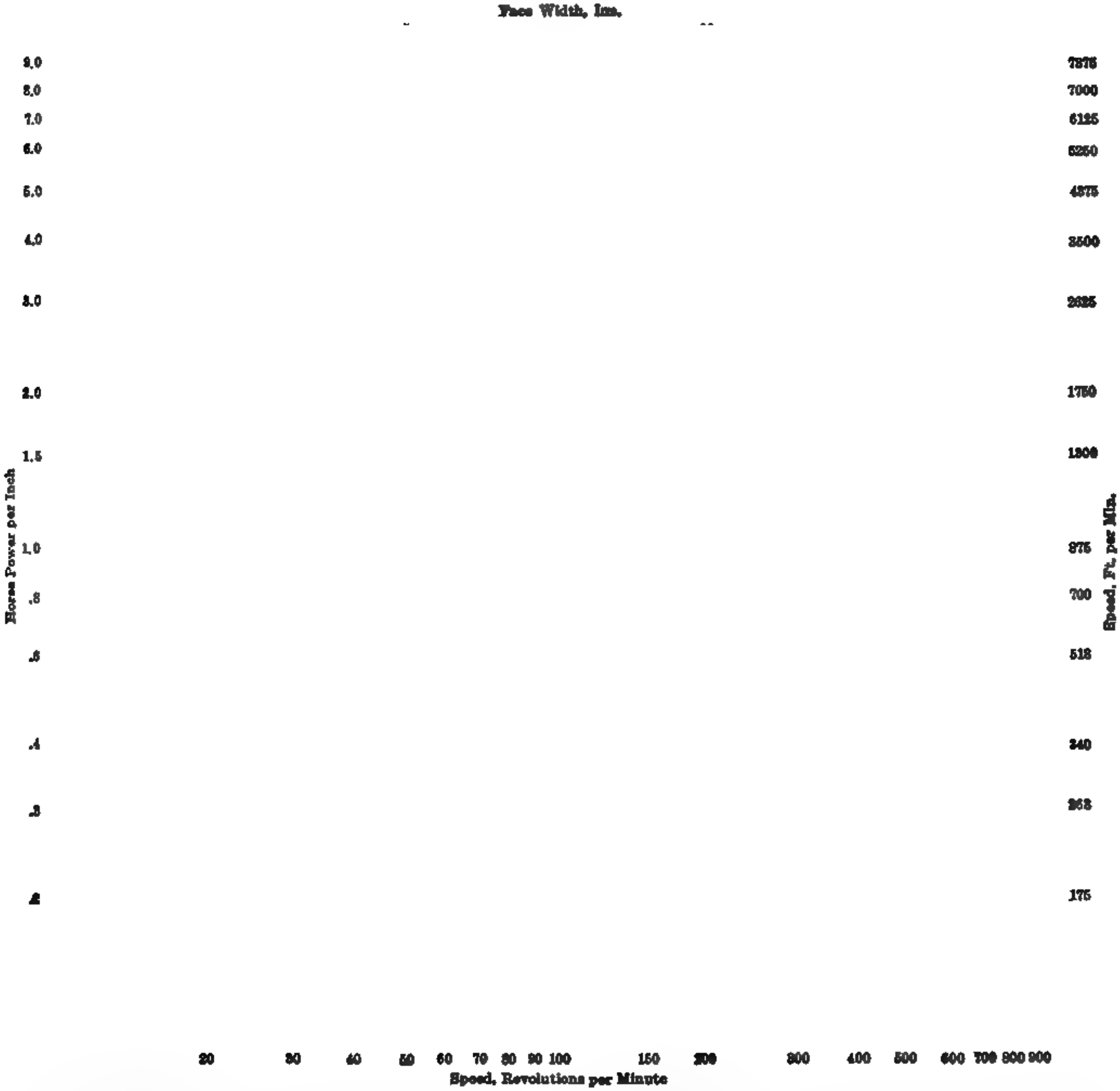
	Pressure, lbs.
Straw fiber	150
Leather fiber	240
Tarred fiber	240
Sulphite fiber	140
Leather	150

The recommended values of the coefficient of friction, which are 60 per cent. of the laboratory results, are given in Table 2.

TABLE 2.—WORKING VALUES OF THE COEFFICIENT OF FRICTION

	Coefficient of friction
Straw fiber and iron	.255
Straw fiber and aluminum	.273
Straw fiber and type metal	.186
Leather fiber and iron	.309
Leather fiber and aluminum	.297
Leather fiber and type metal	.183
Tarred fiber and iron	.150
Tarred fiber and aluminum	.183
Tarred fiber and type metal	.165
Sulphite fiber and iron	.330
Sulphite fiber and aluminum	.318
Sulphite fiber and type metal	.309
Leather and iron	.135
Leather and aluminum	.216
Leather and type metal	.246

The recommended formulas for the working loads in h.p. are given in Table 3
in which d = diameter of friction wheel, ins.,
W = width of face, ins.,
N = r. p. m.



To find peripheral speed, locate the intersection of the vertical line representing the given speed in r.p.m., with the diagonal one representing the given diameter. The horizontal line passing through this point will give the surface speed in ft. per min. on the vertical scale to the right of the chart.

To find the horse power for a given wheel, locate the intersection of the vertical line representing the given speed in r.p.m. with the diagonal line representing the given diameter. Follow the horizontal line passing through this point to the right or left until the intersection between it and the vertical line representing the given width, as shown on the scale at the top of the chart, is reached. The diagonal line passing through this point marked Total Horse Power will represent the required horse power.

To find the face width of a given wheel necessary to transmit a given horse power, the speed being known, locate the intersection of the vertical line representing the given speed in r p.m. with the diagonal line representing the given diameter. Follow the horizontal line passing through this point to the right or left until the intersection between it and the diagonal line representing the required horse power is reached. The vertical line passing through this point will give the width of face in ins. on the scale at the top of the chart.

For other material than straw fiber and iron, multiply the horse power by the following factors:

Sulphite fiber and iron	**1.23**	**Leather and iron**	**.53**
Leather fiber and iron	**1.97**	**Tarred fiber and iron**	**.97**

FIG. 1.—Dimensions of friction wheels of straw fiber and iron.

TABLE 3.—FORMULAS FOR WORKING LOADS

	Horse-power
Straw fiber and iron	.00030 dWN
Straw fiber and aluminum	.00033 dWN
Straw fiber and type metal	.00022 dWN
Leather fiber and iron	.00059 dWN
Leather fiber and aluminum	.00057 dWN
Leather fiber and type metal	.00035 dWN
Tarred fiber and iron	.00029 dWN
Tarred fiber and aluminum	.00035 dWN
Tarred fiber and type metal	.00031 dWN
Sulphite fiber and iron	.00037 dWN
Sulphite fiber and aluminum	.00035 dWN
Sulphite fiber and type metal	.00034 dWN
Leather and iron	.00016 dWN
Leather and aluminum	.00026 dWN
Leather and type metal	.00029 dWN

All usual problems connected with the dimensions and power capacity of friction wheels may be solved by the use of Fig. 1, with which the necessary explanations are given. The chart represents the formula for straw fiber and iron:

$$h.p. = .0003\ dWN$$

In the application of friction gearing, the fibrous wheel must always be the driver; the rolling surfaces should be kept clean or, if this is impossible, the wheels should be increased in size to provide for a lower coefficient of friction due to the presence of dirt; and the pressure should be by positive, inflexible mechanism—springs are not admissable.

The formulas and chart are equally applicable to face friction gearing, with the proviso that it is advisable to make the width of face of the driver and the distance between the driver and the center of the follower such that the variation in the velocity of the two edges of the driver shall not exceed 4 per cent. This may be secured by making the minimum distance between the driver and the center of the follower twelve times the width of the face of the driver. If this distance is made smaller, as it frequently must be, the gearing will work successfully but its power capacity will be decreased because of the fact that the coefficient of friction diminishes if the slip exceeds 4 per cent.

In making friction wheels, one $\frac{3}{4}$-inch bolt should be provided for every 20 sq. in. of disk.

Bevel friction wheels, unless supported at the outer angle, give trouble by failure under the pressure at that point. E. R. PLAISTED (*Amer. Mach., Sept.* 18, 1902) states that a disk of soft wood about $\frac{3}{4}$ in. thick as a backing for the paper at that point obviates the difficulty.

WORM GEARS

For the distinction between lead and pitch, see Lead and Pitch.

The thread profile of worms is most commonly made to the Brown and Sharpe standard which is a direct outgrowth of their gear-tooth standard, the section of a worm and wheel through the axis of the worm being the same as that of a rack and gear in mesh. When this standard is used the following formulas (from the Brown & Sharpe Mfg. Co's. *Formulas in Gearing*) apply, reference being made toFig. 1.

Table 6 of spur-tooth parts by circular pitch (page 90) contains, in the 2d, 9th and 10th columns, a list of worm-tooth parts.

FORMULAS FOR BROWN AND SHARPE STANDARD WORM GEARING

L = lead of worm.
N = number of teeth in gear.
m = turns per inch of worm.
d = diameter of worm.
d' = pitch diameter of worm.
d'' = diameter of hob.
D = throat diameter.
D' = pitch diameter of worm wheel.
B = blank diameter (to sharp corners).
C = distance between centers.
p = diametral pitch.
P = circular pitch for worm wheels or axial pitch for worms.
$\left.\begin{matrix} r' \\ r'' \end{matrix}\right\}$ See Fig. 1.
s = addendum.
t = thickness of tooth at pitch line.
t^n = normal thickness of tooth.
f = clearance at bottom of tooth.
D'' = working depth of tooth.
$D''+f$ = whole depth of tooth.
b = pitch circumference of worm.
v = width of worm thread tool at end.
w = width of worm thread at top and width of hob tool at end.
δ = angle of tooth of worm wheel with its axis, or the angle of thread of worm with a line at right angles to its axis.

If the lead is for single, double, triple, etc., thread, then

$$L = P,\ 2P,\ 3P, \text{ etc.}$$

In multiple-threaded worms and their mating wheels, if the angle δ is more than 15° the tooth parts should be figured on the normal as for spiral gears. In using the formulas for spiral gears, it should be borne in mind that while P is the axial pitch for worms it is the circular pitch for spiral gears.

$\alpha = 60°$ to $90°$

$L = \frac{1}{m}$

$P = \frac{\pi D}{N+2}$

$D' = \frac{NP}{\pi} = \frac{N}{p}$

$D = \frac{N}{p} + 2s$

$b = \pi(d-2s) = \pi d'$

$\tan\delta = \frac{L}{b}$ $\Big\{$ Practical only when width of wheel on wheel-pitch circle is not more than ⅔ pitch diameter of worm.

$t^n = t\cos\delta$

$r' = \frac{d}{2} - 2s$

$r'' = r' + D'' + f$

$C = \frac{D'+d}{2} - s = \frac{D'+d'}{2}$

$B = D + 2\left(r' - r'\cos\frac{\alpha}{2}\right)$ A measurement of sketch is generally sufficient.

$d'' = d + 2f$

$v = .31\,P$

$w = .335\,P$

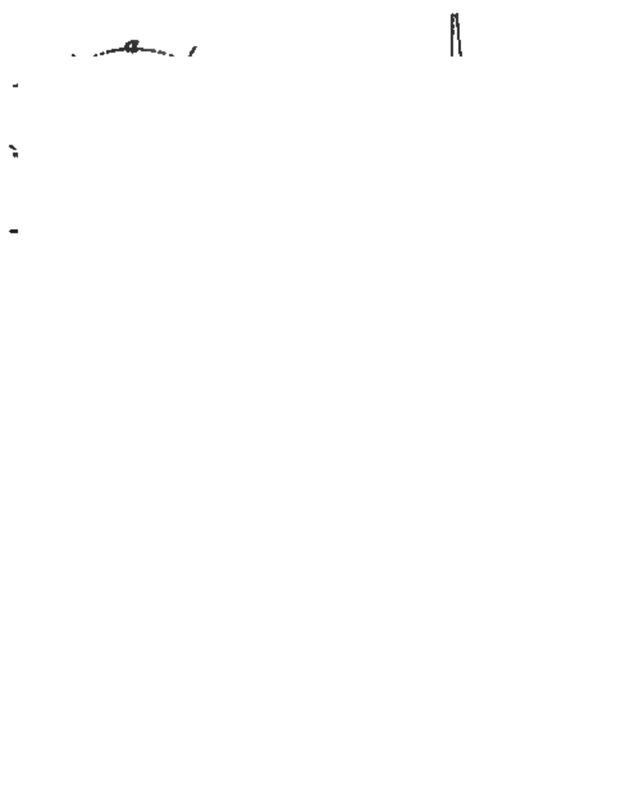


FIG. 1.—Notation of formulas for worm gearing.

The profiles of worm teeth being the same as those of rack teeth, the same interference takes place if the wheel has less than 30 teeth. If the wheel be finished with a hob, the interference will be overcome but at the expense of undercut teeth. Both interference and undercut may be prevented by increasing the throat diameter of the wheel, making the diameter in accordance with the formula:

$$D = \cos^2 14\tfrac{1}{2}° \frac{N}{P} + 4s$$
$$= .937\frac{N}{P} + 4s$$

The increase of throat diameter increases also the center distance, the amount of increase being shown by comparing this value of D with the one previously given. To keep the original center distance,

the outside diameter of the worm must be reduced by the amount the throat diameter of the wheel is increased.

The pitch diameters of circular-pitch worms, Brown and Sharpe standard, may be obtained from Table 1. For larger or smaller worms than those given, add or subtract the required number of inches thus:

Given a worm $4\frac{11}{16}$ ins. outside diameter, $\frac{11}{16}$-in. pitch. From the table $1\frac{11}{16}$ ins. outside diameter, $\frac{11}{16}$ pitch $= 1.375$ ins. pitch diameter. Therefore, $4\frac{11}{16}$ ins. outside diameter, $\frac{11}{16}$-in. pitch $= 1.375 + 3 = 4.375$ ins. pitch diameter. Given a worm $\frac{3}{4}$-in. outside diameter, $\frac{5}{16}$ pitch. From the table $1\frac{3}{4}$ ins. outside diameter, $\frac{5}{16}$ in. pitch $= 1.676$ pitch diameter. Therefore $\frac{3}{4}$ in. outside diameter, $\frac{5}{16}$ ins. pitch must $= 1.676 - 1 = .676$ in. pitch diameter.

Cutting Diametral Pitch Worms

The cutting of diametral pitch worms requires the introduction in the change gear train of the lathe of gears having to one another the ratio of π, for which, for ordinary purposes, the value $\frac{22}{7}$ is a sufficiently close approximation. It gives rise to an error of less than half a thousandth per inch of length of the worm. The formula is:

$$\frac{\text{teeth in screw gear}}{\text{teeth in stud gear}} = \frac{22 \times \text{threads per in. of lead screw}}{7 \times \text{diametral pitch}}$$

Table 2, by E. J. Rantsch (*Amer. Mach., April* 11, 1907) gives the ratio of the gears for ordinary cases on the above basis together with other dimensions of worms of $14\frac{1}{2}$ deg. obliquity. The numerators of the fractions represent the screw gear and the denominators the stud gear. When necessary, both numerator and denominator are to be multiplied by a (the same) number to give actual gears. If gears to satisfy the ratio $\frac{22}{7}$ are not at hand, less accurate ratios are often sufficient, useful ratios in the order of accuracy being as follows:

$$\frac{22}{7} = 3.1429$$

$$\frac{69}{22} = 3.1364$$

$$\frac{47}{15} = 3.1333$$

If the ratio $\frac{22}{7}$ is considered not sufficiently accurate the ratio $\frac{355}{113}$ is adequate to any possible requirement, its decimal value being 3.1415929. The relations of circular and normal pitches are given in Table 3.

Table 2.—Change Gears for Diametral Pitch Worms

Diametral pitch	Single depth	Width of tool point	Width of top of thread	Pitch of lead screw 2	3	4	5	6	7	8	10
2	1.078 in.	.487 in.	.526 in.	22/7	33/7	44/7	55/7	66/7	77/7	88/7	110/7
$2\frac{1}{2}$	.862	.390	.421	88/35	132/35	176/35	44/7	264/35	308/35	352/35	440/35
3	.719	.325	.350	44/21	22/7	88/21	110/21	44/7	22/3	176/21	220/21
$3\frac{1}{2}$	.616	.278	.300	88/49	132/49	176/49	220/49	264/49	44/7	352/49	440/49
4	.540	.243	.263	11/7	33/14	22/7	55/14	33/7	11/2	44/7	55/7
5	.431	.195	.210	44/35	66/35	88/35	22/7	132/35	22/5	176/35	44/7
6	.360	.162	.175	22/21	11/7	44/21	55/21	22/7	11/3	88/21	110/21
7	.308	.139	.150	44/49	66/49	88/49	110/49	132/49	22/7	176/49	220/49
8	.270	.122	.131	11/14	33/28	11/7	55/28	33/14	11/4	22/7	55/14
9	.240	.108	.117	44/63	22/21	88/63	110/63	44/21	22/9	176/63	220/63
10	.216	.097	.105	22/35	33/35	44/35	11/7	66/35	11/5	88/35	22/7
11	.196	.088	.096	4/7	6/7	8/7	10/7	12/7	14/7	16/7	20/7
12	.180	.081	.088	11/21	11/14	22/21	55/42	11/7	11/6	44/21	55/21
14	.154	.069	.075	22/49	33/49	44/49	55/49	66/49	11/7	88/49	110/49
16	.135	.061	.066	11/28	33/56	22/28	55/56	33/28	77/56	11/7	55/28
18	.120	.054	.058	22/63	11/21	44/63	55/63	22/21	11/9	88/63	110/63
20	.108	.048	.053	11/35	33/70	22/35	11/14	33/35	77/70	44/35	11/7
24	.090	.040	.044	11/42	33/84	11/21	55/84	33/42	77/84	22/21	55/42
28	.077	.034	.038	11/49	33/98	22/49	55/98	33/49	11/14	44/49	55/49
32	.067	.030	.033	11/56	33/112	11/28	55/112	33/56	77/112	11/14	55/56
40	.054	.024	.026	11/70	33/140	11/35	11/28	33/70	77/140	22/35	11/14
48	.045	0.20	.022	11/84	33/168	11/42	55/168	33/84	77/168	11/21	55/84

Durability and Efficiency of Worm Gearing

The durability of worm gearing is largely dependent on the angle of the helix with the tangent to the pitch line. In order that a worm gear may be durable, the helix angle should be large—that is, the worm should be a steep pitch screw. This fact is established by theory, by experiment and by experience. The unfortunate experience that many have had with worm gearing is due to bad design and not to any inherent defect of the construction.

An analysis of the worm-gear problem with examples collected from practice by the author (*Amer. Mach., Jan.* 13, 20, 1898, republished as No. 116 of Van Nostrand's Science Series) is the source of much that follows.

The reason why an increase of pitch, other things being equal, or, in other words, an increase of the angle of the thread, gives increased efficiency reduced wear and longer life, will be understood

Table 1.—Pitch Diameters for Circular Pitch Worms

Worm, outside diameter ins.	Pitch in inches: 1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1	1 1/8	1 1/4	1 1/2
	Pitch diameters															
1	.8408	.8011	.7613	.7215	.6817	.6419	.6021	.5623	.5225	.4827	.4430	.4032	.3634	.2838	.2042	.0451
1 1/16	.903	.864	.824	.784	.744	.704	.665	.625	.585	.545	.505	.466	.426	.346	.267	.108
1 1/8	.966	.926	.886	.846	.807	.767	.727	.687	.647	.608	.568	.528	.488	.409	.329	.170
1 3/16	1.028	.989	.949	.909	.869	.829	.790	.750	.710	.670	.630	.591	.551	.471	.392	.233
1 1/4	1.091	1.051	1.011	.971	.932	.892	.852	.812	.772	.733	.693	.653	.613	.534	.454	.295
1 5/16	1.153	1.114	1.074	1.034	.994	.954	.915	.875	.835	.795	.755	.716	.676	.596	.517	.358
1 3/8	1.216	1.176	1.136	1.096	1.057	1.017	.977	.937	.897	.858	.818	.778	.738	.659	.579	.420
1 7/16	1.278	1.239	1.199	1.159	1.119	1.079	1.040	1.000	.960	.920	.880	.841	.801	.721	.642	.483
1 1/2	1.341	1.301	1.261	1.221	1.182	1.142	1.102	1.062	1.022	.983	.943	.903	.863	.784	.704	.545
1 9/16	1.403	1.364	1.324	1.284	1.244	1.204	1.165	1.125	1.085	1.045	1.005	.966	.926	.846	.767	.608
1 5/8	1.466	1.426	1.386	1.346	1.307	1.267	1.227	1.187	1.147	1.108	1.068	1.028	.988	.909	.829	.670
1 11/16	1.528	1.489	1.449	1.409	1.369	1.329	1.290	1.250	1.210	1.170	1.130	1.091	1.051	.971	.892	.733
1 3/4	1.591	1.551	1.511	1.471	1.432	1.392	1.352	1.312	1.272	1.233	1.193	1.153	1.113	1.034	.954	.795
1 13/16	1.653	1.614	1.574	1.534	1.494	1.454	1.415	1.375	1.335	1.295	1.255	1.216	1.176	1.096	1.017	.858
1 7/8	1.716	1.676	1.636	1.596	1.557	1.517	1.477	1.437	1.397	1.358	1.318	1.278	1.238	1.159	1.079	.920
1 15/16	1.778	1.739	1.699	1.659	1.619	1.579	1.540	1.500	1.460	1.420	1.380	1.341	1.301	1.221	1.142	.983

TABLE 3.—RELATION OF CIRCULAR AND NORMAL PITCHES OF WORM WHEELS

By WM. HAUGHTON (*Amer. Mach.*, June 1, 1911)

Pitch diameter of worm ins.	Number of threads: ¼ normal pitch				5/16 normal pitch				⅜ normal pitch				7/16 normal pitch				½ normal pitch			
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
1	.25075	.2530	.2572	.2623	.3143	.3185	.3262	.3363	.3777	.3854	.3983	.4155	.4419	.4540	.4740	.500	.5063	.5246	.5539	.5928
1⅛	.2506	.2525	.2556	.2598	.3137	.3174	.3235	.3315	.3770	.3834	.3935	.4073	.4409	.4509	.4683	.4881	.5050	.5196	.5431	.5743
1¼	.25055	.2520	.2545	.2580	.3136	.3164	.3213	.3279	.3767	.3817	.3907	.4012	.4402	.4484	.4613	.4789	.5040	.5160	.5352	.5611
1⅜	.25042	.2517	.2537	.2566	.31334	.31572	.3197	.3251	.3764	.3805	.3875	.3966	.4397	.4464	.4571	.4724	.5033	.5132	.5293	.5509
1½	.25037	.2515	.2531	.2555	.31318	.3152	.3184	.3233	.3761	.3797	.3856	.3935	.4396	.4449	.4540	.4666	.5028	.5111	.5247	.5431
1⅝	.2503	.2512	.2527	.2547	.3131	.3148	.3176	.3217	.3760	.3791	.3840	.3968	.4391	.4438	.4519	.4625	.5024	.5094	.5211	.5370
1¾	.25025	.25103	.25236	.2541	.31363	.3145	.3170	.3205	.37586	.3785	.3828	.3887	.4389	.4429	.4498	.4591	.5020	.5082	.5183	.5322
1⅞	.25022	.2509	.2520	.2535	.31298	.3142	.3165	.3194	.3758	.3780	.3818	.3869	.4387	.4424	.4482	.4564	.5018	.5071	.5160	.5280
2	.25017	.2508	.2517	.2531	.3129	.3140	.3159	.3185	.3756	.3776	.3809	.3855	.4385	.4417	.4469	.4541	.5016	.5062	.5140	.5247
2⅛	.25017	.2507	.2515	.2528	.3128	.3138	.3155	.3179	.3756	.3773	.3802	.3842	.4384	.4412	.4459	.4523	.5014	.5055	.5125	.5220
2¼	.25015	.2506	.2514	.2524	.3128	.3136	.3152	.3172	.3755	.3771	.3796	.3833	.4383	.4409	.4449	.4507	.5012	.5048	.5111	.5196
2⅜	.25014	.25057	.2512	.2522	.3127	.3136	.3150	.3168	.3754	.3768	.3792	.3825	.4382	.4405	.4442	.4493	.5011	.5045	.5099	.5176
2½	.25012	.2505	.2511	.2520	.3127	.3135	.3147	.3164	.3754	.3767	.3788	.3817	.4382	.4402	.4435	.4482	.5010	.5040	.5090	.5160
2⅝	.2501	.25047	.25102	.2518	.3127	.3134	.3144	.3161	.3753	.3765	.3784	.3811	.4381	.4399	.4430	.4473	.5009	.5037	.5081	.5144
2¾	.2501	.25042	.2509	.2516	.3127	.3133	.3143	.3157	.3753	.3764	.3781	.3805	.4380	.4398	.4425	.4464	.5008	.5033	.5074	.5131
2⅞	.25007	.25037	.25085	.2514	.3126	.3132	.3142	.3155	.3753	.3762	.3779	.3801	.4379	.4395	.4420	.4456	.5008	.5030	.5068	.5121
3	.25007	.25035	.2507	.2514	.3126	.3132	.3139	.3152	.3753	.3762	.3776	.3796	.4379	.4393	.4417	.4449	.5007	.5028	.5063	.5111
3¼	.25007	.2503	.2506	.2512	.3126	.3130	.3138	.3148	.3752	.3760	.3772	.3790	.4379	.4391	.4411	.4439	.5006	.5024	.5054	.5095
3½	.25007	.2502	.2506	.2510	.3126	.3130	.3136	.3145	.3751	.3758	.3769	.3785	.4378	.4389	.4405	.4430	.5005	.5020	.5046	.5081
3¾	.25005	.2502	.2505	.2509	.3126	.3129	.3135	.3142	.3751	.3757	.3767	.3780	.4378	.4387	.4401	.4423	.5004	.5018	.5040	.5072
4	.25005	.2502	.2504	.2508	.3126	.3129	.3133	.3140	.3751	.3757	.3765	.3776	.4377	.4385	.4399	.4417	.5004	.5016	.5036	.5063

Pitch diameter of worm ins.	Number of threads: ⅝ normal pitch				¾ normal pitch				⅞ normal pitch				1 in. normal pitch			
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
1	.6371	.6727	.7278	.7988	.7710	.8310	.9224	1.037	.9083	1.001	1.140	1.310	1.049	1.185	1.382	1.619
1⅛	.6346	.6629	.7079	.7658	.7667	.8147	.8891	.9836	.9014	.9764	1.089	1.231	1.0393	1.1488	1.3115	1.5096
1¼	.6328	.6558	.6925	.7398	.7635	.8028	.8642	.9443	.8965	.9579	1.0524	1.1719	1.032	1.1123	1.258	1.427
1⅜	.6315	.6507	.6816	.7229	.7611	.7939	.8456	.9131	.8927	.9440	1.0238	1.113	1.0263	1.1019	1.2175	1.3630
1½	.6305	.6465	.6725	.7078	.7594	.7870	.8310	.8892	.8901	.9333	1.0148	1.0903	1.0233	1.0863	1.1855	1.3115
1⅝	.6296	.6435	.6658	.6959	.7581	.7815	.8195	.8698	.8879	.9249	.9839	1.0608	1.0188	1.0739	1.1597	1.2705
1¾	.6289	.6410	.6603	.6866	.7571	.7773	.8102	.8542	.8859	.9182	.9695	1.0373	1.0164	1.0641	1.1390	1.2371
1⅞	.6286	.6389	.6559	.6789	.7561	.7739	.8028	.8417	.8845	.9128	.9579	1.0176	1.0142	1.0560	1.1223	1.2087
2	.6280	.6372	.6522	.6727	.7553	.7710	.7966	.8309	.8835	.9083	.9484	1.0114	1.0125	1.0494	1.1079	1.1855
2⅛	.6277	.6358	.6492	.6673	.7546	.7687	.7914	.8222	.8825	.9046	.9401	.9879	1.0112	1.0440	1.0963	1.1569
2¼	.6274	.6348	.6466	.6629	.7542	.7668	.7870	.8147	.8816	.9015	.9333	.9763	1.010	1.0392	1.0863	1.1489
2⅜	.6272	.6336	.6444	.6591	.7537	.7649	.7833	.8083	.8809	.8987	.9277	.9665	1.009	1.0354	1.0778	1.1334
2½	.6270	.6328	.6425	.6559	.7533	.7634	.7802	.8028	.8803	.8963	.9226	.9578	1.008	1.0318	1.0704	1.120
2⅝	.6267	.6321	.6409	.6533	.7531	.7623	.7773	.7981	.8799	.8945	.9182	.9504	1.007	1.0289	1.064	1.111
2¾	.6266	.6315	.6394	.6506	.7528	.7612	.7750	.7939	.8794	.8928	.9145	.9440	1.006	1.0263	1.0586	1.102
2⅞	.6265	.6309	.6385	.6484	.7526	.7602	.7729	.7901	.8791	.8912	.9111	.9386	1.006	1.024	1.537	1.0936
3	.6261	.6305	.6372	.6466	.7524	.7593	.7710	.7870	.8788	.8895	.9081	.9333	1.0057	1.0222	1.0494	1.0863
3¼	.6261	.6296	.6354	.6435	.7520	.7579	.7679	.7816	.8782	.8878	.9034	.9249	1.0048	1.0190	1.0422	1.074
3½	.6260	.6289	.6340	.6409	.7517	.7568	.7655	.7773	.8778	.8859	.8995	.9182	1.004	1.0163	1.0366	1.0641
3¾	.6258	.6285	.6328	.6389	.7515	.7560	.7636	.7740	.8773	.8845	.8965	.9128	1.003	1.0144	1.032	1.056
4	.6257	.6281	.6319	.6372	.7513	.7553	.7619	.7710	.8771	.8834	.8938	.9082	1.003	1.0125	1.028	1.049

from Fig. 2. If *ab* be the axis of the worm and *cd* a line representing a thread, against which a tooth of the wheel bears, it will be seen that if the tooth bears upon the thread by a pressure *P*, that pressure may be resolved into two components, one of which, *ef*, is perpendicular, while the other, *eg*, is parallel to the thread surface. The perpendicular component produces friction between the tooth and the thread. The useful work done during a revolution of the thread is the product of the load *P* and the lead of the worm, while the work lost in friction is the product of the perpendicular pressure *ef*, the coefficient of friction and the distance traversed in a revolution, which is the length of one turn of the thread. Now, if the angle of the thread be doubled, as indicated, the load *P* remaining the same, the new perpendicular component *fh* of *P* will be slightly reduced from the old value *ef*, while the length of a turn of the thread will be slightly increased. Consequently their product and the lost work of friction per revolution will not be much changed. The useful work per revolution will, however, be doubled, because, the pitch being doubled, the distance traveled by *P* in one revolution will be doubled. For a given amount of useful work the amount of work lost is therefore reduced by the increase in the thread angle, and, since the tendency to heat and wear is the immediate result of the lost work, it follows that that tendency is reduced. For small angles of thread the change is very rapid, and continues, though in diminishing degree, until the angle reaches a value not far from 45 deg., when the conditions change and the lost work increases faster than the useful work, an increase of the angle of the thread beyond that point reducing the efficiency.

These general principles have been given mathematical expres-

sion by Prof. J. H. Barr (*Amer. Mach., Jan.* 13, 1898). Assuming frictionless bearings—a condition that is nearly fulfilled by ball bearings—the formula for the efficiency of a worm gear is:

$$e = \frac{\tan \alpha \ (1 - f \tan \alpha)}{\tan \alpha + f}$$

in which

e = efficiency,
α = angle of thread, being the angle dfi of Fig. 2,
f = coefficient of friction.

Assuming a plain step bearing of the collar type having the same mean friction radius as the worm, the formula becomes:[1]

$$e = \frac{\tan \alpha \ (1 - f \tan \alpha)}{\tan \alpha + 2f} \text{ (approximately).}$$

Notation as before.

Note that a worm being essentially a screw these formulas and the following chart, Fig. 3, apply also to the efficiency of screws.

In order to present to the eye a picture of the meaning of these formulas, Fig. 3 has been plotted from them.

The scale at the bottom gives the angles of the thread from 0 to 90 deg., while the vertical scale gives the calculated efficiencies, the values of which have been obtained from the equations and plotted on the diagram. The upper curve is from the first equation, and gives the efficiencies of the worm thread with a frictionless step; while the lower curve, from the second equation, gives the combined efficiency of the worm and step. In the calculations for the diagram it is necessary to assume a value for f, and this has been taken at .05, which is probably a fair mean value,

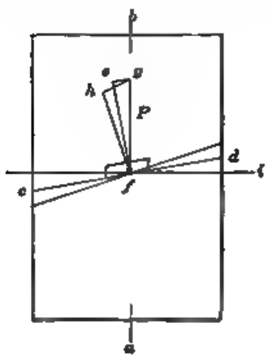


Fig. 2.—The principle of worm gear efficiency.

Fig. 3.—Relation of helix angle and efficiency of worm gearing.

although, since it varies with the rubbing speed, no single curve can represent all conditions.

The curves will be seen to rise to a maximum and then to drop. The values of the helix angle α for maximum efficiency as found from the foregoing equations by the methods of the calculus are:

For a frictionless step bearing, $\alpha = 43° \ 34'$.

For a plain step bearing, $\alpha = 52° \ 49'$.

Of more importance than the angle of maximum efficiency is the general character of the curves, of which the most pronounced peculiarity is the extreme flatness, showing that for a wide range of angles the efficiency varies but little. Thus, for the upper curve there is scarcely any choice between 30 and 60 deg. of angle, and but little drop at 20 deg.

At first sight the lower curve might be thought the more useful of the two, as it includes the effect of the step, but a little consideration will show that this is not the case. For most cases in which worms are used the efficiency of the transmission, as such, is of very little account. What the designer concerns himself with is the question of durability and satisfactory working, and the results to be expected in this respect are best shown by the upper curve, in which high efficiency means a durable worm.

The chief significance of efficiency in this connection is that since lost power is expended in friction and wear, low efficiency means rapid wear and high efficiency the reverse.

The above conclusions are confirmed by the well-known experiments of Wilfred Lewis for Wm. Sellers & Co. (*Trans. A. S. M. E., Vol.* 7). The small crosses plotted on Fig. 3 represent the results of such experiments as developed the same coefficient of friction as that used in plotting the curves from the above equations. The experimental results will be seen to have a very satisfactory agreement with the lower curve with which they should be compared, as the step bearings used by Mr. Lewis were of the plain pattern without balls.

Similar high efficiencies have been obtained repeatedly and most recently by Prof. Wm. K. Kennerson for the Brown & Sharpe Mfg. Co. (*Trans. A. S. M. E., Vol.* 34). Using worms of 45° and 38° 16′ helix angle with ball step bearings on both worms and wheels, Professor Kennerson obtained efficiencies as high as 97½ per cent.

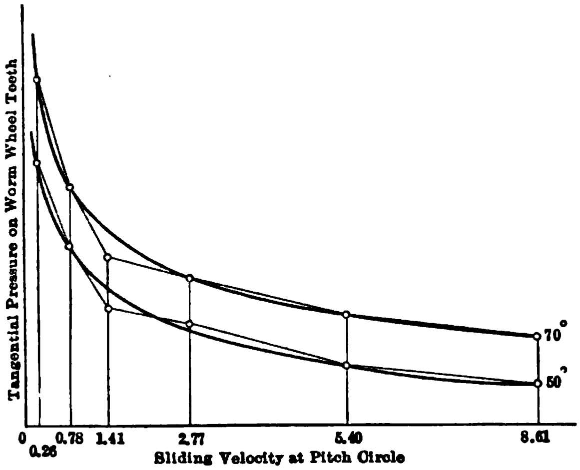
The articles by the author, above referred to, include particulars of 18 worm drives of various helix angles doing heavy duty, some of which were successful while others failed from rapid wear. Summing up the results of the investigation it was found that all worms having helix angles greater than 12° 30′ were successful, that all having angles less than 9° were failures from rapid wear, while between these angles some were successful and some failures. In several cases unsuccessful worms of low angle had been replaced with others of high angle with the result of changing failure to success.

The prevailing materials for worm gearing are hardened steel for the worm and bronze for the worm wheel. Referring to the examples collected by the author and already cited, of eleven successful gears doing heavy duty, five had bronze and six cast-iron gears, and, moreover, numerous other cases of successful cast-iron wheels could be cited. The Sellers planer drive, which is essentially a worm drive and which has been successful through a long series of years, always has the rack of cast-iron. The conditions under which worm gearing operates, especially if an oil bath is used, as it should be,

[1] In both formulas the worm thread is assumed to be square in section. Thread sections in common use affect the results but little.

regardless of the materials, would seem to be especially favorable to the well-known glazing action of cast-iron bearings.

It may be concluded that, given suitable dimensions for the work, worms having helix angles exceeding 15 deg. or better 20 deg. and running in oil baths should transmit continuous loads with good and even high efficiency and long life.



Sliding velocity at the pitch circle of the worm in meters per second; differences of temperature in deg. Cent. Pressures in kilograms.

FIG. 4.—Relation of pressure and velocity at observed differences of temperature in worm gearing.

Load Capacity of Worm Gears

The loads to be carried by worm gearing depend upon the dimensions of the gearing and the speed.

The law connecting the speed, pressure and temperature was disclosed in experiments by PROFS. C. BACH and E. ROSER (*Zeitschrift des Vereines Deutscher Ingenieure*, 1902, *and Amer. Mach.*, *July* 16, 23, 1903). This law is plotted in metric measures in Fig. 4 from which unfortunately, the scale of pressure is omitted. For comparative purposes it may be scaled. The two curves are for observed differences of temperature (centigrade) as noted, between the oil cellar and the surrounding air.

Professors Bach and Roser deduced formulas for the performance of worm gearing from their experiments, these formulas translated into British units being as follows:

$$P = 14.235[c\ \tfrac{5}{9}\ (t_o - t_a) + d]\ bp$$

in which

P = axial thrust on worm, lbs.,
t_o = temperature in oil cellar, Fahr.,
t_a = temperature of surrounding air, Fahr.,
b = breadth of wormwheel teeth measured on the arc at the roots of the teeth, ins.,
p = Pitch (not lead in the case of multiple thread worms) ins.,
$c = \dfrac{13.17}{v} + .4192$,
$d = \dfrac{21{,}476}{v + 541} - 24.92$,
v = circumferential velocity at pitch line of worm, ft. per min.[1]

In the experiments the included profile angle of the worm thread was 29 deg., to which angle the application of the formula is properly limited. Flooded lubrication was used in the experiments and is assumed in the formula.

[1] In the original papers v was given as the sliding velocity at the pitch line, but a checking of the calculations shows that the quantity actually used was the circumferential velocity.

Fig. 5 by PROF. J. B. PEDDLE (*Amer. Mach., Jan.* 23, 1913) has been constructed to give the same results as the Bach and Roser formula. The use of the chart is shown by the example below it. Several trials will usually be required to find suitable proportions of pitch to face.

In the use of the formula or chart, the temperature rise to be permitted must be determined by the designer. There does not seem to be any cause for alarm at a considerable rise if suitable oil be used. In Professors Bach and Roser's experiments the extreme rise was 95 deg. C. (171 deg. Fahr.), while in experiments by PROFESSOR KENNERSON (*Trans. A. S. M. E.*, *Vol.* 34) a rise of 225 deg. Fahr. was experienced repeatedly, and in one case a rise of 322 deg. Fahr., the room temperature ranging between 66 and 88 deg.

In the former experiments the oil used was "an extremely viscous steam cylinder oil" while Professor Kenerson used an oil intended for use with superheated steam. In both sets of experiments the worms were flooded, as they always should be.

There is no doubt that bearings frequently operate at higher temperatures than is commonly believed and without giving trouble. Ordinary bearing oil loses its lubricating qualities at a temperature of about 250 deg. Fahr. See Index.

Professor Kenerson's experiments show that, when subjected to varying loads, worm gearing need not be designed for the greatest load. The final temperature of the oil cellar was not reached until the lapse of two, and in some cases three, hours, while abnormally high loads were carried for an hour and more without failure. The uniform experience was that, while the rise of oil temperature was rapid at the start, it became more gradual as the run continued.

The materials used in Professors Bach and Roser's experiments were unhardened steel for the worm, and bronze for the wheel. In Professor Kenerson's experiments the worms were of case-hardened machinery steel, and the wheels of various grades of bronze, among which no great differences were observed so far as the rise of temperature was concerned.

Good reason exists for doubting the correctness of Professors Bach and Roser's fundamental analysis and hence of the formula and chart based upon it. They are, however, included here because they embody the best existing information at the time of writing.

As a contribution to this chapter, the Hindley Gear Co. (successors to Morse Williams Co.) have placed at my disposal their method of determining the dimensions of their well-known Hindley worms which are based on the most extended experience with this gear extant.

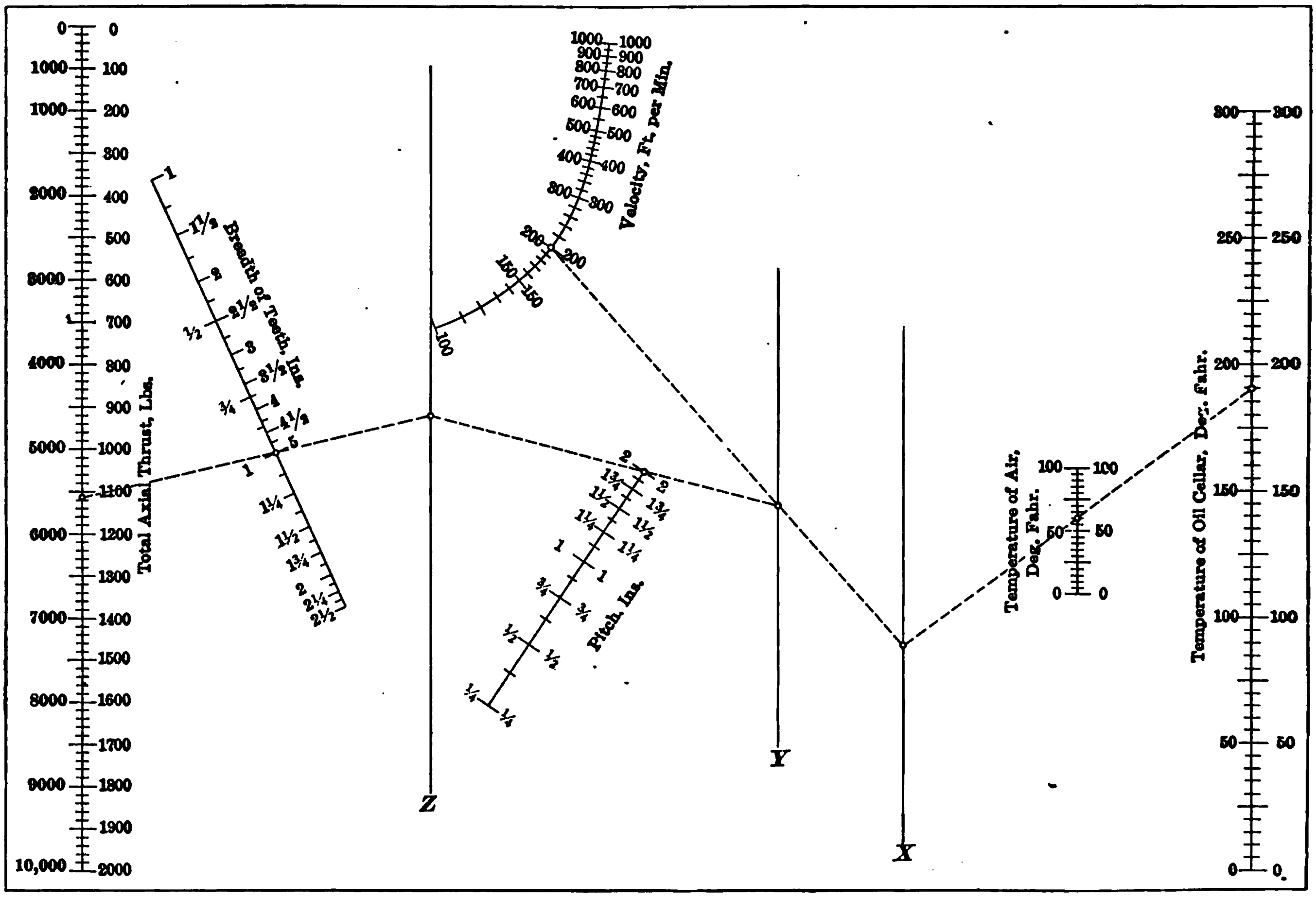
The method is based on the use of a constant for the product of the velocity and unit pressure, this constant being chosen to suit the conditions and being thus largely a matter of experience and only partially capable of reduction to a formula. The initial assumption is that the load is carried by two teeth and that 70 per cent. of the area of these teeth is effective bearing area. The area of the teeth is determined from the drawing and multiplied by .7 when the chosen constant is substituted in the formula

$$pv = C$$

in which

p = pressure on effective area, lbs. per sq. in.,
v = circumferential speed of worm at the throat diameter pitch line, ft. per min.,
C = constant.

For the most unfavorable condition, that is, for a steady load acting continuously and for an indefinite period of time, a conservative value is $C = 250{,}000$. For intermittent loads much larger values may be taken for C—an experience that is directly in line with Professor Kenerson's experimental determination that abnormally high loads may be carried for considerable periods. In general, for intermittent loads C may be increased in the inverse proportion which



Join oil cellar and air temperatures and note intersection with axis *X*; connect this intersection with the velocity and note intersection with axis *Y*. Any two lines from this point and from the thrust which intersect on axis *Z* will give the required pitch and breadth of teeth. For light loads use light faced figures on both thrust and breadth scales; for heavy loads use heavy faced figures on both scales. The example is for oil-cellar temperature = 190 deg., air temperature = 60 deg., velocity = 200 ft. per min., thrust = 1114 and 5570 lbs., giving pitch = 2 ins. and breadth = 1 and 5 ins.

FIG. 5.—Dimensions of worm gears.

the load time bears to the total time. Thus should the load time be one-half the total time, *C* becomes 250,000×2 = 500,000, while should the load time be one-quarter of the total, *C* becomes 250,000×4 = 1,000,000. In extreme cases of intermittent loads applied for short periods and at large intervals, *C* may reach as large a value as 3,500,000.

In the matter of materials the timid are disposed to favor the general use of bronze for the gear. The experience of the Hindley Gear Co. has led them to the following choice of materials.

For pressures per sq. in. of effective area not exceeding 350 lbs. and velocities not exceeding 500 ft. per min., cast-iron for both worm and gear.

For velocities of 1000 ft. per min. and over, the pressure per sq. in. of effective area being 350 lbs. for steady or 500 lbs. for intermittent service, openhearth steel of about 35 points carbon for the worm and bronze for the wheel.

For very heavy thrusts (say 10,000 lbs.) and low velocities (300 ft. per min. or less) openhearth steel as above for the worm and Cramp's special gear bronze for the wheel.

Worm-gear cases and, for that matter, all gear cases, should be provided with vents; if this is not done the expansion of the air by the heat will drive the oil out through the bearings. The action repeats itself every time the gearing is started from the cold state and ultimately empties the case of most of its oil.

HELICAL (COMMONLY MISCALLED SPIRAL) GEARS

Of the load-carrying capacity of helical gears the author has no data.

Note that in what follows the angle of the helix is taken as the angle between the teeth and the tangent to the pitch circle—that is, as the angle *kal*, Fig. 4. There is no uniformity of practice in this notation, some writers using the complement of this angle—that is, the angle *lar*, Fig. 4. This difference of practice should be kept in mind when comparing formulas from different sources.

There is no geometrical difference between worm and helical gearing. This fact is illustrated in Fig. 1 of which the example in the immediate foreground is plainly a case of worm gearing, while that in the far background is as plainly a case of helical gearing. The difference between them is, however, one of degree only, as shown by the intermediate constructions.

Such difference as there is relates to the method of production. Worms are commonly cut with threading tools in a lathe, the pitch with which we are chiefly concerned being that parallel with the axis —the *axial pitch*—which, in the case of the worm wheels, becomes the circular pitch, precisely as in spur gears. The section through the worm center line of a worm and gear in mesh is the same as the section of a rack and gear in mesh. Helical gears, on the other hand, are cut with cutters in a milling machine, the pitch with which we are chiefly concerned being that perpendicular to the teeth—the *normal pitch.*

Helical Gears of 45°. Helix Angle on Shafts at Right Angles by Calculation

may be made for most cases by the use of Table 1 by E. J. Kearney *Calculations of helical gears of 45° helix angle on shafts at right angles* (*Amer. Mach., June* 29, 1911).

The law connecting the durability and efficiency with the helix angle of worm gearing, which see, applies also to helical gearing. For all practical purposes the angle of maximum durability and efficiency is 45 deg. Helical gears having this helix angle are also the easiest of all to calculate and to make. They are, hence, deservedly popular. The speed ratio of such gears is the same as that of spur gears—that is, the speeds are inversely as the diameters and as the number of teeth, the numbers of teeth being proportional to the diameters, and the pitch-line speeds of mating gears are equal. Such gears should be used whenever possible, there being, in fact, little reason for using other angles except in cases when the required speed ratio cannot be obtained with 45 deg. gears on the given center distances.

The calculation of helical gears not found in Table 1 begins with finding the length of the normal helix, that is, the length of a line equal to the normal pitch multiplied by the number of teeth. The normal pitch of helical gears is determined by the cutter used and is the same as the circular pitch of spur gears. In spur gears the circular pitch multiplied by the number of teeth equals the circumference, from which it is plain that, in the calculations, the normal helix of helical gears is analogous to and takes the place of the circumference in spur gears. Similarly, in spur gears, $\frac{\text{circumference}}{\pi}=\text{diameter}$, and in helical gears, $\frac{\text{length of normal helix}}{\pi}=$ a quantity which has no name but which is analogous to, and, in the calculations, takes the place of, the diameter in spur gears.

The author has shown (*Amer. Mach., Nov.* 28, 1901. Van Nostrand's Science Series No. 116) that for 45-deg. helical gears:

$$\frac{l_1}{\pi}=\frac{1.4142C}{1+\frac{r_1}{r_2}} \qquad (a)$$

in which l_1 = length of normal helix of driver, ins.,
C = distance between centers, ins.,
r_1 = r.p.m. of driver.
r_2 = r.p.m. of follower.

$\frac{l_1}{\pi}$ being the quantity which takes the place of the diameter in spur-gear calculations as already noted.

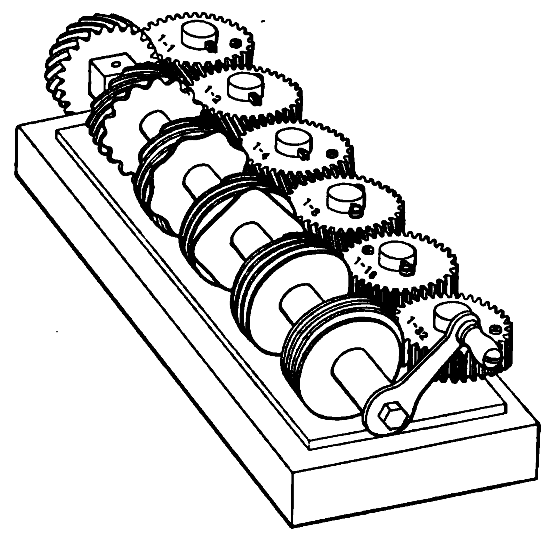


Fig. 1.—Helical and worm gears.

The author has also shown (same references) that for 45 deg. gears:

$$d_1=\frac{\frac{l_1}{\pi}}{.7071} \qquad (b)$$

in which d_1 = diameter of driver, ins.

The use of these formulas is best shown by an example:

Let $C=4\frac{15}{32}=4.468$ ins.

and $\frac{r_1}{r_2}=4$

Inserting these values in (*a*) we have:

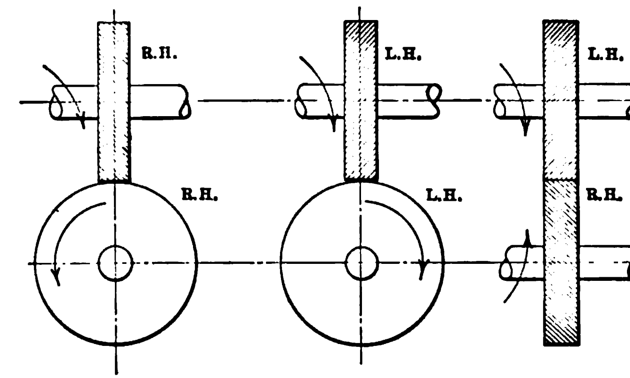
$$\frac{l_1}{\pi}=\frac{1.4142\times4.468}{1+4}$$
$$=1.2637 \text{ ins.}$$

Assuming next that a 6 diametral pitch cutter is to be used, we multiply this quantity by the diametral pitch to find the number of teeth, precisely as we multiply the diameter of a spur gear by the diametral pitch and obtain:

$$\text{number of teeth in driver}=1.2637\times6$$
$$=7.58$$

TABLE 2.—DIMENSIONS OF 45-DEG. HELICAL GEARS ON SHAFTS AT RIGHT ANGLES

Pitch of cutter	Pitch diameter	Pitch of spiral in inches to one turn	Number of teeth in spur same curvature	Outside diameter	Thickness of tooth at pitch line (normal)	Depth of tooth	Clearance	Circular pitch (normal)
	Multiply by number of teeth in spiral gear			Add to p.d.				
2	.70710	2.22142	2.828	1.0000	.7854	1.0785	.0785	1.5708
2¼	.62855	1.97464	2.828	.8888	.6981	.9587	.0699	1.3963
2½	.56566	1.77707	2.828	.8000	.6283	.8628	.0628	1.2566
2¾	.51425	1.61556	2.828	.7273	.5712	.7844	.0572	1.1424
3	.47140	1.48094	2.828	.6666	.5236	.7190	.0524	1.0472
3½	.40406	1.26939	2.828	.5714	.4488	.6163	.0449	.8976
4	.35355	1.11071	2.828	.5000	.3927	.5393	.0393	.7854
5	.28283	.88853	2.828	.4000	.3142	.4314	.0314	.6283
6	.23570	.74047	2.828	.3333	.2618	.3595	.0262	.5236
7	.20203	.63469	2.828	.2857	.2244	.3081	.0224	.4488
8	.17677	.55534	2.828	.2500	.1963	.2696	.0196	.3927
9	.15714	.49367	2.828	.2222	.1745	.2397	.0175	.3491
10	.14143	.44431	2.828	.2000	.1571	.2157	.0157	.3142
11	.12856	.40388	2.828	.1818	.1428	.1961	.0143	.2856
12	.11785	.37024	2.828	.1666	.1309	.1798	.0131	.2618
14	.10101	.31733	2.828	.1429	.1122	.1541	.0112	.2244
16	.08836	.27759	2.828	.1250	.0982	.1348	.0098	.1963
18	.07855	.24677	2.828	.1111	.0873	.1198	.0088	.1745
20	.07071	.22214	2.828	.1000	.0785	.1079	.0079	.1571
22	.06428	.20194	2.828	.0909	.0714	.0980	.0071	.1428
24	.05892	.18510	2.828	.0833	.0654	.0898	.0065	.1309
26	.05437	.17081	2.828	.0769	.0604	.0829	.0060	.1208
28	.05050	.15865	2.828	.0714	.0561	.0770	.0056	.1122
30	.04713	.14806	2.828	.0666	.0524	.0719	.0053	.1047
32	.04425	.13901	2.828	.0625	.0491	.0674	.0050	.0982
36	.03929	.12343	2.828	.0555	.0436	.0599	.0043	.0785
40	.03533	.11099	2.828	.0500	.0393	.0539	.0039	.0873
48	.02944	.09249	2.828	.0417	.0327	.0449	.0033	.0654



EXAMPLE

Let it be desired to construct a pair of spiral gears with 35 teeth in the gear and 16 teeth in the pinion, using a 10 pitch cutter.

$.14143 \times 35 = 4.950$ = pitch diameter.

$4.950 + .200 = 5.150$ = outside diameter.

$.44431 \times 35 = 15.550$ = pitch in inches to one turn of spiral.

NOTE.—A slight variation in one turn makes no practical difference, hence the ordinary change gears furnished with a universal miller will usually be found sufficient.

$2.828 \times 35 = 98.980$ = number of teeth in spur with same curvature.

Looking at Brown and Sharpe spur-gear cutter list, we see that 99 is between 55 and 134, therefore we select a No. 2 cutter.

In a similar manner using 16 as a multiplier we obtain the data for the pinion.

$$\frac{4.950 + 2.262}{2} = 3.606 = \text{center distance}$$

	Gear	Pinion
Number of teeth	35	16
Pitch diameter	4.950	2.262
Outside diameter	5.150	2.462
Pitch in inches to one turn	15.550	7.108
Angle of spiral	45°	45°
Pitch of cutter	10	10
No. of cutter	2	3
Whole depth of tooth	.216	.216
Angle of shafts	90°	90°
Center distance of shafts	3.606	3.606

Helix Angle of the Helical Gear

Locate the intersec' of the lines for the helix angle and the number of teeth. The number in the area in which the intersection falls is the cutter number required.

Fig. 2.—Cutters for helical gears.

This number is fractional as it almost invariably is. A fractional number of teeth is, of course, impossible and the obtaining of such a number shows that the assumed conditions are impossible and that they must be changed. More specifically, the center distance must be increased to admit 8 teeth or reduced to provide for 7. Adopting 8 teeth, the result is to change the length of the normal helix from the trial value first found. As in spur gears:

$$\text{diameter} = \frac{\text{number of teeth}}{\text{diameter pitch}}$$

So in helical gears:

$$\frac{\text{true normal helix}}{\pi} = \frac{\text{number of teeth.}}{\text{diametral pitch}}$$

$$= \frac{8}{6} = 1.333 \text{ ins.}$$

instead of the trial value 1.2637 ins., as first found. Inserting this true value in (*b*) we have:

$$d_1 = \frac{1.333}{.7071} = 1.885 \text{ ins.}$$

as the diameter of the driver. Since the numbers of teeth and the diameters of the two gears are inversely as the speeds we have:

Number of teeth in follower $= 8 \times 4 = 32$

Diameter of follower $= d_2 = 1.885 \times 4 = 7.540$ ins.

Finally $C = \frac{d_1 + d_2}{2} = \frac{1.885 + 7.540}{2} = 4.7125$ ins.

= corrected center distance that must be used.

To find the outside diameter of the blank add $\frac{2 \text{ ins.}}{\text{diametral pitch}}$ to the pitch diameter.

To find the Brown and Sharpe cutter to be used, multiply the number of teeth in the gear to be cut by 2.83 ($=\sec.^3 45°$) and select a cutter for the resulting number of teeth or use the chart, Fig. 2, by A. E. Larsson (*Amer. Mach., July 2*, 1908), the use of which is explained below it.

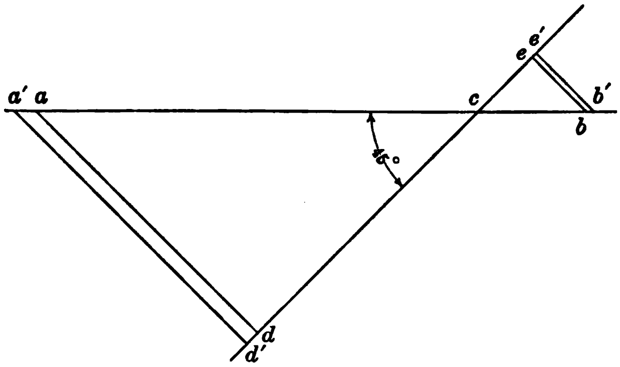
Helical Gears of 45 Deg. on Shafts at Right Angles by Graphics

The results may be checked by a graphical construction, Fig. 3, the use of which is explained below it.

Helical Gears of any Helix Angle on Shafts at Right Angles by Calculation

The solution of helical-gear problems for helix angles other than 45 deg. depends upon the same principle—finding the length of the normal helix to contain the required number of teeth—the process always involving a trial and a final solution. A clear understanding of the methods involves a knowledge of fundamental principles.

Fig. 4 is a conventional representation of a helical gear with its pitch cylinder prolonged. One of the teeth is also prolonged to make a complete turn around the cylinder, the resulting curve, *abcdef*, being the *tooth helix*. For purposes of calculation this helix is defined by the angle *kal* between a tooth and the tangent to the pitch cylinder. For shop purposes it is more commonly defined by the length *af* of



Lay down ab = twice the assumed center distance.

$= 2 \times 4.468 = 8.936$ ins.

Divide ab at c into two parts, the lengths of which are in the ratio of the speeds, that is

$$cb : ca :: 1 : 4$$

or $\quad cb = 1/5 \times 8.936 = 1.7872$ **ins.**

and $\quad ca = 4/5 \times 8.936 = 7.1488$ **ins.**

Draw de at an angle of 45 deg. with ab and draw ad and be at right angles with de, giving cd and ce which equal the

$$\frac{\text{trial lengths of normal values}}{\pi}$$

Scale cd and ce and multiply them by the diametral pitch, that is:

Scale length $1.26 \times 6 = 7.56 =$ Trial number of teeth in driver.

Scale length $5.04 \times 6 = 30.24 =$ Trial number of teeth in follower.

The results being fractional, increase (or decrease) them to the nearest whole numbers having the given ratio of 1 to 4, that is, increase them to 8 and 32. Divide these numbers by the diametral pitch

$$8/6 = 1.333 \text{ ins.}$$
$$32/6 = 5.333 \text{ ins.}$$

and lay down cd' = 5.333 ins. and ce' = 1.333 ins. Draw d'a' and e'b' perpendicular to de and we have:

ca' = diameter of gear = 7.540 ins.

c'b' = diameter of pinion = 1.885 ins.

a'b' = twice the center distance = $9.425 \times$ ins.

FIG. 3.—Graphical construction for helical gears of 45 deg. helix angle.

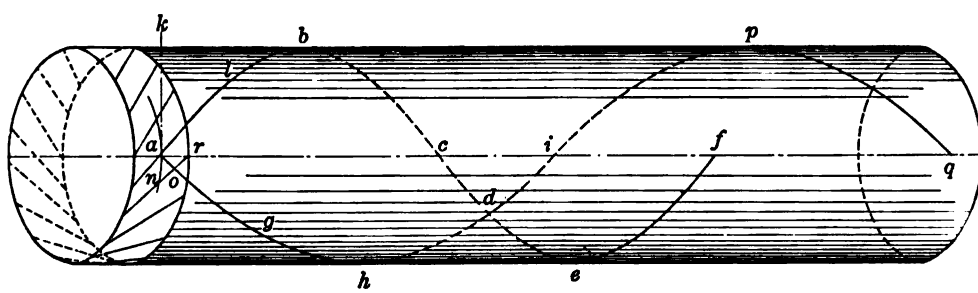


FIG. 4.

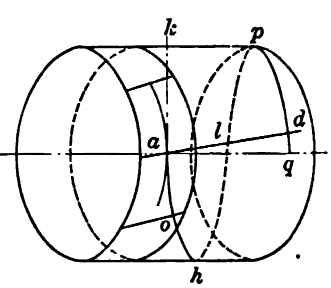


FIG. 5.

FIGS. 4 and 5.—Tooth and normal helices of helical gears.

a complete turn around the cylinder—that is, by giving the pitch of the helix.

The *normal helix, aghdipq,* is also drawn in. The *normal pitch* is the distance *ao*, *an* being the *circular pitch.* The normal pitch multiplied by the number of teeth must equal the length *aghd* of the normal helix between its intersections with the tooth helix—not the length *aghipq* of a complete turn around the cylinder. That this is true may be seen by reference to Fig. 5 in which the angle *kal* is nearly a right angle. It is apparent from this illustration that the length of the normal helix from *a* to *d* takes in all the teeth and that *ao*, multiplied by the number of teeth, must equal *ahpd* and not *ahpq*. This length *ahpd* is always less than *ahpq*, and usually much less. Fig. 6 *A* is a development of the gear end of Fig. 5 on a reduced scale, *ad* being the developed length of the normal helix. Fig. 6*B* and Fig. 6 *C* show how with the same circumferential pitch and the same number of teeth but a reduced value of the angle *kal*, the length of the normal helix, which cuts all the teeth, grows shorter until it may make but a small part of a complete turn around the cylinder. It is clear that in all cases the line *ad* cuts all the teeth precisely as does the circumference *aa*, which goes completely around the cylinder. It is also clear that if the normal pitch is decided upon at the start, a diameter of cylinder and a helix angle must be found such that the normal pitch, multiplied by the number of teeth, shall equal the length of the normal helix between two intersections with the tooth helix.

It is natural to ask: Why not employ the circumferential pitch and so deal directly with the circumference instead of the normal helix? Because we do not know what it is. The normal pitch is determined by the cutter used, while the circumferential pitch depends also upon the helix angle, and until this angle is known the circumferential pitch is not known.

In the extreme case of a helical gear in which the helix angle is so small that the gear becomes a single thread worm, as in Fig. 7, points *o* and *d* of Fig. 5 coincide and the length of the helix between *a* and *d* becomes the normal pitch. It is, however, true as before that the normal pitch, multiplied by the number of teeth, which is now one, is still equal to the length of the normal helix betweed two intersections with the tooth helix.

A glance at Fig. 6 will show that in gears of the same diameter the length of the normal helix[1] grows shorter as the angle *kal* grows less, and hence that it and its gear will contain successively fewer and fewer teeth of the same normal pitch. That is to say, the number of teeth in a gear varies with the helix angle as well as with the diameter and *the number of teeth in two gears of the same normal pitch is not necessarily proportional to the diameters.* In fact, it is never so proportional, except when the angle *kal* is equal to 45 deg. *The diametral pitch of the cutters and the diameter of the gear thus do not determine the number of teeth.*

Fig. 8 illustrates the simplest possible case of a pair of helical gears. The shafts are at right angles, the gears are of equal size and the tooth helix has an angle *kal* of 45 deg. Such a pair of gears will obviously run at the same speed—that is, have a speed ratio of 1—and as obviously both will have the same number of teeth. Unlike spur gears, there are two ways in which the speed ratio of such a pair of helical gears may be varied. First, the diameters of the gears may be changed, as with spur gears, the angle of the tooth helix remaining unchanged, as in Fig. 9; and second, the angle of the helix may be changed, the diameters of the gears remaining unchanged, as in Fig. 10. These methods act in very different ways. The first method is

[1] "Length of normal helix" is to be understood as meaning the length of that helix between two intersections with the same tooth helix.

analogous to the procedure with spur gears. As with spur gears the circumferential or pitch-line speed of the two gears remains, as before the change, equal, but the length of the circumference of the two gears is unequal and the larger one thus makes a less number of revolutions than the smaller one. The second method is entirely unlike anything seen in connection with spur gears. By it the pitch-line speeds of the two gears are made unequal, and hence, while their diameters are equal, the lower one revolves the more slowly. This points out another fundamental difference between helical and spur gears: With helical gears, unless the helix angle is 45 deg., *the pitch-line speeds of two mating gears are not the same.*

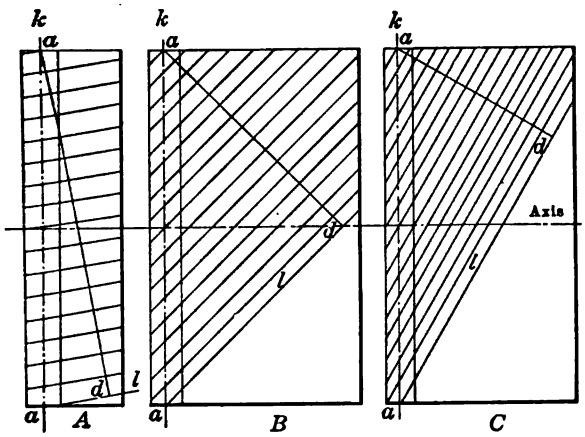


FIG. 6.—Developed helical gears.

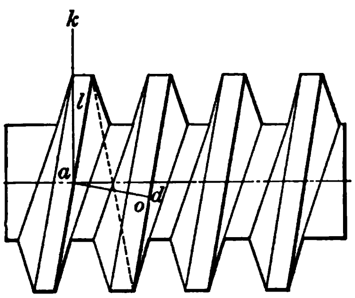


FIG. 7.—Normal helix of a worm.

The two methods of changing the speed ratio shown in Figs. 8 and 9 may be combined. That is, part of the desired change in speed may be obtained by changing the diameters of the gears and the remainder by changing the angle of the helix. Given the speed ratio and the diameter of one of the gears, we may assume a helix angle and find a diameter for the second gear to go with it which shall give the desired speed ratio and, having done this, a second angle may be assumed and a second diameter be found. There are thus an indefinite number of combinations of angles and diameters which will give the required speed ratio. Note, however, that with the diameter of one gear fixed, every change in the diameter of the other changes the distance between centers, and that the lengths of both normal helixes must be exact multiples of the normal pitch of the teeth. The problem of designing spiral gears thus consists of finding a pair which shall have a given center distance, helix angles which can be cut with the means at hand, and such a normal pitch that stock cutters can be used.

Geometrically speaking, there is a wide range of choice in the helix angle. As regards the desirability of different angles from the standpoint of durability, the conditions are essentially the same as in worm gearing, in which the most favorable angle for durability is not far from 45 deg. There is, however, but a trifling increase in wear down to 30 deg., no serious increase down to 20 deg., and no destructive increase down to about 12 deg. Where gears are to transmit considerable power, the best results should attend the use of angles between 30 and 45 deg., while angles as low as 20 deg. may be used in case of need, and as low as 12 deg. if the gears are to run in an oil bath or do light work only. The angle may also be increased above 45 deg. by similar amounts and with similar results.

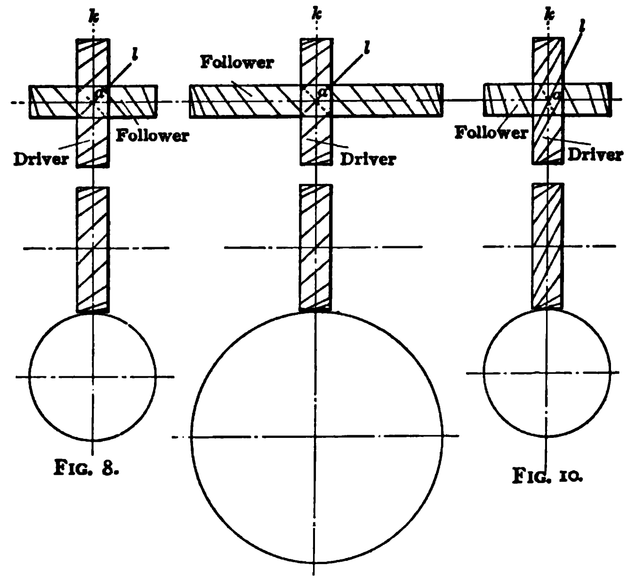


FIG. 8.

FIG. 10.

FIG. 9.

FIGS. 8 to 10.—The speed ratio of helical gears.

The author has shown (*Amer. Mach., Nov.* 21, 1901; *Van Nostrand's Science Series No.* 116) that for helical gears having shafts at right angles:

$$\frac{r_2}{r_1}=\frac{d_1}{d_2}\tan\alpha \qquad (c)$$

in which r_1 = r.p.m. of driver,
r_2 = r.p.m. of follower,
d_1 = diameter of driver, ins.,
d_2 = diameter of follower, ins.,
α = helix angle of driver, deg.,
= angle *kal* of Figs. 5, 6 and 7 measured on the driver.

Note that formula (*c*) differs from the corresponding formula for spur gears only by the introduction of the factor tan α.

In any actual case the center distance and the speeds are given and the diameters and helix angle must be found. We may assume a ratio for the diameters and find the angle, or we may assume an angle and find the ratio of diameters. It is desirable to assume the angle first, as on it depends, largely, the durability of the gears. To do this the above formula may be more conveniently written:

$$\frac{d_2}{d_1}=\frac{r_1}{r_2}\tan\alpha \qquad (d)$$

The author has also shown (same references) that

$$d_1=\frac{2C}{\frac{r_1}{r_2}\tan\alpha+1} \qquad (e)$$

in which C = distance between centers, ins.

Having assumed a value for α and substituted its tangent and the ratio of the desired speeds in (*e*), we find a value for d_1, and, having found d_1, d_2 may obviously be found by subtracting d_1 from $2C$.

Such a solution is complete in a geometrical sense, and if it were feasible to make a cutter to suit each case, it would be complete in a

practical sense also. When, however, we go a step further and find the length of the normal helixes, the probabilities are all against their being exact multiples of the pitch of any stock cutter. The solution so obtained must therefore be considered as provisional and be modified to suit the cutters to be used.

The author has also shown (same references) that

$$l_1 = c_1 \sin \alpha$$
$$= \pi d_1 \sin \alpha$$

or

$$\frac{l_1}{\pi} = d_1 \sin \alpha \qquad (f)$$

and that

$$l_2 = c_2 \cos \alpha$$
$$= \pi d_2 \cos \alpha$$

or

$$\frac{l_2}{\pi} = d_2 \cos \alpha \qquad (g)$$

in which c_1 = circumference of driver, ins.,
c_2 = circumference of follower, ins.,
d_1 = diameter of driver, ins.,
d_2 = diameter of follower, ins.,
l_1 = length of normal helix of driver between intersections with tooth helix, ins.,
l_2 = length of normal helix of follower between intersections with tooth helix, ins.,
α = tooth helix angle of driver, deg.

Note that (f) and (g) give the lengths of the normal helixes divided by π and not their actual lengths. This is done because, in dealing with diametral pitch cutters the calculations are made less laborious as has been explained in connection with 45-deg. gears. Dividing (f) by (g) gives:

$$\frac{l_1}{l_2} = \frac{d_1 \sin \alpha}{d_2 \cos \alpha}$$
$$= \frac{d_1}{d_2} \tan \alpha \qquad (h)$$

Comparing (c) with (h) proves what is almost self-evident, that the lengths of the normal helixes are to each other inversely as the number of revolutions, and hence that *a pitch which will exactly divide the short helix will also divide the long one* and that *the numbers of teeth in the gears are inversely as the speeds.*

The use of the formulas is best shown by an example:

Assume that $$\frac{\text{r.p.m. of driver}}{\text{r.p.m. of follower}} = \frac{r_1}{r_2} = 4$$

and

$$\text{center distance} = C = 4\tfrac{15}{32}$$
$$= 4.468 \text{ ins.}$$

We are, at the start, entirely at sea regarding the whole matter; but as an angle of 30 deg. is favorable to durability we may use it as a trial angle and see what it will lead to. Finding the tangent of 30 deg. in a table and substituting it and the value of $\frac{r_1}{r_2}$ in (e) we obtain:

$$d_1 = \frac{2 \times 4.468}{4 \times .57735 + 1}$$
$$= 2.7 \text{ ins.}^{1}$$

Obviously $d_1 + d_2 = 2C$ or

$$d_2 = 2C - d_1$$

that is, $$d_2 = 2 \times 4.468 - 2.7$$
$$= 6.236 \text{ ins.}$$

From (f) we find $$\frac{l_1}{\pi} = 2.7 \times .5$$
$$= 1.35 \text{ ins.}$$

and from (g) $$\frac{l_2}{\pi} = 6.236 \times .866$$
$$= 5.4 \text{ ins.}$$

[1] For a method of greatly abbreviating the calculations from this point on, in most cases, see Table 2 of Real Diametral Pitches of Helical Gears.

These values of d_1, d_2, $\frac{l_1}{\pi}$, and $\frac{l_2}{\pi}$ are the provisional values belonging with 30 deg. for α. We must next find if these lengths of the normal helixes will contain exact whole numbers of teeth. Assume that 6 diametral pitch cutters are to be used. With spur gears,

$$\text{diameter} \times \text{diametral pitch} = \frac{\text{circumference}}{\pi} \times \text{diametral pitch}$$
$$= \text{No. of teeth}$$

and so with helical gears,

$$\frac{\text{length of normal helix}}{\pi} \times \text{diametral pitch} = \text{No. of teeth}$$

Performing the multiplications we have:

$$\frac{l_1}{\pi} \times 6 = 1.35 \times 6$$
$$= 8.1 \text{ teeth}$$

and

$$\frac{l_2}{\pi} \times 6 = 5.4 \times 6$$
$$= 32.4 \text{ teeth}$$

The provisional normal helixes thus contain 8.1 and 32.4 teeth of the desired pitch, and as these numbers are impossible, we take the nearest whole numbers having the desired ratio of 1 to 4, namely, 8 and 32. That is, we decide to make the gears smaller and so shorten the normal helixes until they contain exactly 8 and 32 teeth—the result being also to reduce the center distance from the assumed value.

To determine how much to reduce the diameters we must first find the reduced lengths of the normal helixes, which must be such that:

$$\frac{l_1}{\pi} \times 6 = 8$$

or

$$\frac{l_1}{\pi} = \frac{8}{6}$$
$$= 1.333 \text{ ins.}$$

and

$$\frac{l_2}{\pi} \times 6 = 32$$

or

$$\frac{l_2}{\pi} = 5.333 \text{ ins.}$$

Knowing these corrected values of $\frac{l_1}{\pi}$ and $\frac{l_2}{\pi}$, it is easy to find the new diameters thus: The ratio between the provisional and final diameters is the same as that between the lengths of the provisional and final helixes, which latter is 8.1 to 8 or, its equal, 32.4 to 32. That is:

$$\frac{\text{final diameter}}{\text{provisional diameter}} = \frac{8}{8.1}$$

or final diameter = provisional diameter $\times \frac{8}{8.1}$

or

$$\text{final } d_1 = 2.7 \times \frac{8}{8.1}$$
$$= 2.667 \text{ ins.}$$

and

$$\text{final } d_2 = 6.236 \times \frac{8}{8.1}$$
$$= 6.159 \text{ ins.}$$

and $d_1 + d_2 = 2.667 + 6.159 = 8.825$ ins. = twice the new center distance.

These calculations may be greatly abbreviated in most cases by the use of Table 2 by WM. HAUGHTON (*Amer. Mach., Sept.* 7, 1911). In this table MR. HAUGHTON has given a series of numbers (which he calls *real diametral pitches*) having the same relation to the circular pitches of helical gears that the usual diametral pitches have to the circular pitches of spur gears. We have the well known relation:

$$\frac{\text{No. of teeth in a spur gear}}{\text{diametral pitch}} = \text{pitch diameter}$$

And so with this table:

$$\frac{\text{No. of teeth in a helical gear}}{\text{real diametral pitch}} = \text{pitch diameter}$$

The use of the table is best shown by applying it to the example already worked out, in which

$$\frac{\text{revolutions of driver}}{\text{revolutions of follower}} = 4$$

Trial center distance $= 4.468$ ins.
Helix angle $= 30$ deg.
Diametral pitch of cutter $= 6$

We first apply formula (2), for which a column of tangents will be found in Table 2, the result being as before:

trial diameter of driver $= 2.7$ ins.

We now consult the Table 2 and, opposite 30 deg. helix angle and below 6 pitch, we find 3 as the real diametral pitch. Now:

No. of teeth $=$ pitch diameter $\times$ real diametral pitch

$$= 2.7 \times 3 = 8.1$$

This result being fractional and hence impossible, we reduce it to 8 and then find

$$\text{final diameter of driven} = \frac{\text{No. of teeth}}{\text{real diametral pitch}}$$

$$= \frac{8}{3} = 1.667 \text{ ins.}$$

The speed ratio being 1 to 4 the motive gear must have

$$8 \times 4 = 32 \text{ teeth}$$

Looking in the table again for the real diametral pitch of the mating gear, we find it to be 5.196 and, as before,

$$\text{final diameter of follower} = \frac{\text{No. of teeth}}{\text{real diametral pitch}}$$

$$= \frac{32}{5.196} = 6.159 \text{ ins.}$$

Note that for depth of tooth and diameter of blank the diametral pitch as given in the top line of the table is to be used.

The special values of helix angles 26 deg. 34 min. and 63 deg. 26 min. are for gears of equal diameter with a speed ratio of 2.

By an adjustment of the angle a it is possible to solve the problem for the assumed center distance. In this, helical gears posshss a property not shared by spurs of diametral pitch, which can only be made of such diameters as will contain an exact whole number of teeth. The lack of this property has not been found of moment with spur gears and it would seem that its possession is of correspondingly small value with helical gears. For this reason the author has omitted its consideration here. Those who desire to learn its use are referred to Worm and Spiral Gearing—(Van Nostrand's Science Series No. 116) by the author.

Helical Gears of any Helix Angle by Graphics

The above determinations may be made or checked by a graphical construction, Fig. 11, the use of which is explained below it.

To find the outside diameter of the blank, add $\dfrac{2 \text{ ins.}}{\text{diametral pitch}}$ to the pitch diameter.

To find the Brown and Sharpe cutter to be used divide the number of teeth in the gear to be used by $\sin^3 \alpha$ and select a cutter for the resulting number of teeth or use the chart, Fig. 2.

The pitches of the tooth helixes are found by the formulas:

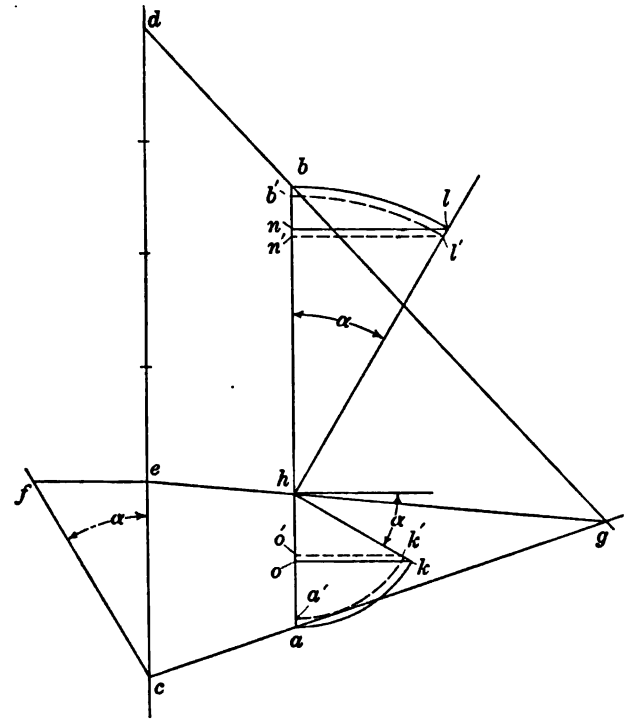
pitch of tooth helix of driver $= \pi d_1$ 1 $\tan \alpha$
pitch of tooth helix of follower $= \pi d_2 \cos \alpha$

α being taken from the driver in both cases.

Helical Gears of any Helix Angle on Shafts at any Angle

The calculation of helical gears on shafts at other angles than 90 *deg.* brings in the angle between the shafts.

Let r_1 = r.p.m. of driver.
r_2 = r.p.m. of follower.
d_1 = diameter of driver, ins.
d_2 = diameter of follower, ins.
α = helix angle of driver, deg.
ϕ = angle between the shafts, that is, the lesser of the supplementary angles, deg.
C = trial distance between centers, ins.



Lay off ab = 2C = $8\frac{15}{16}$ inches. At any convenient distance lay off the indefinite line cd parallel to ab. At c lay off the provisional angle α = 30 degrees. Draw ef at any convenient point perpendicular to cd. Take ef in the dividers and step it off from e toward d as many times as will represent the ratio of the desired speed of the driver divided by that of the follower. That is, in the present case, lay off ef 4 times above e and thus obtain d. Draw ca and db and extend them till they meet at g.[1] Draw ge, giving ah and bh, which are provisional diameters of driver and follower respectively. Draw hp perpendicular to ab, at h lay down hk and hl to repeat α, and from h strike arcs ak and bl. Draw ko and nl perpendicular to ab and we have the provisional values.

$$\text{ah} = \text{d}_1$$
$$\text{bh} = \text{d}_2$$
$$\text{ho} = \frac{\text{l}_1}{\pi}$$
$$\text{hn} = \frac{\text{l}_2}{\pi}$$

Scale ho and hn and multiply them by the diametral pitch number – 6. If the results are not whole numbers, as they usually are not, select the nearest whole numbers having the desired speed ratio, and they are the final numbers of teeth. Divide these numbers by the diametral pitch number to obtain the final values of $\frac{\text{l}_1}{\pi}$ and $\frac{\text{l}_2}{\pi}$ and lay them down as ho' and hn'. Draw o'k' and l'n' and k'a' and l'b', giving:

a'h = the final d_1,
b'h = the final d_2,
a'b' = twice the new center distance.

Fig. 11.—Graphical solution of helical gears of any helix angle on shafts at right angles.

Formula (e) becomes for this case:

$$d_1 = \frac{2C}{\dfrac{r_1 \sin \alpha}{r_2 \sin (\alpha + \phi)}} \qquad (i)$$

As before $d_2 = 2\,C - d_1$.

[1] Had cd been taken shorter than ab, g would have fallen to the left of the diagram, but the construction would otherwise have been unchanged.

TABLE 2.—REAL DIAMETRAL PITCHES OF HELICAL GEARS

Helix angle α		Normal diametral pitches																										Tangent	
		2		2½		3																10		12			
Driver	Follower	Driver	Follower	Driver	Follower	Driver	Follower		lower		lower		lower		lower		lower		lower		lower	Driver	Follower	Driver	Follower	Driver	Follower
10°	80°	.3473	1.9696	.4341	2.4620	.5209	2.9544	.6078	3.4468	.6946	3.9392	.8682	4.9240	1.0419	5.9089	1.2155	6.8937	1.3892	7.8785	1.5628	8.8633	1.7365	9.8481	2.0838	11.8177	.1763	5.6713
11°	79°	.3816	1.9633	.4770	2.4541	.5724	2.9449	.6678	3.4357	.7632	3.9265	.9540	4.9081	1.1449	5.8898	1.3358	6.8714	1.5265	7.8530	1.7173	8.8347	1.9081	9.8163	2.2900	11.7796	.1944	5.1446
12°	78°	.4158	1.9563	.5198	2.4454	.6237	2.9344	.7277	3.4235	.8316	3.9126	1.0395	4.8907	1.2475	5.8689	1.4554	6.8470	1.6633	7.8252	1.8712	8.8033	2.0791	9.7815	2.4949	11.7378	.2126	4.7046
13°	77°	.4499	1.9487	.5624	2.4359	.6748	2.9231	.7873	3.4103	.8998	3.8975	1.1247	4.8718	1.3497	5.8462	1.5746	6.8206	1.7996	7.7950	2.0245	8.7693	2.2495	9.7437	2.6994	11.6924	.2309	4.3315
14°	76°	.4838	1.9406	.6048	2.4257	.7258	2.9109	.8467	3.3396	.9677	3.8812	1.2096	4.8515	1.4515	5.8218	1.6934	6.7921	1.9354	5.7624	2.1773	8.7327	2.4192	9.7030	2.903	11.6436	.2493	4.0108
15°	75″	.5176	1.9319	.6470	2.4148	.7765	2.8978	.9059	3.3807	1.0353	3.8637	1.2941	4.8296	1.5529	5.7956	1.8117	6.7615	2.0706	7.7274	2.3294	8.6934	2.5882	9.6593	3.1058	11.5912	.2679	3.7321
16°	74°	.5513	1.9225	.6891	2.4031	.8269	2.8838	.9647	3.3644	1.1026	3.8450	1.3782	4.8063	1.6538	5.7676	1.9295	6.7288	2.2051	7.6901	2.4808	8.6513	2.7564	9.6126	3.3077	11.5351	.2867	3.4874
17°	73°	.5847	1.9126	.7309	2.3907	.8771	2.8689	1.0233	3.3470	1.1695	3.8252	1.4618	4.7815	1.7542	5.7378	2.0466	6.6941	2.3390	7.6504	2.6313	8.6067	2.9237	9.5630	3.5084	11.4576	.3057	3.2709
18°	72°	.6180	1.9021	.7725	2.3776	.9271	2.8532	1.0816	3.3287	1.2361	3.8042	1.5451	4.7553	1.8541	5.7064	2.1631	6.6574	2.4722	7.6085	2.7812	8.5595	3.0902	9.5106	3.7082	11.4127	.3249	3.0777
19°	71°	.6511	1.8910	.8139	2.3638	.9767	2.8366	1.1395	3.3093	1.3023	3.7821	1.6278	4.7276	1.9534	5.6731	2.2790	6.6186	2.6046	7.5642	2.9301	8.5097	3.2557	9.4552	3.9068	11.3462	.3443	2.9042
20°	70°	.6840	1.8794	.8550	2.3492	1.0261	.8191	1.1971	3.2889	1.3681	3.7588	1.7101	4.6984	2.0521	5.6381	2.3941	6.5778	2.7362	7.5175	3.0782	8.4572	3.4202	9.3969	4.1042	11.2763	.3640	2.7475
21°	69°	.7167	1.8672	.8959	2.3339	1.0751	2.8007	1.2543	3.2675	1.4335	3.7343	1.7918	4.6679	2.1502	5.6015	2.5086	6.5351	2.8670	7.4686	3.2253	8.4022	3.5837	9.3358	4.3004	11.2030	.3839	2.6051
22°	68°	.7492	1.8544	.9365	2.3179	1.1238	2.7815	1.3111	3.2451	1.4984	3.7087	1.8730	4.6359	2.2477	5.5631	2.6223	6.4903	2.9969	7.4174	3.3715	8.3446	3.7461	9.2718	4.4953	11.1262	.4040	2.4751
23°	67°	.7815	1.8410	.9768	2.3012	1.1722	2.7615	1.3675	3.2217	1.5629	3.6820	1.9536	4.6025	2.3444	5.5230	2.7351	6.4435	3.1258	7.3640	3.5166	8.2845	3.9073	9.2050	4.6888	11.0460	.4245	2.3559
24°	66°	.8135	1.8271	1.0168	2.2839	1.2202	2.7406	1.4236	3.1974	1.6270	3.6542	2.0337	4.5677	2.4404	5.4813	2.8472	6.3948	3.2539	7.3084	3.6607	8.2219	4.0674	9.1355	4.8809	10.9626	.4452	2.2460
25°	65°	.8452	1.8126	1.0565	2.2658	1.2679	2.7189	1.4792	3.1721	1.6905	3.6252	2.1131	4.5315	2.5357	5.4379	2.9583	6.3442	3.3810	7.2505	3.8036	8.1568	4.2262	9.0631	5.0714	10.8757	.4663	2.1445
26°	64°	.8567	1.7976	1.0701	2.2470	1.2851	2.6964	1.4993	3.1458	1.7535	3.5952	2.1918	4.4939	2.6302	5.3927	3.0686	6.2915	3.5070	7.1903	3.9453	8.0891	4.3837	8.9879	5.2604	10.7855	.4877	2.0503
26° 34′	63° 26′	.8945	1.7888	1.1181	2.2360	1.3417	2.6832	1.5653	3.1304	1.7890	3.5776	2.2362	4.4720	2.6834	5.3664	3.1307	6.2610	3.578	7.1553	4.0252	8.0497	4.472	8.944	5.3669	10.7329	.50004	1.99986
27°	63°	.908	1.782	1.135	2.2275	1.362	2.673	1.589	3.1185	1.816	3.564	2.270	4.455	2.724	5.340	3.178	6.237	3.632	7.128	4.086	8.019	4.540	8.910	5.448	10.692	.5095	1.9626
28°	62°	.9389	1.7659	1.1737	2.2074	1.4084	2.6488	1.643	3.090	1.8779	3.5318	2.3473	4.4175	2.8168	5.2977	3.2863	6.1806	3.7558	7.0636	4.2252	7.9465	4.695	8.829	5.6336	10.5954	.5317	1.8807
29°	61°	.9696	1.7492	1.212	2.1865	1.4544	2.6239	1.6968	3.0612	1.9392	3.4985	2.424	4.3731	2.9089	5.2477	3.3937	6.1223	3.8785	6.997	4.3633	7.8716	4.848	8.7462	5.8177	10.4954	.5543	1.8040
30°	60°	1.00	1.732	1.250	2.1651	1.500	2.5981	1.750	3.0311	2.00	3.464	2.500	4.330	3.00	5.196	3.500	6.0622	4.00	6.9282	4.500	7.7943	5.00	8.660	6.00	10.3923	.5774	1.7321
31°	59°	1.030	1.7143	1.2876	2.1429	1.545	2.5715	1.8026	2.990	2.060	3.4287	2.5752	4.2859	3.090	5.143	3.6052	6.000	4.120	6.8574	4.6354	7.7145	5.150	8.572	6.1805	10.286	.6009	1.6643
32°	58°	1.0598	1.696	1.3248	2.120	1.5898	2.544	1.8547	2.968	2.1197	3.3922	2.6496	4.240	3.1795	5.088	3.7094	5.9363	4.2394	6.7844	4.7693	7.6324	5.299	8.480	6.359	10.1766	.6249	1.6003
33°	57°	1.0893	1.6773	1.3616	2.0967	1.6339	2.516	1.9062	2.9353	2.1786	3.3547	2.7232	4.1933	3.2678	5.0320	3.8125	5.8707	4.357	6.7094	4.9018	7.548	5.4464	8.3867	6.5357	10.064	.6494	1.5399
34°	56°	1.1184	1.6581	1.398	2.0726	1.6776	2.487	1.957	2.902	2.237	3.316	2.796	4.1452	3.355	4.9742	3.9143	5.8033	4.4735	6.6323	5.0327	7.4614	5.5919	8.2904	6.7103	9.9485	.6745	1.4826
35°	55°	1.1472	1.6383	1.434	2.048	1.7207	2.4574	2.0075	2.867	2.2943	3.2766	2.868	4.0957	3.4415	4.9149	4.015	5.734	4.5886	6.5532	5.1622	7.3723	5.7358	8.1915	6.883	9.8298	.7002	1.4281
36°	54°	1.1756	1.618	1.4695	2.0225	1.7634	2.427	2.0573	2.8316	2.3512	3.2361	2.939	4.0451	3.5267	4.8541	4.1145	5.6631	4.7023	6.4721	5.2901	7.2812	5.8779	8.0902	7.0535	9.7082	.7265	1.3764
37°	53°	1.2036	1.5973	1.5045	1.9966	1.8055	2.3959	2.1064	2.7952	2.4073	3.1946	3.0091	3.9932	3.6109	4.7918	4.2127	5.5905	4.8146	6.3891	5.4164	7.1878	6.0182	7.9864	7.2218	9.5837	.7536	1.3270
38°	52°	1.2313	1.576	1.5391	1.970	1.847	2.364	2.1548	2.758	2.4626	3.152	3.0783	3.940	3.694	4.728	4.3096	5.5161	4.9253	6.3041	5.409	7.092	6.1566	7.8801	7.3879	9.4561	.7813	1.2799
39°	51°	1.2586	1.5543	1.5733	1.9429	1.888	2.3314	2.2026	2.720	2.5173	3.1086	3.1466	3.8857	3.7759	4.6629	4.4052	5.440	5.0346	6.2152	5.6639	6.9943	6.2932	7.7715	7.5518	9.3258	.8098	1.2349
40°	50°	1.2856	1.5321	1.607	1.9151	1.9284	2.2981	2.2498	2.6811	2.5712	3.0642	3.2139	3.8302	3.8567	4.5962	4.4995	5.3623	5.1423	6.1283	5.7851	6.8944	6.4279	7.6604	7.7135	9.1925	.8391	1.1918
41°	49°	1.3121	1.5094	1.6401	1.8868	1.9682	2.2641	2.2962	2.6415	2.6242	3.0188	3.2803	3.7735	3.9364	4.5283	4.5924	5.2830	5.2485	6.0377	5.9045	6.7924	6.5606	7.5471	7.8727	9.0565	.8693	1.1504
42°	48°	1.3383	1.4863	1.6728	1.8578	2.0074	2.2294	2.3419	2.601	2.6765	2.9726	3.3456	3.7157	4.0148	4.4588	4.6839	5.202	5.353	5.9451	6.0222	6.6883	6.6913	7.4314	8.0296	8.9177	.9004	1.1106
43°	47°	1.364	1.4627	1.705	1.8284	2.046	2.1940	2.387	2.5597	2.728	2.9254	3.410	3.6567	4.092	4.3881	4.774	5.1194	5.456	5.8508	6.138	6.5821	6.820	7.3135	8.184	8.7762	.9325	1.0724
44°	46°	1.3893	1.4387	1.7366	1.7983	2.084	2.1580	2.4313	2.5177	2.7786	2.8774	3.4733	3.5967	4.1680	4.316	4.8626	5.0354	5.5573	5.7547	6.2519	6.474	6.9466	7.1934	8.3359	8.6321	.9657	1.0355
45°	45°	1.4142		1.7678		2.1213		2.4749		2.8284		3.5355		4.2427		4.9498		5.6569		6.364		7.0711		8.4853		1.0000	1.0000

Formulas for use:

Pitch diameter = number of teeth divided by real pitch.

Lead of spiral = circumference on pitch line multiplied by the tangent of angle.

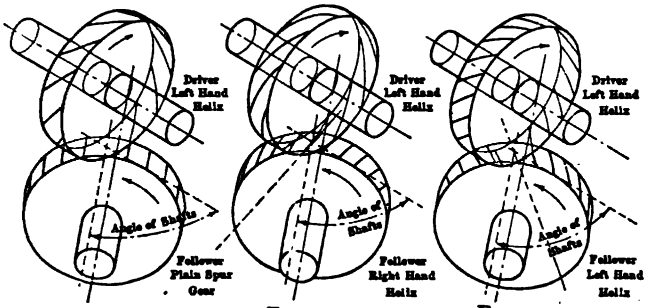


FIG. 12. FIG. 13. FIG. 14.

FIGS. 12 to 14.—Helices of gears on shafts at other than right angles.

As before these values of d_1 and d_2 are to be treated as trial values and tested and adjusted for exact whole numbers of teeth. For the driver, formula (*f*) applies directly, but not so with (*g*) for the follower. If the shafts are at right angles the helix angles of mating gears are compliments and the cosine of one may be used for the sine of the other, as in (*g*). With the shafts at other angles, this is no longer true, and it is necessary to find the angle of the follower by the relation:

$$\text{Sum of helix angles} = 180° - \text{shaft angle}$$

Calling the helix angle of the follower β, we have in place of (*g*):

$$\frac{l_2}{\pi} = d_2 \sin \beta. \qquad (j)$$

So also, in finding the pitch of the tooth helix, the cotangent of the angle of the driver cannot be used for the tangent of the angle of the follower, the formulas becoming:

$$\text{pitch of tooth helix of driver} = \pi\, d_1 \tan \alpha$$

$$\text{pitch of tooth helix of follower} = \pi d_2 \tan \beta$$

The helixes of gears on shafts at right angles are always of the same hand but, with shafts at other angles, the relation given above for the sum of the helix angles makes it possible that one of the gears may be a spur, or its helix may be of opposite hand to its mate. This is more clearly shown in Figs. 12, 13 and 14, by H. B. McCABE (*Amer. Mach., Oct.* 11, 1906). In Fig. 12 the driver has a left-hand helix with an angle equal to the shaft angle and the follower in a plain spur. In Fig. 13 the angle of the driver has been reduced and the helix of the follower is *right* hand, while in Fig. 14 the angle of the driver has been increased and the helix of the follower is *left* hand

PLANETARY (EPICYCLIC) GEARS

The action of epicyclic or planetary trains of gearing may be determined from the following collection of formulas by F. J. Bostock (*Amer. Mach., May* 16, 1907).

An epicyclic gear is one that revolves around the center of another with which it is in mesh. The formulas that follow begin with simple and lead up to more complex arrangements.

Example 1.—If in Fig. 1 R and N are two gears in mesh, r and n being their respective numbers of teeth, their bearings being fixed, then:

$$\frac{\text{velocity of driven gear } N}{\text{velocity of driver gear } R}=\frac{r}{n};$$

$$\text{or } N\text{'s velocity}=R\text{'s velocity}\times\frac{r}{n}$$

If, however, R revolve in a positive direction, N must revolve in the opposite, that is, in a negative direction.

$$\therefore N\text{'s velocity}=-R\text{'s velocity}\times\frac{r}{n} \tag{a}$$

In all these calculations it is essential that great care be taken in order to obtain the correct sign of the resulting velocity.

Example 2.—An intermediate gear I is placed in contact with both N and R, Fig. 2. The effect will be that of giving N motion in the same direction as R.

$$\therefore N\text{'s velocity}=R\text{'s velocity}\times\frac{r}{n} \tag{b}$$

Simple Epicyclic Train

Example 3.—Two gears, F and N, are in mesh, the centers of which are on the arm R, which is capable of revolving around the center of F. It is required to find the velocity ratio between R and N when R revolves around the fixed gear F; Fig. 3 shows the arrangement. The gear N is subject to two motions due to the following two conditions:

a. The fact of its being fixed to the arm R.

b. The fact that it is in contact with the gear F.

We will therefore in the first place suppose that they are not in gear, and that N cannot rotate on the arm R. Then if R makes one revolution around F it is obvious that N must also make one revolution around F, as in Fig. 4.

$\therefore N$*'s* velocity due to condition a, $=R$*'s* velocity,
the direction being the same as R*'s*.

Secondly, if instead of R making one revolution around F in a + direction, we cause F to make one in the opposite, that is, negative direction, we shall have exactly the same effect. Therefore place F and N in mesh, and fix the arm R, as in Fig. 5.

Then if F makes -1 revolution, N will make $+\frac{f}{n}$ revolutions. (According to equation (*a*).

But -1 of $F=+1$ of R.

$$\therefore 1 \text{ revolution of } R=\frac{f}{n} \text{ revolutions of } N,$$

$$\text{or,}\qquad N\text{'s velocity, due to condition } b=\frac{f}{n}\times R\text{'s velocity.}$$

By addition we obtain the total impulses given to N, that is:

$$N\text{'s velocity}=R\text{'s velocity}+\frac{f}{n}\times R\text{'s velocity}$$

$$=R\text{'s velocity}\times\left(1+\frac{f}{n}\right). \tag{c}$$

Epicyclic Train with an Idler

Example 4.—If an intermediate gear I be inserted between F and N, as in Fig. 6, we have a similar case to the above, but the intermediate gear has the effect of changing the direction of revolution of N (equation b), due to its contact with F through I.

$$\therefore N\text{'s velocity}=R\text{'s velocity}\times\left(1-\frac{f}{n}\right). \tag{d}$$

It will be seen that if $f=n$, N will not have any motion of rotation at all; and it will have a positive one if $f<n$ and negative if $f>n$. Thus by the adjustment of f and n one can obtain great reduction in speed by means of few moving parts.

Simple Epicyclic Train with Internal Gear

Example 5.—Instead of the driven gear N being external, it might have been internal, as shown in Fig. 7. The effect will be the same as inserting an intermediate gear in Example 3, giving the same result as case 4, namely:

$$N\text{'s velocity}=R\text{'s velocity}\times\left(1-\frac{f}{n}\right). \tag{e}$$

In this case $n>f$.

$\therefore$ The final direction is always +.

Internal Gear Epicyclic Train with Intermediate Gear

Example 6.—Fig. 8 shows a still further modification of this condition, I being an intermediate gear. The result is:

$$N\text{'s velocity}=R\text{'s velocity}\times\left(1+\frac{f}{n}\right). \tag{f}$$

The Same Train with the Internal Gear Driving

Example 7.—With the above type, one often arranges the outer internal gear to be the driver, imparting motion to the arm carrying the intermediate gear. See Fig. 9.

We have seen by equation 6 that:

$$\frac{N\text{'s velocity (driven)}}{R\text{'s velocity (driver)}}=\frac{1}{1+\frac{f}{r}}$$

$$\therefore N\text{'s velocity}=R\text{'s velocity}\div\left(1-\frac{f}{r}\right)$$

$$=R\text{'s velocity}\times\left(\frac{r}{r+f}\right) \tag{g}$$

The latter two examples constitute what is known as the "Sun and Planet" gear, which is largely used in many mechanisms. All the above examples show "simple" gearing, but they can be compounded with great advantage.

Compound Gears in Fixed Bearings

Example 8.—Gears compounded together are shown in Figs. 10 and 11, 11 being a diagram of 10. One repeats the well-known rule that:

$$\frac{\text{velocity of driven gear}}{\text{velocity of driver gear}}=\frac{\text{product of number of teeth of driver gears}}{\text{product of number of teeth of driven gears}}$$

$$\text{or,}\qquad N\text{'s velocity}=R\text{'s velocity}\times\frac{r\times m}{s\times n} \tag{h}$$

The direction is the same as N's namely, +.

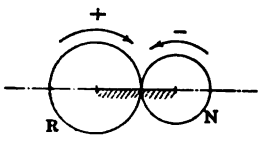


FIG. 1.—Simple pair of gears in fixed bearings.

Eq. 1 $N's\ V. = -R's\ V. \times \frac{r}{n}$

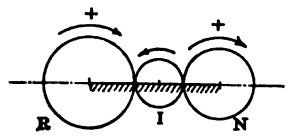


FIG. 2.—Gears in fixed bearings with an idler.

Eq. 2 $N's\ V. = R's\ V. \times \frac{r}{n}$

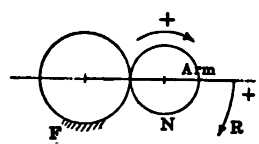


FIG. 3.—Simple epicyclic train.

Eq. 3 $N's\ V. = R's\ V. \times \left(1 + \frac{f}{n}\right)$

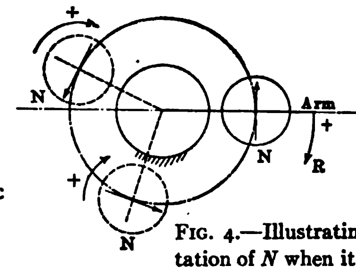


FIG. 4.—Illustrating rotation of N when it is revolved about the center of F.

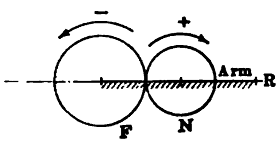


FIG. 5.—Second stage in deriving equation 3; arm assumed to be fixed, F turned backward.

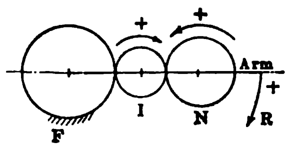


FIG. 6.—Epicyclic train with an idler.

Eq. 4 $N's$ velocity $= R's$ velocity $\times \left(1 - \frac{f}{n}\right)$

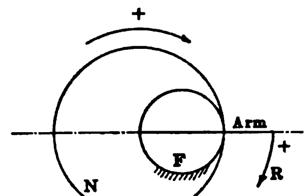


FIG. 7.—Simple epicyclic train with internal gear.

Eq. 5 $N's$ velocity $= R's$ velocity $\times \left(1 - \frac{f}{n}\right)$

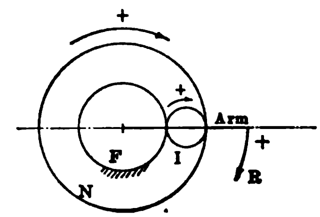


FIG. 8.—Internal gear train with intermediate gear; the arm driving.

Eq. 6 $N's$ velocity $= R's$ velocity $\times \left(1 + \frac{f}{n}\right)$

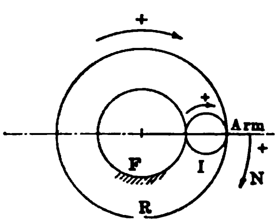


FIG. 9.—Same train as Fig. 8 but with the internal gear driving.

Eq. 7 $N's\ V. = R's\ V. \times \left(\frac{r}{r+f}\right)$

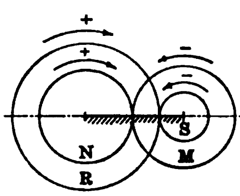


FIG. 10.—Compounded gears in fixed bearings

Eq. 8 $N's\ V. = R's\ V \times \frac{rm}{sn}$

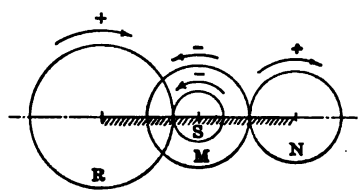


FIG. 11.—Compounded gears in fixed bearings. See equation 8, Fig. 10.

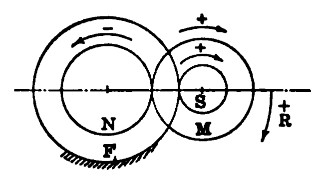


FIG. 12.—Compounded epicyclic train.

Eq. 9 $N's\ V. = R's\ V. \times \left(1 - \frac{fm}{sn}\right)$

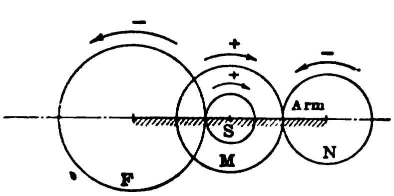


FIG. 13.—Second stage in deriving equation; 9 arm assumed to be fixed, F turned backward.

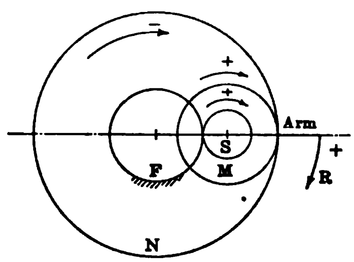


FIG. 14.—Compound epicyclic train with one internal gear.

Eq. 10 $N's\ V. = R's\ V. \times \left(1 + \frac{fm}{sn}\right)$

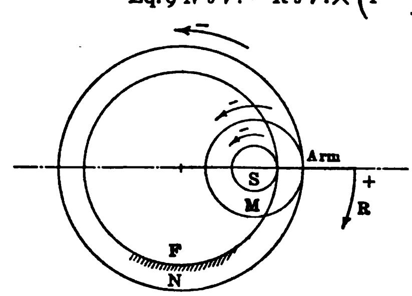


FIG. 15.—Compound epicyclic train with two internal gears.

See Eq. 9, same as Fig. 12.

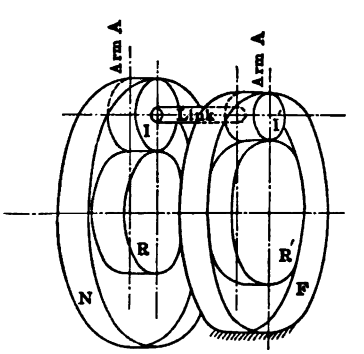


FIG. 16.—An epicyclic train consisting of two central gears, one arm carrying two planetary gears, and two internal gears, one of which is fixed.

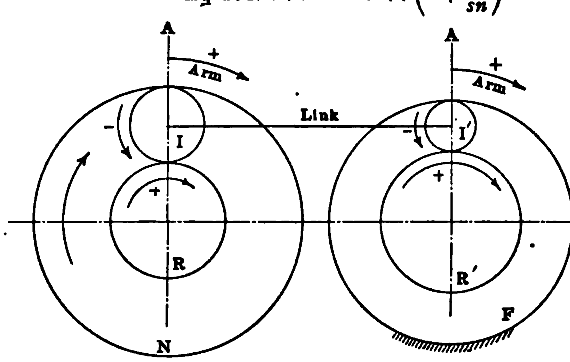


FIG. 17.—Diagram of the train of Fig. 16.

Eq. 11. $N's V. = R's V. \times \left(\frac{r'n - rf}{n(r'+f)}\right)$.

NOTATION

R — Denotes Driving Gear, or in some cases, Arm.
r — Number of Teeth in Driving Gear.
N — Denotes Driven Gear or Arm.
n — Number of Teeth in Driven Gear.
I, S and M denote Intermediate Gears.
F — Denotes Fixed Gear.
f — Number of Teeth in it.
V — Angular Velocity.
Denotes part which is Fixed.

FIGS. 1 to 17.—Planetary gear trains with corresponding velocity ratio formulas.

Compound Epicyclic Train without Internal Gear

Example 9.—We will now arrange to fix one of the gears F, and by means of the arm R revolve the others around it, thereby causing N to revolve as shown in Figs. 12 and 13. As before, we will assume the gears M and S to be out of mesh, so that when the arm R, carrying with it the gear N, makes one revolution around F, N must also make one revolution relatively to F. Also when they are in mesh, the arm R being fixed and F makes one revolution in a negative direction (see Fig. 13), N will make $-\frac{fm}{sn}$ revolutions (equation h).

Now the total motion imparted to N must be the sum of these two, namely:

$$1 \text{ revolution of } R = 1-\frac{fm}{sn} \text{ revolutions of } N,$$

or

$$N\text{'s velocity} = R\text{'s velocity} \times \left(1-\frac{f\times m}{s\times n}\right). \qquad (i)$$

Compound Epicyclic Train with One Internal Gear

Example 10.—Fig. 14 shows a slight modification of the last case, N being an internal instead of an external gear. Obviously the only difference will be in the direction of N's motion, that is:

$$N\text{'s velocity} = R\text{'s velocity} \times \left(1+\frac{fm}{sn}\right). \qquad (j)$$

Compound Epicyclic Train with Two Internal Gears

Example 11.—A further modification, however, is one in which both F and N are internal gears, Fig. 15, the effect of such being a change of sign in the equation.

$$\therefore N\text{'s velocity} = R\text{'s velocity} \times \left(1-\frac{fm}{sn}\right). \qquad (i)$$

The type shown in Figs. 12 and 15 is, perhaps, one of the best methods of obtaining a good reduction of speed in an easy and cheap manner.

There are several combinations of the examples shown, but as they are all somewhat similar we will take another typical case as a guide for future calculations.

An Epicyclic Train Consisting of Two Central Gears, One Arm Carrying Two Planetary Gears, and Two Internal Gears, One of Which Is Fixed

Example 12.—The writer has successfully used the arrangement shown in Figs. 16 and 17, in which R and R' are two spur gears mounted on one shaft; I and I' are two "planet" pinions, while F and N are two internal gears, the former being fixed. R and R' are made to revolve, which has the effect of giving N a very slow speed

Finding the Velocity Ratio

As this is somewhat complicated, we will work it out in stages:

1. Obtain the revolutions of the arm A when R' makes one revolution, F, of course, being fixed.
2. Obtain N's revolutions when the arm A is fixed and R makes one revolution.
3. Assume R fixed, and that the arm makes one revolution; obtain, then, N's revolutions.
4. Then if N makes so many revolutions to one of the arm, as given by stage 3, we can by proportion obtain how many will be caused by the amount given by stage 1.
5. Add the results of 2 and 4 together, and obtain the motion given to N by one revolution of R, which is the desired result.

Working the above out we obtain:

1. When F is fixed and R' makes one revolution, the arm A must make $+\frac{r'}{r'+f'}$. (According to equation (g))
2. R makes one revolution, arm A being fixed; then N must make $-\frac{r}{n}$ revolutions. (According to equation (b).) Negative sign used because of the internal gear.)
3. When R is fixed and arm A makes one revolution, N will make $+\left(1+\frac{r}{n}\right)$ revolutions. (According to equation (f).)
4. With one revolution of arm, N makes $1+\frac{r}{n}$ revolutions, from stage 3;

$\therefore$ with $\frac{r'}{r'+f}$ revolutions of the arm, as derived in stage 1, N will make

$$\left(1+\frac{r}{n}\right)\times\left(\frac{r'}{r'+f}\right).$$

5. The aggregate is the sum of the effects derived in stages 4 and 2, namely, to one of R, N makes:

$$\left(1+\frac{r}{n}\right)\times\left(\frac{r'}{r'+f}\right)+\left(-\frac{r}{n}\right)$$
$$=\frac{(n+r)r'}{n(r'+f)}-\frac{r}{n}$$
$$=\frac{rr'+r'n-rr'-rf}{n(r'+f)}$$
$$=\frac{r'n-rf}{n(r'+f)}.$$

The final direction of revolution of N will depend upon the relation which $r'n$ bears to rf; if the former be greater, then the direction will be positive (+), and *vice versa*. The formula for this combination is, then:

$$N\text{'s velocity} = R\text{'s velocity} \times \left(\frac{r'n-rf}{n(r'+f)}\right). \qquad (k)$$

Some Numerical Examples in Epicyclic Gearing

In order to illustrate the above examples we will take one or two cases.

If in example and Fig. 3, $f=30$, $n=25$, then to one revolution of R, N will make $\left(1+\frac{f}{n}\right)=1+\frac{30}{25}=2\frac{1}{5}$ revolutions.

It will be obvious that with $f=n$, N would revolve at twice the speed of R.

In the type shown in Fig. 7, $f=60$, $n=65$.

then

$$\frac{\text{Velocity of } N}{\text{Velocity of } R}=\frac{1-\frac{f}{n}}{1}=\frac{1-\frac{60}{65}}{1}=\frac{5}{65}=\frac{1}{13}.$$

The arrangement of Fig. 12 is much used. Let $n=60$, $f=61$, $s=40$, $m=41$.

Then the velocity ratio between N and R is $1-\frac{fm}{sn}:1$

$$=1-\frac{61\times 41}{40\times 60}=1-\frac{2501}{2400}$$

= say 1 : 24, in a minus direction.

Illustrating example 12, Fig. 16, let $r=90$, $r'=91$, $f=120$, $n=121$.

$$\frac{\text{Velocity of } N}{\text{Velocity of } R}=\frac{r'n-rf}{n(r'+f)}=\frac{91\times 121-90\times 120}{121(91+120)}$$
$$\frac{11011-10800}{121\times 211}=\frac{211}{121\times 211}=\frac{1}{121}.$$

ROPES

Important differences exist between British and American practice in rope driving. Thus British engineers prefer three strand cotton rope and the multiple-wrap system, while American engineers, with few exceptions, have adopted four strand manilla rope and the continuous-wrap system. This diversity of practice, based on extended experience in both cases, is difficult to reconcile or explain.

An obvious advantage of the multiple-wrap system is that the ropes give out one at a time and, as the failure of a single rope does not cripple the system, delays due to failure are lessened. This is at least partially offset by the fact that ropes never fail without giving warning. An equally undoubted advantage of the continuous wrap system with its weighted idler is its flexibility as regards center distance, inclination from the horizontal, direction of rotation and the passage of obstructions by the use of guide pulleys. Manilla rope is also most suitable for situations involving exposure to weather conditions or to dampness.

The rival claims of cotton *vs.* manilla rope relate chiefly to cost and durability, regarding which no accurate data exist. Whatever the explanation, British engineers consider the superiority of cotton rope as proven beyond question.

The Plymouth Cordage Co., who install both systems, consider the British system best when the driving and driven sheaves are of about equal diameters, the shafts enough out of the vertical to prevent the slack side from falling too much out of the sheave grooves, the shafts between 30 and 125 ft. apart, the load fairly uniform, the speed not excessive and the drive protected from the weather. In general, the multiple system inclines to the larger and the continuous system to the smaller installations. Cotton rope is, however, better for small interior drives which take the place of belts.

The continuous system should generally be used when shafts are nearer together than 30 ft., as, in the multiple system, a small amount of stretch will so decrease the initial tension that the centrifugal force carries the rope out of its groove and quickly diminishes its driving capacity.

With shallow grooves to avoid chafing due to the necessary side lead of the continuous system, sheaves may be run on 10 ft. centers while, without supporting idlers, the shafts may be placed as much as 150 ft. apart. With idlers the distance may be increased almost indefinitely.

Rope drives up to 4000 h.p. capacity are in operation, drives of between 1000 and 2000 h.p. being fairly numerous.

The comparative first cost of rope and belt drives, according to the Plymouth Cordage Co., may be determined by assuming the belt to cost about two and one-half times as much as manilla rope of equal capacity and the rope sheaves to cost about one-third more than belt pulleys—the advantage of ropes increasing with the distance between shafts. The average life of manilla rope of good proportions may be taken as eight years, the life of belts being materially longer.

The most economical speed to run the rope, taking into consideration the first cost and durability, is given as about 4500 ft. per min. Lower speed increases the durability, while higher speed, within certain limits, reduces the first cost.

The diameter of manilla ropes used in the United States for heavy drives ranges between 1 and $1\frac{3}{4}$ ins., while the speeds used run up to 5000 ft. per min. The following charts and tables are from a paper on Rope Driving by C. W. Hunt (*Trans. A. S. M. E., Vol.* 12).

The relative first cost of manilla rope as related to the speed may be obtained from Fig. 1, the cost of a rope running at 80 ft. per sec. being taken as 100. The first cost for other speeds is in proportion to the ordinates for those speeds. Thus if a rope is to run at 60 ft. per sec., its cost will be $\frac{112}{100}$ of that for a speed of 80 ft.

The rule for the load to be carried by manilla ropes is that the stress on the tight side in lbs. shall be equal to 200 times the square of the diameter in ins. Table 1 gives the horse-power suitable for usual sizes of rope on this basis.

The horse power including the effect of centrifugal force in relation to the speed is given in Fig. 2.

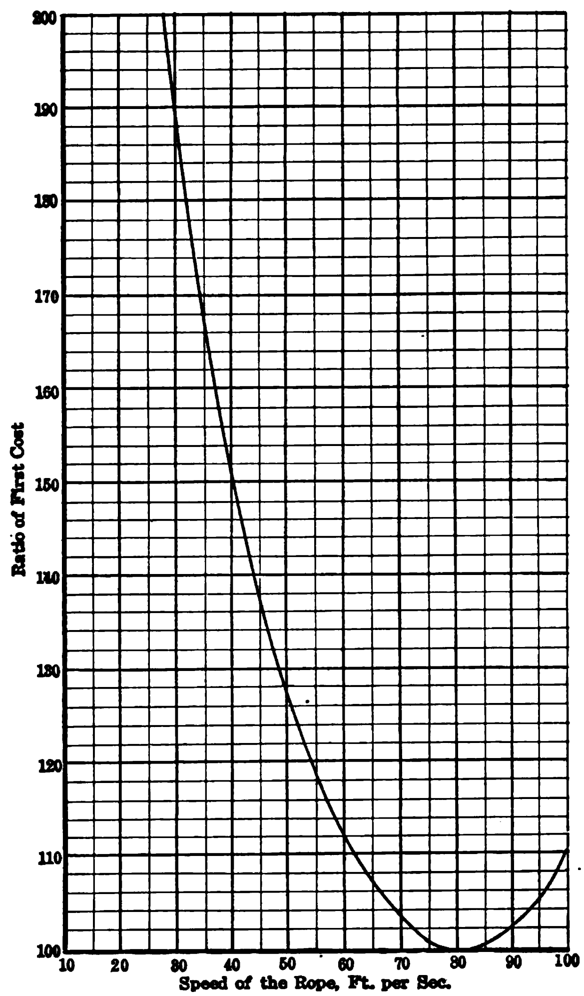


Fig. 1.—Relative first cost of rope at various speeds.

The tension on the slack part of the rope, when transmitting the amount of power given in Table 1, may be obtained from Table 2. The use of this tension is in determining the weight to be applied to the idler in order to obtain the necessary adhesion.

The sag of the rope (drive horizontal) when transmitting the amounts of power given in Table 1 may be obtained from Table 3. This sag is the same at all speeds for the driving part, but is variable for the slack part.

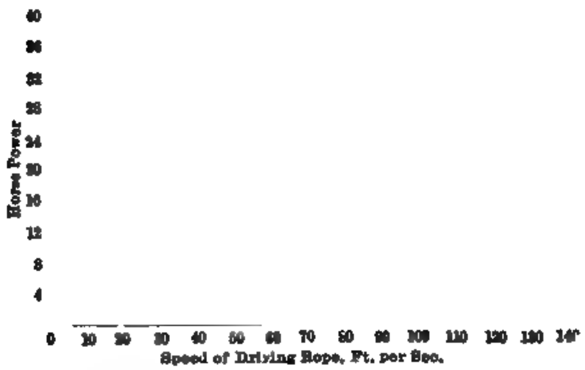


FIG. 2.—Horse-power of Manilla rope, including the effect of centrifugal force.

TABLE 1.—HORSE-POWER OF MANILLA ROPE

Diam. of rope ins.	Speed of the rope in ft. per min.											Diam. of smallest pulley or idler in ins.
	1500	2000	2500	3000	3500	4000	4500	5000	6000	7000	8400	
½	1.45	1.9	2.3	2.7	3.0	3.2	3.4	3.4	3.1	2.2	0	20
⅝	2.3	3.2	3.6	4.2	4.6	5.0	5.3	5.3	4.9	3.4	0	24
¾	3.3	4.3	5.2	5.8	6.7	7.2	7.7	7.7	7.1	4.9	0	30
⅞	4.5	5.9	7.0	8.2	9.1	9.8	10.8	10.7	9.3	6.9	0	36
1	5.8	7.7	9.2	10.7	11.9	12.8	13.6	13.7	12.5	8.8	0	42
1¼	9.2	12.1	14.3	16.8	18.6	20.0	21.2	21.4	19.5	13.8	0	54
1½	13.1	17.4	20.7	23.1	26.8	28.8	30.6	30.8	28.2	19.8	0	60
1¾	18.0	23.7	28.2	32.8	36.4	39.2	41.5	41.8	37.4	27.6	0	72
2	23.2	30.8	36.8	42.8	47.6	51.2	54.4	54.8	50.0	35.2	0	84

TABLE 2.—TENSION ON THE SLACK PART OF THE ROPE

Speed of rope in ft. per sec.	Diameter of the rope and pounds tension on the slack rope								
	½	⅝	¾	⅞	1	1¼	1½	1¾	2
20	10	27	40	54	71	110	162	216	283
30	14	29	42	56	74	115	170	226	296
40	15	31	45	60	79	123	181	240	315
50	16	33	49	65	85	132	195	259	339
60	18	36	53	71	93	145	214	285	373
70	19	39	59	78	101	158	236	310	406
80	21	43	64	85	111	173	255	340	445
90	24	48	70	93	122	190	279	372	487

TABLE 3.—SAG OF THE ROPE BETWEEN PULLEYS

Distance between pulleys in ft.	Driving rope All speeds		Slack side of rope					
			80 ft. per sec.		60 ft. per sec.		40 ft. per sec.	
	Ft.	Ins.	Ft.	Ins.	Ft.	Ins.	Ft.	Ins.
40	0	4	0	7	0	9	0	11
60	0	10	1	5	1	8	1	11
80	1	5	2	4	1	10	3	3
100	2	0	3	8	4	5	5	2
120	2	11	5	3	6	3	7	4
140	3	10	7	2	8	9	9	9
160	5	1	9	3	11	3	14	0

Ordinary manilla rope should not be used. American manilla transmission rope is always laid up with internal lubricant which is essential to long life.

The cross-sections of rope sheaves used by the Plymouth Cordage Co. are shown in Figs. 3, 4 and 5. Fig. 3 shows the usual section, Fig. 4 being used to avoid side chafing when the sheaves are close together, under which circumstances the higher webs are not needed as there is no tendency for the rope to jump the grooves. Fig. 5 shows the section used for idler sheaves.

The diameter of sheaves for manilla rope should not be less than 40 diameters of the rope. Cotton rope being more flexible, the sheaves for it may be smaller. The British rule for cotton-rope sheaves provides a minimum of 30 rope diameters. The same rule for diameters should be observed with idler as with driving sheaves, as it is the bending of the rope on the sheave that does the damage.

For dimensions of rope sheave arms see Dimensions of Pulley Arms.

The horse-power of cotton ropes, according to British practice, is given in Table 4 by EDWARD KENYON (*Trans. South Wales Ins. of*

FIG. 3.

FIG. 5.

FIGS. 3 to 5.—Cross-sections of rope sheaves.

Engrs., 1909). British practice with cotton ropes does not hesitate to adopt speeds of 7000 ft. per min., whereas American practice regards 5000 ft. as the economical limit with manilla ropes. According to Mr. Kenyon, the angle between the driving faces of the groove should not exceed 40 deg. in order to prevent the rolling over of the ropes in their grooves which reduces their life at least one-third.

For the efficiency of rope driving see below.

Manilla Rope for Hoisting

The proper working loads of hoisting rope, according to C. W. HUNT (*Trans. A. S. M. E., Vol.* 23) are well settled by extended experience. Table 5 gives Mr. Hunt's figures for the working loads and the sheave diameters under various conditions. The terms in the captions of the columns have the following meanings:

Slow: Derrick, crane and quarry work; speed from 50 to 100 ft. per min.

Medium: Wharf and cargo hoisting; 150 to 300 ft. per min.

Rapid: 400 to 800 ft. per min.

The efficiency to be expected from hoisting blocks is given in Table 6 from some experiments made by Robert Grimshaw and quoted by Mr. Hunt. The blocks experimented upon had a 6-fold purchase, the three upper sheaves having roller bearings and the three

TABLE 4.—HORSE-POWER OF THREE STRAND COTTON ROPES

Rope diameters	1"	1⅛"	1¼"	1⅜"	1½"	1⅝"	1¾"	1⅞"	2"
Minimum diameter of smallest pulley	2 ft. 6 ins.	2 ft. 10 ins.	3 ft. 2 ins.	3 ft. 5 ins.	3 ft. 9 ins.	4 ft. 1 in.	4 ft. 5 ins.	4 ft. 9 ins.	5 ft.
Velocity in ft. per min.									
1000	3.3	4.1	5.1	6.1	7.4	8.6	10	11.5	13
1100	3.6	4.5	5.6	6.7	8.1	9.5	11	12.6	14.3
1200	3.9	4.9	6.1	7.3	8.9	10.3	12	13.8	15.6
1300	4.2	5.3	6.6	8	9.7	11.2	13	14.9	16.9
1400	4.6	5.7	7.1	8.6	10.4	12	14	16	18.3
1500	4.9	6.1	7.6	9.2	11.1	12.9	15	17.2	19.6
1600	5.2	6.6	8.1	9.8	11.9	13.8	16	18.4	20.9
1700	5.5	7	8.6	10.4	12.6	14.6	17	19.5	22.2
1800	5.8	7.4	9.1	11	13.4	15.5	18	20.7	23.5
1900	6.2	7.8	9.6	11.6	14.1	16.3	19	21.8	24.8
2000	6.5	8.2	10.1	12.2	14.9	17.2	20	22.9	26.1
2100	6.8	8.6	10.6	12.8	15.6	18.1	21	24.1	27.4
2200	7.1	9.	11.1	13.4	16.3	18.9	22	25.3	28.7
2300	7.5	9.5	11.6	14	17.1	19.8	23	26.4	30
2400	7.8	9.9	12.1	14.7	17.8	20.7	24	27.6	31.3
2500	8.1	10.3	12.6	15.3	18.5	21.5	25	28.7	32.6
2600	8.4	10.7	13.1	15.9	19.2	22.4	26	29.9	33.9
2700	8.7	11.1	13.6	16.5	20	23.3	27	31	35.2
2800	9.1	11.5	14.1	17.1	20.8	24.1	28	32.2	36.5
2900	9.4	11.9	14.6	17.7	21.5	25	29	33.3	37.8
3000	9.7	12.3	15.1	18.3	22.3	25.8	30	34.5	39.1
3100	10	12.7	15.6	18.9	23	26.7	31	35.6	40.4
3200	10.4	13.2	16.2	19.6	23.8	27.6	32	36.8	41.8
3300	10.7	13.6	16.7	20.2	24.6	28.4	33	37.9	43.1
3400	11	14	17.2	20.8	25.3	29.3	34	39.1	44.4
3500	11.3	14.4	17.7	21.4	26	30.1	35	40.2	45.7
3600	11.7	14.8	18.2	22	26.7	31	36	41.4	47
3700	12	15.2	18.7	22.6	27.5	31.9	37	42.5	48.3
3800	12.3	15.6	19.2	23.2	28.2	32.7	38	43.7	49.6
3900	12.6	16	19.7	23.8	29	33.6	39	44.8	50.9
4000	13	16.4	20.2	24.5	29.7	34.5	40	46	52.2

Rope diameters	1"	1⅛"	1¼"	1⅜"	1½"	1⅝"	1¾"	1⅞"	2"
Minimum diameter of smallest pulley	2 ft. 6 ins.	2 ft. 10 ins.	3 ft. 2 ins.	3 ft. 5 ins.	3 ft. 9 ins.	4 ft. 1 in.	4 ft. 5 ins.	4 ft. 9 ins.	5 ft.
Velocity in ft. per min.									
4100	13.3	16.9	20.7	25.1	30.4	35.3	41	47.1	53.5
4200	13.6	17.3	21.2	25.7	31.2	36.2	42	48.3	54.8
4300	13.9	17.7	21.7	26.3	31.9	37	43	49.4	56.1
4400	14.3	18.1	22.2	26.9	32.7	37.9	44	50.6	57.4
4500	14.6	18.5	22.7	27.5	33.4	38.8	45	51.7	58.7
4600	14.9	18.9	23.2	28.1	34.2	39.6	46	52.9	60
4700	15.2	19.3	23.7	28.7	34.9	40.5	47	54	61.3
4800	15.6	19.8	24.2	29.4	35.7	41.4	48	55.2	62.7
4900	15.9	20.2	24.7	30	36.4	42.2	49	56.3	64
5000	16.2	20.6	25.3	30.6	37.1	43.1	50	57.5	65.3
5100	16.5	21	25.8	31.2	37.9	43.9	51	58.6	66.6
5200	16.9	21.4	26.3	31.8	38.6	44.8	52	59.8	67.9
5300	17.2	21.8	26.8	32.4	39.4	45.7	53	60.9	69.2
5400	17.5	22.2	27.3	33	40.1	46.5	54	62.1	70.5
5500	17.8	22.6	27.8	33.6	40.9	47.4	55	63.2	71.8
5600	18.2	23.1	28.3	34.3	41.6	48.3	56	64.4	73.1
5700	18.5	23.5	28.8	34.9	42.3	49.1	57	65.5	74.4
5800	18.8	23.9	29.3	35.5	43.1	50	58	66.7	75.7
5900	19.1	24.3	29.8	36.1	43.8	50.8	59	67.8	77
6000	19.5	24.7	30.3	36.7	44.6	51.7	60	69	78.3
6100	19.8	25.1	30.8	37.3	45.3	52.6	61	70.1	79.6
6200	20.1	25.5	31.3	37.9	46.1	53.4	62	71.3	80.9
6300	20.[illegible]	25.9	31.8	38.5	46.8	54.3	63	72.4	82.2
6400	20.8	26.4	32.4	39.2	47.6	55.2	64	73.6	83.6
6500	21.1	26.8	32.9	39.8	48.3	56	65	74.7	84.9
6600	21.4	27.2	33.4	40.4	49	56.9	66	75.9	86.2
6700	21.7	27.6	33.9	41	49.8	57.7	67	77	87.5
6800	22.1	28	34.4	41.6	50.5	58.5	68	78.2	88.8
6900	22.4	28.4	34.9	42.2	51.3	59.4	69	79.3	90.1
7000	22.7	28.8	35.4	42.8	52	60.3	70	80.5	91.4

TABLE 5.—WORKING LOADS FOR MANILLA HOISTING ROPE

Diameter of rope, ins.	Ultimate strength, lbs.	Working load, lbs.			Minimum diameter of sheaves, ins.		
		Rapid	Medium	Slow	Rapid	Medium	Slow
1	7,100	200	400	1,000	40	12	8
1⅛	9,000	250	500	1,250	45	13	9
1¼	11,000	300	600	1,500	50	14	10
1⅜	13,400	380	750	1,900	55	15	11
1½	15,800	450	900	2,200	60	16	12
1⅝	18,800	530	1,100	2,600	65	17	13
1¾	21,800	620	1,250	3,000	70	18	14

TABLE 6.—EFFICIENCY OF BLOCK AND FALL

Net load on tackle, weight raised	Theoretical amount required to raise the net weight	Actual power required	Extra power required over the theoretical	
600 lbs.	100 lbs.	158 lbs.	58 lbs.	58 %
800 lbs.	133.3 lbs.	198 lbs.	64.3 lbs.	48 %
1,000 lbs.	166.7 lbs.	243 lbs.	76 lbs.	45.8 %
1,200 lbs.	200 lbs.	288 lbs.	88 lbs.	44 %

lower ones plain, solid bushings. The rope was 3 strand of 3¾-ins. circumference. The sheaves were of 8 ins. diameter.

Splicing Manilla Rope

The following particulars regarding the splicing of rope and the various forms of knots are taken, by permission, from the publications of the C. W. Hunt Co.

The splice in a transmission rope is not only the weakest part of the rope, but is the first to fail when the rope is worn out. If the splice is not strong, the rope will fail by breakage or pulling out of the splice. If the rope is larger at the splice, the projecting parts will wear on the pulleys, and the rope fail from the cutting off of the strands.

Do not put in a "short splice," or an ordinary "long splice," or get an old sailor to do the work, but have a handy man follow implicitly the directions given herein for a splice in a four-strand rope.

For splicing, add to the net length the following amount for making a splice:

1 in. diameter......... 12 ft.	1¾ ins. diameter......... 18 ft.
1¼ ins. diameter......... 14 ft.	2 ins. diameter.........20 ft.
1½ ins. diameter. .... 16 ft.	

The splicing of a 1¾-in. rope is shown in Figs. 6 to 9. Begin by tieing a piece of twine, 9 and 10, around the rope to be spliced, about six feet from each end. Then unlay the strands of each end back to the twine. Butt the ropes together, and twist each corresponding pair of strands loosely, to keep them from being tangled, as shown in Fig. 6.

The twine 10 is now cut, and the strand 8 unlaid, and strand 7 carefully laid in its place for a distance of four and a half feet from the junction. The strand 6 is next unlaid about one and a half feet, and strand 5 laid in its place. The ends of the cores are now cut off so they just meet. Unlay strand 1 four and a half feet, laying strand 2 in its place. Unlay strand 3 one and a half feet, laying in strand 4. Cut all the strands off to a length of about twenty inches, for conve-

nience in manipulation. The rope now assumes the form shown in Fig. 7, with the meeting points of the strands three feet apart.

Each pair of strands is successively subjected to the following operation:

From the point of meeting of the strands 8 and 7, unlay each one three turns; split both the strands 8 and the strand 7 in halves as far back as they are now unlaid, and the end of each half strand

FIG. 6.

FIG. 7.

FIG. 8.

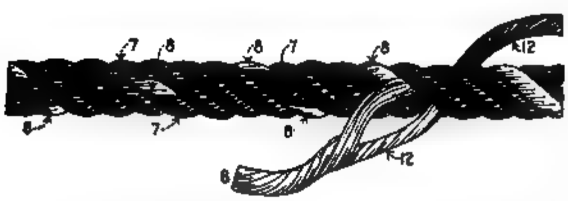


FIG. 9.

FIGS. 6 to 9.—Splicing Manilla rope.

whipped with a small piece of twine. The half of the strand 7 is now laid in three turns, and the half of 8 also laid in three turns. The half strands now meet and are tied in a simple knot, 11, Fig. 8, making the rope at this point its original size.

The rope is now opened with a marlin spike, and the half strand of 7 worked around the half strand of 8 by passing the end of the half strand 7 through the rope, as shown in the engraving, drawn taut, and again worked around this half strand until it reaches the half strand 13 that was laid not in. This half strand 13 is now split, and the half strand 7 drawn through the opening thus made, and then tucked under the two adjacent strands, as shown in Fig. 9. The other half of the strand 8 is now wound around the other half strand 7 in the same manner. After each pair of strands has been treated in this manner, the ends are cut off at 12, leaving them about four inches long. After a few days' wear they will draw into the body of the rope or wear off, so that the locality of the splice can scarcely be detected.

Knots

A great number of knots have been devised, of which a few only are illustrated, but those selected are the most frequently used. In Fig. 10 they are shown open, or before being drawn taut, in order to show the position of the parts. The names usually given to them are:

A. Bight of a rope.
B. Simple or Overhand Knot.
C. Figure 8 Knot.
D. Double Knot.
E. Boat Knot.
F. Bowline, first step.
G. Bowline, second step.
H. Bowline completed.
I. Square or Reef Knot.
J. Sheet Bend or Weaver's Knot.
K. Sheet Bend, with a toggle.
L. Carrick Bend.
M. Stevedore Knot completed.
N. Stevedore Knot commenced.
O. Slip Knot.
P. Flemish Loop.
Q. Chain Knot, with toggle.
R. Half-hitch.
S. Timber-hitch.
T. Clove-hitch.
U. Rolling-hitch.
V. Timber-hitch and Half-hitch.
W. Blackwall-hitch.
X. Fisherman's Bend.
Y. Round Turn and Half-hitch.
Z. Wall Knot commenced.
AA. Wall Knot completed.
BB. Wall Knot Crown commenced.
CC. Wall Knot Crown completed.

The principle of a knot is that no two parts, which would move in the same direction if the rope were to slip, should lie alongside of and touch each other.

The bowline is one of the most useful knots, it will not slip, and after being strained is easily untied. It should be tied with facility by every one who handles rope. Commence by making a bight in the rope, then put the end through the bight and under the standing part, as shown in *G*, then pass the end again through the bight, and haul tight.

The square or reef knot must not be mistaken for the "granny" knot that slips under a strain. Knots *H*, *K*, and *M* are easily untied after being under strain. The knot *M* is useful when the rope passes through an eye and is held by the knot, as it will not slip, and is easily untied after being strained.

The timber hitch, *S*, looks as though it would give way, but it will not; the greater the strain the tighter it will hold. The wall knot looks complicated, but is easily made by proceeding as follows: Form a bight with strand 1, and pass the strand 2 around the end of it, and the strand 3 round the end of 2, and then through the bight of 1, as shown in the engraving *Z*. Haul the ends taut when the appearance is as shown in the engraving *AA*. The end of the strand 1 is now laid over the center of the knot, strand 2 laid over 1, and 3 over 2, when the end of 3 is passed through the bight of 1, as shown in the engraving *BB*. Haul all the strands taut, as shown in the engraving *CC*.

The efficiency of knots, as determined at the Massachusetts Institute of Technology, is given in Table 7. The efficiency compares the strength of the knots with the full strength of the rope.

Wire Rope

Standard wire rope for hoisting purposes is composed of 6 strands and a hemp center with 19 wires to the strand. Extra pliable hoisting rope is of two constructions, one composed of 8 strands and a hemp center with 19 wires to the strand and the other of 6 strands and a hemp center with 37 wires to the strand. Standard coarse laid rope for haulage is composed of 6 strands and a hemp center with 7 wires to the strand. Ropes are made of Swedish iron, cast steel,

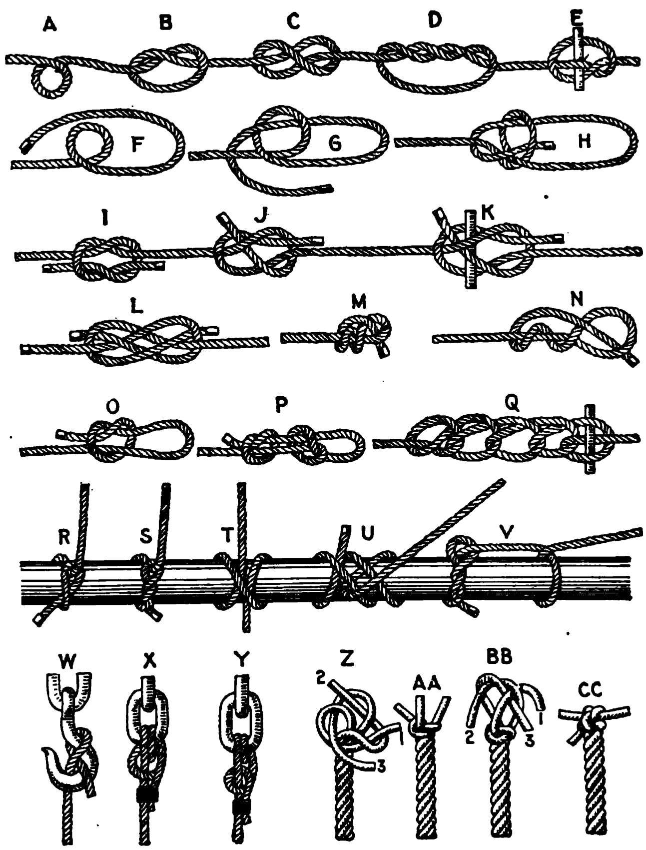


FIG. 10.—Knots, hitches and bends.

extra strong cast steel, plough steel and improved plough steel. Special tiller rope, hawsers, ship's rigging rope, etc., are also made of them. Particulars may be obtained of the makers.

Sweedish iron rope is soft, tough and pliable and is especially adopted for passanger elevators and similar service where the tendency to abrasion is slight, the speed high, the loads moderate and the arrangement of the sheaves such as to produce severe bending stresses in the rope. Other materials are used when it is desired to obtain increased strength or security without increasing the diameter. Cast-steel rope is used for general hoisting. Coarse laid rope is much stiffer than standard hoisting rope and requires larger sheaves. A higher factor of safety should be used as the breaking of one or two wires materially reduces the strength. Tables 8–11 give the particulars of the leading brands as made by the John. A Roebling's Sons Co. and bring out clearly the progressive increase of strength. The diameter of a wire rope is that of a true circle enclosing the rope.

Splicing Wire Rope

Wire rope is susceptible of almost perfect splicing and the operation is so simple that it may be learned in an hour by any mechanic who is at all skillful in the use of ordinary tools. For all kinds of transmission rope the long splice is used and should not be less than 16 ft. in length for ½ in. rope and increasing to 30 ft. for the larger sizes.

Where the splicing must be done in position, rope blocks are used

TABLE 7.—EFFICIENCY OF KNOTS

Eye-splice over an iron thimble	Short splice in the rope	Timber hitch, round turn, and half-hitch	Bowline slip knot, clove hitch	Square knot, weavers' knot, sheet bend	Flemish loop, over-hand knot	Rope dry, average of four tests from the same coil as the knots
90	80	65	60	50	45	100

to draw the wire rope taut, as in Fig. 11, care being taken to make fast far enough from the ends to leave plenty of room for the splice and the men who make it. If possible, it is better to hold the rope taut, mark the splice on both ends, by securely winding with No. 20 annealed-iron wire, throw it off the sheaves and make the splice on the floor or staging, as may be most convenient.

The strands of both ends are unlaid, back to the points wound with wire, the hemp core cut off and the ends brought together with the strands interlaced, Fig. 12. Any strand, as *a*, is unlaid and closely followed by the corresponding strand 1 of the other end of the rope which is pressed closely into the groove left by the unlaid strand. The unwinding of one strand and the inwinding of the other are continued until all but about 12 ins. of strand are inlaid, when *a* is cut off at the same length with a sharp chisel. See Fig. 13. Strands 4 and *d* are next treated in the same way and the process is repeated with each pair of strands until all are laid and cut as in Fig. 14.

Around each point where the free strands cross, a few turns of stout twine are made and the length of the splice is bent and worked in all directions until the tension in all the strands is equal and the rope as flexible there as elsewhere. If this is not done and there is more tension in some of the strands than in others when a stress is put on the rope, these strands will pull into the rope, making a bad-looking and weak splice.

Next, the open or free ends of the 12 strands are carefully trimmed and served or wound with fine wire, and two rope and stick clamps, Fig. 15, are secured to the rope, one on each side of an end crossing, as in Fig. 18, for the purpose of aiding in tucking the strand ends into the middle of the rope.

There are two ways of tucking in these ends. They are first straightened with a mallet. The long ends of the rope-clamp handles are twisted in opposite directions, separating the strands and exposing the hemp core, which is cut off and pulled out between the points to where the tucked-in strands will reach and the ends forced into the place formerly occupied by the core.

This is most easily done with the aid of a marine spike, which is passed over the strand which is to be tucked and under two strands of the rope, Fig. 16, and moved along the rope spirally following the lay and forcing the free end into the core space, Fig. 17.

In the other method the strands are more widely separated by untwisting the rope with the clamps, Fig. 19, slipping the free end in between the strands and correcting slight kinks by the use of a mallet.

The order in which the ends are tucked in is immaterial. Some operators prefer to tuck all the ends pointing in one direction before any of those pointing the opposite way, while others finish each pair of ends in series.

If the foregoing directions are intelligently followed the splice will be uniform with the rest of the rope, of nearly equal strength throughout, and after a few hours' use it will be almost impossible to detect the splice. (F. L. JOHNSON *Power, Jan.* 30, 1912).

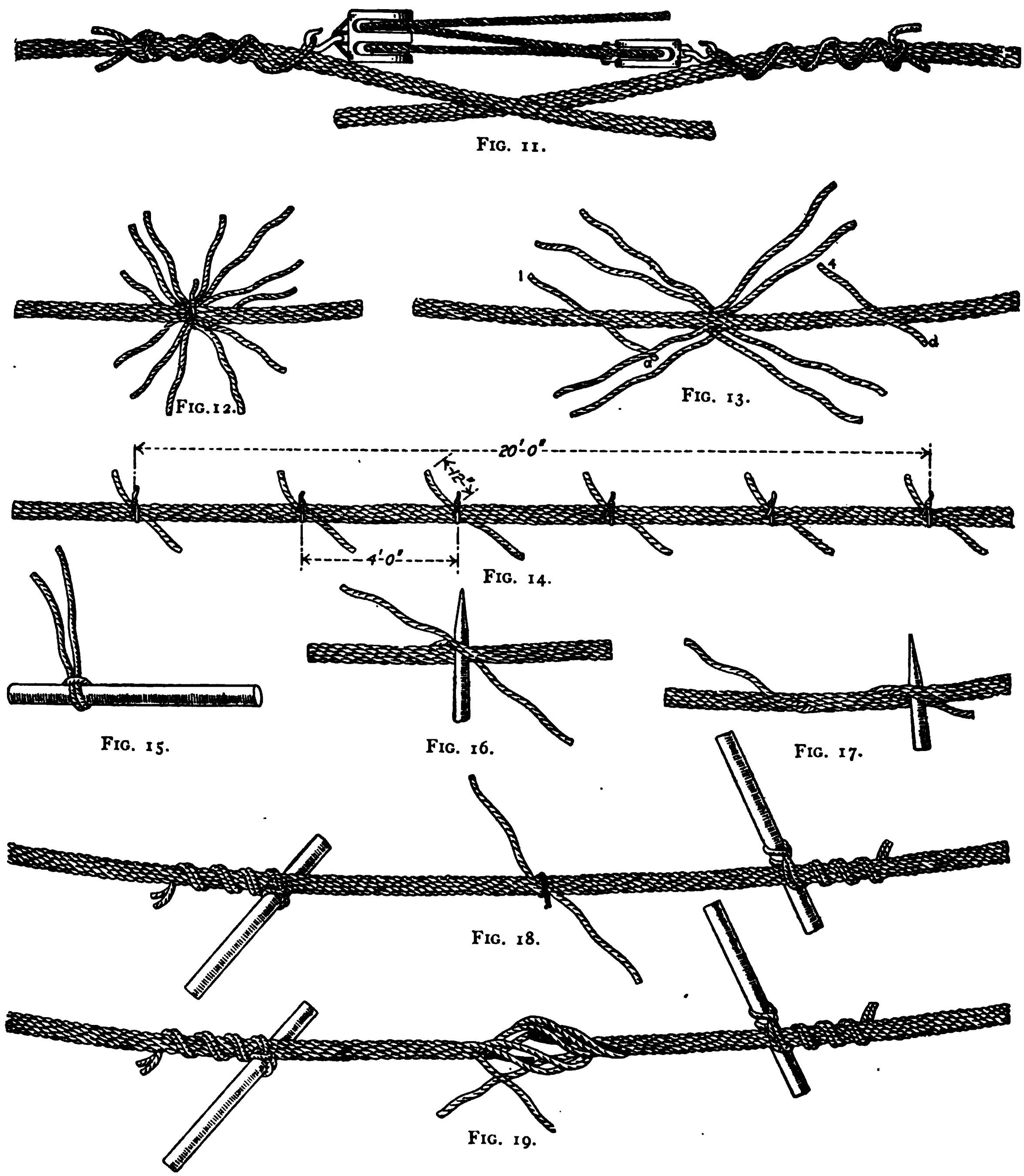


FIG. 11. FIG. 12. FIG. 13. FIG. 14. FIG. 15. FIG. 16. FIG. 17. FIG. 18. FIG. 19.

FIGS. 11 to 19.—Splicing wire rope.

Hoisting Drums

A superior construction of large hoisting drums, including the leading dimensions, by the Nordberg Mfg. Co. (*Am. Mach., Sept.* 21, 1899) is shown in Fig. 20. The customary practice with such drums is to have a number of spiders, four or five, at different points of the shaft for supporting the drum. These drums are, however, of considerable length and great stiffness, and the long shafts, instead of supporting the drum, should be supported by the drum. The drum shown rests on two spiders only, one at each end, the shaft being supported by tension rods from the shell of the drum. The detail drawing of the drum, Fig. 20, shows plainly the mode of construction. In Fig. 21 the position of the spiders is represented before the longitudinal rods A, Fig. 22, are put in. The deflection of the shaft is considerable, making the distance x between spiders on top less than x_1 on bottom. This deflection also causes the weight to come entirely on the edges of the bearings nearest to the drum. In erecting the drum, rods A are first put in place, and their tension is so adjusted as to make the faces BB, Fig. 22, perfectly parallel and bearings e level. Then the drum shell is put in place, and lastly the diagonal tension rods are so adjusted as to take out the deflection in center of shaft, as shown by d, Fig. 22. The shell and spiders are of abundant strength to transmit the whole power of the engines. There is a reel on each end, mounted on shafts inside the drum, for taking up the slack rope.

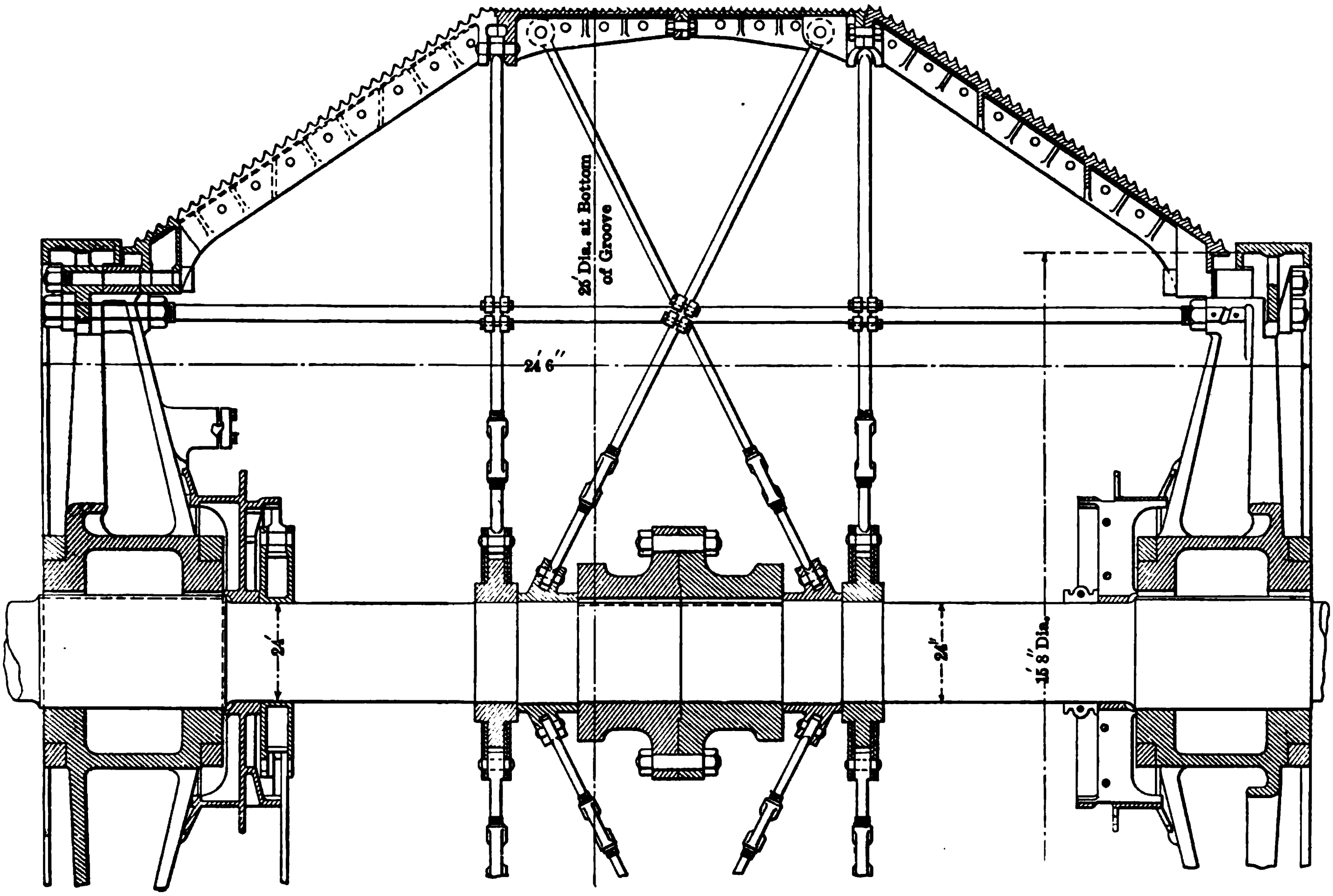


FIG. 20.—Nordberg construction of large hoisting drums.

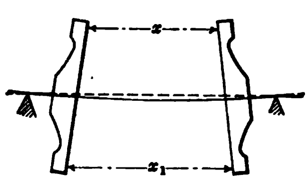


FIG. 21.

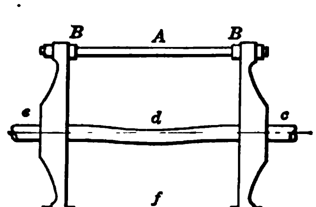


FIG. 22.

FIGS. 21 and 22.—Method of erecting the hoisting drum shown in Fig. 20.

Durability of Wire Rope

The durability of wire ropes formed the subject of a paper by DANIEL ADAMSON (*Proc. I. M. E.* 1912) from which the following is taken.

If the wires are too large they are stressed considerably when passing over the pulleys, and accordingly the material is quickly fatigued and the wires break. Smaller wires, on the other hand, are more quickly worn through by rubbing against the pulleys and against their neighbors in the body of the rope.

Investigations of the durability to be expected from consideration of working stresses lead to calculated results that are never experienced and that cannot reasonably be expected, and it must be taken for granted that abrasion is the principal factor in limiting the life of wire ropes.

Comparing two ropes of equal size, one from wires half the diameter of the other, when the rope of finer wires is passing over the pulley, there being four times as many wires in it, the pressure at each point of contact between the rope and the pulley and beween the individual wires of the rope may be assumed to be one-quarter of what it is in the rope of larger wires. The wires being of half the diameter the damage done to them by contact, even under this lower pressure, will be at least half as much as occurs to the coarser wires in the other rope, and this half damage done to a wire of one-quarter the sectional area will result in the cutting through of the wire in half the time, so that the effect of abrasion upon the rope of finer wires will be twice as great. If a smaller pulley be used for the rope of finer wires, as suggested by some authorities, the pressure at the points of contact and the stress due to bending will be proportionately increased, so that it may reasonably be expected that with a pulley diameter bearing the same proportion to the diameter of the wires, the life of the rope with fine wires will be one-quarter of that of the rope of coarser wires working over a pulley of correspondingly increased diameter.

Mr. Adamson quotes from experiments by A. S. BIGGART (*Proc. I. C. E.*, *Vol.* 101) on apparatus consisting of two pulleys around which the rope under trial was passed, the lower pulley being weighted to give the required tension on the rope which was passed, under a normal working load, to and fro over the pulleys until breakage ensued. Experiments were repeated with different diameters of pulleys and different makes of rope.

TABLE 8.—STANDARD HOISTING ROPE COMPOSED OF 6 STRANDS AND A HEMP CENTER WITH 19 WIRES TO THE STRAND

Swedish Iron

Diameter, ins.	Approximate circumference, ins.	Approximate weight per ft., lbs.	Approximate strength, tons of 2000 lbs.	Proper working load, tons of 2000 lbs.	Diameter of drum or sheave in ft. advised
2¾	8⅝	11.95	111	22.2	17
2½	7⅞	9.85	92	18.4	15
2¼	7⅛	8	72	14.4	14
2	6¼	6.30	55	11	12
1⅞	5⅞	5.55	50	10	12
1¾	5½	4.85	44	8.8	11
1⅝	5	4.15	38	7.5	10
1½	4¾	3.55	33	6.5	9
1⅜	4⅜	3	28	5.5	8.5
1¼	4	2.45	22.8	4.56	7.5
1⅛	3½	2	18.6	3.72	7
1	3	1.58	14.5	2.90	6
⅞	2¾	1.20	11.8	2.36	5.5
¾	2¼	.89	8.5	1.70	4.5
⅝	2	.62	6	1.20	4
9/16	1¾	.50	4.7	.94	3.5
½	1½	.39	3.9	78	3
7/16	1¼	.30	2.9	.58	2.75
⅜	1⅛	.22	2.4	.48	2.25
5/16	1	.15	1.5	.30	2
¼	¾	.10	1.1	.22	1.50

Extra Strong Cast Steel

Diameter, ins.	Approximate circumference, ins.	Approximate weight per ft., lbs.	Approximate strength, tons of 2000 lbs.	Proper working load, tons of 2000 lbs.	Diameter of drum or sheave in ft. advised
2¾	8⅝	11.95	243	48.6	11
2½	7⅞	9.85	200	40	10
2¼	7⅛	8	160	32	9
2	6¼	6.3	123	24.6	8
1⅞	5⅞	5.55	112	22.4	8
1¾	5½	4.85	99	19.8	7
1⅝	5	4.15	83	16.6	6½
1½	4¾	3.55	73	14.6	6
1⅜	4⅜	3	64	12.8	5½
1¼	4	2.45	53	10.6	5
1⅛	3½	2	43	8.6	4½
1	3	1.58	34	6.80	4
⅞	2¾	1.20	26	5.20	3½
¾	2¼	.89	20.2	4.04	3
⅝	2	.62	14	2.80	2½
9/16	1¾	.50	11.2	2.24	2¼
½	1½	.39	9.2	1.84	2
7/16	1¼	.30	7.25	1.45	1¾
⅜	1⅛	.22	5.30	1.06	1½
5/16	1	.15	3.50	.70	1¼
¼	¾	.10	2.43	.49	1

Cast Steel

Diameter, ins.	Approximate circumference, ins.	Approximate weight per ft., lbs.	Approximate strength, tons of 2000 lbs.	Proper working load, tons of 2000 lbs.	Diameter drum or sheave in ft. advised
2¾	8⅝	11.95	211	42.2	11
2½	7⅞	9.85	170	34	10
2¼	7⅛	8	133	26.6	9
2	6¼	6.30	106	21.2	8
1⅞	5⅞	5.55	96	19	8
1¾	5½	4.85	85	17	7
1⅝	5	4.15	72	14.4	6½
1½	4¾	3.55	64	12.8	6
1⅜	4⅜	3.	56	11.2	5½
1¼	4	2.45	47	9.4	5
1⅛	3½	2	38	7.6	4½
1	3	1.58	30	6	4
⅞	2¾	1.20	23	4.6	3½
¾	2¼	.89	17.5	3.5	3
⅝	2	.62	12.5	2.5	2½
9/16	1¾	.50	10	2	2¼
½	1½	.39	8.4	1.68	2
7/16	1¼	.30	6.5	1.30	1¾
⅜	1⅛	.22	4.8	.96	1½
5/16	1	.15	3.1	.62	1¼
¼	¾	.10	2.2	.44	1

Plough Steel

Diameter, ins.	Approximate circumference, ins.	Approximate weight per ft., lbs.	Approximate strength, tons of 2000 lbs.	Proper working load, tons of 2000 lbs.	Diameter of drum or sheave in ft. advised
2¾	8⅝	11.95	275	55	11
2½	7⅞	9.85	229	46	10
2¼	7⅛	8	186	37	9
2	6¼	6.3	140	28	8
1⅞	5⅞	5.55	127	25	8
1¾	5½	4.85	112	22	7
1⅝	5	4.15	94	19	6½
1½	4¾	3.55	82	16	6
1⅜	4⅜	3	72	14	5½
1¼	4	2.45	58	12	5
1⅛	3½	2	47	9.4	4½
1	3	1.58	38	7.6	4
⅞	2¾	1.20	29	5.8	3½
¾	2¼	.89	23	4.6	3
⅝	2	.62	15.5	3.1	2½
9/16	1¾	.50	12.3	2.4	2¼
½	1½	.39	10	2	2
7/16	1¼	.30	8	1.6	1¾
⅜	1⅛	.22	5.75	1.15	1½
5/16	1	.15	3.8	.76	1¼
¼	¾	.10	2.65	.53	1

Improved Plough Steel

Diameter, ins.	Approximate circumference, ins.	Approximate weight per ft., lbs.	Approximate strength, tons of 2000 lbs.	Proper working load, tons of 2000 lbs.	Diameter of drum of sheave in ft. advised
2¾	8⅝	11.95	315	63	11
2½	7⅞	9.85	263	53	10
2¼	7⅛	8	210	42	9
2	6¼	6.30	166	33	8
1⅞	5⅞	5.55	150	30	8
1¾	5½	4.85	133	27	7
1⅝	5	4.15	110	22	6½
1½	4¾	3.55	98	20	6
1⅜	4⅜	3	84	17	5½
1¼	4	2.45	69	14	5

Diameter, ins.	Approximate circumference, ins.	Approximate weight per ft., lbs.	Approximate staength, tons of 2000 lbs.	Proper working load, tons of 2000 lbs.	Diameter of drum of sheave in ft. advised
1⅛	3½	2	56	11	4½
1	3	1.58	45	9	4
⅞	2¾	1.20	35	7	3½
¾	2¼	.89	26.3	5.3	3
⅝	2	.62	19	3.8	2½
9/16	1¾	.50	14.5	2.9	2¼
½	1½	.39	12.1	2.4	2
7/16	1¼	.30	9.4	1.9	1¾
⅜	1⅛	.22	6.75	1.35	1½
5/16	1	.15	4.50	.9	1¼
¼	¾	.10	3.15	.63	1

TABLE 9.—EXTRA PLIABLE HOISTING ROPE COMPOSED OF 8 STRANDS AND A HEMP CENTER WITH 19 WIRES TO THE STRAND

Cast Steel

Diameter, ins.	Approximate circumference, ins.	Approximate weight per ft., lbs.	Approximate strength, tons of 2000 lbs.	Proper working load, tons of 2000 lbs.	Diameter drum or sheave in ft. advised
1½	4¾	3.19	58	11.6	3.75
1⅜	4¼	2.70	51	10.2	3.5
1¼	4	2.20	42	8.4	3.2
1⅛	3½	1.80	34	6.8	2.83
1	3	1.42	26	5.2	2.5
⅞	2¾	1.08	20	4	2.16
¾	2¼	.80	15.3	3.06	1.83
⅝	2	.56	10.9	2.18	1.75
9/16	1¾	.45	8.7	1.74	1.5
½	1½	.35	7.3	1.46	1.33
7/16	1¼	.27	5.7	1.14	1.16
⅜	1⅛	.20	4.2	.84	1
5/16	1	.13	2.75	.55	.83
¼	¾	.09	1.80	.36	.75

Plough Steel

Diameter, ins.	Approximate circumference, ins.	Approximate weight per ft., lbs.	Approximate strength, tons of 2000 lbs.	Propper working load, tons of 2000 lbs.	Diameter of drum or sheave in ft. advised
1½	4¾	3.19	74	14.8	3.75
1⅜	4¼	2.70	64	12.8	3.5
1¼	4	2.20	52	10.4	3.2
1⅛	3½	1.80	43	8.6	2.83
1	3	1.42	33	6.6	2.5
⅞	2¾	1.08	26	5.2	2.16
¾	2¼	.80	20	4	1.83
⅝	2	.56	14	2.8	1.75
9/16	1¾	.45	11.6	2.32	1.50
½	1½	.35	8.7	1.74	1.33
7/16	1¼	.27	6.90	1.38	1.16
⅜	1⅛	.20	5.12	1.02	1
5/16	1	.13	3.35	.67	.83
¼	¾	.09	2.25	.45	.75

Extra Strong Cast Steel

Diameter, ins.	Approximate circumference, ins.	Approximate weight per ft., lbs.	Approximate strength, tons of 2000 lbs.	Proper working load, tons of 2000 lbs.	Diameter of drum or sheave in ft. advised
1½	4¾	3.19	66	13	3.75
1⅜	4¼	2.70	57	11	3.5
1¼	4	2.20	47	9.4	3.2
1⅛	3½	1.80	38	7.6	2.83
1	3	1.42	29.7	5.9	2.5
⅞	2¾	1.08	23	4.6	2.16
¾	2¼	.80	17.6	3.5	1.83
⅝	2	.56	12.4	2.5	1.75
9/16	1¾	.45	10.1	2	1.5
½	1½	.35	8	1.6	1.33
7/16	1¼	.27	6.30	1.26	1.16
⅜	1⅛	.20	4.66	.93	1
5/16	1	.13	3.05	.61	.83
¼	¾	.09	2.02	.40	.75

Improved Plough Steel

Diameter, ins.	Approximate circumference, ins.	Approximate weight per ft., lbs.	Approximate strength, tons of 2000 lbs.	Proper working loads, tons of 2000 lbs.	Diameter of drum or sheave in ft. advised
1½	4¾	3.19	80	16	3.75
1⅜	4¼	2.70	68	13	3.5
1¼	4	2.20	56	11	3.2
1⅛	3½	1.80	46	9.2	2.83
1	3	1.42	36	7.2	2.5
⅞	2¾	1.08	28	5.6	2.15
¾	2¼	.80	22	4.4	1.83
⅝	2	.56	15	3	1.75
9/16	1¾	.45	12	2.4	1.5
½	1½	.35	9.5	1.9	1.33

The effect of oiling the ropes was found to be very beneficial, increasing the life of a given rope by two or three times. Experiments were also made to ascertain the effect on the life of a rope of running it over pulleys so arranged that the rope was subjected to reverse stresses, Fig. 23. The results obtained from this series of experiments showed that, generally, the life of a rope working under such conditions was only one-half as long as a similar rope bent in one direction only.

The experiments show that when the first wire breaks, the rope may be assumed to have passed through one-half of its life, and as no one knowingly works a rope until it breaks entirely, then the breakage of even a few wires is a sign that a rope should be carefully watched and replaced by a new one at an early opportunity.

The effect of varying the proportions of diameter of pulley to diameter of rope is one of the most important features to be noticed. Speaking generally, Mr. Biggart's experiments show that increasing the diameter of the pulleys by an amount equal to two circumferences of the rope will double the life of the rope. This is approximately correct for all the varieties of rope and conditions experimented with, and may therefore be taken as equally correct for all the varying conditions under which cranes are worked. It is very remarkable that so simple a rule should evolve from such numerous and varied experiemnts.

These conclusions enable one to express a definite value for the effect upon the durability of ropes, of the various arrangements of pulleys that are commonly adopted in overhead cranes, some of which are illustrated in Figs. 24 to 30. Assuming that Fig. 25, in which the ropes make three bends in working, namely, one at the upper drum and one on each side of the lower pulley, *i.e.*, at entering and leaving, is the arrangement most frequently adopted in practice, and representing the anticipated life of the rope under these conditions by 100, then the relative lives of the ropes in each of the other arrangements indicated will be shown in Table 12.

If it be desired to design each of the above arrangements of pulleys so that the ropes shall have equal durability, then the ratio of the drum diameters to rope circumferance (if the law mentioned above is to be relied upon) must be increased, as shown in Table 13.

TABLE 12.—COMPARISON OF ANTICIPATED LENGTH OF LIFE OF ROPES ARRANGED AS SHOWN IN FIGS. 5 TO 11

Fig. No.	Number of bends	Relative life of rope
24	1	300
25	3	100
26	3[1]	75
27	7	43
28	11	27
29	7[1]	37½
30	11[1]	25

[1] Including one reverse bend which is twice as effective in wearing out the rope.

TABLE 13.—REQUIRED INCREASE IN DIAMETERS OF ROPE DRUMS (MEASURED IN TERMS OF CIRCUMFERENCE OF ROPE) REQUIRED TO GIVE EQUAL DURABILITY

Fig. No.	Increase over diameter called for by Fig. 25
26	1 circumference of rope
27	2½ circumferences of rope
28	4 circumferences of rope
29	3 circumferences of rope
30	4 circumferences of rope

TABLE 10. EXTRA PLIABLE HOISTING ROPES COMPOSED OF 6 STRANDS AND A HEMP CENTER WITH 37 WIRES TO THE STRAND

Cast Steel

Diameter, ins.	Approximate circumference, ins.	Approximate weight per ft., lbs.	Approximate strength, tons of 2000 lbs.	Proper working load, tons of 2000 lbs.	Diameter of drum or sheave in ft. advised
2 3/4	8 5/8	11.95	200	40	....
2 1/2	7 7/8	9.85	160	32	....
2 1/4	7 1/8	8	125	25	....
2	6 1/4	6.30	105	21	....
1 3/4	5 1/2	4.85	84	17	....
1 5/8	5	4.15	71	14	....
1 1/2	4 3/4	3.55	63	12	3.75
1 3/8	4 1/4	3	55	11	3.5
1 1/4	4	2.45	45	9	3.2
1 1/8	3 1/2	2	34	7	2.83
1	3	1.58	29	6	2.5
7/8	2 3/4	1.20	23	5	2.16
3/4	2 1/4	.89	17.5	3.5	1.83
5/8	2	.62	11.2	2.2	1.75
9/16	1 3/4	.50	9.5	1.9	1.5
1/2	1 1/2	.39	7.25	1.45	1.33
7/16	1 1/4	.30	5.5	1.1	1.16
3/8	1 1/8	.22	4.2	.84	1

Plough Steel

Diameter, ins.	Approximate circumference, ins.	Approximate weight per ft., lbs.	Approximate strength, tons of 2000 lbs.	Proper working load, tons of 2000 lbs.	Diameter of drum or sheave in ft. advised
2 3/4	8 5/8	11.95	265	53	....
2 1/2	6 7/8	9.85	214	43	....
2 1/4	7 1/8	8	175	35	....
2	6 1/4	6.30	130	26	....
1 3/4	5 1/2	4.85	108	22	....
1 5/8	5	4.15	90	18	....
1 1/2	4 3/4	3.55	80	16	3.75
1 3/8	4 1/4	3	68	14	3.5
1 1/4	4	2.45	55	11	3.2
1 1/8	3 1/2	2	44	9	2.83
1	3	1.58	35	7	2.5
7/8	2 3/4	1.20	27	5	2.16
3/4	2 1/4	.89	21	4	1.83
5/8	2	.62	14	3	1.75
9/16	1 3/4	.50	11.5	2.3	1.5
1/2	1 1/2	.39	9.25	1.85	1.33
7/16	1 1/4	.30	7.2	1.4	1.16
3/8	1 1/8	.22	5.1	1	1

Extra Strong Cast Steel

Diameter, ins.	Approximate circumference, ins.	Approximate weight per ft., lbs.	Approximate strength, tons of 2000 lbs.	Proper working load, tons of 2000 lbs.	Diameter of drum or sheave in ft. advised
2 3/4	8 5/8	11.95	233	47	....
2 1/2	7 7/8	9.85	187	37	....
2 1/4	7 1/8	8	150	30	....
2	6 1/4	6.30	117	23	....
1 3/4	5 1/2	4.85	95	19	....
1 5/8	5	4.15	79	16	....
1 1/2	4 3/4	3.55	71	14	3.75
1 3/8	4 1/4	3	61	12	3.5
1 1/4	4	2.45	50	10	3.20
1 1/8	3 1/2	2	39	8	2.83
1	3	1.58	32	6.4	2.5
7/8	2 3/4	1.20	25	5	2.16
3/4	2 1/4	.89	19	3.8	1.83
5/8	2	.62	12.6	2.5	1.75
9/16	1 3/4	.50	10.5	2.1	1.5
1/2	1 1/2	.39	8.25	1.65	1.33
7/16	1 1/4	.30	6.35	1.27	1.16
3/8	1 1/8	.22	4.65	.93	1

Improved Plough Steel

Diameter, ins.	Approximate circumference, ins.	Approximate weight per ft., lbs.	Approximate strength, tons of 2000 lbs.	Proper working load, tons of 2000 lbs.	Diameter of drum or sheave in ft. advised
2 3/4	8 5/8	11.95	278	55	....
2 1/2	7 7/8	9.85	225	45	....
2 1/4	7 1/8	8	184	37	....
2	6 1/4	6.30	137	27	....
1 3/4	5 1/2	4.85	113	23	....
1 5/8	5	4.15	95	19	....
1 1/2	4 3/4	3.55	84	17	3.75
1 3/8	4 1/4	3	71	14	3.50
1 1/4	4	2.45	58	11	3.20
1 1/8	3 1/2	2	46	9.2	2.83
1	3	1.58	37	7.4	2.50
7/8	2 3/4	1.20	29	5.8	2.16
3/4	2 1/4	.89	23	4.6	1.83
5/8	2	.62	16	3.2	1.75
9/16	1 3/4	.50	12.5	2.5	1.50
1/2	1 1/2	.39	9.75	1.9	1.33
7/16	1 1/4	.30	7.50	1.5	1.15
3/8	1 1/8	.22	5.30	1.06	1

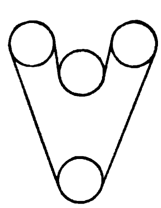
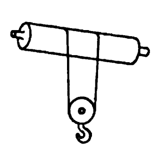
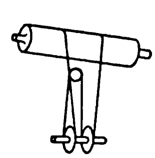


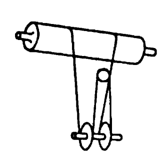
FIG. 23.



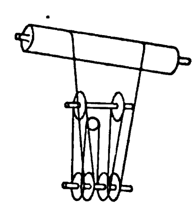
One Bend
FIG. 24.



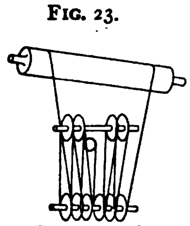
Three Bends
FIG. 25.



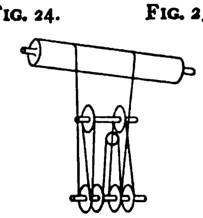
Three Bends, One Reverse
FIG. 26.



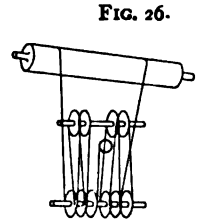
Seven Bends
FIG. 27.



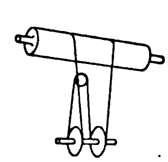
Eleven Bends
FIG. 28.



Seven Bends, One Reverse
FIG. 29.



Eleven Bends, One Reverse
FIG. 30.



Three Bends, Large Bottom Pulleys
FIG. 31.

FIGS. 23 to 31.—Various arrangements of wire ropes on cranes.

Table 11.—Standard Coarse Laid Rope for Haulage and Transmission Composed of 6 Strands and a Hemp Center with 7 Wires to the Strand

Swedish Iron

Diameter, ins.	Approximate circumference, ins.	Approximate weight, per ft., lbs.	Approximate strength, tons of 2000 lbs.	Proper working load, tons of 2000 lbs.	Diameter of drum or sheave in ft. advised
1 1/2	4 3/4	3.55	32	6.4	16
1 3/8	4 1/4	3	28	5.6	15
1 1/4	4	2.45	23	4.6	13
1 1/8	3 1/2	2	19	3.8	12
1	3	1.58	15	3	10 1/2
7/8	2 3/4	1.20	12	2.4	9
3/4	2 1/4	.89	8.8	1.7	7 1/2
11/16	2 1/8	.75	7.3	1.5	7 1/4
5/8	2	.62	6	1.2	7
9/16	1 3/4	.50	4.8	.96	6
1/2	1 5/8	.39	3.7	.74	5 1/2
7/16	1 1/2	.30	2.6	.52	4 1/2
3/8	1 1/4	.22	2.2	.44	4
5/16	1	.15	1.7	.34	3 1/2
9/32	7/8	.125	1.2	.24	3

Extra Strong Cast Steel

Diameter, ins.	Approximate circumference, ins.	Approximate weight per ft. lbs.	Approximate strength, tons of 2000 lbs.	Proper working load, tons tons of 2000 lbs.	Diameter of drum or sheave in ft. advised
1 1/2	4 3/4	3.55	73	14.6	11
1 3/8	4 1/4	3	63	12.6	10
1 1/4	4	2.45	54	10.8	9
1 1/8	3 1/2	2	43	8.6	8
1	3	1.58	35	7	7
7/8	2 3/4	1.20	28	5.6	6
3/4	2 1/4	.89	21	4.2	5
11/16	2 1/8	.75	16.7	3.3	4 3/4
5/8	2	.62	14.5	2.9	4 1/2
9/16	1 3/4	.50	11	2.2	4
1/2	1 5/8	.39	8.85	1.8	3 1/2
7/16	1 1/2	.30	6.25	1.25	3
3/8	1 1/4	.22	5.25	1.05	2 3/4
5/16	1	.15	3.95	.79	2 1/4
9/32	7/8	.12 1/2	2.95	.59	1 3/4

Cast Steel

Diameter, ins.	Approximate circumference, ins.	Approximate weight per ft. lbs.	Approximate strength, tons of 2000 lbs.	Proper working load, tons tons of 2000 lbs.	Diameter of drum or sheave in ft. advised
1 1/2	4 3/4	3.55	63	12.6	11
1 3/8	4 1/4	3	53	10.6	10
1 1/4	4	2.45	46	9.2	9
1 1/8	3 1/2	2	37	7.4	8
1	3	1.58	31	6.2	7
7/8	2 3/4	1.20	24	4.8	6
3/4	2 1/4	.89	18.6	3.7	5
11/16	2 1/8	.75	15.4	3.1	4 3/4
5/8	2	.62	13	2.6	4 1/2
9/16	1 3/4	.50	10	2	4
1/2	1 5/8	.39	7.7	1.5	3 1/2
7/16	1 1/2	.30	5.5	1.1	3
3/8	1 1/4	.22	4.6	.92	2 3/4
5/16	1	.15	3.5	.70	2 1/4
9/32	7/8	.12 1/2	2.5	.50	1 3/4

Plough Steel

Diameter, ins.	Approximate circumference, ins.	Approximate weight per ft., lbs.	Approximate strength, tons of 2000 lbs.	Proper working load, tons of 2000 lbs.	Diameter of drum or, sheave in ft. advised
1 1/2	4 3/4	3.55	82	16.4	11
1 3/8	4 1/4	3	72	14.4	10
1 1/4	4	2.45	60	12	9
1 1/8	3 1/2	2	47	9.4	8
1	3	1.58	38	7.6	7
7/8	2 3/4	1.20	31	6.2	6
3/4	2 1/4	.89	23	4.6	5
11/16	2 1/8	.75	18	3.6	4 3/4
5/8	2	.62	16	3.2	4 1/2
9/16	1 3/4	.50	12	2.4	4
1/2	1 5/8	.39	10	2	3 1/2
7/16	1 1/2	.30	7	1.4	3
3/8	1 1/4	.22	5.9	1.2	2 3/4
5/16	1	.15	4.4	.88	2 1/4
9/32	7/8	.125	3.4	.68	1 3/4

Improved Plough Steel

Diameter, ins.	Approximate circumference, ins.	Approximate weight per ft., lbs.	Approximate strength, tons of 2000 lbs.	Proper working load, tons of 2000 lbs.	Diameter of drum or sheave in ft. advised
1 1/2	4 3/4	3.55	90	18	11
1 3/8	4 1/4	3	79	16	10
1 1/4	4	2.45	67	13	9
1 1/8	3 1/2	2	52	10	8
1	3	1.58	42	8.4	7
7/8	2 3/4	1.20	33	6.6	6
3/4	2 1/4	.89	25	5	5
11/16	2 1/8	.75	20	4	4 3/4
5/8	2	.62	17 1/2	3.5	4 1/2
9/16	1 3/4	.50	13	2.6	4
1/2	1 5/8	.39	11	2.2	3 1/2
7/16	1 1/2	.30	7 3/4	1.5	3
3/8	1 1/4	.22	6 1/2	1.3	2 1/2

It is quite usual for purchasers to specify in their inquiries that the diameters of the pulleys and drums must bear a certain relation to the diameter of the rope, but this stipulation is not sufficient in itself without some consideration being also given to the arrangement of the rope and pulleys.

If the generally accepted ratio of seven circumferences, or twenty-two diameters, of the rope for the diameter of the barrel be assumed as suitable for the drum and pulleys as in Fig. 25, then the diameters for the other figures, to give equal durability, should be as shown in Table 14.

To make the comparisons quite fair between the different arrangements it must now be pointed out that, owing to the increased number of falls of rope adopted in Figs. 27 and 29, the size of the rope may be reduced as shown in Table 15 while retaining the same factor of safety.

Combining the figures given in Tables 14 and 15 will give drum and pulley diameters as shown in Table 16.

The noticeable feature in the last table is that whether two, four, or six falls are adopted, the diameter of the drum and pulleys should

TABLE 14.—RATIO OF DIAMETER OF PULLEYS AND DRUMS TO CIRCUMFERENCE OF ROPE TO GIVE EQUAL DURABILITY

Fig. No.	Ratio of pulley and drum diameter to rope circumference
24	4 to 1
25	7 to 1
26	8 to 1
27	9.5 to 1
28	11 to 1
29	10 to 1
30	11 to 1

TABLE 15.—RELATIVE ROPE CIRCUMFERENCE ALLOWING FOR SMALLER ROPES DUE TO INCREASED NUMBER OF FALLS

Fig. No.	Number of falls	Relative rope circumference
24	2	140
25	4	100
26	4	100
27	8	70
28	12	57
29	8	70
30	12	57

remain about the same if the ropes are to have equal durability (compare Figs. 27 and 28 with Fig. 25). It is clear that very large proportions are necessary to insure a reasonable life for ropes on cranes with many falls of rope. Reference to Fig. 26 and Fig. 29 in Table 16 shows the increase that should be made in the diameter of the drum and pulleys if a reverse bend occurs in the run of the rope.

In Fig. 25, as already mentioned, the ropes make two bends at the lower pulleys to one at the drum, and therefore, if the lower pulleys are made of the same diameter as the drum, they will be responsible for two-thirds of the wear and tear of the rope. It is usually difficult to increase the diameter of the working barrel or drum of a crane,

TABLE 16.—DRUM AND PULLEY DIAMETERS RESULTING FROM A COMBINATION OF TABLES 14 AND 15, AND STILL ASSUMING THAT 100 REPRESENTS THE CONDITION IN FIG. 25

Fig. No.	Ratio of pulley and drum diameter to rope circumference according to Table 14	Relative circumference of rope as per Table 15	Resultant pulley and drum diameter assuming Fig. 25 = 100
24	4	140	80
25	7	100	100
26	8	100	114
27	9½	70	95
28	11	57	90
29	10	70	100
30	11	57	90

because to do so affects the ratio of the gearing and also requires a much larger framework with a correspondingly greatly increased cost of manufacture, but if it is agreed, as a result of Mr. Biggart's experiments, that increasing the diameter of the pulley, over which a loaded rope passes, by an amount equal to twice the circumference of the rope, reduces the evil effects of bending the rope round it to one-half, then a simple means of improving the durability of crane ropes is immediately at the disposal of the designer, namely, to increase the diameter of the pulleys in the blocks, leaving the drums of the original size, as indicated by Fig. 31. This alteration can usually be effected without serious alteration of the design, and may even be carried out on existing cranes.

The result of increasing the diameter of the pulleys, as shown by Fig. 31, by an amount equal to two circumferences of the rope, will be that the effect of the double bend around the lower pulley is halved, and the resultant effect of the three bends will be equal to

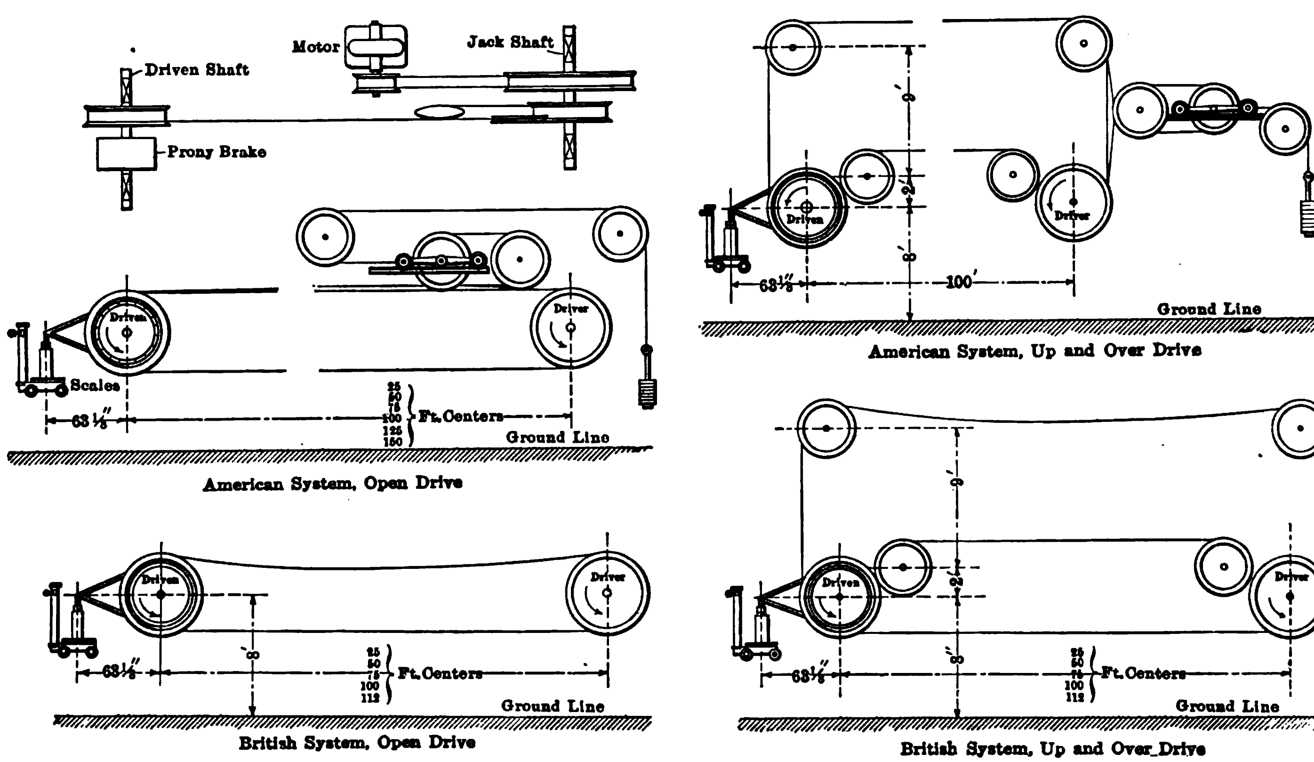
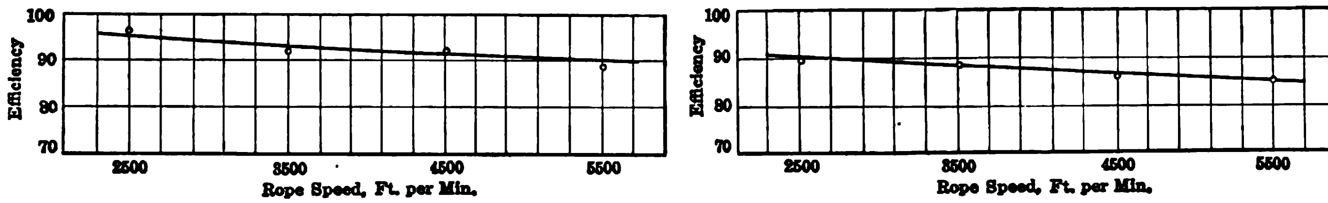
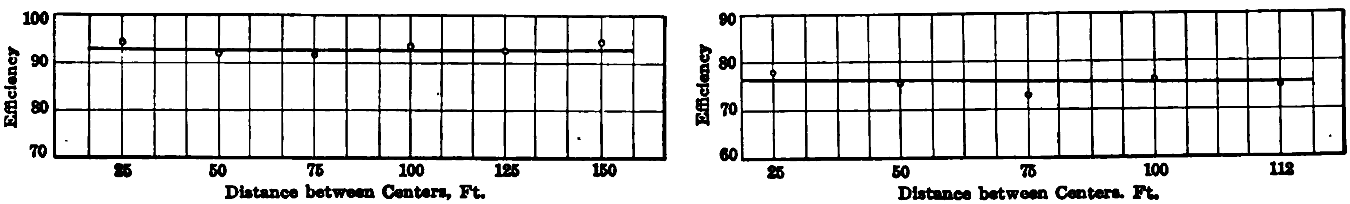


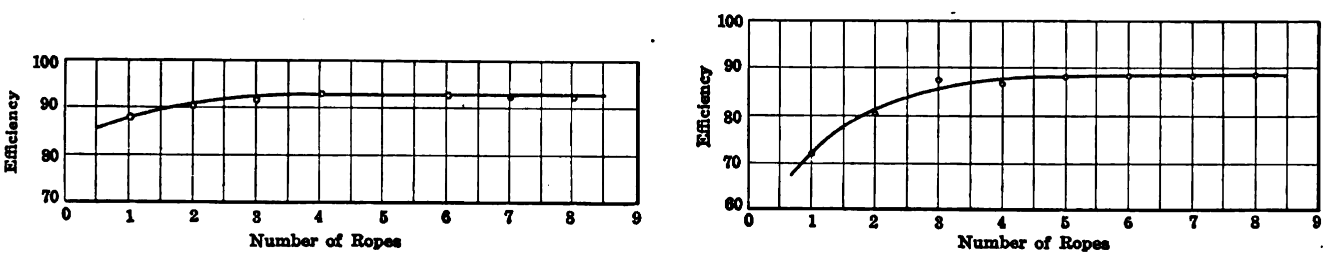
FIG. 32.—Arrangements of rope drives in efficiency tests.



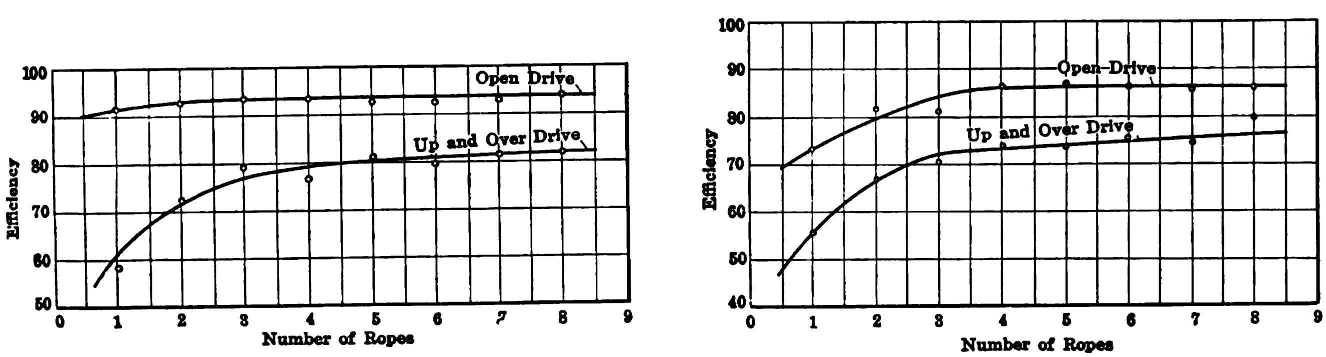
Open drive, 50-ft. centers, 6 ropes, ¾ load.
Relation of efficiency and speed.



Open drive, 7 ropes, American, 6 ropes British, 4500-ft. speed, ¾ load.
Relation of efficiency and distance between centers.



Open drive, American, 50-ft. centers, 4500-ft. speed, British 75-ft. centers, 3500-ft. speed.
Relation of efficiency and number of ropes.



100-ft. centers, 4500-ft. speed, ¾ load.
Comparison of efficiencies of open and up-and-over-drives.

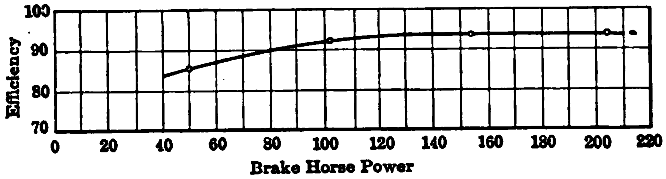
FIG. 33.—American efficiencies of British and American systems of rope driving.

two only and the relative life of the rope will be increased by 50 per cent., or the drum diameter might be reduced by an amount equal to 1.2 times the circumference of the rope with a corresponding reduction in the size of the framework of the crab or winch, while still retaining a relative life for the rope equal to Fig. 25. In this case the diameter of the lower pulleys would only require to be about one circumference of the rope larger than the original size of Fig. 25.

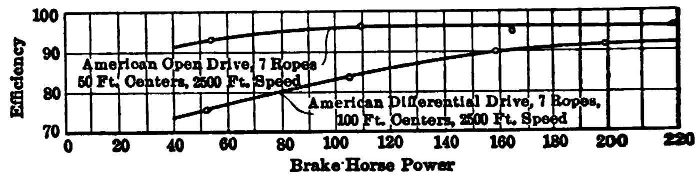
In making the foregoing comparisons of diameters of drum and pulleys with different arrangements of rope it has been assumed that the hook is raised to the full height available at each lift. This, however, is not the case in actual practice, the majority of loads not being raised one-half this height.

This consideration brings to light another great advantage of Fig. 31 as compared with any of the others. Where, as is usually the case, the average height of lift in a shop does not reach half the maximum available, then that portion of the rope which passes under the lower pulley does not reach the upper drum, and accordingly is only subject to the wearing action of the two bends at the lower pulley. If, therefore, the effect of the bends at the lower pulley is reduced to one-half, by the proposed increase in diameter of the pulley, then the actual life of the rope will be doubled, instead of only being increased by 50 per cent. as was first assumed.

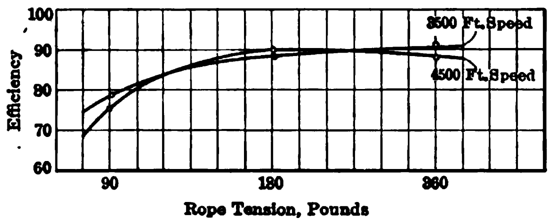
Where there are more than two falls of rope, as in Figs. 27 and 28, the effect of increasing the diameter of the pulleys by an amount equal to two circumferences of the rope is also very marked, reducing the effect of the seven bends in Fig. 27 to four and a half, with corresponding increase in the lift of the ropes. This shows up the fault of those designers who adopt large drums (in order to ob-



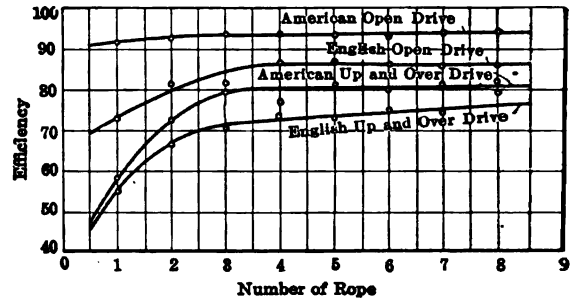
American open drive 4500 ft. speed 75 ft. centers 5 ropes.
Relation of efficiency and load.



Comparison of efficiencies of exact and differential drives.



American open drive, 100 ft. centers, 1 rope, full load.
Relation of efficiency and rope tension.



100 ft. centers, 4500 ft. speed, ¾ load.
Comparison of efficiencies of four general plans
of rope driving under uniform conditions.

FIG. 34.—Various data from rope-drive efficiency tests.

tain the great length of rope entailed by high lifts) and are yet content to make the pulleys of small sizes, when they could enormously increase the durability of the rope by the adoption of larger pulleys at little extra cost.

When the rope makes a reverse bend at the barrel, as in Figs. 26 29 and 30, the barrel ought to be increased in diameter to counteract the effect of the reverse bend. Thus, if in each of these cases the diameter of the drums were made larger by an amount equal to two circumferences of the rope, the durability of the rope would be equal to Figs, 25, 27 and 29 respectively.

The "lay" of the strands and the lubrication of the rope when in use have each a considerable effect upon durability, Mr. Biggart's experiments showed Lang's lay ropes to have more than double the life of those of ordinary lay, and ropes that are oiled last more than twice as long as when this precaution is neglected, as already mentioned. The superiority shown by Lang's lay naturally gives rise to the question as to why it is not exclusively used. The explanation given by rope makers is that such ropes must be very carefully handled to avoid "kinks," and also they are found to be more liable to "spin."

Efficiency of Rope Driving

Very complete tests of the efficiency of rope driving were made by E. H. Ahara at the works of the Dodge Mfg. Co. (*Journal A. S. M. E., Aug.* 1913). Both the British and American systems were tested and in each case the open and the up and over arrangements were included, the meaning of these terms being sufficiently explained by Fig. 32, which illustrates the constructions tested. The losses of the motor, jack shaft and intermediate drive were eliminated by taking preliminary readings from the prony brake applied to the jack shaft under all the various loads and speeds. One-inch manilla rope was used in all the tests. All bearings were of the ring-oiled babbitted type.

The results of the tests are shown in Figs. 33 and 34. Most of the charts are self-explanatory, but, regarding the one relating to exact and differential drives, it should be explained that in the former the grooves of any one sheave were as nearly as possible of the same diameter, while in the latter the diameter of each groove was approximately $\frac{1}{32}$ in. less than the preceeding groove, the eighth groove being ¼ in. smaller in diameter than the first one. The limitation of the tests of the British system to 112 ft. center distance was due to the dragging of the slack ropes on the ground when that distance was exceeded.

CHAINS

The leading types of chains used for power transmission are shown in Figs. 1 to 9, while Table 1 by H. E. Hayward, engineer of experiments and tests, Link Belt Co. (*Amer. Mach.*, *Aug.* 28, *Sept.* 4, 1913) gives the uses to which they are put and the limiting speeds under which they should run.

Crane Chains

The strength of open and stud link crane and cable chains formed the subject of an elaborate investigation and analysis by Profs. G. A. Goodenough and L. E. Moore (*University of Illinois Bulletin No.* 18). The authors conclude that the unit stresses on which the formulas of Unwin, Weisbach and Bach are based are much in excess of the values regarded as permissible in machine construction using reasonable factors of safety. The formulas proposed by the authors for the strength of chain links are:

$P = .4\ d^2 s$ (open).

$P = .5\ d^2 s$ (stud).

in which P = load, lbs.,

d = diameter of bar, ins.,

s = permissible unit stress, lbs. per sq. in.

The following conclusions are of interest as bearing upon certain general opinions held by engineers in regard to chains. "The introduction of a stud in the link equalizes the stresses throughout the link, reduces the maximum tensile stresses about 20 per cent. and reduces the excessive compressive stress at the end of the link about 50 per cent.

"The stud-link chain of equal dimensions will, within the elastic limit, bear from 20 to 25 per cent. more load than the open-link chain. The ultimate strength of the stud-link chain is, however, probably less than that of the open-link chain.

"In the formulas for the safe loading of chains given by the leading authorities on machine design, the maximum stress to which the link is subjected seems to be underestimated and the constants are such as to give maximum stresses of from 30,000 to 40,000 lbs. per square inch for full load."

The loading of hoisting chains, as practiced by the Illinois Steel Co., is given in Table 2. The loads given are uniformly one-tenth the breaking loads. This company requires all chains to be annealed at least every six months.

Table 2.—The Loading of Hoisting Chains

Size, ins.	Safe load, lbs.	Size, ins.	Safe load, lbs.
1/4	305	1 1/2	10,525
3/8	690	1 5/8	12,350
1/2	1,230	1 3/4	14,325
5/8	1,920	1 7/8	16,450
3/4	2,765	2	18,715
1	4,925	2 1/4	22,440
1 1/8	5,925	2 1/2	27,705
1 1/4	7,310	2 3/4	33,530
1 3/8	8,830	3	39,895

The lay-out of sprockets for crane chains is thus explained by A. W. Jenks, Chief Engr. Vulcan Iron Works (*Amer. Mach.*, *March* 24, 1910).

Cable chain is either hand-made or machine-made. The machine-made, which is the cheaper quality, will be found very accurate in pitch and shape of links. The hand-made varies a little in pitch of links and the links will be found to vary considerably in shape, especially in the weld, which is at the end of the link.

When it is intended to use a hand-made chain, the sprocket casting to be used is sent to the chainmaker who makes the chain over the wheel, fitting each link into place; thus making what is known as hand-made wheel chain. However, regardless of which chain is to be used, it is preferable to adopt the sizes given in manufacturers' catalogs for machine-made chain, as the proportions of the links have been adopted after long experience.

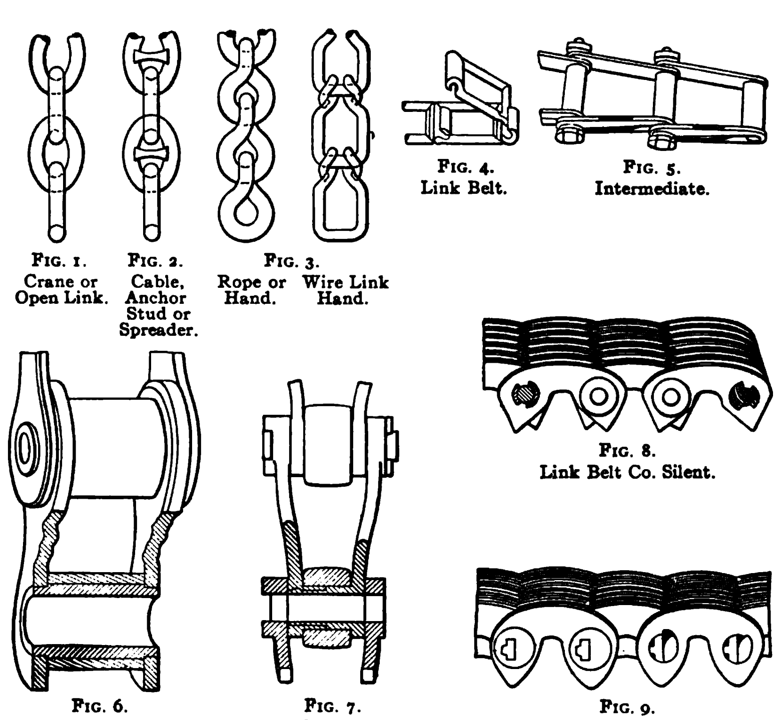


Fig. 1. Crane or Open Link. Fig. 2. Cable, Anchor Stud or Spreader. Fig. 3. Rope or Hand. Wire Link Hand. Fig. 4. Link Belt. Fig. 5. Intermediate. Fig. 6. Roller. Fig. 7. Roller. Fig. 8. Link Belt Co. Silent. Fig. 9. Morse Silent.

Figs. 1 to 9.—Leading types of chains.

Should the chain be on hand, it is wise to measure over some 18 or 20 links, and obtain an average pitch per link. Extreme accuracy must be used in all operations or the sprocket will not turn out right.

The following example, Fig. 10, covers the design of a 10-tooth or 10-pocket sprocket for 1-in. chain. It is first necessary to obtain the dimensions A and B. The chain catalogue gives the length of link W for 1-in. chain, as 4 3/4 ins. and the width of link w as 3 1/2 ins. From the length we find that $A = 3\frac{3}{4}$ ins. and $B = 1\frac{3}{4}$ ins. The points XX are the pin centers upon which the links revolve when passing over the sheave.

The next step is to find the pitch diameter D, which passes through these points XX as follows:

Let N = number of teeth or of pockets in whole wheel.

$= 10$

A = distance, center to center of link length.

$= 3\frac{3}{4}$ ins.

B = distance, center to center between two links.

$= 1\frac{3}{4}$ ins.

$$\text{Angle } y = \frac{180°}{N}$$

$$= \frac{180°}{10} = 18°$$

$$\text{Tan angle } z = \frac{\sin y}{\frac{B}{A} + \cos y}$$

$$= \frac{.30902}{\frac{1.75}{3.75} + .95106} = .21797$$

The tangent .21797 corresponds to an angle of 12° 17′ 46″, the sine of which = .21297.

$$\text{Pitch diameter} \quad = \frac{A}{\sin z} = \frac{3.75}{.21297} = 17.6081 \text{ ins.}$$

As the pockets are not to be machined, allow ample clearance; make the pocket at least ¼ in. longer than the link, which will leave the width of tooth at the center line of the wheel ½ in. The groove in the center of the wheel which accommodates the vertical links, should be amply wide and deep enough to permit the links to fall into a natural position.

It must be noted that the tooth as drawn, is not the shape of the tooth at the side of the central groove, but at the center line. The patternmaker will find it easier to work from this imaginary place, but explain with a note, so that he will not misunderstand. The shape of the tooth faces can be found by considering that each link lifts on its center X, until it comes in line with the next link, when the two lift on the second center, until in line with the third and so on. However, make the tooth somewhat thinner than the contour thus found, or it will be difficult to get the chain into the wheel.

TABLE 1.—TYPES AND USES OF CHAINS

Type	Classes	Where used	Highest speed for general use, ft. per min.	Description and Qualifications
Crane or open link	Open link	Cranes, dredges hoists and slings,	150 for 1 in. and larger	Used for heavy loads moving at low speeds, rough work; power applied by drums, one end of chain *secured to drum*, or by *pocket wheels*, both ends of chain free.
	Spreader or stud	anchors, moorings,	100 on capstans	
	hand chains	hand hoists	350	For application of hand power to hoists, etc., used endless, runs on *pocket wheels* and *rag wheels*.
Detachable	Ewart with hook joint	Power transmission, elevators and conveyors	600	Used for power transmission at moderate speeds. Buckets and flights are attached to chain by means of "attachment links" when chain is used for elevating and conveying.
"Link Belt"	Closed joint	Same as Ewart for dirty places	600	Used chiefly for power transmission in dirty and gritty locations. Made, in best type, with hardened-steel pins and bushings.
Roller	Machine made	Power transmission		Machine-made steel roller chains are much more accurate than malleable-iron chains and will run at higher speeds and loads than "Link Belt." Rollers give better sprocket action than solid joint. Wheels are cut and good machine-made roller chain may be compared with cut gearing.
	Cast malleable and steel	Elevators and conveyors	600	Malleable-iron rollers with telescoped mal.-iron end bars (tubular),the halves of end bars telescoping one into the other. Steel pin passes through tubular end bar. Substantial and durable.
High speed silent	Extended bearing	Power transmission	1200	Bearing surface of joint is given maximum possible area through use of segmental hardened-steel bushings extending throughout entire width of chain and bearing on a cylindrical pin which is free to rotate. Shape of link and wheel tooth is such that elongation is compensated for and sprocket action is very gentle, allowing chain to run quietly at high speeds. Superior to cut gearing at equal speeds and loads. Runs on cut sprocket wheels of special form.
	Rocker joint			Link form substantially similar to the above with the same quiet running and compensating action but with each joint provided with specially designed roller bearing.
	Plain bearing			Link form similar to above, joint bearing formed by round hole in link with round pin, affording half the bearing surface given by extended bearing construction in chains of equal width and equal diameter pin.

It is advisable to use at least five decimal places in all quantities the result is very greatly affected by their absence.

Having the pitch diameter, the wheel can be laid out. It is desirable to make a full-sized layout, if only as a check upon the computations.

Divide the circle into 10 parts for the 10 teeth. But two or three links need be drawn to obtain all necessary dimensions for the shape of the pocket. The horizontal chain link is a chord of 3¾ ins. length on the pitch circle; the vertical link is a chord of 1¾ ins. length. The axis of the vertical link passes through the centers XX of the two adjacent horizontal links. The diameter E across the flats can now be measured and this is really the most important dimension of the wheel.

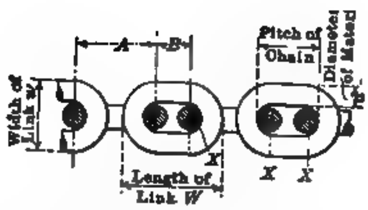
It would be wise, as a check, to space off alternate chords of 1¾ ins. and 3¾ ins. around the entire wheel. Of course, there should be 10 of each. The distance E can also be found by computing the cosine of angle z with radius = 8.80405 ins. or half the calculated pitch diameter and deducing from the result ½ the diameter of the link material.

Wheels are sometimes made with pockets for the vertical links, but it is preferable not to use them, as, when the wheel becomes a little worn, the vertical link has a tendency to pry the horizontal link from its bearing.

It is better that the chain be too tight than too loose, as some stretch will occur in the first few days of operation.

Care must be used in the calculations; approximations will not do, as the errors multiply.

The attachment of chains to hoisting drums by the common method shown in Fig. 11 is pointed out by G. E. FLANAGAN (*Amer. Mach., Oct.* 23, 1902) to be defective. The fault lies in drilling the hole for the spur of the chain anchor in the groove, in place of through solid metal beyond the chain groove as shown in Fig. 12. The first method subjects the anchor spur to a bending stress several times greater than is done by the second, and may cause the failure of the connection. Crane drums should be so proportioned that from one-half to a full coil of chain will remain upon the drum with the hook in the lowest position in which it is possible for it to sustain a load, and in this case only a fraction of the full stress will come upon the anchor;



A = pitch of chain $+ d$ $H = d$
B = pitch of chain $- d$ $J = w + \frac{d}{3} + \frac{1}{16}$ in.
D = pitch diameter $K = d + \frac{d}{5}$
E = diameter across flats $L = \frac{d}{3}$
$F = d \times .75$ $M = d \times .375$
$G = d$

FIG. 10.—Proportions of sprockets for cable chains.

but although this requirement may be observed when the machine is first installed, conditions may be changed afterward, as by digging deeper pits in the foundry, and thus the entire load may come directly upon the chain anchor, which ought to be fully capable of meeting such an emergency. It will be noted that Fig. 12 requires a link of extra length on the end of the chain in order to reach the spur in the position shown. Figs. 11 and 12 are both open to the objection that the drum may be rotated far enough to bring the pull of the chain as shown in dotted lines of Fig. 12. In this case the load will not come upon the spur at all, but principally upon the nearest tap bolt, and it will be increased by a leverage depending upon the distance from the chain to this bolt. A better arrangement than either is shown in Fig. 13.

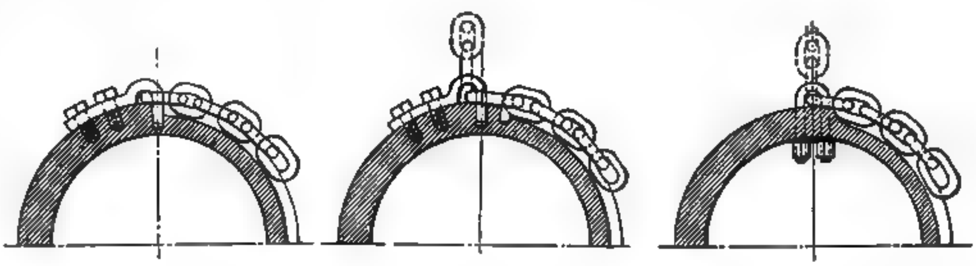


FIG. 11. FIG. 12. FIG. 13.

FIGS. 11 to 13.—Anchors for crane chains.

Fig. 14 shows a section of a grooved drum, with proportions. The thickness of the metal below the bottom of the groove is determined by treating the drum as a hollow cylindrical beam with the load concentrated in the middle. The links should not bottom in the groove.

Hand chains are used endless, hanging from hoists, etc., with the lower loop free. The rims of the wheels over which the chain passes are called rag wheels. These rims are simple in design, usually having a V-groove with about 60 deg. included angle, with ridges cast radially along the inner sides of the V to provide gripping points for the chain links.

The Ewart Chain

The Ewart chain with hook joint, Fig. 4, is used for power transmission at moderate speeds. According to Mr. HAYWARD (*Amer. Mach., Aug.* 28, 1913) the thickness of the tooth of the sprocket wheel must be such as to give the hook portion of the link some freedom between teeth. If the tooth space were made to conform to the shape of the hook, the slightest stretching of the chain under load or through wear would cause the chain to ride up on the flanks of the teeth, and eventually the chain would be broken by riding over the crowns. Conversely, the wider the tooth space the greater the amount of stretch or wear that can take place before riding. On wheels of a large number of teeth the space may be considerably wider than on small wheels, as the wear on the tooth flanks is more distributed, each tooth coming into action once in each revolution of the wheel. The wide tooth space is also desirable in large wheels because it permits of the greatest possible elongation of the chain before the increased pitch of the chain causes it to ride.

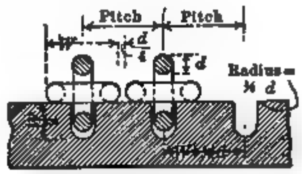


FIG. 14.—Section of chain drum.

The working load which may be applied to a given link belt depends in each case upon speed of chain, cleanliness of location, and character of the load. The best method of rating a chain is by applying a factor, varying with the speed, to the average breaking strength of the given chain. Table 3 gives factors that have been determined by exhaustive experiment and by use:

TABLE 3.—SPEEDS AND WORKING LOADS FOR EWART CHAINS

Chain speed in ft. per min.	To obtain working load divide average ultimate strength by
0–200	6
200–300	8
300–400	10
400–500	12
500–600	16
600–700	20

If the chain is to be subjected to shock or is to work in a gritty place, especially in elevators and conveyors where coarse or gritty materials are being handled, the factors must be increased beyond those given in the table. In lookingover the catalog of manufacturers of link belting the wide range of sizes and types often makes the selection of chain difficult; the following suggestions may simplify the problem.

In selecting chains for power transmission, first determine the

diameter of the small sprocket wheel, keeping it as small as possible. Select a chain of medium pitch with the proper breaking strength and find the number of teeth in the sprocket wheel of the diameter selected. If possible use wheels with more than eight teeth; smaller wheels cause rapid wear on both wheels and chain through the large angle of articulation when the links enter and leave the wheel.

Note that a chain of medium pitch is desirable for ordinary transmission purposes. Short-pitch chains will not permit of sufficient tooth space in the wheel to allow for much elongation of the chain through wear, also the joints are greater in number in the same length of chain, and the same amount of wear per joint will cause greater elongation than in medium pitch. Short-pitch chains are designed primarily for applications where backlash of the chain on the wheel is objectionable. The long-pitch chains are only desirable for transmission purposes where the chain speed is very low.

At equal chain speeds on equal-diameter sprockets the long-pitch chains hammer on the sprocket wheels much harder than the medium pitch, make much more noise, and, strength being equal with medium pitch, do not make as durable a drive. Long-pitch chains are applicable principally to elevator and conveyor work. Where the chain speeds, compared with ordinary power transmissions, are low, seldom exceeding 200 ft. per min., the diameters of the sprocket wheels are governed by consideration of the material being handled rather than the energy transmitted.

Cast-iron sprocket wheels in the rough are likely to be irregular in tooth spacing. Unequal shrinkage, rapping of the pattern in molding and inaccuracies in the pattern often cause the wheels to be incorrect in pitch and diameter. In an ordinary gray-iron casting the best way to secure a good wheel is to cast a little large in diameter and grind the periphery down to properly fit a piece of standard chain.

Hard-rim sprocket wheels are furnished by some manufacturers. These are cast from special iron in such a manner as to make the rims and teeth extremely hard and tough while the hubs are soft for boring and keyseating. These wheels when properly made, are a decided economy in spite of their slightly higher price, as they last several times longer than the gray-iron wheels. Care should be given to their selection, however, as they are subject to some troubles not present in the ground soft-iron wheels.

The face of the wheels, both on the teeth and on the root diameter, should be parallel to the bore of the wheel and the edges of the wheels should be free from fins or other projections. The iron is too hard to grind over the entire face of the wheel, and projections will cut the chain to pieces very quickly.

The design of a sprocket wheel for link belting is not a difficult matter, but the patterns are expensive and the production of a good wheel in the foundry requires considerable skill. For the use of those who wish to make their own wheels, the following is offered:

Having determined the number of teeth desired and the pitch of the chain, the next step is to find the diameter of the pitch circle, Fig. 15. Careful measurements should then be made of the chain to determine the dimensions shown in Fig. 16. The root diameter of the wheel, as shown in Fig. 17, is the pitch diameter less $2\times A$. The flanks of the sprockets or teeth may be made straight from just above the root line to a little above the pitch line and should be inclined at such an angle as to give the hook of the link ample clearance in leaving and entering the wheel.

It is not necessary to make the tooth at its base conform to the shape of the hook. The straight-tooth flanks will cast more regularly and will form a more uniform bearing for the chain than if the hook and tooth forms were similar. The thickness of the tooth at the pitch circle determines the amount of wear and stretch that may take place in the chain before it begins to ride the wheel; each tooth comes into action once in a revolution of the wheel, and the wear on a tooth is proportional to the number of times it comes into action. Therefore, the wear on a wheel with a small number of teeth is greater than on one with a large number, making heavy teeth necessary in small wheels. Furthermore, the number of links in mesh with a wheel of a small number of teeth is less than in a wheel with a large number, making it possible to reduce the clearance without taking from the useful life of the chain.

Table 4 expresses this difference in percentages of the available tooth space in the chain, as shown in *B–D*, Fig. 16

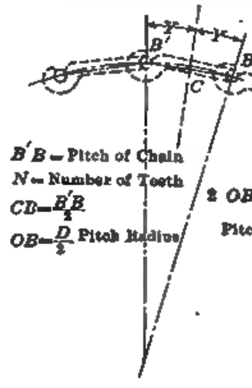


FIG. 15.
Pitch Circle Diameter

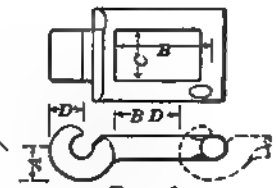


FIG. 16.
Proportions of Links

FIG. 17.
Root Diameter

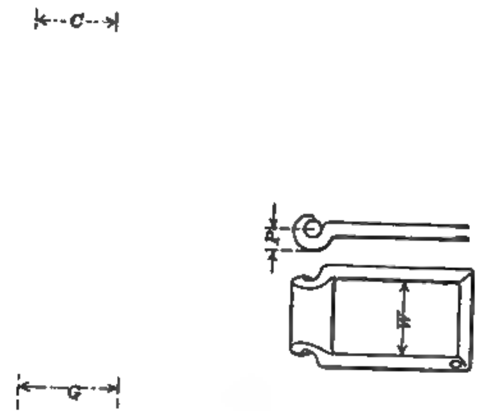


FIG. 18.—Section of rim.

FIGS. 15 to 18.—Laying out sprockets for the Ewart chain.

TABLE 4.—NUMBER AND THICKNESS OF TOOTH

Number of teeth in wheel	Per cent. thickness of teeth at pitch line
8 to 12	75–80 of tooth space in chain
13 to 20	70 of tooth space in chain
21 to 35	65 of tooth space in chain
36 to 60	55–60 of tooth space in chain

The straight flank of the tooth may be continued to nearly the total height of the tooth and then curved over to form a flat crown, or a rounded crown may be used as shown in the dotted lines in Fig. 14.

Fig. 18 shows a section of a typical sprocket-wheel rim. The dimensions may be expressed approximately in terms of the dimensions of the link, thus:

$B = W - \frac{1}{16}W$ up to $\frac{1}{4}$ in., which is sufficient for wide chains

$$C = \frac{W}{2}$$

$$H = 2.5P$$

$$D = .7W$$

$$E = 1.5W$$

$$F = \frac{W}{3}$$

$$G = .6W$$

These proportions are necessarily approximate. The wide range of designs and sizes makes it impossible to formulate a satisfactory method that will apply to all cases.

The proper use of the Ewart chain has been explained by S. B. Peck, Vice President Link Belt Co. (*Amer. Mach., May* 14, 1908) as follows:

When considering the relative merits of different methods of running chain-drives, the drive should be considered as a whole, and the action noted at four points: *a*, entering point on the driver; *b*, releasing point on the driver; *c*, entering point on the driven; *d*, releasing point on the driven, as shown by *abcd*, in Fig. 19.

In this discussion the action at a point is said to be good when all the articulation or bending takes place in the joint of the chain, Fig. 20. The action is said to be bad when, in bending, the link rubs on the sprocket, producing wear on the sprocket and outside or external wear on the hook, Fig. 21.

In Fig. 26 we have the driver large, the driven small; hence *b* and *d* are the teeth in action. Chain runs bar first; action at *a* good, at *b* bad, at *c* good, at *d* bad.

In Fig. 27 we have the same sprockets as in Fig. 26, but the chain runs hook first. Here the action at *a* is bad, but the fact that the hook is not in contact with a tooth face at this point makes the consequent wear of little account. The action at *b* is good. The action at *c* is bad, but this is on the slack side of the chain and this bad action causes no wear. The action at *d* is good.

It is thus seen that there are two very bad points (*b* and *d*) where the chain runs bar first and only one serious trouble (*a*) when running hook first. Therefore, always run hook first in sucd a case.

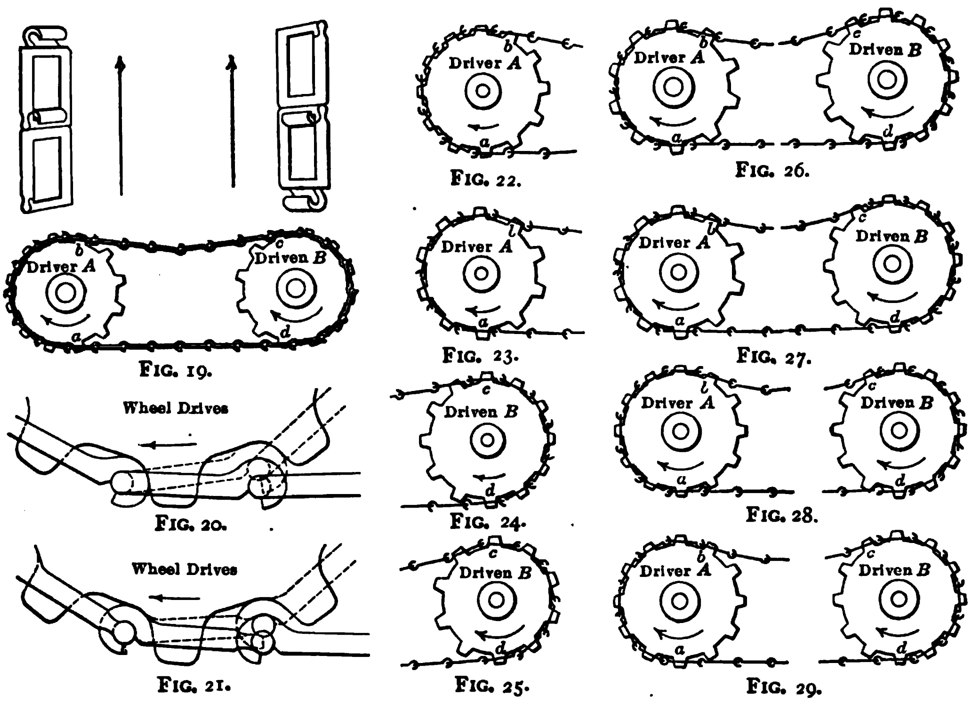


Fig. 19. Fig. 20. Fig. 21. Fig. 22. Fig. 23. Fig. 24. Fig. 25. Fig. 26. Fig. 27. Fig. 28. Fig. 29.

Figs. 19 to 29.—The correct use of Ewart chain.

Another fact is also to be remembered: There is never more than one tooth in action at any one time. No matter how carefully the chain and sprocket may be made, as soon as the load comes on, there is a change caused by stretch and wear.

We can predetermine which tooth shall be in action by making the pitch of the wheel either larger or smaller than the pitch of the chain. Thus, on the driver, Fig. 22, the wheel pitch being smaller than the chain pitch, the entering tooth does all the work. In Fig. 23 the conditions are reversed: the wheel pitch is the larger and the releasing tooth does the work. On the driven the same thing holds, except that here conditions are reversed.

When the wheel pitch is smaller than the chain pitch the releasing tooth does the work, Fig. 24, and when the wheel pitch is larger than the chain pitch the entering tooth does all the work, Fig. 25.

For the best work the pitch of the driver should be larger than the pitch of the chain, Fig. 23, and the pitch of the driven should be smaller than that of the chain, as in Fig. 24. The releasing teeth *b* and *d* are, therefore, the working teeth, and the chain can seat at *a* and *c* quietly and take the load gradually as the wheel revolves.

Having considered the question of wheels, we will now regard the drive as a whole to determine whether the chain shall be run bar first or hook first.

Consider a drive when the sprockets are such as are usually furnished: These are ground to fit the new chain; when the latter stretches, both driver and driven are small as compared to it, and teeth *a* and *d* are now in action.

In Fig. 28 we have such a pair of wheels with the chain running bar first. The action at *a* is good; at *b* it is bad, but as there is no tension on the chain at this point, this is not objectionable. At *c* the action is good; at *d* it is bad. In this case, therefore, it would seem that the wear would be confined to the driven wheel and this is so in actual practice.

Only wear is on driven, caused by the bad action at *d*, this forms hook on *d* and breaks chain.

Observe the same wheels with the chain running hook first, Fig. 29. The action at *a* is bad; at *b* it is good; at *c* it is bad, but not objectionable, because, as before, there is no tension at this point; action at *d* is good. Thus all the wear would seem to be on the driver as a result of the action at *a*. This is found to be the case; hence both theory and practice show that with the chain running *bar* first *driven* wheel wears, while with chain running *hook* first *driver* wheel wears.

Now, it is found that because the wear at *d*, running bar first, is caused by the link slipping up the tooth, it tends to undercut and form a hook and thus break the chain. On the other hand, the wear at *a*, when running hook first, is caused by the link slipping down the tooth, and the wheel will wear out completely without endangering the chain. It has also been proved that the driver, running hook

first, lasts several times as long as the driven wheel when running bar first. As the driven wheel is in nearly every case much larger than the driver and the consequent wear on each tooth is less, it would seem that if the chain were run so as to wear the driven, the wear on the two wheels would be equalized. This would be poor practice for the reason that the driver, being smaller, is more cheaply replaced, and the repair account will, therefore, be less running hook first.

In elevators, the head wheel acts as a driver, and the foot wheel simply as an idler, because it is doing no work. Therefore, run the chain bar first so as to favor the driver. On conveyers one wheel is always an idler, comparatively speaking, and the same reasoning holds as for elevators: the chain should run *bar first* in all cases.

These remarks apply equally well to all closed-end pin chains; the closed end corresponds to the hook and the pin end to the bar of the Ewart chain.

In general, therefore, on drives run *hook* first. On elevators and conveyers run *bar* first.

The Roller Chain

A chain construction, which is intermediate between the Ewart type detachable-link belting and roller chain, is shown in Fig. 5. The links are joined by a steel pintle, which passes the tubular portion of the link. A cotter pin in one end of the pintle makes it easy to disconnect the chain by withdrawing the pintle.

A machine-made roller chain is shown, Fig. 6. This chain will run at a much higher speed than link-belt type of chains, and also produce better sprocket action.

Malleable-iron roller chains are made in a range of sizes from 3 to 24 ins. in pitch, and have a wide field of application, being suitable for transmission and for elevators and conveyors. It is essential that the side bars of the links be held rigidly in line with each other in order that the load may be equally distributed between them and also that they be held apart in order to insure free rotation of the roller. These features are secured in the construction shown in Fig. 7.

Roller chains are very generally used for driving motor trucks. In this service the factors of safety employed are much greater than in stationary practice, on account of shocks, heavy momentary loads, and the gritty conditions under which the chains must operate. The working loads vary from $\frac{1}{10}$ to $\frac{1}{20}$ of the breaking strength of the chains. As a rule, the factor increases with the capacity of the truck.

Roller-chain sprocket wheels for accurately made chain are always made with cut teeth. Cutters may be obtained from many tool manufacturers, and there is a wide range of tooth forms in use, some

TABLE 5.—DATA FOR THE DESIGN OF MORSE SILENT CHAIN DRIVES

Notes

1. Number of teeth = T.
   Exact outside diameter = D.
   When T has less than 20 teeth, D = pitch diameter.
   When T has more than 20 teeth, D = pitch diameter + (2 × addendum).
2. Use sprockets having an odd number of teeth whenever possible.
3. When specially authorized, a larger number of teeth than shown may be cut in large sprocket.
4. Thickness of sprocket rim, including teeth, should be at least 1.2 times the chain pitch.
5. The number of grooves in the sprocket, their width and distance apart, varies according to pitch and widthof chain. In every case leave the designing and turning ot these grooves to the Morse Chain Company.
6. The width of the sprocket should be $\frac{1}{8}$ to $\frac{1}{4}$ in. greater on small drives, and $\frac{1}{4}$ tc $\frac{1}{2}$ in. greater on large drives than nominal width of the chain.
7. An even number of links in the chain and an odd number of teeth in the wheels are desirable.
8. Horizontal drives preferred; tight chain on top desirable for short drives without center adjustment.
9. Adjustable wheel centers desirable for horizontal drives and necessary for vertical drives.
10. Avoid vertical drives.
11. Allow a side clearance for chain (parallel to axis of sprockets and measured from nominal width of chain) equal to the pitch.
12. Maximum linear velocity for commercial service 1200 to 1600 ft. per minute.

These data are for use in preliminary design. Engineering features should always be submitted to the Morse Chain Company for approval before ordering.

Pitch, ins.	$\frac{3}{8}$	$\frac{4}{10}$[1]	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{9}{10}$	$1\frac{2}{10}$	$1\frac{1}{2}$	2	3	4
Minimum number of teeth:											
Small sprocket driver	13	13	13	13	13	15	15	17	17	17	19
Small sprocket driven	17	17	17	17	21	25	29	29	31	35	37
Desirable number of teeth in driver sprockets.	15–17	15–17	15–17	17–21	17–21	17–23	17–23	17–27	17–31	19–31	21–31
Maximum number of teeth in sprockets. (See Note 3).	75	85	99	109	115	125	129	129	129	131	131
Desirable number of teeth in driven sprockets.	35–55	35–55	55–75	55–75	55–85	55–95	55–105	55–115	55–115	55–115	55–115
To find pitch diameter of wheel multiply number of teeth by (ins.).	.1195	.127	.159	.199	.239	.2865	.382	.477	.636	.955	1.2732
Addendum. For outside diameter of sprockets 20 to 130 T. (See Note 1), ins.	0	0	.05	.06	.075	.09	.12	.15	.20	.30	.80
Maximum r.p.m.	3000	2600	2400	1800	1200	1100	800	600	400	250	100
Tension per inch width chain, lbs:											
Small sprocket driver	40	45	80	100	120	150	200	270	450	750	900
Small sprocket driven	30	35	65	80	95	120	160	210	350	600	700
Radial clearance beyond tooth required for chain, ins.	.37	.40	.50	.62	.75	.90	1.2	1.5	2.0	3.0	4.00
Approximate weight of chain per inch wide, 1 ft. long, lbs.	.7	.75	1.00	1.20	1.50	1.80	2.50	3.00	4.00	6.00	8.00
C for solid pinions	......	.....	.0045	.0063	.009	.013	.023	.035	.058	.145	
C for armed sprockets	......	.....	.16	.25	.35	.45	.7	1.0	2.0	4.0	

APPROXIMATE WEIGHTS FOR SOLID AND ARMED SPROCKETS

T = number of teeth.
F = face in inches.
C = constant in pounds per inch in face per tooth as per table.
Weight of armed sprocket = $T \times F \times C$.
Add 25 per cent. for split and 50 per cent. for spring and split sprockets.
Weight of solid pinion = $T^2 \times (F+1) \times C$.

[1]. Note (1) does not apply. Pitch and out-side diameter are the same or equal.

of which are poor in design. A type which was common and is still in use but which should be avoided, has for the bottom of the tooth space an arc of a circle of the same radius as the roller, with the flanks of the teeth made with just sufficient relief to allow the rollers to enter and leave the wheel. Any elongation occurring as soon as a load is put on the chain, will cause the rollers to strike the tooth flanks in entering the wheel. On account of the absence of clearance the teeth rapidly wear hook-shaped, making the drive noisy and short-lived.

A wheel with a flat bottom in the tooth spaces and considerable clearance, like the wheel described for Ewart-type link belting, is much better than the above. The bottom of the tooth space should be joined to the tooth space by a fillet of a radius equal to the radius of the roller, and the tooth should be relieved on the flanks to allow the roller easy entrance and exit. The objection to this type lies in the fact that each link slips back on leaving the sprocket wheel,

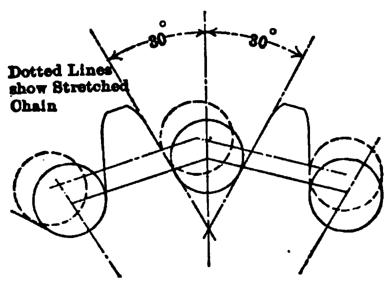


FIG. 30.—Link belt star wheel adapted to roller chain.

or advances on entering, an amount equal to the elongation after the chain stretches, causing noise and undue wear on the tooth flanks and contributing to the ultimate destruction of the chain.

The latest and most advanced type, combining simplicity and smooth chain action, is shown in Fig. 30. The tooth space is made with the bottom the same radius as the roller, and the flanks of the teeth slope back at an angle of 30 deg. with a radius drawn through the center of the roller. The flanks of the teeth are slightly relieved near the crown and the teeth are made higher than in other types. Stretch of the chain is compensated for by the rollers rising on the tooth flanks, giving smooth and comparatively quiet action.

The High Speed Silent Chain

The preliminary design of Morse silent chain drives may be made in accordance with Table 5, which has been supplied by the Morse Chain Co.

It should be borne in mind that the silent chain is practically a flexible rack, and gives a positive drive. Its use, therefore, is undesirable where the necessity of some slip exists, such as would be found in driving punching presses where the accumulated speed of the balance wheel does the work of each stroke. In drives having an infrequent shock load, due to accident or lack of uniformity of material to be worked, a safety or shearing pin sprocket is fitted as a safeguard.

In regularly intermittent service, such as air-compresser driving, a spring sprocket wheel is always desirable and sometimes necessary. In drives subject to sudden overloading and heavy shock, a shearing or safety pin is often fitted. These chains are regularly used for transmitting loads up to 1500 h.p.

The following observations explain more fully some of the information given in the table:

The limitation of the desirable number of teeth in the large sprocket, given in the fourth line of the table, is intended to give a reasonable provision for the increased pitch of the chain due to use. As is well known, the chain gradually engages the sprocket at increased diameters as its pitch increases, and, with too large sprockets, the reduced ratio of pitch to diameter reduces this provision below the desirable limit. The call in note 7 for an even number of links in the chain is intended to eliminate the special link which an odd number of links require. This can usually be brought about by a slight adjustment

TABLE 6.—CAPACITY OF LINK BELT SILENT CHAINS

Speeds in ft per min.			500	600	700	800	900	1000	1100	1200	1300	1400	1500
Nom. pitch	Nom. width	Face of wheel	H.p	H.p	H.p	H.p	H.p	H.p	H.p	H.p	H.p	H.p	H.p
	1/2	1	.58	.66	.72	.78	.82	.88	.91	.95			
	3/4	1 1/4	.87	.98	1.07	1.16	.122	1.30	1.38	1.42			
	1	1 1/2	1.16	1.31	1.43	1.55	1.63	1.73	1.82	1.89			
3/8"	1 1/4	1 3/4	1.45	1.64	1.79	1.91	2.04	2.18	2.28	2.36			
	1 1/2	2	1.74	1.97	2.15	2.30	2.45	2.60	2.73	2.83			
	2	2 1/2	2.32	2.62	2.86	3.08	3.27	3.46	3.64	3.78			
	3	3 1/2	3.48	3.91	4.28	4.61	4.89	5.22	5.46	5.67			
	1/2	1 1/8	.84	.95	1.04	1.11	1.19	1.27	1.33	1.38	1.42		
	3/4	1 3/8	1.26	1.40	1.56	1.70	1.79	1.91	1.99	2.07	2.13		
	1	1 5/8	1.68	1.89	2.08	2.25	2.34	2.54	2.65	2.76	2.84		
1/2"	1 1/2	2 1/8	2.52	2.91	3.12	3.44	3.57	3.88	3.98	4.14	4.25		
	2	2 5/8	3.37	3.82	4.17	4.48	4.77	5.10	5.30	5.52	5.68		
	3	3 5/8	5.05	5.73	6.25	6.75	7.15	7.60	7.95	8.29	8.50		
	4	4 5/8	6.73	7.64	8.30	9.00	9.53	10.1	10.6	11.1	11.3		
	1	1 3/4	2.22	2.51	2.74	2.96	3.15	3.33	3.50	3.64	3.75		
	1 1/4	1 7/8	2.77	3.15	3.41	3.71	3.93	4.18	4.37	4.54	4.70		
	1 1/2	2 3/16	3.33	3.76	4.12	4.43	4.72	5.00	5.25	5.45	5.62		
5/8"	2	2 11/16	4.43	5.02	5.47	5.91	6.30	6.67	7.00	7.28	7.50		
	3	3 11/16	6.65	7.52	8.22	8.88	9.45	10.0	10.05	10.09	11.2		
	4	4 3/4	8.86	10.0	10.9	11.8	12.6	13.3	14.0	14.5	15.0		
	6	6 3/4	13.3	15.0	16.4	17.7	18.9	20.0	21.0	21.8	22.5		
	1	1 7/8	2.85	3.22	3.51	3.78	4.05	4.37	4.48	4.65	4.82		
	1 1/4	2 1/8	3.56	3.98	4.39	4.70	5.06	5.30	5.60	5.78	6.02		
	1 1/2	2 3/8	4.27	4.85	5.27	5.67	6.10	6.40	6.72	6.98	7.23		
3/4"	2	2 7/8	5.68	6.42	7.03	7.56	8.10	8.55	8.95	9.31	9.63		
	3	3 7/8	8.55	9.63	10.5	11.4	12.1	18.8	13.4	14.0	14.5		
	4	4 7/8	10.4	12.8	14.0	15.1	16.3	17.3	17.9	18.6	19.3		
	6	6 7/8	17.1	19.3	21.1	22.8	24.3	25.7	26.8	27.9	28.9		
	2	2 3/4	7	7.91	8.65	9.33	10	10.5	10.9	11.4	11.8		
	2 1/2	3 1/4	9	10.1	11.1	12.0	12.9	13.5	14.1	14.7	15.2		
	3	3 3/4	11	12.4	13.6	14.6	15.7	16.5	17.2	18	18.6		
1"	4	4 3/4	15	16.9	18.6	20	21.5	22.5	23.5	24.6	25.4		
	5	5 3/4	19	21.5	23.5	25.2	27.2	28.7	29.7	31.1	32.1		
	6	6 11/16	23	26	28.5	30.5	32.9	34.5	36	37.6	38.9		
	8	8 11/16	31	34.9	38.4	41.2	44.3	46.3	48.5	50.7	52.4		
	2	3	9.7	11.0	11.9	13	13.8	14.6	15.3	15.9	16.4	16.7	
	3	4	15.3	17.3	18.7	20.3	21.7	22.9	24.2	25	25.7	26.5	
	4	5	20.8	23.5	25.5	27.6	29.6	31.2	32.6	34.1	35.1	36.2	
1 1/4"	5	6	26.3	29.8	32.3	35.1	37.5	39.7	41.6	43.2	44.5	45.8	
	6	7	31.8	36.2	39.1	42.7	45.3	48.2	50.3	52.2	53.8	55.5	
	8	9	42.8	48.5	52.7	57.2	61.2	64	67.8	70.3	72.5	74.6	
	10	11	54.1	61.3	66.5	72.2	77.1	81.2	85.6	88.7	91.4	94.1	
	3	4	20.1	22.7	24.7	26.9	28.7	30.3	31.8	33	34	35	35.7
	4	5	27.5	31.1	33.7	36.6	39.1	41.2	43.4	45	46.4	48	48.7
	5	6	34.8	39.3	42.7	46.3	49.5	52.3	55	57	58.7	60.7	61.6
1 1/2"	6	7	42.2	47.6	51.8	56.3	60	63.4	66.5	69	71.1	73.5	74.7
	8	9	56.7	64.2	69.7	75.7	81	85.2	89.7	93	95.8	99	101
	10	11	71.4	80.7	87.7	95.2	102	107	113	117	121	124	127
	12	13	86	97.3	106	115	123	129	136	141	145	150	152
	6	7 1/16	56.1	63.5	69	75	80	84.3	88.8	92	94.8	97.5	99.6
	8	9 1/16	75.7	85.6	93	101	108	114	120	124	128	131	134
2"	10	11 1/16	95.2	107	117	126	136	143	151	156	161	165	169
	12	13 1/16	114	129	141	153	164	172	182	188	194	199	204
	14	15 1/16	134	152	165	179	191	201	212	220	227	233	240
	16	17 1/16	154	174	189	205	220	231	243	252	260	267	273
	6	7 1/2	73	82.7	90	98	104	110	116	120	124	127	130
	8	9 1/2	100	113	123	133	143	150	158	164	169	174	178
2 1/2"	10	11 1/2	126	143	155	168	180	190	200	207	213	220	224
	12	13 1/2	153	173	188	204	218	230	242	251	259	266	272
	14	15 1/2	179	204	220	240	255	270	284	294	303	313	318
	16	17 1/2	206	235	253	274	294	310	326	338	348	359	365

of the center distance. The call in the same note for an odd number of teeth in the sprockets is intended to provide a hunting-tooth effect, by which all parts of the sprocket-tooth faces are successively engaged by the links. While this is a desirable it is not an essential feature, and when exact speed ratios which call for an even number of teeth are required, such teeth may be used without hesitation.

The preliminary design of link belt silent chain drives may be made in accordance with Tables 6 and 7 which have been supplied by the Link Belt Co.

For cases in which it is necessary to calculate the exact distance between the shafts, in order that the silent chain may run without slack, the following formula by H. S. PIERCE, of the Link Belt Co., may be used:

$$L = (2C \cos A) + \left(\frac{90+A}{180}PN\right) + \left(\frac{90-A}{180}Pn\right)$$

in which L = length of chain, ins.,

C = center distance, ins.,

N = number of teeth, large wheel,

n = number of teeth, small wheel,

D = diameter of large wheel, ins.,

d = diameter of small wheel, ins.,

P = pitch of chain, ins.,

A = angle whose $\sin = \dfrac{D-d}{2C}$.

The formula is solved by repeated assumptions of the center distance C until a value of C is found which makes L an exact multiple of the pitch.

In applying silent chains of any type, the following suggestions should be considered:

Drives should not be vertical if such arrangement can be avoided; if vertical or nearly so make the center distance between shafts short; a long vertical chain will tend to drop away from the teeth of the lower wheel, causing bad chain action.

Provide means for adjusting the distance between shafts. This will facilitate the installation of the chain and will, in some cases, prolong the life of the drive by making it possible to take up wear. On extremely short center drives the adjustability is not as essential if care is exercised to make the center distance such as will keep the chain without slack.

Do not run chains tight; initial tension is not necessary, and it increases the bearing pressures in both chain and shaft bearings.

TABLE 7.—DATA FOR THE DESIGN OF LINK BELT SILENT CHAIN DRIVES

This table is based on standard practice and is to be used for preliminary design only. Consult the makers before final design is adopted.

Pitch of chain	Approx. diam. of sprocket, multiply number of teeth by	Total diam. of chain on wheel, add to wheel diam.	Face of wheel, add to width of chain	Driving wheel				Driven wheel		Minimum advisable centers for general use
				Minimum No. of teeth	Maximum No. of teeth	Max r.p.m. for min. No. of teeth	Chain speed max. r.p.m. and min. No teeth.	Minimum No. of teeth	Maximum No. of teeth	
3/8″	.12	3/8″	1/4″	17	100	2260	1200′	19	150	Diameter of large wheel plus ½ diam. of small wheel
1/2″	.16	1/2″	3/8″	17	100	1835	1300′	19	150	
5/8″	.20	5/8″	1/4″	17	100	1467	1300′	19	150	
3/4″	.24	3/4″	7/8″	17	100	1223	1300′	19	150	
1″	.32	1″	7/8″	17	100	918	1300′	21	150	
1 1/4″	.40	1 1/4″	1″	17	100	791	1400′	21	150	
1 1/2″	.48	1 1/2″	1″	17	100	705	1500′	21	150	
2″	.637	2″	1 3/8″	19	100	474	1500′	23	150	
2 1/2″	.796	2 1/2″	1 1/2″	19	100	378	1500′	23	150	

The retarding moments of band brakes may be obtained from the formula by E. R. DOUGLASS (*Amer. Mach., Dec.* 19, 1901):

$$M=P(K-1)\frac{d}{2} \qquad (a)$$

in which M = retarding moment, lb.-ins.
P = force pulling free end of brake, lbs.,
d = diameter of brake drum, ins.,
K = a factor such that
com. log K = .00758 fc,
f = coefficient of friction,
c = angle of drum embraced by band, degrees,
all as shown in Fig. 1.

The value of M may be found from Fig. 2, which is plotted for $P=1$ and $d=10$. To use the chart find M for the arc of contact and the assumed value of f and multiply the result by the value of P and by the ratio of $\frac{d}{10}$. The plotted values of f are:

Cork inserts on metals,	f = about .33
Leather on metals,	f = about .4
Leather on wood,	f = about .3
Metals on wood,	f = about .2
Metals on metals,	f = about .2

The pulling force at the free end of the strap being known, that at the fixed end may be found from Fig. 4.

The width of the brake band may be found from the following formulas, adapted from those by R. A. GREENE (*Amer. Mach., Oct.* 8, 1908) which give the practice of the Browning Engineering Co.,

$$F=\frac{4MX}{d^2S} \qquad (b)$$

in which F = width of drum face, ins.,
M = retarding moment as found from (a) or Fig. 2,
X = a factor from Table 1,
d = diameter of drum, ins.,
S = a limiting factor which should not exceed 65.

Assume a brake drum of diameter $d=30$ ins., a pulling force $P=1500$ lbs., an arc of contact of 260 deg. and a metal strap on a metal drum. From Fig. 2 we find, for a pulling force of 1 lb. and a drum diameter of 10 ins., a value of M of 7.3, giving for the actual case

$$M=7.3\times1500\times\frac{30}{10}$$
$$=32{,}850 \text{ lb.-ins.}$$

From Table 1 we find the value of X to be 1.68 and hence from (b)

$$F=\frac{4\times32850\times1.68}{900\times65}$$
$$=3.8 \text{ ins. or, say, 4 ins.}$$

Next we check against the speed by the formula:

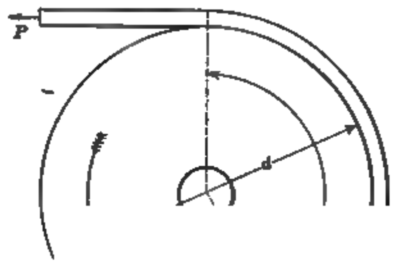
$$R=S\times d\times.262\times\text{r.p.m.} \qquad (c)$$

in which R = a factor which should not exceed 54,000 according to Yale and Towne practice, or 60,000 according to Brown Hoist practice. If r.p.m. = 100 we find:

$$R=65\times30\times.262\times100$$
$$=51{,}090$$

which, being below the limiting value, we conclude that the brake will answer although near the limit. Had the value of R gone above the limit it would have been necessary to assume a wider drum and by substitution in (b) found a smaller value of S which, substituted in (c), would have brought the value of R below the limit.

The differential hand brake, Fig. 3, is more used in Europe than in the United States, where some attempts to use it have met with failure because its principle was not correctly grasped. In Fig. 3, a represents the weight to be lifted by the rope drum b. At c is the brake drum, the ends of the brake strap beng at d and e. The direction of motion when lowering the load being that shown by the arrow,



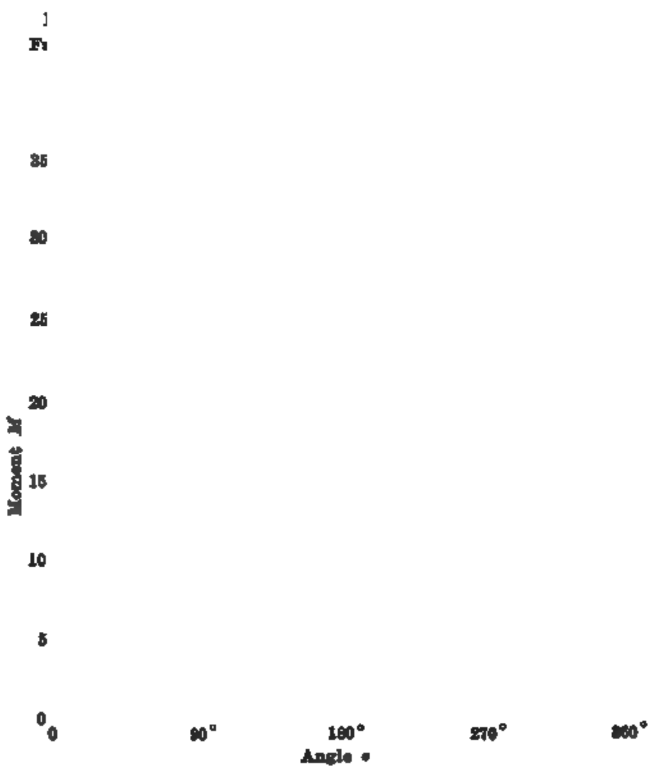


FIG. 2.—The retarding moment of band brakes.

it follows that the end d of the strap carries the load, this end being commonly attached to the frame and the application of the brake being made by tightening e, which is attached for that purpose to the end f of a bell crank which is operated by the brake lever g. With the differential brake, however, the end d is attached to an additional arm h of the bell crank, the action being that the tension in d tends to

tighten e and thus apply the brake, and it is on the ratio between the arms f and h that the design hinges.

The two ends of the brake strap are under the same conditions as the slack and the tight sides of a belt. It is obvious that if the strain on d be, for example, twice that on e, and if the length of h be slightly more than half that of f, the brake will apply itself when the drum is released from the engine which hoisted the load, and if the load is to be lowered at all the brake must be released by hand. On the other hand, if the length of h be slightly less than half of f the brake will not apply itself, the action of the tension on d serving merely to reduce the pressure which must be applied to g in order to hold the load.

TABLE 1.—FACTORS FOR THE WIDTH OF BAND BRAKES

Degrees	X		
	$f = .2$	$f = .3$	$f = .4$
180	2.14	1.64	1.40
195	2.03	1.56	1.35
210	1.93	1.50	1.30
240	1.76	1.40	1.23
250	1.72	1.37	1.21
260	1.68	1.35	1.19
270	1.64	1.32	1.18
280	1.60	1.30	1.17
290	1.57	1.28	1.15
300	1.54	1.26	1.14

TABLE 2.—COEFFICIENTS FOR DIFFERENTIAL BAND BRAKES

Fraction of circumference embraced by brake strap	Value of coefficient of friction			
	.18	.28	.33	.47
	Ratio between arms			
.5	1.76	2.41	2.82	4.38
.6	1.97	2.87	3.47	5.88
.7	2.21	3.43	4.27	7.90
.8	2.47	4.09	5.26	10.62

Table 2 is the work of H. A. Vezin and represents the practice of the F. M. Davis Iron Works Co. (*Amer. Mach., Nov.* 23, 1905). It gives the ratios which the arms f and h must bear to each other in order to give the limiting condition between those described; that is, in order that the brake may be self applying, the arm h must be slightly longer, and in order that it may need help applied to lever g, it must be slightly shorter than the figures given by the table.

For coefficients of friction other than those used by Mr. Vezin, Mr. Brown's chart for the ratio of the tensions, Fig. 4, may be used, as it gives the same results as Mr. Vezin's table.

Certain band-brake calculations are more readily made with the aid of Fig. 4, by JAS. A. BROWN (*Amer. Mach., Apr.* 19, 1906), than with Fig. 2. The relation of the tensions in the two ends of the strap is given by the formula:

$$\log. K = \log. \frac{T_2}{T_1} = 2.729\, fn.$$

in which T_1 = lesser tension,
T_2 = greater tension,
f = coefficient of friction
n = fractional part of drum embraced by strap.

The value of $K = \frac{T_2}{T_1}$ may be obtained directly from Fig. 4. Find the product of $f \times n$ on the right-hand vertical, trace horizontally to the curve and then down and read the value of K. For example, with a coefficient of friction of .33 and one-half the circumference in contact, the product is .165, giving a value for K of 2.82. This chart is also useful in belt and other calculations relating to wrapping friction.

The retarding moment of block brakes, Fig. 5, after wear has brought about uniform contact, may be found from the formula (E. R. DOUGLASS, *Amer. Mach., Dec.* 26, 1901):

$$M = \tfrac{1}{2} JQdf$$

in which M = retarding moment for one block, lb.-ins.,
Q = force pressing block on drum, lbs.,
d = diameter of brake drum, ins.,
f = coefficient of friction

$$J = \frac{1}{1 - \frac{1}{3}\sin^2\left(\frac{b}{2}\right)}$$

b being the angle subtended by one block, degrees. For more than one block multiply by the number of blocks. Values of J may be taken directly from Fig. 5. Values of f have been given in the discussion of band brakes.

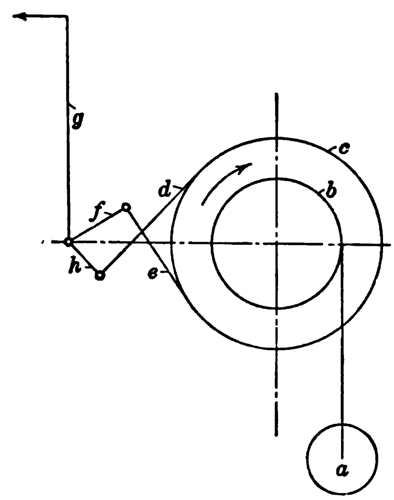


FIG. 3.—The principle of the differential band brake.

The retarding moments of axial brakes, Fig. 6, after wear has taken place, may be found from the formula (E. R. DOUGLASS, *Amer. Mach., Dec.* 26, 1901):

$$M = Xf\frac{d + d'}{4}$$

in which M = retarding moment for one block, lb.-ins.,
X = force pressing block on disk, lbs.,
d = external diameter, ins.,
d' = internal diameter, ins.,
f = coefficient of friction.

For more than one block, as in the Weston multiple disk brake, multiply by the number of friction surfaces in contact. Values of f have been given in the discussion of band brakes.

The surfae cof brake drums should also be sufficient to provide for the dissipation of the heat generated without undue rise of temperature, and this obviously depends on the frequency of the service. No comprehensive study of this subject has been made so far as the author is aware. According to E. R. DOUGLASS (*Amer. Mach., Dec.* 26, 1901) for such brakes as are used on electric cranes, where the work is severe and constant, good results are obtained with a provision of 1 sq. in. of wood or leather frictional surface for every 200 or 250 ft.-lbs. of energy to be absorbed. When the brake is less often called into service these figures may be much exceeded. The brakes of railway cars, which operate under the most favorable conditions for keeping cool, are of metal and are used less frequently, are required in extreme conditions to absorb as much as 20,000 ft.-lbs. per sq. in. of brake shoe. This service tears off and ignites the metal and the shoes must be frequently replaced. According to P. M. HELDT (*Horseless Age, Aug.* 28, 1912), hub brakes of automobiles (pleas-

ure cars) should have 1 sq. in. of surface for each 15 lbs. weight of the car, while on commercial vehicles the ratio should be 1 sq. in. for each 30 lbs. weight of car. Assuming 20 and 10 miles per hour respectively, as the average speeds to be dealt with in stopping the cars, these figures give 240 and 120 ft.-lbs. of energy per sq. in. of surface.

The band brake is not so much used on large hoisting engines as

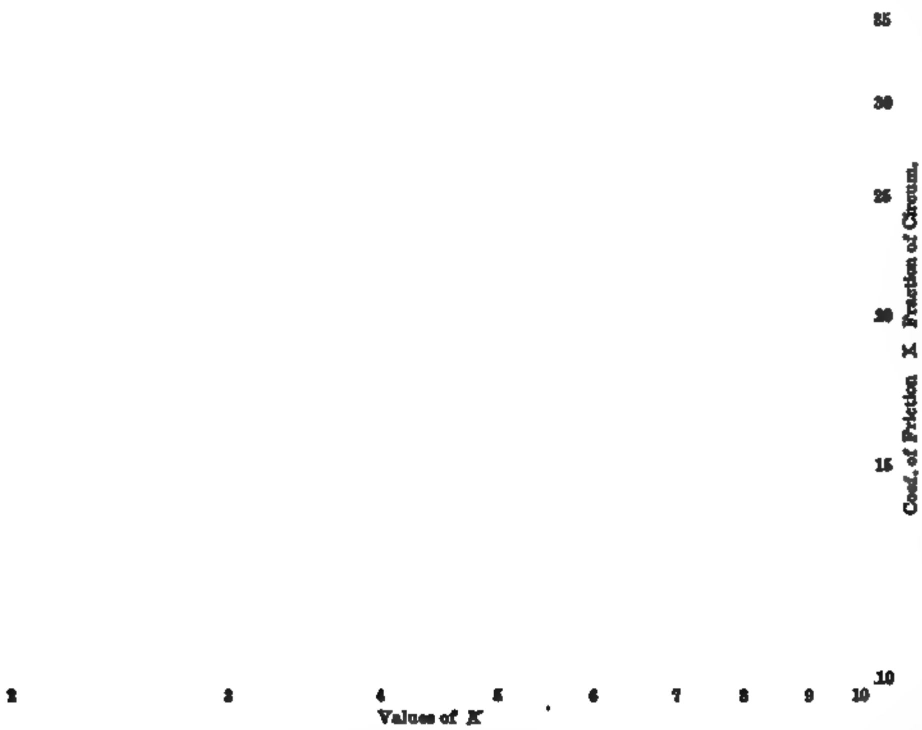


FIG. 4.—The ratio of the tensions in band brakes.

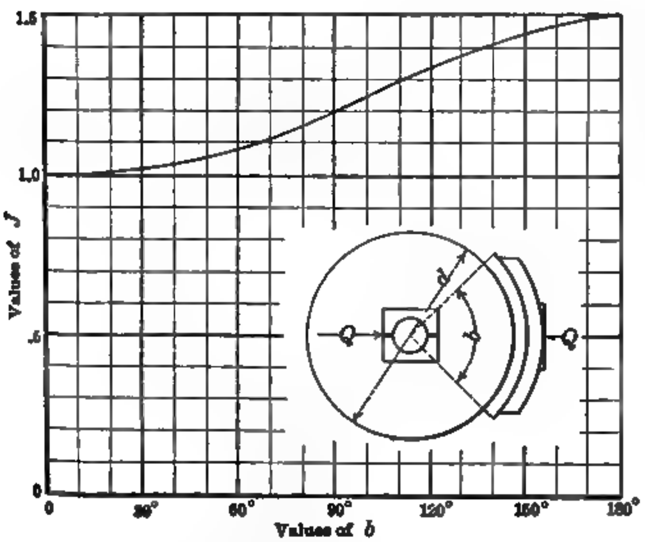


FIG. 5.—The coefficient of block brakes.

formerly, partly because of the fact that it completely surrounds the drum and interferes with the free dissipation of the heat and partly because it does not positively withdraw from the drum when loosened, but consumes power and generates heat during the hoisting movement. The block brake withdraws positively and leaves a large portion of the drum exposed, thus favoring the dissipation of the heat.

A Superior Hoisting Brake

A superior and very large brake is shown, in principle, in Fig. 7. It was applied by the Nordberg Mfg. Co. to the main hoisting engine of the Tamarack Mining Co. (*Amer. Mach., Sept. 21, 1899*), a brake being applied to each end of the hoisting drum.

The brake consists of a pair of jaws AA_1, adapted to grip the brake wheel, the surfaces in contact being basswood and cast-iron. The jaws are supported by a pair of carriers or anchors, BB_1, which, as they have to prevent the jaws from partaking in the rotation of the drum when the brake is applied, have to be securely anchored into the foundation. The jaw A carries a pair of levers CC_1, the short ends of which connect by means of rods DD_1 to end of jaw A, while the long arms connect to a lever E by means of two rods FF_1. E carries a weight G sufficiently heavy to furnish the braking power needed. By a steam device H and lever I the weight can be applied or released. Lever E is held in the jaw A, but as the pins on which rod F connects to lever E are equidistant each side of the fulcrum, there is no reaction due to the forces in rods FF_1 on jaw A. All parts shown on Fig. 10, except steam device and rock shaft for lever I, are in duplicate at the two ends of the drum. In order to secure a parallel motion of the jaws to prevent the lower portion gripping first, the parallel rod J is used. Its action is plain if it is stated that its length is equal to that of the carrier B, making the action of the brake like a parallel motion vise. K, K_1, K_2, K_3, are

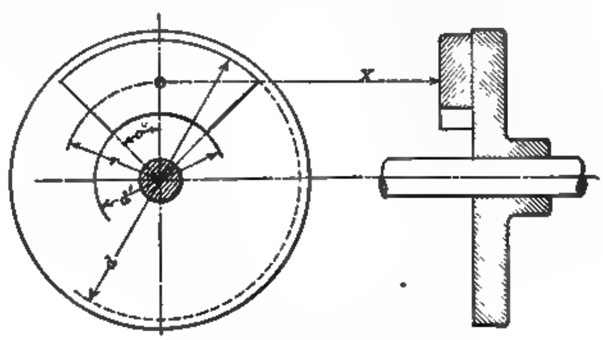


FIG. 6.—Axial brake notation.

set screws to limit the motion of the brake jaws. The steam device H is single acting, the steam releasing while the weight sets the brake. The connection to hand lever is by a floating lever, whereby the piston is caused to follow the motion of the hand lever. It is plain that this type of brake is always ready to go on, even if the steam should fail, and it is thus as reliable as a hand brake.

It is, in fact, more reliable, as it will not allow the engine to be started without steam being first turned on. Accidents have happened from the opposite arrangement.

FIG. 7.—Nordberg arrangement of large block brakes.

Automatic Brakes

One of many constructions of automatic load brakes is shown in Fig. 8, by A. D. WILLIAMS (*Amer. Mach., Aug.* 20, 1903). In action, the tendency of the load to run down locks the brake, which revolves freely in the hoisting direction. When lowering, the motor counteracts this tendency of the load to lock the brake and the load cannot move until its action on the brake is overcome by that due to the motor. The brake absorbs the acceleration due to gravity on the load, so that it drops at a speed determined by the motor.

Fig. 8 shows a brake of the Weston type. *B* is a ratchet ring free to revolve when hoisting, but held by dogs or pawls from turning in the lowering direction. *A* is a retaining ring for holding the pawl ring on the rim of the clutch jaw *K*. Inside of *B*, between *K* and the thrust collar *H*, are eight washers, four of which, marked *F*, are of brass and are free to turn in either direction. The remaining four washers are steel, two, marked *D*, being keyed to the pawl ring *B* by the feather *C*, and two, marked *E*, being keyed to the collar *H* by the feather *G*. The clutch jaw *K* is keyed to the shaft *P* and held in place by the nut *N*. The thrust collar *H* is secured to a flange on the pinion *M*. A clutch jaw to mate with *K* is formed on one end of the pinion *M*, and its bore is threaded to fit the screw cut on the shaft. The enlarged portion of the shaft beyond the pinion is a bearing whose far end takes the thrust due to the screw. There is a bearing beyond the nut *N* also.

The action of this brake is due to the downward pull of the load on the pinion. The shaft being stationary, presses the thrust collar tight against the washers, so that the whole brake is locked from turning in the lowering direction. Any motion in this direction causes the dogs to engage with the ratchet ring, and no further downward motion is possible unless the motor is started to lower. Upon starting the motor to lower it turns shaft *P* and relieves th pressure on the washers, and as soon as the motor overcomes th pressure sufficiently to permit the load to revolve the washers *E*, i will fall. The friction between the washers overcomes any tendenc of the load to accelerate. The chamber in which the washers ar enclosed must be kept flooded with oil, but, nevertheless, considerabl heat is developed. In hoisting the disks are clamped by the screw and the whole brake revolves.

For brass and steel washers the pressure between the surface should not exceed 100 lbs. per sq. in.

When designing a brake of this character (G. F. DODGE, *Amer. Mach., Oct.* 29, 1903) the maximum load is known from the capacity of the crane. Assuming some reasonable values for the outer an inner radii of the disks, within the limits of clearance we have at ou command, we find the average radius of the disks. Dividing th moment of the load by this radius, we obtain the pull in pounds tha the disks must resist at this radius and if we divide this by *N* time the coefficient of friction (which for lubricated surfaces may be take at .05), we obtain the axial pressure necessary to hold the load, *N* representing the number of rubbing surfaces and being assume such that the pressure per square inch of disk is within reasonabl limits. The axial pressure having been determined, it is then lef to find the angle of the helical cam such that it will produce a littl more than this pressure under the influence of the load upon the pinion, the excess being but a trifle more than sufficient to offset the friction between the helical cams. Too small a pitch simply adds to the work done in lowering and a consequent generation of heat.

The Prony Brake

A construction of Prony brake (due to Professor Sweet) which has come into large use, is shown in Figs. 9 and 10. The brake drum is provided with internal flanges about 2 ins. high, forming an annular trough for the water which absorbs the heat. Supply and waste pipes are provided as shown in Fig. 9, the latter having its end

FIG. 8.—The Weston multiple-washer load brake.

.attened to act as a scoop. When in action the centrifugal force auses the water to revolve with the drum.

The location of the tension screw and of the gap in the brake band hould be at the bottom as in Fig. 10, and not at the top as in Fig. 9. E. J. ARMSTRONG, Chf. Engr., Ball Engine Co., *Amer. Mach., Aug.* 19, 900.) Thus located, it is subjected to the initial wrapping tension nly, and is free from the irregular gripping action. This makes it easible to introduce a spring as shown, which, in turn, accomplishes he purpose of the more complex compensating devices of which many have been made. Mr. Armstrong, who has had much experince with Prony brakes, finds no difficulty in maintaining loads of 200 h.p. for indefinite periods and with a greatly improved degree of steadiness.

The area of the brake surface of Prony brakes may be deduced from the experiences of Mr. Armstrong and of the Union Gas Engine Co., which latter company also has a complete outfit of brakes for testing its engines (*Amer. Mach., July* 27, 1905). In both cases the brakes are of the Sweet pattern, Figs. 9 and 10, and the brake blocks are of maple.

One of the Union Gas Engine Co's. brakes having a brake drum 30 ins. diameter by 20 ins. face has been found capable of absorbing continuously 140 h.p. at 350 r.p.m., while, under continuous work, it took fire when loaded with 150 h.p.

Mr. Armstrong has a brake with a drum 48 ins. diameter by 16 ins. face, which absorbs 200 h.p. "without very much trouble from the blocks catching fire. At 225 h.p. it takes fire every minute or two and 260 h.p. is the absolute limit with one man handling a garden hose and devoting himself to putting out the fires." The blocks are kept well greased with tallow.

E. H. Waring has pointed out (*Amer. Mach., Nov.* 30, 1905) that the ultimate capacity of the Prony brake is measured by the capacity of the water to absorb the heat, which is measured by the drum surface alone. When operating below the ultimate capacity, the brake absorbs power in proportion to its speed but, as an increase of speed does not increase the surface in contact with the water, such increase, after the capacity is reached, while adding to the work put into the brake, does not add to its capacity to absorb it.

With Mr. Armstrong's brake 200 h.p. are obviously about equivalent (perhaps a little more than equivalent) to 150 h.p. with the Union Gas Engine Co's. brake. The Armstrong brake has a total drum surface of $\frac{48\times 16\times 3.1416}{144} = 16.755$ sq. ft. which, divided by 200, gives .0838 sq. ft. per h.p. Similarly the Union Gas Engine Co's. brake has a drum surface of $\frac{30\times 20\times 3.1416}{144} = 13.08$ sq. ft. which, divided by 150, gives .0872 sq. ft. per h.p. From these data we may fairly conclude that .09 sq. ft. per h.p. marks the limit where firing is imminent, and this value is further fortified by Mr. Waring's value of .1.

Mr. Armstrong considers that the amount of block surface exerts an influence. His brake has 25 3×16-in. blocks—covering almost exactly one-half the circumference. This figure for the Union Gas Engine Co's. brake is unknown. The concordance of the figures, however, is a clear index of their reliability.

For brakes without water cooling no data are available, but, obviously, the constant should be greatly increased. It is, in fact, practically impossible to absorb much power continuously without water cooling.

Bass, beech, poplar, and maple are found more satisfactory for brake blocks than harder woods.

The Rope Brake

The rope brake has found much favor of late years for small and medium capacities. It is very flexible as regards capacity and is practically free from chattering.

For the area of the drum surface in relation to capacity, the author has no special data. The limiting figures already given for the Prony brake (.09 sq. ft. per h.p.) will serve as a guide, remembering that, as ropes naturally take fire more readily than wood blocks, some increase in the surface provided is advisable.

Additional data for the design of rope brakes are given by PROF. J. C. SMALLWOOD (*Power, May* 28, 1912) as follows:

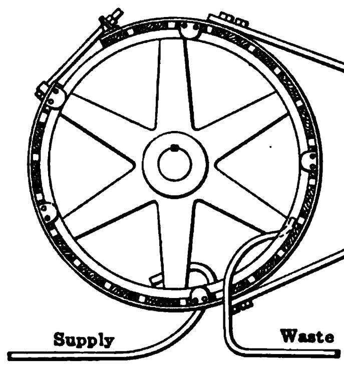


FIG. 9.

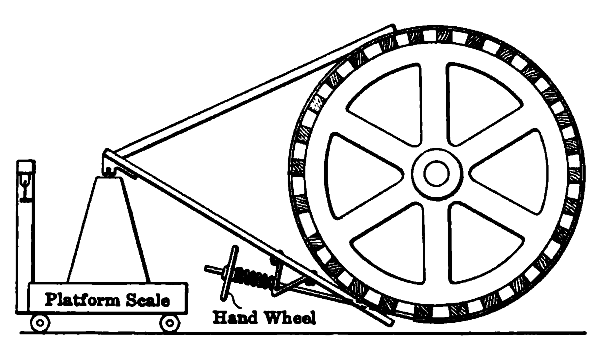


FIG. 10.

FIGS. 9 and 10.—Customary and improved constructions of the Prony brake.

The simplest form of rope brake consists of a rope wrapped around a fly-wheel or pulley on the shaft the power of which is to be measured, as in Fig. 11. The ends of the rope are attached to some stationary apparatus through spring balances for measuring the pull which is created by previously tightening the rope. The difference between the tensions on the two ends is the net force overcome. To vary this force, it is necessary only to change the initial tightness of the rope.

The horse-power is obtained from the formula

$$\text{B.h.p.} = \frac{2\times \text{radius}\times 3.1416\times \text{force}\times \text{r.p.m.}}{33000}$$

or

$$\text{B.h.p.} = .00019\times \text{radius}\times \text{r.p.m.}\times \text{force}$$

Strictly speaking, the radius of the brake should be taken as the radius of the wheel plus the radius of the rope, but, in most cases, the radius of the wheel only is sufficiently accurate.

It is seen that since only the difference between the rope tensions is needed it is not necessary to measure them separately. Separate measurenent, however, allows a form of brake which is easier to make, although it is not so convenient to use.

The form of brake shown in Fig. 11 may be applied to a vertical shaft, and the rope tensions are measured by the spring balances *SS*, the turnbuckle being provided to vary the tensions. This brake is suitable for small torques and high rotative speeds such as yielded by an electric motor or a steam turbine. For large torques, at least one of the spring balances must be replaced by a measuring device having a larger capacity.

The force on the slacker side of the rope is generally small and therefore a spring balance is sufficient to measure it. The use of two spring balances, even with small torques, is objectionable as they are apt to allow bodily motion of the rope. This may result in chattering.

In Fig. 12 one of the balances and the turnbuckle are dispensed with by using dead weights on the rope extremity having the greater

tension. The horse-power is varied by adding or removing these weights. This brake has the advantage than an increase of length of the rope does not affect the brake load since such an increase would be accompanied by a lowering of the weights only, the tensions remaining the same.

The weights should be provided with a stop P, to prevent an accidentally excessive friction from raising or throwing them. The spring balance of Fig. 12 may be replaced by another but smaller set of dead weights. Then, when weight is added to one side, enough should also be added to the other to produce equilibrium. It is an awkward matter, however, to do this nicely, as unavailable subdivisions of weights are at times required.

The construction of Fig. 12 may be modified by using a single heavy weight on the left side and a set of small weights on the right. The heavy weight rests on a platform scales so that the rope tension on this side is the difference between the platform-scale reading and the weight on the scales. The brake load is varied by changing the weights on the right side. This device has been found very satisfactory in that fine regulation may be secured with very uniform resistance. In this case the net force overcome at the rim of the flywheel is the heavy weight minus the sum of the scale reading and the small weights.

In some cases there is not room to hang weights from the fly-wheel, in which case a brake of the type shown in Fig. 13 is applicable. The heavier tension is measured by the platform scales, being transmitted from the rope end by a lever L, having equal arms. The fulcrum is preferably a triangular piece of steel in order to avoid friction, but may be a pin, as shown. The handwheel is used to make fine adjustments of the load by tightening the rope; the turnbuckle may be used for coarse adjustments.

Fig. 14 shows a form of brake in which the difference of tensions is measured directly from a single scale reading. The ends of the rope are attached to the cross pieces CC of a wooden frame which rests on a platform scales. It is important that stops SS be provided to prevent the lifting of the framework, if it is possible for the engine to reverse. It should be noted that the weight of the frame should be carefully determined with the ropes detached, and this weight subtracted from the readings taken during operation.

Generally, if a brake load is to be carried by an engine for any length of time, some provision must be made to withdraw the heat generated by the friction to prevent the rope charring or catching fire. This is usually accomplished by feeding water into the trough formed by internal flanges on the rim of the fly-wheel or pulley to which the brake is attached. Usually, another pipe with a scoop-like entrance is arranged to withdraw the heated water. Very often, however, this is unnecessary, since, if boiling is allowed, the water supply may be adjusted so as to equal the evaporation; that is, evaporation disposes of the water without allowing the rope to get too hot. If the fly-wheel is not too small for the power absorbed, this means of disposal will be found effective.

Air radiation sometimes provides ample cooling when the brake pulley is large in comparison to the power absorbed. This is particularly the case when the brake load is to be applied for a short time only.

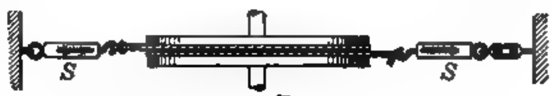


FIG. 11.

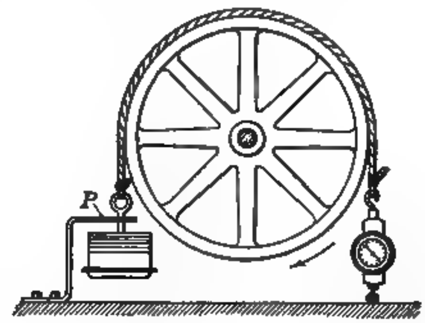


FIG. 12.

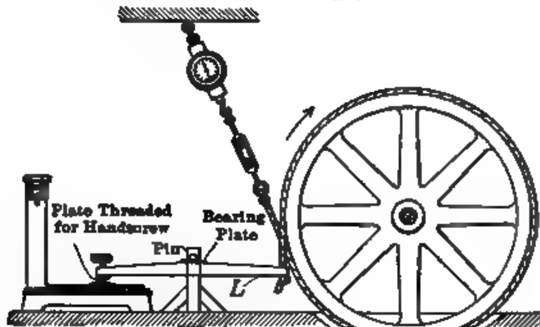


FIG. 13.

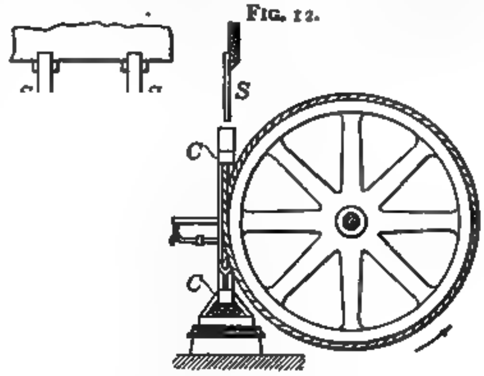


FIG. 14.

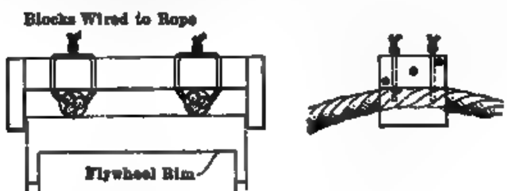


FIG. 15.

FIGS. 11 to 15.—Rope brakes.

Some safeguard is needed generally to prevent the rope from slipping off the pulley. The rope, like a belt, tends to run to the center of a crowned pulley, and where only a single or half turn is used on such a pulley no other provision need be made to keep it on. For large powers, where a number of ropes are necessary, it is well to provide some other means to accomplish this purpose.

An externally flanged pulley will do this simply; if one is not available, blocks like that shown in Fig. 15 may be fitted to the rope. This shows a block suitable to a single turn, or less, of double rope. Enough of these blocks should be fastened to the rope to hold it securely to the wheel.

Except for light powers, it is better to use double rope, as this provides more surface to resist wear without altering the desired relation of the tensions.

It is preferable to attach the device for adjusting the rope tension to that end upon which the tension is smaller, namely, the end which points in the direction of rotation. The preference is made because, the force being less, it requires less effort to change it.

If two spring balances are used the springs should be of different stiffness; otherwise their vibrations are likely to synchronize and a chattering will result.

The design is usually adapted to the size of pulley or fly-wheel at hand. It is first necessary to ascertain if the available pulley is large enough.

To determine the size and number of ropes, there must first be found the net force which the brake must handle; that is, the force which will be indicated upon the scale, or the difference of the forces if two scales are used. To find this:

Multiply the horse-power by 5250 and divide the product by the radius of the wheel in feet and the number of revolutions per minute. The final quotient will be the net tension.

It is well here to emphasize the meanings of the terms *net force* and *rope tensions*. The net force is the effective force overcome by the engine. The rope tensions are the forces existing in the ends of the rope and their difference equals the net force. The tension in the tight end of the rope is therefore greater than the net force, and the rope must be strong enough to carry this tension.

Table 3.—Ratio of Tensions in Rope Brakes. Calculated for a Coefficient of Friction of .4

Number of turns rope	Ratio of greater tension to net force	Ratio of greater to lesser tension
½	1.40	3.51
¾	1.18	6.59
1	1.09	12.3
1¼	1.05	23.2
1½	1.02	43.4

The relative values of the rope tensions depend upon the number of turns around the wheel and the condition of the rubbing surfaces. Table 3 gives quantities that will reduce the calculations for design in the general case.

The data apply to well worn manila rope on smooth pulleys. This table gives the ratios of the brake forces for various numbers of rope turns. From it is seen, for instance, that with a half turn, the greater rope tension is 1.4 times the net force and 3.51 times the lesser tension. It follows that to find the greatest tension resisted by the rope:

Multiply the net force by the figure in the second column of Table 3, corresponding to the number of turns in the first column.

Using this result, a suitable rope to carry the load may be selected from Table 4. This gives the working strength of good manila rope of three strands, a factor of safety of about five being used. The rope is listed according to its largest diameter.

Table 4.—Strength of Ropes

Diameter, ins.	Working strength, lbs.
½	300
⅝	450
¾	700
⅞	1000
1	1300
1⅛	1700
1¼	2100

In selecting the rope provision should be made for the weakening effect of wear.

FRICTION CLUTCHES

Every small and medium-sized planer is an illustration of the perfection of action of a shifting belt acting as a friction clutch when properly proportioned and under loads that are not too great. The shifting belt, when applied to lathe and other machine-tool counter-shafts, is not satisfactory because its speed is too low, leading to the loss of that smartness and promptness of action characteristic of planer belts. Were counter-shaft belts driven at the speeds of planer belts, their action would be just as satisfactory.

The most satisfactory analysis of friction clutches known to the author is that by JOHN EDGAR (*Amer. Mach., June* 29, 1905). Mr. Edgar takes as his design constant the product of the coefficient of friction and the unit pressure between the surfaces and thereby eliminates preliminary assumptions of that most uncertain factor, the value of the coefficient of friction. His constant is, of course, equal to the unit tractive force of the surfaces, this way of looking at it giving a more tangible idea of its meaning. The analysis was originally offered for expanding ring clutches in which an internal split metal ring is expanded against the interior surface of a surrounding metal drum, but it is applicable to nearly all types, materials and duties, provided the constant is obtained from successful clutches of the type and subject to the duty in question. Mr. Edgar's formula is:

$$\text{h.p.} = C\frac{d^2b \times \text{r.p.m.}}{4c120}$$

in which

C = Edgar's constant = coefficient of friction $\times$ radial pressure, lbs. per sq. in. = tractive force of friction surfaces, lbs. per sq. in.,

d = diameter of friction surfaces, ins.,

b = width of friction surfaces, ins.

This formula may be solved for several types of clutches by Fig. 1 by PROF. J. B. PEDDLE (*Amer. Mach., Aug.* 8, 1912) the use of which is as follows:

Join the given horse power with the suitable value of Edgar's constant and note the intersection with axis I; join this intersection with the given r.p.m. and note the intersection with axis II. Any values of diameter and breadth lying in a line passing through the intersection on axis II will transmit the given horse power at the given speed.

The example is for h.p. = 40, Edgar's constant = 50; r.p.m. = 100; giving, for diameter = 10 ins., breadth = 3.23 ins. or, for diameter = 20 ins., breadth = 8 ins.

The values of Edgar's constant given on the chart have been obtained as follows:

For expanding ring clutches, metal on metal, Mr. Edgar compared actual clutches with the formula and found the value of C to range between 50 and 100. Table 1 of dimensions of actual clutches of the same type is supplied by C. L. UTCHER (*Amer. Mach., June* 24, 1909), column 11 giving Mr Edgar's constant, having been added by the author. In all of Mr. Utcher's cases the expanding rings are of cast-iron while the rings into which they expand are of cast-iron or low carbon steel (about 35 points carbon). Mr. Utcher says that "in case 9 clutch fails on very heavy cuts which are quite within the capacity of the machine otherwise" and for this reason the average has also been given with this clutch omitted. The average value thus obtained agrees quite closely with Mr. Edgar's lower value, while the other cases, excluding 9, indicate that his higher value is admissible, especially as Mr. Utcher says, "from careful observation of the condition of the clutches after several years' running, I can assert that, without doubt, any of the clutches is capable of transmitting the full power, as given in column 2, for an indefinitely long period of time."

TABLE 1.—DIMENSIONS OF EXPANDING RING CLUTCHES

Case	H. p. of belt	Maximum h. p. of cut	R. p. m. of of clutch	Diameter of clutch, ins.	Width of clutch, ins.	Surface of clutch, sq. ins.	Surface speed of clutch, ft. per min.	Tangential force of resistance R	Radial force to produce R $F=fR$ $f=.1$	Pressure on surfaces of clutch, lbs. per sq. in.	Edgar's constant from column 2
Columns	1	2	3	4	5	6	7	8	9	10	11
1	8	4	600	3	1	9.5	430	310	3,100	325	32.5
2	8	4	300	5	1	16	390	335	3,350	210	21
3	16	8	400	6	1¼	24	630	420	4,200	180	18
4	24	12	400	7	1¼	28	730	540	5,400	200	20
5	8	4	30	8½	1½	40	67	1,970	19,700	495	49.5
6	8	4	30	9	1¼	35	71	1,860	18,600	525	52.5
7	16	8	25	10	1½	47	65	4,000	40,000	850	85
8	24	12	20	12	1½	57	63	6,300	63,000	1,115	111
9	....	7½	6	16	1¼	63	25	9,900	99,000	1,570	157
Average of all											61
Average omitting 9.											48.7

In Mr. Utcher's clutches the surface of the rings was interrupted by grooves about a quarter of an inch in width cut transversely in order to permit the lubricant to escape. Experiment shows, he says, that this practice increases the driving power of the clutch about 20 per cent. for the same applied pressure. C. W. Hunt (*Trans. A. S. M. E., Vol.* 30) cut such grooves ¼ in. wide by $\frac{1}{16}$ in. deep in increasing numbers and found a progressive improvement in the prompt engagement of the clutch until the grooves were spaced a little more than 1 in. apart.

The diameter of clutches of this type should be large in proportion to the width, and the expansion ring should be stiff enough to prevent its expansion by centrifugal force. The pressure should be applied by some form of toggle or bell crank mechanism by which the pressure increases rapidly at the end of the movement. Thus equipped, Mr. Utcher says that in his clutches "in no case is the span of move-

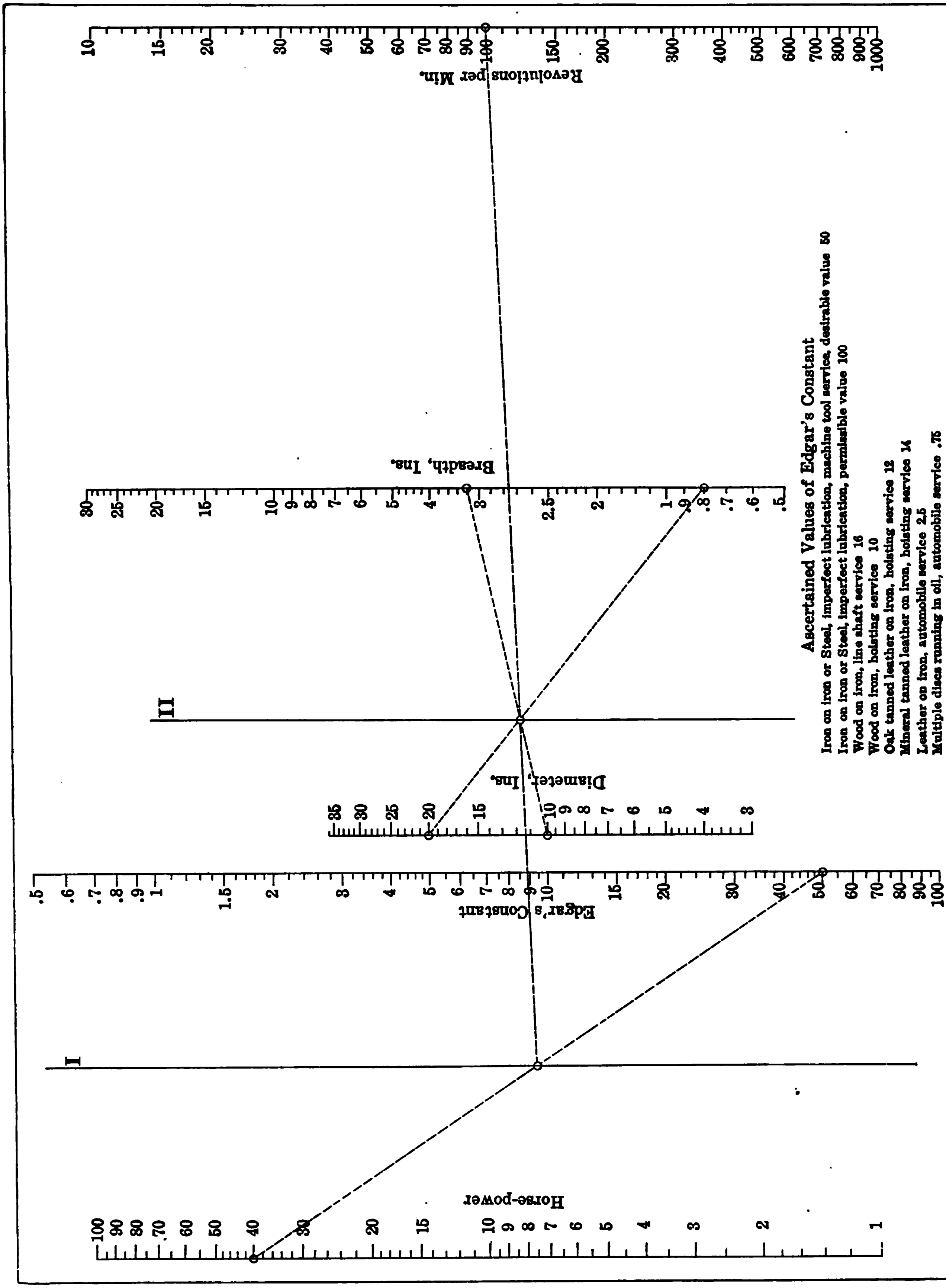


FIG. 1.—Dimensions of friction clutches.

ment more than 15 ins., nor the pressure needed more than can be comfortably applied by one hand."

Within the limits of uncertainty of the value of the coefficient of friction, the value of the tangential force required to push apart the ends of the expanding ring may be obtained by the relation that exists between radial and tangential forces, as in steam boilers, for example. Thus we have

$$\text{separating force, lbs.} = \frac{C\,d\,b}{2\,f}$$

f being the coefficient of friction and the remaining notation as before. For the imperfectly lubricated metal surfaces used in clutches of this type, f may be taken at .1.

For clutches with jaws of wood working with drums of cast-iron and intended for occasional engagement (line shaft and similar service) Henry Souther (*Trans. A. S. M. E., Vol.* 30) gives dimensions of four clutches by the Dodge Mfg. Co. of powers ranging between 25 and 98 h.p. at 100 r.p.m. The wood blocks were of maple and, with the dimensions substituted in Mr. Edgar's formula, the clutches yield values of C of 16.4, 15.9, 14.1, and 16.9 respectively, or an average value of 15.8, for which we may use 16.

When the wood blocks do not embrace the entire circumference, suitable correction must be made for the value of b when substituting in the formula or when using the chart. Thus, if the blocks embrace one-half the circumference, the value to be used for b is one-half the actual width of the blocks, and so for other fractions of the circumference embraced by the blocks.

For the cone clutch it should be remembered when applying the formula, that the pressure factor in C is the normal pressure per sq. in., and that, for d, the mean friction diameter is to be used. The same values of C are applicable for the same materials and services.

For iron on iron surfaces, values of C have been given and those for other surfaces follow and have been incorporated in Professor Peddle's chart.

For cone clutches having wood on iron surfaces and used under the conditions of frequent service, two hoisting clutches by the Lidgerwood Mfg. Co. were examined by the author and gave values for C of 9.1 and 10.4 respectively, of which the mean is 9.75 or, say, 10.

For leather faced cone clutches with the opposite engaging surface of metal and used under the conditions of frequent service, a hoisting clutch by the C. W. Hunt Co. was examined by the author and, under successful operating conditions, gave, for C, a value of 14. On one occasion this size of clutch was overloaded and the leather facing failed. The value of C for this condition works out at $20\frac{1}{2}$, indicating that, for the materials used, 14 is a safe value. Mineral tanned leather is used by the C. W. Hunt Co. for clutch facings, and is found to be more serviceable than oak tanned leather. For the latter material a smaller value would seem appropriate and, in the absence of other data, we may, for it, take 12 as a safe value.

For leather faced cone clutches with the opposite engaging surface of metal, under the conditions of automobile service, six automobile clutches of medium and moderate powers (4 cylinders, of which the largest was $5 \times 5\frac{1}{2}$ ins.) were examined by the author, calculated not rated horse-powers being used. The resulting values of C were 2.01, 3.32, 2.83, 2.4, 2.85, and 2.26, the average being 2.61 or say 2.5.

For the axial pressure required to engage cone clutches the author has shown (*Amer. Mach., Aug.* 8, 1912) that

$$\text{axial pres.} = \frac{C\pi d b \sin \phi}{f} \text{[1]}$$

ϕ being the angle, degrees, between the axis and the conical surface, and this formula is true for any material and any service, suitable values of C and f being used. The following values of f are fair average values. Iron on iron dry .2; iron on iron imperfectly lubricated .1; wood on iron, .2, leather on iron, .3; cork inserts on iron, 33.

Professor Peddle's second chart, Fig. 2, may be used in place of this formula.

The values of the axial pressure obtained from the formula or chart being those which will just drive, some surplus should be added.

The angle of the cone is of importance as regards freedom of disengagement. For metal on metal surfaces, the dividing angle between sticking and non-sticking is about 6 or 7 deg. measured between the axis and the conical surface, according to A. J. Shaw (*Amer. Mach., June* 11, 1887). For free disengagement, the angle should be not less than 10 deg. For wood on metal surfaces the angle should not be less than 20 deg. (C. W. Hunt, *Trans., A. S. M. E., Vol.* 30). A common angle for the leather and metal surfaces of automobile clutches is $12\frac{1}{2}$ deg., but such clutches are always held in engagement by a spring and disengaged by foot pressure. According to C. W. Hunt (*Trans. A. S. M. E., Vol.* 30), leather faced cone clutches with angles of 18 to 20 deg. are fitted with an operating device for disengagement.

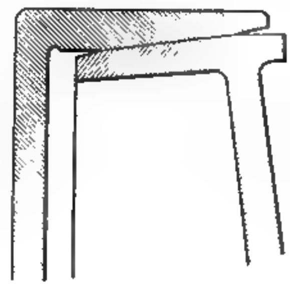


Fig. 3.—Incorrect construction of cone clutch.

Fig. 4.—Correct construction of cone clutch.

A defective construction of cone clutch is illustrated in Fig. 3 which shows the effect of wear. The correct construction is shown in Fig. 4. The extension of the male and female ends are at an equal angle from the acting surface, the result being an avoidance of the shoulder and an increase of surface as the parts wear.

Cone clutches should have holes provided for the escape of air from between the cones.

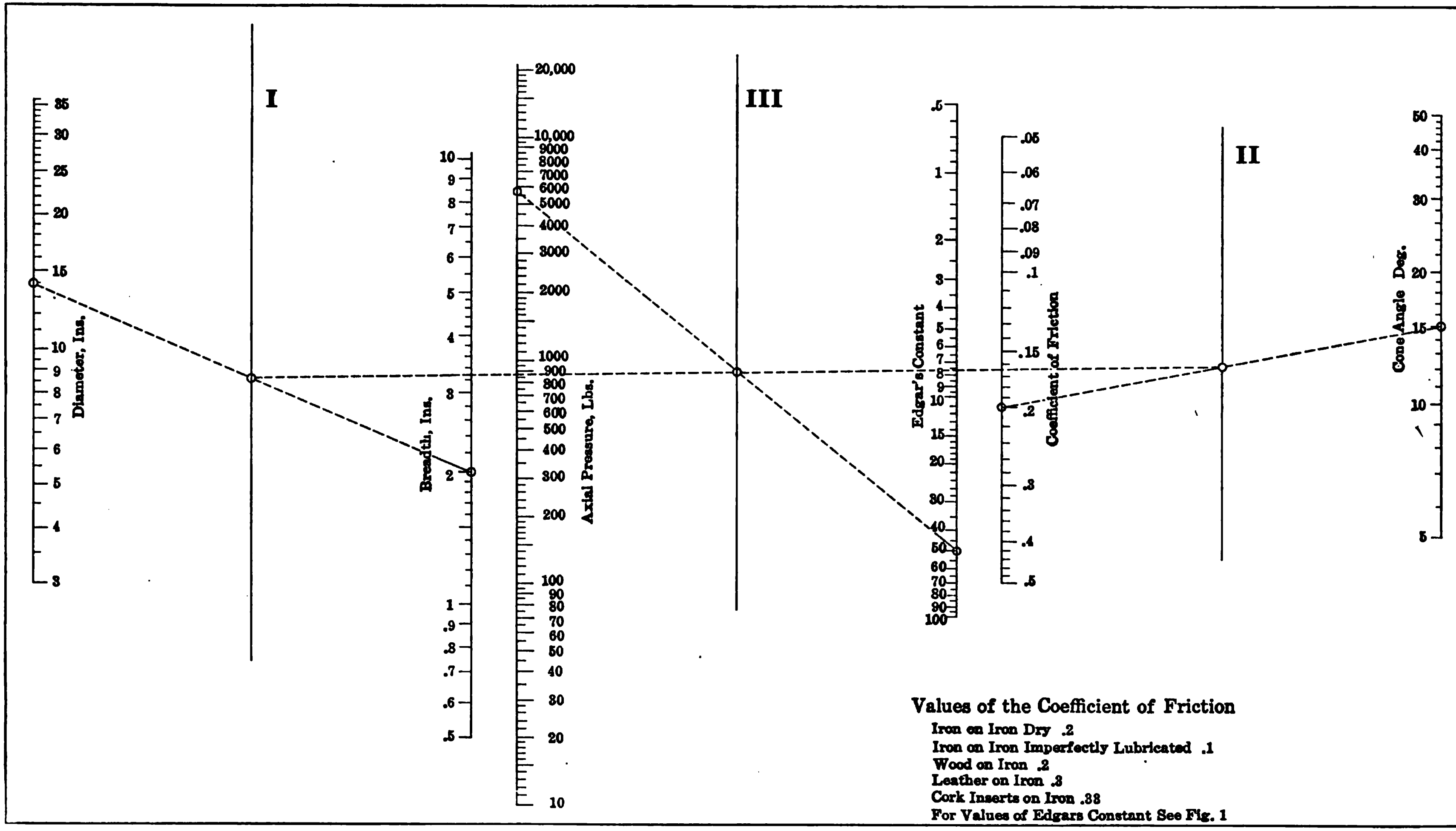
For Weston (multiple disk) automobile clutches running in oil, four clutches were examined by the author, three having six and one four cylinders, without finding any difference of practice characteristic of the number of cylinders used. The highest powered car examined had six $4\frac{3}{4} \times 5\frac{1}{2}$ in. cylinders. The clutches had from 31 to 51 disks, of outside diameters ranging between 8 and 12 ins., with face widths of $\frac{3}{4}$ to 1 in.

In this type of clutch the mean diameter of the friction surfaces is to be treated as d in the formula, while for b the actual radial width multiplied by the number of rubbing contacts, that is, the number of disks—not twice the number of disks—is to be used. The width given by the chart is this product, which is to be divided by the number of rubbing contacts to obtain the width of the disks. Should large clutches run beyond the scale of the chart, the horse-power may be divided by 2 and the resulting number of rubbing contacts be multiplied by 2. As these clutches run in a bath of oil a low value of C is to be expected.

The resulting values of C were .888, 1.09, .565, and .565, the average being .777 or, say, .75. The highest value was found for the highest powered car and one of the lowest values for the lowest powered car, although the second lowest value was found for the next to the highest powered car.

Multiple disk clutches should have narrow rubbing surfaces—preferably not over $\frac{1}{16}$ the diameter, though this ratio is, in some

[1] The author is convinced that the formula in which a factor, the coefficient of friction times the cosine of the angle, is introduced to provide for overcoming the endwise sliding friction of the cones on each other, is erroneous. It is a matter of common experience that a shaft, when in motion in its bearings, may be traversed endwise by a force so small as to be negligible, and it would seem that the same action takes place in a clutch.



Join diameter with breadth and note intersection on axis I; join the suitable coefficient of friction with the angle between side and axis of cone and note intersection on axis II; join intersections on axis I and II and note intersection on axis III. A line through this intersection and the suitable value of Edgar's constant will give the axial thrust.

In the example solved, diameter = 14 ins., breadth = 2 ins., cone angle = 15 deg., coefficient of friction = .20, Edgar's constant = 50; giving axial thrust = 5700 lbs.

FIG. 2.—Axial thrust on cone clutches.

clutches, as low as $\frac{1}{16}$. With wide surfaces the unsatisfactory action of step bearings is introduced. Moreover, with wide surfaces, the bite of the central feather and the uncertainty of disengagement are increased. For both reasons the results are more satisfactory if the required surface is obtained by increasing the number of disks instead of the width of the surfaces.

The Scotch coil clutch, Fig. 5, which has been used in Great Britain for transmitting 8000 horse-power, requires a different analysis from other types. H. H. NACHMAN (*Amer. Mach., Apr.* 1, 1909) discusses the construction as follows: The clutch consists of a steel

FIG. 5.—Scotch coil clutch applied to a hoisting drum.

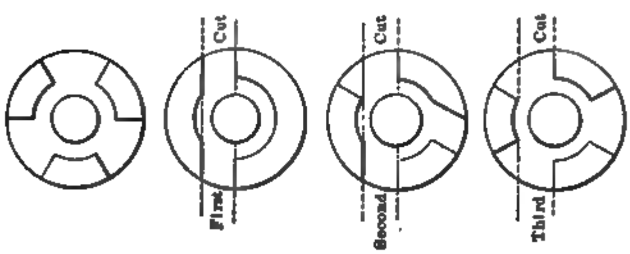


FIG. 6. FIG. 7. FIG. 8. FIG. 9.

FIGS. 6 to 9.—Milling a claw clutch with an odd number of teeth.

coil wound on a chilled cast-iron drum. At each end of the coil a head is formed. The head at the large end is attached to the pulley or shaft that is to be set in motion, while that at the smaller end of the coil serves as a point of application of a force, which produces an initial pull on the coil to wind it on the drum, thus gripping it firmly. The motion of the drum is thus transmitted to whatever is attached to the large end of the coil.

The friction of the coil on the drum is the same as that of a rope or belt on a pulley. That is, the relation of tension at the small end of the coil to that at the large end may be found from the equation

$$\frac{P}{Q}=e^{f\alpha},$$

in which

P = pull at large end of coil,
Q = pull at small end of coil,
e = base of natural logarithms = 2.718,
f = coefficient of friction between coils and drum,
α = angle subtended by coil in radian measure,
= 6.283 for each turn of coil.

The table below gives values of $\frac{P}{Q}$ for various values of α on the assumption that f for steel on cast-iron, lubricated, is .05.

α	$\frac{P}{Q}$	α	$\frac{P}{Q}$
1 turn	1.37	5 turns	4.81
2 turns	1.87	6 turns	6.58
3 turns	2.57	7 turns	8.60
4 turns	3.51	8 turns	12.33

If D is the diameter of the drum in ins., and N the r.p.m., then

$$\text{h. p.}=\frac{\pi DNP}{33000\times 12}=.00000793\ DNP.$$

As an example let it be required to find the horse-power capable of being transmitted by a clutch in which a coil of six turns is wound upon a drum 24 ins. in diameter The shaft makes 200 r.p.m. and a pull of 500 lbs. is applied at the small end of the coil.

From the table we find

$$P=6.58\ Q=6.58\times 500=3290 \text{ lbs.}$$

and

$$\text{h. p.}=.00000793\times 24\times 200\times 3290=125.$$

This type of clutch will transmit motion in one direction only.

The Lane friction clutch, which is in wide use in the United States on mine hoists employs the same principle of wrapping friction. The strap, however, goes but once (effectively a little less than once) around the drum and is lined with wood blocks. The relation of the tensions on the two ends of the strap may be obtained from Mr Brown's chart, Fig. 4, of the section on brakes, which see, using a coefficient of friction of .2 for wood on iron.

The contracting band clutch used on some automobiles is subject to the same analysis as the Lane clutch, from which it differs only in materials and dimensions, provided the coefficient of friction be known.

Claw clutches are more cheaply made if they have an odd number of teeth (PROFESSOR SWEET, *Amer. Mach., Mar.* 17, 1910). To mill a clutch with an even number of teeth it is necessary to set the milling machine twice: first to cut one side of the teeth and then the other; but to cut an odd number of teeth it is necessary to have only a plain mill, thick enough so that twice through will cut the wide part of the gap, and thin enough so that it will pass through the small part. This will be best understood by referring to Figs. 6, 7, 8 and 9.

Fig. 6 shows a finished three-tooth clutch; Fig. 7 a single cut with a proper thickness milling cutter set with one side exactly central; Fig. 8 the second cut, which finishes one tooth; and Fig. 9, the third cut, which finishes the other two teeth, completing the job.

A clutch with any odd number of teeth can be finished in the same way, and if one side of the cutter is exactly central the clutch will be a mechanical fit.

CAMS

The following methods for laying out cams, except when otherwise specified, are those of C. F. SMITH (*Amer. Mach*, 1905)

The usual motion given by a cam, when the cam roller is mounted on a radius arm, is that given by a crank and connecting rod to a cross head, the length of the radius arm being the equivalent of that of the connecting rod When the roller is mounted on a slide the motion becomes that of a crank and slotted cross head (Scotch yoke).

For high speed cams this motion requires modification as will be explained later.

The use of a spring for effecting the motion in one direction is sometimes desirable. In some cases the spring performs the return, while in others it performs the operating movement. When a spring performs the return movement, *the failure of the spring to act* leaves its driven part in the operating position and a wreck may be the result, while, if the spring performs the operating movement, a failure to act leaves the driven part withdrawn from the operating position and serious consequences are less probable. A conspicuous illustration of springs performing the operating movement is found in the linotype, adoption of the plan being due to considerations of safety.

An objectionable feature of springs, especially for the return movement, is that the effort of extending or compressing them is added to the effort required to do the work. Except that the milling cutter must be kept to size, there is no greater difficulty in making a cam effect both movements than one.

Omitting consideration of unusual cases, *cams are of two types* drum or barrel and face or radial, of which the former have the advantage that they are of smaller diameter for a given angle of cam groove, and the latter that their action on the roller is more perfect.

Laying Out Drum Cams

To lay out a drum cam operating a roller mounted on a slide, proceed as in Fig. 1, in which the rectangle o, B, C, 12 represents the development of that part of the cam's periphery in which the movement is to take place. The movement of the roller is shown full size by the line o B, and this movement is to be performed while the cam turns through an arc of which BC is the development. Upon A o equal and parallel to B o, a semicircle, called the throw-circle, is drawn equal in diameter to the movement of the roller and this semicircle is divided into any number of equal parts—in this case 12. The developed arc BC is then divided into the same number of equal parts and projecting lines from the points on the semicircle give by their intersections with the corresponding verticals, points on the center line of the cam groove as indicated by the small circles. While the cam turns from B to C the movement of the roller is precisely the same as would be given by a crank turning through the semicircle o, 6, 12. The return movement may or may not be performed in the same interval of time, but its groove will be laid out in the same way. Should the movements be performed in the same time the movement of the roller is precisely the same as that of a cross head, but with a pause at each end of the stroke, and, should they be performed in different times, the same will be true except that one movement will be made more quickly than the other.

The angle of the tangent with the center line of the groove should not exceed about 30 degrees, as indicated in Fig. 1.[1] In laying out the drawing the distances oB and BC are given as has been stated, and it is desirable to avoid the necessity of laying out the curve in order to determine this angle, and this determination may be made in several ways. Thus with the angle equal to 30 degrees, there is a constant ratio of 2.72 between the length of the lines Bo and BC. Bo being given by the conditions of the problem, it is only necessary to multiply it by 2.72 in order to obtain the length which BC must have in order to make the final angle 30 degrees. The length of BC, considered as a fraction of the entire circumference, is also known from the conditions, and from this the entire circumference and *the diameter of the cam may be quickly determined.* Thus, BC being 2.72 times the throw and occupying say n degrees of the circumference, we have:

$$\text{circumference} = 2.72 \times \text{throw} \times \frac{360}{n}$$

or

$$\text{diameter} = \frac{2.72 \times \text{throw} \times 360}{3.1416 \times n}$$
$$= \frac{312 \times \text{throw}}{n}, \text{ approximately.}$$

The length of BC may also be determined with a protractor from the fact that, with the angle of the tangent to the cam curve equal to 30 deg., the angle Co 12 will be equal to 20 deg. 12 min. or, for practical purposes, 20 deg. This ratio of 2.72, and this angle of 20 degrees, are strictly correct for drum cams, of which the rollers move in straight lines only, but they may be used for cams of which the rollers are guided by radius arms without important error. For face cams these constants are modified, as will be explained later.

It will be observed also that near the point of tangency the curve and tangent coincide very closely and we may take advantage of this to obtain *a graphical method which is accurate enough.* Having divided the end circle, lay out the triangle abc, the height being projected from the throw circle and the hypothenuse being drawn at 30 degrees, when the base gives the length of one of the divisions of the base line. The same result may, of course, be obtained from the triangle cde. The chief use of the constants is in the preliminary layout of the chart, as will be explained later. When laying out the curves on the individual cam drawings the graphical method is preferable.

If the roller is supported, by a radius arm, as it usually is, the procedure is slightly modified as shown in Fig. 2. The base line and throw circle are divided as before, but instead of drawing perpendiculars from the base line divisions, arcs of circles are struck through them with a radius equal to that of the proposed radius arm, as indicated by the line ab and centers c, d, e, etc., and the intersections of these arcs with the projection lines from the divisions of the throw circle form the locating points for the center line of the cam groove as indicated by the small circles. The angle of this line at its middle should, as in the former case, be not greater than about 30 degrees, as shown, and is predetermined as in Fig. 1.

Laying Out Face Cams

When the groove is upon the side or face of the cam disk the procedure is shown in Figs. 3 and 4. Fig. 3 corresponds with Fig. 1

[1] Some designers increase this figure, going even to 60 degrees for light work. The monotype—a conspicuous example of high class cam construction—employs angles as high as 37 degrees.

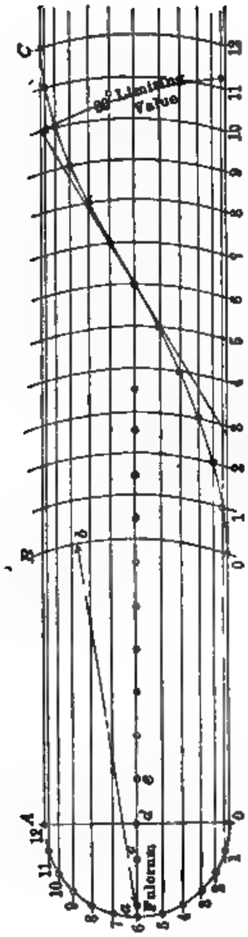


FIG. 2.—Drum cam, crank motion, for operating a pivoted lever.

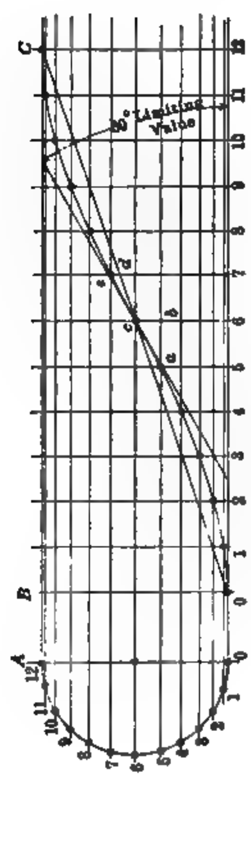


FIG. 1.—Drum cam, crank motion, for operating a slide.

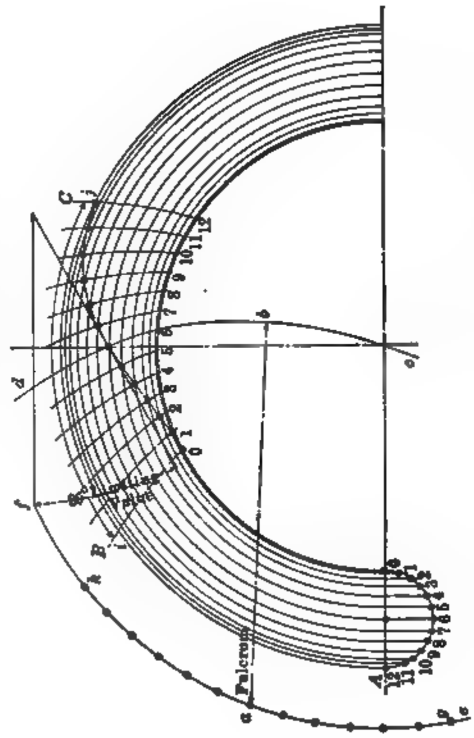


FIG. 4.—Face cam, crank motion, for operating a pivoted lever.

FIG. 3.—Face cam, crank motion, for operating a slide.

FIGS. 1 to 4.—Laying out the center lines of cam grooves.

in that the roller is mounted upon a slide which moves radially with the cam disk. The cam is here required to move the roller the distance oB while turning through the angle BC. The throw circle 0, 6, 12 is drawn with a diameter equal to the radial movement of the roller and is divided as before into equal parts. The cam angle is similarly divided and radius lines 0, 1, 2, 3, etc., are drawn. The division points of the circle are projected to its diameter and arcs struck from the center of the cam and from the feet of the projection lines give by their intersection with the radial lines the points of the center line of the cam groove; the limiting angle appearing as before. In determining this angle for this style of cam the lower triangle *abc* should be used in preference to the upper one *cde*, as the tangency is closer below the middle point than above it. As with the drum cam this angle is regulated by varying the diameter of the cam disk.

In laying down this style of cam upon the chart the figure B, 0, 12, C must, of course, be represented by a rectangle of which the length is properly made equal to the outer arc BC. The constants which have been given for drum cams become for this construction 3.23 and 17 deg. 15 min., or, for practical purposes, 17 deg.; that is, the length of the rectangle should be at least 3.23 times its height and the angle of its diagonal with its base should not exceed 17 deg., showing that face cams must be larger than the drum style in order to have an equally favorable angle.

While the constants given above apply to face cams, the extreme radius of a face cam groove is not much less than the radius of the piece, there being but a wall of metal and the radius of the roller between, and the convenience of a uniform outside diameter of cams is so great that this may be neglected, the curves of the chart being laid out as though the cams were all to be of the drum style and then make both face and drum cams of the same outside diameter. The face cam grooves will thus be slightly steeper than if they were of the drum style, but in only a few, and probably in no case, will the angle extend 30 deg. Were this angle to be materially exceeded in one of the face cams, especially in an important one doing heavy work, the whole set should be redesigned and enlarged.

In Fig. 4 the roller is carried by a radius arm of a length *ab*. An arc *cd* is so struck as to pass through the center of the cam shaft if possible. Sometimes this is impossible, but the practice should be departed from only in case of necessity. The center *a* located, the arc *ef* is struck from the center of the cam disk and from centers located on it and with a radius equal to the length of the arm the arcs Bo and $C12$ are struck, such that the arc *ij* is the angle of cam movement during which the roller movement is to take place. The arc *ef* between the extreme centers *gh* of the arcs Bo and $C12$ is then divided as in previous cases and arcs 0, 1, 2, 3, etc., are drawn from the division points as centers. The throw circle 0, 6, 12 is then drawn and the cam curve is quickly located. The limiting angle is again shown.

Two-step Cams

Face cams giving a movement in two steps are laid

out as in Fig. 5. The driven crank arm is centered at *A*, its extreme positions being *B* and *C* with an intermediate dwell at *D*. The first movement is from *B* to *D* and the second from *D* to *C*. The cam roller is carried by an arm pivoted to the end of the crank arm and having a forked end which straddles a square guide block which rides on the cam shaft.

The extreme throw is laid out on the horizontal center line and the lines *B* and *C* are drawn from *A* to give the same movement on each side of the vertical center line. The arc *EF* is drawn and the position *D* of the arm at the intermediate dwell is laid down. Chords *GH* are drawn and on them throw circles are drawn each of which is divided into parts as before, the division points being projected to the arc *EF*. The extreme positions of the arm which carries the roller are drawn in at *I* and *J*. The extreme positions of the roller are at *K* and *L*, and an arc *KL* tangent to *J* will give approximately the path of the roller which may thus be treated as though pivoted at the center *M* of the arc *KL*, which does not conform to the recommendation that it should pass through the center of the cam shaft, as it is impossible to make it so conform. With radius *FK* and centers at the points of the arc *EF* projected from the division points of the throw circle on chord *G*, the positions of the circles 0, 1, 2, 3, etc., from the cam-shaft center are obtained and they are drawn. Points *P*, *Q* are found on the arc *MN* from which to strike arcs 0_a and 12_a to include the angle of movement of the cam within which the first movement of the roller is to take place, and the arc *PQ* is then divided as before and arcs 1_a, 2_a, 3_a, etc., are drawn, the intersections of which with the correspondingly numbered circles drawn from the cam-shaft center give the outline of the center line of the cam for th movement. The required length of dwell *RS*, for which th curve is of course a circle, is then laid out and the profile f second movement is then determined in the same way fro throw circle drawn on chord *H*, but of this details need not be The limiting angle of 30 deg. appears in both cases.

The return movement, not shown, is laid out from a third circle of which the diameter is equal to the sum of those already used.

The above methods are sufficient to lay out any single-roller drum or disk cam of the types illustrated of which the time and extent of the movement only are determined beforehand.

FIG. 5.—Face cam, crank motion, for a double-throw fork lever.

An Example of a Face Cam

The layout of an actual face cam is shown in Fig. 6, a portion of the machine frame being also shown. The cam is to move a slide *a* by means of a fulcrumed lever *b*, the position of the roller center being at *c* on an arc which is laid out to pass through the center of the cam shaft, as already advised in connection with Fig. 4.

The diameter of the cam disk being known or assumed, we draw the outer circle of the cam and lay down the thickness of the outer retaining wall, and the radius of the roller, and draw the arc *no*. The radial movement of the roller *x* is laid down as shown giving the arc *pq* as the inner limit of movement of the roller center The throw circle is next drawn in and divided as has been explained and arcs from the feet of the perpendiculars from the division points are drawn. The arc through the center of the throw circle gives, by its intersection with the arc struck from the fulcrum *h* of the lever as a center, the location of the mid-position *c* of the roller. In the present case this happens also to be the middle of the return curve of the cam, but this is accidental.

Assuming or knowing that the working movement of the roller must be made during an angle α of movement of the cam, we draw an arc *de* from the center of the cam shaft as a center and passing through the center *h* of the lever fulcrum, and take its radius *hi* in the tram and find centers *f*, *g* from which to strike arcs *jk* and *lm* passing through the cam shaft center and spanning the angle α as shown. These arcs, by their intersections with *no* and *pq*, determine the positions of the cam curve for beginning and ending

the movement. Dividing the arc fg as before, we find points 1, 2, 3, etc., from which to strike arcs 1, 2, 3, etc., which, by their intersections with the arcs from the feet of the perpendiculars through the divisions of the throw circle, give the successive positions of the roller center. From these centers circles having a radius equal to that of the roller define the cam groove. The return movement is laid out in the same way and from the same stroke circle, and is thus the same as the acting movement, although the two cam curves are very different. Those portions of the groove which represent dwells

The surface of the zinc may be blackened for the purpose (Wm. V. Lowe, *Amer. Mach., Feb.* 27, 1908) by the use of a solution of four ounces of sulphate of copper in one pint of water to which about 10 drops of nitric acid have been added. Clean any oil from the zinc before coating, then pour the solution over it and distribute it with a piece of waste. The color is governed by the nitric acid. Add acid until the color is right. After blacking rub the surface with an oily rag. This makes the color a more intense black. It should be dead, without luster.

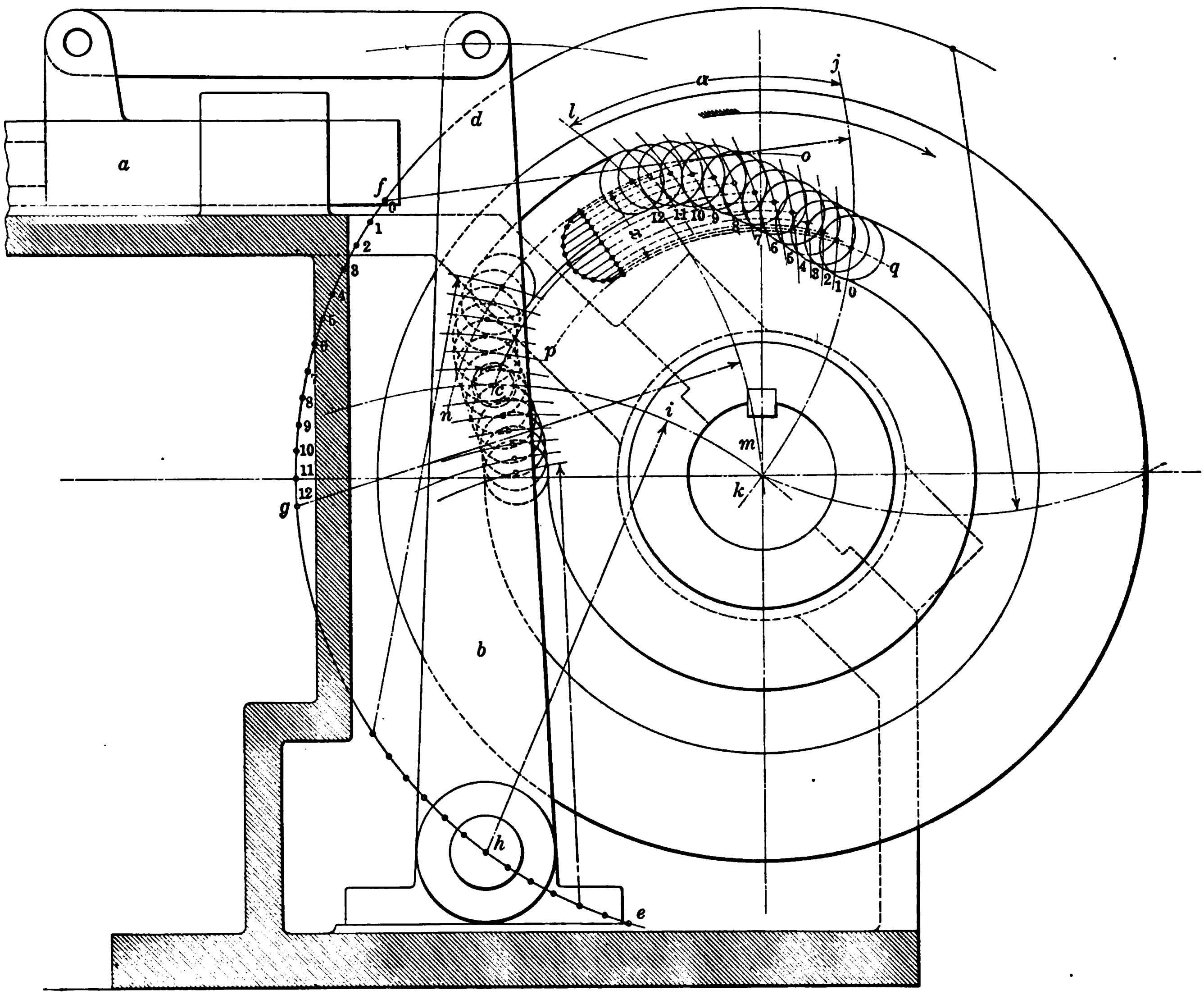


Fig. 6.—Laying out a face cam.

of the roller are, of course, arcs of circles having the center of the cam shaft as centers, and are drawn in.

Making the Templet

To make the templet proceed as in Fig. 7.

A piece of thin sheet metal, for which zinc in very suitable, is tacked down on the drawing as shown by the shaded outlines. This sheet is previously cut to a shape which shall fall within the pitch line of the cam groove but without the inner border of the groove. Its form is easily determined by laying a piece of tracing paper over the drawing and then drawing freehand a line which shall mark the desired outline of the zinc. The paper is then trimmed to this line and is used as a templet to which to cut the zinc.

With the zinc tacked over the drawing the horizontal and vertical center lines are carefully drawn with a fine, sharp scriber and then with a pair of dividers having fine, sharp points, those portions of the roller circles which are covered up by the zinc are redrawn on the zinc. With an irregular curve the bounding line of these arcs is then drawn, though not shown in the illustration, the circular portion of the groove which represents dwells of the cam roller being drawn with the dividers from the center of the zinc as located by the center lines.

Making the Former

The outline completed, the metal is carefully dressed down by hand to the outline and the templet is then placed upon the former blank

which has previously been faced off. The center lines are drawn on the former blank and marked as on the drawing and templet and the outline is then transferred to the blank by a fine, sharp scriber, and the former is then dressed down to this line, the bulk of the metal being removed in a milling machine. The circular portions of the outline are easily followed exactly and the other portions are followed as closely as possible, after which they are dressed down to the outline as carefully as possible by hand.

Fig. 8 illustrates the laying out of a drum cam. To make the horizontal measurements on this drawing, it is necessary to translate the positions given in the chart by degrees into inches of circumference and, dividing the entire circumference of the chart by the number of 5-deg. divisions, we find the length of each one of them.

The movement of the roller should take place during 12 5-deg. intervals which, translated into ins., are laid down as is the verti-

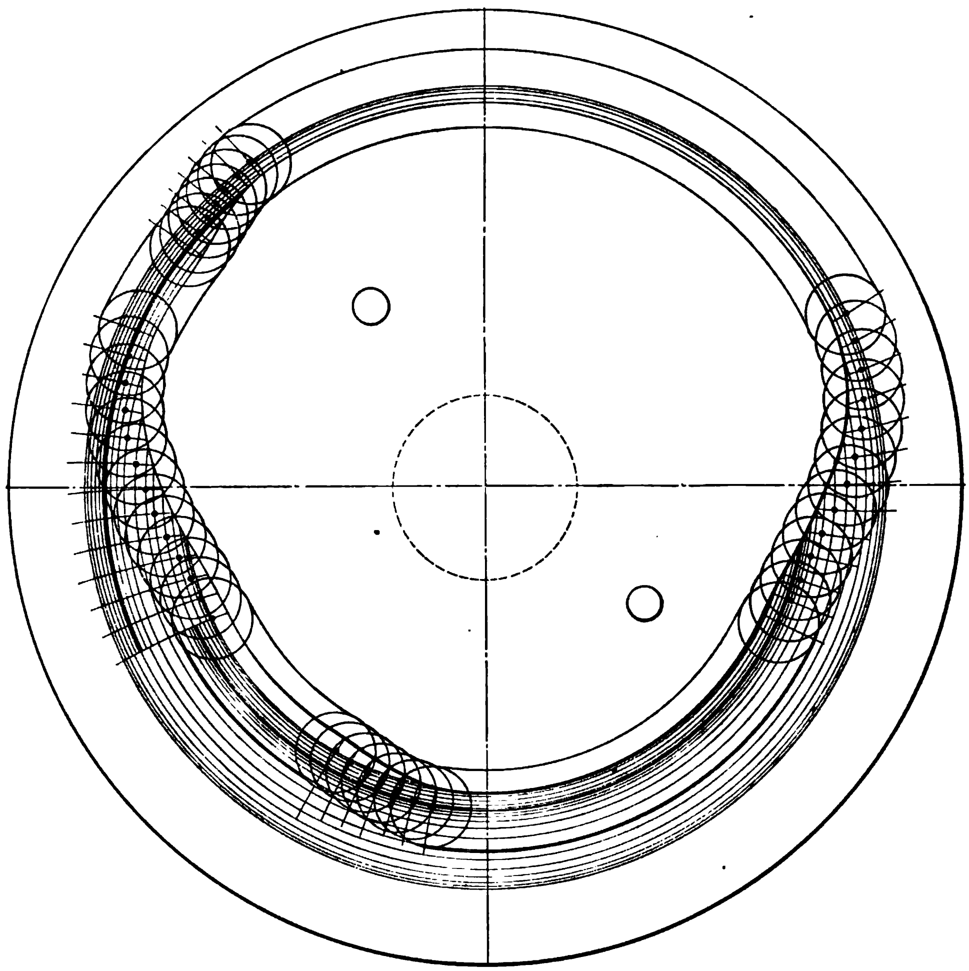


FIG. 7.—Transferring the cam profile to the templet.

cal movement giving the rectangle within which the curve is to be laid down and to find it we have only to follow the method given in Fig. 2. The throw circle is drawn and divided as before. Arcs *ab* and *cd* are drawn through the corners of the rectangle with the length of the radius arm as a center, giving the centers *ef*. The line *ef* is then divided and the intermediate arcs are drawn, the intersections of which with the parallels through the divisions of the throw circle give points on the pitch line of the groove from which circles with a radius equal to that of the roller define the groove. The dwell before the return movement takes place is then laid down and the return curve is drawn in the same way.

The error introduced by the hand work is not important as, the circular portions being exact, the movement given by the cam is exact, the slight error due to the hand work being in the *rate* of the movement cnly.

Laying Out Drum Cams

The laying out of drum cams c nnot be done quite as directly as that of face cams, as the layout must be a development and not a projection drawing. After laying out the profile, it is transferred to a sheet zinc templet, in the same manner as a face cam, which is then wrapped around the former on which the outline is scribed as in the case of a face cam. To provide for any minute discrepancy between the length of the templet and the circumference of the former, it is important that the place selected for the joint in the templet shall be within a straight part of the groove. Were it within an inclined part the result of such a discrepancy would be a jog when the templet is wrapped around the former, but by selecting a straight portion this is avoided.

A feature of drum cams which should not be overlooked is the dividing up of the arc of motion, as shown in Fig. 9, in which the lever is laid out as it should be with the arc of motion divided by both horizontal and vertical center lines.

Conical rollers are frequently used for drum cams, the rollers being laid out as are the pitch lines of bevel gears. This practice is of doubtful value as it leads to an end thrust which cramps the roller and leads to wear under heavy loads. Straight rolls are preferable

and to reduce the theoretically imperfect rolling action, they should be short—not much longer than half their diameter.

The cam should run away from the fulcrum—not toward it.

The diameter of the roller pin should not exceed one-half that of the roller in order to reduce the tendency of the roller to stick on the pin and thus wear flats.

bodies. As falling bodies experience no shock in starting, so cam motions laid out to conform to the same law experience no shock in starting and the same is true for stopping if the retardation is made uniform like the acceleration reversed.

The simplest method of laying out the gravity cam curve is that given in Fig. 10, in which the drawing of the parabola is avoided, by A

FIG. 8.—Laying out a drum cam.

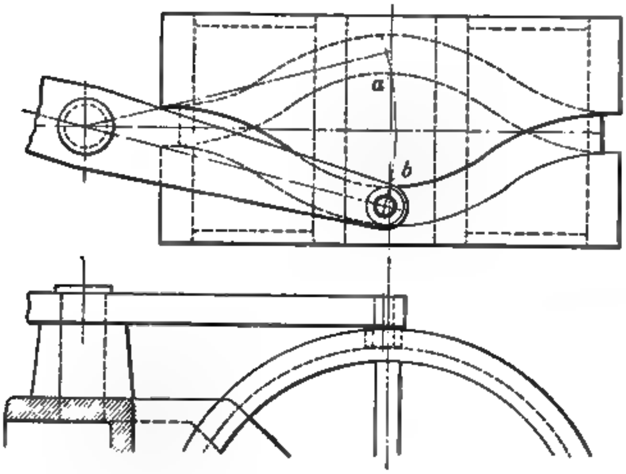


FIG. 9.—Correct division of the arc of motion.

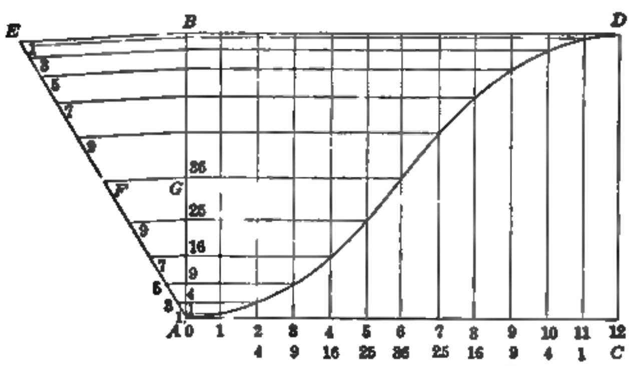


FIG. 10.—Laying out cams to the gravity base curve.

B. LENFEST (*Amer. Mach., April* 13, 1905). Divide the base line AC into equal parts as for the throw circle. Draw a line AE at any convenient angle and of indefinite length and lay off on it, to a convenient scale, distances from A to F and from E to F proportional to the odd numbers 1, 3, 5, 7, 9, 11; connect E to B or F to the middle point G of AB and draw from points 1, 3, 5, 7, 9, lines parallel to EB or FG, intersecting AB at 1, 4, 9, 16, 25, 36. Project these points thus found on AB to verticals from points 1, 4, 9, 16, 25, 36, on AC, and draw the curve (5) through these points thus located on the verticals.

The Cam Chart

The cam chart, by which the proper timing and coordination of cams is obtained, is such a large subject that its elements only can be presented here. The usual procedure in designing a cam-operated machine is to begin at its operating point, determining first the movements required and the general location of the cams and connections and then to lay out the chart in accordance with the required movements. A portion of such a chart is shown in Fig. 11. A base line is drawn representing the assumed circumference of the cams, which is subject to correction should it be found impossible to get all the movement into a circumference without increasing the cam groove angles beyond the limiting value. Needless to say, the process involves a good deal of trial and error work.

The base line is divided into 5-deg. intervals of which only 22 are shown in the illustration. A zero line common to all the movements is drawn at the left, the cams being treated at this stage as though all the rollers were upon the same line and had a common zero. It is simpler at this stage to treat all cams as though of the drum type.

The extent of movement of the cam rollers are laid down vertically and full size. The point in the revolution when each movement must be begun or completed is laid down and a rise of the line from the base line represents this movement. The constants that have been given enable the preliminary layouts to be quickly made, though, if the movements are at all crowded, the chart curves must be laid out from the throw circles and radius arms, as has been explained in connection with the laying out of the cams.

The movements are individually simple, being the simple shifting of a lever. The laying out of cams thus becomes a matter entirely separate from and subsequent to the design of the machine as a whole. The operating parts and their movements and the location of the cam shaft being determined and the connecting levers laid down, the matter, so far as it relates to individual cams, reduces itself to the moving of these levers at the right times and by the right amounts. The chart deals with these movements only, without regard to their direction or the connecting mechanism.

Levers of Unequal Length

When the cam lever arms are of unequal length (the cam end being the shorter) the Lanston Monotype Machine Company, employs

High Speed Cams

For high speed cams the throw circle is not a satisfactory base curve. It is well known that the crank motion, instead of being an easy, is a harsh one. In the center of the movement of an engine cross head, where the velocity is highest, the acceleration is zero while at the centers where the velocity is zero, the acceleration is at its maximum. Moreover, as the center is turned, the acceleration is abruptly changed from a negative to a positive maximum. For these reasons if there is the slightest lost motion pounding and noise result.

The ideal base curve for high speed cams (note that the dividing line between low and high speed cams cannot be drawn) is the gravity curve (parabola) which differs from the circle as a base curve for cams in that the acceleration given by it is uniform as with falling

the method shown in Fig. 12 (*Amer. Mach., Dec.* 14, 1905) for laying out the cam curves. The chart is laid out for the full movement and then the line *kl* representing this full movement is divided, *kn* being the cam movement. Point *m* being assumed at convenience, lines *lm* and *nm* are drawn. Points on the chart curve being then projected to the line *lm* and from the intersections down to *nm* the heights of the last intersections above the base line give the distances to be used in laying out the pitch curve of the cam. A reverse method obviously applies to the reversed arrangement of the arms.

More Accurate Methods

Increased accuracy of the former is obtained by the Lanston Monotype Machine Company by the use of an iron drawing board, Fig. 13, by which the cam outline is laid out directly on the former blank and the errors due to a transfer are avoided. The drawing board contains a pocket of a depth equal to the thickness of the former and a stump of the same diameter as the hole in the former. The board has a protractor ruled on its surface which enables angular lines to be ruled on the former which is coppered for the purpose. The heights of the cam curve as obtained from the chart are laid out on the blank which is then dressed down to the scribed outline.

The radii representing dwells are made to micrometer measurements from the hole in the former, the inaccuracies of the hand work thus affecting the rate of movement only.

Charts with Separate Base Lines

Charts with different base lines for the various cams are preferred by some designers. An example of this lay out is shown in Fig. 14.

Constructive Details

An objectionable arrangement of drum cams is shown in Figs. 15 and 16 (E. LAWRENZ, *Amer. Mach., Oct.* 12, 1911). Not only is an unnecessary side thrust put on the lever and its bearing, but Fig. 16 shows the cutting of such a cam to be difficult if proper contact with the roller is to be obtained—a difficulty which is still greater if the cam is to drive the roller in both directions. Fig. 17 shows the correct form with proper contact between cam and roller.

Conjugate Cams

The inertia of the roller gives rise to serious wear of closed cams at high speeds. The direction of rotation of the roller on its pin is reversed twice during each revolution of the cam at the points where the roller changes contact from one side of the groove to the other. At 140 r. p. m. this action on the monotype was found so destructive that with hardened rollers and steel pieces inserted in the cams at the places where the greatest wear developed, the usual life of some of the cams did not exceed six months.

The double or conjugate system of cams was invented to meet this difficulty by J. SELLERS BANCROFT (*Amer. Mach., Dec.* 14, 1905). A pair of conjugate cams, *a* and *b*, Fig. 18, are keyed to a pair of shafts which are so geared as to run at the same speed and in the same direction as indicated by the arrows. The roller *c* lies between them and is driven in one direction by one cam and in the opposite by the other. It will be observed that with this arrangement the direction of rotation of the roller on its pin is never changed. Its speed, of course, varies with the diameter of the cam surface acting at the moment, and to this extent its inertia comes into play to induce sliding, but such changes in speed are small in com-

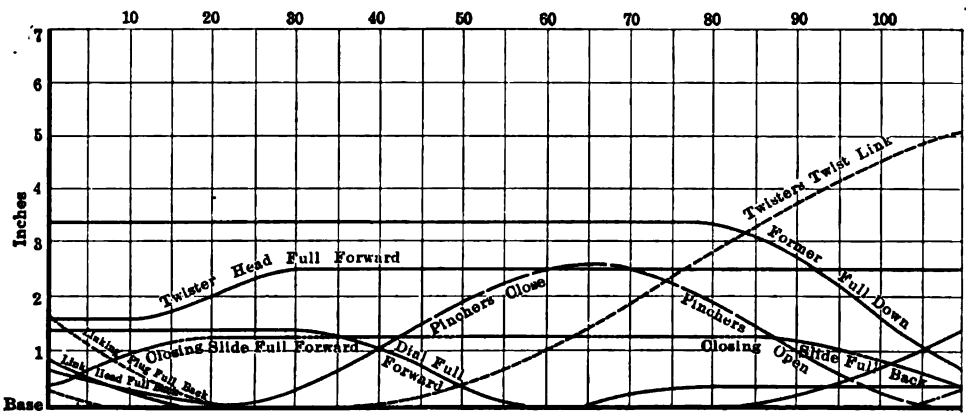


FIG. 11.—A portion of a cam chart

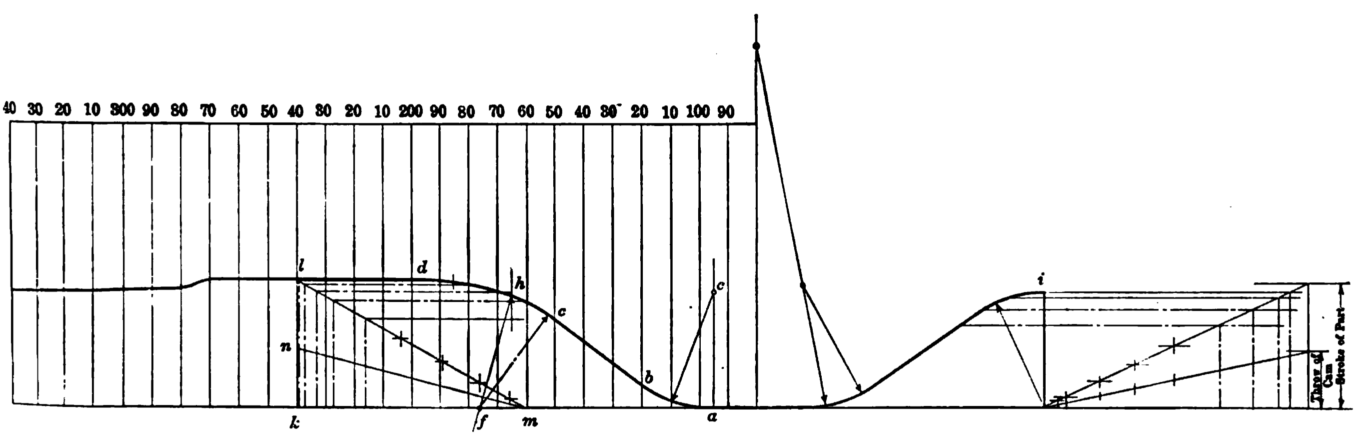


FIG. 12.—Construction for lever arms of unequal length.

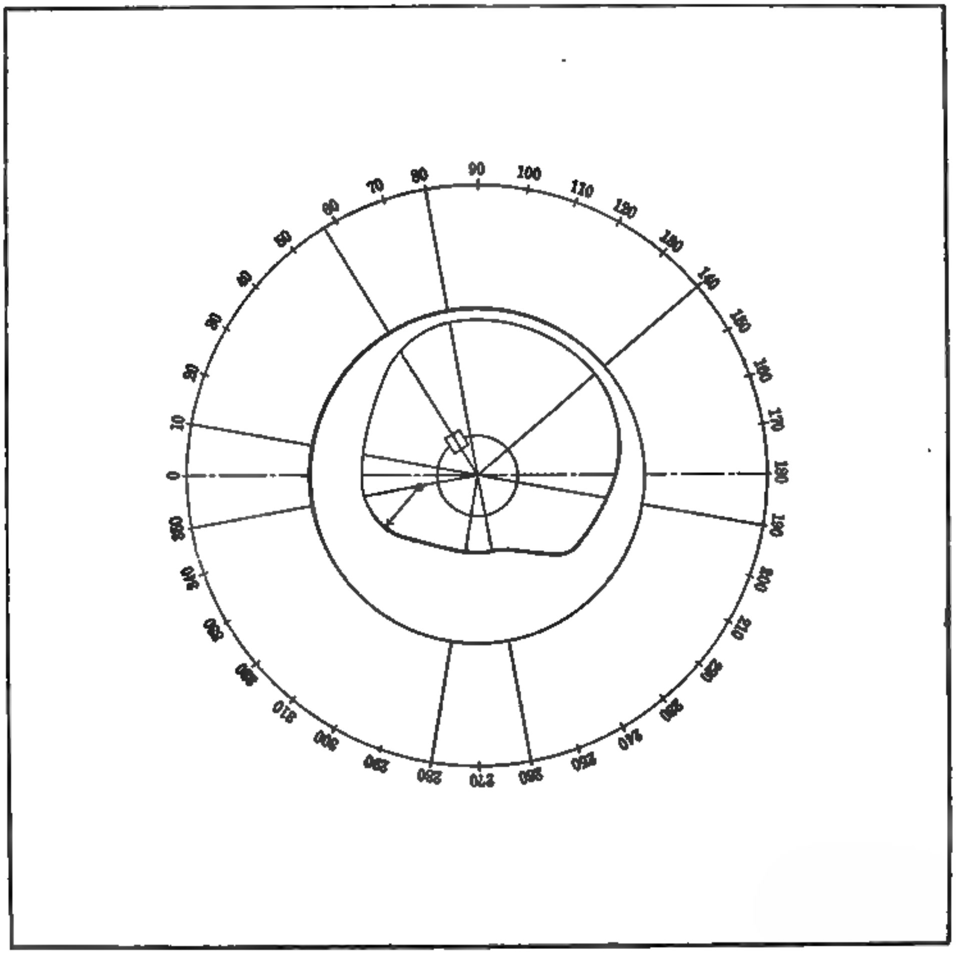


FIG. 13.—Iron drawing board for laying out formers.

FIG. 14.—Chart with a different base line for each cam.

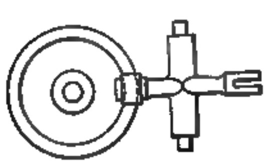


FIG. 15.

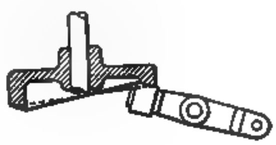


FIG. 16.

FIGS. 15 and 16.—Incorrect cam-lever arrangements.

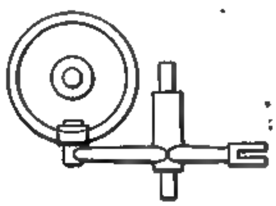


FIG. 17.—Correct cam-lever arrangement.

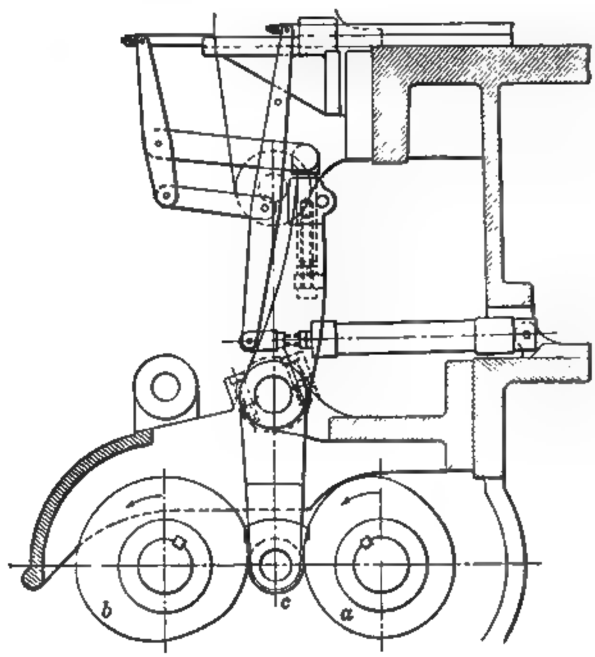


FIG. 18.—Conjugate cam system of the monotype.

parison with reversal and, moreover, they are always gradual, whereas the reversal with the usual style of cam is abrupt.

Such cams of cast-iron are more than twelve times as durable as the old style of face cams with steel inserts.

To insure the cams being true conjugates they are cut on a special machine, one cutter acting simultaneously on both cams.

The reversal of the roller does not take place with cams of which the movement in one direction is made by a spring, and hence such constructions are free from wear due to such reversal; on the other hand, the spring construction introduces other difficulties at high speeds.

Spring Adjustments

The varying resistance of springs due to their extension and the resulting varying pressure on the roller may be approximately compensated by the method shown in Figs 19 and 20, by E. LAWRENZ (*Amer. Mach., Oct.* 12, 1911).

The cam lever AB with the roller at A swings through the arc AA_1, the spring S being connected to a third arm OC, so arranged that, as the spring is extended and its resistence increased, the effective lever arm is reduced in the same proportion that the extension (not the total length) in increased. Thus Fig. 20, the free length of the spring, being $DE = DE_1$, for position OC_1 the extension is E_1C_1 and the effective lever arm is OF_1, while for position OC the extension is EC and the lever arm OF. To make the compensation (which is exact at the extreme positions and approximate at intermediate points) it is only necessary to make

$$OF \times EC = OF_1 \times E_1C_1$$

To find the required extensions draw a diagram, Fig. 21, in which $ab = OF$ and $ac = OF_1$. Through any convenient point d on the base line, draw db and dc and extend them. Locate efg' such that g is equal to the difference between DC and DC_1 when $ef = E_1C_1$ and $eg = EC$.

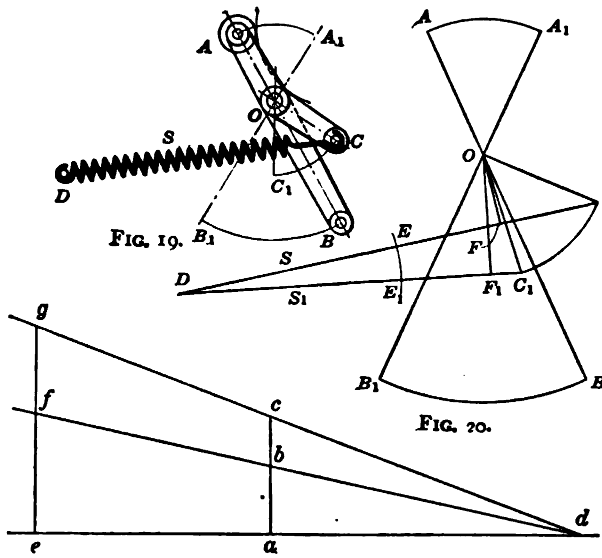


FIG. 19.

FIG. 20.

FIG. 21.

FIGS. 19 to 21.—Equalizing the Spring Pressure on Cams.

This construction is most appropriate with cams laid out on the gravity curve system, in which the acceleration of the driven piece being uniform, a uniform force is suitable. With cams laid out on the throw circle system the acceleration is at a maximum at the beginning of the movement and the natural action of a spring in giving the greatest force at the point where the acceleration is greatest is suitable. The acceleration becomes a factor, however, only at speeds such that the inertia of the parts is a factor and at speeds below this point the construction becomes appropriate for cams laid out from a throw circle.

SPRINGS

The use of spring tables or charts is greatly facilitated by preliminary calculation of the data, which may be made graphically as explained by B. C. Batcheller, Chief Engr. New York Pneumatic Service Co. (*Amer. Mach., Aug.* 3, 1911) as follows:

In designing machinery in which a spring is required, the designer usually knows the length of movement of the spring, the force that the spring should exert at some point in its movement, say at the beginning, and the variation in force that is allowable throughout its movement. These are his fundamental data and before he can use the spring formula or tables he must ascertain by computation the total compression or extension of the spring from the free length. In case the spring is free at one end of its movement, no such computation is required, but such is not usually the case.

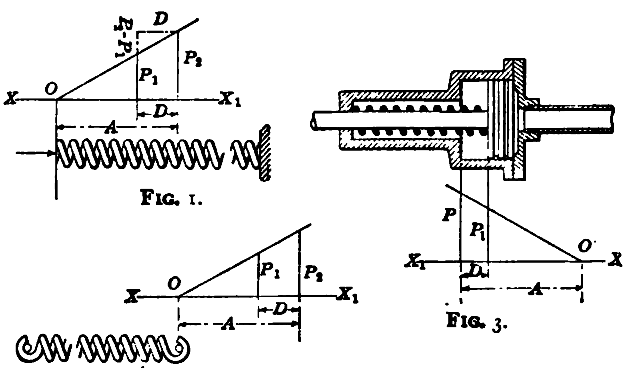


FIG. 1.

FIG. 2.

FIG. 3.

FIGS. 1 to 3.—The preliminary design of springs.

Fig. 1 represents a spring, to be used in compression, exerting a minimum force P_1, a maximum force P_2, with a movement in length D. We wish to know the total amount of compression A. On the horizontal line XX, erect two perpendiculars, P_1 and P_2, of as many units in length as the respective forces, and distant apart, D. Through the upper ends of the two perpendiculars, P_1 and P_2, draw an inclined line, producing it until it intersects the base line at O. The distance A from the point O to the perpendicular P_2, is the total amount of compression of the spring. By similar triangles:

$$\frac{A}{D}=\frac{P_2}{P_2-P_1},$$

therefore

$$A=\frac{P_2D}{P_2-P_1}.$$

Knowing A and P_2 we are now prepared to use the formula or tables of springs above referred to.

Fig. 2 shows a similar diagram for a spring in extension. It is obvious that P_1 can never equal P_2; in other words, we cannot make a spring to exert a constant force through a sensible length of movement, and the less the difference between P_1 and P_2, the greater must be the total amount of compression or extension. Such a diagram and computation should be made of every spring, no matter how insignificant, for they give a clear idea of the limitations of the case in hand.

Example.—Required, a spring to move a piston in one direction, that is moved in the opposite direction by a definite fluid pressure, Fig. 3.

Let $P_1=150$ lbs.,
$P_2=175$ lbs.,
and $D=1\frac{1}{4}$ ins.

$$A=\frac{175\times 1\frac{1}{4}}{175-150}=8\frac{3}{4} \text{ ins.}$$

We must have a spring of sufficient length, diameter, number of coils and size of wire to bear a total compression of $8\frac{3}{4}$ ins., and exert a force of 175 lbs. under this maximum compression, without injury to the spring.

Helical (Commonly Miscalled Spiral) Springs

The carrying capacity and deflection of helical springs of round wire, in tension or compression, may be determined from the established formulas:

$$W=.3927\frac{Sd^3}{D}$$

$$F=8\frac{PD^3N}{Gd^4}$$

For square wire there is some variation in the coefficients given by different authorities. Square wire is disappearing from the best practice as it should—the circular section being the more suitable. The formulas recommended, if square wire is to be used, are:

$$W=.444\frac{Sd^3}{D}$$

$$F=5.65\frac{PD^3N}{Gd^4}$$

in which

W = carrying capacity, lbs.,
S = fiber stress, lbs. per sq. in.,
d = diameter of round or side of square wire, ins.,
D = mean diameter of coil, ins.,
F = deflection of spring, ins.,
G = torsional modulus of elasticity,
P = load, lbs.,
N = number of coils.

These formulas ignore certain secondary stresses, and the proportions of the springs must be such as to make these stresses negligible if the results given by the formulas are to agree with the facts. Thus the larger the coil in relation to the diameter of the wire the better. In no case should the ratio of these diameters be less than 5. Again the smaller the helix angle the closer will the calculated results agree with the facts. The formulas for deflection again presuppose that the correct torsional modulus of elasticity for the material used is employed. The formulas will again give more accurate results for tempered steel than for piano-wire springs, in which latter internal stresses complicate the conditions.

Uniformity of practice in the matter of fiber stress is not, of course, to be expected. From discussions that have appeared in the columns of the *American Machinist* and elsewhere the following stresses appear to be safe and conservative:

For small springs of hard-drawn piano wire there is good warrant for stresses up to 100,000 lbs. per sq. in. For springs of tempered steel the following stresses may be used:

Diameter of steel, ins.	Stress, lbs. per sq in.
Up to $\frac{1}{8}$	75,000
$\frac{1}{4}$	70,000
$\frac{3}{8}$	60,000
$\frac{1}{2}$	50,000

It should, however, be said that some large users of springs limit the stress to 40,000 lbs. while, on the other hand, the Pennsylvania R. R. uses stresses of 60,000 to 70,000 lbs. All these figures are for springs subject to moderate shock. For heavy shock they should be reduced. Phosphor-bronze may be stressed to 15,000 lbs. and brass wire to 5000 lbs., the figures for brass being the least well established of all.

The torsional modulus of elasticity for steel, according to American investigators, averages about 12,600,000, while British experimenters give the smaller value, 11,000,000. It has been repeatedly proven that this constant has the same value for tempered and untempered steel. The effect of tempering is to raise the elastic limit and ultimate strength, without changing the modulus. The result is that while a tempered spring will carry a much heavier load without permanent set, the rate of deflection is unchanged. The value of the modulus for phosphor-bronze is 6,200,000 and for brass 3,400,000, the figures for brass being, again, less well established than the others. Considering the miscellaneous compositions that go by the name of "brass" definite values for the fiber stress and modulus are not to be looked for.

The accompanying charts, Figs. 4 and 5, by Prof. J. B. Peddle (*Amer. Mach., Aug.* 15, 1912) give the same results as the above formulas and enable calculations for helical springs to be made with great facility. The use of the charts is explained below them.

The charts may obviously be worked in any convenient direction in accordance with the given and required quantities.

In ordinary cases several trials must be made before a spring of the required strength and deflection is found, and it is in the convenience of the charts for making these trials that their best feature lies.

Of course, good sense must be used in all such work. Thus in the case of a compression spring it may easily happen that the deflection given by the chart will more than close the spring—an impossible condition of course. This must be watched for and a spring be chosen which will not be an absurdity. The charts are equally applicable to both extension and compression springs—no initial tension being understood in the case of extension springs.

The deflection of conical helical springs may be obtained from the formula (G. M. Strombeck *Amer. Mach., Feb.* 1, 1912):

$$f = 2NP\frac{D_1^3 + D_1^2D_2 + D_1D_2^2 + D_2^3}{Gd^4}$$

in which f = total deflection under load P, ins.,
N = number of coils,
P = load, lbs.,
D_1 = largest diameter of coil to center of wire, ins.,
D_2 = smallest diameter of coil to center of wire, ins.,
G = torsional modulus of elasticity,
d = diameter of wire, ins.

To design a double or triple helical spring (two or more concentric springs) each individual spring to carry an equal part of the total load, proceed as follows, (O. A. Thelin *Amer. Mach., Dec.* 27, 1906):

Let P = total load, lbs.
N = number of individual springs,
D = pitch diameter of outer coil, ins.,
d = diameter of wire of outer coil, ins;
then $\frac{P}{N}$ = load per spring, lbs.

Design the outer coil for load $\frac{P}{N}$, draw a line perpendicular to the axis of the spring through the center of the cross-section and offset .3d to each side of this line on the axis, as in Fig. 6. From these two points draw tangents to the circular section of the coil d, when any coil d_2, d_3, etc., tangent to the two lines will also carry $\frac{P}{N}$ lbs. load.

Theoretically, the two tangents will not be straight lines, but form the curve of a cubic parabola. The difference is, however, slight and need not be considered for practical purposes.

Helical Springs in Torsion

The strength and deflection of helical springs in torsion are usually calculated from the equations for the bending of straight beams. While these equations are inexact for the conditions, they are much simpler than those based on the curved-beam theory and, for springs of the usual proportions of wire and coil diameters, they lead to errors that are unimportant.

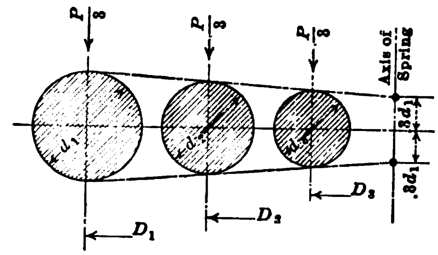


Fig. 6.—The design of multiple springs.

The formulas are:

$$\left.\begin{aligned} M &= \frac{\pi d^3}{32}S \\ \alpha &= \frac{3667\ MDN}{Ed^4} \end{aligned}\right\} \text{for round wire}$$

$$\left.\begin{aligned} M &= \frac{\pi d^3}{6}S \\ \alpha &= \frac{2160\ MDN}{Ed^4} \end{aligned}\right\} \text{for square wire}$$

in which M = twisting moment, lb.-ins.
d = diameter of round or side of square wire, ins.,
S = fiber stress, lbs per sq. in.,
α = angle of twist, deg.
D = mean diameter of coil, ins.,
N = number of coils,
E = tension (not torsion) modulus of elasticity.

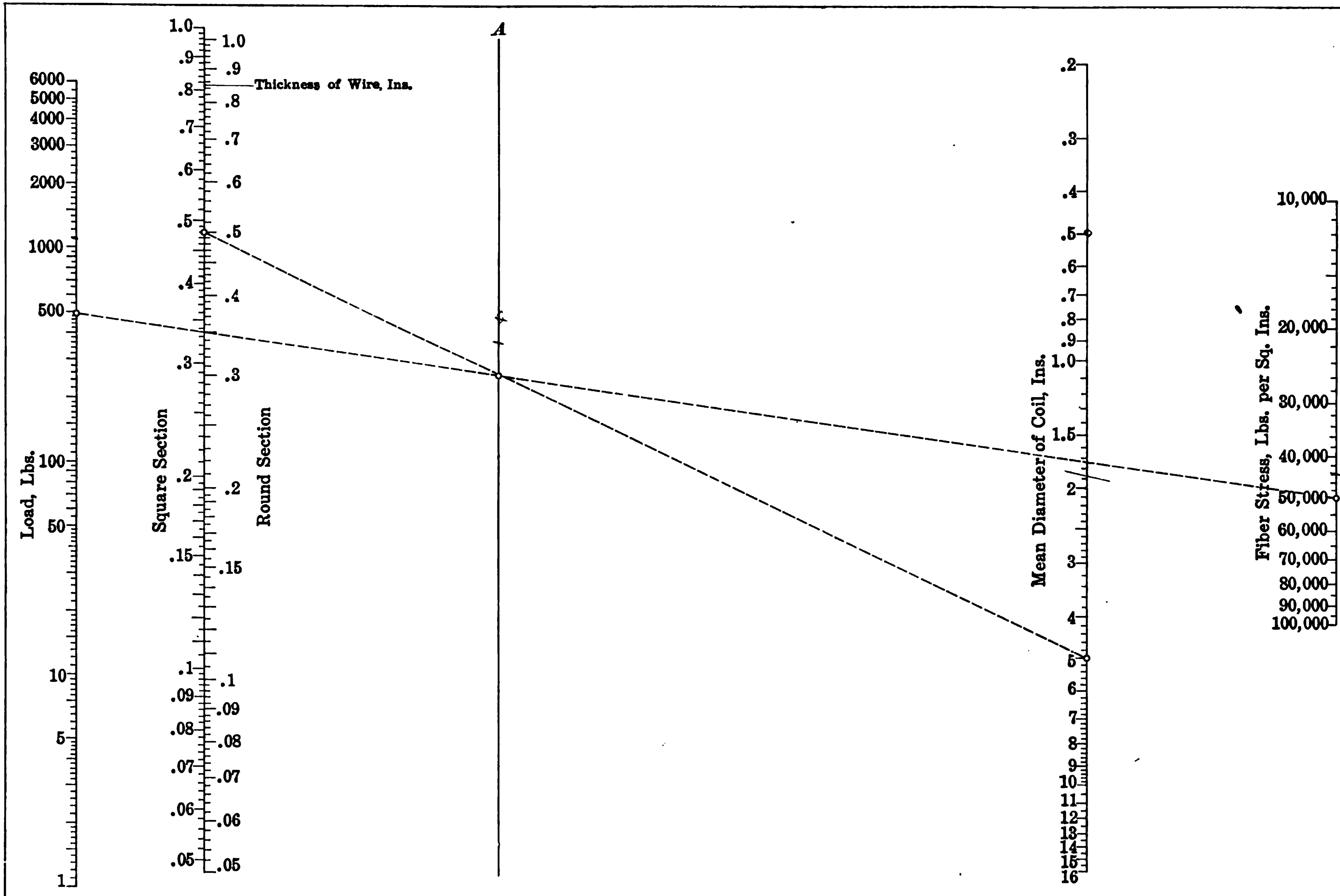
For springs loaded in this manner square wire is more appropriate than round.

The accompanying charts, Figs. 7 and 8, by Prof. J. B. Peddle (*Amer. Mach., June* 19, 1913) are based on the above formulas. Instructions for use will be found below them. Safe fiber stresses may be taken at from 80,000 to 100,000 lbs. per sq. in. for tempered and from 30,000 to 40,000 for untempered steel. For hard-drawn spring brass wire stresses of 15,000 to 20,000 lbs. per sq. in. may be used. For steel the usual values of the modulus of elasticity are to be used. For spring brass wire, the values of the modulus, according to Professor Peddle, range between 13,000,000 and 14,800,000, with an average of 14,000,000.

With a little calculation, the charts are applicable to wire of rectangular sections other than square.

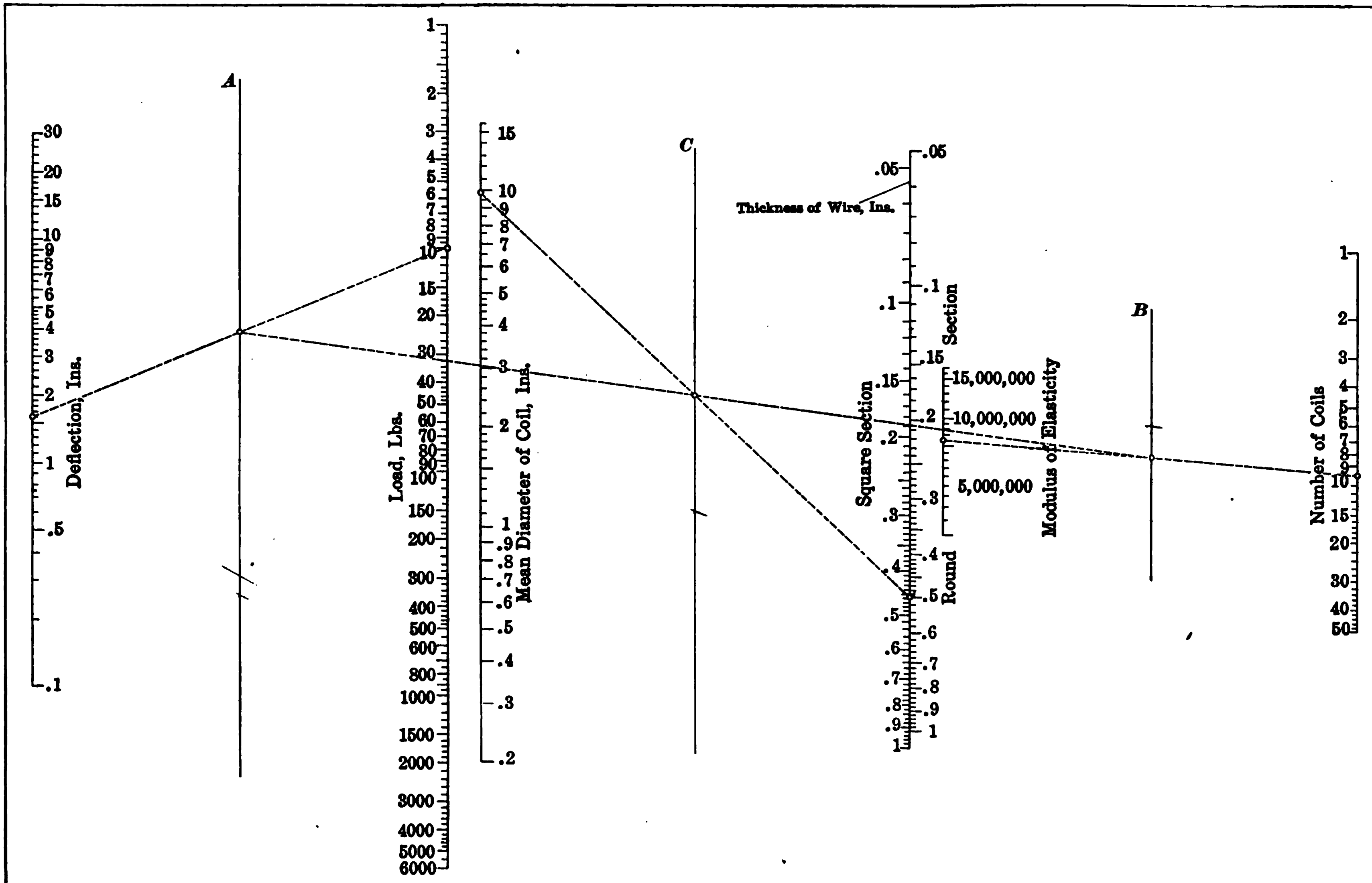
Assume that a wire .2 in. thick (perpendicular to the axis of the coil) is to be used, the load being 120 lb.-ins. and the fiber stress 60,000 lbs. per sq. in. First find the load which may be carried by a square wire .2 in. on a side, namely 80 lb.-ins. For other widths, parallel with the axis of the coil, the load is proportional to the width, so that for a load of 120 lb.-ins. the required width is .2 in. $\times \frac{120}{80} = .3$ in.

The deflection, on the other hand, will vary inversely as the width, Hence, in the example shown on the deflection chart, if we use a wire .2×.3 in., instead of a wire .2 in. square, the deflection will be ⅔ of a revolution instead of one revolution with the number of coils given. Or if we must have a deflection of one revolution, it will be necessary to increase the number of coils by 50 per cent., which would mean 37½ coils instead of 25.



Connect the load with the fiber stress and note intersection with axis *A*. Any line through this intersection will give a combination of thickness of wire and diameter of coil which will carry the given load with the given fiber stress. The example solved is for load = 500 lbs. and fiber stress = 50,000 lbs. per sq. in., giving diameter of coil = 5 ins., and round wire = .5 in. diam., or square wire = .48 in thick.

FIG. 4.—Carrying capacity of helical springs in tension and compression.



Connect the desired deflection with the load and note the intersection with axis A, connect the number of coils with the modulus of elasticity and note the intersection with axis B, connect the intersections and note intersection with axis C. Any line through this intersection will give a combination of thickness of wire and diameter of coil that will give the required deflection under the given load. The example solved is for load = 10 lbs., deflection = 1.6 ins., number of coils = 10, modulus = 8,000,000; giving diameter of coil = 10 ins. and round wire .5 ins. diameter.

FIG. 5.—Deflection of helical springs in tension and compression.

Elliptic and Semi-elliptic Springs

The strength and deflection of elliptic and semi-elliptic springs may be determined from the formulas:

$$P=\frac{nbt^2f}{3L}$$

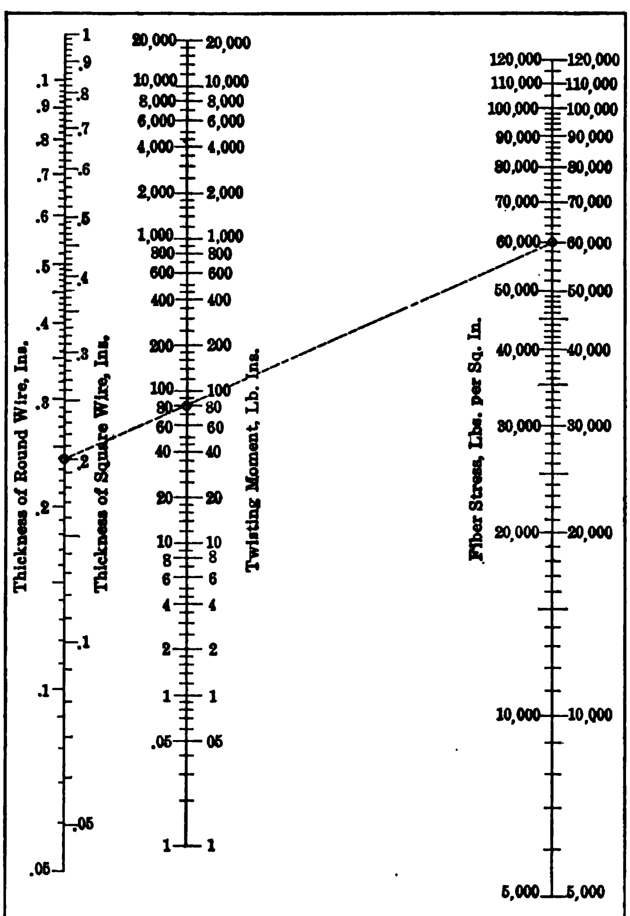
$$D=\frac{4L^2f}{tE}K \quad \text{full elliptic}$$

$$D=\frac{2L^2f}{tE}K \quad \text{semi-elliptic}$$

in which P = safe load, lbs.,
n = number of leaves (total for semi-elliptic and for one side of full elliptic),
b = breadth of leaves, ins.,
t = thickness of leaves, ins.
f = safe fiber stress, lbs. per sq. in.,
L = free length or projection of one end from center band, ins.
D = deflection, ins.,
E = modulus of elasticity.

$$K=\frac{1}{(1-r)^3}\left[\frac{1-r^2}{2}-2r(1-r)-r^2\log_e r\right]$$

$$r=\frac{\text{No. of full length leaves}}{\text{total No. of leaves}}$$



Connect the given twisting moment with the desired fiber stress. The line extended gives the required thickness of wire. The example shows that a square wire .2 in. thick will carry a twisting moment of 80 lb.-ins. under a fiber stress of 60,000 lbs. per sq. in.

FIG. 7.—Carrying capacity of helical springs in torsion.

In these formulas, by Prof. J. B. Peddle, the equivalent plate spring is assumed of the trapezoidal form, Fig. 9, instead of, as usual, the triangular form, Fig. 10. For structural reasons there must be at least one full-length blunt-ended leaf, and the assumption of a triangular equivalent, when the number of leaves is few and r, therefore, large, leads to errors which may equal 10 or 12 per cent. and even more if there is more than one full-length blunt leaf.

It is necessary, in order that the comparison between the ideal and actual springs should hold good, to have the points of the shortened leaves tapered in width or in thickness, or both, so as to make the transmission from one leaf to the next one gradual. If this is not done and the leaves are blunt-ended, the sides of the ideal plate spring would have to be stepped instead of straight. In the absence of a definite knowledge of the constants of the material, it may be assumed that for the usual spring steels the safe fiber-stress will lie between 75,000 and 100,000 lbs. per sq. in. The stresses to be used for thin leaves will usually approach the higher values, while those for thick ones will be lower. E will usually be found to lie between 25,000,000 and 30,000,000, though both lower and higher values than these are given by some authorities.

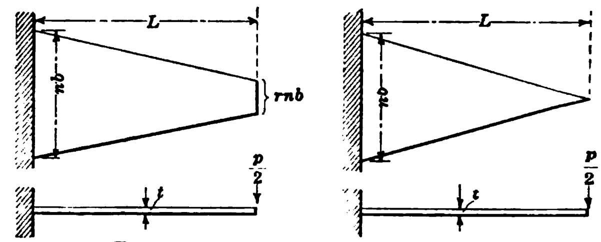


FIG. 9. FIG. 10.

FIGS. 9 and 10.—Equivalent plate springs.

The accompanying charts, Figs. 11 and 12, also by PROFESSOR PEDDLE (*Amer. Mach., Apr.* 17, 1913) give the same results as the formulas and eliminate the laborious calculations due to the complex form of the expression for K. The use of the charts is explained below them.

When comparing the calculated with the actual deflection of leaf springs, it must be remembered that the friction between the leaves introduces a disturbing factor, the effect of which cannot be calculated.

The strength and deflection of flat (single leaf) springs may be determined from the formulas of Table 1, by R. A. BRUCE (*Amer. Mach., July* 19, 1900).

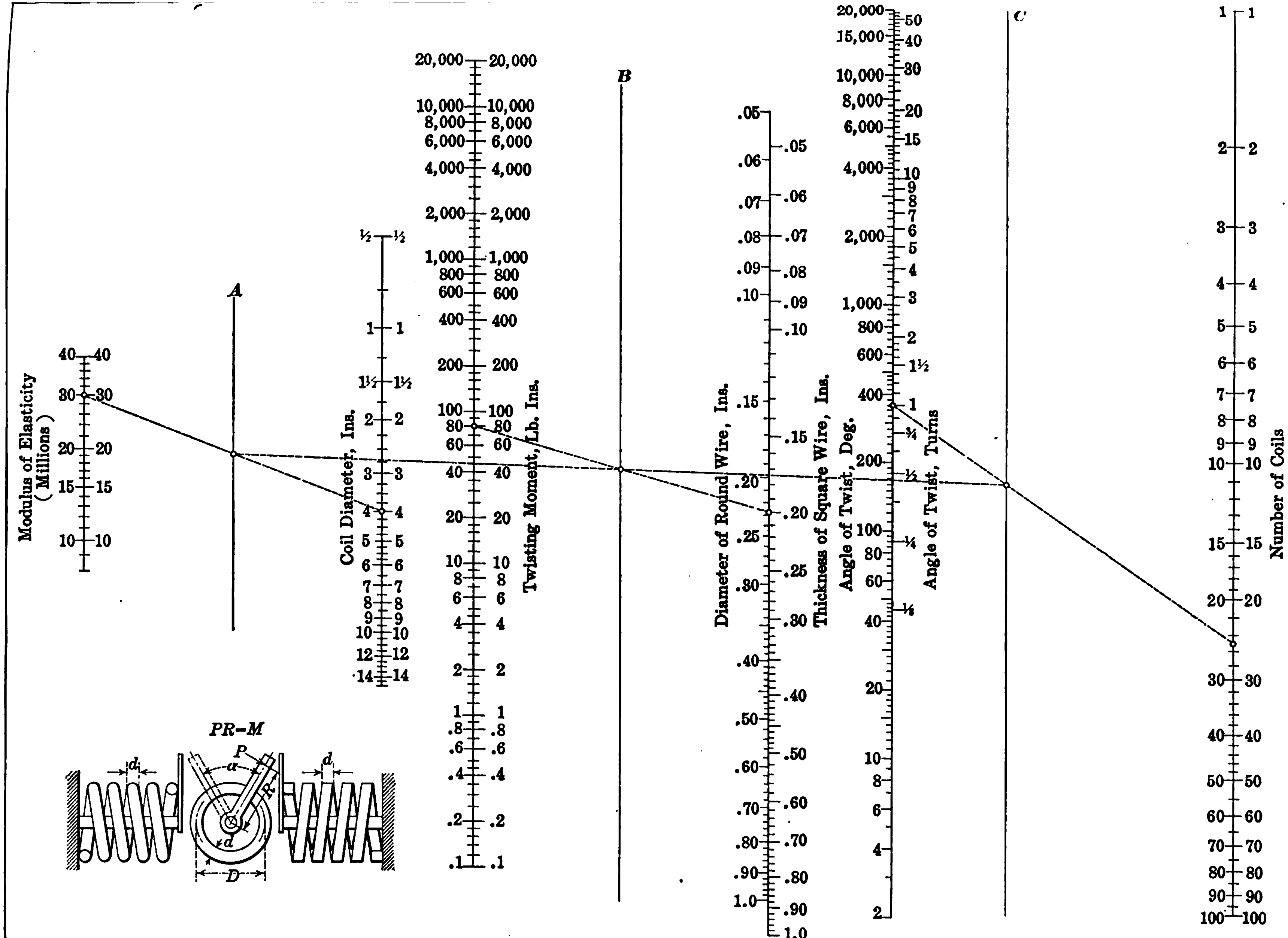
Assuming the length to be determined by circumstances and the load and deflection to be given, the simplest method of proceedure is to first settle upon the proper depth t in order to secure the requisite deflection. The formula for this purpose is

$$t=a\times\frac{l^2}{\delta} \qquad (a)$$

in which l = length,
δ = deflection,
t = thickness,

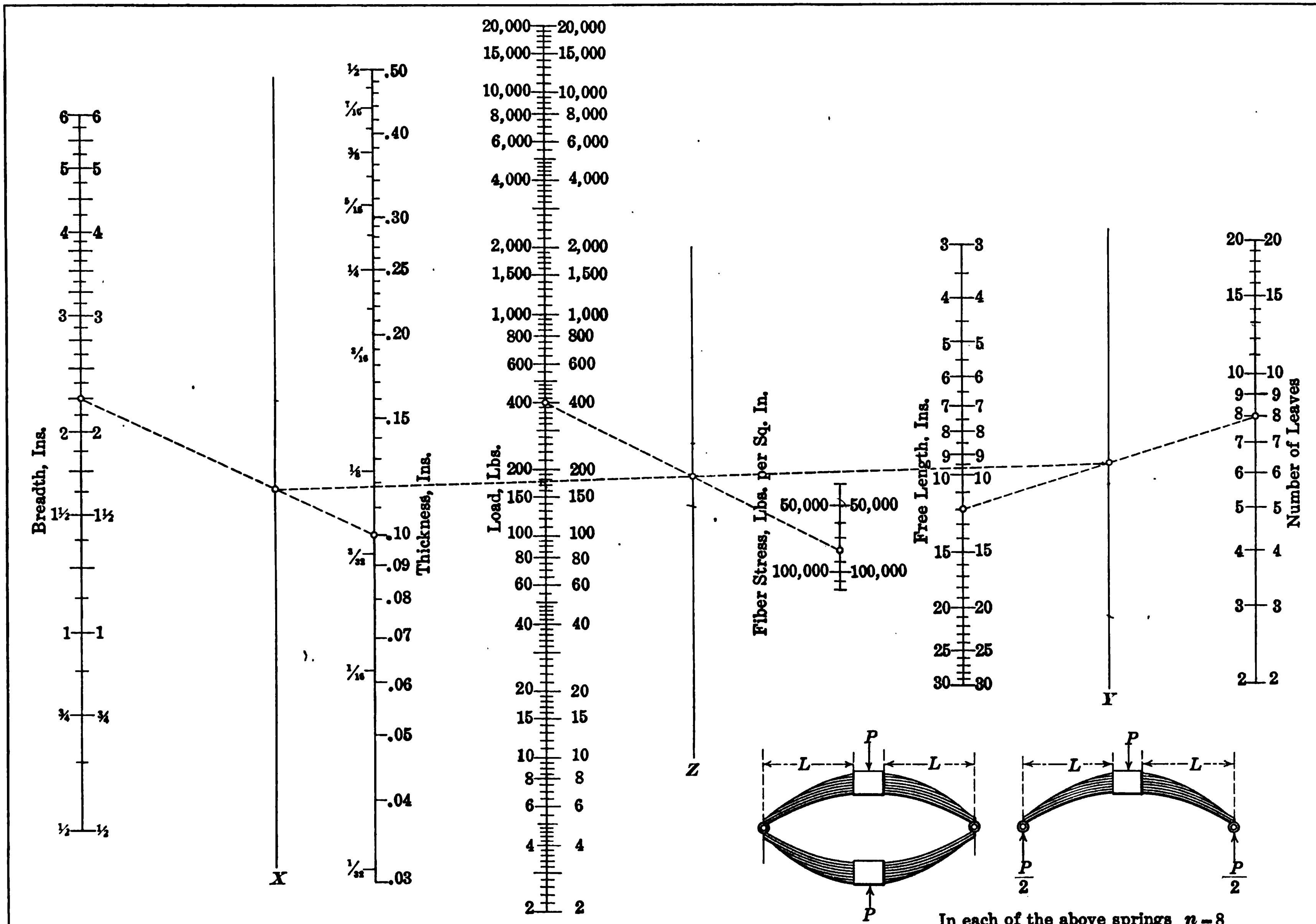
all in inches.

The value of the multiplier a depends upon the safe stress f and the modulus of elasticity multiplied by a number which varies according to the type of spring adopted. The general value of a is given for each type of spring in the column under the heading a, but inasmuch as a good all-round value for f is 60,000 and for E 30,000,000, a second column has been added, giving the numerical value of a for these values of the stress and modulus. They may be confidently used for general work, and in cases where springs are not subject to alternate bending in opposite directions. The thickness having



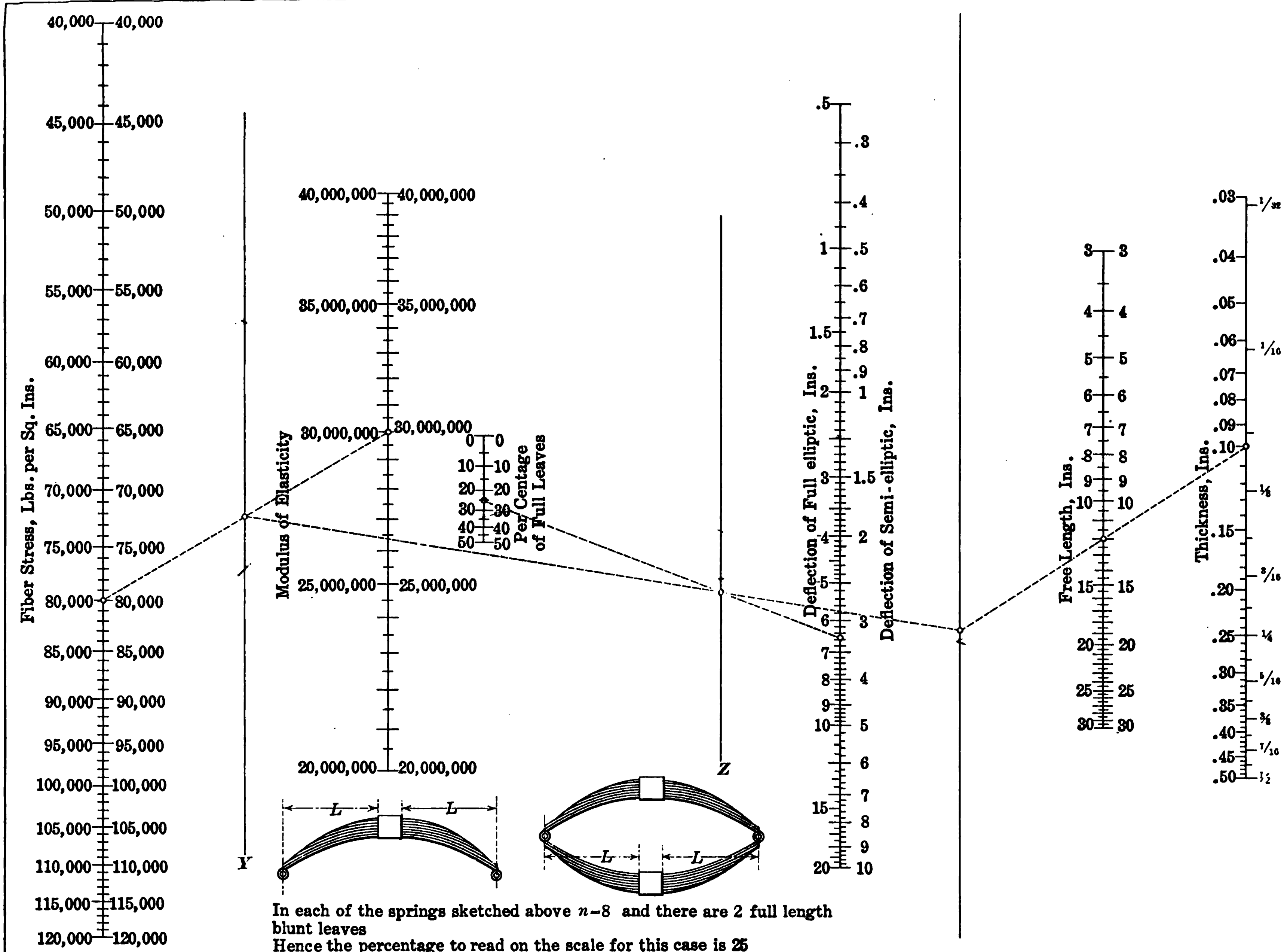
Connect the modulus of elasticity with the coil diameter and note intersection with axis A; connect the twisting moment with the thickness of wire and note intersection with axis B; join the intersections and extend the line to intersect axis C. A line through this intersection will connect the angle of twist with the required number of coils. The example shows that a square wire of steel, having a modulus of elasticity of 30,000,000, measuring .2 in. on the side, wound into a spring of 4 ins. mean diameter and 25 coils, will twist one full turn under a twisting moment of 80 lb.-ins.

FIG. 8.—Deflection of helical springs in torsion.



Connect the breadth with the thickness and note intersection with axis X, connect the free length (L of the sketches) with the number of leaves and note intersection with axis Y. Join intersections on X and Y and note intersection with axis Z. A line through this intersection and the selected fiber stress will give the safe load. The example solved is for breadth = $2\frac{1}{4}$ ins., thickness = $\frac{1}{10}$ ins., free length = 12 ins., fiber stress = 80,000 lbs. per sq. in.; giving load = 400 lbs.

FIG. 11.—Carrying capacity of elliptic and semi-elliptic springs.



Connect the fiber stress with the modulus of elasticity and note intersection with axis X. Connect the free length (L of the sketches) with the thickness and note intersection with axis Y. Connect intersections on X and Y and note intersection with axis Z. A line through this intersection and the percentage of full length leaves will give the deflection. The example solved is for fiber stress = 80,000 lbs. per sq. in.; modulus of elasticity = 30,000,000; thickness = $\frac{1}{10}$ in.; free length = 12 ins.; percentage of full length leaves = 25; giving deflection = 6.56 ins. for full and 3.28 ins. for semi-elliptic springs.

FIG. 12.—Deflection of elliptic and semi-elliptic springs.

been determined, the breadth in ins. is next found by making use of the formula,

$$b = c \times \frac{Wl}{t^2} \qquad (b)$$

in which W = maximum load, lbs.,
t = depth, ins.,
l = length, ins.

The value of c for the general case is given under the first column marked at the head c. For ordinary stêel springs a second column of values of c has been added, the values assigned to f and E being the same as before. The principal dimensions of the spring are therefore easily settled. In order to find the cubic volume of the spring, multiply the product of l, b and t by the number given under the column marked v. A useful check on the work is to find the energy to be absorbed by the spring by multiplying the deflection by half the maximum load. If this quantity be divided by the number given under the heading R, the result should be equal to the volume of the spring in cubic inches. The first column lettered R gives the general value of the resilience per cubic inch for a spring of a particular type and the second gives the resilience when the stress is 60,000 lbs. per sq. in. and the modulus of elasticity is 30,000,000.

The maximum load and the deflection for a given load can be found by transposing formulas (a) and (b) as follows

$$\delta = a\frac{l^2}{t} \qquad (c)$$

$$W = \frac{bt^2}{cl} \qquad (d)$$

Materials of and Miscellaneous Information on Springs

Ordinary carbon steel used for springs, according to C. A. Tupper (*Amer. Mach., Mar.* 24, 1910), has about the following chemical composition: Carbon .95 to 1.05 per cent.; manganese .025 to .040 per cent.; silicon .12 to .15 per cent.; phosphorus and sulphur not over .03 per cent. each. The elastic limit to be expected from such steel varies so much with the heat treatment and the methods of tempering used that general statements are without value. The highest figures observed were given in a paper read before the International Society for Testing Materials, September 9, 1909, for steel of approximately the above characteristics hardened in water at 1425 deg. Fahr. and drawn to 750 deg. Fahr. The diameter of the test piece was .994 in. It showed an elastic limit of 240,800 lbs., with modulus of elasticity 29,220,000 and broke under a deflection at the middle of .744 in. It is apparent that the allowable limits of specifications for finished springs are rising at a very rapid rate, and what the immediate future will bring can only be conjectured. In practice, however, the elastic limit actually necessary is very far below the extreme figures just cited. For special alloy steels the chemical composition varies widely, particularly in relation to the carbon and manganese contents, which may range considerably lower.

Fuller, though older, information regarding the carbon content of steel for springs is found in a paper read by Wm. Metcalf before the American Society for Testing Materials, 1903, as follows:

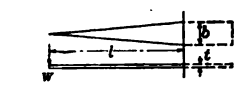
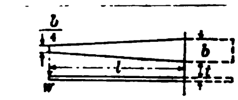
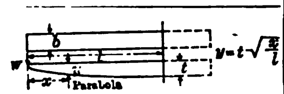
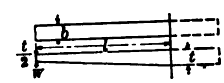
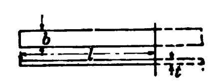
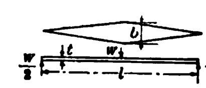
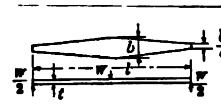
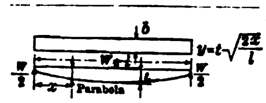
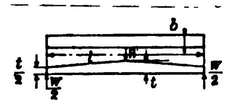
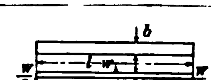
The lower carbons should be put into the larger bars, because the large bars are the most difficult to harden safely, and the difficulty increases in a geometrical ratio with the increase in carbon. A good rule is to put .70 to .90 carbon into bars of more than 1 in. diameter; bars from 1 to ¾ in., .90 to 1.10 carbon; bars from ¾ to ½ ins., 1.10 to 1.20 or even 1.30 carbon, and little rods below ½ in. any high carbon up to as much as 1.45.

Regarding hardening and tempering, Mr. Metcalf says that steel of .60 to .90 carbon may be hardened in water; with about .90 carbon, a film of oil may be used on the water. From .90 to 1.10 carbon, about 4 or 5 ins. of oil may be used on the water, and for higher steel oil should be used and kept cool by an external tank of

Table 1.—Strength and Deflection of Flat Springs

Notation

f = safe stress, lbs. per sq. in.,
E = modulus of elasticity,
R = resilience or energy, in.-lbs. per cu.in.,
W = maximum load on spring, lbs.,
δ = maximum deflection, ins.,
l = length of spring, ins.,
t = thickness or depth in ins. $= a \times \frac{l^2}{\delta}$
b = breadth in ins. $= c \times \frac{Wl}{t^2}$
V = cu. ins. in (useful part of) spring $= v \times lbt$.

Types of spring used	General				For f = 60,000 E = 30,000,000		
	a	c	R	v	a	c	R
	$\frac{f}{E}$	$\frac{6}{f}$	$\frac{f^2}{6E}$	$\frac{1}{2}$	$\frac{2}{1000}$	$\frac{1}{10000}$	20
	$.87\frac{f}{E}$	$\frac{6}{f}$	$\frac{.70f^2}{6E}$	$\frac{5}{8}$	$\frac{1.75}{1000}$	$\frac{1}{10000}$	14
	$\frac{4}{3}\frac{f}{E}$	$\frac{6}{f}$	$\frac{f^2}{6E}$	$\frac{2}{3}$	$\frac{8}{3}\times\frac{1}{1000}$	$\frac{1}{10000}$	20
	$1.09\frac{f}{E}$	$\frac{6}{f}$	$\frac{.725f^2}{6E}$	$\frac{3}{4}$	$\frac{2.18}{1000}$	$\frac{1}{10000}$	14.52
	$\frac{2f}{3E}$	$\frac{6}{f}$	$\frac{.33f^2}{6E}$	1	$\frac{4}{3}\times\frac{1}{1000}$	$\frac{1}{10000}$	6.66
	$\frac{f}{4E}$	$\frac{6}{4f}$	$\frac{f^2}{6E}$	$\frac{1}{2}$	$\frac{1}{2}\times\frac{1}{1000}$	$\frac{1}{40000}$	20
	$\frac{.87f^2}{4E}$	$\frac{6}{4f}$	$\frac{.70f^2}{6E}$	$\frac{5}{8}$	$\frac{7}{16}\times\frac{1}{1000}$	$\frac{1}{40000}$	14
	$\frac{1}{3}\frac{f}{E}$	$\frac{6}{4f}$	$\frac{f^2}{6E}$	$\frac{2}{3}$	$\frac{2}{3}\times\frac{1}{1000}$	$\frac{1}{40000}$	20
	$\frac{1.09f}{4E}$	$\frac{6}{4f}$	$\frac{.725f^2}{6E}$	$\frac{3}{4}$	$\frac{.54}{1000}$	$\frac{1}{40000}$	14.52
	$\frac{f}{6E}$	$\frac{6}{4f}$	$\frac{.33f^2}{6E}$	1	$\frac{1}{3}\times\frac{1}{1000}$	$\frac{1}{40000}$	6.66

cold circulating water, or by a coil of pipe inside of the tank with cold water running through.

Tempering must be suited to the carbon; .70 to .80 carbon will require very little drawing; .90 to 1.10 may require the oil to flash, and for higher carbons the oil may be burned off. Above 1.30 carbon a heat that barely begins to show color will generally give a good spring temper. In tempering, as in hardening, good sense and good judgment are the best guides.

The desirable composition of steel for helical springs according to the specifications of the Pennsylvania Railroad is as follows:

Carbon	1.00 per cent.
Manganese	.25 per cent.
Phosphorus, not above	.05 per cent.
Silicon	.35 per cent.
Sulphur, not above	.03 per cent.

In case the carbon is found to be below .90 per cent., the manganese above .50 per cent., the phosphorus above .07 per cent. and the silicon below .25 or above .50 per cent., the springs represented by the sample or samples will be rejected. Springs made from bars three-eighths of an inch or less in diameter need not conform to the chemical limits above, but such springs will be rejected if the carbon is below .50 per cent., the manganese above 1.00 per cent. and the phosphorus above .11 per cent.

The desirable composition of steel for elliptical springs, according to the specifications of the Pennsylvania Railroad, is as follows:

Carbon	1.00 per cent.
Phosphorus, not above	.03 per cent.
Manganese, not above	.25 per cent.
Silicon, not above	.15 per cent.
Sulphur, not above	.03 per cent.
Copper, not above	.03 per cent.

Springs, however, will be accepted which on analysis show the metal to contain:

Carbon, not below .90 per cent. or not above	1.10 per cent.
Phosphorus, not above	.05 per cent.
Manganese, not above	.50 per cent.
Silicon, not above	.25 per cent.
Sulphur, not above	.05 per cent.
Copper, not above	.05 per cent.

For additional information on steel for springs see Steel.

Particulars regarding the behavior of springs and the practice of the Westinghouse Electric and Mfg. Co. are given by R. A. PEEBLES, Research Engineer of the company (*Amer. Mach., May* 2, 1912).

The drawing usually specifies the free height of the spring, the number of turns, the size of wire or bar, and either the outside or inside diameter of the spring according to where it is to be used. The specification provides, however, that the manufacturer, in order to obtain the desired combination of load and compression, may vary the size of wire used, provided the fiber stress figured from the size substituted be not more than 10 per cent. greater than the stress figured from the size specified on the drawing. Imperfect workmanship is responsible for the greater number of poor springs, and in the majority of cases the faults seem so slight that it is often difficult to convince the manufacturer that they are the cause of the discrepancies.

It will be found that of springs which are accurately designed those give most trouble which have the smallest number of turns or the smallest diameter in proportion to the size of the wire. This is because the source of most of the inaccuracy is in the two end turns which are set up against the adjacent turns and ground or hammered flat to fit the spring seat.

The end of the wire in the set up end turn should no more than touch the next turn as shown in Fig. 13, and not lap against it as in Fig. 14. In a large proportion of the springs tested the end turns lap as in Fig. 14, often to the extent of ¼ to ⅜ of a turn at each end. This robs the spring of ½ to ¾ of an active turn, thus increasing both the compression per turn and the corresponding load at the required compression.

It will easily be seen that this is a serious matter in a spring of say, four to six turns and is sufficient in itself to throw the spring outside the requirements even of a specification which permits a variation of 10 per cent. from the specified load. In the case of some large springs tested the load at a given compressed length was found to vary from 6000 to 14,000 lbs. The load required was 9000 lbs. These springs were from $\frac{15}{16}$ in. diameter bar and had only 3½ turns. In some of them the end turns did not touch the adjacent turn at the tip. These were the low ones. In others the end turns lapped nearly a half turn.

By far the larger number of springs which fail to pass are too strong. Springs are seldom below the requirements. This was explained by one spring maker on the ground that springs are usually accepted by railway companies even when considerably over the specified loads. Since the work done for railway companies forms a large

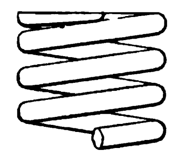


FIG. 13.

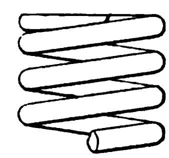


FIG. 14.

FIGS. 13 and 14.—Correct and incorrect constructions of springs.

part of their business, the spring maker usually aims high. This is given for what it is worth; as a matter of fact, we have seen that th ordinary defects of manufacture are such as would increase the load at a given deflection.

In a discussion on springs (*Trans. A. S. M. E. Vol.* 23) A. S. Cary referred to the many different methods of making springs, the various ways of preparing the wire for them, their treatment during manufacture and their treatment after they leave the spring machine.

Thus springs are made from hard drawn or hard rolled wire which receives no tempering treatment after being coiled. Wire made hard by working owes this quality of hardness to the many internal stresses it contains. Such springs have a comparatively limited elastic limit and are easily fatigued, although the resistance to extension or compression, within the elastic limit, is considerably greater than with any other kind of wire of the same size.

A hard drawn or hard rolled *steel* spring can be much improved by a process invented and patented by Mr. Cary's father, by which the spring, after being formed and pressed, is heated to a temperature between 400 and 700 deg. Fahr. and then rapidly cooled in a blast of cold air. A spring of this kind, after this treatment, seems to have the internal stresses which were introduced during the coiling and pressing processes removed; its elastic limit is materially increased and it is less easily fatigued.

In making *compression* springs by this process it is found necessary to coil them to a considerably greater pitch than is found in the finished spring, and then they must be subjected to a sudden overload beyond their original elastic limit which reduces their pitch considerably, and then we obtain a fairly efficient spring, but one inferior to a tempered steel spring.

The best and most durable springs that can be made are formed from comparatively high carbon soft steel wire which, after being finished, are hand tempered—that is, they are first heated in a charcoal fire to a cherry red or slightly higher, and then plunged into a liquid bath, which is generally one of oil. They are then carefully polished (over more or less of their surface) and held above the charcoal fire until the required temper color appears, which color differs with the various qualities of steel used.

There are many variations of this process, differing in small particulars, but so delicate are the different manipulations if uniform results in any considerable number of these springs are to be obtained, that the process has been almost entirely abandoned by spring manufacturers, the best of whom have adopted specially prepared tempered wire for their spring stock, and after forming and machining their springs they are tempered by the Cary process.

I might add here that my most uniform results in hand tempering springs were obtained by heating and afterwards drawing them by passing an electric current through them. The wire composing such hand tempered springs is, if they are properly made, free from all internal stresses when the spring is at rest, and in properly proportioned compression springs there is no setting or decrease in pitch after the coils are closed tightly one upon the other.

Another method of making springs is to take steel wire or bars and heat them to a lower temperature than a welding heat, then coil them hot on the arbor, and before they have an opportunity to cool below a dull red throw them into an oil bath. The quality of steel used in this process is sufficiently low in carbon to make it unnecessary to draw the temper after hardening, but such springs are not to be classed as high grade. Most of the heavy car springs are made this way.

In plunging red hot springs into their cooling bath great care must be exercised. If they are slowly immersed sideways, one side of the spring will often be tempered harder than the other, because of the different temperatures of the opposite sides of the spring when they are immersed. A similar result is sometimes obtained when long springs are slowly immersed endwise, one end of the spring being found harder than the other.

Experience has taught that the most serviceable extension springs are those coiled in such a manner as to have their coils, before extension, press so closely together as to require the application of a certain initial load before the coils begin to open. This initial set is obtained by using hard drawn or tempered spring wire and delivering it to the arbor on which the spring is formed in a twisted manner—that is, by twisting or revolving the wire around its own axis the same as the strands of a rope are twisted firmly together. It has been found that the Cary process of tempering does not affect this initial torsional strain in extension springs, although the hand tempering process, where the spring is heated to redness, destroys it.

Springs for use in salt water should be made of phosphor-bronze in order to avoid corrosion. According to the Brass World 1907, if the mixture is rightly made this material cannot be surpassed by anything except steel, but if not, it is inferior to yellow brass.

Experience has taught that if the phosphorus in rolled metal exceeds .05 per cent., the bronze is injured.

The greatest variation in rolled or drawn phosphor-bronze is caused by the tin content. A bronze which contains only 3 per cent. of tin is inferior to one which contains 8 per cent., although both may be phosphor-bronze. On account of the difficulty in rolling or drawing phosphor-bronze containing a high tin content, manufacturers will substitute a lower percentage if it is possible to do it.

A good spring should contain only copper, tin and a very small quantity of phosphorus.

Those who have had trouble with phosphor-bronze springs should ascertain whether their troubles are not caused by the absence of the necessary amount of tin, or the presence of zinc. The temper is produced by cold-rolling.

The terms lead and pitch, as applied to screw threads, are not always clearly defined, the result being confusion in the case of multiple-thread screws. A further confusion arises from a loose use of the word pitch in the case of single-thread screws which advance the nut somewhat near 1 in. per turn. Thus, while the term 8-pitch means, clearly enough, ⅛ in. pitch, the expression 1½ pitch is not clear because it is not known whether the screw is of one and a half turns per inch or of 1½ in. per turn. This form of expression has no proper application to screw threads and should be discontinued. The best form of expression, because of its universal application, is ⅛ in. pitch, 1½ in. pitch, etc. If the pitch is an aloquot part of an inch, for example ⅛, the expression 8 threads per inch is satisfactory, as it cannot lead to confusion.

The confusion between the terms lead and pitch should lead to the general, as it already has to considerable, adoption of the usage of these terms by the Brown and Sharpe Mfg. Co. By this usage, the advance of a screw to one turn is the *lead*, which is the only term

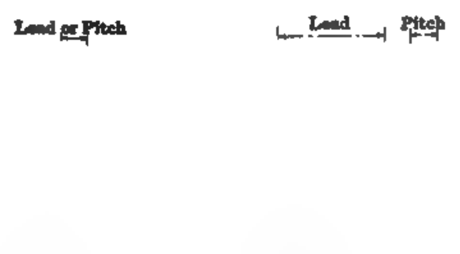


FIG. 1.—Single thread. FIG. 2.—Multiple thread.
Lead and Pitch of Screws and Worms.

they ever use to designate this advance. *The turns to an inch* are obtained by dividing 1 in. by the *lead*. Conversely, the quotient of 1 in. divided by the *number of turns to an inch* is the *lead*. In other words, the product of the *lead* multilplied by *the turns to an inch* is always equal to 1 *in*.

The term pitch has been limited to designate the distance between two consecutive threads or between two consecutive teeth. Divide 1 in. by the pitch and the quotient will be the threads to an inch. Divide 1 in. by number of threads to an inch and the quotient will be the pitch.

The product of the *pitch* multiplied by the number of threads to an inch is always equal to 1 in.

The distinction for single,- and multiple-thread worms or screws is shown in Figs. 1 and 2, from the former of which it will be seen that, for single,- but not for multiple-thread screws, pitch and lead are dentical (O. J. BEALE, *Amer. Mach., July* 18, 1907).

Screw Thread Standards

There is no standard V thread and the continuance of that construction is a simple nuisance. The taps and dies of different makers are not alike and will not interchange, while none of them agree with the theoretical or paper "standard." Under these circumstances it is impossible to give tables of dimensions of any value and for this reason such tables are here omitted. American tap and die makers are making a united effort to retire the V thread in favor of the U. S. Standard and their efforts should have the support of all.

Friction of and Resultant Load on Screws and Bolts

The friction of screw threads formed the subject of experiments by PROF. ALBERT KINGSBURY (*Trans. A. S. M. E., Vol.*, 17). The experiments were made on a set of square threaded screws and nuts of the following dimensions:

Outside diameter of screw	1.426 ins.
Inside diameter of nut	1.278 ins.
Mean diameter of thread	1.352 ins.
Pitch of thread	⅛ ins.
Effective depth of nut	$1\frac{1}{16}$ ins.

The conclusions are that for metallic screws in good condition, turning at extremely slow speeds, under any pressure up to 14,000 lbs. per sq. in. of bearing surface and freely lubricated before application of the pressure, the following coefficients may be used·

Lubricant	Coefficients of friction		
	Min.	Max.	Mean
Lard oil	.09	.25	.11
Heavy machinery oil (mineral)	.11	.19	.143
Heavy machinery oil and graphite in equal volumes.	.03	15	.07

Note that the experiments measured the friction of the thread surfaces alone—the friction of the step being eliminated by the construction of the apparatus.

The screws tested were made of various materials—mild steel, wrought iron, cast-iron, cast bronze and case hardened mild steel, and the nuts of mild steel, wrought iron, cast-iron, and cast brass. No material difference was found due to these materials.

In use, these coefficients should be substituted in the formula by which they were calculated as follows:

$$Q = P\frac{p + f\pi d}{\pi d - fp}$$

in which Q = tangential force necessary to turn the nut applied at the mean radius of the thread, lbs.

P = total axial load, lbs.
p = pitch of thread, ins.
d = mean diameter of thread, ins.
f = coefficient of friction.

For the efficiency of screws as affected by the helix angle of the thread, see Efficiency of Worm Gears. The same formulas apply to both constructions.

The resultant stress on bolts due to the initial stress resulting from tightening the nut and the addition of a load, such as the steam pressure on a cylinder head, depends on the relative elasticity of the bolt and the connected parts and is usually indeterminate. There are two extreme conditions between which actual cases usually lie. The extreme conditions are represented by Figs. 3 and 4.

Case I.—Elastic Bolt and Non-elastic Block.—Let the bolt be represented by a powerful spring, as in Fig. 2. Let the block be fixed and let the nut be screwed up to produce an initial strain of 5000 lbs., and then let the stirrup with its weight of 5000 lbs. be added. Under these conditions, the added weight will not increase the strain, because if it should the spring bolt would stretch under it, the block being non-elastic would not follow, the lower washer would leave contact with the block, and the supposed increased strain would instantly relieve itself—hence there can be no such increase.

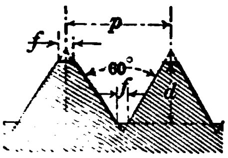
TABLE 1.—U. S. STANDARD BOLTS AND NUTS

Diameter		Thread		Area		Strength at 17,500 lbs.			Strength at 10,000 lbs.			Thickness		Short diameter of hexagon and square	Across corners		Weight of 100				Steel	Depth of thread	
						Tension 17,500 lbs. at root of thread	Shearing 10,000 lbs.		Tension 10,000 lbs. at root of thread	Shearing 7000 lbs.							Hexagon		Square		Weight of 100 round rods 1 in. long		
Of bolt	At root of thread	Per inch	Length of	Of bolt	At root of thread		Full bolt	At root of thread		Full bolt	Root of thread	Head	Nut		Hexagon	Square	Heads, lbs.	Nuts, lbs.	Heads, lbs.	Nuts, lbs.		Single	Double
1/4	.185	20	3/4-1	.049	.027	471	491	270	270	343	189	1/4	1/4	1/2	37/64	45/64	1.5	1.2	1.8	1.4	1.40	.0325	.0650
5/16	.240	18	7/8-1	.077	.045	795	767	450	450	539	315	19/64	5/16	19/32	11/16	27/32	1.9	2	2.3	2.4	2.17	.0361	.0722
3/8	.294	16	7/8-1	.110	.068	1,187	1,104	680	680	770	476	11/32	3/8	11/16	51/64	31/32	4	3.2	4.6	3.7	3.12	.0406	.0812
7/16	.344	14	1 - 1 1/4	.150	.093	1,633	1,503	930	930	1,050	651	25/64	7/16	25/32	29/32	1 7/64	5.6	4.8	6.8	5.6	4.25	.0464	.0928
1/2	.400	13	1 - 1 1/4	.196	.126	2,200	1,963	1,260	1,260	1,370	882	7/16	1/2	7/8	1 1/64	1 1/4	8.2	6.7	9.6	7.7	5.55	.0500	.1000
9/16	.454	12	1 1/4- 2	.248	.162	2,837	2,485	1,620	1,620	1,740	1,133	31/64	9/16	31/32	1 1/8	1 3/8	10.8	9	12.8	10.8	7.04	.0542	.1084
5/8	.507	11	1 1/4- 2	.307	.202	3,532	3,068	2,020	2,020	2,150	1,415	17/32	5/8	1 1/16	1 15/64	1 1/2	14.7	12	17	14.3	8.69	.0590	.1108
3/4	.620	10	1 3/8- 2	.442	.302	5,285	4,418	3,020	3,020	3,090	2,120	5/8	3/4	1 1/4	1 29/64	1 49/64	24	19.5	27.6	23.3	12.5	.0650	.1300
7/8	.731	9	1 3/8- 2 1/4	.601	.419	7,338	6,013	4,190	4,190	4,210	2,940	23/32	7/8	1 7/16	1 21/32	2 1/32	36.3	29.8	42	35.7	17.1	.0722	.1444
1	.837	8	1 3/4- 2 1/2	.785	.551	9,643	7,854	5,510	5,510	5,500	3,860	13/16	1	1 5/8	1 7/8	2 19/64	52.5	43	60	52.6	22.1	.0812	.1624
1 1/8	.940	7	2 1/4- 3	.994	.693	12,129	9,940	6,930	6,930	6,950	4,850	29/32	1 1/8	1 13/16	2 3/32	2 9/16	73	60	84	71.5	28.2	.0928	.1856
1 1/4	1.065	7	2 1/2- 3	1.227	.890	15,573	12,272	8,900	8,900	8,600	6,230	1	1 1/4	2	2 5/16	2 53/64	98	80	112.4	96.2	34.8	.0928	.1856
1 3/8	1.160	6	2 1/2- 3 1/2	1.485	1.054	18,447	14,849	10,540	10,540	10,400	7,380	1 3/32	1 3/8	2 3/16	2 17/32	3 3/32	128	105.3	145	126.7	42	.1083	.2166
1 1/2	1.284	6	3 - 4 1/2	1.767	1.294	22,642	17,671	12,940	12,940	12,400	9,050	1 3/16	1 1/2	2 3/8	2 3/4	3 23/64	164	134	190	161.6	55.7	.1083	.2166
1 5/8	1.389	5 1/2	3 - 4 1/2	2.074	1.515	26,511	20,739	15,150	15,150	14,500	10,600	1 9/32	1 5/8	2 9/16	2 31/32	3 5/8	202	166.7	235	206.7	58.7	.1181	.2362
1 3/4	1.491	5	3 - 5	2.405	1.745	30,522	24,053	17,450	17,450	16,850	12,200	1 3/8	1 3/4	2 3/4	3 3/16	3 57/64	255	206.6	295	256.4	68	.1300	.2600
1 7/8	1.616	5	3 1/2- 6	2.761	2.049	35,858	27,612	20,490	20,490	19,300	14,350	1 15/32	1 7/8	2 15/16	3 25/64	4 5/32	306	252.5	355	313.5	78.2	.1300	.2600
2	1.712	4 1/2	3 1/2- 6	3.142	2.300	40,252	31,416	23,000	23,000	22,000	16,100	1 9/16	2	3 1/8	3 39/64	4 27/64	364	306.8	432	380.2	89	.1444	.2888
2 1/4	1.962	4 1/2	4 - 7	3.976	3.021	52,873	39,761	30,210	30,210	27,800	21,200	1 3/4	2 1/4	3 1/2	4 3/64	4 61/64	526	434.8	609	540.6	112.6	.1444	.2888
2 1/2	2.176	4	4 - 7	4.909	3.716	65,035	49,087	37,160	37,160	34,400	26,000	1 15/16	2 1/2	3 7/8	4 15/32	5 31/64	735	609.7	838	751.9	138.6	.1625	.3250
2 3/4	2.426	4	4 1/2- 8	5.940	4.62	80,843	59,396	46,200	46,200	41,600	32,300	2 1/8	2 3/4	4 1/4	4 29/32	6 1/64	943	797.4	1,090	987.2	168.5	.1625	.3250
3	2.629	3 1/2	4 1/2- 9	7.069	5.428	94,985	70,686	54,280	54,280	49,500	38,000	2 5/16	3	4 5/8	5 11/32	6 35/64	1,220	1,025.6	1,400	1,269	200	.1857	.3714
3 1/4	2.879	3 1/2	5 -10	8.295	6.509	113,911	82,958	65,090	65,090	58,100	45,600	2 1/2	3 1/4	5	5 49/64	7 5/64	1,535	1,290	1,770	1,597.5	235	.1857	.3714
3 1/2	3.100	3 1/2	5 1/2-11	9.621	7.549	132,109	96,211	75,490	75,490	67,400	52,900	2 11/16	3 1/2	5 3/8	6 13/64	7 39/64	1,900	1,605.1	2,200	1,984.1	272.5	.2000	.4000
3 3/4	3.318	3	6 -12	11.044	8.641	151,221	110,447	86,410	86,410	77,000	60,500	2 7/8	3 3/4	5 3/4	6 5/8	8 1/8	2,320	1,968.5	2,695	2,439	311.2	.2166	.4332
4	3.567	3	6 -12	12.566	9.993	174,876	125,664	99,930	99,930	88,000	70,000	3 1/16	4	6 1/8	7 5/64	8 21/32	2,820	2,302.2	3,260	2,958.6	356.1	.2166	.4332

TABLE 2.—TAP DRILLS FOR U. S. FORM OF THREAD (Pratt & Whitney Co.)

Standard Sizes		Diameters from 1/16 to 17/64 in.											
Size, ins.	Size, of drill	Diam., ins.	No. of threads to the inch	Exact diameter bottom of thread, ins.	Gage No. of drill	Diam., ins.	No. of threads to the inch	Exact diameter bottom of thread, ins.	Gage No. of drill	Diam., ins.	No. of threads to the inch	Exact diameter bottom of thread, ins.	Gage No. of drill
1/4	12	1/16	60	.041	57	7/32	24	.164	19	11/64	32	.131	29
5/16	D	1/16	64	.042	56	7/32	28	.172	16	11/64	36	.136	28
3/8	N	3/32	48	.067	50	7/32	32	.178	14	11/64	40	.139	27
7/16	S	3/32	50	.068	50	7/32	36	.183	12	13/64	24	.149	24
1/2	27/64	3/32	56	.071	49	1/4	18	.178	14	13/64	28	.157	20
9/16	31/64	3/32	60	.072	48	1/4	20	.185	12	13/64	32	.162	19
5/8	17/32	1/8	40	.093	41	1/4	22	.190	10	13/64	36	.167	18
3/4	5/8	1/8	44	.096	40	1/4	24	.196	8	15/64	24	.180	13
7/8	23/32	1/8	48	.098	39	1/4	26	.200	6	15/64	28	.188	10
1	53/64	5/32	32	.116	31	5/64	56	.055	53	15/64	32	.194	8
1 1/8	15/16	5/32	36	.120	1/8	5/64	60	.056	53	15/64	36	.198	7
1 1/4	1 1/16	5/32	40	.124	30	7/64	40	.077	46	17/64	18	.193	9
1 3/8	1 11/64	3/16	24	.133	29	7/64	44	.080	45	17/64	20	.201	5
1 1/2	1 19/64	3/16	28	.141	27	7/64	48	.082	44	17/64	24	.211	3
1 5/8	1 13/32	3/16	30	.144	26	9/64	32	.100	37				
1 3/4	1 1/2									17/64	26	.216	2
1 7/8	1 5/8	3/16	32	.147	25	9/64	36	.105	15	17/64	32	.225	1

Table 3.—U. S. (Sellers) Standard Screw Thread



Formulas

$$p = \text{pitch} = \frac{1}{\text{No. threads per inch}}$$

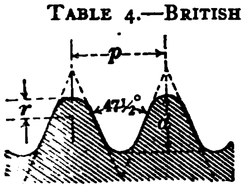
$$d = \text{depth} = p \times .64952$$

$$f = \text{flat} = \frac{p}{8}$$

Diameter of screw, ins.	Threads per in.	Diameter at root of thread, ins.	Width of flat, ins.
1/4	20	.185	.0062
5/16	18	.2403	.0069
3/8	16	.2938	.0078
7/16	14	.3447	.0089
1/2	13	.4001	.0096
9/16	12	.4542	.0104
5/8	11	.5069	.0114
*11/16	10	.5576	.0125
3/4	10	.6201	.0125
*13/16	9	.6682	.0139
7/8	9	.7307	.0139
*15/16	8	.7751	.0156
1	8	.8376	.0156
1 1/8	7	.9394	.0179
1 1/4	7	1.0644	.0179
1 3/8	6	1.1585	.0208
1 1/2	6	1.2835	.0208
1 5/8	5 1/2	1.3888	.0227
1 3/4	5	1.4902	.0250
1 7/8	5	1.6152	.0250
2	4 1/2	1.7113	.0278
2 1/8	4 1/2	1.8363	.0278
2 1/4	4 1/2	1.9613	.0278
2 3/8	4	2.0502	.0313
2 1/2	4	2.1752	.0313
2 5/8	4	2.3002	.0313
2 3/4	4	2.4252	.0313
2 7/8	3 1/2	2.5038	.0357
3	3 1/2	2.6288	.0357
3 1/4	3 1/2	2.8788	.0357
3 1/2	3 1/4	3.1003	.0385
3 3/4	3	3.3170	.0417
4	3	3.5670	.0417
4 1/4	2 7/8	3.7982	.0435
4 1/2	2 3/4	4.0276	.0455
4 3/4	2 5/8	4 2551	.0476
5	2 1/2	4.4804	.0500

*The 11/16, 13/16 and 15/16 are usually made 11, 10 and 9 threads per inch respectively, but under the Sellers' Formula, strictly followed, they should be 10, 9 and 8 respectively.

Table 4.—British Association Standard Screw Thread



Formulas

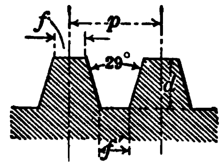
$$p = \text{pitch}$$

$$d = \text{depth} = p \times .6$$

$$r = \text{radius} = \frac{2 \times p}{11}$$

No.	Diameter, mm.	Pitch, mm.	No.	Diameter, mm.	Pitch, mm.
0	6.0	1.00	7	2.5	.48
1	5.3	.90	8	2.2	.43
2	4.7	.81	9	1.9	.39
3	4.1	.73	10	1.7	.35
4	3.64	.66	12	1.3	.28
5	3.2	.59	14	1.0	.23
6	2.8	.53	16	.79	.19

Table 5.—Acme Standard Screw Thread



Formulas

$$p = \text{pitch} = \frac{1}{\text{No. threads per inch}}$$

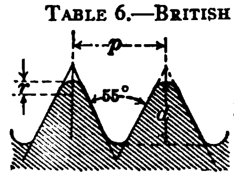
$$d = \text{depth} = \tfrac{1}{2}p + .010$$

$$f = \text{flat on top of thread} = p \times .3707$$

$$f' = \text{flat on bottom of thread} = p \times .3707 - .0052$$

Pitch	No. of threads per inch.	Depth of thread	Width at top of thread	Width at bottom of thread	Space at top of thread	Thickness at root of thread
2	1/2	1.010	.7414	.7362	1.2586	1.2637
1 7/8	8/15	.9475	.6950	.6897	1.1799	1.1850
1 3/4	4/7	.8850	.6487	.6435	1.1012	1.1064
1 5/8	8/13	.8225	.6025	.5973	1.0226	1.0277
1 1/2	2/3	.7600	.5560	.5508	.9439	.9491
1 7/16	16/23	.7287	.5329	.5277	.9046	.9097
1 3/8	8/11	.6975	.5097	.5045	.8652	.8704
1 5/16	16/21	.6662	.4865	.4813	.8259	.8311
1 1/4	4/5	.635	.4633	.4581	.7866	.7918
1 3/16	16/19	.6037	.4402	.4350	.7472	.7525
1 1/8	8/9	.5725	.4170	.4118	.7079	.7131
1 1/16	16/17	.5412	.3938	.3886	.6686	.6739
1	1	.510	.3707	.3655	.6293	.6345
15/16	1 1/15	.4787	.3476	.3424	.5898	.5950
7/8	1 1/7	.4475	.3243	.3191	.5506	.5558
13/16	1 3/13	.4162	.3012	.2960	.5112	.5164
3/4	1 1/3	.385	.2780	.2728	.4720	.4772
11/16	1 5/11	.3537	.2548	.2496	.4327	.4379
2/3	1 1/2	.3433	.2471	.2419	.4194	.4246
5/8	1 3/5	.3225	.2316	.2264	.3934	.3986
9/16	1 7/9	.2912	.2085	.2033	.3539	.3591
1/2	2	.260	.1853	.1801	.3147	.3199
7/16	2 2/7	.2287	.1622	.1570	.2752	.2804
2/5	2 1/2	.210	.1482	.1430	.2518	.2570
3/8	2 2/3	.1975	.1390	.1338	.2359	.2411
1/3	3	.1766	.1235	.1183	.2098	.2150
5/16	3 1/5	.1662	.1158	.1106	.1966	.2018
2/7	3 1/2	.1528	.1059	.1007	.1797	.1849
1/4	4	.1350	.0927	.0875	.1573	.1625
2/9	4 1/2	.1211	.0824	.0772	.1398	.1450
1/5	5	.110	.0741	.0689	.1259	.1311
3/16	5 1/3	.1037	.0695	.0643	.1179	.1232
1/6	6	.0933	.0617	.0565	.1049	.1101
1/7	7	.0814	.0530	.0478	.0899	.0951
1/8	8	.0725	.0463	.0411	.0787	.0839
1/9	9	.0655	.0413	.0361	.0699	.0751
1/10	10	.060	.0371	.0319	.0629	.0681
1/16	16	.0412	.0232	.0180	.0392	.0444

Table 6.—British (Whitworth) Standard Screw Thread



Formulas

$$p = \text{pitch} = \frac{1}{\text{No. thds. per. in.}}$$

$$d = \text{depth} = p \times .64033$$

$$r = \text{radius} = p \times .1373$$

Diam. ins.	No. thrds. per in.	Diam. ins.	No. thrds. per in.	Diam. ins.	No. thrds. per in.	Diam. ins.	No. thrds. per in.
1/4	20	7/8	9	2	4 1/2	3 1/4	3 1/4
5/16	18	15/16	9	2 1/8	4 1/2	3 3/8	3 1/4
3/8	16	1	8	2 1/4	4	3 1/2	3 1/4
7/16	14	1 1/8	7	2 3/8	4	3 5/8	3 1/4
1/2	12	1 1/4	7	2 1/2	4	3 3/4	3
9/16	12	1 3/8	6	2 5/8	4	3 7/8	3
5/8	11	1 1/2	6	2 3/4	3 1/2	4	3
11/16	11	1 5/8	5	2 7/8	3 1/2		
3/4	10	1 3/4	5	3	3 1/2		
13/16	10	1 7/8	4 1/2	3 1/8	3 1/2		

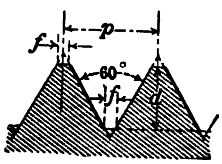
TABLE 7.—CONSTANTS FOR FINDING THE DIAMETER AT THE BOTTOM OF SCREW THREADS OF U. S. FORM (PRATT & WHITNEY CO.)

C = constant for number of threads per inch.
D = outside diameter.
D^1 = diameter at bottom of thread.

$$D^1 = D - C.$$

Threads per inch	Constant	Threads per inch	Constant
64	.02030	16	.08119
60	.02165	14	.09279
56	.02320	13	.09993
50	.02598	12	.10825
48	.02706	11½	.11296
44	.02952	11	.11809
40	.03248	10	.12990
36	.03608	9	.14434
32	.04059	8	.16238
30	.04330	7	.18558
28	.04639	6	.21651
27	.04812	5½	.23619
26	.04996	5	.25981
24	.05413	4½	.28868
22	.05905	4	.32476
20	.06495	3½	.37115
18	.07217	3	.43301

TABLE 8.—INTERNATIONAL AND FRENCH METRIC STANDARD SCREW THREADS



Formulas $\begin{cases} p = \text{pitch} \\ d = \text{depth} = p \times .64952 \\ f = \text{flat} = \dfrac{p}{8} \end{cases}$

Diameter of screw, mm.	Pitch, mm.	Diameter at root of thread, mm.	Width of flat, mm.
3	.5	2.35	.06
4	.75	3.03	.09
5	.75	4.03	.09
6	1.0	4.70	.13
7	1.0	5.70	.13
8	1.0	6.70	.13
8	1.25	6.38	.16
9	1.0	7.70	.13
9	1.25	7.38	.16
10	1.5	8.05	.19
11	1.5	9.05	.19
12	1.5	10.05	.19
12	1.75	9.73	.22
14	2.0	11.40	.25
16	2.0	13.40	.25
18	2.5	14.75	.31
20	2.5	16.75	.31
22	2.5	18.75	.31
24	3.0	20.10	.38
26	3.0	22.10	.38
27	3.0	23.10	.38
28	3.0	24.10	.38
30	3.5	25.45	.44
32	3.5	27.45	.44
33	3.5	28.45	.44
34	3.5	29.45	.44
36	4.0	30.80	.5
38	4.0	32.80	.5
39	4.0	33.80	.5
40	4.0	34.80	.5
42	4.5	36.15	.56
44	4.5	38.15	.56
45	4.5	39.15	.56
46	4.5	40.15	.56
48	5.0	41.51	.63
50	5.0	43.51	.63
52	5.0	45.51	.63
56	5.5	48.86	.69
60	5.5	52.86	.69

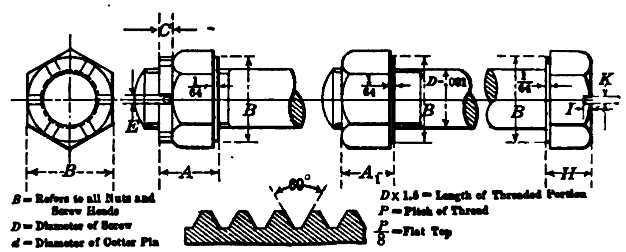


TABLE 9.—S. A. E. SCREW STANDARD

Dimensions.—All dimensions in inches.

Finish.—All heads and nuts to be semi-finish.

Material.—For all screws and nuts—steel; tensile strength, not less than 100,000 lbs. per square inch; elastic limit, not less than 60,000 lbs. per square inch.

Screws are to be left soft. Screw heads are to be left soft. The plain nuts are to be left soft. The castle nuts are to be case-hardened.

It is assumed that where screws are to be used in soft material, such as cast-iron, brass, bronze or aluminum, the United States standard pitches will be used.

Tolerance.—The body diameter of the screws shall be one-thousandth (.001″) inch less than the nominal diameter, with a plus tolerance of zero and a minus tolerance of two-thousandths (.002″) inch.

The nuts shall be a good fit without perceptible shake.

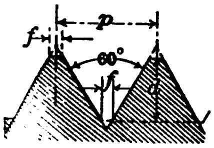
The tap shall be between two-thousandths (.002″) inch and three-thousandths (.003″) inch large.

D	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	7/8	1	1 1/8	1 1/4	1 3/8	1 1/2
P	28	24	24	20	20	18	18	16	16	14	14	12	12	12	12
A	7/32	17/64	21/64	23/64	7/16	31/64	35/64	39/64	41/64	49/64	1	1 1/32	1 1/4	1 13/32	1 1/2
A_1	9/32	21/64	27/64	3/8	9/16	39/64	43/64	23/32	49/64	55/64	7/8	63/64	1 3/32	1 13/64	1 5/16
B	7/16	1/2	9/16	5/8	3/4	7/8	15/16	1	1 1/16	1 1/4	1 7/16	1 5/8	1 13/16	2	2 3/16
C		3/32	1/8	1/8	5/32	5/32	1/4	1/4	1/4	1/4	1/4	5/16	5/16	3/8	3/8
E	3/64	3/64	1/8	1/8	1/8	5/32	5/32	5/32	5/32	5/32	5/32	7/32	7/32	1/4	1/4
H	3/16	15/64	9/32	21/64	3/8	27/64	31/64	35/64	9/16	43/64	3/4	27/32	15/16	1 1/32	1 1/8
I	3/32	3/32	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8	5/32	5/32	1/4	1/4
K	1/16	1/16	3/32	3/32	3/32	3/32	3/32	3/32	3/32	3/32	3/32	3/32	3/32	1/16	1/16
d	1/16	1/16	3/32	3/32	3/32	1/8	1/8	1/8	1/8	1/8	1/8	11/64	11/64	11/64	11/64

SIZES OF TAPS	DRILL SIZES
1/4 in. × 28 threads	7/32 in.[1]
5/16 in. × 24 threads	17/64 in.
3/8 in. × 24 threads	21/64 in.
7/16 in. × 20 threads	3/8 in.
1/2 in. × 20 threads	7/16 in.
9/16 in. × 18 threads	1/2 in.
5/8 in. × 18 threads	35/64 in.
11/16 in. × 16 threads	39/64 in.
3/4 in. × 16 threads	43/64 in.
7/8 in. × 14 threads	25/32 in.
1 in. × 14 threads	29/32 in.
1 1/8 in. × 12 threads	1 1/64 in.
1 1/4 in. × 12 threads	1 9/64 in.
1 3/8 in. × 12 threads	1 17/64 in.
1 1/2 in. × 12 threads	1 25/64 in.

[1] No. 5 Drill gage.

TABLE 10.—A. S. M. E. STANDARD MACHINE SCREWS, U. S. STANDARD FORM OF THREAD



Formulas $\begin{cases} p = \text{pitch} = \dfrac{1}{\text{No. thds. per in.}} \\ d = \text{depth} = p \times .64952 \\ f = \text{flat} = \dfrac{p}{8} \end{cases}$

Basic size		Outside diam.		Pitch diam.		Root diam.	
No.	O.D.-T.P.I.	Min.	Max.	Min.	Max.	Min.	Max.
0	.060-80	.0572	.0600	.0505	.0519	.0410	.0438
1	.073-72	.0700	.0730	.0625	.0640	.0520	.0550
2	.086-64	.0828	.0860	.0742	.0759	.0624	.0657
3	.099-56	.0955	.0990	.0857	.0874	.0721	.0758
4	.112-48	.1082	.1120	.0966	.0985	.0808	.0849
5	.125-44	.1210	.1250	.1082	.1102	.0910	.0955
6	.138-40	.1338	1380	.1197	.1218	.1007	.1055
7	.151-36	.1466	.1510	.1308	.1330	.1097	.1149
8	.164-36	.1596	.1640	.1438	.1460	.1227	.1279
9	.177-32	.1723	.1770	.1544	.1567	.1307	.1364
10	.190-30	.1852	.1900	.1660	.1684	.1407	.1467
12	.216-28	.2111	.2160	.1903	.1928	.1633	.1696
14	.242-24	.2368	.2420	.2123	.2149	.1807	.1879
16	.268-22	.2626	.2680	.2358	.2385	.2013	.2090
18	.294-20	.2884	.2940	.2587	.2615	.2208	.2290
20	.320-20	.3144	.3200	.2847	.2875	.2468	.2550
22	.346-18	.3402	.3460	.3070	.3099	.2649	.2738
24	.372-16	.3660	.3720	.3284	.3314	.2810	.2908
26	.398-16	.3920	.3980	.3544	.3574	.3070	.3168
28	.424-14	.4178	.4240	.3745	.3776	.3204	.3312
30	.450-14	.4438	.4500	.4005	.4036	.3464	.3572

NOTE.—Maximum sizes are standard.

TABLE 11.—A. S. M. E. STANDARD SPECIAL SCREWS, U. S. STANDARD FORM OF THREAD

Basic size		Outside diam.		Pitch diam.		Root diam.	
No.	O.D.-T.P.I.	Min.	Max.	Min.	Max.	Min.	Max.
1	.073-64	.0698	.0730	.0612	.0629	.0494	.0527
2	.086-56	.0825	.0860	.0727	.0744	.0591	.0628
3	.099-48	.0952	.0990	.0836	.0855	.0678	.0719
4	.112-40	.1078	.1120	.0937	.0958	.0747	.0795
	.112-36	.1076	.1120	.0918	.0940	.0707	.0759
5	.125-40	.1208	.1250	.1067	.1088	.0877	.0925
	.125-36	.1206	.1250	.1048	.1070	.0837	.0889
6	.138-36	.1336	.1380	.1178	.1200	.0967	.1019
	.138-32	.1333	.1380	.1154	.1177	.0917	.0974
7	.151-32	.1463	.1510	.1284	.1307	.1047	.1104
	.151-30	.1462	.1510	.1269	.1294	.1017	.1077
8	.164-32	.1593	.1640	.1414	.1437	.1177	.1234
	.164-30	.1592	.1640	.1399	.1423	.1147	.1207
9	.177-30	.1722	.1770	.1529	.1553	.1277	.1337
	.177-24	.1718	.1770	.1473	.1499	.1158	.1229
10	.190-32	.1853	.1900	.1674	.1697	.1437	.1494
	.190-24	.1848	.1900	.1603	.1629	.1287	.1359
12	.216-24	.2108	.2160	.1863	.1889	.1547	.1619
14	.242-20	.2364	.2420	.2067	.2095	.1688	.1770
16	.268-20	.2624	.2680	.2327	.2355	.1948	.2030
18	.294-18	.2882	.2940	.2550	.2579	.2129	.2218
20	.320-18	.3142	.3200	.2810	.2839	.2389	.2478
22	.346-16	.3400	.3460	.3024	.3054	.2550	.2648
24	.372-18	.3662	.3720	.3330	.3359	.2909	.2998
26	.398-14	.3918	.3980	.3485	.3516	.2944	.3052
28	.424-16	.4180	.4240	.3804	.3834	.3330	.3428
30	.450-16	.4440	.4500	.4064	.4094	.3590	.3688

NOTE.—Maximum sizes are standard.

There is a fairly widespread feeling that the differences between the maximum and minimum sizes of the above tables are too large.

TABLE 12.—TAP DRILLS FOR MACHINE SCREW TAPS (Pratt & Whitney Co.)

Size of tap	No. of threads	Size of drill	Size of tap	No. of threads	Size of drill
2	48	51	12	24	19
2	56	50	13	20	19
2	64	49	13	24	15
3	40	49	14	20	16
3	48	48	14	22	13
3	56	44	14	24	9
4	32	48	15	18	13
4	36	45	15	20	10
4	40	44	15	24	6
5	30	44	16	16	13
5	32	43	16	18	10
5	36	41	16	20	6
5	40	40	16	24	2
6	30	41	17	16	7
6	32	37	17	18	4
6	36	36	17	20	2
6	40	33	18	16	3
7	28	35	18	18	2
7	30	34	18	20	A
7	32	31	19	16	I
8	24	34	19	18	B
8	30	30	19	20	D
8	32	30	20	16	C
9	24	30	20	18	E
9	28	29	20	20	H
9	30	28	22	16	H
9	32	27	22	18	J
10	24	28	24	14	K
10	28	26	24	16	L
10	30	24	24	18	N
10	32	24	26	14	N
11	24	24	26	16	O
11	28	21	28	14	Q
11	30	19	28	16	S
12	20	24	30	14	T
12	22	20	30	16	V

These drills will give a thread near enough for all practical purposes, but not a *full* thread.

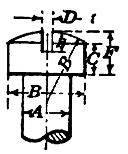
TABLE 13.—TAP DRILLS FOR A. S. M. E. STANDARD MACHINE SCREW TAPS (Pratt & Whitney Co.)

Size of tap	No. of threads	Size of drill	Size of tap	No. of threads	Size of drill
0	80	.0465	9	32	.1405
1	64	.055	10	24	.140
1	72	.0595	10	30	.152
2	56	.0670	10	32	.154
2	64	.070	12	24	.166
3	48	.076	12	28	.173
3	56	.0785	14	20	.182
4	36	.080	14	24	.1935
4	40	.082	16	20	.209
4	48	.089	16	22	.213
5	36	.0935	18	18	.228
5	40	.098	18	20	.234
5	44	.0995	20	18	.257
6	32	.1015	20	20	.261
6	36	.1065	22	16	.272
6	40	.110	22	18	.281
7	30	.113	24	16	.295
7	32	.116	24	18	.302
7	36	.120	26	14	.316
8	30	.1285	26	16	.323
8	32	.1285	28	14	.339
8	36	.136	28	16	.348
9	24	.1285	30	14	.368
9	30	.136	30	16	.377

The diameter given for each hole to be tapped allows for a practical clearance at the root of the thread of the screw and will not impose undue strain upon the tap in service.

TABLE 14.—A. S. M. E. STANDARD PROPORTIONS OF MACHINE SCREW HEADS

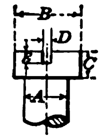
OVAL FILLISTER HEAD SCREWS



A = diameter of body
$B = 1.64A - .009$ = diam. of head and rad. for oval
$C = .66A - .002$ = height of side
$D = .173A + .015$
$E = \frac{1}{2}F$ = depth of slot
$F = .134B + C$ = height of head

A	B	C	D	E	F
.060	.0894	.0376	.025	.025	.0496
.073	.1107	.0461	.028	.030	.0609
.086	.132	.0548	.030	.036	.0725
.099	.153	.0633	.032	.042	.0838
.112	.1747	.0719	.034	.048	.0953
.125	.196	.0805	.037	.053	.1068
.138	.217	.089	.039	.059	.1180
.151	.2386	.0976	.041	.065	.1296
.164	.2599	.1062	.043	.071	.1410
.177	.2813	.1148	.046	.076	.1524
.190	.3026	.1234	.048	.082	.1639
.216	.3452	.1405	.052	.093	.1868
.242	.3879	.1577	.057	.105	.2097
.268	.4305	.1748	.061	.116	.2325
.294	.4731	.192	.066	.128	.2554
.320	.5158	.2092	.070	.140	.2783
.346	.5584	.2263	.075	.150	.3011
.372	.601	.2435	.079	.162	.3240
.398	.6437	.2606	.084	.173	.3469
.424	.6863	.2778	.088	.185	.3698
.450	.727	.295	.093	.201	.4024

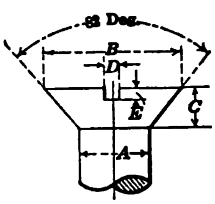
FLAT FILLISTER HEAD SCREWS



A = diam. of body
$B = 1.64A - .009$ = diam. of head
$C = .66A - .002$ = height of head
$D = .173A + .015$ = width of slot
$E = \frac{1}{2}C$ = depth of slot

A	B	C	D	E
.060	.0894	.0376	.025	.019
.073	.1107	.0461	.028	.023
.086	.132	.0548	.030	.027
.099	.153	.0633	.032	.032
.112	.1747	.0719	.034	.036
.125	.196	.0805	.037	.040
.138	.217	.0890	.039	.044
.151	.2386	.0976	.041	.049
.164	.2599	.1062	.043	.053
.177	.2813	.1148	.046	.057
.190	.3026	.1234	.048	.062
.216	.3452	.1405	.052	.070
.242	.3879	.1577	.057	.079
.268	.4305	.1748	.061	.087
.294	.4731	.1920	.066	.096
.320	.5158	.2092	.070	.104
.346	.5584	.2263	.075	.113
.372	.601	.2435	.079	.122
.398	.6437	.2606	.084	.130
.424	.6863	.2778	.088	.139
.450	.727	.295	.093	.147

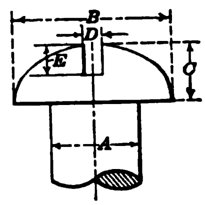
FLAT HEAD SCREWS



A = diameter of body
$B = 2A - .008$ = diameter of head
$C = \dfrac{A - .008}{1.739}$ = depth of head
$D = .173A + .015$ = width of slot
$E = \frac{1}{2}C$ = depth of slot

A	B	C	D	E
.060	.112	.029	.025	.010
.073	.138	.037	.028	.012
.086	.164	.045	.030	.015
.099	.190	.052	.032	.017
.112	.216	.060	.034	.020
.125	.242	.067	.037	.022
.138	.262	.075	.039	.025
.151	.294	.082	.041	.027
.164	.320	.090	.043	.030
.177	.346	.097	.046	.032
.190	.372	.105	.048	.035
.216	.424	.120	.052	.040
.242	.472	.135	.057	.045
.268	.528	.150	.061	.050
.294	.580	.164	.066	.055
.320	.632	.179	.070	.060
.346	.682	.194	.075	.065
.372	.732	.209	.079	.070
.398	.788	.224	.084	.075
.424	.840	.239	.088	.080
.450	.892	.254	.093	.085

ROUND HEAD SCREWS



A = diam. of body
$B = 1.85A - .005$ = diam. of head
$C = .7A$ = height of head
$D = .173A + .015$ = width of slot
$E = \frac{1}{2}C + .01$ = depth of slot

A	B	C	D	E
.060	.106	.042	.025	.031
.073	.130	.051	.028	.035
.086	.154	.060	.030	.040
.099	.178	.069	.032	.044
.112	.202	.078	.034	.049
.125	.226	.087	.037	.053
.138	.250	.096	.039	.058
.151	.274	.105	.041	.062
.164	.298	.114	.043	.067
.177	.322	.123	.046	.071
.190	.346	.133	.048	.076
.216	.394	.151	.052	.085
.242	.443	.169	.057	.094
.268	.491	.187	.061	.103
.294	.539	.205	.066	.112
.320	.587	.224	.070	.122
.346	.635	.242	.075	.131
.372	.683	.260	.079	.140
.398	.731	.278	.084	.149
.424	.779	.296	.088	.158
.450	.827	.315	.093	.167

TABLE 15.—S. A. E. LOCK WASHER STANDARDS

AUTOMOBILE HEAVY (A. H.)

For General Use

Temper.—After compression to flat, reaction shall be sufficient to indicate necessary spring power, and on a subsequent compression to flat the lock washer shall manifest no appreciable loss in reaction.

Toughness.—Forty-five per cent. of the lock washer, including one end, shall be firmly secured in a vise, and 45 per cent., including the other end, shall be secured firmly between parallel jaws of a wrench. Movement of the wrench at right angle to helical curve shall twist the lock washer through 45 deg. without sign of fracture; and shall twist the lock washer entirely apart within 135 deg.

The outside diameters of lock washers shall coincide practically with the long diameters of S. A. E. Standard nuts, which are approximately the short diameters of United States Standard nuts.

The inside diameters of the lock washers shall be from 1/64 in. to 1/32 in. larger than bolt diameters.

All lock washers shall be parallel-faced sections; and bulging or malformed ends must be avoided.

Bolt diameter	Lock washer section	Bolt diameter	Lock washer section
3/16 in.	1/16 in.×1/16 in.	11/16 in.	1/4 in.×1/4 in.
1/4 in.	5/64 in.×5/64 in.	3/4 in.	1/4 in.×1/4 in.
5/16 in.	1/8 in.×1/8 in.	7/8 in.	21/64 in.×21/64 in.
3/8 in.	1/8 in.×1/8 in.	1 in.	5/16 in.×5/16 in.
7/16 in.	9/64 in.×9/64 in.	1 1/8 in.	5/16 in.×5/16 in.
1/2 in.	11/64 in.×11/64 in.	1 1/4 in.	3/8 in.×3/8 in.
9/16 in.	11/64 in.×11/64 in.	1 3/8 in.	3/8 in.×3/8 in.
5/8 in.	13/64 in.×13/64 in.	1 1/2 in.	7/16 in.×7/16 in.

AUTOMOBILE LIGHT (A. L.)

For Optional use Against Soft Metal

Bolt diameter	Lock washer section
3/16 in.	1/16 in.×3/64 in.
1/4 in.	5/64 in.×1/16 in.
5/16 in.	1/8 in.×3/32 in.
3/8 in.	1/8 in.×3/32 in.
7/16 in.	11/64 in.× 1/8 in.
1/2 in.	11/64 in.× 1/8 in.
9/16 in.	11/64 in.×5/32 in.
5/8 in.	11/64 in.×5/32 in.
11/16 in.	1/4 in.×3/16 in.
3/4 in.	1/4 in.×3/16 in.
7/8 in.	21/64 in.×3/16 in.
1 in.	5/16 in.× 1/4 in.

Case II.—Elastic Block and Non-elastic Bolt.—Let the block be now represented by a spring, as in Fig. 4. As before, let the nut be screwed up to produce an initial strain of 5000 lbs., and then let the stirrup and weight be added. Obviously the bolt is now loaded with a strain of 10,000 lbs., because, unlike the first case, the second load has in no way affected the first.

In actual cases the situation is more involved, as both block and bolt are elastic, and the question becomes one of difference of elasticity; but the conditions and the final effect vary between the two cases shown as extremes. It is sometimes possible, but more often impossible, to say which extreme is most nearly approximated, but when the whole matter hinges on such obscure conditions, the only safe course is the conservative one, to regard the initial load as part of the final strain.

TABLE 16.—TENSILE AND SHEARING STRENGTHS OF S. A. E. STANDARD BOLTS AND NUTS. BY JOSEPH A. ANGLADA

Bolt			Areas		Tensile strength			Shearing strength			
								Full Bolt		Bottom of thread	
Nominal diameter	Threads per inch	Diameter, bottom of thread	Bolt	Bottom of thread	At 20,000 lbs. per sq. in.	At 25,000 lbs. per sq. in.	At 30,000 lbs. per sq. in.	At 15,000 lbs. per sq. in.	At 22,500 lbs. per sq. in.	At 15,000 lbs. per sq. in.	At 22,500 lbs. per sq. in.
1/4	28	.2037	.0491	.0325	651	814	977	737	1,105	488	731
5/16	24	.2584	.0767	.0525	1,050	1,312	1,575	1,151	1,726	788	1,181
3/8	24	.3209	.1104	.0808	1,617	2,021	2,426	1,656	2,484	1,212	1,818
7/16	20	.3626	.1503	.1132	2,264	2,830	3,396	2,255	3,382	1,698	2,547
1/2	20	.4351	.1963	.1486	2,972	3,726	4,459	2,945	4,417	2,229	3,344
9/16	18	.4904	.2485	.1888	3,777	4,722	5,666	3,728	5,591	2,832	4,248
5/8	18	.5529	.3068	.2400	4,800	6,000	7,200	4,602	6,903	3,600	5,400
11/16	16	.6064	.3712	.2888	5,776	7,220	8,664	5,568	8,352	4,332	6,198
3/4	16	.6680	.4418	.3514	7,028	8,785	10,542	6,627	9,941	5,271	7,907
7/8	14	.7823	.6013	.4816	9,633	12,141	14,449	9,020	13,529	7,224	10,836
1	14	.9073	.7854	.6463	12,926	16,158	19,380	11,781	17,672	9,695	14,542

TABLE 17.—TAP DRILLS FOR STANDARD PIPE TAPS

(No Reamers Required)

Nominal size	Thds. per in.	Dbl depth of th'd	Tap drill	Outside diameter
		inches		
1/8	27	.048	11/32	.405
1/4	18	.072	29/64	.540
3/8	18	.072	19/32	.675
1/2	14	.093	23/32	.840
3/4	14	.093	15/16	1.050
1	11 1/2	.113	1 5/32	1.315
1 1/4	11 1/2	.113	1 31/64	1.660
1 1/2	11 1/2	.113	1 23/32	1.900
2	11 1/2	.113	2 3/16	2.375
2 1/2	8	.162	2 11/16	2.875
3	8	.162	3 5/16	3.500
3 1/2	8	.162	2 13/16	4.000
4	8	.162	4 5/16	4.500
4 1/2	8	.162	4 13/16	5.000
5	8	.162	5 3/8	5.563
6	8	.162	6 7/16	6.625
7	8	.162	7 7/16	7.625
8	8	.162	8 7/16	8.625
9	8	.162	9 7/16	9.625
10	8	.162	10 9/16	10.750
11	8	.162	11 15/16	12.000
12	8	.162	12 15/16	13.000

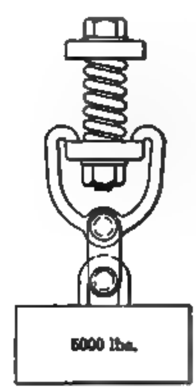


FIG. 3. FIG. 4.

FIGS. 3 and 4.—Resultant stress on bolts due to initial and applied loads.

Taper Bolts

The taper bolt system of securing parts together, universally used for the exacting conditions of locomotive work, deserves wider use in other fields than it has received. Following are the particulars of the Baldwin Locomotive Works standard (*Amer. Mach., May* 22, 1902) which has been in entirely satisfactory use since 1884:

All bolts in this system are fitted in reamed gages made of cast-iron and of suitable length and section; the gages are known as 9, 12, 18, 24 and 30-in. blocks. A bolt 9 ins. in length—measured from under the head to the point—is taken as a starting point. This bolt (shown in Fig. 5 at *C*) is exactly 1 in. in diameter at the point, and, consequently, at a taper of $\frac{1}{16}$ in. per foot it is $1\frac{3}{64}$ ins. in diameter under the head.

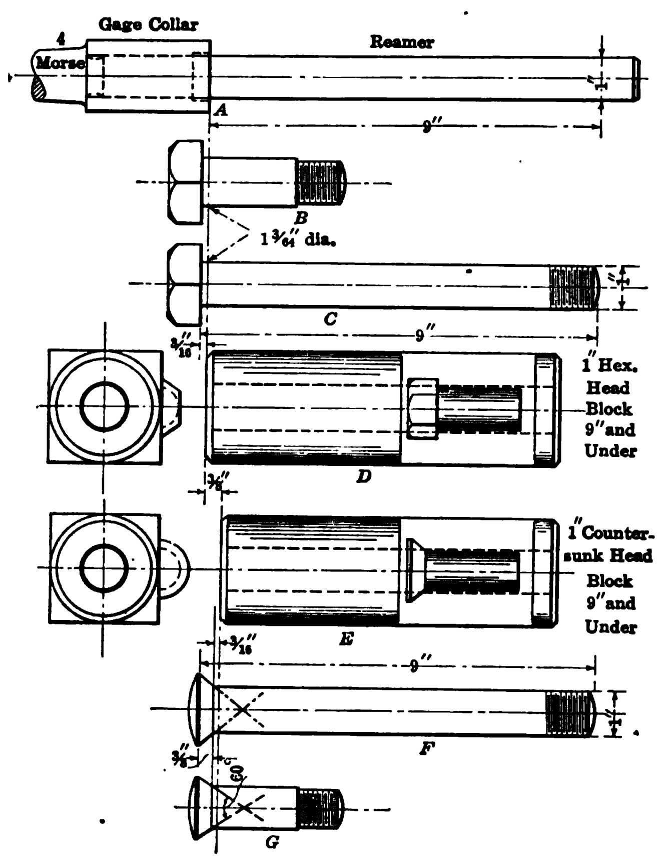


FIG. 5.—The Baldwin Locomotive Works standard taper bolt system.

The diameter at the small end and the angle of taper being the same, the amount of taper for the 12-in. bolt is $\frac{1}{16}$, for the 18-in. bolt $\frac{3}{32}$, for the 24-in. bolt, $\frac{1}{8}$, and for the 30-in. bolt, $\frac{5}{32}$ in. The large end of the hole of the various blocks has the following diameters:

Length	Diameter
9 ins.	$1\frac{3}{64}$ ins.
12 ins.	$1\frac{1}{16}$ ins.
18 ins.	$1\frac{21}{32}$ ins.
24 ins.	$1\frac{1}{8}$ ins.
30 ins.	$1\frac{5}{32}$ ins.

All bolts 9 ins. and under in length —as *B* and *C*—are fitted in the 9-in. block, all from 9 to 12 ins. in the 12-in. block, all from 12 to 18 ins. in the 18-in. block, all from 18 to 24 ins. in the 24-in. block, all from 24 to 30 ins. in the 30-in. block.

A reamer of length suitable to ream a hole for a 30-in. bolt would answer for any bolt of lesser length, but would be too clumsy.

In practice it is found more convenient to have reamers for each gage division, and these are known as 9, 12, 18, 24 and 30-in. hand and machine reamers. The flutes of all reamers are made long enough to allow for 3 ins. wear.

Gage collars, as shown at *A*, reamed and counterbored, are driven on the upper part of the flutes and under shank or head, and coming exactly to the top of block when the reamer is inserted, insure a hole in the work the same size as the hole in the block.

All holes being reamed standard, the allowance for snug driving fits is made by fitting all bolts to stand out of the gage blocks $\frac{3}{16}$ in., which has been found sufficient.

A taper of more than $\frac{1}{16}$ in. per ft. offers no advantage, but has the fault of making a long bolt too large under the head.

Thus far only the hexagon and square head bolts have been considered. Two other kinds are used, the countersunk and the round counterbore head bolts. The countersunk head bolt, shown at *F* and *G*, is used in places where a hexagon head would interfere. The included angle of this head is 60 deg. and the head is $\frac{3}{8}$ in. thick. This style of bolt requires a gage block, *E*, so made that the standard plug gage will stand out $\frac{3}{8}$ in. The counterbore head bolt is used where a very strong concealed head is desired, the head usually driving snugly in the counterbore. This style bolt is fitted in the hexagon head-bolt gage. The hole for this bolt body when reamed is made the size of the bolt under the head.

The same reamer is used for all bolts, and the same allowance for drive, viz., $\frac{3}{16}$ in., is made for all styles. The blocks, as shown in the illustration, have cast on the side a descriptive shape which aids the workman in finding the one desired, whether hexagon or countersunk.

This system recommends itself in that it contains but few standards for each nominal diameter of bolt and provides for a multitude of lengths.

To preserve the sizes, a set of master plugs is kept in the tool-room. When the gage blocks are worn they are easily restored to standard by re-reaming and facing off the top to suit the plug gage.

The gage blocks for regular diameters have the following lengths:

Diameter, ins.	Length, ins.
$\frac{1}{4}$, $\frac{3}{8}$	9
$\frac{1}{2}$, $\frac{5}{8}$, $\frac{3}{4}$	9 and 12
$\frac{7}{8}$	9, 12 and 18
1, $1\frac{1}{8}$, $1\frac{1}{4}$, $1\frac{3}{8}$, $1\frac{1}{2}$	9, 12, 18, 24 and 30

Split Nuts

The interference of split nuts with lead screws may be determined by the method shown in Fig. 6 by H. S. FULLERTON (*Mchy. June*, 1912). Parallel with the parting line of the half nuts, draw the line *AB* at a distance *K* from the center line such that:

$$K = \frac{l}{2\pi \tan \theta}$$

in which, l = lead of thread, ins.,

θ = angle between side of thread and a perpendicular to the axis of the screw.

For the Acme thread, in which $\theta = 14\frac{1}{2}$ deg., the formula becomes:

$$K = \frac{8l}{13}$$

Tangent to the outside and root diameters, draw perpendiculars cutting AB at m and n. Draw radial lines to these points cutting the outside and root diameters, respectively, at r and s. These are the interference points of the respective diameters. Points for intermediate diameters may be located in like manner and a curve drawn through them. For all practical purposes a straight line drawn through the interference points located on the outside and root diameters will be found a sufficiently close approximation. The illustration shows in correct proportions an end elevation of a pair of nuts for a screw 4 ins. in diameter with 1-in. lead single Acme threads. The dot-and-dash lines above are the interference curves for 2-, 3-, and 4-in. lead Acme threads.

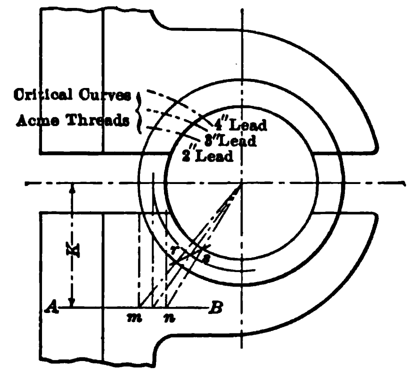


FIG. 6.—Finding the interference between screws and split nuts.

WIRE AND SHEET METAL GAGES

The multiplication of wire and sheet metal gages is the source of much confusion and has become an intolerable nuisance. The suggestion has been repeatedly made that the entire gage system be abandoned and that sizes be specified by their decimal values. The difficulty in carrying out this plan is that the gage sizes must be adhered to in order to obtain the material from stock and this compels constant reference to the gage tables. The following plan, due to the Westinghouse Electric and Mfg. Co. (*Amer. Mach., Apr.* 14, 1904) while keeping to the gage sizes eliminates all reference to the gage numbers. After nine years of use it is found to accomplish the desired object.

The existing standard dimensions of gaged materials are not changed, but the gage names, the conflicting and arbitrary gage numbers, and the commercial gage plates for identifying materials are discarded. The same actual dimensions as hitherto indicated by gage numbers continue in use but are expressed in decimal parts of the inch, the unnecessary refinements found in many of the commercial gage equivalent tables being, however, dropped.

All material that has heretofore been known by gage number, as No. 20 B. & S. sheet copper, is now known by decimal thickness only, as .032 sheet copper.

Throughout the business of the company, and on all drawings, drawing lists, specifications, bills of material and correspondence, decimal dimensions are used instead of gage numbers.

In the shop and storerooms all material that was formerly gaged is now measured with the micrometer or limit gages, and is specified, ordered, marked and carried in stock by decimal thicknesses instead of by gage numbers.

Drawings, drawing lists, specifications, bills of material, etc., made before the adoption of this plan, and specifying gage numbers were not changed except as found convenient by the engineering department.

The extreme refinements shown by the fifth and sixth decimal places have been dropped, not more than three significant figures being used in specifying sizes. By significant figures is meant all figures to the right of ciphers after the decimal point. For example, U. S. Standard No. 2 sheet gage is given as .265625. The Westinghouse decimal for this gage is .266.

Materials that were formerly purchased by gage numbers are now purchased of the same dimensions expressed in decimals.

It should be especially noted that, with the exception of twist drills, the above changes do not affect finished articles of any kind such as are kept in stock by manufacturers and known to the trade by gage numbers.

In Table 1 will be found a column giving American screw gage sizes. This has been inserted for convenience in selecting sizes of machine screws and wood screws, and it should be especially noted that the gage numbers are retained for these sizes.

Tables 1 and 2 are for use in those departments whence specifications for material emanate. Columns 1 to 10, Table 2, are for reference in connection with Table 1 which serves as an index to Table 2. In each column the decimals coincide with a series of size equivalents commercially known by gage name and number. Table 2 ignores all gage names and numbers; the dimensions are not numbered in any way, all read from the largest down, and are so arranged with respect to one another that the same (approximate) dimensions are on the same horizontal line. This arrangement makes it easy to choose in one column a dimension coinciding closely with a dimension in any other column.

The reference numbers in Table 1 indicate in which column of Table 2 to look for commercial diameters or thickness of any given material.

Example.—For commercial diameters of steel spring wire refer to Table 1 opposite *Wire, Spring* and, under *Steel* (*S*) read 3, showing that the commercial sizes of steel spring wire are to be found in column 3 of Table 2. Similarly Table 1 shows that the sizes of brass or phosphor-bronze spring wire are to be found in column 1 of Table 2.

THE WESTINGHOUSE METHOD OF ABANDONING SHEET METAL AND WIRE GAGES

TABLE 1.—INDEX TO COLUMN HEADLINES OF TABLE 2

	Index to columns 1 to 10	Cp.	Br.	S.	Ph. bz.	W.I.	Ger. Sil.	Ld.	Alloy	Aln	Pl.	Zn
Sheet or plate	Commercial	1	1	5	1	5	1		1	1	1	6
	Planished			5		5						
	Galvanized			5		5						
	Tinned			5		5						
	Terne			5								
	Spring		1	2	1							
Wire	Bare	1	1	3	1	3	1			1	1	1
	Insulated	1								1		
	Galvanized			3		3						
	Tinned[4]			3								
	Spring		1	3	1							
	Music			4								
	Resistance						1		1		1	
	Annealed			3		3						
Rod	Commercial	1	1		1[2]							
	Cold rolled	1	1	8[1]								
	Drill			8								
Tube	Seamless[3]	2	2		2		2			2		
	Brazed		1									

Twist drills: 9. Coppered steel wire: 3.
All cable, lamp cord and fuse wire: 1.

[1] ⅜ in. dia. up can be had in fractions.
[2] ½ in. dia. and larger, in fractions.
[3] All seamless tubing may be specified by diameters instead of thickness of wall.
[4] Tinned steel banding wire: special diameters.

Explanatory.—With the exception of twist drills, the following instructions to draftsmen do *not* affect *finished articles* such as are known to the trade by gage numbers. For example: Machine and wood screws will continue to be specified by the American screw gage numbers.

Instructions to draftsmen.—When specifying material of "gage" thickness, do *not* give gage name or number, but specify in decimals, thus:

.036 Sheet copper. .162 Brass rod.

Amer. screw gage	
000	.0315
00	.0447
0	.0578
1	.0710
2	.0842
3	.0973
4	.110
5	.124
6	.137
7	.150
8	.163
9	.176
10	.189
11	.203
12	.216
13	.229
14	.242
15	.255
16	.268
17	.282
18	.295
19	.308
20	.321
21	.334
22	.347
23	.361
24	.374
25	.387
26	.400
27	.413
28	.426
29	.439
30	.453
31	.466
32	.479
33	.492
34	.505

TABLE 2.—GAGE SIZES IN DECIMALS

1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10
		.490		.500	.500	.505											.100		
						.492											.106	.107	
						.479											.103	.104	
.460	.454	.460	.460	.469		.466				.102	.109	.106	.102		.100	.0973	.101	.102	.100
						.453											.099	.0995	
		.430		.438		.439											.097	.0980	
						.426											.095	.0960	.095
.410	.425	.394	.410	.406		.413	.413	.413										.0935	
							.404	.404		0907	.095	.0915	.090	.0938	.090		.092	.0890	.090
						.400	.397	.397					.086				.088	.0860	
						.387	.386	.386									.085	.0820	.085
						.374	.377	.377		.0808	.083	.0800	.082	.0781	.080	.0842	.081	.0810	.080
.365	.380	.363	.365	.375	.375	.361	.368	.368					.078				.079	.0785	
							.358	.358									.077		
	.340	.331		.344		.347	.348	.348					.074				.075	.0760	.075
							.339	.339		.0720	.072	.0720	.071	.0703	.070	.0710	.072	.0730	.070
						.334	.332	.332									.069	.0700	
.325			.325	.313		.321	.323	.323					.067				.066	.0670	
							.316	.316		.0641	.065	.0625	.063	.0625	.060		.063	.0635	.065
	.300	.307				.308	.302	.302					.059					.0595	.060
							.295	.295		.0571	.058	.0540	.055	.0563	.055	.0578	.058	.0550	.055
.289	.284	.283	.289	.281		.295	.290	.290									.055		
						.282	.281	.281		.0508	.049		.051	.0500	.050		.050	.0520	.050
							.277	.277					.048						
							.272	.272		.0453		.0475	.046	.0438	.045	.0447	.045	.0465	.045
				.266		.268	.266	.266					.044					.0430	
							.261	.261					.042				.042	.0420	
.258	.259	.263	.258	.250	.250	.255	.257	.257									.041	.0410	
							.250	.250		.0403	.042	.0410	.040		.040		.040	.0400	.040
							.246	.246									.039	.0390	
		.244				.242	.242	.242	.240				.038				.038	.0380	
							.238	.238									.037	.0370	
							.234	.234		.0359	.035	.0348	.036	.0375	.036		.036	.0360	.036
.229	.238	.225	.229	.234		.229	.227	.228	.220				.034	.0344			.035	.0350	
	.220			.219			.219	.221									.033	.0330	
						.216	.212	.213		.0320	.032	.0318	.032	.0313	.032	.0315	.032	.0320	.332
								.209									.031	.0310	
							.207	.206					.030				.030		
.204	.203	.207	.204	.203		.203	.204	.204	.200								.029	.0293	
							.201	.201		.0285	.028	.0286	.028	.0281	.028		.027	.0280	.028
							.199	.199					.026				.026	.0260	
							.197	.196		.0254	.025	.0258	.024	.0250	.024		.024	.0250	.025
							.194	.194										.0240	
		.192					.191	.181		.0226	.022	.0230	.022	.0219			.023	.0225	.022
						.189	.188	.189									.022	.0210	
							.185	.185		.0201	.020	.0204	.020	.0188	.020		.020	.0200	.020
.182	.180	.177	.182	.188		.176	.182	.182	.180	.0179	.018	.0181	.018	.0172	.018		.018	.0180	.018
							.180	.180				.0173							
							.178	.177		.0159	.016	.0162	.016	.0156	.016		.016	.0160	.016
							.175					.0150					.015		
				.172			.172	.173		.1042	.014	.0140	.014	.0141	.014		.014	.0145	.014
							.168	.170				.0135							
							.164	.166		.0126	.013	.0128	.013	.0125	.012		.013	.0135	.012
.162	.165	.162	.162			.163	.161	.161	.165		.012		.012						
								.159		.0113	.010	.0118	.011	.0109					
				.156			.157	.157		.0100		.0104	.010	.0102	.010				.010
							.155	.154				.0095		.0094					
							.153	.152		.0089	.009	.0090	.009	.0086					
							.151	.150				.0085							
						.150	.148	.147		.0080	.008	.0080	.0085	.0078	.008				.008
							.146					.0075							
.144	.148	.148	.144	.141			.143	.144	.150	.0071	.007	.0070		.0070					
							.139	.141						.0066					
						.137	.134	.136	.135	.0063				.0063	.006				.006
.129	.134	.135	.129	.125	.125	.124	.127	.129	.125	.0056									
	.120	.121					.120	.120		.0050	.005								
								.116		.0045									
.114			.114	.109		.110	.115	.113	.110	.0040	.004								.004
							.112	.111		.0035									
							.110	.110		.0031									.002

The following are the names of the gages to which the dimensions in columns 1 to 10 agree; in some cases the same gage is known by more than one name.
1. Brown & Sharpe; American Standard Wire. 2. Birmingham; Stubb's Iron Wire. 3. National; Rœbling's; Washburn & Moen; American Steel Wire Co. 4. Down to .102, Brown & Sharpe; .090 and down, Trenton or Wolff's Music Wire. 5. United States Standard. 6. Zinc. 7. American Screw. 8. Stubb's Steel Wire. 9. Morse Twist Drill and Steel Wire. 10. Master Mechanics' Decimal.

TABLE 3.—LIST OF NINE DIFFERENT STANDARD GAUGES USED IN THE UNITED STATES

No. of gage	American or Brown and Sharpe iron wire	Birmingham or Stubb's iron wire	Washburn and Moen iron wire	Imperial wire gage	U. S. Standard for plate (iron and steel)	Stubb's steel wire	Twist drill and steel wire	Washblrn and Moen music wire	Wood and machine screws	No. of gage
8–0							This gage from one to three thousanths larger than same Nos. of Stubb's steel wire gage.	.0083		8–0
7–0					.500			.0087		7–0
6–0				.464	.469			.0095		6–0
5–0				432	.438			.010		5–0
4–0	.460	.454	.394	.400	.406			.011		4–0
3–0	.410	.425	.363	.372	.375			.012	.032	3–0
2–0	.365	.380	.331	.348	.344			.013	.045	2–0
0	.325	.340	.307	.334	.313			.014	.058	0
1	.289	.300	.283	.300	.281	.227	.228	.016	.071	1
2	.258	.284	.263	.276	.266	.219	.221	.017	.084	2
3	.229	.259	.244	.252	.250	.212	.213	.018	.097	3
4	.204	.238	.225	.232	.234	.207	.209	.019	.110	4
5	.182	.220	.207	.212	.219	.204	.206	.020	.124	5
6	.162	.203	.192	.192	.203	.201	.204	.022	.137	6
7	.144	.180	.177	.176	.188	.199	.201	.023	.150	7
8	.128	.165	.162	.160	.172	.197	.199	.024	.163	8
9	.114	.148	.148	.144	.156	.194	.196	.026	.176	9
10	.102	.134	.135	.128	.141	.191	.194	.027	.189	10
11	.091	.120	.121	.116	.125	.188	.191	.028	.203	11
12	.081	.109	.106	.104	.109	.185	.189	.030	.216	12
13	.072	.095	.092	.092	.094	.182	.185	.031	.229	13
14	.064	.083	.080	.080	.078	.180	.182	.033	.242	14
15	.057	.072	.072	.072	.070	.178	.180	.035	.255	15
16	.051	.065	.063	.064	.063	.175	.177	.036	.268	16
17	.045	.058	.054	.056	.056	.172	.173	.038	.282	17
18	.040	.049	.048	.048	.050	.168	.170	.040	.295	18
19	.036	.042	.041	.040	.044	.164	.166	.041	.308	19
20	.032	.035	.035	.036	.038	.161	.161	.043	.321	20
21	.028	.032	.032	.032	.034	.157	.159	.046	.334	21
22	.025	.028	.029	.028	.031	.155	.157	.048	.347	22
23	.023	.025	.026	.024	.028	.153	.154	.051	.360	23
24	.020	.022	.023	.022	.025	.151	.152	.055	.374	24
25	.018	.020	.020	.020	.022	.148	.150	.059	.387	25
26	.016	.018	.018	.018	.019	.146	.147	.063	.400	26
27	.0141	.016	.0173	.0164	.0171	.143	.144	.066	.413	27
28	.0126	.014	.0162	.0149	.0156	.139	.141	.072	.426	28
29	.0112	.013	.015	.0136	.014	.134	.136	.076	.439	29
30	.010	.012	.014	.0124	.0125	.127	.129	.080	.453	30
31	.0089	.010	.0132	.0116	.0109	.120	.120		.466	31
32	.0079	.009	.0128	.0108	.0101	.115	.116		.479	32
33	.007	.008	.0118	.010	.0093	.112	.113		.492	33
34	.0063	.007	.0104	.0092	.0085	.110	.111		.505	34
35	.0056	.005	.0095	.0084	.0078	.108	.110		.518	35
36	.005	.004	.009	.0076	.007	.106	.1065		.532	36
37	.0044			.0068	.0066	.103	.104		.545	37
38	.0039			.006	.0062	.101	.1015		.558	38
39	.0035			.0052		.099	.0995		.571	39
40	.0031			.0048		.097	.098		.584	40
41						.095	.096		.597	41
42						.092	.094		.611	42
43						.088	.089		.624	43
44						.085	.086		.637	44
45						.081	.082		.650	45
46						.079	.081		.663	46
47						.077	.079		.676	47
48						.075	.076		.690	48
49						.072	.073		.703	49
50						.069	.070		.716	50

LETTER SIZES STUBB'S STEEL WIRE

A	.234	I	.272	R	.339
B	.238	J	.277	S	.348
C	.242	K	.281	T	.358
D	.246	L	.290	U	.368
E	.250	M	.295	V	.377
F	.257	N	.302	W	.386
G	.261	O	.316	X	.397
H	.266	P	.323	Y	.404
		Q	.332	Z	.413

TABLE 4.—STANDARD DECIMAL GAGE

Standard decimal gage in ins.	Thickness in fractions of an inch	Weight per square foot in pounds, avoirdupois: Iron, basis—480 lbs. per cu. ft.	Steel, basis—489.6 lbs. per cu. ft.
.002	1-500	.08	.0816
.004	1-250	.16	.1632
.006	3-500	.24	.2448
.008	1-125	.32	.3264
.010	1-100	.40	.4080
.012	3-250	.48	.4896
.014	7-500	.56	.5712
.016	2-125(1/64+)	.64	.6528
.018	9-500	.72	.7344
.020	1-50	.80	.8160
.022	11-500	.88	.8976
.025	1-40	1.00	1.0200
.028	7-250	1.12	1.1424
.032	4-125(1/32+)	1.28	1.3056
.036	9-250	1.44	1.4688
.040	1-25	1.60	1.6320
.045	9-200	1.80	1.8360
.050	1-20	2.00	2.0400
.055	11-200	2.20	2.2440
.060	3-50 (1/16 −)	2.40	2.4480
.065	13-200	2.60	2.6520
.070	7-100	2.80	2.8560
.075	3-40	3.00	3.0600
.080	2-25	3.20	3.2640
.085	17-200	3.40	3.4680
.090	9-100	3.60	3.6720
.095	19-200	3.80	3.8760
.100	1-10	4.00	4.0800
.110	11-100	4.40	4.4880
.125	1-8	5.00	5.1000
.135	27-200	5.40	5.5080
.150	3-20	6.00	6.1200
.165	33-200	6.60	6.7320
.180	9-50	7.20	7.3440
.200	1-5	8.00	8.1600
.220	11-50	8.80	8.9760
.240	6-25	9.60	9.7920
.250	1-4	10.00	10.2000

The Standard Decimal Gage has been adopted by the Association of American Steel Manufacturers, the American Railway Master Mechanics' Association and by about seventy-two of the principal railroads of the United States, Canada and Mexico. The decimal system of gaging was recommended by the American Institute of Mining Engineers in 1877 and by the American Society of Mechanical Engineers in 1895.

TABLE 5.—SIZES OF NUMBERS OF THE U. S. STANDARD GAGE FOR SHEET AND PLATE IRON AND STEEL

Be it enacted by the Senate and House of Representatives of the United States of America in Congress assembled:

That for the purpose of securing uniformity the following is established as the only gage for sheet and plate iron and steel in the United States of America, namely:

Number of gage	Approximate thickness in fractions of an inch	Approximate thickness in decimal parts of an inch	Weight per square foot in ounces avoirdupois	Weight per square foot in pounds avoirdupois
0000000	1-2	.5	320	20.00
000000	15-32	.46875	300	18.75
00000	7-16	.4375	280	17.50
0000	13-32	.40625	260	16.25
000	3-8	.375	240	15.00
00	11-32	.34375	220	13.75
0	5-16	.3125	200	12.50
1	9-32	.28125	180	11.25
2	17-64	.265625	170	10.625
3	1-4	.25	160	10.00
4	15-64	.234375	150	9.375
5	7-32	.21875	140	8.75
6	13-64	.203125	130	8.125
7	3-16	.1875	120	7.5
8	11-64	.171875	110	6.875
9	5-32	.15625	100	6.25
10	9-64	.140625	90	5.625
11	1-8	.125	80	5.00
12	7-64	.109375	70	4.375
13	3-32	.09375	60	3.75
14	5-64	.078125	50	3.125
15	9-128	.0703125	45	2.8125
16	1-16	.0625	40	2.5
17	9-160	.05625	36	2.25
18	1-20	.05	32	2
19	7-160	.04375	28	1.75
20	3-80	.0375	24	1.50
21	11-320	.034375	22	1.375
22	1-32	.03125	20	1.25
23	9-320	.028125	18	1.25
24	1-40	.025	16	1.
25	7-320	.021875	14	.875
26	3-160	.01875	12	.75
27	11-640	.0171875	11	.6875
28	1-64	.015625	10	.625
29	9-640	.0140625	9	.5625
30	1-80	.0125	8	.5
31	7-640	.0109375	7	.4375
32	13-1280	.01015625	6½	.40625
33	3-320	.009375	6	.375
34	11-1280	.00859375	5½	.34375
35	5-640	.0078125	5	.3125
36	9-1280	.00703125	4½	.28125
37	17-2560	.006640625	4¼	.265625
38	1-160	.00625	4	.25

"And on and after July first, eithteen hundred and ninety-three, the same and no other shall be used in determining duties and taxes levied by the United States of America on sheet and plate iron and steel. But this act shall not be construed to increase duties upon any articles which may be imported.

"SEC. 3. That in the practical use and application of the standard gage hereby established a variation of two and one-half per cent. either way may be allowed.

Approved March 3, 1893."

HYDRAULICS AND HYDRAULIC MACHINERY

For tabulated values of barometric pressures at various altitudes in ins. of mercury, lbs. per sq. in, and ft. of water see Barometric Pressure.

For the relations of British and American measures of capacity see Weights and Measures.

TABLE 1.—HYDRAULIC CONSTANTS

Weight, Volume and Pressure of Water

1 cu. in.	= 0.03608 lb.
1 cu. ft.	= 62.355 lbs.
1 cu. ft.	= 7.481 U. S. gals.
1 U. S. gal.	= 231. cu. ins.
1 U. S. gal.	= 8.34 lbs.
1 U. S. gal.	= 0.1337 cu. ft.
1 lb.	= 0.016037 cu. ft.
1 lb.	= 27.712 cu. ins.
1 lb.	= 0.1199 U. S. gal.
100 ft. head of water	= 43.31 lbs. per sq. in.
100 lbs. per sq. in	= 230.9 ft. head.

Value of 1 Atmosphere of Pressure

Lbs. per sq. in.	Ft. head of water	Ins. of mercury.
14.7	33.947	30

Temperature 62 deg. Fahr.

TABLE 2.—PRESSURE PER SQUARE INCH CONVERTED INTO FEET HEAD OF WATER

Lbs. per sq. in.	Feet head	Lbs. per sq. in.	Feet head	Lbs. per sq. in.	Feet head
1	2.31	55	126.99	180	415.61
2	4.62	60	138.54	190	438.71
3	6.93	65	150.08	200	461.78
4	9.24	70	161.63	225	519.51
5	11.54	75	173.17	250	577.24
6	13.85	80	184.72	275	634.95
7	16.16	85	196.26	300	692.69
8	18.47	90	207.81	325	750.41
9	20.78	95	219.35	350	808.13
10	23.09	100	230.90	375	865.89
15	34.63	110	253.98	400	923.58
20	46.18	120	277.07	500	1154.46
25	57.72	125	288.62	....	....
30	69.27	130	300.16	....	....
35	80.81	140	323.25	....	....
40	92.36	150	346.34	....	....
45	103.90	160	369.43	....	....
50	115.45	170	392.52	....	....

The fundamental formula for the flow of water under the action of gravity is the same as that for the law of falling bodies, viz:

$$v = \sqrt{64.4h}$$
$$= 8\sqrt{h} \text{ nearly}$$

in which v = velocity of efflux, ft. per sec.,
h = pressure head, ft.

The theoretical volume of water discharged is equal to the velocity multiplied by the area of the orifice. The actual volume is equal to the theoretical volume multiplied by a coefficient of discharge which varies with the nature of the orifice. The following values of this coefficient are from Clark's Manula of Rules, Tables and Data:

Nature of orifice	Coefficient of discharge.
Thin plate	0.62
Cylinder at least 2 diameters in length	0.82
Converging cone, length = 2½ diameters	0.95
Contracted vein, length = ½ diameter of orifice, smallest diameter = .785 diameter of orifice.	1.00
Diverging cone, length = 9 diameters	1.46

The spouting velocity, discharge and horse-power of water jets, together with the proper diameters of impulse water-wheels, may be obtained from Fig. 1, by R. A. BRUCE (*Amer. Mach.*, Jan. 5, 1899). The use of the chart is shown by the example below it.

The assumption is made that the speed of an impulse wheel should be 50 per cent. of the speed of the jet. This is, of course, sometimes departed from for various practical reasons.

The velocities given by the actual velocity curve are 95 per cent. of those given by the theoretical velocity curve. If the reader prefers to make his own allowances from theoretical results, it is only necessary to trace the theoretical velocity curve and then proceed as in the example.

TABLE 3.—FEET HEAD OF WATER CONVERTED INTO PRESSURE PER SQUARE INCH

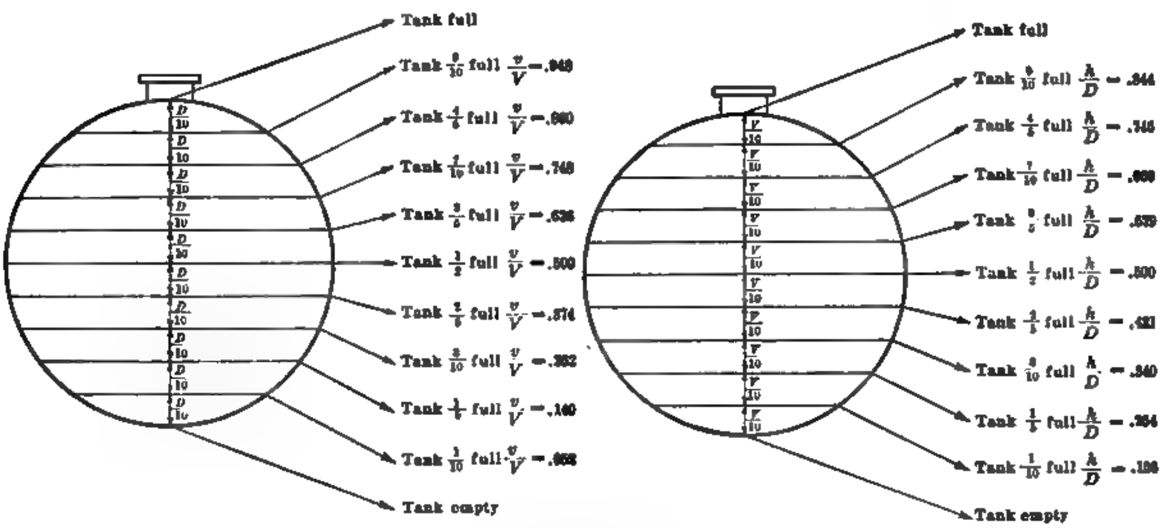
Feet head	Lbs. per sq. in.	Feet head	Lbs. per sq. in.	Feet head	Lbs. per sq. in.
1	.43	55	23.82	190	82.29
2	.87	60	25.99	200	86.62
3	1.30	65	28.15	225	97.45
4	1.73	70	30.32	250	108.27
5	2.17	75	32.48	275	119.10
6	2.60	80	34.65	300	129.93
7	3.03	85	36.81	325	140.75
8	3.40	90	38.98	350	151.58
9	3.90	95	41.14	375	162.41
10	4.33	100	43.31	400	173.24
15	6.50	110	47.64	500	216.55
20	8.66	120	51.97	600	259.85
25	10.83	130	56.30	700	303.16
30	12.90	140	60.63	800	346.47
35	15.16	150	64.96	900	389.78
40	17.32	160	69.29	1000	433.09
45	19.49	170	73.63	....	....
50	21.65	180	77.96	....	....

The Flow of Water in Pipes

Exact results must not be expected in calculations of the flow of water in pipes. In addition to the different results given by the different formulas is the ever present question of the condition of the pipe, which is scarcely capable of exact expression and which renders unimportant the differences between the results given by different formulas. The formulas are intended to apply directly to a standard condition of smoothness and cleanness and, since pipe-lines are almost certain to become foul in time, the calculated diameters should be increased. An addition of about 15 per cent. to the calculated diameter will provide for an extreme condition of roughness.

Assuming a head of 440 ft., trace vertically from A to E, thence horizontally to H, where read 159 ft. per sec., actual velocity, or trace to F and thence to G where read 168 ft. per sec. theoretical velocity. Trace horizontally from E to K on 2¼-in. diagonal, thence vertically to M and read 274 lbs. per sec. discharge of 2¼-in. nozzle under 440 ft. head. Trace vertically from A to B, thence horizontally to C on 2¼-in. diagonal, thence down to D, where read 220 horse-power for a 2¼-in. nozzle under 440 ft. head. Trace from A to E to N on 18-ft. diameter line, thence vertically and read 85 r.p.m. for an 85-ft. impulse wheel under 440 ft. head.

Fig. 1.—Spouting velocity, discharge and horse-power of water jets.



Tank divided into 10 parts of equal height.

Tank divided into 10 parts of equal volume.

V = total volume of tank,
v = volume occupied by liquid,
D = diameter of tank,
h = height of liquid in tank.

TABLE 4.—CAPACITY OF HORIZONTAL CYLINDRICAL TANKS

Table 5.—Theoretical Spouting Velocity and Actual Discharge of Water Through a Clean Square-edged Orifice of 1 Sq. In. Area, the Latter Calculated for a Coefficient of Discharge of .6

Head, ft.	Theoretic velocity, ft. per sec.	Discharge of orifice of 1 sq. in. area, cu. ft. per. min.	Head, ft.	Theoretic velocity, ft. per sec.	Discharge of orifice of 1 sq. in. area, cu. ft. per. min.	Head, ft.	Theoretic velocity, ft. per sec.	Discharge of orifice of 1 sq. in. area, cu. ft. per. min.
1/8	2.835	.709	9	24.061	6.01	51	57.31	14.33
1/4	4.010	1.003	9 1/2	24.720	6.18	52	57.87	14.47
3/8	4.911	1.228	10	25.362	6.34	53	58.42	14.60
1/2	5.671	1.417	10 1/2	25.988	6.50	54	58.97	14.74
5/8	6.340	1.59	11	26.600	6.65	55	59.52	14.83
3/4	6.946	1.737	11 1/2	27.198	6.80	56	60.05	15.01
7/8	7.502	1.89	12	27.783	6.94	57	60.59	15.14
1	8.020	2.00	12 1/2	28.356	7.08	58	61.12	15.28
1 1/8	8.507	2.126	13	28.917	7.23	59	61.64	15.41
1 1/4	8.967	2.242	13 1/2	29.468	7.367	60	62.16	15.54
1 3/8	9.404	2.351	14	30.009	7.50	61	62.68	15.69
1 1/2	9.823	2.46	14 1/2	30.540	7.64	62	63.19	15.80
1 5/8	10.224	2.556	15	31.062	7.77	63	63.70	15.92
1 3/4	10.610	2.65	15 1/2	31.576	7.88	64	64.20	16.05
1 7/8	10.982	2.75	16	32.081	8.02	65	64.70	16.18
2	11.342	2.83	16 1/2	32.579	8.14	66	65.20	16.30
2 1/8	11.691	2.92	17	33.068	8.26	67	65.69	16.42
2 1/4	12.030	3.	18	34.027	8.50	68	66.18	16.54
2 3/8	12.360	3.09	19	34.959	8.74	69	66.66	16.67
2 1/2	12.681	3.17	20	35.89	8.96	70	67.15	16.79
2 5/8	12.994	3.249	21	36.78	9.18	71	67.62	16.91
2 3/4	13.300	3.325	22	37.64	9.40	72	68.10	17.02
2 7/8	13.599	3.40	23	38.49	9.61	73	68.57	17.14
3	13.90	3.47	24	39.32	9.82	74	69.03	17.26
3 1/8	14.177	3.54	25	40.13	10.02	75	69.50	17.38
3 1/4	14.459	3.614	26	40.92	10.22	76	69.96	17.49
3 3/8	14.734	3.69	27	41.70	10.42	77	70.42	17.60
3 1/2	15.004	3.75	28	42.47	10.62	78	70.88	17.72
3 5/8	15.270	3.82	29	43.22	10.80	79	71.33	17.83
3 3/4	15.531	3.88	30	43.96	10.98	80	71.78	17.95
3 7/8	15.783	3.94	31	44.68	11.17	81	72.23	18.05
4	16.040	4.01	32	45.40	11.35	82	72.67	18.17
4 1/4	16.534	4.13	33	46.10	11.57	83	73.11	18.28
4 1/2	17.013	4.25	34	46.79	11.70	84	73.55	18.38
4 3/4	17.480	4.37	35	47.48	11.86	85	73.99	18.50
5	17.934	4.48	36	48.15	12.04	86	74.42	18.61
5 1/4	18.377	4.59	37	48.81	12.20	87	74.85	18.71
5 1/2	18.809	4.70	38	49.47	12.36	88	75.28	18.82
5 3/4	19.232	4.81	39	50.12	12.53	89	75.91	18.93
6	19.645	4.91	40	50.76	12.69	90	76.13	19.03
6 1/4	20.050	5.01	41	51.39	12.85	91	76.56	19.13
6 1/2	20.448	5.11	42	52.01	13.00	92	76.97	19.24
6 3/4	20.837	5.21	43	52.62	13.16	93	77.39	19.35
7	21.219	5.30	44	53.23	13.31	94	77.81	19.45
7 1/4	21.595	5.40	45	53.84	13.46	95	78.21	19.55
7 1/2	21.964	5.50	46	54.43	13.61	96	78.63	19.66
7 3/4	22.327	5.58	47	55.02	13.75	97	79.04	19.76
8	22.685	5.67	48	55.60	13.90	98	79.24	19.86
8 1/4	23.036	5.75	49	56.18	14.05	99	79.85	19.96
8 1/2	23.383	5.84	50	56.75	14.18	100	80.25	20.06

Table 6.—Capacity of Round Cisterns and Tanks in Cubic Feet and U. S. Gallons, from the National Tube Co's. Book of Standards

Diameter, ft. ins.	Cu. ft. per ft. depth	Gals. per ft. depth	Diameter, ft. ins.	Cu. ft. per ft. depth	Gals. per ft. depth	Diameter, ft. ins.	Cu. ft. per ft. depth	Gals. per ft. depth
1 0	.785	5.87	5 8	25.22	188.66	19 0	283.53	2120.9
1 1	.922	6.89	5 9	25.97	194.25	19 3	291.04	2177.1
1 2	1.069	8.00	5 10	26.73	199.92	19 6	298.65	2234.0
1 3	1.227	9.18	5 11	27.49	205.67	19 9	306.35	2291.7
1 4	1.396	10.44	6 0	28.27	211.51	20 0	314.16	2350.1
1 5	1.576	11.79	6 3	30.68	229.50	20 3	322.06	2409.2
1 6	1.767	13.22	6 6	33.18	248.23	20 6	330.06	2469.1
1 7	1.969	14.73	6 9	35.78	267.69	20 9	338.16	2529.6
1 8	2.182	16.32	7 0	38.48	287.88	21 0	346.36	2591.0
1 9	2.405	17.99	7 3	41.28	308.81	21 3	354.66	2653.0
1 10	2.640	19.75	7 6	44.18	330.48	21 6	363.05	2715.8
1 11	2.885	21.58	7 9	47.17	352.88	21 9	371.54	2779.3
2 0	3.142	23.50	8 0	50.27	376.01	22 0	380.13	2843.6
2 1	3.409	25.50	8 3	53.46	399.88	22 3	388.82	2908.6
2 2	3.687	27.58	8 6	56.75	424.48	22 6	397.61	2974.3
2 3	3.976	29.74	8 9	60.13	449.82	22 9	406.49	3040.8
2 4	4.276	31.99	9 0	63.62	475.89	23 0	415.48	3108.0
2 5	4.587	34.31	9 3	67.20	502.70	23 3	424.56	3175.0
2 6	4.909	36.72	9 6	70.88	530.24	23 6	433.74	3244.6
2 7	5.241	39.21	9 9	74.66	558.51	23 9	443.01	3314.0
2 8	5.585	41.78	10 0	78.54	587.52	24 0	452.39	3384.1
2 9	5.940	44.43	10 3	82.52	617.26	24 3	461.86	3455.0
2 10	6.305	47.16	10. 6	86.59	647.74	24 6	471.44	3526.6
2 11	6.681	49.98	10 9	90.76	678.95	24 9	481.11	3598.9
3 0	7.069	52.88	11 0	95.03	710.90	25 0	490.87	3672.0
3 1	7.467	55.86	11 3	99.40	743.58	25 3	500.74	3745.8
3 2	7.876	58.92	11 6	103.87	776.99	25 6	510.71	3820.3
3 3	8.296	62.06	11 9	108.43	811.14	25 9	520.77	3895.6
3 4	8.727	65.28	12 0	113.10	846.03	26 0	530.93	3971.6
3 5	9.168	68.58	12 3	117.86	881.65	26 3	541.19	4048.4
3 6	9.621	71.97	12 6	122.72	918.00	26 6	551.55	4125.9
3 7	10.085	75.44	12 9	127.68	955.09	26 9	562.00	4204.1
3 8	10.559	78.99	13 0	132.73	992.91	27 0	572.56	4283.0
3 9	11.045	82.62	13 3	137.89	1031.5	27 3	583.21	4362.7
3 10	11.541	86.33	13 6	143.14	1070.8	27 6	593.96	4443.1
3 11	12.048	90.13	13 9	148.49	1110.8	27 9	604.81	4524.3
4 0	12.566	94.00	14 0	153.94	1151.5	28 0	615.75	4606.2
4 1	13.095	97.96	14 3	159.48	1193.0	28 3	626.80	4688.8
4 2	13.635	102.00	14 6	165.13	1235.3	28 6	637.94	4772.1
4 3	14.186	106.12	14 9	170.87	1278.2	28 9	649.18	4856.2
4 4	14.748	110.32	15 0	176.71	1321.9	29 0	660.52	4941.0
4 5	15.321	114.61	15 3	182.65	1366.4	29 3	671.96	5026.6
4 6	15.90	118.97	15 6	188.69	1411.5	29 6	683.49	5112.0
4 7	16.50	123.42	15 9	194.83	1457.4	29 9	695.13	5199.9
4 8	17.10	127.95	16 0	201.06	1504.1	30 0	706.86	5287.7
4 9	17.72	132.56	16 3	207.39	1551.4	30 3	718.69	5376.2
4 10	18.35	137.25	16 6	213.82	1599.5	30 6	730.62	5465.4
4 11	18.99	142.02	16 9	220.35	1648.4	30 9	742.64	5555.4
5 0	19.63	146.88	17 0	226.98	1697.9	31 0	754.77	5646.1
5 1	20.29	151.82	17 3	233.71	1748.2	31 3	766.99	5737.5
5 2	20.97	156.83	17 6	240.53	1799.3	31 6	779.31	5829.7
5 3	21.65	161.93	17 9	247.45	1851.1	31 9	791.73	5922.6
5 4	22.34	167.12	18 0	254.47	1903.6	32 0	804.25	6016.2
5 5	23.04	172.38	18 3	261.59	1956.8	32 3	816.86	6110.6
5 6	23.76	177.72	18 6	268.80	2010.8	32 6	829.58	6205.7
5 7	24.48	183.15	18 9	276.12	2065.5	32 9	842.30	6301.5

The formulas proposed by Wm. Cox, (*Amer. Mach., Oct.* 4, 25, 1894) give, quite closely, the same results as the more cumbersome formulas by Weisbach. Mr. Cox's formulas are as follows:

Let D = discharge, cu. ft. per min..
d = diameter of pipe, ins.,
V = velocity of discharge, ft. per sec.,
L = length of pipe, ft.,
H = head required to produce velocity V, ft.,

Then $$D = .3275 V d^2 \qquad (1)$$

whence $$d = \sqrt{\frac{D}{.3275 V}} \qquad (1a)$$

and $$V = \frac{D}{.3275 d^2} \qquad (1b)$$

Also $$4V^2 + 5V - 2 = \frac{1200 dH.}{L} \qquad (2)$$

Putting $4V^2+5V-2=K$, (2) becomes,

$$K=\frac{1200\,dH}{L} \tag{2a}$$

whence $$d=\frac{KL}{1200\,H} \tag{2b}$$

$$H=\frac{KL}{1200d} \tag{2c}$$

and $$L=\frac{1200dH}{K} \tag{2d}$$

Formulas (2)-(2d) apply to clean smooth cast-iron pipes. To make them and the tables applicable to lapped and rivetted pipes, 1000 must be used instead of 1200 in formula (2) and its transpositions, while for seamless wrought-iron pipes with flush joints the constant 1500 should be used.

The velocities in pipe-lines seldom exceed 6 ft. per sec. in low and medium head-water power plants. In high-head plants the velocity sometimes reaches 13 ft. per sec. For the mere delivery of water no such restriction holds.

TABLE 7.—DISCHARGE FROM PIPES IN CU. FT. PER MIN. WITH VELOCITY = 1 FT. PER SEC.

Diam., ins.	Cubic ft.	Diam., ins.	Cubic ft.	Diam., ins.	Cubic ft.
1	0.32725	17	94.575	33	356.37
2	1.3090	18	106.03	34	378.30
3	2.9452	19	118.14	35	400.88
4	5.2360	20	130.90	36	424.11
5	8.1812	21	144.32	37	448.00
6	11.781	22	158.39	38	472.55
7	16.035	23	173.11	39	497.75
8	20.944	24	188.50	40	523.60
9	26.507	25	204.53	41	550.11
10	32.725	26	221.22	42	577.27
11	39.597	27	238.56	43	605.09
12	47.124	28	256.56	44	633.56
13	55.305	29	275.22	45	662.68
14	64.141	30	294.52	46	692.46
15	73.631	31	314.49	47	722.90
16	83.776	32	335.10	48	753.98

All other velocities in strict proportion.

Table 7 gives the discharge in cu. ft. per min. from pipes of 1 to 48 ins. diameter for a uniform velocity of 1 ft. per sec. and from it the discharge for any other velocity may be obtained by simple proportion thus:

What diameter of pipe will discharge 500 *cu. ft. per min. with a velocity of* 4 *ft. per sec?*

The proportionate discharge for a velocity of 1 ft. per sec. would be $\frac{500}{4}=125$ cu. ft., and from Table 7 we see that this would require a pipe 20 ins. diameter.

Table 8 gives values of $K=4V^2+5V-2$ corresponding to velocities V from 1 to 20 ft. per sec. Its use is best shown by examples:

Given a pipe 12 *ins. diameter,* 3000 *ft. long and* 20 *ft. head; what will be the velocity of discharge and the discharge?*

By equation (2a) we have

$$K=\frac{12\times20\times1200}{3000}=96,$$

which, according to Table 8, corresponds to a velocity of 4.4 ft. per sec., nearly. Now, from Table 7, we have

$$\text{Discharge}=47.124\times4.4=207.3 \text{ cu. ft. per min.}$$

Given a pipe 2400 *ft. long, with a head of* 80 *ft., what must be its diameter to produce a velocity of discharge of* 5 *ft.?*

Taking from Table 8 the value of K corresponding to a velocity of 5 ft., and inserting it in equation (2b), we have

$$d=\frac{123\times2400}{80\times1200}=3 \text{ ins.}$$

Given a 24*-in. pipe* 1800 *ft. long, what head is required to produce a velocity of discharge of* 8 *ft. per sec.?*

Taking from Table 8 the value of K corresponding to a velocity of 8 ft., and inserting it in equation (2c), we have

$$\text{Head}=\frac{294\times1800}{24\times1200}=18.4 \text{ ft.}$$

What diameter of pipe 4500 *ft. long will discharge* 4020 *cu. ft. per min., with a head of* 24 *ft.?*

In this problem neither the velocity nor the diameter of the pipe are given, so that we must proceed by trial. We will assume, therefore, a *trial* diameter of 40 ins., and inserting this in equation (2a), we have

$$K=\frac{40\times24\times1200}{4500}=256$$

which, according to Table 8, corresponds to a velocity of 7.4 ft. per sec. nearly. Now by Table 7 the discharge of a 40-in. pipe with this velocity is

$$D=523.6\times7.4=3874.64 \text{ cu. ft.}$$

This diameter is, therefore, clearly not enough, as there is a shortage of 4020—3874.64 = 145.36 cu. ft. From Table 7 we now see that a 41-in. pipe will discharge 26.5 cu. ft. per unit of velocity more than a 40-in. pipe; therefore, with the same velocity of 7.4 ft., we have

$$26.5\times7.4=196.1 \text{ cu. ft.,}$$

which is more than the previous shortage, so that a 41-in. pipe is amply large enough to satisfy the requirements of the problem.

This problem is probably the one whose solution is most frequently called for and is the most tedious to solve. The solution here given is believed to be the simplest that has been offered.

What head is required to discharge 1000 *cu. ft. per. min. from a* 30*-in. pipe* 3000 *ft. long?*

From Table 7 we find that the velocity of discharge must be $\frac{1000}{294.5}=3.4$ ft. per sec. Taking from Table 8 the value of K corresponding to this velocity and inserting it in equation (2c), we have

$$\text{Head}=\frac{61.24\times3000}{30\times1200}=5.1 \text{ ft.}$$

TABLE 8.—VALUES OF $K=4V^2+5V-2$

V	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	V
0.0	......	......	......	......	0.64	1.50	2.44	3.46	4.56	5.74	0.0
1.0	7.00	8.34	9.76	11.26	12.84	14.50	16.24	18.06	19.96	21.94	1.0
2.0	24.00	26.14	28.36	30.66	33.04	35.50	38.04	40.66	43.36	46.14	2.0
3.0	49.00	51.94	54.96	58.06	61.24	64.50	67.84	71.26	74.76	78.34	3.0
4.0	82.00	85.74	89.56	93.46	97.44	101.50	105.64	109.86	114.16	118.54	4.0
5.0	123.00	127.54	132.16	136.86	141.64	146.50	151.44	156.46	161.56	166.74	5.0
6.0	172.00	177.34	182.76	188.26	193.84	199.50	205.24	211.06	216.96	222.94	6.0
7.0	229.00	235.14	241.36	247.66	254.04	260.50	267.04	273.66	280.36	287.14	7.0
8.0	294.00	300.94	307.96	315.06	322.24	329.50	336.84	344.26	351.76	359.34	8.0
9.0	367.00	374.74	382.56	390.46	398.44	406.50	414.64	422.86	431.16	439.54	9.0
10.0	448.00	456.54	465.16	473.86	482.64	491.50	500.44	509.46	518.56	527.74	10.0
11.0	537.00	546.34	555.76	565.26	574.84	584.50	594.24	604.06	613.96	623.94	11.0
12.0	634.00	644.14	654.36	664.66	675.04	685.50	696.04	706.66	717.36	728.14	12.0
13.0	739.00	749.94	760.96	772.06	783.24	794.50	805.84	817.26	828.76	840.34	13.0
14.0	852.00	863.74	875.56	887.46	899.44	911.50	923.64	935.86	948.16	960.54	14.0
15.0	973.00	985.54	998.16	1010.86	1023.64	1036.50	1049.44	1062.46	1075.56	1088.74	15.0
16.0	1102.00	1115.34	1128.76	1142.26	1155.84	1169.50	1183.24	1197.06	1210.96	1224.94	16.0
17.0	1239.00	1253.14	1267.36	1281.66	1296.04	1310.50	1325.04	1349.66	1354.36	1369.14	17.0
18.0	1384.00	1398.94	1413.96	1429.06	1444.24	1459.50	1474.84	1490.26	1505.76	1521.34	18.0
19.0	1537.00	1552.74	1568.56	1584.46	1600.44	1616.50	1632.64	1648.86	1665.16	1681.54	19.0
20.0	1698.00	1714.54	1731.16	1747.86	1764.64	1781.50	1798.44	1815.46	1832.56	1849.74	20.0

To make the calculations in U. S. gallons instead of cubic feet, formula (1) becomes:

$$G = 2.45\ Vd^2 \qquad (3)$$

in which G = discharge, gals. per min.,
V = velocity of discharge, ft. per sec.,
d = diameter of pipe, ins.

Table 9 gives this discharge for pipes from 1 to 48 ins. dimaeter for a uniform velocity unit of 1 ft. per sec. It is used in precisely the same manner as Table 7.

To find the velocity head, that is, the head required to produce any given velocity of discharge, use the formula:

$$V_h = .0155\ V^2 \qquad (4)$$

in which V_h = the head in ft. required to produce the velocity V. Similarly, to find the pressure head, that is, the pressure required to produce any given velocity of discharge, use the formula:

$$V_p = .00673\ V^2 \qquad (5)$$

in which V_p = the pressure in lbs. per sq. in. required to produce the velocity V.

Table 9.—Discharge From Pipes in U. S. Gals. per Min. With Velocity = 1 Ft. per Sec.

Diam., ins.	Gallons	Diam., ins.	Gallons	Diam., ins.	Gallons
1	2.448	17	707.47	33	2665.9
2	9.792	18	793.15	34	2829.9
3	22.032	19	883.73	35	2998.8
4	39.168	20	979.20	36	3172.6
5	61.200	21	1079.6	37	3351.3
6	88.128	22	1184.8	38	3534.9
7	119.95	23	1295.0	39	3723.4
8	156.67	24	1410.0	40	3916.8
9	198.29	25	1530.0	41	4115.1
10	244.80	26	1654.8	42	4318.3
11	296.21	27	1784.6	43	4526.3
12	352.51	28	1919.2	44	4739.3
13	413.71	29	2058.8	45	4957.2
14	479.81	30	2203.2	46	5180.0
15	550.80	31	2352.5	47	5407.6
16	626.69	32	2506.7	48	5640.2

All other velocities in strict proportion.

Table 10 gives a list of heads in ins. and ft. with their equivalent pressures in lbs. per sq. in. and also the corresponding velocities in ft. per sec. produced by these heads or pressures.

It is often more convenient to base all calculations upon pressure in lbs. instead of heads in ft. To reduce heads in ft. to pressures in lbs. per sq. in., multiply the head by .4331 or consult Table 3. So likewise equation (2a) becomes:

$$K = \frac{2768\ dP}{L} \qquad (6a)$$

in which P = pressure, lbs. per sq. in. corresponding to head H of (2a), the remaining notation being as in (2a).

Table 11 for the loss of head due to friction in lapped and rivetted pipe has been calculated by the Pelton Water Wheel Co. from Cox's formulas, except that the factor 1200 is replaced by 1000 as directed by Mr. Cox.

A graphical method of making pipe-line calculations for the flow of water is given in Fig. 2, by Walter R. Clark, Mech. Engr. of the Bridgeport Brass Co. (*Amer. Mach., July* 8, 1909). Large pressure losses are included in the scale in order to adapt the chart to pipe-lines for high-pressure hydraulic machine service, in which much larger losses are permissible than in others because, under the high pressures, a large absolute loss is still a small percentage loss.

The chart represents the following formulas which were deduced from Ellis and Howland's tables:

$$Q = 2.45\ VD^2$$
$$F = \frac{.03Q^2}{D^5}$$

in which Q = discharge, gals. per min.,
V = velocity, ft. per sec.,
D = inside diameter, ins.,
F = friction loss, lbs. per sq. in. for each 100 ft. of clean straight iron pipe, the values given being approximately true for V greater than 3.

The use of the chart is shown by the example below it.

The full lines refer to pipe of which the nominal and actual diameters are the same, while the dotted lines refer to standard pipe. To use the chart for extra and double extra strong pipe, determine the actual diameter from the full lines and then refer to Tables 12 and 13 for the pipe having its diameter nearest to that given by the chart.

Table 10.—Velocities With Corresponding Heads and Pressures

Vel. head, ins.	Vel. pressure, lbs. per sq. in.	Vel., ft. per sec.	Vel. head, ft.	Vel. pressure. lbs. per sq. in.	Vel., ft. per sec.
.186	.0067	1.00	1.255	.5451	9.00
.744	.0269	2.00	1.550	.6730	10.00
1.000	.0361	2.32	2.000	.8670	11.35
1.674	.0606	3.00	2.307	1.0000	12.19
2.000	.0722	3.27	3.000	1.3005	13.90
2.976	.1077	4.00	4.000	1.7340	16.05
3.000	.1084	4.01	4.614	2.0000	17.24
4.000	.1445	4.63	5.000	2.1675	17.94
4.650	.1682	5.00	6.000	2.6010	19.66
5.000	.1806	5.18	6.920	3.0000	21.12
6.000	.2167	5.67	7.000	3.0345	21.23
6.696	.2423	6.00	8.000	3.4680	22.70
7.000	.2529	6.13	9.000	3.9015	24.07
8.000	.2890	6.55	9.227	4.0000	24.38
9.000	.3251	6.95	10.000	4.3350	25.38
9.114	.3298	7.00	11.534	5.0000	27.26
10.000	.3612	7.32	13.841	6.0000	29.86
11.000	.3974	7.65	16.148	7.0000	32.26
11.904	.4307	8.00	18.454	8.0000	34.48
12	.4335	8.02	20.761	9.0000	36.58

The friction losses given by the chart do not include those due to water flowing into the pipe from another vessel or *vice versa*, nor to ells or tees, for which latter see below.

The chart may also be used in other ways. Thus if the pipe diameter and velocity are given, we find the intersection of the two diagonals representing pipe size and velocity and read down from their intersection to get gals. per min.; up to get cu. ft. per min.; and to the right to get lbs. pressure drop per 100 ft. of pipe.

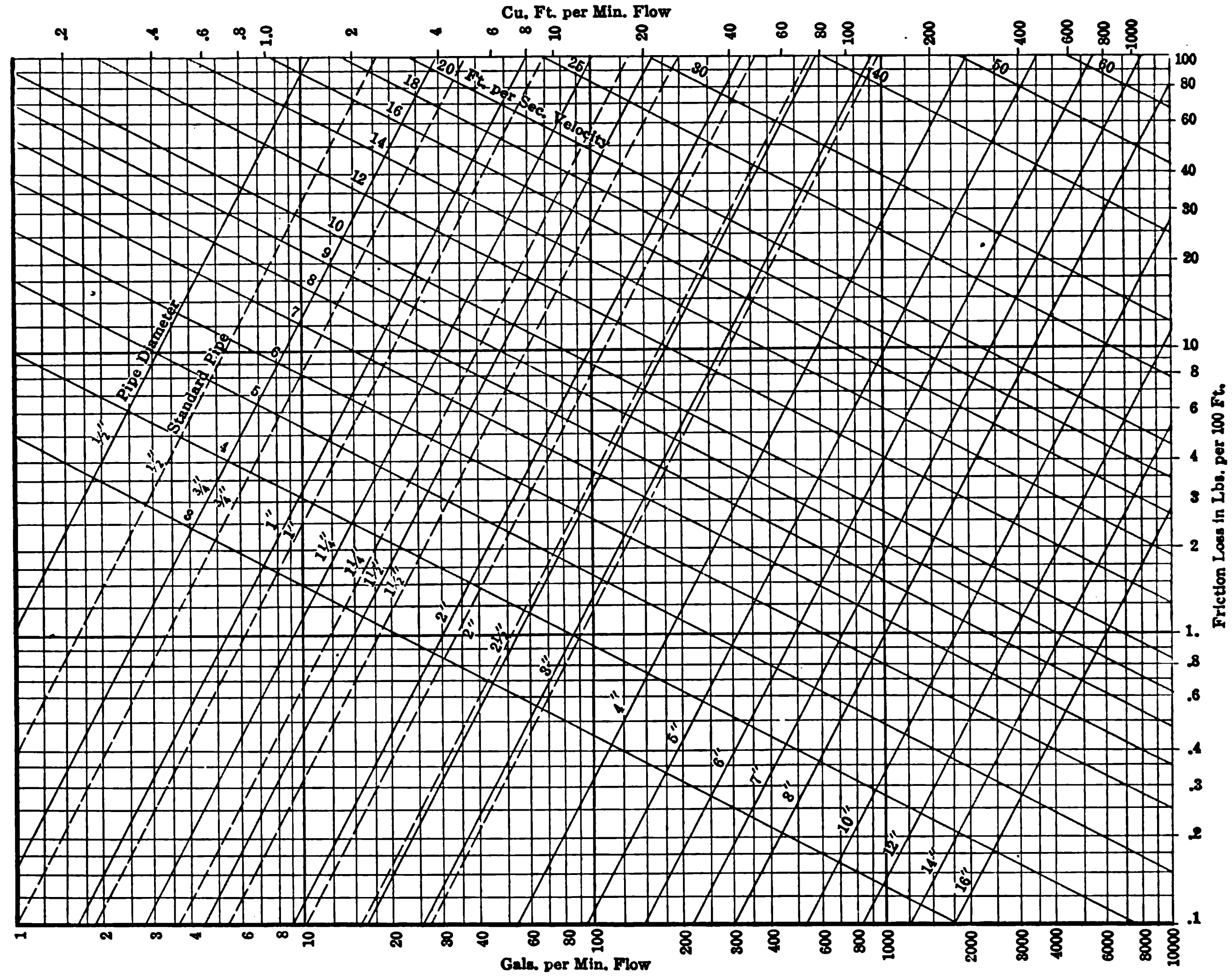
For the equation of main and branch lines of commercial pipe, including standard, extra and double extra strong by their actual inside diameters, see Index.

The resistance of pipe fittings to the flow of water formed the subject of experiments by Prof. F. E. Giesecke (*Domestic Engineering, Nov.* 2, 1912). The information sought was for use in the design of apparatus for warming buildings by hot water and hence the fittings tested did not exceed 2 ins. nominal diameter and the observed velocities of flow did not exceed 1 ft. per sec. In spite of these limitations the experiments are the best of which the author has knowledge—the more so as the concordance of the results indicates that they may be applied materially beyond the limits of the observations without sensible error.

TABLE 11.—LOSS OF HEAD BY FRICTION PER 100 FT. LENGTH OF PIPE

Diam. ins.	6		7		8		9		10		11		12		13		14		15		16		18	
Vel. in ft. per sec.	Loss of head in feet	Cubic feet per min.	Loss of head in feet	Cubic feet per min.	Loss of head in feet	Cubic feet per min.	Loss of head in feet	Cubic feet per min.	Loss of head in feet	Cubic feet per min.	Loss of head in feet	Cubic feet per min.	Loss of head in feet	Cubic feet per min.	Loss of head in feet	Cubic feet per min.	Loss of head in feet	Cubic feet per min.	Loss of head in feet	Cubic feet per min.	Loss of head in feet	Cubic feet per min.	Loss of head in feet	Cubic feet per min.
2.0	.39	23.5	.33	32.0	.30	41.9	.26	53.0	.23	65.4	.21	79	.198	94	.183	110	.169	128	.158	147	.147	167	.132	212
2.2	.46	25.9	.40	35.3	.35	46.1	.31	58.3	.28	72.	.25	87	.234	103	.216	121	.200	141	.187	162	.175	184	.156	233
2.4	.54	28.2	.46	38.5	.41	50.2	.36	63.6	.32	78.5	.29	95	.273	113	.252	133	.234	154	.218	176	.205	201	.182	254
2.6	.63	30.6	.54	41.7	.47	54.4	.42	68.9	.37	85.1	.34	103	.315	122	.290	144	.270	167	.252	191	.236	218	.210	275
2.8	.72	32.9	.61	44.9	.54	58.6	.48	74.2	.43	91.6	.39	111	.360	132	.332	156	.308	179	.288	206	.270	234	.240	297
3.0	.81	35.3	.69	48.1	.61	62.8	.54	79.5	.48	98.2	.44	119	.407	141	.375	166	.349	192	.325	221	.306	251	.271	318
3.2	.91	37.7	.78	51.3	.68	67.0	.60	84.8	.54	105	.49	127	.457	151	.422	177	.392	205	.366	235	.343	268	.305	339
3.4	1.02	40.0	.87	54.5	.76	71.2	.68	90.1	.61	111	.55	134	.510	160	.471	188	.438	218	.408	250	.383	284	.339	360
3.6	1.13	42.4	.96	57.7	.84	75.4	.75	95.4	.67	118	.61	142	.566	169	.522	199	.485	231	.452	265	.425	301	.377	382
3.8	1.25	44.7	1.07	60.9	.93	79.6	.83	101	.74	124	.68	150	.624	179	.576	210	.535	243	.499	280	.468	318	.416	403
4.0	1.37	47.1	1.17	64.1	1.02	83.7	.91	106	.82	131	.74	158	.685	188	.632	221	.587	256	.548	294	.513	335	.456	424
4.2	1.49	49.5	1.28	67.3	1.12	87.9	.99	111	.89	137	.81	166	.749	198	.691	232	.641	269	.598	309	.561	352	.499	445
4.4	1.62	51.8	1.39	70.5	1.22	92.1	1.08	116	.97	144	.88	174	.815	207	.751	243	.698	282	.651	324	.611	368	.542	466
4.6	1.76	54.1	1.51	73.7	1.32	96.3	1.17	122	1.05	150	.96	182	.883	217	.815	254	.757	295	.707	339	.662	385	.588	488
4.8	1.90	56.5	1.63	76.9	1.43	100.	1.27	127	1.14	157	1.04	190	.954	226	.881	265	.818	308	.763	353	.715	402	.636	509
5.0	2.05	58.9	1.76	80.2	1.54	105	1.37	132	1.23	163	1.12	198	1.028	235	.949	276	.881	321	.822	368	.770	419	.685	530
5.2	2.21	61.2	1.89	83.3	1.65	109	1.47	138	1.32	170	1.20	206	1.104	245	1.020	287	.947	333	.883	383	.828	435	.736	551
5.4	2.37	63.6	2.03	86.6	1.77	113	1.57	143	1.41	177	1.28	214	1.183	254	1.092	298	1.014	346	.947	397	.888	452	.788	572
5.6	2.53	65.9	2.17	89.8	1.89	117	1.68	148	1.51	183	1.37	222	1.26	264	1.167	309	1.083	359	1.011	412	.949	469	.843	594
5.8	2.70	68.3	2.31	93.0	2.01	121	1.80	154	1.61	190	1.46	229	1.34	273	1.245	321	1.155	372	1.078	427	1.011	486	.899	615
6.0	2.87	70.7	2.46	96.2	2.15	125	1.92	159	1.71	196	1.56	237	1.43	283	1.325	332	1.229	385	1.148	442	1.076	502	.957	636
7.0	3.81	82.4	3.26	112.0	2.85	146	2.52	185	2.28	229	2.07	277	1.91	330	1.75	387	1.630	449	1.520	515	1.430	586	1.270	742

Diam. ins.	20		22		24		26		28		30		33		36		39		42		45		48	
Vel. in ft. per sec.	Loss of head in feet	Cubic feet per min.	Loss of head in feet	Cubic feet per min.	Loss of head in feet	Cubic feet per min.	Loss of head in feet	Cubic feet per min.	Loss of head in feet	Cubic feet per min.	Loss of head in feet	Cubic feet per min.	Loss of head in feet	Cubic feet per min.	Loss of head in feet	Cubic feet per min.	Loss of head in feet	Cubic feet per min.	Loss of head in feet	Cubic feet per min.	Loss of head in feet	Cubic feet per min.	Loss of head in feet	Cubic feet per min.
2.0	.119	262	.108	316	.098	377	.091	442	.084	513	.079	589	.073	712	.066	848	.061	995	.057	1155	.053	1325	.050	1508
2.2	.140	288	.127	348	.116	414	.108	486	.099	564	.093	648	.085	785	.078	933	.072	1094	.067	1270	.063	1456	.059	1658
2.4	.164	314	.149	380	.136	452	.126	531	.116	616	.109	707	.100	855	.091	1018	.084	1194	.079	1385	.073	1590	.069	1809
2.6	.189	340	.171	412	.157	490	.145	575	.134	667	.126	766	.115	927	.104	1100	.097	1294	.090	1500	.084	1721	.079	1960
2.8	.216	366	.195	443	.180	528	.165	619	.153	718	.144	824	.131	1000	.119	1188	.111	1394	.103	1617	.096	1855	.090	2110
3.0	.245	393	.222	475	.204	565	.188	663	.174	770	.163	883	.148	1070	.135	1273	.125	1442	.117	1730	.109	1987	.102	2260
3.2	.275	419	.249	507	.229	603	.211	708	.195	821	.182	942	.167	1140	.152	1367	.141	1591	.131	1845	.122	2120	.115	2410
3.4	.306	445	.278	538	.255	641	.235	752	.218	872	.204	1001	.186	1210	.169	1442	.157	1690	.146	1961	.136	2250	.128	2560
3.6	.339	471	.308	570	.283	678	.261	796	.242	923	.226	1060	.206	1282	.188	1527	.174	1790	.162	2079	.151	2382	.142	2715
3.8	.374	497	.340	601	.312	716	.288	840	.267	974	.249	1119	.226	1355	.207	1612	.191	1891	.178	2190	.166	2515	.156	2865
4.0	.410	523	.373	633	.342	754	.315	885	.293	1026	.273	1178	.248	1425	.228	1697	.210	1990	.195	2310	.182	2650	.171	3016
4.2	.449	550	.408	665	.374	791	.345	929	.320	1077	.299	1237	.270	1495	.249	1782	.229	2091	.213	2422	.198	2780	.186	3165
4.4	.488	576	.444	697	.407	829	.375	973	.348	1129	.325	1296	.295	1568	.271	1866	.250	2190	.232	2540	.216	2910	.203	3318
4.6	.529	602	.482	728	.441	867	.407	1017	.378	1180	.353	1355	.321	1640	.294	1951	.271	2290	.252	2658	.235	3045	.220	3470
4.8	.572	628	.521	760	.476	905	.440	1062	.409	1231	.381	1414	.346	1710	.318	2036	.293	2389	.270	2770	.254	3180	.238	3619
5.0	.617	654	.561	792	.513	942	.474	1106	.440	1283	.411	1472	.374	1780	.342	2121	.316	2490	.294	2885	.273	3310	.256	3770
5.2	.662	680	.602	823	.552	980	.510	1150	.473	1334	.441	1531	.403	1852	.368	2206	.342	2590	.317	3000	.296	3442	.278	3920
5.4	.710	707	.645	855	.591	1018	.546	1194	.507	1385	.473	1590	.430	1922	.394	2291	.364	2689	.338	3115	.315	3578	.295	4071
5.6	.758	733	.690	887	.632	1055	.583	1239	.542	1437	.506	1649	.453	1995	.421	2376	.393	2790	.374	3230	.340	3710	.319	4222
5.8	.809	759	.735	918	.674	1093	.622	1283	.578	1488	.540	1708	.495	2065	.450	2460	.419	2886	.389	3348	.363	3840	.340	4373
6.0	.861	785	.782	950	.717	1131	.662	1327	.615	1539	.574	1767	.520	2140	.479	2545	.441	2986	.408	3461	.382	3970	.358	4524
7.0	1.143	916	1.040	1109	.953	1319	.879	1548	.817	1796	.762	2061	.693	2495	.636	2968	.586	3484	.545	4030	.509	4638	.476	5277



Example: 200 gals. per min. are to be transmitted 500 ft. with an allowable loss of pressure of 50 lbs. per sq. in. From 200 gals. on the base line trace vertically to the intersection with the line for a pressure loss of 10 lbs. per sq. in. The intersection is found near the diagonal for 2 1/2 in. pipe which is nearest the required size. The intersection also falls near the diagonal for 12 ft. per sec. velocity showing the velocity to be about that figure. If preferred read cu. ft. on the top scale in place of gals. on the bottom. Full lines refer to pipe of which the nominal and actual diameters are the same; dotted lines refer to standard pipe.

FIG. 2.—Flow of water in pipes.

TABLE 12.—ACTUAL INTERNAL DIAMETERS OF EXTRA STRONG PIPE

Nominal size, ins.	Internal diameter, ins.	Nominal size, ins.	Internal diameter, ins.
1/8	.215	4	3.826
1/4	.302	4 1/2	4.290
3/8	.423	5	4.813
1/2	.546	6	5.761
3/4	.742	7	6.625
1	.957	8	7.625
1 1/4	1.278	9	8.625
1 1/2	1.500	10	9.750
2	1.939	11	10.750
2 1/2	2.323	12	11.750
3	2.900	13	13.000
3 1/2	3.364	14	14.000

TABLE 13.—ACTUAL INTERNAL DIAMETERS OF DOUBLE EXTRA STRONG PIPE

Nominal size, ins.	Internal diameter, ins.	Nominal size, ins.	Internal diameter, ins.
1/2	.252	3 1/2	2.728
3/4	.434	4	3.152
1	.599	4 1/2	3.580
1 1/4	.896	5	4.063
1 1/2	1.100	6	4.897
2	1.503	7.	5.875
2 1/2	1.771	8	6.875
3	2.300		

The resulting data, as related to elbows, are given in graphic form in Fig. 3, while the accompanying table gives ratios for other fittings.

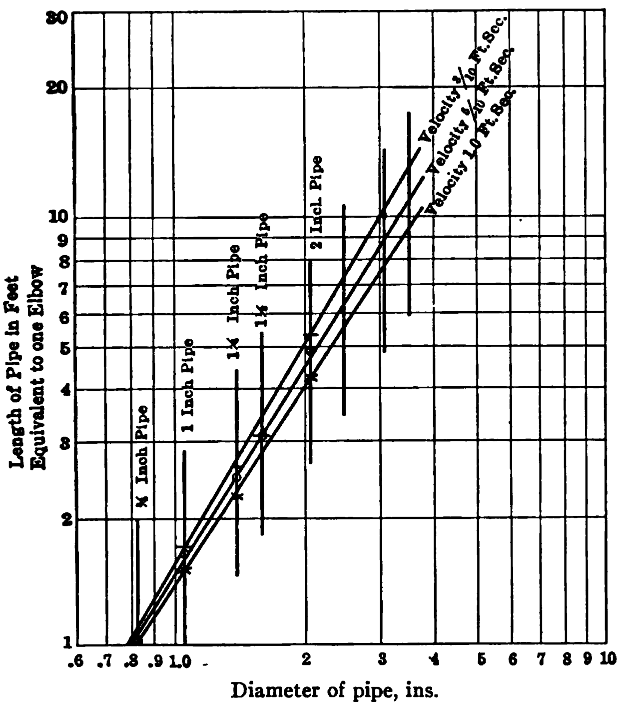
In laying pipe-lines for flow due to gravity it should not be forgotten that no part of the line must rise above the hydraulic grade line. That is to say, referring to Fig. 4, if ABC represents a pipe-line, then its hydraulic grade is the straight line AC joining its two extremities, and no part of the pipe-line must rise above this grade. Therefore, before laying pipes through rough country, it is necessary to have a profile of the ground so as to be sure that the pipe, when laid, shall conform to this requirement. Neglect of this indispensable condition will lead to an interrupted or diminished flow.

Should the pipe at any point rise above the hydraulic grade line,

it becomes in effect a siphon and subject to the uncertainties of a siphon.

Hydraulic Press Cylinders and Rams

A reaction has taken place against the high pressures (5000 to 6000 lbs. per sq. in.) which were favored at one time for hydraulic machinery. Such pressures are now used only when necessary and are obtained locally by intensifiers from a lower service pressure. When subject to free choice, the general service pressures used range



1 – 45 deg. elbow equals	½ – 90 deg. elbow
1 – open return bend equals	1 – 90 deg. elbow
1 – gate valve equals	½ – 90 deg. elbow
1 – globe valve equals	12 – 90 deg. elbows
1 – tee equals	2 – 90 deg. elbows

FIG. 3.—Resistance of pipe fittings to the flow of water.

between 1000 and 1500 lbs. per sq. in. for which, so far as possible, the operating machines are designed.

The thickness of hydraulic press cylinders may be determined from Fig. 5 by PROF. A. L. JENKINS (*Amer. Mach., Mar.* 31, 1910) which plots Barlow's and Lamé's formulas. Experiments by Professor Goodman, of Leeds, England, confirm the substantial correctness of Barlow's formula, which is to be preferred. The extensive use of Lamé's formula leads the author to include its chart line for those who, using it, prefer to continue to do so, and for those who wish to make comparisons. It should be said also that Merriman's formula gives identical results with Barlow's and that Burr's and Lanza's formulas give identical results with Lamé's. Following are Barlow's and Lamé's formulas:

$$\frac{S}{P}=\frac{R}{T}+1 \qquad \text{Barlow.}$$

$$T=R\left(\sqrt{\frac{S+P}{S-P}}-1\right) \qquad \text{Lamé.}$$

in both of which S = fiber stress, lbs. per sq. in.,
P = hydraulic pressure, lbs. per sq. in.,
R = inside radius of cylinder, ins.
T = thickness of cylinder, ins.

The use of the chart is best shown by an example. Required the thickness of a cylinder 20 ins. internal diameter, subjected to a pressure of 1000 lbs. per sq. in., the fiber stress on the material being 6000 lbs. per sq. in., giving $\frac{S}{P}=\frac{6000}{1000}=6$. Find 6 in the base line, trace upward to the diagonal line for Barlow's formula and then to the left, where read 5 for the value of $\frac{R}{T}$. The value of R being 10 gives $\frac{R}{T}=\frac{10}{T}=5$ or $T=2$ ins. Or, using the line for Lamé's formula, we find $\frac{R}{T}=5.5$, that is, $\frac{R}{T}=\frac{10}{T}=5.5$ or, $T=1.82$ ins.

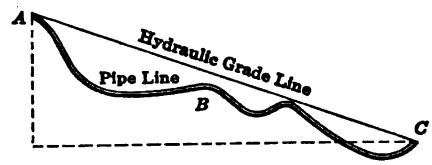


FIG. 4.—Precaution in pipe laying.

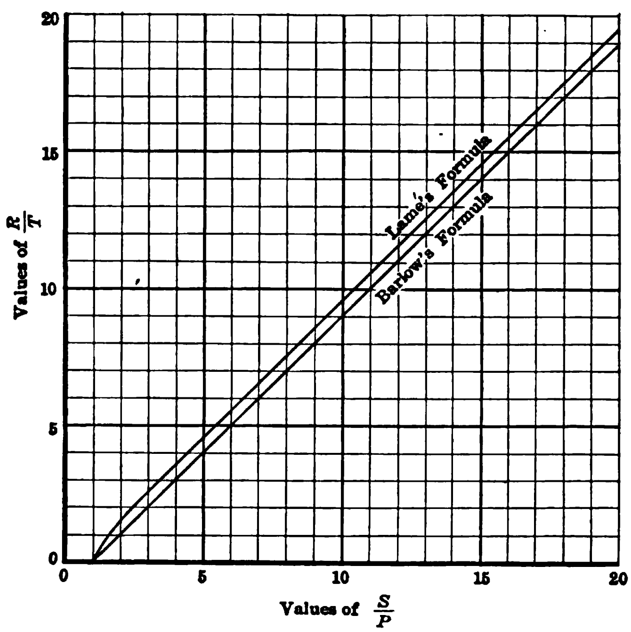


FIG. 5.—Thickness of hydraulic press cylinders.

Professor Jenkins recommends a fiber stress of 6000 lbs. per sq. in. for cast-iron and 13,000 to 18,000 for steel cylinders. The former figure seems rather high unless air-furnace iron be used, which, indeed, it should be to avoid the porosity of cupola iron.

The common construction by which the radius of the closed end of a hydraulic cylinder is made equal to the diameter of the cylinder, is a mistaken application of the fact that the stress per sq. in. due to internal pressure in a sphere is one-half the longitudinal stress in a cylinder of the same diameter. The theory of such stresses is based on the supposition of a complete hemisphere and is not true for lesser segments. This application of the theory leads to bending stresses at the junction of the end with the barrel. Such stresses can be avoided only by making the end of the cylinder a complete hemisphere as shown in Fig. 12. When necessary to save room, some designers use an inside radius of three-fourths, a fillet of one-fourth the diameter of the cylinder and an end thickness equal to that of the cylinder walls.

The thickness of hydraulic press rams, centrally and eccentrically loaded, may be determined from Fig. 6, by PROFESSOR JENKINS

(*Amer. Mach.*, *Dec.* 8, 1910). For centrally loaded rams, the relation is expressed by the formula:

$$\frac{R}{T}=\frac{1}{1-\sqrt{1-1.75\frac{P}{S}}}$$

to the left for the value of $\frac{R}{T}$ when, R being known, T is quickly found. For pressures less than 2000 lbs. per sq. in., the chart gives values for T that are small compared with those used in practice, due to the fact that the assumption of central loading can seldom be made.

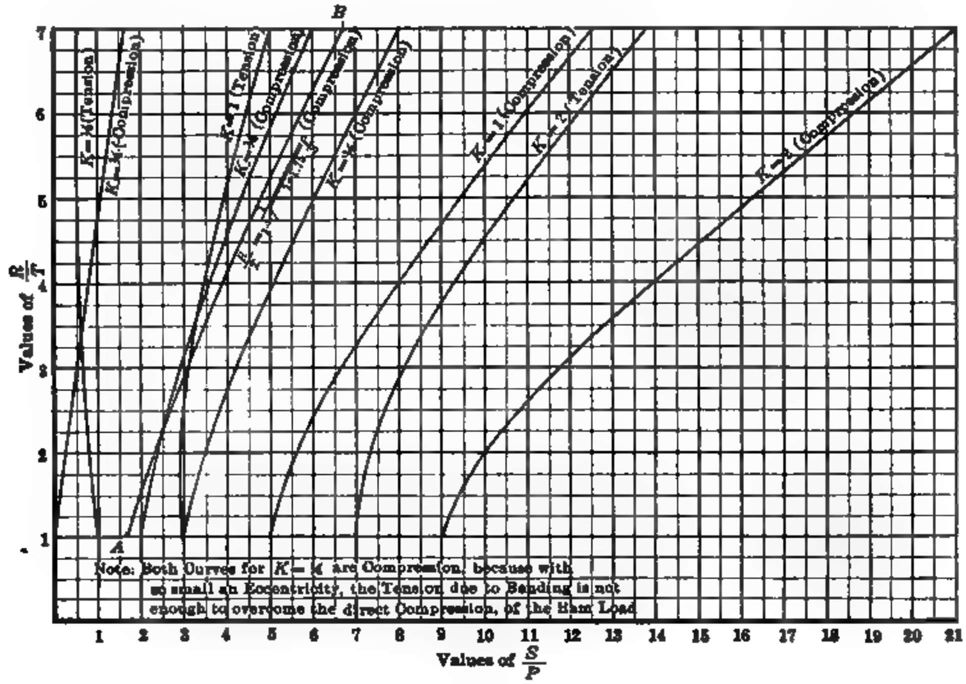


FIG. 6.—Thickness of hydraulic press rams.

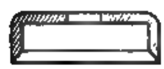


FIG. 7.
Cup Packing

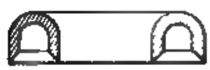


FIG. 8.
U Packing

FIG. 9.
Hat Packing

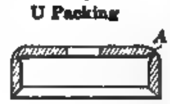


FIG. 10.
Failure of Packing

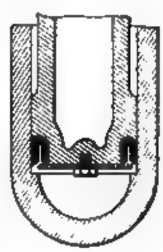


FIG. 11.
End U Packing

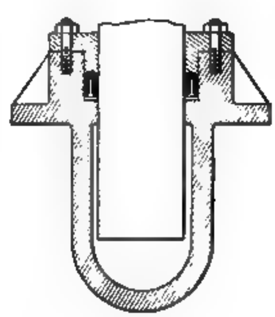


FIG. 12.
Neck U Packing

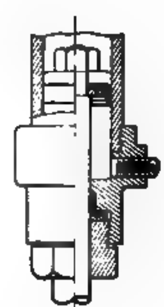


FIG. 13.
Cup and Hat Packing

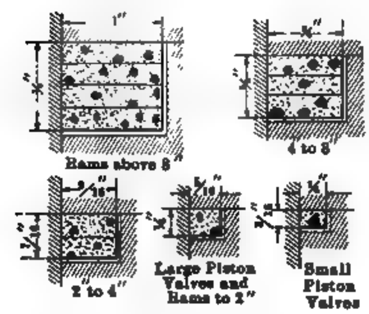


FIG. 14.
Leather ring plunger packing.

FIG. 16.
Leather ring flange packing.

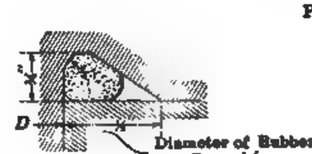


FIG. 15.
Rapleff flange packing.

FIGS. 7 to 16.—High-pressure hydraulic packings.

in which R = outside radius of ram, ins.,
T = thickness of ram, ins.,
P = pressure on ram, lbs. per sq. in.,
S = stress on ram, lbs. per sq. in.

This equation is plotted in the curve AB, Fig. 6. Enter the chart by the assumed value of $\frac{P}{S}$, trace upward to the curve AB and then

It is, therefore, best to assume an eccentric load for which the formula is:

$$\frac{S}{P}=\frac{1}{1-\left(1-\frac{T}{R}\right)^2}\pm\frac{4K}{1-\left(1-\frac{T}{R}\right)^4}$$

in which
$$K=\frac{\text{eccentricity of load, ins.}}{R}$$

the remaining notation being as in the last formula, while the plus sign relates to the compressive and the negative to the tensile stress.

The use of the formula is best shown by an example:

Let $P = 2000$ lbs. per sq. in.,
$R = 5$ ins.,
$T = 1\frac{2}{3}$ ins.,
eccentricity of load $= 10$ ins.,
giving $K = \frac{10}{5} = 2$

Enter the chart with $\frac{R}{T} = \frac{5}{1\frac{2}{3}}$
$= 3$ with sufficient accuracy, and trace to the right for the curves for $K = 2$ and then down, finding $\frac{S}{P} = 8.2$ for tension and 11.75 for compression, or, $S = 8.2 \times 2000 = 16{,}400$ for tension and $11.75 \times 2000 = 23{,}500$ for compression.

Hydraulic Packing

The present tendency, for work of a rugged character, is to adopt the stuffing box in place of all forms of leather packing in hydraulic machinery. The chief reason lies in the easier renewal and the reduced liability of stuffing-box packing to injury from rusty and scored rams and cylinders. Security against injury from these causes makes necessary the use of brass or copper linings with leather packings, a precaution that is not necessary with the stuffing box. Leather, however, still has, and probably always will have, application in packings for valves and in places where it can be protected from injury.

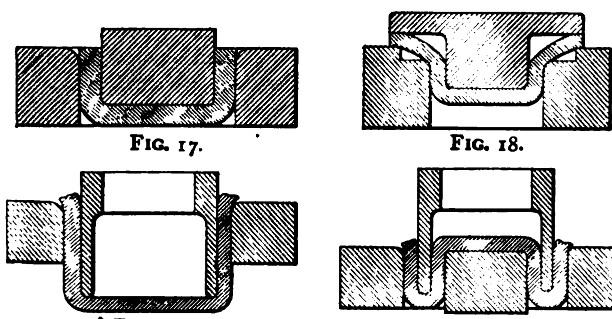


FIG. 17. FIG. 18.

FIG. 19. FIG. 20.

FIGS. 17 to 20.—Molds for making hydraulic cupped leathers.

Figs. 7, 8 and 9 show the cup, U and hat forms of hydraulic leathers, Figs. 11, 12 and 13 showing applications. Fig. 10 shows the manner in which the leathers fail at the bend A. In order to reduce this tendency it is important to shape the part against which the bend of the leather fits to the shape of the leather as shown in Figs. 11, 12 and 13. The depth of the packing has no effect on its tightness or friction and, to save unnecessary stress on the leather when forming it, a moderate depth is best. PROF. A. LEWIS JENKINS (*Amer. Mach., Sept.* 22, 1910) gives the dimensions of Table 14 for U and cup leathers.

Some constructors prefer the leather-ring form of packing to the cupped leather. This construction with dimensions is shown in Fig. 14. It has been used successfully for pressures of 6000 lbs. per sq. in. Cylinder packing is turned to fit the cylinder and ram packing is bored to fit the ram, other dimensions being left rough. Multiple leathers are stitched or cemented together.. The pressure water must be given free access to the spaces behind and below the ring and, contrary to what might be expected, this construction can only be used for pressure in one direction—double-acting cylinders requiring a ring for each direction of pressure. Obviously too, it will not make the double joint of Fig. 12. If used in this construction a joint must be made under the gland flange and for this the Rapieff joint, Fig. 15, is well tested and very suitable (see Rapieff joint).

The action of the leather-ring packing is undoubtedly the same as that of the cupped form, that is, the pressure behind and below the

TABLE 14.—DIMENSIONS OF HYDRAULIC CUPPED LEATHER PACKINGS

	End U-packing				U-wall, or neck packing				Cup packing		Stock	
Diam. of ram	D	O	H	T	D	O	H	K	D	H	Diam.	Thick.
4"	2½"	4"	¾"	1/16"	4"	5½"	1"	1/16"	4"	1"	6"	1/16"
6"	4½"	6"	1"	1/16"	6"	7½"	1 1/16"	1/16"	6"	1¼"	8½	1/16"
8"	6½"	8"	1¼"	1/16"	8"	9½'	1¼"	1/16"	8"	1½"	11"	1/16
10"	8½"	10"	1¼"	¼"	10"	11½"	1⅜"	⅛"	10"	1½"	13"	¼"
12"	10½"	12"	1½"	¼"	12"	13½"	1½"	⅛"	12"	1½"	15"	¼"
13"	11½"	13"	1½"	¼"	13"	14½"	1½"	⅛"	13"	1¾"	16½"	¼"
14"	12½"	14"	1½"	¼"	14"	15½"	1⅝"	⅛ "	14"	1¾"	17½"	¼"
18"	16½"	18"	1⅝"	¼"	18"	19½"	1⅝"	1/16"	18"	2"	22"	¼"
21"	19½"	21"	1⅝"	¼"	21"	22½"	1⅝"	1/16"	21"	2"	25"	¼"
23"	21½"	23"	1⅝"	¼"	23"	24½"	1⅝"	1/16"	23"	2"	27"	¼"
24"	22½"	24"	1⅝"	¼"	24"	25½"	1⅝"	1/16"	24"	2"	28"	¼"

leather forces it toward the joint, as Fig. 10 shows it to do with cupped packing. This makes it suitable for use as a flange packing as shown in Fig. 16, in which it is not pinched in place but is free to be forced into the joint after the manner of the Rapieff construction.

Leather packings are usually made of hard, close-grained, oak-tanned leather, though some prefer Vim leather. They should be shaved to uniform thickness and soaked in warm water to make them pliable. Concensus of opinion favors the flesh side for the wearing side. After drying, they should be soaked for a half hour in warm tallow or paraffin Fig. 17 (*Mechanical World*, 1905) shows the form of the mold for cup, Fig. 18 for hat, and Figs. 19 and 20 for U leathers. In making U leathers, after the first operation, Fig. 19, the leather should stand an hour or more when the center block is inserted, the middle part is forced partly back and the whole allowed to dry. The mold should be forced together with a vice or press, the use of a central bolt being inadvisable as the leather draws away from the hole and sometimes tears.

The proportions of stuffing boxes for high-pressure hydraulic work may be determined from Fig. 21 by PROFESSOR JENKINS (*Amer. Mach., Sept.* 22, 1910). The diameter of the bolts should be figured for a stress at the root of the thread of from 10,000 to 15,000 lbs. per sq. in., remembering that the studs need carry no considerable stress due to screwing up and that the load due to the pressure is only that for the ring area of the cavity.

There is a diversity of practice regarding the shape of the bottom of the stuffing box and the end of the gland, some claiming that the taper shown by the dotted lines leads to an undue pressure against the ram if leakage around and outside the packing is to be prevented, while others claim that the flat construction requires such pressure of the gland as to cause the packing to harden and score the ram. The construction shown in the small view of Fig. 21 would seem to be the most logical.

Friction of the Hydraulic Cup Leather Packing

The friction of hydraulic cup leather packing formed the subject of a thorough and painstaking investigation with specially constructed apparatus by John Hick of Bolton, England, and published in a pamphlet by E. & F. N. Spon in 1867. Experiments were made on

$\frac{3}{4}$-, 4-, and 8-in. rams under a great variety of pressures ranging between 200 and 6000 lbs. per sq. in., and showed the following general results:

The total friction increases with the pressure.

At constant pressure per sq. in. the total friction increases in direct proportion with the diameter of the ram, that is to say:

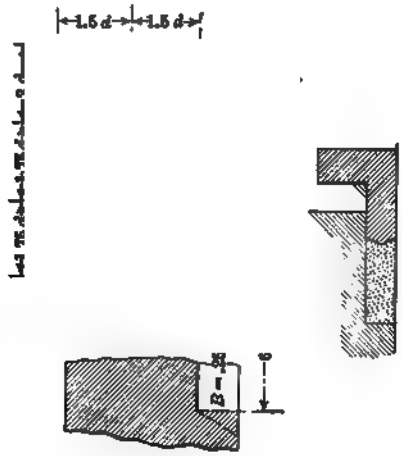


Fig. 21.—Dimensions of high-pressure hydraulic stuffing boxes.

Fig. 22.—High-pressure hydraulic stop valve.

At constant pressure per sq. in. the percentage lost by friction varies inversely with the diameter.

The depth of the leather does not affect the friction of the ram.

The experiments resulted in the following formula:

Total friction, lbs. $= C \times$ ram diam., ins. $\times$ pres. per sq. in., lbs.

The value of C for new, hard, badly lubricated leathers was .0471 and for well-lubricated leathers in good condition .0314.

Mr. Hick concludes the account of his experiments with Table 15 of the frictional resistance as a percentage of the total pressure on rams. These losses seem very small but they are well authenticated.

Table 15.—The Frictional Resistance of Cup Leather Hydraulic Packing

Diameter, inches	Friction, per cent	Diameter, inches	Friction, per cent
2	2.0	12	0.33
3	1.33	13	0.30
4	1.0	14	0.28
5	0.80	15	0.26
6	0.66	16	0.25
7	0.57	17	0.23
8	0.50	18	0.22
9	0.44	19	0.21
10	0.40	20	0.20
[illegible]	0.38		

Friction of Hydraulic Stuffing Boxes

The friction of hydraulic stuffing boxes using braided hemp packing, compressed rather hard with fair lubrication and plunger condition, formed the subject of experiments made at the Pencoyd Iron Works, and reported by Walter Ferris (*Amer. Mach., Feb.* 3, 1898). The apparatus used was a hydraulic intensifier having rams of $17\frac{3}{4}$, $14\frac{1}{2}$, and 8 ins. diameter used in various combinations. Initial pressures of 285, 335, 350 and 475 and terminal pressures of 750, 1450 and 1510 lbs. per sq. in. were used.

Unlike the cup leather, the stuffing box is subject to the outside pressure of the gland bolts and with the gland screwed down hard enough to hold high pressures, an increased percentage loss was naturally found when lighter pressures were used. With the packing compressed only enough to prevent leakage, Mr. Ferris finds the formula:

Total friction, lbs. $= .2 \times$ ram diam., ins. $\times$ pres. per sq. in., lbs.

to fairly represent the most efficient performance to be expected from machines having a single ram.

For the percentage of loss Mr. Ferris deduces from this:

$$\frac{\text{Total friction, lbs.}}{\text{Total pressure, lbs.}} = \frac{.255}{\text{diam. ram, ins.}}$$

showing the percentage of loss to vary inversely with the diameter and, since low pressures involve large diameters, he advocates low pressures, with intensifiers where necessary.

For intensifiers, which have two rams, he deduces the formula:

$$p_2 = p_1 \frac{A - .2D}{a + .2d}$$

in which p_1 = initial pressure, lbs. per sq. in.,
p_2 = intensified pressure, lbs. per sq. in.,
A = area of large ram, sq. ins.,
a = area of small ram, sq. ins.,
D = diameter of large ram, ins.,
d = diameter of small ram, ins.

Table 16 compares the results obtained with varying initial pressures under the same adjustment of the stuffing box with the results to be expected from this formula and brings out the progressive effect of an undue tightening of the gland bolts.

Table 16.—Ferris's Experimental Results Compared with His Formula for the Friction of Hydraulic Intensifiers Fitted with Hemp Packed Stuffing Boxes

Initial pressure, lbs. per sq. in.	475	350	335	285
Intensified pressure by experiment, lbs. per sq. in.	1450	1510	1450	750
Intensified pressure by formula, lbs. per sq. in.	1433	1643	1572	860
Ratio $\frac{\text{experiment}}{\text{formula}}$	1 01	.92	.92	.87

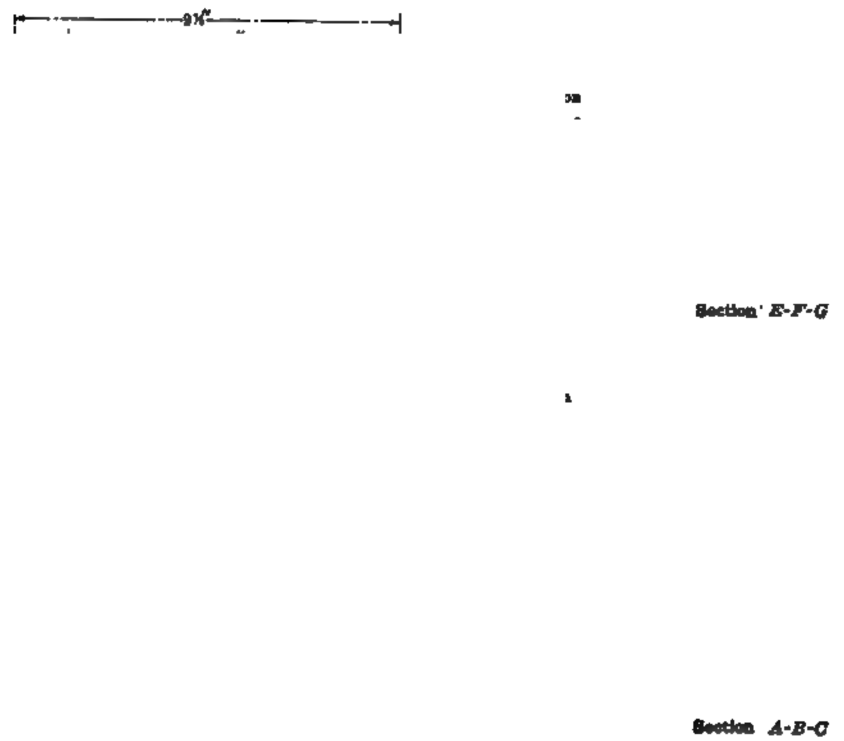


FIG. 23.—Quick return hydraulic valve.

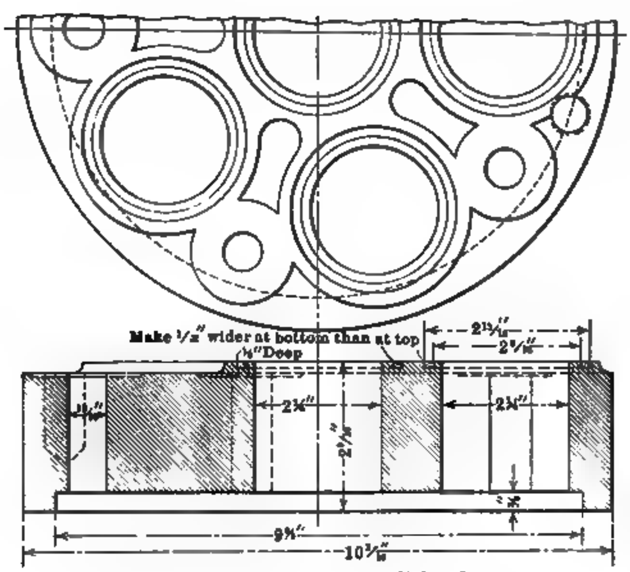


FIG. 24.- Leather Rings in Valve Seat

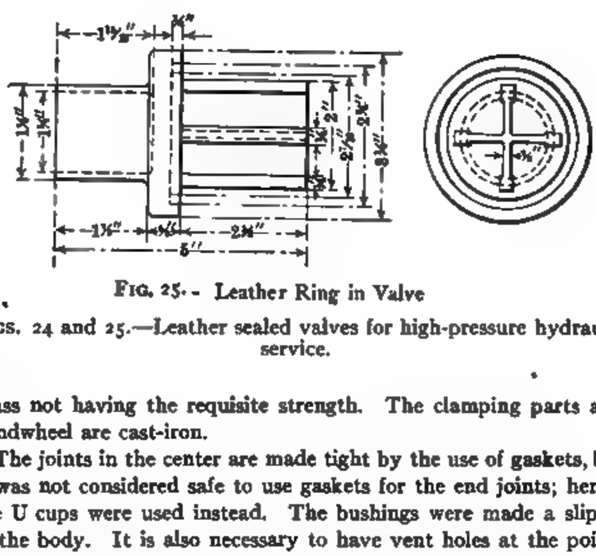


FIG. 25.- Leather Ring in Valve

FIGS. 24 and 25.—Leather sealed valves for high-pressure hydraulic service.

High-pressure Hydraulic Valves and Fittings

For high-pressure flange joints, see Index.

A valve for high-pressure (1500 *lbs.*) *hydraulic service* is shown in Fig. 22, by JAS. CLARK (*Amer. Mach., Aug.* 10, 1911).

The body *B* is a steel casting. The inlet and outlet are 2 ins. in diameter. The bushings *C* are 3½ ins. inside diameter and were made from drawn brass tube ⅜ in. thick. They are provided with 16 inlet ports *d* and 16 outlet ports *e*, each ⅜×⅜ in. The cup seats, distance pieces, and screw are brass, while the stem *F* is soft steel, brass not having the requisite strength. The clamping parts and handwheel are cast-iron.

The joints in the center are made tight by the use of gaskets, but it was not considered safe to use gaskets for the end joints; hence, the U cups were used instead. The bushings were made a slip-fit in the body. It is also necessary to have vent holes at the points *g* and *h*, so that in case a cup should leak, the balance would not then be destroyed.

In service these valves have proven absolutely tight.

A quick return hydraulic valve, intended to reduce the time required for the return stroke of hydraulic machinery, is shown in Fig. 23 (*Amer. Mach., Apr.* 8, 1897). The valve is used successfully in a large steel works under pressures as high as 4000 lbs. per sq. in. The construction involves a main valve operated by the water pressure, which latter is controlled by the usual small hand-operated valve, which becomes, with this arrangement, a supplementary valve.

The valve is attached at any convenient point, preferably directly to the hydraulic cylinder at *a*. The high-pressure water enters

from below at *b*. At the top, at orifice *c*, is a connection to a small hand valve, which controls the action of the piston *d*, which forms the head of and actuates the main valve. The water is discharged from the main cylinder through the valve and the opening *e*, the valve, as shown, being in position for discharging the main cylinder.

In operation, when the water above the piston *d* is discharged by a small hand valve, the water at *b* forces the valve up at once, closing the exhaust and introducing the high-pressure water into the main cylinder. A reverse operation forces the piston *d* down, shutting off the water from the main cylinder, and simultaneously opening the exhaust *via* the outlet *e*. It will be observed that the valve is

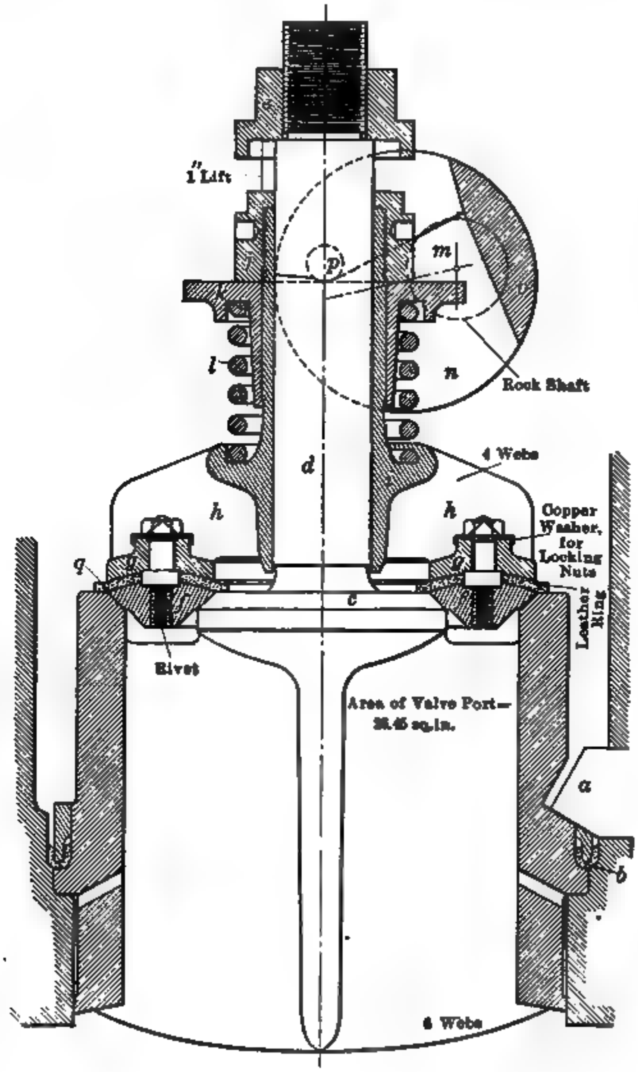


FIG. 26.—Poppet valve for high-pressure hydraulic service, Riedler system.

metal-seated, no live packing being necessary except that shown in the small piston *d*.

A leather-seated valve for high-pressure hydraulic pump service, as used at the Pencoyd Iron Works under 500 lbs. pressure, is shown in Figs. 24 and 25 (*Amer. Mach., July* 14, 1898).

A groove about ½ in. square, but slightly dovetailed at the bottom, is cut in the face, in which is inserted the ring of leather, this ring, as inserted, standing slightly above the surrounding surface, but, of course, soon becoming flattened under the pressure until this projection above the surface is scarcely appreciable. Fig. 24 shows the construction as applied to the seat, while Fig. 25 shows it applied to a valve.

This construction is practically a complete preventative of the tendency of high-pressure water to cut and score the valve seats.

Valves for high-pressure hydraulic service as used on the pumping engines of the Pope Tube Mills (Riedler system) are shown in Fig. 26 (*Amer. Mach., Oct.* 27, 1898). The illustration shows both suction and discharge valves which are identical. The engine operates at 60 r.p.m. and under 1500 lbs. per sq. in. pressure with entire smoothness, indicator cords under those conditions being almost entirely free from oscillations. The special feature of the Riedler system of valves is that while they are closed positively by mechanism they are opened by fluid pressure—air or water as the case may be. The valve seat

FIG. 27.
Rapid Coupler for Hydraulic Connection

FIG. 28.
Hydraulic Hand Union

FIG. 31.

FIG. 32.

FIGS. 27 to 32.—Rapid and flexible hydraulic pipe connections.

will be seen to be inserted and to be held down in place by the conical ended pins *a*, while it is packed against leakage by the cupped leather *b*. By the aid of six webs it carries a central seat *c*, the opening being thus annular. From the seat *c* rises the guiding stem *d*, on which the valve slides as it opens, the stop *e* limiting its movement. The valve proper *f* is a ring and carries above it a second ring *g*, having four cross webs *h* cast in one with it, the sleeve *i*, also in one with *h*, surrounding the stem *d*. The sleeve *i* has a threaded cap *j* screwed down to a defined position, and serving on its lower side as a stop for the flange *k*, which is pressed upward against *j* by a strong spring *l*. The rock shaft *m* is the valve-closing shaft. Two disks, one of which is seen at *n*, stand one each side of the valve stem *d*, their simultaneous action being secured by the connecting tie *o*, while each disk carries a pin *p*, which, when moving downward, acts on the flange *k* and closes the valve through the medium of the spring *l*. At first sight there would appear to be still a spring action here, but in point of fact the spring is only a safeguard. It is of a strength

sufficient to cause the flange *k* and the sleeve *i* and with them the valve, to move as one piece. The action is strictly mechanical, and the valve moves with the mechanism as though the spring were not there, *but* should a particle of foreign substance lodge under the valve, and so prevent its seating, or should the valve be set wrong, during the trial adjustments, the spring would give way and prevent damage. Without the provision of the spring, disaster might follow on slight causes.

Another novel feature of this valve is seen in the leather ring *q*, placed between the valve *f* and the ring *g*. The tendency of high-pressure water to take advantage of any slight leak through a valve and cut the valve and seat into channels is well known, and this leather ring is provided to stop it. The valve is not leather packed in the usual sense, in that it has a metal to metal joint, being in fact a perfect valve from usual standards without the leather ring, which simply serves to seal the joint and prevent the water from finding its way through any incipient leaks that may be present, and thus prevent grooving and channeling the valve and seat.

Flexible hydraulic fittings, designed originally for a testing room but useful in other locations, are shown in Figs. 27–32 by F. S. Bunker (*Amer. Mach., June* 22, 1911). The rapid-connection coupling, Fig. 27, has its upper half connected into the stop valve on the main-line tap while the lower half is fitted to the pipe *A* of any of the flexible or semi-flexible joints, Figs. 29, 30 and 31. When connecting Fig. 27, it is only necessary to push the central pipe into the main housing and give it a quarter turn which seats the cross pin in the hook *H*. Fig. 32 is a swivel joint which, used in pipes *A* and *B* of Figs. 29, 30 and 31, makes them all universal. Fig. 28 shows a union joint.

A swivel pipe joint for hydraulic pipe work, compiled from actual experience, is supplied by U. Peters (*Amer. Mach., June* 11, 1903) and shown in Fig. 33 and the accompanying table of dimensions.

An air-chamber charging device used by the Nordberg Mfg. Co. in connection with their large pumping engines is shown in Fig. 34 (*Amer. Mach., Mar.* 26, 1903).

The main globe casting, which is of 13 ins. interior diameter, is located near the water end of the engine and is connected at *a* with one end of one of the pump cylinders and at *b* with the air space in the air chamber. The connection at *a* being with a pump cylinder, the interior of the globe is subject to alternate pressure and suction. During the suction stroke the globe is filled with air by the valve *d*, and during the pressure stroke this air is expelled through the valve *e* and the pipe *b* to the air chamber. The valve *c* is introduced to control the ingress and egress of the water. On the one hand air must not enter so freely as to more than fill the globe and then escape to the pump cylinder, and on the other the globe must be so nearly full of water that the rise during the pressure stroke will expel the air. With free egress of the water it might easily escape in such volume that, opposed by the compressed air above it, it would not rise to a point where the air would be expelled. On the contrary, there is no danger of having too much water in the globe, as the return of the water from pipe *b* is prevented by the valve *e*, any drop of the water level insuring the entering of a fresh supply of air. To insure working conditions it is essential then that the water shall enter the globe more freely than it escapes from it. Valve *c* is therefore an obstruction valve only—that is, it has a hole through it for the water to escape. During the pressure stroke this valve opens freely and the water enters and expels the air, but during the suction stroke the water can only escape through the hole and the flow outward being more restricted than that inward, it is made certain that the volume of water escaping from the globe will not be in excess of that which will again completely fill it and expel the air on the succeeding pressure stroke.

An air-chamber charging device for high-pressure work by Walter Ferris is shown in Fig. 35 (*Amer. Mach., Nov.* 2, 1899). It consists of a vertical barrel of pipe, connected at the bottom with *one* water cylin-

Nominal Size	3/4"	1"	1 1/4"	Nominal Size	3/4"	1"	1 1/4"	Nominal Size	3/4"	1"	1 1/4"	Nominal Size	3/4"	1"	1 1/4"
A	5/8	3/4	1	*Q*	1 3/4	2	2 1/4	*R*	6	7	8	*a*	6 1/4	7 1/8	8
L	7 5/16	8 5/16	9 3/8	*Y*	1/2	7/8	1	*E*	1 1/4	1 1/4	1 3/8	*b*	1 5/16	1 5/16	1 3/8
M	2 5/8	2 3/4	3	*X*	5/8	5/8	3/4	D_1	1 3/4	1 7/8	2	*n*	1/16	1/16	1/16
N	3	3 1/4	3 3/4	*K*	2 3/4	2 7/8	3 3/8	D_2	2 5/16	2 1/2	2 3/8	*d*	1	1 1/8	1 1/8
O	3 11/16	3 3/4	3 13/16	*G*	1 5/16	1 11/16	1 3/4	D_3	2 1/2	2 3/4	2 3/4	*e*	5/16	11/32	3/8
D	4 1/4	4 7/16	4 1/2	*H*	1 1/8	1 1/4	1 1/2	*S*	1 1/2	1 1/2	1 1/2	*f*	1/4	1/4	1/4
I	1 3/4	2 1/8	2 3/8	*B*	2 5/8	2 3/4	3 7/8	*T*	3 1/2	3 3/4	3 3/4	*g*	3/8	5/8	3/8
C	3 7/16	3 5/8	4	*U*	3/4	1	1 1/8	*J*	3 3/4	3 3/4	4	*h*	5/16	5/16	5/16
F	1	1 1/4	1 3/8	*V*	1	1 1/8	1 1/4	*Z*	1/4	1/8	5/16	*i*	1/8	1/8	1/8
P	2 11/16	2 3/4	3	*W*	3/4	15/16	1					*t*	5/16	5/16	5/16

Fig. 33.—Dimensions of swivel pipe joints for high-pressure hydraulic service.

der of the main pressure pump, and ending at the top in a branch carrying two check valves, one inward and one outward opening. The connection to the main pressure pump is preferably—though not necessarily—double; a large pipe with a check valve opening toward the main pump, and a small pipe without a check valve. This is to partially equalize the amount of water passing from the main pump to the air pump under (say) 500 lbs. pressure per sq. in. with the amount returning to the main pump during its suction stroke, at a pressure of 5 or 10 lbs. Hence, during the suction stroke the water from the air pump comes into the main pump through both the 1½-in. and ½-in. pipes, but it is expelled during the delivery stroke through the ½-in. pipe only, as the check valve in the 1½-in. pipe closes. It is absolutely necessary to the successful working of this pump that it should have *no* appreciable clearance spaces. During the delivery stroke of the main pump the water must rise into the air pump and fill it up to the upper check valve V_2, expelling every particle of air. Some water goes through the check valve, too, but that does no harm. Then, during the suction stroke the

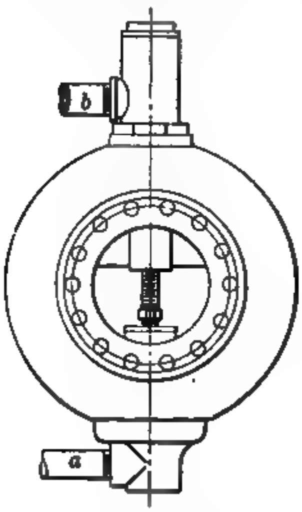


FIG. 34.—Air chamber charging device.

valve V_2 closes, the water is drawn back to the main pump, followed by air which enters through the check valve V_1, and which is expelled during the next delivery stroke. The branch carrying the two check valves V_1 and V_2 is inclined, in order to prevent the trapping of air under the caps of the valves or in other recesses. For the same reason the nipple *a* is tapped into the cap *b* at the highest point. In proportioning the pipes *c* and *d* it is also necessary to be sure that more water will leave the main pump during each delivery stroke than will come back during the next suction stroke. Otherwise the barrel of the air pump will soon be pumped dry and fail to work.

Long suction pipes should be fitted with air chambers. It is not the pressure but the energy of the moving water that causes water hammer and, except for the increased diameter of the pipe and the resulting reduced velocity of the water, air chambers are about as necessary on suction as on pressure lines. F. M. Wheeler has pointed out (*Trans. A. S. M. E., Vol.* 14) that suction chambers are frequently wrongly connected. The air chamber should be so placed that whenever the column of water is stopped or checked by the action of the pump it can flow on past the pump suction chamber or valves to the air chamber, so that its energy can be expended directly on the confined air. The chamber should not be so placed that the water passes under or past a right angle opening into it. Mr. Wheeler cites Fig. 36 as a bad and Figs. 37, 38 and 39 as good arrangements.

The siphon, although the simplest of mechanical appliances, is subject to many vagaries and frequently refuses to operate. LESTER ALLEN (*Amer. Mach., Sept.* 21, 1893) after much experimental work, explains many of these actions and the means of overcoming them as follows:

The siphon shown in Fig. 41 is clearly inoperative. The discharge from the free end *b* is so much greater than the inflow at *a* that the liquid in the branch *cb* runs out, admitting air and stopping the action almost immediately, unless the siphon be so small that the leg *cb*, through its capilarity, holds the liquid column from breaking up In general, through the effect of capilarity, small siphons will often work even though large ones constructed in the same way will not.

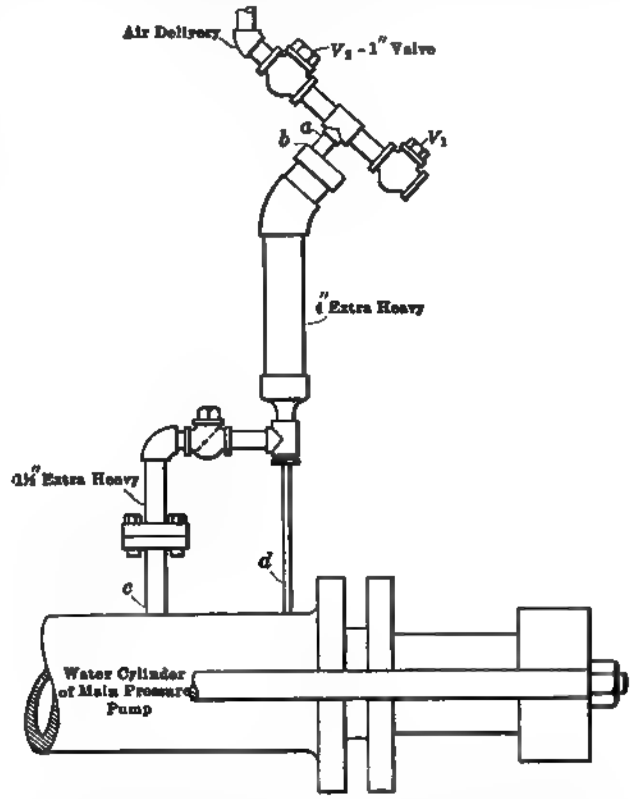


FIG. 35.—Air chamber charging device for high pressures.

In Fig. 42 is shown a reverse construction in which the supply end is made much larger than the discharge end, the latter being still free as before. Here the supply always being more than sufficient for satisfying the capacity of the discharge end, the siphon, if operating upon a liquid entirely free from absorbed gas and non-volatile, will be continuously operative so long as the liquid in *A* is maintained at a level that will keep the end *a* submerged.

Fig. 43 illustrates how the inoperative form of siphon, shown in Fig. 41, may be made operative by submerging the discharge end in the liquid of the receiving tank *B*. With a non-volatile liquid containing no absorbed gas, the discharge of the siphon, thus constructed and arranged, would be continuous, so long as both ends are kept submerged.

A practical conclusion from what has been said is, that siphons will be more certain in their action when the supply end is larger than the discharge end, and when the discharge end is submerged. A regulating valve or cock, at or near the discharge end of the siphon, may be used to adjust the discharge and regulate it into proper relation with the supply. When this appliance is used, the pipe may be made of equal size throughout; and, in such case, the discharge end may be left free without cessation of flow, when the liquid to be siphoned is non-volatile and free from absorbed gas. The supply end may even be smaller than the discharge end when the regulating valve is used.

When the supply end of a siphon is only a little lower than the

highest point of the bend, and when, also, the level of discharge is very much lower than the level of supply, if the discharge end be left free, and the caliber of the tube be of moderate size, the flow through it will be so free, and the supply will be so copious in proportion to the discharge, that, if the water flowing through the pipe be pure, the siphon may act satisfactorily for a long period. If, on the contrary, the difference of level of the supply and discharge be quite small, and the caliber of the tube be large, the longer leg, if the discharge end be left free and unregulated by a valve, will be very apt to run out and render the siphon inoperative. It is scarcely necessary to add that any obstruction to inflow, such as the accumulation of silt in the supply end, or in any depressed portion of the tube, such as would form a trap wherein deposits of obstructive substances may collect, would produce a similar result. In such cases, a siphon, when refilled, may act for a time; but it slowly becomes inoperative provided the discharge end is not submerged, because the longer leg more or less gradually empties itself. This may be guarded against by a strainer placed over the supply end, but in many cases, the floating impurities in the liquid to be siphoned are so fine, that any strainer sufficiently fine to intercept them might itself become a sufficient obstruction to render the siphon inoperative.

Siphons which are to operate in water containing free or unabsorbed gas, should have a supply tank of sufficient size to permit the gas to rise and escape at the surface and the pipe should draw from the bottom of the tank. Dissolved air, which is always present in water, has a tendency to separate under the reduced pressure in the bend. Under active action, it is commonly carried along with the water and does no harm but, if the action be stopped by closing one end only, the air will collect at the bend and eventually make the siphon inoperative. On the other hand, if both ends be stopped, the separation of the air is prevented. Hence it is clear that siphons which are to be used intermittently should be provided with means for stopping both ends when the action is suspended.

The discharge of a siphon may be calculated from the formula:

$$V = \sqrt{2g(h'-h}$$

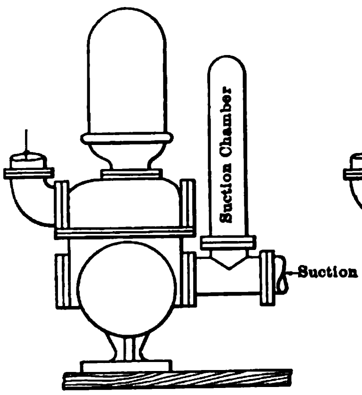


FIG. 36.

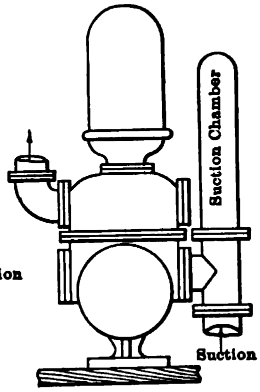


FIG. 37.

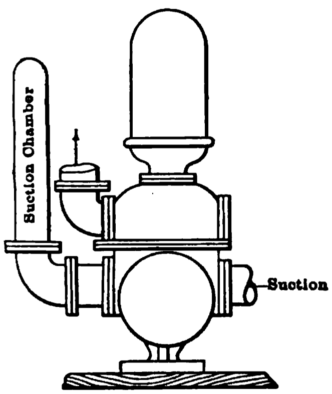


FIG. 38.

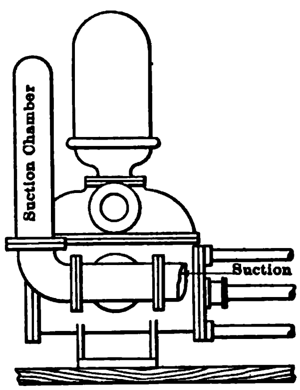


FIG. 39.

FIGS. 36 to 39.—Correct and incorrect arrangements of suction air chambers.

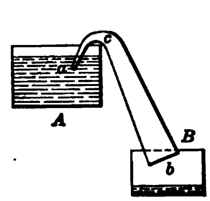


FIG. 41.

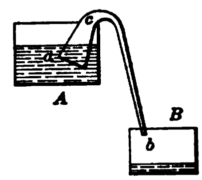


FIG. 42.

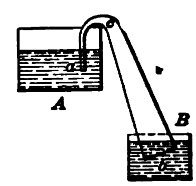


FIG. 43.

FIGS. 41 to 43.—Operative and inoperative siphons.

TABLE 17.—PERCENTAGE OF THE TOTAL AMOUNT OF WATER SUPPLIED TO HYDRAULIC RAMS WHICH IS DELIVERED TO VARIOUS ELEVATIONS

	Elevation of discharge above delivery valve at ram													
Working head	15 ft.	18 ft.	21 ft.	24 ft.	27 ft.	30 ft.	35 ft.	40 ft.	45 ft.	50 ft.	60 ft.	70 ft.	80 ft.	90 ft.	100 ft.
2 ft.	.0724	.0533	.0402	.0307	.0235	.0181	.0112	.0063	.0027	....	....	....	....	....	....
3 ft.	.1327	.1020	.0807	.0651	.0530	.0441	.0326	.0243	.0181	.0132	.0063	.0017	....	....	....
4 ft.	.1960	.1535	.1234	.1020	.0854	.0724	.0560	.0441	.0348	.0281	.0180	.0112	.0063	.0027	....
5 ft.	.2614	.2068	.1686	.1404	.1189	.1020	.0807	.0652	.0533	.0441	.0307	.0217	.0150	.0099	.0063
6 ft.	.3282	.2614	.2146	.1800	.1535	.1327	.1063	.0870	.0724	.0608	.0441	.0325	.0243	.0180	.0132
7 ft.	.3960	.3170	.2614	.2203	.1885	.1640	.1327	.1096	.0920	.0782	.0580	.0441	.0340	.0264	.0205
8 ft.	.4647	.3733	.3090	.2614	.2248	.1960	.1595	.1327	.1121	.0960	.0724	.0560	.0441	.0351	.0281
9 ft.	.5341	.4303	.3572	.3030	.2614	.2285	.1868	.1561	.1327	.1142	.0870	.0682	.0545	.0441	.0360
10 ft.	.6040	.4877	.4058	.3450	.2984	.2614	.2145	.1800	.1535	.1327	.1020	.0807	.0651	.0533	.0441
11 ft.	.6745	.5459	.4549	.3874	.3357	.2947	.2425	.2041	.1746	.1514	.1172	.0934	.0760	.0627	.0524
12 ft.	.7453	.6040	.5043	.4302	.3733	.3282	.2708	.2285	.1960	.1704	.1327	.1063	.0870	.0723	.0608
13 ft.	.8166	.6627	.5540	.4732	.4112	.3620	.2994	.2532	.2177	.1896	.1483	.1194	.0983	.0821	.0694
14 ft.	.8881	.7217	.6040	.5166	.4494	.3960	.3282	.2780	.2395	.2090	.1640	.1327	.1096	.0920	.0782
15 ft.	.9600	.7809	.6543	.5601	.4877	.4303	.3572	.3030	.2614	.2285	.1800	.1460	.1211	.1020	.0870
16 ft.	....	.8404	.7048	.6040	.5263	.4647	.3863	.3282	.2835	.2482	.1960	.1595	.1327	.1121	.0960
17 ft.	....	.9001	.7555	.6480	.5650	.4993	.4157	.3535	.3058	.2680	.2123	.1731	.1444	.1223	.1050
18 ft.	....	.9600	.8064	.6921	.6040	.5341	.4451	.3790	.3282	.2880	.2286	.1868	.1561	.1327	.1142
19 ft.	....	....	.8574	.7364	.6430	.5690	.4746	.4046	.3507	.3081	.2449	.2006	.1680	.1430	.1262
20 ft.	....	....	.9086	.7808	.6823	.6040	.5042	.4303	.3733	.3282	.2614	.2145	.1800	.1535	.1327
21 ft.	....	....	.9600	.8254	.7217	.6392	.5340	.4561	.3960	.3486	.2780	.2286	.1920	.1640	.1420
22 ft.	....	....	....	.8701	.7612	.6745	.5640	.4820	.4188	.3688	.2947	.2425	.2041	.1746	.1514
23 ft.	....	....	....	.9150	.8007	.7098	.5940	.5080	.4417	.3892	.3114	.2567	.2163	.1853	.1609
24 ft.	....	....	....	.9600	.8404	.7453	.6241	.5341	.4657	.4097	.3282	.2708	.2085	.1960	.1704

15, to 113 Ft. Head

100, to 300 Ft. Head

From the line for gallons per minute or hour trace to the right to the line for the head and down to the line for the efficiency of the pump where read brake horse-power.

FIG. 40.—Power and capacity of water pumps.

in which V = velocity, ft. per sec.,

h = difference of level between bend and surface of the supply tank, ft.,

h' = difference of level between bend and surface of the receiving tank, ft.,

g = acceleration of gravity = 32.2.

The performance of hydraulic rams is subject to conditions which are usually unknown; for example, the adjustment of the delivery valve. Under these circumstances no universal figures for the performance can be given, but Table 17 (*Amer. Engr., March* 18, 1886) shows what may reasonably be expected. Should the delivery pipe be long, the friction head should be added to the static head.

PIPE AND PIPE JOINTS

TABLE 1.—DIMENSIONS OF COMMERCIAL DRAWN PIPE

From the National Tube Co.'s Book of Standards

Standard

The permissible variation in weight is 5 per cent. above and 5 per cent. below. Taper of threads is ¾ in. diameter per ft. length for all sizes.

Size, ins.	Diameters, ins.		Thickness, ins.	Weight per foot, lbs.		Threads per inch	Couplings		
	External	Internal		Plain ends	Threads and couplings		Diameter, ins.	Length, ins.	Weight, lbs.
⅛	.405	.269	.068	.244	.245	27	.562	⅞	.029
¼	.540	.364	.088	.424	.425	18	.685	1	.043
⅜	.675	.493	.091	.567	.568	18	.848	1⅛	.070
½	.840	.622	109	.850	.852	14	1.024	1⅜	.116
¾	1.050	.824	.113	1.130	1.134	14	1.281	1⅝	.209
1	1.315	1.049	.133	1.678	1.684	11½	1.576	1⅞	.343
1¼	1.660	1.380	.140	2.272	2.281	11½	1.950	2⅛	.535
1½	1.900	1.610	.145	2.717	2.731	11½	2.218	2⅜	.743
2	2.375	2.067	.154	3.652	3.678	11½	2.766	2⅝	1.208
2½	2.875	2.469	.203	5.793	5.819	8	3.276	2⅞	1.720
3	3.500	3.068	.216	7.575	7.616	8	3.948	3⅛	2.498
3½	4.000	3.548	.226	9.109	9.202	8	4.591	3⅜	4.241
4	4.500	4.026	.237	10.790	10.889	8	5.091	3⅝	4.741
4½	5.000	4.506	.247	12.538	12.642	8	5.591	3⅞	5.241
5	5.563	5.047	.258	14.617	14.810	8	6.296	4⅛	8.091
6	6.625	6.065	.280	18.974	19.185	8	7.358	4⅛	9.554
7	7.625	7.023	.301	23.544	23.769	8	8.358	4⅜	10.932
8	8.625	8.071	.277	24.696	25.000	8	9.358	4⅝	13.905
8	8.625	7.981	.322	28.554	28.809	8	9.358	4⅝	13.905
9	9.625	8.941	.342	33.907	34.188	8	10.358	5⅛	17.236
10	10.750	10.192	.279	31.201	32.000	8	11.721	6⅛	29.877
10	10.750	10.136	.307	34.240	35.000	8	11.721	6⅛	29.877
10	10.750	10.020	.365	40.483	41.132	8	11.721	6⅛	29.877
11	11.750	11.000	.375	45.557	46.247	8	12.721	6⅛	32.550
12	12.750	12.090	.330	43.773	45.000	8	13.958	6⅛	43.098
12	12.750	12.000	.375	49.562	50.706	8	13.958	6⅛	43.098
13	14.000	13.250	.375	54.568	55.824	8	15.208	6⅛	47.152
14	15.000	14.250	.375	58.573	60.375	8	16.446	6⅛	59.493
15	16.000	15.250	.375	62.579	64.500	8	17.446	6⅛	63.294

Extra Strong

The permissible variation in weight is 5 per cent. above and 5 per cent. below.

Size, ins.	Diameters, ins.		Thickness, ins.	Weight per ft. plain ends, lbs.
	External	Internal		
⅛	.405	.215	.095	.314
¼	.540	.302	.119	.535
⅜	.675	.423	.126	.738
½	.840	.546	.147	1.087
¾	1.050	.742	.154	1.473
1	1.315	.957	.179	2.171
1¼	1.660	1.278	.191	2.996
1½	1.900	1.500	.200	3.631
2	2.375	1.939	.218	5.022
2½	2.875	2.323	.276	7.661
3	3.500	2.900	.300	10.252
3½	4.000	3.364	.318	12.505
4	4.500	3.826	.337	14.983
4½	5.000	4.290	.355	17.611
5	5.563	4.813	.375	20.778
6	6.625	5.761	.432	28.573
7	7.625	6.625	.500	38.048
8	8.625	7.625	.500	43.388
9	9.625	8.625	.500	48.728
10	10.750	9.750	.500	54.735
11	11.750	10.750	.500	60.075
12	12.750	11.750	.500	65.415
13	14.000	13.000	.500	72.091
14	15.000	14.000	.500	77.431
15	16.000	15.000	.500	82.771

Double Extra Strong

The permissible variation in weight is 10 per cent. above and 10 per cent. below.

Size, ins.	Diameters, ins.		Thickness, ins.	Weight per ft. plain ends, lbs.
	External	Internal		
½	.840	.252	.294	1.714
¾	1.050	.434	.308	2.440
1	1.315	.599	.358	3.659
1¼	1.660	.896	.382	5.214
1½	1.900	1.100	.400	6.408
2	2.375	1.503	.436	9.029
2½	2.875	1.771	.552	13.695
3	3.500	2.300	.600	18.583
3½	4.000	2.728	.636	22.850
4	4.500	3.152	.674	27.541
4½	5.000	3.580	.710	32.530
5	5.563	4.063	.750	38.552
6	6.625	4.897	.864	53.160
7	7.625	5.875	.875	63.079
8	8.625	6.875	.875	72.424

Bursting and Collapsing Strength of Pipe

Extended experiments on the bursting strength of pipe for the National Tube Co. by PROF. R. T. STEWART (*Trans. A. S. M. E., Vol.* 34) led the professor to advise the use of Barlow's formula for all ordinary calculations, with the proviso that the fiber stress be obtained as explained below. Barlow's formula is as follows:

$$\frac{S}{P}=\frac{R}{T}+1$$

in which S = fiber stress, lbs. per sq. in.,
P = internal pressure, lbs. per sq. in.,
R = inside radius of pipe, ins.,
T = thickness of pipe wall, ins.

Professor Stewart's recommendation regarding the fiber stress is that it be determined from the formulas:

Table 2.—British Standard Pipe Threads

Threads of standard Whitworth form and straight. The joint is made on the incomplete taper threads made by the die mouth.

Nominal inside diameter pipe, ins.	Approximate outside diameter pipe, ins.	Gage diameter top of thread, ins.	Depth of thread	Core diameter	Number of threads per inch
1/8	13/32	.383	.0230	.337	28
1/4	17/32	.518	.0335	.451	19
3/8	11/16	.656	.0335	.589	19
1/2	27/32	.825	.0455	.734	14
5/8	29/32	.902	.0455	.811	14
3/4	1 1/16	1.041	.0455	.950	14
7/8	1 7/32	1.189	.0455	1.098	14
1	1 11/32	1.309	.0580	1.193	11
1 1/4	1 11/16	1.650	.0580	1.534	11
1 1/2	1 29/32	1.882	.0580	1.766	11
1 3/4	2 5/32	2.116	.0580	2.000	11
2	2 3/8	2.347	.0580	2.23	11
2 1/4	2 5/8	2.587	.0580	2.471	11
2 1/2	3	2.960	.0580	2.844	11
2 3/4	3 1/4	3.210	.0580	3.094	11
3	3 1/2	3.460	.0580	3.344	11
3 1/4	3 3/4	3.700	.0580	3.584	11
3 1/2	4	3.950	.0580	3.834	11
3 3/4	4 1/4	4.200	.0580	4.084	11
4	4 1/2	4.450	.0580	4.334	11
4 1/2	5	4.950	.0580	4.834	11
5	5 1/2	5.450	.0580	5.334	11
5 1/2	6	5.950	.0580	5.834	11
6	6 1/2	6.450	.0580	6.334	11
7	7 1/2	7.450	.0640	7.322	10
8	8 1/2	8.450	.0640	8.322	10
9	9 1/2	9.450	.0640	9.322	10
10	10 1/2	10.450	.0640	10.322	10
11	11 1/2	11.450	.0800	11.290	8
12	12 1/2	12.450	.0800	12.290	8
13	13 3/4	13.680	.0800	13.520	8
14	14 3/4	14.680	.0800	14.520	8
15	15 3/4	15.680	.0800	15.520	8
16	16 3/4	16.680	.0800	16.520	8
17	17 3/4	17.680	.0800	17.520	8
18	18 3/4	18.680	.0800	18.520	8

$$S=\frac{40,000}{n} \text{ for butt-welded steel pipe}$$

$$=\frac{50,000}{n} \text{ for lap-welded steel pipe}$$

$$=\frac{60,000}{n} \text{ for seamless steel tubes}$$

$$=\frac{28,000}{n} \text{ for wrought-iron pipe}$$

in which S = working or safe fiber stress, lbs. per sq. in.,
n = factor of safety based on ultimate strength.

Some of the results of Professor Stewart's tests are given in Table 5.

The strength of tubes against collapsing pressure formed the subject of exhaustive tests by Prof. R. T. Stewart (*Trans. A. S. M. E., Vol.* 27). Over 500 tubes, provided by the National Tube Co., were tested, the diameters ranging between 3 and 10 ins., outside, and of all commercial thicknesses obtainable, the material being Bessemer steel, lap welded. The first result of the tests was to show that the collapsing pressure decreases as the length of the tube increases up to a length equal to about 6 diameters, beyond which there is no further material decrease in the collapsing pressure with increase of length. Beyond that length the collapsing pressure is given by the formulas:

Table 3.—Length of Commercial Drawn Pipe for 1 Sq. Ft. of Surface

From the National Tube Co.'s Book of Standards

Size ins.	External diameter, ins.	Standard weight pipe: Thickness, in.	Standard weight pipe: Length of pipe in ft. per sq. ft. of External surface	Standard weight pipe: Length of pipe in ft. per sq. ft. of Internal surface	Extra strong pipe: Thickness, in.	Extra strong pipe: Length of pipe in ft. per sq. ft. of External surface	Extra strong pipe: Length of pipe in ft. per sq. ft. of Internal surface	Double extra strong pipe: Thickness, in.	Double extra strong pipe: Length of pipe in ft. per sq. ft. of External surface	Double extra strong pipe: Length of pipe in ft. per sq. ft. of Internal surface
1/8	.405	.068	9.431	14.199	.095	9.431	17.766	....	......	......
1/4	.540	.088	7.073	10.493	.119	7.073	12.648	....	......	......
3/8	.675	.091	5.658	7.747	.126	5.658	9.030	....	......	......
1/2	.840	.109	4.547	6.141	.147	4.547	6.995	.294	4.547	15.157
3/4	1.050	.113	3.637	4.635	.154	3.637	5.147	.308	3.637	8.801
1	1.315	.133	2.904	3.641	.179	2.904	3.991	.358	2.904	6.376
1 1/4	1.660	.140	2.301	2.767	.191	2.301	2.988	.382	2.301	4.263
1 1/2	1.900	.145	2.010	2.372	.200	2.010	2.546	.400	2.010	3.472
2	2.375	.154	1.608	1.847	.218	1.608	1.969	.436	1.608	2.541
2 1/2	2.875	.203	1.328	1.547	.276	1.328	1.644	.552	1.328	2.156
3	3.500	.216	1.091	1.245	.300	1.091	1.317	.600	1.091	1.660
3 1/2	4.000	.226	.954	1.076	.318	.954	1.135	.636	.954	1.400
4	4.500	.237	.848	.948	.337	.848	.998	.674	.848	1.211
4 1/2	5.000	.247	.763	.847	.355	.763	.890	.710	.763	1.066
5	5.563	.258	.686	.756	.375	.686	.793	.750	.686	.940
6	6.625	.280	.576	.629	.432	.576	.663	.864	.576	.780
7	7.625	.301	.500	.543	.500	.500	.576	.875	.500	.650
8	8.625	.277	.442	.473	.500	.442	.500	.875	.442	.555
8	8.625	.322	.422	.478	....	......	......	....	......	......
9	9.625	.342	.396	.427	.500	.396	.442	....	......	......
10	10.750	.279	.355	.374	.500	.355	.391	....	......	......
10	10.750	.307	.355	.376	....	......	......	....	......	......
10	10.750	.365	.355	.381	....	......	......	....	......	......
11	11.750	.375	.325	.347	.500	.325	.355	....	......	......
12	12.750	.330	.299	.315	.500	.299	.325	....	......	......
12	12.750	.375	.299	.318	....	......	......	....	......	......
13	14.000	.375	.272	.288	.500	.272	.293	....	......	......
14	15.000	.375	.254	.268	.500	.254	.272	....	......	......
15	16.000	.375	.238	.250	.500	.238	.254	....	......	......

$$P=50,210,000\left(\frac{t}{d}\right)^2 \qquad (a)$$

$$\text{and } P=86,670\frac{t}{d}-1386 \qquad (b)$$

in which P = collapsing pressure, lbs. per sq. in.,
d = outside diameter, ins.,
t = thickness, ins.

Formula (a) is to be used when the ratio $\frac{t}{d}$ is less than .023 and formula (b) when this ratio exceeds that figure.

The dimensions of Bessemer steel lap-welded tubes of a greater length than six diameters against collapsing pressure may also be determined from Fig. 1, by Professor Stewart (*Trans. A. S. M. E., Vol.* 27).

In order to condense the size of the chart, the curve is broken into two parts, XX and YY; YY being the upper portion of XX transferred to the left and then dropped down, the break in the curve corresponding to a collapsing pressure of 2080 lbs. and a thickness divided by diameter of .040. The scales for the portion XX are at the lower and right-hand margins, while those for the portion YY are at the upper and left-hand margins.

The use of the chart is best shown by an example: Find the probable collapsing pressure of a tube having an external diameter equal to 6 ins. and a thickness of wall equal to .203 in.

TABLE 4.—TEST PRESSURE OF COMMERCIAL DRAWN PIPE

From the National Tube Co.'s Book of Standards

Standard: Size, ins.	Weight per foot complete, lbs.	Test pressure, lbs. Butt	Lap	Extra Strong: Size, ins.	Weight per foot plain ends, lbs.	Test pressure, lbs. Butt	Lap
⅛	.245	700		⅛	.314	700	.
¼	.425	700	.....	¼	.535	700	.
⅜	.568	700	.....	⅜	.738	700	...
½	.852	700	...	½	1.087	700	...
¾	1.134	700	.	¾	1.473	700	......
1	1.684	700	......	1	2.171	700	
1¼	2.281	700	1000	1¼	2.996	1500	......
1½	2.731	700	1000	1½	3.631	1500	2500
2	3.678	700	1000	2	5.022	1500	2500
2½	5.819	800	1000	2½	7.661	1500	2000
3	7.616	800	1000	3	10.252	1500	2000
3½	9.202	......	1000	3½	12.505	....	2000
4	10.889	......	1000	4	14.983	...	2000
4½	12.642	....	1000	4½	17.611	...	1800
5	14.810	......	1000	5	20.778	.	1800
6	19.185	....	1000	6	28.573	....	1800
7	23.769	....	1000	7	38.048	....	1500
8	25.000	...	800	8	43.388	....	1500
8	28.809	....	1000	9	48.728	..	1500
9	34.188	......	900	10	54.735	....	1200
10	32.000	....	600	11	60.075	......	1100
10	35.000	....	800	12	65.415	..	1100
10	41.132	...	900	13	72.091	.	1000
11	46.247	....	800	14	77.431	....	1000
12	45.000	..	600	15	82.771	....	1000
12	50.706	.....	800				
13	55.824	......	700				
14	60.375	......	700				
15	64.500	....	600				

In addition to the above test, on sizes ⅛ in. to 1 in. inclusive, the pipe is jarred with a hammer while under pressure.

Double Extra Strong

Size, ins.	Weight per foot plain ends, lbs.	Test pressure, lbs. Butt	Lap
½	1.714	700	....
¾	2.440	700	......
1	3.659	700	.....
1¼	5.214	2200	....
1½	6.408	2200	3000
2	9.029	2200	3000
2½	13.695	2200	3000
3	18.583	....	3000
3½	22.850	......	2500
4	27.541	.....	2500
4½	32.530	....	2000
5	38.552	......	2000
6	53.160	......	2000
7	63.079	....	2000
8	72.424	......	2000

TABLE 5.—BURSTING TEST PRESSURES OF COMMERCIAL DRAWN PIPE (From the National Tube Co.'s Book of Standards)

The column marked "See note above" gives the number burst by failure of material not at weld.

C—Clavarino conditions.

B—Birnie conditions.

	Size, ins.	Number of pieces burst	Nominal external diameter, ins.	Average thickness of walls, ins.	Bursting pressures, lbs. per sq. in. Minimum	Maximum	Average	Head condition	See note above	Average fiber stress by Barlow's formula	Class of material
Steel, butt-welded	⅛	10	.405	.066	11,840	17,320	14,266	*C*	1	44,011	Standard pipe
	¼	10	.540	.085	8,830	14,680	12,206	*C*	1	38,645	Standard pipe
	⅜	10	.675	.088	5,850	13,030	10,330	*C*	1	30,272	Standard pipe
	½	10	.840	.101	11,380	16,310	14,038	*C*	0	58,163	Standard pipe
	¾	10	1.050	.109	7,150	9,150	8,020	*C*	0	38,657	Standard pipe
	1	10	1.315	.131	4,500	8,800	6,990	*C*	0	35,085	Standard pipe
	1¼	10	1.660	.139	4,400	7,300	5,808	*C*	0	34,603	Standard pipe
	1¼	15	1.660	.140	5,500	11,900	7,700	*C*	1	45,215	[illegible]
	1½	10	1.900	.143	3,000	6,100	4,960	*C*	0	33,031	Standard pipe
	2	11	2.375	.149	3,830	6,060	4,951	*C*	0	40,485	Standard pipe
	2½	10	2.875	.198	4,310	5,740	5,134	*C*	0	37,351	Standard pipe
	3	10	3.500	.204	4,650	6,370	5,398	*C*	0	46,234	Standard pipe
	1¼	10	1.660	.180	7,910	14,280	10,514	*C*	0	48,922	Extra strong
	2	10	2.375	.213	7,250	8,940	8,238	*C*	0	45,935	Extra strong
	2	10	2.375	.220	6,160	8,920	7,661	*C*	0	41,347	Extra strong
	2	10	2.375	.445	8,500	18,314	14,992	*C*	0	40,023	XX strong
						General average				41,686	
Steel, lap-welded	2	10	2.375	.155	4,890	7,940	6,645	*C*	1	50,962	Standard pipe
	2	10	2.375	.182	4,860	10,060	7,361	*C*	0	47,889	Standard pipe
	3	10	3.500	.210	3,830	8,200	6,368	*C*	7	53,560	Standard pipe
	4	10	4.500	.232	4,810	5,680	5,249	*C*	1	51,462	Standard pipe
	5	10	5.563	.258	3,410	5,260	4,538	*C*	1	48,882	Standard pipe
	6	5	6.625	.275	2,450	5,210	4,088	*C*	0	49,286	Standard pipe
	6	5	6.625	.275	3,170	4,760	3,666	*B*	0	44,106	Standard pipe
	10	5	10.750	.349	3,560	4,730	4,290	*C*	1	66,080	Standard pipe
	10	5	10.750	.347	2,770	3,940	3,396	*B*	2	52,692	Standard pipe
	2	10	2.375	.218	2,500	9,870	7,909	*C*	0	43,254	Extra strong
	2	10	2.000	.108	5,100	6,560	6,062	*C*	7	55,607	Boiler tubes
	3	10	3.000	.112	3,220	4,860	3,967	*C*	1	52,957	Boiler tubes
	4	5	4.000	.135	3,640	4,070	3,840	*C*	2	56,978	Boiler tubes
	4	5	4.000	.136	3,720	4,040	3,914	*B*	1	57,440	Boiler tubes
						General average				52,225	
Steel, seamless	2	10	2.000	.098	5,420	6,590	6,052	*C*	10	6	oiler tubes
	3	10	3.000	.112	3,940	4,730	4,272	*C*	10	5	ioler tubes
	4	6	4.000	.134	4,160	4,440	4,318	*C*	6	6	oiler tubes
	4	4	4.000	.134	4,250	4,440	4,328	*B*	4	6	oiler tubes
						General average				6	
Iron, butt-welded	1¼	10	1.660	.136	2,880	6,290	5,283	*C*	3	3	tandard pipe
	1¼	10	1.660	.136	3,640	5,680	4,891	*C*	1	2	tandard pipe
	2	10	2.375	.156	2,930	4,250	3,687	*C*	2	2	tandard pipe
	1¼	10	1.660	.188	2,770	7,330	5,895	*C*	1	2	xtra strong
						General average				2..	
Iron, lap-welded	2	10	2.375	.152	2,400	3,940	3,213	*C*	1	25,122	Standard pipe
	2	10	2.375	.207	5,530	7,120	6,349	*C*	8	36,461	Extra strong
						General average				30,792	

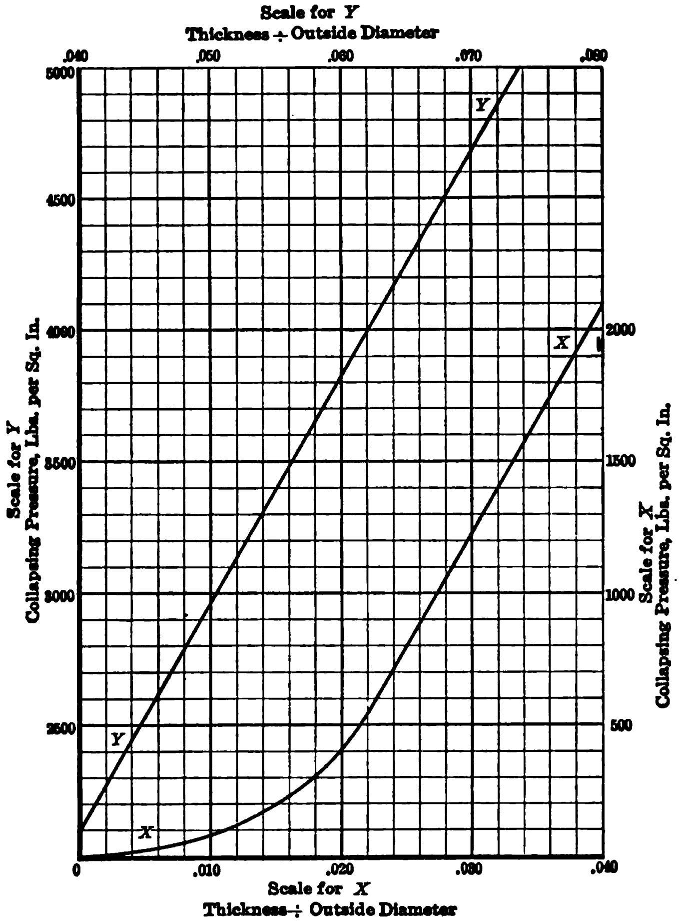
Dividing the outside diameter by the thickness of wall we get equal .0338. Since this value is less than .04 we look for it the scale at the lower margin of the chart and then trace upward ntil the line *XX* is reached; then trace to the right and read from e scale of probable collapsing pressures 1540 lbs. per sq. in. This the probable collapsing pressure for a length of 20 ft., but is also bstantially correct for any length greater than about six diameters, 3 ft. for a 6-in. tube, between transverse joints tending to hold e tube to a circular form.

A second chart by Professor Stewart, Fig. 2, shows the relation the probable collapsing pressure to the plain-end weight, while e preceding chart shows its relation to the thickness of wall. his chart should be used in calculations relating to collapsing pressure when the plain-end weight is either given or required, while the preceding chart should be used when the thickness of wall is given or required.

Example.—Find the probable collapsing pressure of a 6⅝ (7 O. D.) in. casing whose plain-end weight is 17 lbs. per ft.

Dividing the plain-end weight in lbs. per ft. by the square of the outside diameter in in., we get $\frac{w}{d^2}$, equal .347. Finding this value on the scale at the lower margin of Fig. 2 we trace vertically until the line *XX* is reached, then horizontally toward the right and read 1525 lbs. per sq. in. as the probable collapsing pressure required.

While this value is for a 20-ft. length of tube, as in the preceding chart, it may be used without substantial error for any length greater than about six diameters, or in this case 3½ ft., between joints tending to hold the tube to a circular form.



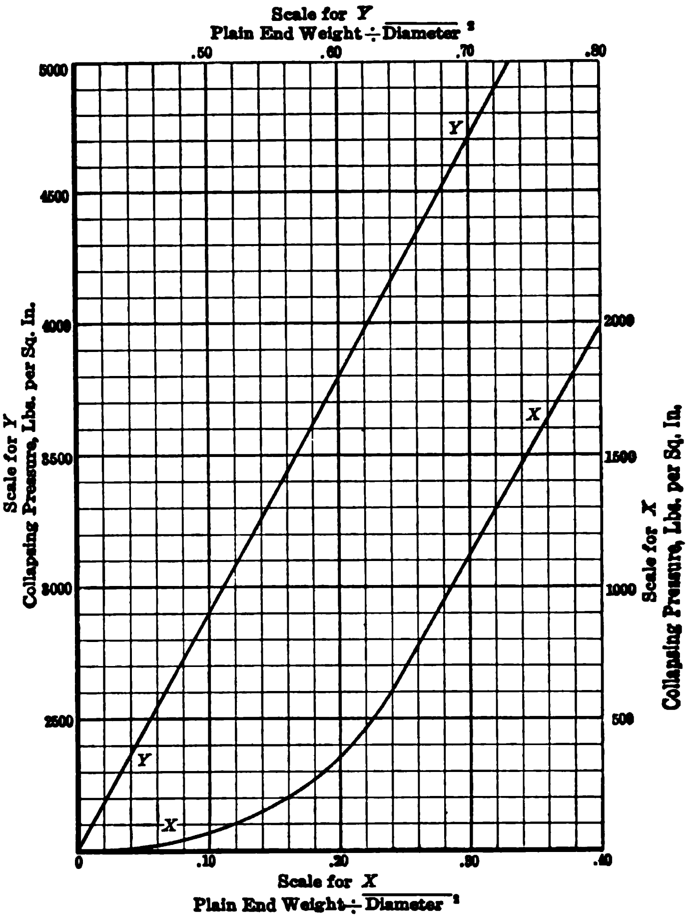


FIG. 1.—Plotted in terms of thickness. FIG. 2.—Plotted in terms of weight.

FIGS. 1 and 2.—The strength of Bessemer steel tubes against collapsing pressure.

Professor Stewart's paper contains the following observations: The apparent fiber stress under which the different tubes failed varied from about 7000 lbs. for the relatively thinnest to 35,000 lbs. per sq. in. for the relatively thickest walls. Since the average yield point of the material was 37,000 and the tensile strength 58,000 lbs. per sq. in., it would appear that the strength of a tube subjected to a fluid collapsing pressure is not dependent alone upon either the elastic limit or ultimate strength of the material constituting it.

The experiments show that the element of greatest weakness in a commercial lap-welded tube is its departure from roundness, even when this departure is comparatively small, as was the case with the tubes tested. The thinnest portion of wall, while in itself an element of weakness, is wholly subordinate to out-of-roundness in its influence upon the collapsing strength of commercial lap-welded tubes.

The weld is not an element of weakness for tubes subjected to external fluid pressure.

The factor of safety, he concludes, should be determined from the following considerations:

For the most favorable practical conditions, namely, when the tube is subjected only to stress due to fluid pressure and only the most trivial loss could result from its failure, a factor of safety of three would appear sufficient.

When only a moderate amount of loss could result from failure use a factor of four.

When considerable damage to property and loss of life might result from a failure of the tube, then use a factor of safety of six.

When the conditions of service are such as to cause the tube to become less capable of resisting collapsing pressure, such as the thinning of wall due to corrosion, the weakening of the material due to over-heating, the creating of internal stress in the wall of the tube due to unequal heating, vibration, etc., the above factors of safety should be increased in proportion to the severity of these actions.

Additional experiments on the strength of tubes against collapsing pressure were made by PROFS. A. P. CARMAN and M. L. CARR (*University of Illinois Bulletin*, 1906). These experiments relate more especially to drawn brass and to cold-drawn seamless steel tubes. They confirm Professor Stewart's conclusion that the influence of the length on the strength ceases at about 6 diameters.

The dimensions of drawn brass and of cold-drawn seamless steel tubes of a greater length than 6 diameters against collapsing pressure may be determined from Figs. 3 and 4, which are from the published record of these experiments. The charts are to be used in the same manner as Professor Stewart's charts for lap-welded Bessemer steel tubes.

The bursting strength of cast-iron elbows and tees formed the subject of a series of tests at the Case School of Applied Science by S. M. CHANDLER (*Amer. Mach., March* 8, 1906) and the results are given in Table 6. Three samples of each size were tested and the individual and average results are given.

Equation of Pipes

For the equation of pipes, that is, finding the number of small pipes having the same frictional resistance as one large one, the most

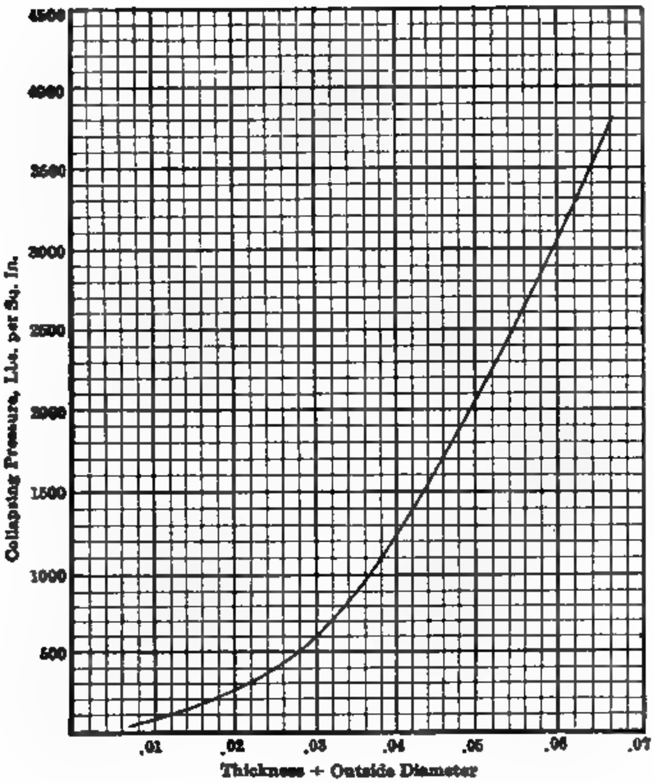


FIG. 3.—The strength of drawn brass tubes against collapsing pressure.

Collapsing Pressure, Lbs. per Sq. In.

Thickness ÷ Outside Diameter

FIG. 4.—The strength of cold drawn steel tubes against collapsing pressure.

TABLE 6.—BURSTING STRENGTH IN LBS. PER SQ. IN. OF STANDARD SCREWED GRAY IRON ELBOWS AND TEES

Size	Elbows			Average	Size	Tees			Average
$2\frac{1}{2}$	3500	3200	3400	3400	$1\frac{1}{4}$	3400	3300	3300	3333
3	2400	2600	2100	2500	$1\frac{1}{2}$	3400	3200	2800	3300
$3\frac{1}{2}$	2100	1700	2400	2250	2	2500	2800	2500	2600
4	2800	2500	2500	2600	$2\frac{1}{2}$	2400	2100	2500	2450
$4\frac{1}{2}$	2000	2600	2600	2600	3	1400	1900	1800	1850
5	2600	2500	2500	2533	$3\frac{1}{2}$	1200	1500	1800	1650
6	2600	2200	2300	2367	4	1800	2100	1700	1867
7	1800	2100	1900	1950	$4\frac{1}{2}$	1100	1400	1400	1400
8	1700	1600	1700	1667	5	1700	1300	1500	1600
9	1800	1800	1900	1833	6	1400	1500	1100	1450
10	1800	1700	1600	1700	7	1400	1400	1500	1433
12	1100	1200	900	1150	8	1200	1400	1390	1350
					9	1300	1400	1200	1300
					10	1100	1300	1200	1200
					12	1100	1000	1100	1067

accurate formula, according to PROF. G. F. GEBHARDT (*Power, June*, 1907) is:

$$n=\frac{d^2\sqrt{d_1+3.6}}{d_1^2\sqrt{d+3.6}}$$

in which d = diameter of larger pipe,

d_1 = diameter of smaller pipe,

n = number of small pipes equivalent to one large one.

Table 7 has been calculated from this formula. The table gives the equation of standard drawn pipes, and of pipes of which the nominal and actual diameters are the same. Instructions for use are given above the table.

The equation of extra strong and double extra strong pipes is given in Tables 8 and 9 by H. D. Nitchie, an engineer of the Watson Stillman Company (*Power, Aug.* 3, 1909). Instructions for use are given above the tables.

Cast-iron and Riveted Pipe

The thickness of cast-iron pipe, in the smaller diameters, is determined chiefly by the foundry consideration of the least thickness which it is desirable to cast. A critical examination of existing formulas and prevailing practice was made by P. H. Baerman in a paper read before the Engineers' Club of Philadelphia in 1882. The resulting formula for the least thickness was:

Thickness, ins. = .3 in. + .015 × diameter, ins.

For water pipe this gives an excess of strength for heads up to 300 ft. and diameters up to 10 ins. For cases beyond those conditions Mr. Baerman gives the formula:

Thickness, ins. = .00015 × head, ft. × diameter, ins.

The ultimate strength of cast-iron is taken at 18,000 lbs. per sq in. and the factor of safety at $12\frac{1}{2}$. For any given case the thickness should be calculated from both formulas and the greatest resulting thickness be used.

The dimensions of the American Water Works Association standard cast-iron pipe are given in Tables 10 and 11, of the Abendroth and Root spiral rivited pipe in Table 12, of Abendroth and Root flanged fittings in Table 13 and of the Pelton Water Wheel Co.'s riveted hydraulic pipe in Table 18.

Standard Flanged Fittings

The dimensions of flanged pipe fittings, according to the 1912 joint standard of the National Association of Steam and Hot Water

Table 7.—The Equation or Equalization of Pipes

Follow the line for one size of pipe to the column for the other; the figures at the intersection give the number of small pipes equivalent to one large one. Use the upper right-hand portion of the table for standard drawn pipe and the lower left-hand portion for pipe of which the nominal and actual diameters are the same.

Standard Drawn Pipes

Dia.	½	¾	1	1½	2	2½	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	Dia.
½		2.27	4.88	15.8	31.7	52.9	96.9	205	377	620	918	1,292	1,767	2,488	3,014	3,786	4,904	5,927	7,321	8,535	9,717	½
¾	2.60		2.05	6.97	14.0	23.3	42.5	90.4	166	273	405	569	779	1,096	1,328	1,668	2,161	2,615	3,226	3,761	4,282	¾
1	7.55	2.90		3.45	6.82	11.4	20.9	44.1	81.1	133	198	278	380	536	649	815	1,070	1,263	1,576	1,837	2,092	1
1½	24.2	9.30	3.20		1.26	3.34	6.13	13.0	23.8	39.2	58.1	81.7	112	157	190	239	310	375	463	539	614	1½
2	54.8	21.0	7.25	2.26		1.67	3.06	6.47	11.9	19.6	29.0	40.8	55.8	78.5	95.1	119	155	187	231	269	307	2
2½	102	39.4	13.6	4.23	1.87		1.83	3.87	7.12	11.7	17.4	24.4	33.4	47.0	56.9	71.5	92.6	112	138	161	184	2½
3	170	65.4	22.6	7.03	3.11	1.66		2.12	3.89	6.39	9.48	13.3	20.9	23.7	31.2	39.1	50.6	61.1	75.5	88.0	100	3
4	376	144	49.8	15.5	6.87	3.67	2.21		1.84	3.02	4.48	6.30	8.61	12.1	14.7	18.5	23.9	28.9	35.7	41.6	47.4	4
5	686	263	90.9	28.3	12.5	6.70	4.03	1.83		1.65	2.44	3.43	4.69	6.60	8.00	10.0	13.0	15.7	19.4	22.6	25.8	5
6	1,116	429	148	46.0	20.4	10.9	6.56	2.97	1.63		1.48	2.09	2.85	4.02	4.86	6.11	7.91	9.56	11.8	13.8	15.6	6
7	1,707	656	226	70.5	31.2	16.6	10.0	4.54	2.49	1.51		1.41	1.93	2.71	3.28	4.12	5.34	6.45	7.97	9.31	10.6	7
8	2,435	936	322	101	44.5	23.8	14.3	6.48	3.54	2.18	1.43		1.35	1.93	2.33	2.92	3.79	4.57	5.67	6.60	7.52	8
9	3,335	1,281	440	137	60.8	32.5	19.5	8.85	4.85	2.98	1.95	1.37		1.41	1.71	2.14	2.77	3.35	4.14	4.83	5.50	9
10	4,393	1,688	582	181	80.4	42.9	25.8	11.7	6.40	3.93	2.57	1.80	1.32		1.21	1.52	1.97	2.38	2.94	3.43	3.91	10
11	5,642	2,168	747	233	103	55.1	33.1	15.0	8.22	5.05	3.31	2.32	1.70	1.28		1.26	1.63	1.88	2.43	2.83	3.22	11
12	7,087	2,723	938	293	129	69.2	41.6	18.8	10.3	6.34	4.15	2.91	2.13	1.61	1.26		1.30	1.57	1.93	2.26	2.58	12
13	8,657	3,326	1,146	358	158	84.5	50.7	23.0	12.6	7.75	5.07	3.56	2.60	1.98	1.53	1.22		1.21	1.49	1.74	1.98	13
14	10,600	4,070	1,403	438	193	103	62.2	28.2	15.4	9.48	6.21	4.35	3.18	2.41	1.88	1.50	1.22		1.24	1.44	1.64	14
15	12,824	4,927	1,698	530	234	125	75.3	34.1	18.7	11.5	7.52	5.27	3.85	2.92	2.27	1.81	1.48	1.21		1.17	1.35	15
16	14,978	5,758	1,984	619	274	146	88.0	39.9	21.8	13.4	8.78	6.15	4.51	3.41	2.66	2.12	1.73	1.42	1.18		1.14	16
17	17,537	6,738	2,322	724	320	171	103	46.6	25.6	15.7	10.3	7.20	5.27	3.99	3.11	2.47	2.03	1.66	1.37	1.17		
18	20,327	7,810	2,691	840	371	198	119	54.1	29.6	18.2	11.9	8.35	6.11	4.63	3.60	2.87	2.35	1.92	1.59	1.36	1.16	
20	26,676	10,249	3,532	1,102	487	260	157	70.9	38.9	23.9	15.6	10.9	8.02	6.07	4.73	3.76	3.08	2.52	2.08	1.78	1.52	
24	42,624	16,376	5,644	1,761	778	416	250	113	62.1	38.2	25.0	17.5	12.8	9.70	7.55	6.01	4.92	4.02	3.32	2.84	2.43	
30	75,453	28,990	9,990	3,117	1,378	736	443	201	110	67.6	44.2	31.0	22.7	17.2	13.4	10.7	8.72	7.14	5.88	5.03	4.30	
36	120,100	46,143	15,902	4,961	2,193	1,172	705	319	175	108	70.4	49.3	36.1	27.3	21.3	16.9	13.9	11.3	9.37	8.01	6.85	
42	177,724	68,282	23,531	7,341	3,245	1,734	1,044	473	259	159	104	73.0	53.4	40.5	31.5	25.1	20.5	16.8	13.0	11.9	10.1	
48	249,351	95,818	33,020	10,301	4,554	2,434	1,465	663	363	223	146	102	75.0	56.8	44.2	35.2	28.8	23.5	19.4	16.6	14.2	
Dia.	½	¾	1	1½	2	2½	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	

Actual Internal Diameters

Fitters and the American Society of Mechanical Engineers, are given in Tables 15 and 16 to which the following explanatory notes apply:

Standard or extra heavy reducing elbows carry same dimensions center to face as regular elbows of largest straight size.

Standard or extra heavy tees, crosses and laterals, reducing on run, carry same dimensions face to face as largest straight size.

If flanged fittings for lower working pressures than 125 lbs. are made, they shall conform in all dimensions, except thickness of shell, to this standard, and shall have the guaranteed working pressure cast on each fitting. Flanges for these fittings must be standard dimensions.

Where long-turn fittings are specified, it has reference only to elbows, which are made in two center to face dimensions, to be known as "elbows" and "longturn" elbows, the latter being used only when so specified.

All standard weight fittings must be guaranteed for 125 lbs. and extra heavy fittings for 250 lbs. working pressure, and each fitting must have some mark cast on it indicating the maker and guaranteed working steam pressure.

All extra heavy fittings and flanges to have a raised surface $\frac{1}{16}$ in. high inside of bolt holes for gasket.

Standard weight fittings and flanges to be plain faced.

Bolts to be ⅛ in. smaller in diameter than bolt holes.

Bolt holes should straddle center lines.

Size of all fittings scheduled indicates inside diameter of ports. For outside diameter pipe use corresponding size of inside diameter fittings.

The face to face dimension of a reducer, either straight or eccentric, shall be equal to the diameter of the large flange.

Square-head bolts with hexagonal nuts are recommended.

Twin ells, double branch ells, side outlet ells, side outlet tees and four-way tees, whether straight sizes or reducing, carry same dimensions center to face and face to face as regular ells and tees.

Bull-head tees or tees increasing on outlet will have same center to face and face to face dimensions as a straight fitting of the size of the outlet.

Up to and including the 4-ins. size, center to face and face to face dimensions of reducing fittings will be the same as that of a straight fitting of the larger opening. (See table for reducing fittings larger than 4 ins.)

Steel flanges, fittings and valves are recommended for superheated steam.

In connection with the report of the standardization committee, the accompanying chart, Fig. 5, showing the stresses in the bolts at the base of the thread due to a pressure in the pipe of 250 lbs. per square inch appears. The chart shows the extent to which these stresses increase with the sizes of the pipe in the manufacturers' standard, and the dotted line shows the departure made from the latter in order to keep the stresses down to a safe limit.

Pipe Joints

The common gasket joint is a constant source of trouble and a poor thing at best. When used, its security will be greatly increased if the gasket does not extend beyond the bolts. Still greater security may be obtained by facing the flanges slightly concave, with which

construction the gasket must be entirely within the bolts. The types of joint shown below apply to most conditions and should be used in preference to the common construction.

Professor Sweet's joint for the cylinder covers of steam engines is shown in the section on steam engines (See Cylinder Cover Joints). It has also been adopted by the Ball Engine Co. with entire success.

The joint is metal to metal and without grinding, the surfaces being ordinary tooled surfaces. The only, and a necessary, precaution is to make the joint *narrow*—not over ⅜ in. wide.

The narrow metal to metal joint is also entirely successful for high pressure air as will be shown later.

The Rapieff joint used with invariable success for the numerous joints of the Zalinski dynamite gun and its air plant, where it regularly withstood pressures of 2000 lbs. per sq. in., is shown in Figs. 6–14. It is thus described by B. C. Batcheller, Chf. Engr. Amer. Pneumatic Service Co. (*Amer. Mach., Apr.* 23, 1908). Just inside the bolt circle a groove of peculiar shape, *abc* Fig. 6, is turned in the face of each flange into which a ring of round rubber cord is laid and the flanges are bolted up metal to metal. The combined cross-sectional area of the grooves is made slightly less than the sectional area of the rubber cord, resulting in compression of the rubber, the surplus flowing into the narrow space *d*, which is about 1/16 in. wide, and opens into the interior of the pipe.

The fluid pressure acts against the rubber, tending to force it outward and, putting the entire ring of rubber under static pressure, seals the joint at *c*. Thus the higher the pressure the tighter is the joint.

The rubber gasket ring is shown in Fig. 7. It is made from rubber cord that can be bought by the yard and

Table 8.—The Equation of Equalization of Extra Strong Pipe

Follow the line for one size to the column for the other and at the intersection find the number of the smaller pipes equivalent to one of the larger

Pipe size	¼ in.	⅜ in.	½ in.	¾ in.	1 in.	1¼ ins.	1½ ins.	2 ins.	2½ ins.	3 ins.	3½ ins.	4 ins.	4½ ins.	5 ins.	6 ins.	Pipe size
Int. area	.068	.139	.231	.452	.71	1.271	1.753	2.935	4.209	6.569	8.856	11.45	14.18	18.193	25.97	Int. area
Ins.																Ins.
¼	1	2.05	3.4	6.7	10.4	18.8	25.8	43.	62.	96.5	130.	168.5	208.	267.	382	¼
⅜		1	1.66	3.26	5.1	9.2	12.6	21.1	30.3	47.3	63.8	82.5	102.	131.	186	⅜
½			1	1.96	3.08	5.5	7.6	12.7	18.2	28.4	38.4	47.5	61.5	78.8	112	½
¾				1	1.57	2.82	3.9	6.5	9.3	14.5	19.6	25.3	31.3	40.2	57.4	¾
1					1	1.79	2.47	4.14	5.93	9.25	12.5	16.1	20	25.6	36.5	1
1¼						1	1.38	2.3	3.31	5.16	6.96	8.4	11.1	14.3	20.4	1¼
1½							1	1.67	2.4	3.74	5.05	6.54	8.1	10.4	14.8	1½
2								1	1.43	2.24	3.02	3.91	4.84	6.2	8.9	2
2½									1	1.56	2.1	2.72	3.36	4.31	6.15	2½
3										1	1.35	1.75	2.16	2.77	3.95	3
3½											1	1.29	1.6	2.05	2.92	3½
4												1	1.24	1.59	2.26	4
4½													1	1.28	1.83	4½
5														1	1.37	5
6															1	6
	¼ in.	⅜ in.	½ in.	¾ in.	1 in.	1¼ ins	1½ ins	2 ins.	2½ ins	3 ins.	3½ ins	4 ins.	4½ ins	5 ins.	6 in.	

Table 9.—The Equation or Equalization of Double Extra Strong Pipe

Pipe size	⅜ in.	½ in.	¾ in.	1 in.	1¼ ins.	1½ ins.	2 ins.	2½ ins.	3 ins.	3½ ins.	4 ins.	4½ ins.	5 ins.	6 ins.	Pipe size
Int. area	.042	.047	.139	.271	.615	.93	1.744	2.419	4.097	5.794	7.724	10	12.96	18.66	Int. area
Ins.															Ins
⅜	1	1.12	3.32	6.45	14.6	22.1	41.5	57.5	95.6	137	184	236	308	444	⅜
½		1	2.96	5.77	13.1	19.7	37.2	51.5	87	123	164	213	276	398	½
¾			1	1.95	4.43	6.7	12.5	17.4	29.4	41.6	55.5	72	93.5	134	¾
1				1	2.27	3.42	6.45	8.95	15.1	21.4	28.5	37	47.9	69	1
					1	1.51	2.84	3.94	6.65	9.4	12.5	16.3	21.1	30.4	1¼
1¼															
1½						1	1.88	2.6	4.4	6.21	8.3	10.8	14	20	1½
2							1	1.39	2.34	3.31	4.42	5.74	7.45	10.7	2
2½								1	1.69	2.39	3.19	4.1	5.33	7.72	2½
3									1	1.41	1.88	2.44	3.16	4.55	3
										1	1.33	1.73	2.24	3.22	3½
3½															
4											1	1.3	1.68	2.42	4
4½												1	1.3	1.87	4½
5													1	1.44	5
6														1	6
	⅜ in.	½ in.	¾ in.	1 in.	1¼ ins	1½ ins	2 ins.	2½ ins	3 ins.	3½ ins	4 ins.	4½ ins	5 ins.	6 ins.	

Table 10.—American Water Works Association Standard Cast-iron Pipe for Fire Lines and other High-pressure Service

Adopted May 12, 1908

The weights are per length to lay 12 ft., including standard sockets; proportionate allowance to be made for any variation.

Nominal inside diameter, ins.	Class E, 500-ft. head, 217 lbs. pressure			Class F, 600-ft. head, 260 lbs. pressure			Class G, 700-ft. head, 340 lbs. pressure			Class H, 800-ft. head, 347 lbs. pressure			Nominal inside diameter, ins.
	Thickness, ins.	Weight per Foot	Weight per Length	Thickness, ins.	Weight per Foot	Weight per Length	Thickness, ins.	Weight per Foot	Weight per Length	Thickness, ins.	Weight per Foot	Weight per Length	
6	.58	41.7	500	.61	43.3	520	.65	47.1	565	.69	49.6	595	6
8	.66	61.7	740	.71	65.7	790	.75	70.8	850	.80	75.0	900	8
10	.74	86.3	1035	.80	92.1	1105	.86	100.9	1210	.92	106.7	1280	10
12	.82	113.8	1365	.89	122.1	1465	.97	135.4	1625	1.04	143.8	1725	12
14	.90	145.0	1740	.99	157.5	1890	1.07	174.2	2090	1.16	186.7	2240	14
16	.98	179.6	2155	1.08	195.4	2345	1.18	219.2	2620	1.27	232.5	2790	16
18	1.07	220.4	2645	1.17	238.4	2860	1.28	267.1	3205	1.39	286.7	3440	18
20	1.15	263.0	3155	1.27	286.3	3435	1.39	320.8	3850	1.51	344.6	4135	20
24	1.31	359.6	4315	1.45	392.9	4715	....	....	....	....	....	....	24
30	1.55	521.7	6260	1.73	585.4	7025	....	....	....	....	....	....	30
36	1.80	725.0	8700	2.02	820.0	9840	....	....	....	....	....	....	36

Table 11.—American Water Works Association Standard Cast-iron Pipe

Adopted May 12, 1908

The weights are per length to lay 12 ft., including standard sockets; proportionate allowance to be made for any variation.

Nominal inside diameter, ins.	Class A, 100-ft. head, 43 lbs. pressure			Class B, 200-ft. head, 86 lbs. pressure			Class C, 300-ft. head, 130 lbs. pressure			Class D, 400-ft. head, 173 lbs. pressure			Nominal inside diameter, ins.
	Thickness, ins.	Weight per Foot	Weight per Length	Thickness, ins.	Weight per Foot	Weight per Length	Thickness, ins.	Weight per Foot	Weight per Length	Thickness, ins.	Weight per Foot	Weight per Length	
4	.42	20.0	240	.45	21.7	260	.48	23.3	280	.52	25.0	300	4
6	.44	30.8	370	.48	33.3	400	.51	35.8	430	.55	38.3	460	6
8	.46	42.9	515	.51	47.5	570	.56	52.1	625	.60	55.8	670	8
10	.50	57.1	685	.57	63.8	765	.62	70.8	850	.68	76.7	920	10
12	.54	72.5	870	.62	82.1	985	.68	91.7	1,100	.75	100.0	1,200	12
14	.57	89.6	1,075	.66	102.5	1,230	.74	116.7	1,400	.82	129.2	1,550	14
16	.60	108.3	1,300	.70	125.0	1,500	.80	143.8	1,725	.89	158.3	1,900	16
18	.64	129.2	1,550	.75	150.0	1,800	.87	175.0	2,100	.96	191.7	2,300	18
20	.67	150.0	1,800	.80	175.0	2,100	.92	208.3	2,500	1.03	229.2	2,750	20
24	.76	204.2	2,450	.89	233.3	2,800	1.04	279.2	3,350	1.16	306.7	3,680	24
30	.88	291.7	3,500	1.03	333.3	4,000	1.20	400.0	4,800	1.37	450.0	5,400	30
36	.99	391.7	4,700	1.15	454.2	5,450	1.36	545.8	6,550	1.58	625.0	7,500	36
42	1.10	512.5	6,150	1.28	591.7	7,100	1.54	716.7	8,600	1.78	825.0	9,900	42
48	1.26	666.7	8,000	1.42	750.0	9,000	1.71	908.3	10,900	1.96	1050.0	12,600	48
54	1.35	800.0	9,600	1.55	933.3	11,200	1.90	1141.7	13,700	2.23	1341.7	16,100	54
60	1.39	916.7	11,000	1.67	1104.2	13,250	2.00	1341.7	16,100	2.38	1583.3	19,000	60
72	1.62	1283.4	15,400	1.95	1545.8	18,550	2.39	1904.2	22,850	....	....	....	72
84	1.72	1633.4	19,600	2.22	2104.2	25,250	....	....	....	....	....	....	84

Table 12.—Dimensions, Strength and Weight of Abendroth & Root Black Spiral Riveted Pipe

Diameter, ins.	Thickness, B. W. gage	Approximate bursting pressure, lbs. per sq. in.	Plain end pipe Weight per 100 ft.	With A. & R. flanges, bolts and gaskets Weight per 100 ft.	With Root bolted joint complete Weight per 100 ft.
3	22	1060	115	139	153
	20	1325	147	171	185
	18	1860	205	229	243
4	20	1000	195	227	247
	18	1390	273	305	325
	16	1845	360	392	412
5	20	795	242	282	304
	18	1100	340	380	402
	16	1480	448	488	510
6	18	930	385	433	475
	16	1220	508	556	598
	14	1580	653	701	743
	12	2060	858	906	948
7	18	790	446	510	540
	16	1060	588	652	682
	14	1340	755	819	849
	12	1780	992	1056	1086
8	18	690	507	587	604
	16	945	669	749	766
	14	1180	860	940	957
	12	1540	1130	1210	1227
9	16	820	753	873	863
	14	1040	967	1087	1077
	12	1380	1271	1391	1381
10	16	740	835	963	1025
	14	945	1071	1199	1261
	12	1024	1408	1536	1598
11	16	670	916	1060	1122
	14	860	1176	1320	1382
	12	1120	1546	1690	1752
12	16	615	1003	1163	1215
	14	790	1287	1447	1499
	12	1025	1692	1852	1904
	10	1265	2080	2240	2292

Diameter, ins.	Thickness, B. W. gage	Approximate bursting pressure, lbs. per sq. in.	Plain end pipe Weight per 100 ft.	With A. & R. flanges, bolts and gaskets Weight per 100 ft.	With Root bolted joint complete Weight per 100 ft.
13	16	570	1106	1274	1346
	14	730	1420	1588	1660
	12	950	1866	2034	2106
	10	1165	2294	2462	2534
14	16	530	1199	1399	1465
	14	675	1539	1739	1805
	12	890	2022	2222	2288
	10	1090	2486	2686	2752
15	14	630	1649	1889	1973
	12	825	2167	2407	2491
	10	1015	2664	2904	2988
16	14	590	1771	2051	2149
	12	770	2327	2607	2705
	10	950	2861	3141	3239
18	14	525	1974	2334	2394
	12	690	2593	2953	3013
	10	850	3188	3548	3608
20	14	470	2180	2556	2608
	12	620	2863	3239	3201
	10	760	3521	3897	3949
22	14	430	2390	2830	2830
	12	565	3140	3580	3580
	10	695	3860	4300	4300
24	14	395	2604	3108	3084
	12	515	3421	3925	3901
	10	635	4216	4720	4696
26	12	475	3558	4718	4090
	10	580	4380	5540	4912
28	12	440	3894	5274	4478
	10	545	4720	6100	5304
30	12	410	4115	5531	4755
	10	510	5063	6479	5703

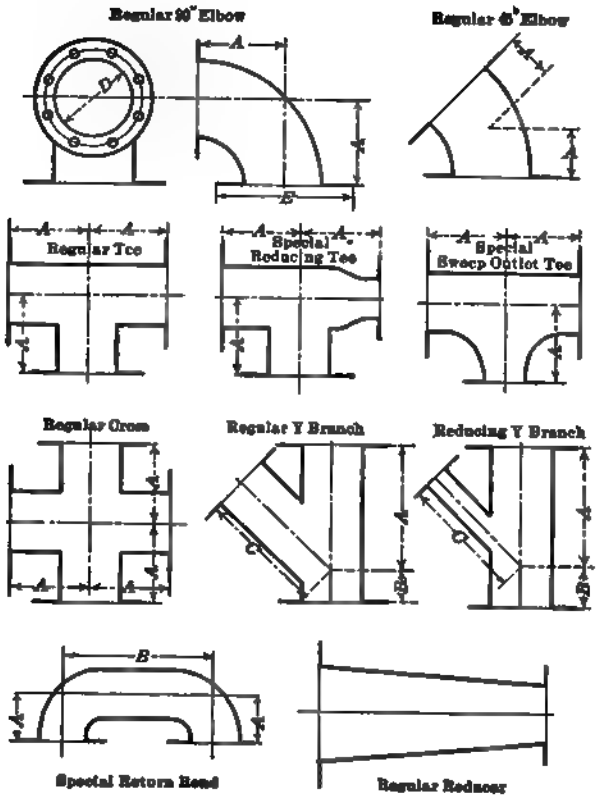


TABLE 13.—CENTER TO FACE MEASUREMENTS OF ABENDROTH & ROOT FLANGED FITTINGS

Spaces Filled from Center to Face
Dimensions in inches

Inside diameter, ins.	90° elbows, tees and crosses	45° elbows	Y-branches			Return bends	
D	A	A	A	B	C	A	B
3	3½	2¼	9	2½	9	3½	7½
4	4½	2 13/16	11	2¾	11	4½	8¾
5	5½	3⅛	12	3	12	5½	10½
6	6½	3⅜	13½	3¼	13½	6½	12½
7	7½	4	15	4¼	15	7½	14½
8	8½	4 13/16	17	5	17	8½	16½
9	9½	5 13/16	18½	5¼	18½	9½	18½
10	10½	5½	21	5½	21	10½	20½
11	11	5¾	22½	5¾	22½	11	22
12	12½	6¼	24	6	24	12½	24½
13	13	5⅞	26	6¼	26	13	26
14	13½	5⅝	27½	6½	27½	14	28
15	15	5¼	29½	6¾	29½	15	30
16	16	6⅝	31½	7	31½	16	32
18	18	7⅝	35	7½	35	18	36
20	20	10⅝	38½	8	38½	20	40
22	22	11	41	9	41	22	44
24	24	12	44	10	44	24	48
26	26	...	...	...	...	...	...
28	28	...	...	...	...	...	...
30	30	...	...	...	...	...	...

Length of Reducers

Inside Diameter..	4	5	6	7	8	9	10	11	12	13	14	16	18	20
Total length...	23	23	23	22	22	22	33	33	33	33	32	32	32	32

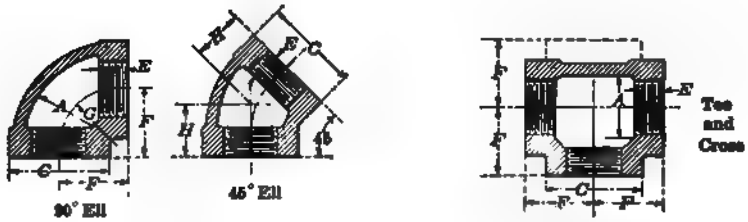


TABLE 14.—CAST-IRON SCREWED PIPE FITTINGS FOR PRESURES UP TO 100 LBS. PER SQ. IN.

Walworth Mfg. Co.'s Standard

Dimensions in inches

Pipe dimensions: Nominal inside dia. of pipe	Threads	For all fittings: Inside dia. A	B	Dia. of bead C	D	Length for thd. E	Cen. to face F	90° ell rad. G	45° ell, face to face H	45° Ys. Face to face J	45° Ys. Cen. to face K
¼	18	9/16		1		⅜	⅞	⅝	9/16	...	...
⅜	18	¾		1⅛		7/16	1	9/16	⅝	2 5/16	1 9/16
½	14	⅞		1 5/16		½	1⅛	11/16	11/16	2⅝	1¾
¾	14	1⅛		1⅝		9/16	1 5/16	13/16	13/16	2⅞	2
1	11½	1⅜		2 1/16		⅝	1½	1	15/16	3¼	2½
1¼	11½	1¾		2½		11/16	1 13/16	1¼	1 1/16	3⅝	3
1½	11½	2		2¾		11/16	2	1½	1 3/16	4¼	3½
2	11½	2½		3¼		¾	2¼	1 15/16	1¼	5¼	4
2½	8	2 15/16		4¼		1	2¾	1⅞	1⅜	6¼	5
3	8	3 9/16		4⅝		1	3 5/16	2 5/16	1½	7⅜	5⅞
3½	8	4 1/16		5¼		1 1/16	3 11/16	2½	2 3/16	8⅜	6⅜
4	8	4½		6		1⅛	4	2 13/16	2⅜	9⅜	7¼
4½	8	5		6 9/16		1⅛	4 5/16	3 1/16	2 9/16	10⅜	7⅞
5	8	5 9/16		7⅛		1¼	4 11/16	3½	2 11/16	11½	8⅞
6	8	6 11/16		8¼		1¼	5 5/16	4	2 15/16	13½	10
7	8	7 11/16		9¼		1⅜	6 5/16	4¾	3¼	14⅞	11½
8	8	8¾		10¼		1⅜	6 13/16	5 5/16	3 7/16	16⅞	13
9	8	9¾		12¼		1½	7½	5 15/16	3¾	19	14¾
10	8	10¾		13¼		1⅝	8¼	6 9/16	4 3/16	20⅞	16
12	8	12¾		15¼		1¾	9 5/16	7⅞	4¾	24	19

Key to the Dimensions Given in the Tables

A = size of pipe line,
B = center to face, elbows,
C = center to face, 45-deg. elbows,
D = center to face, long turn elbows,
E = center to face, tees and crosses,
F and F^1 = center to face, laterals or Y branches,
G = face to face, tees and crosses,
H = face to face, laterals or Y branches,
I = Diameter of flange, all fittings,
J = Thickness of flange, all fittings,
K = Diameter of bolt circle, all fittings,
L = Number of bolt holes, all fittings,
M = Diameter of bolt holes, all fittings.

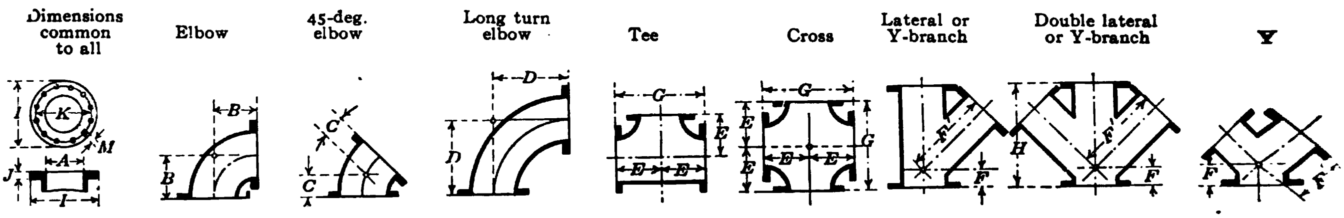


Table 15.—Standard Weight Flanged Fittings

Dimensions in inches

A	B	C	D	E	F & F'	G	H	I	J	K	L	M
1	3½	2	5½	3½	2 & 6*	7	8	4	7/16	3	4	9/16
1¼	3¾	2	6	3¾	2¼ & 6¼	7½	8½	4½	½	3⅜	4	9/16
1½	4	2¼	6½	4	2½ & 6¾	8	9¼	5	9/16	3⅞	4	⅝
2	4½	2½	7	4½	2½ & 8	9	10½	6	⅝	4¾	4	¾
2½	5	2¾	7½	5	2½ & 9½	10	12	7	11/16	5½	4	¾
3	5½	3	8	5½	3 & 10	11	13	7½	¾	6	4	¾
3½	6	3½	9	6	3 & 11½	12	14½	8½	13/16	7	4	¾
4	6½	4	10	6½	3 & 12	13	15	9	15/16	7½	8	¾
4½	7	4¼	11	7	3 & 12	14	15½	9¼	15/16	7¾	8	¾
5	7½	4½	12	7½	3½ & 13½	15	17	10	15/16	10	8	⅞
6	8	5	13	8	3½ & 14½	16	18	11	1	9½	8	⅞
7	8½	5½	14½	8½	4 & 16½	17	20½	12½	1 1/16	10¾	8	⅞
8	9	6	16	9	4½ & 17½	18	22	13½	1⅛	11¾	8	⅞
9	10	6½	18	10	4½ & 19½	20	24	15	1⅛	13¼	12	⅞
10	11	7	20	11	5 & 20½	22	25½	16	1 3/16	14¼	12	1
12	12	7½	22	12	5½ & 24½	24	30	19	1¼	17	12	1
14	14	7½	24	14	6 & 27	28	33	21	1⅜	18¾	12	1⅛
15	14½	8	26	14½	6 & 28½	29	34½	22¼	1⅜	20	16	1⅛
16	15	8	28	15	6½ & 30	30	36½	23½	1 7/16	21¼	16	1⅛
18	16½	8½	30	16½	7 & 32	33	39	25	1 9/16	22¾	16	1¼
20	18	9½	32	18	8 w 35	36	43	27½	1 11/16	25	20	1¼
22	20	10	34	20	8½ & 37½	40	46	29½	1 13/16	27¼	20	1⅜
24	22	11	36	22	9 & 40½	44	49½	32	1⅞	29½	20	1⅜
26	23	13	39	23	9½ & 43½	46	53	34¼	2	31¾	24	1⅝
28	24	14	42	24	10½ & 46½	48	57	36½	2 1/16	34	28	1⅝
30	25	15	45	25	11 & 49½	50	60½	38¾	2⅛	36	28	1⅝

Table 16.—Extra Heavy Flanged Fittings

Dimensions in inches

A	B	C	D	E	F & F'	G	H	I	J	K	L	M
1	4½	2½	5½	4½	2¾ & 7½*	8½	9¾	4½	11/16	3¼	4	⅝
1¼	4½	2½	6	4½	2¾ & 7¼	9	9¾	5	¾	3¾	4	⅝
1½	4½	2¾	6½	4½	2½ & 8½	10	11	6	13/16	4½	4	¾
2	5	3	7	5	2½ & 9½	11	12	6½	⅞	5	4	¾
2½	5½	3½	7½	5½	2½ & 10½	12	13	7½	1	5⅞	4	⅞
3	6	3½	8	6	3 & 11½	13	14½	8¼	1⅛	6⅝	8	¾
3½	6½	4	9	6½	3 & 12½	14	15½	9	1 3/16	7¼	8	⅞
4	7	4½	10	7	3 & 13½	15	16½	10	1¼	7⅞	8	⅞
4½	7½	4½	11	7½	3½ & 14½	16	18	10½	1 5/16	8½	8	⅞
5	8	5	12	8	3½ & 16	18	19½	11	1⅜	9¼	8	⅞
6	9	5½	13	9	4 & 17½	19	21½	12½	1 7/16	10⅝	12	⅞
7	9½	5¾	14½	9½	4½ & 19	21	23½	14	1½	11⅞	12	1
8	10½	6	16	10½	5 & 20½	22	25½	15	1⅝	13	12	1
9	11	6½	18	11	5 & 22¾	24	27¾	16¼	1¾	14	12	1⅛
10	12	7	20	12	5½ & 25¼	27	30¾	18¼	1⅞	15¼	16	1⅛
12	13½	8	22	13½	6 & 27¾	30	33¾	20¾	2	17¾	16	1¼
14	15	9½	24	15	6½ & 31½	31	37¾	23½	2⅛	20¼	20	1¼
15	15½	10	26	15½	6½ & 33¼	33	39¾	25½	2 3/16	21½	20	1¼
16	16½	10½	28	16½	7½ & 34½	35½	42	26	2¼	22½	20	1⅜
18	17¾	11	30	17¾	8 & 37¾	38½	45¾	28½	2⅜	24¾	24	1⅜
20	19¼	11½	32	19¼	8½ & 40¾	41	49¾	31	2½	27	24	1⅜
22	20½	12	34	20½	9½ & 43¾	45	53¼	33	2⅝	29¼	28	1⅝
24	22½	13	36	22½	10 & 47½	....	57½	36	2¾	32	28	1⅝

Asterisk (*) indicates center to face dimensions of run on laterals or double laterals, each way. The smaller dimension is also the center to face dimension of a Y on the short end. All other dimensions on Y's are the same as for elbows of the same size. A reducing Y carries same dimensions as a straight size Y of the size of the largest opening in the fitting in question.

All dimensions are in inches.

made into rings as required. A splice is shown at f, which is made by cutting the cord obliquely and joining the ends with rubber cement. The ring should have the same diameter as the grooves in the face of the flanges. Rubber cord ½ in. diameter is large enough for the largest joints, and it is not convenient to use cord much less than ¼ in. in diameter. The rubber should be of good quality, and preferably what is known in the trade as "pure gum." When a joint is made in a horizontal pipe, the rubber ring can be held in the groove of one flange by rubber cement when the joint is put together. The surface bc may have an angle of 60 deg.

When alignment of the pipe sections is required it is readily obtained by the construction shown in Fig. 8. Fig. 9 shows a modified form and Fig. 11 an application to a cylinder head. The rubber rings, of which sections are shown, may be cut from flat sheets. In making the joint shown in Fig. 11, the ring should be stretched over the head to hold it in place, talc powder being used to prevent its sticking.

Two or more joints at as many shoulders on the same piece may be made with this joint as shown in Fig. 13 in which a lantern A is bolted to an annular casting B. Joint C is like Fig. 6 and joint D like Fig. 11.

This joint is used with complete success for low pressures in the Batcheller pneumatic postal tubes, in which the pressure seldom exceeds 5 lbs. per sq. in. A rectangular groove, Fig. 14, ½ in. wide by ¼ in. depth, is turned in one flange and a tongue ⅜ in. wide by $\frac{5}{32}$ in. high on the face of the opposite flange. The outside diameter of the tongue fits the outside diameter of the groove. A rubber ring, as shown, ⅛ in. wide and ¼ in. thick, is laid in the groove and the flanges are bolted together. The rubber ring is compressed to a thickness of $\frac{3}{32}$ in., the surplus flowing into the space provided by

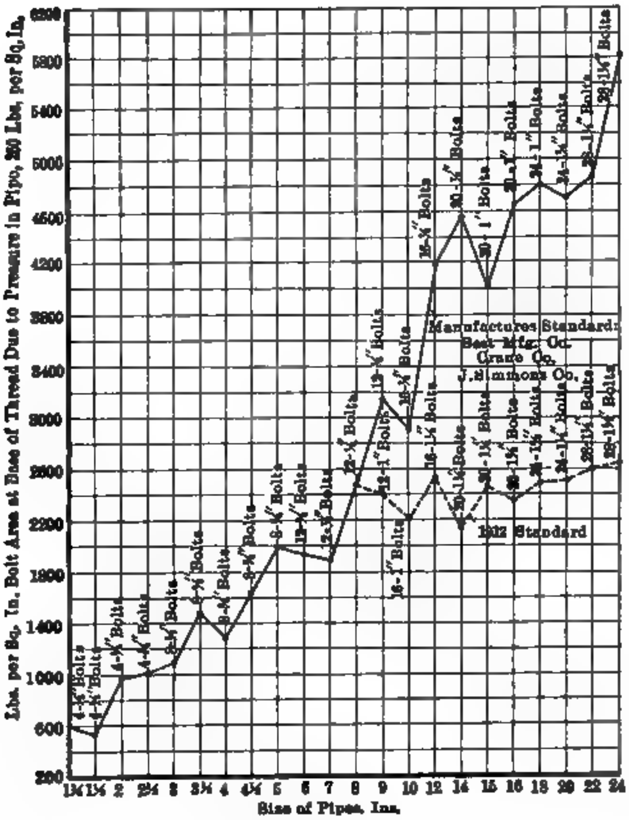


FIG. 5.—Stress at root of thread of bolts for flanged pipe fittings under a pressure of 250 lbs. per sq. in.

making the tongue narrower than the groove. The tongue and groove of this form are easily machined and alignment of the sections is insured.

This joint is also regularly used by the Nordberg Mfg. Co. for mine-pumping plants where the pressure is heavy and the service severe.

Pipe joints and threaded unions for high-pressure air, according to H. V. HAIGHT, Chf. Engr. Canadian Ingersoll-Rand Company (*Amer. Mach., Apr.* 23, 1908) should have metal to metal joints with narrow faces and are preferably of the ball and socket type. Regarding the actual joint, the rule is the higher the pressure the narrower the joint. Fig. 15 shows details of a ball and socket joint used for pressures up to 1000 lbs. per sq. in. The radius at the end of the pipe is slightly less than in the socket, giving line contact. The thread not being subject to air pressure is made straight and the flange screwed on by hand. The ball and socket feature makes the joint tight even if the parts are not in perfect alignment. Extra heavy pipe was used in order to have sufficient thickness after threading.

A high-pressure flange union is shown in Fig. 16. It permits a movement of 5 deg. in any direction, as indicated in the smaller illustration. The recesses *a* are for calking with lead should it be necessary, which it seldom is. Fig. 17 shows a type of fitting used in the United States Navy for torpedo service and for air pressures up to 3000 lbs. per sq. in. Note especially the knife-edge joints.

Fig. 18 shows a fitting for connecting copper tubing. The tube is swelled outward and the end pinched between the nipple and swivel, the former being turned to an angle of 30 deg. with the center line.

Flange joints for high-pressure hydraulic work are shown in Figs. 19 and 20 and Table 18 by U. PETERS (*Amer. Mach., Apr.* 18, 1901).

Fig. 19 shows a joint for bored or seamless drawn steel pipes

FIG. 8.

FIG. 7.

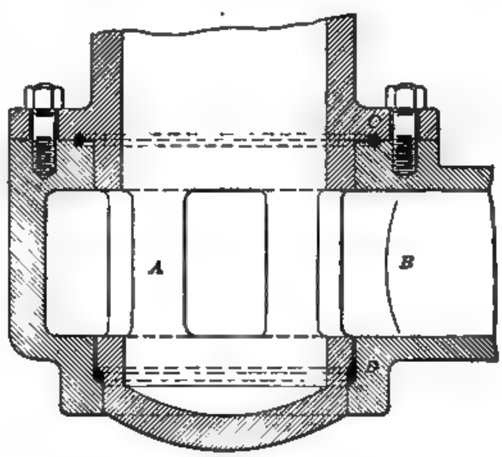


FIG. 13.

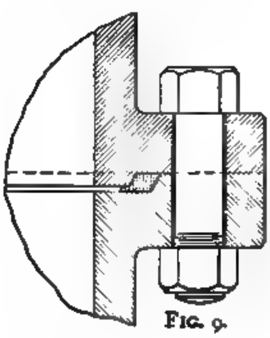


FIG. 9.

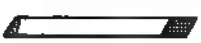


FIG. 10.

FIG. 12.

FIG. 14.

FIGS. 6 to 14.—The Rapieff pipe joint.

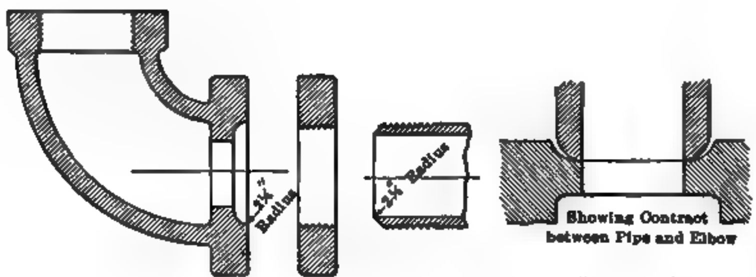


FIG. 15-Details of an Elbow and Companion Flange of a Ball and Socket Joint

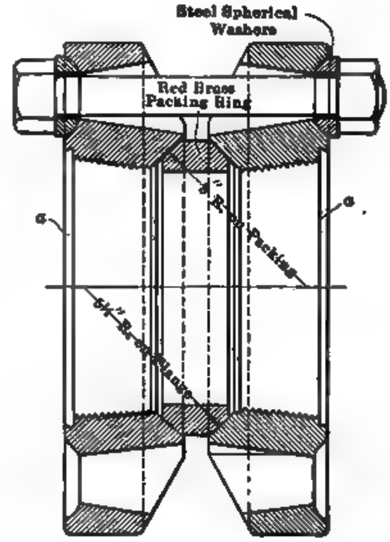
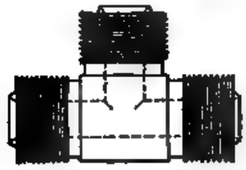


FIG. 18.-Tank Connection for ¾" Inch Copper Tube



Union for Copper Tube Connection

FIG. 17-Fittings used in U.S. Torpedo Service

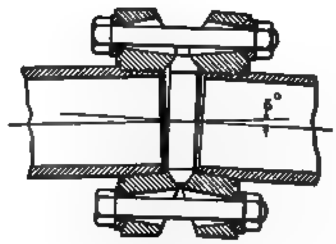


FIG. 16.-Flange Union for 1000 Pounds Air Pressure

FIGS. 15 to 18.—Pipe fittings for high-pressure air.

from 3 to 8 ins. and larger diameters with ring-grooved male and female ends for the copper gasket. The flanges and bolts are also of steel, and the accompanying table gives some dimensions for the various sizes. The table is made up from connections actually in use and approved. They stood a trial test of 7 gross tons per sq. in. without showing any sign of leakage, if properly connected.

The pipes and flanges are threaded either by the United States standard of 8 threads, or by the Whitworth screw standard with 6 threads per inch.

For smaller pipes the connection shown in Fig. 20 is applied. Such pipes are generally called by the catalog names of extra heavy or double extra heavy steel or wrought-iron pipes. As given in the tables of the various makers, they are of different dimensions, for pressures from 500 to 7000 lbs. per sq. in. The flanges are usually of forged or cast steel and of different shapes, corresponding to the number of bolts from oval-like, triangular and square to round, and it would take too much space to tabulate all these dimensions. More difficult to determine than the size of the pipes for heavy pressure is the size of the flange bolts. A formula is therefore here given:

$$\frac{4000 \text{ to } 5000\ (D^2-d^2)}{\text{number of bolts}} = \text{safe tensile strength of bolt.}$$

The factor 5000 is used for higher pressures. The length of thread for cast-steel and wrought-iron flanges may be made:

$$f = 2.25\ (D\text{-}d)$$

and for cast-iron flanges:

$$f = 2.50\ (D\text{-}d)$$

$f_1 = f + \frac{1}{16}$ in. and $e = \frac{1}{16}$ to $\frac{1}{4}$ in. The thickness of the copper gasket is usually not over $\frac{1}{8}$ in.

For high-pressure superheated steam or hydraulic work S. D. LOVEKIN, Chief Engr. New York Shipbuilding Company (*Amer. Mach., June* 8, 1905), considers the joint shown in Fig. 21 superior to all others. The faces are serrated and for steam a plain gasket of annealed copper is placed between them. For hydraulic work dealing with pressures up to 6000 lbs. per sq. in. Mr. Lovekin uses lead gaskets.

The Van Stone or Walmaco pipe joint for high-pressure (250 lbs.) steam is shown in Fig. 22. In making this joint the flange is slipped on the pipe, the pipe is brought to a red heat, the end is rolled over against the smooth face of the flange by a special machine which insures perfect contact and, finally, the pipe is placed in a lathe and a light cut is taken from the face which is to make the joint. Rubber gaskets are used for pressures up to 125 lbs. and copper gaskets for higher pressures. In some cases the ends are ground together. The advantages of the joint are: The pipe is not weakened by threads; the joint is made between the ends of the pipe; the flanges simply act as collars to hold the ends of the pipe in contact; the flanges swivel, thus greatly reducing the labor of erecting the work.

Table 19 gives the dimensions of this joint as made by the Walworth Mfg. Co.

Pipe Markings

The standard pipe markings of the American Society of Mechanical Engineers (*Trans. A. S. M. E.*, Vol. 33) are as follows:

In the main engine rooms of plants which are well lighted, and where the functions of the exposed pipes are obvious, all pipes shall be painted to conform to the color scheme of the room; and if it is

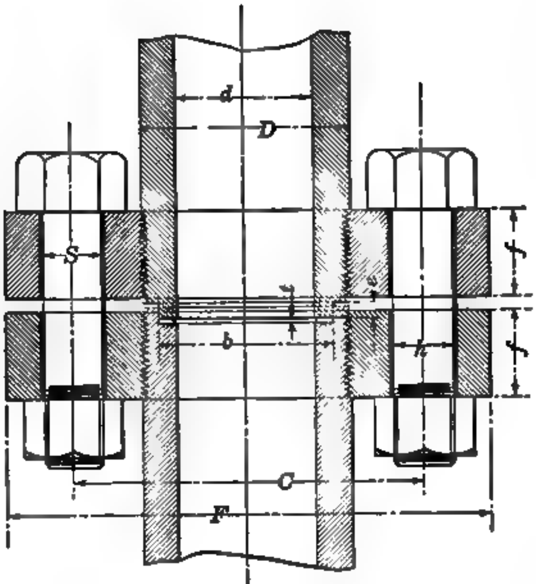


FIG. 19.

TABLE 18.—DIMENSIONS OF HIGH-PRESSURE HYDRAULIC PIPE FLANGES.

Dimensions in Inches						For 6 Bolts			For 8 Bolts		
d	D	b	e	t	f	F	C	S	F	C	S
3	4½	3.6	¼	⅛	1⅞	9¾	7¼	1⅛			
3¼	4⅞	3.9	¼	⅛	2	10¾	7⅞	1⅜			
3½	5¼	4.2	5/16	⅛	2¼	11¾	8¾	1½			
4	6	4.8	5/16	⅛	2 5/16	12¾	9⅝	1⅝	11¼	8¾	1¼
4½	6¾	5.4	⅜	⅛	2½	14	10½	1¾	13	10	1½
5	7½	6.0	⅜	⅛	2¾	15¾	11⅝	2	14¼	11	1⅝
6	9	7.2	⅜	⅛	3	18¼	13¾	2¼	16¾	13	1⅞
7	10½	8.4	⅜	⅛	3¼	20¾	15¾	2½	18¾	14¾	2
8	12	9.6	⅜	⅛	3½	23¼	17¾	2¾	21¼	16¾	2¼

$$h = S + \tfrac{1}{16}d$$

FIG. 21.—Pipe joint for high-pressure superheated steam or hydraulic work.

desirable to distinguish pipe systems, colors shall be used only on flanges and on valve fitting flanges.

In all other parts of the plant, such as boiler house, basements, etc., all pipes (exclusive of valves, flanges, and fittings) except the fire system, shall be painted black, or some other single, plain, durable, inexpensive color.

All fire lines (suction and discharge) including pipe lines, valve flanges and fittings, shall be painted red throughout.

The edges of all flanges, fittings or valve flanges on pipe lines larger than 4 inches inside diameter, and the entire fittings, valves and flanges on lines 4 inches inside diameter and smaller, shall be painted the following distinguishing colors:

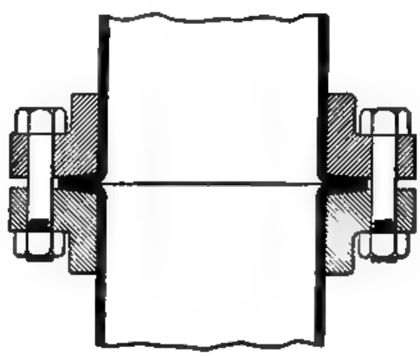


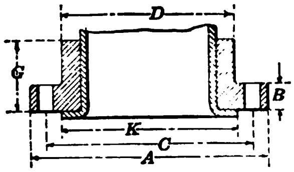
FIG. 22.—The Van Stone pipe joint.

DISTINGUISHING COLORS TO BE USED ON VALVES, FLANGES AND FITTINGS

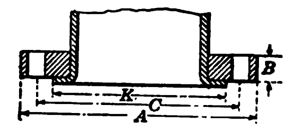
Steam division	High pressure—white. Exhaust steam—buff.
Water division	Fresh water, low pressure—blue Fresh water, high pressure boiler feed lines—blue and white. Salt water piping—green.

TABLE 19.—THE VAN STONE PIPE JOINT FOR PRESSURES UP TO 250 LBS. PER SQ. IN.

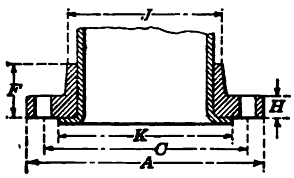
Dimensions in inches



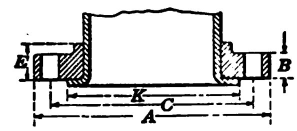
LONG HUB FLANGES
made from cast-iron or ferrosteel



SHORT HUB FLANGES
made from malleable iron or cast steel



LONG HUB FLANGES
made from rolled steel



RING FLANGES
made from malleable iron, cast steel or rolled steel

Size	4	4 1/2	5	6	7	8	9	10	12	14	15	16	18	20	22	24
D. Diameter of hub, cast-iron or ferrosteel	6 3/16	6 11/16	7 3/8	8 1/2	9 5/8	10 3/8	11 5/8	13 1/4	15 3/8	16 5/8	17 7/8	19 1/4	21 1/2	23 5/8	26	28 1/4
J. Diameter of hub, rolled steel	5 3/4	6 1/4	6 5/8	7 5/8	9	10 3/8	11 1/2	12 1/2	14 3/8	16 1/16	17 1/16	18 3/16	20 3/8	22 1/2	24 3/4	27
A. Diameter of flanges	10	10 1/2	11	12 1/2	14	15	16	17 1/2	20	22 1/2	23 1/2	25	27	29 1/2	31 1/2	34
B. Thickness of flange, cast-iron ferrosteel, cast steel, or malleable iron.	1 1/8	1 3/16	1 3/8	1 7/16	1 1/2	1 5/8	1 3/4	1 7/8	2	2 1/8	2 3/16	2 1/4	2 3/8	2 1/2	2 5/8	2 3/4
H. Thickness of flange, rolled steel, with long hub	1 1/8	1 1/4	1 1/4	1 1/4	1 5/16	1 3/8	1 7/16	1 1/2	1 5/8	1 3/4	1 13/16	1 7/8	2	2 1/8	2 1/4	2 7/16
K. Diameter of lap	6 5/8	7 1/4	7 3/4	9	10	11	12 1/4	13 1/2	15 1/4	17	18	19	21 1/2	23 1/2	25 1/2	27 1/2
C. Diameter of bolt circle	7 5/8	8 1/2	9 1/4	10 5/8	11 7/8	13	14	15 1/4	17 3/4	20	21	22 1/2	24 1/2	26 3/4	28 3/4	31 1/4
Number of bolts	8	8	8	12	12	12	12	16	16	20	20	20	24	24	28	28
Size of bolts	3/4	3/4	3/4	3/4	7/8	7/8	7/8	7/8	7/8	7/8	1	1	1	1 1/8	1 1/8	1 1/8
G. Height of flange, cast-iron or ferrosteel	3 3/4	3 15/16	4 1/8	4 1/4	4 7/16	4 5/8	4 13/16	4 15/16	5 3/16	5 1/2	5 5/8	6	6 1/4	6 1/2	6 7/8	7 1/4
F. Height of flange, rolled steel, with long hub	3 1/8	3 1/4	3 1/4	3 1/4	3 3/8	3 1/2	3 5/8	3 3/4	4	4 3/8	4 1/2	4 3/4	5	5 1/8	5 1/4	6 1/4
E. Height of flange, cast steel or malleable iron	1 3/4	1 15/16	1 7/8	2	2 1/16	2 3/16	2 1/4	2 3/8	2 5/16	2 11/16	2 13/16	2 7/8	3 1/16	3 1/4	3 7/16	3 5/8

Oil division	Delivery and discharge—brass or bronze yellow.
Pneumatic division	All pipes—gray.
Gas division	City lighting service—aluminum. Gas engine service—black, red flanges.
Fuel oil division	All piping—black.
Refrigerating system	White and green stripes alternately on flanges and fittings, body of pipe being black.
Electric lines and feeders	Black and red stripes alternately on flanges and fittings, body of pipe being black.

The standard pipe markings of the ships of the United States Navy are given in Fig. 23 (*Amer. Mach., Nov.* 26, 1908).

Outside of machinery spaces all pipes, except pneumatic pipes, are painted white (the general color of neighboring work). The contents of each pipe are indicated by distinctive color bands placed upon the flanges, or at intervals between the flanges, or in both places, as shown.

The valves also are painted distinctive colors, indicating the contents of the pipe.

The direction of flow of the contents of the pipes is indicated by a narrow color band (red or black) painted around the center of the band that indicates the contents of the pipe.

In general, the narrow black band indicates the flow toward the motive power of the system, or toward an auxiliary, and the red band indicates the flow away from the motive power or auxiliary. Except ventilation pipes in the coal bunkers, under the fire and engine rooms and store-room floors in double bottoms and wing passages, all pipes are painted the color of the compartment and retain their distinctive bands.

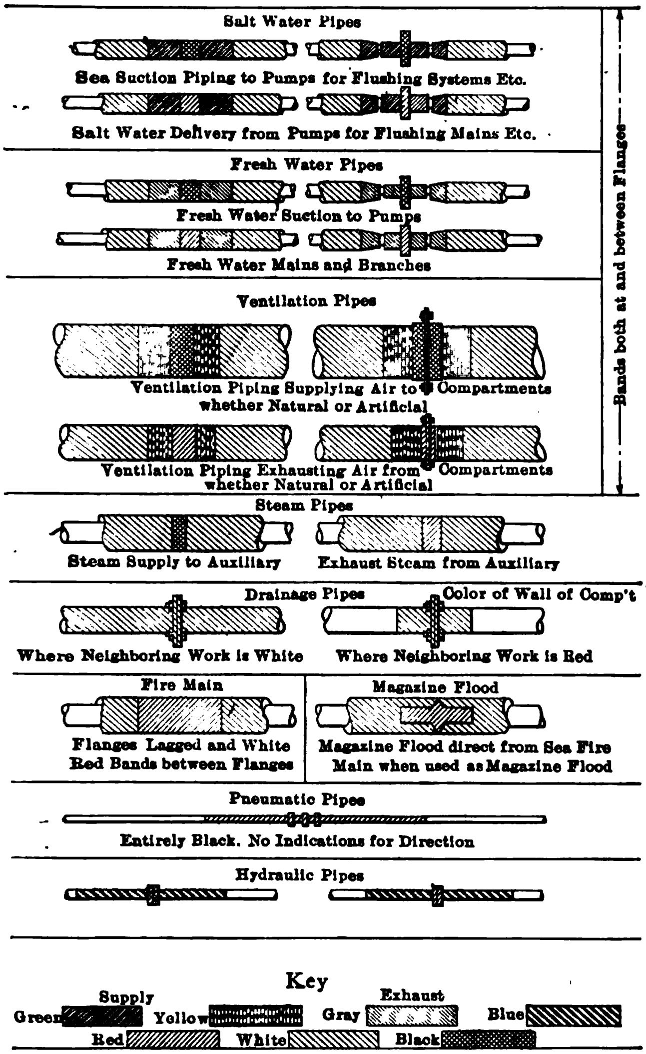


FIG. 23.—Standard pipe markings of the United States Navy.

TABLE 14.—DIMENSIONS, STRENGTH AND WEIGHT OF PELTON WATER WHEEL CO.'S DOUBLE RIVETED HYDRAULIC PIPE

Diameter of pipe, ins.	Thickness of material, U. S. standard gage	Equivalent thickness, ins.	Head in feet that pipe will safely stand	Weight per lineal foot, lbs.	Diameter of pipe, ins.	Thickness of material, U. S. standard gage	Equivalent thickness, ins.	Head in feet that pipe will safely stand	Weight per lineal foot, lbs.
3	18	.05	810	2.25	18	12	.109	295	25.25
4	18	.05	607	3.00	18	11	.125	337	29.00
4	16	.062	760	3.75	18	10	.14	378	32.50
5	18	.05	485	3.75	18	8	.171	460	40.00
5	16	.062	605	4.50	20	16	.062	151	16.00
5	14	.078	757	5.75	20	14	.078	189	19.75
6	18	.05	405	4.25	20	12	.109	265	27.50
6	16	.062	505	5.25	20	11	.125	304	31.50
6	14	.078	630	6.50	20	10	.14	340	35.00
7	18	.05	346	4.75	20	8	.171	415	45.50
7	16	.062	433	6.00	22	16	.062	138	17.75
7	14	.078	540	7.50	22	14	.078	172	22.00
8	16	.062	378	7.00	22	12	.109	240	30.50
8	14	.078	472	8.75	22	11	.125	276	34.50
8	12	.109	660	12.00	22	10	.14	309	39.00
9	16	.062	336	7.50	22	8	.171	376	50.00
9	14	.078	420	9.25	24	14	.078	158	23.75
9	12	.109	587	12.75	24	12	.109	220	32.00
10	16	.062	307	8.25	24	11	.125	253	37.50
10	14	.078	378	10.25	24	10	.14	283	42.00
10	12	.109	530	14.25	24	8	.171	346	50.00
10	11	.125	607	16.25	24	6	.20	405	59.00
10	10	.14	680	18.25	26	14	.078	145	25.50
11	16	.062	275	9.00	26	12	.109	203	35.50
11	14	.078	344	11.00	26	11	.125	233	39.50
11	12	.109	480	15.25	26	10	.14	261	44.25
11	11	.125	553	17.50	26	8	.171	319	54.00
11	10	.14	617	19.50	26	6	.20	373	64.00
12	16	.062	252	10.00	28	14	.078	135	27.25
12	14	.078	316	12.25	28	12	.109	188	38.00
12	12	.109	442	17.00	28	11	.125	216	42.25
12	11	.125	506	19.50	28	10	.14	242	47.50
12	10	.14	567	21.75	28	8	.171	295	58.00
13	16	.062	233	10.50	28	6	.20	346	69.00
13	14	.078	291	13.00	30	12	.109	176	39.50
13	12	.109	407	18.00	30	11	.125	202	45.00
13	11	.125	467	20.50	30	10	.14	226	50.50
13	10	.14	522	23.00	30	8	.171	276	61.75
14	16	.062	216	11.25	30	6	.20	323	73.00
14	14	.078	271	14.00	30	$\frac{1}{4}$	.25	404	90.00
14	12	.109	378	19.50	36	11	.125	168	54.00
14	11	.125	433	22.25	36	10	.14	189	60.50
14	10	.14	485	25.00	36	$\frac{3}{16}$	.187	252	81.00
15	16	.062	202	11.75	36	$\frac{1}{4}$	.25	337	109.00
15	14	.078	252	14.75	36	$\frac{5}{16}$	.312	420	135.00
15	12	.109	352	20.50	40	10	.14	170	67.50
15	11	.125	405	23.25	40	$\frac{3}{16}$	.187	226	90.00
15	10	.14	453	26.00	40	$\frac{1}{4}$	.25	303	120.00
16	16	.062	190	13.00	40	$\frac{5}{16}$	.312	378	150.00
16	14	.078	237	16.00	40	$\frac{3}{8}$	.375	455	180.00
16	12	.109	332	22.25	42	10	.14	162	71.00
16	11	.125	379	24.50	42	$\frac{3}{16}$	.187	216	94.50
16	10	.14	425	28.50	42	$\frac{1}{4}$	.25	289	126.00
18	16	.062	168	14.75	42	$\frac{5}{16}$	.312	360	158.00
18	14	.078	210	18.50	42	$\frac{3}{8}$	.375	435	190.00

MINOR MACHINE PARTS

Tapers

The most commonly used standard tapers are the Morse and the Brown & Sharpe. The former has, nominally, a taper of ⅝ in. per ft. measured on the diameter, as are all tapers, but having been established before the days of accurate measurements, the different numbers range between .6 and .63 in. per ft. The Brown & Sharpe taper is ½ in. per ft., except the No. 10, of which the taper is .5161 in. per ft.

The most desirable taper is the Jarno, which is .6 in. per ft. or .05 in. per in., which latter figure brings out its desirable features most clearly. The number system, instead of being arbitrary as with others, indicates the dimensions, the number of any taper being equal to the diameter in tenths of an inch at the small end, the diameter in eighths of an inch at the large end, and the length in halves of an inch. Again, the length is equal to five times the small diameter or four times the large diameter, any one of the dimensions being the key to the others. The leading machine tool builders who use this taper are the Pratt & Whitney Co. and the Norton Grinding Co. The Reed taper is the same as the Jarno, but without the convenient relationship of numbers, diameters and length. The Sellers taper is ¾ in. per ft. In this taper the customary driving tang of twist drills and boring bars is omitted and in its place the socket is provided with a key and the shank with a keyway to fit. Unlike the tang, this key has ample driving power and eliminates the well known trouble due to the twisting off of the usual tang. There is nothing to prevent its use with other tapers. The sockets being fitted with keys, it is only necessary to mill the keyways in twist drill shanks and thereby get rid of a universal nuisance.

FIG. 1.—Taper of steam-hammer piston-rod ends.

The following tables give dimensions of these various tapers, all dimensions being in inches.

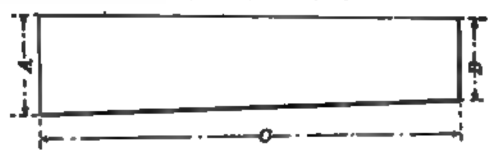
TABLE 1.—THE BROWN & SHARPE TAPER

Dimensions in inches

Number of taper	Diam. of plug at small end *D*	Plug depth *P*	Depth of hole *H*	Keyway from end of spindle *K*	Length of keyway *L*	Width of keyway *W*	Length of arbor tongue *T*	Thickness of arbor tongue *t*	Taper per foot
1	.20	15/16	1 1/16	15/16	3/8	.135	3/16	1/8	.500
2	.25	1 3/16	1 5/16	1 11/64	1/2	.166	1/4	5/32	.500
3	.312	2	2 1/8	1 15/32	5/8	.197	5/16	3/16	.500
4	.35	1 1/4	1 3/8	1 41/64	11/16	.228	11/32	7/32	.500
5	.45	1 3/4	1 7/8	1 15/16	3/4	.260	3/8	1/4	.500
6	.50	2 3/8	2 1/2	2 29/64	7/8	.291	7/16	9/32	.500
7	.60	3	3 1/8	2 61/64	15/16	.322	15/32	[illegible]	.500
8	.75	3 9/16	3 11/16	3 15/32	1	.353	1/2	11/32	.500
9	.90	4	4 1/8	3 7/8	1 1/8	.385	9/16	3/8	.500
9	.90	4 1/4	4 3/8	4 1/8	1 1/8	.385	[illegible]	3/8	.500
10	1.0446	5	5 1/8	4 27/32	1 5/16	.447	21/32	7/16	.5161
10	1.0446	5 11/16	5 13/16	5 15/32	1 5/16	.447	21/32	7/16	.5161
11	1.25	6 3/4	6 7/8	6 13/32	1 5/16	.447	21/32	7/16	.500
12	1.50	7 1/8	7 1/4	6 15/16	1 1/2	.510	3/4	1/2	.500
13	1.75	7 3/4	7 7/8	7 5/16	1 1/2	.510	3/4	1/2	.500
14	2	8 1/4	8 3/8	8 3/32	1 11/16	.572	27/32	9/16	.500
15	2.25	8 3/4	8 7/8	8 19/32	1 11/16	.572	27/32	9/16	.500
16	2.50	9 1/4	9 3/8	9	1 7/8	.635	15/16	5/8	.500
17	2.75	9 3/4	9 7/8	......	......	..	...	......	.500
18	3	10 1/4	10 3/8	......	......	......	......	......	.500

TABLE 2.—THE JARNO TAPER

Dimensions in inches



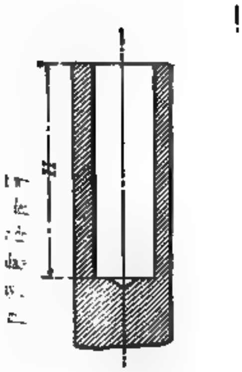
Taper per ft. = .6 in.
Taper per in. = .05 in.

$$\text{Dia. of large end} = \frac{\text{No. of taper}}{8}$$

$$\text{Dia. of small end} = \frac{\text{No. of taper}}{10}$$

$$\text{Length of taper} = \frac{\text{No. of taper}}{2}$$

Number	A	B	C
2	.250	.20	1
3	.375	.30	1½
4	.500	.40	2
5	.625	.50	2½
6	.750	.60	3
7	.875	.70	3½
8	1.000	.80	4
9	1.125	.90	4½
10	1.250	1.00	5
11	1.375	1.10	5½
12	1.500	1.20	6
13	1.625	1.30	6½
14	1.750	1.40	7
15	1.875	1.50	7½
16	2.000	1.60	8
17	2.125	1.70	8½
18	2.250	1.80	9
19	2.375	1.90	9½
20	2.500	2.00	10



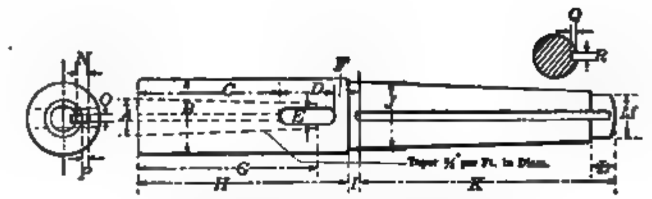


Table 5.—The Sellers Taper

Dimensions in inches

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1/4	1	1 13/16	3/8	1/4	5/32	2 5/16	2 1/8	1/4	11/16	3	1/4	3/16	5/32	5/32	5/32	[illegible]	1/4
3/8	1	1 13/16	3/8	1/4	5/32	2 5/16	2 3/8	1/4	3/4	3 1/2	5/16	5/16	5/32	5/32	[illegible]	5/32	1/4
11/16	1 5/16	2 1/8	1/2	5/16	1/4	2 11/16	3 1/4	1/4	1 1/8	4 1/8	3/8	3/8	11/32	5/32	5/32	1/4	5/16
3/4	1 1/2	3 5/16	1	3/8	5/16	3 11/16	4 3/8	1/4	1 1/8	5 1/4	3/8	11/16	11/32	5/32	5/32	5/32	1/4
1 1/8	2	4 1/8	1	3/8	3/8	4 1/2	5 1/4	1/4	1 1/4	6 1/4	5/16	1 1/4	11/16	11/16	1/4	5/16	1/4
1 5/8	2 7/8	5 3/8	1 1/8	3/8	3/8	6 1/2	7 1/8	1/4	2	8	1/4	1 1/2	11/16	11/16	5/16	5/16	5/16
2 1/8	3 1/2	7 1/2	1 3/8	1/2	..	9 1/2	..	.	.	.	..	..	..	11/16	1/2	...	..

Table 3.—The Morse Taper

Dimensions in inches

No. of Taper	Diam plug at small end	Standard plug depth	Depth of hole	End of spin. to keyway	Length of keyway	Width of keyway	Length of tongue	Diam. of tongue	Thickness of tongue	Rad. of mill for tongue	Rad. of tongue a	Shank depth	Whole length of shank	Taper per foot	Diam. at end of socket	Diam. of point of shank	Taper per inch
	D	P	H	K	L	W	T	d	t	R	a	S	B		A		
1	.369	2 1/8	2 3/16	2 1/16	3/4	.213	3/16	.33	11/64	[illegible]	.05	2 1/8	2 7/16	.600	.475	.356	.05
2	.572	2 9/16	2 5/8	2 1/2	7/8	.26	1/4	17/32	1/4	1/4	.06	2 5/8	3 1/16	.602	.7	.556	.05016
3	.778	3 3/16	3 1/4	3 1/16	1 3/16	.322	5/16	3/4	5/16	[illegible]	.08	3 5/16	3 7/8	.602	.938	.759	.05016
4	1.02	4 1/16	4 1/8	3 7/8	1 1/4	478	3/8	31/32	15/32	5/16	.10	4 1/8	4 7/8	.623	1.231	.997	.05191
5	1.475	5 3/16	5 1/4	4 15/16	1 1/2	.635	1/2	1 13/32	5/8	3/8	.12	5 1/4	6	.630	1 748	1 446	.0525
6	2.116	7 1/4	7 3/8	7	1 3/4	.76	3/4	2	3/4	1/2	.15	8	8 9/16	.626	2.494	2.077	.05216

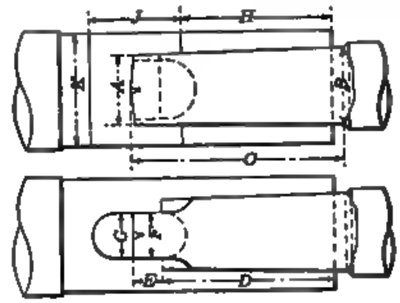


Table 6.—The Standard Tool Co.'s Taper for Twist Drill Shanks

Dimensions in inches

Number of taper	Diameter small end of shank	Diameter large end of shank	Total length of shank	Depth of hole in socket	Length of tongue to mill of socket hole	Thickness of tongue
	A	B	C	D	E	F
1	.378	.484	2 1/4	1 7/8	1/2	1/8
2	.587	.706	2 1/4	1 15/16	5/16	1/4
3	.800	.941	2 15/16	2 1/2	1/2	1/4
4	1.050	1.244	3 1/2	3	[illegible]	1/2
5	1.515	1.757	4 1/2	3 3/4	1/2	1
6	2.160	2.501	6 1/4	5	3/4	1 1/4
7	2 815	3 283	9	7 3/4	1	1 1/2

Number of taper	Width of keyway	End of socket to keyway	Length of keyway	Diameter of socket	Taper per foot	Taper per inch
	G	H	J	K		
1	.263	1 1/2	3/4	15/16	.600	.0500
2	.388	1 3/4	1	1 1/16	.602	.05016
3	.520	2	1 1/4	1 5/16	.602	05016
4	.645	2 15/16	1 1/2	1 13/16	.623	.05191
5	1 020	3 1/2	2	2 9/16	.630	.0525
6	1 270	4 1/2	2 1/4	2 3/4	.626	05216
7	1.520	7	3	......	.625	.05208

Table 4.—The Reed Taper

Table 7.—Tapers per Foot and Corresponding Angles

Taper per ft.	Included angle	Angle with center line	Taper per ft.	Included angle	Angle with center line
1/8 in.	0°–36′	0°–18′	1 in.	4°–46′	2°–23′
1/4 in.	1°–12′	0°–36′	1 1/2 in.	7°–09′	3°–35′
5/16 in.	1°–30′	0°–45′	1 3/4 in.	8°–20′	4°–10′
3/8 in.	1°–47′	0°–54′	2 in.	9°–31′	4°–46′
7/16 in.	2°–05′	1°–02′	2 1/2 in.	11°–54′	5°–57′
1/2 in.	2°–23′	1°–12′	3 in.	14°–15′	7°–08′
3/4 in.	3°–35′	1°–47′	3 1/2 in.	16°–36′	8°–18′
15/16 in.	4°–28′	2°–14′	4 in.	18°–55′	9°–28′

Split-ring expanding mandrels will hold well, and at the same time release freely when the nut is loosened, if given a taper of 3 ins. per ft. measured on the diameter.

The taper required in steam-hammer piston-rod ends and similar pieces in order to permit separation and yet hold the parts together without keys, as determined at the Crescent Steel Works and shown in Fig. 1, is 1 in. per ft. measured on the diameter. The taper should have a length of 3 diameters. The enlarged end prevents breakage within the head.

Taper Pins and Their Correct Use

The diameter of drills for Pratt & Whitney taper pins may be obtained from Table 9 by C. Talbot (*Amer. Mach., Jan.* 28, 1912) The drill sizes given are to be used when the diameter of the work and the length of the pin are the same. If the pin is to be cut off, use drill *C* or *D* according to the end cut off.

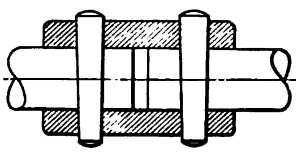


Fig. 2.

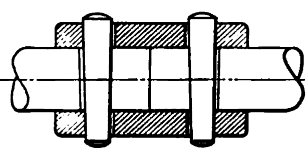


Fig. 3.

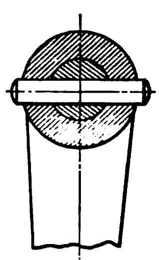


Fig. 4.

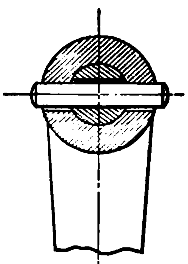


Fig. 5.

Figs. 2 to 5.—Correct and incorrect use of taper pins.

Correctly used, taper pins are capable of far wider application than they have received. When used under alternating stresses they should be given a key draft—opposing pins taking the alternating stresses. A pair of rods joined together as shown in Fig. 2 will, if subjected to alternating pull and thrust, invariably work loose at the pins while, if made as in Fig. 3, they will give no trouble. Again crank arms subject to alternating stresses and pinned to a shaft as in Fig. 4 will work loose, while, if pinned as in Fig. 5, they will not.

The essential feature is the key draft which may be obtained by filing out the holes as shown, or, in the case of Fig. 3, the holes may be reamed a little too deep for a seat without the key draft and then a thin shim—even a piece of paper of substantial thickness—may be placed within the coupling and between the rods. The amount of opening required is but slight, the only essential being that there is enough to insure that each pin pulls positively in one direction only. A suitable diameter for the pins is one-third the diameter of the male member—two or more pins being used if the stresses call for them.

Table 8.—Total Taper from Taper per Foot

Length of tapered portion, ins.	Taper ins. per foot										
	1/16	3/32	1/8	1/4	3/8	1/2 [1]	.6 [2]	5/8 [3]	3/4 [4]	1	1 1/4
1/32	.0002	.0002	.0003	.0007	.0010	.0013	.0016	.0016	.0020	.0026	.0033
1/16	.0003	.0005	.0007	.0013	.0020	.0026	.0031	.0033	.0039	.0052	.0065
1/8	.0007	.0010	.0013	.0026	.0039	.0052	.0062	.0065	.0078	.0104	.0130
3/16	.0010	.0015	.0020	.0039	.0059	.0078	.0094	.0098	.0117	.0156	.0195
1/4	.0013	.0020	.0026	.0052	.0078	.0104	.0125	.0130	.0156	.0208	.0260
5/16	.0016	.0024	.0033	.0065	.0098	.0130	.0156	.0163	.0195	.0260	.0326
3/8	.0020	.0029	.0039	.0078	.0117	.0156	.0187	.0195	.0234	.0312	.0391
7/16	.0023	.0034	.0046	.0091	.0137	.0182	.0219	.0228	.0273	.0365	.0456
1/2	.0026	.0039	.0052	.0104	.0156	.0208	.0250	.0260	.0312	.0417	.0521
9/16	.0029	.0044	.0059	.0117	.0176	.0234	.0281	.0293	.0352	.0469	.0586
5/8	.0033	.0049	.0065	.0130	.0195	.0260	.0312	.0326	.0391	.0521	.0651
11/16	.0036	.0054	.0072	.0143	.0215	.0286	.0344	.0358	.0430	.0573	.0716
3/4	.0039	.0059	.0078	.0156	.0234	.0312	.0375	.0391	.0460	.0625	.0781
13/16	.0042	.0063	.0085	.0169	.0254	.0339	.0406	.0423	.0508	.0677	.0846
7/8	.0046	.0068	.0091	.0182	.0273	.0365	.0437	.0456	.0547	.0729	0911
15/16	.0049	.0073	.0098	.0195	.0293	.0391	.0469	.0488	.0586	.0781	.0977
1	.0052	.0078	.0104	.0208	.0312	.0417	.050	.0521	.0625	.0833	.1042
2	.0104	.0156	.0208	.0417	.0625	.0833	.100	.1042	.125	.1667	.2083
3	.0156	.0234	.0312	.0625	.0937	.1250	.150	.1562	.1875	.250	.3125
4	.0208	.0312	.0417	.0833	.125	.1667	.200	.2083	.250	.3333	.4167
5	.0260	.0391	.0521	.1042	.1562	.2083	.250	.2604	.3125	.4167	.5208
6	.0312	.0469	.0625	.125	.1875	.250	.300	.3125	.375	.500	.625
7	.0365	.0547	.0729	.1458	.2187	.2917	.350	.3646	.4375	.5833	.7292
8	.0417	.0625	.0833	.1667	.250	.3333	.400	.4167	.500	.6667	.8333
9	.0469	.0703	.0937	.1875	.2812	.375	.450	.4687	.5625	.750	.9375
10	.0521	.0781	.1042	.2083	.3125	.4167	.500	.5208	.625	.8333	1.0417
11	.0573	.0859	.1146	.2292	.3437	.4583	.550	.5729	.6875	.9167	1.1458
12	.0625	.0937	.125	.250	.375	.500	.600	.625	.750	1.000	1.250
13	.0677	.1016	.1354	.2708	.4062	.5417	.650	.6771	.8125	1.0833	1.3542
14	.0729	.1094	.1458	.2917	.4375	.5833	.700	.7292	.875	1.1667	1.4583
15	.0781	.1172	.1562	.3125	.4687	.625	.750	.7812	.9375	1.250	1.5625
16	.0833	.125	.1667	.3333	.500	.6667	.800	.8333	1.000	1.3333	1.6667
17	.0885	.1328	.1771	.3542	.5312	.7083	.850	.8854	1.0625	1.4167	1.7708
18	.0937	.1406	.1875	.3750	.5625	.750	.900	.9375	1.125	1.500	1.875
19	.0990	.1484	.1979	.3958	.5937	.7917	.950	.9896	1.1875	1.5833	1.9792
20	.1042	.1562	.2083	.4167	.625	.8333	1.000	1.0417	1.250	1.6667	2.0833
21	.1094	.1641	.2187	.4375	.6562	.875	1.050	1.0937	1.3125	1.750	2.1875
22	.1146	.1719	.2292	.4583	.6875	.9167	1.100	1.1458	1.375	1.8333	2.2917
23	.1198	.1797	.2396	.4792	.7187	.9583	1.150	1.1979	1.4375	1.9167	2.3958
24	.125	.1875	.250	.500	.750	1.000	1.200	1.250	1.500	2.000	2.500

[1] Brown & Sharpe Taper (except No. 10).
[2] Jarno & Reed Tapers.
[3] Nominally Morse Taper.
[4] Sellers Taper.

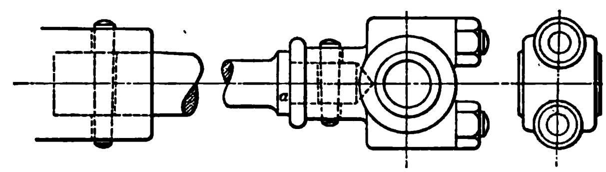


Fig. 6.—Eccentric rod construction.

Fig. 6 shows a cheap, neat and entirely successful eccentric-rod construction of which the author has made many. The rod proper is made from round bar stock, without any forge work whatever. At the left-hand end, where it enters the boss on the eccentric strap, it is turned to the largest even size which the stock will hold up to. The shoulder *a* at the right-hand end, is likewise made as large as the bar will allow, and the taper between the two ends provides an appropriate shape. The eye is a simple cast affair in halves, held

TABLE 9.—REAMER DRILLS FOR TAPER PINS

Length	No.	0	1	2	3	4	5
	A	.156	172	193	.219	250	2 89
.750	C	No. 27	No. 21	No. 15	No. 5	No. A	No. 1
	D	No. 29	No. 24	No. 17	No. 8	No. 1	No. P
1.000	C	No. 28	No. 22	No. 16	No. 6	No. A	No. I
	D	No. 30	No. 26	No. 19	No. 11	No. 2	No. G
1 250	C	No. 28	No. 23	No. 17	No. 7	No. 1	No. H
	D	No. 31	No. 28	No. 20	No 12	No. 3	No. F
1.500	C	No. 29	No 25	No. 17	No. 8	No. 1	No. H
	D	No. 32	No. 29	No. 22	No. 14	No. 3	No. ¼
1.750	C	No. 30	No. 24	No. 18	No. 9	No. 1	No H
	D	No. 33	No 30	No. 24	No. 16	No. 4	No. D
2.000	C		No. 26	No. 19	No 10	No. 2	No. G
	D		No. 31	No. 26	No. 11	No. 5	No. C
2 250	C			No. 19	No. 11	No. 2	No. G
	D			No. 28	No. 19	No. 8	No. B
2 500	C				No. 12	No. 2	No. F
	D				No. 20	No. 10	No. 1
2.750	C				No. 13	No. 3	No. ¼
	D				No. 22	No. 13	No. 2
3 000	C				No. 14	No. 3	No. ¼
	D				No. 24	No. 14	No. 2

Length	No.	6	7	8	9	10
	A	341	.409	492	.591	.706
.750	C	No. P				
	D	No. O				
1.000	C	No. P	No. W			
	D	No. O	No. V			
1.250	C	No. O	No. W	[illegible]		
	D	No. N	No. V	[illegible]		
1.500	C	No. O	No. W	[illegible]	[illegible]	[illegible]
	D	No. N	No. U	[illegible]	[illegible]	[illegible]
1.750	C	No. O	No. W	[illegible]	[illegible]	[illegible]
	D	No. M	No. U	[illegible]	[illegible]	[illegible]
2.000	C	No. N	No. V	[illegible]	[illegible]	[illegible]
	D	No. L	No. T	[illegible]	[illegible]	[illegible]
2.250	C	No. N	No. V	[illegible]	[illegible]	[illegible]
	D	No. L	No. T	[illegible]	[illegible]	[illegible]
2 500	C	No. N	No. V	[illegible]	[illegible]	[illegible]
	D	No. K	No. S	[illegible]	[illegible]	[illegible]
2.750	C	No. N	No. U	[illegible]	[illegible]	[illegible]
	D	No. J	[illegible]	[illegible]	[illegible]	[illegible]
3.000	C	No. N	No. U	[illegible]	[illegible]	[illegible]
	D	No. I	No. R	[illegible]	[illegible]	[illegible]
3.250	C	No. M	No. U	[illegible]	[illegible]	[illegible]
	D	No. H	No. Q	[illegible]	[illegible]	[illegible]
3.500	C	No. M	No. S	[illegible]	[illegible]	[illegible]
	D	No. G	[illegible]	[illegible]	[illegible]	[illegible]
3.750	C	No. M	No. T	[illegible]	[illegible]	[illegible]
	D	No. F	No. P	[illegible]	[illegible]	[illegible]
4.000	C	No. L	No. T	[illegible]	[illegible]	[illegible]
	D	¼	No. O	[illegible]	[illegible]	[illegible]
4.250	C			[illegible]	[illegible]	[illegible]
	D			[illegible]	[illegible]	[illegible]
4.500	C			[illegible]	[illegible]	[illegible]
	D			[illegible]	[illegible]	[illegible]
4.750	C				[illegible]	[illegible]
	D				[illegible]	[illegible]
5.000	C				[illegible]	[illegible]
	D				[illegible]	[illegible]
5.250	C				[illegible]	[illegible]
	D				[illegible]	[illegible]
5.500	C					[illegible]
	D					[illegible]
5 750	C					[illegible]
	D					[illegible]
6.000	C					[illegible]
	D					[illegible]

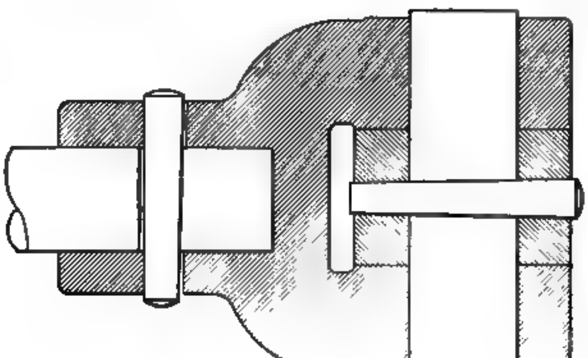


FIG. 7.—Knuckle joint construction.

together with studs and having a socket into which the end of the rod enters It may be of polished brass though painted cast-iron is better. Wear may be taken up by filing the joint, or paper may be inserted in the joint before boring the hole.

Fig. 7 shows a knuckle joint put together in the same way which serves its purpose just as well as much more expensive constructions. Like the eccentric rod, the head is preferably of cast-iron.

Dovetails and T Slots

The leading dimensions of dovetail slides and gibs may be determined in accordance with Fig. 8, and the formulas which accompany it, by JOHN RICHARDS (*A Manual of Machine Construction*). Two methods of arranging the adjusting screws are shown, of which the one at *A*, with an angular point, is correct, and the one at *B*, with flat end, is wrong. The screw *A* exerts its power on the line *m*, pressing the surfaces together at *n*. The one at *B* exerts its force parallel to the faces at *n*, and by forcing the gib into the corner opens the joint at *n*, defeating the very purpose intended.

Either of the methods shown in Fig. 9 is preferable to the set-screw plan. The one at *a*, consisting of a wedge the whole length of the joint, or two wedges, one at each end, can be applied in nearly all cases and is reliable in every way. There is full contact of all surfaces, and the rigidity of the joint is not impaired by the gib.

The one at *e* is also reliable, but not so rigid as the other and requires more width for the saddle, which is frequently objectionable.

Fig. 10 shows a kind of joint, designed by Mr. Richards and employed very successfully for the cross slides of engine lathes. Twisting strains are successfully resisted because of the width between the fulcra at *a* and *e*, and the surfaces are well protected from chips and dirt. Some layers of thin paper are placed in the joint at *c*, so the screws can be set up hard and leave means of compensation.

Exact dimensions of dovetail slides and gibs may be determined from Table 10 which shows the amount to be added or subtracted in dimensioning dovetail slides and their gibs, for the usual angles up to 60 deg. The column for 45-deg. dovetails is omitted, as *A* and *B* are alike for this angle.

In the application of the table, assuming a base with even dimensions, as in Fig. 11, to obtain the dimensions *x* and *y* of the slide Fig. 12, allowing for the gib ¼ in. thick, the perpendicular depth of the

dovetail being $\frac{5}{8}$ in., and the angle 60 deg., look under column A for $\frac{5}{8}$ in. and find opposite $B = .360$ in., which subtracted from 2 ins. gives 1.640 ins., the dimension x. To find y, first get the dimension 1.640 ins., then look under the column for 60-deg. gibs, and find D (for $C = \frac{1}{4}$ in.) to be .289 in., which, added to 1.64, gives 1.929 ins. as the value of y.

$a = \frac{5A}{4}$ or thicker

$b = \frac{6A}{5}$ or thicker

$c = 2A$ or wider

$d = \frac{A}{5}$

$e = \frac{A}{4}$ to $\frac{A}{2}$

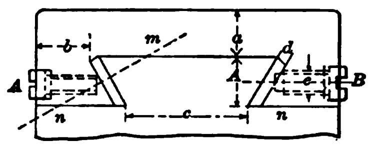


FIG. 8

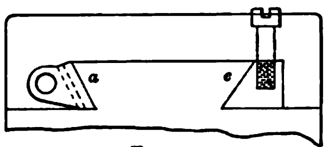


FIG. 9.

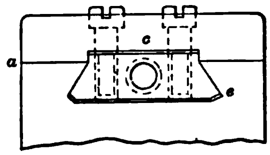


FIG. 10.

FIGS. 8 to 10.—Dovetail slides.

TABLE 10.—DOVETAIL SLIDES AND GIBS
Dimensions in inches

A	B (60°)	B (55°)	B (50°)	C	D (60°)	D (55°)	D (50°)	D (45°)
$\frac{1}{32}$	.018	.022	.027	$\frac{1}{8}$	.144	.152	.163	.176
$\frac{1}{16}$	.036	.044	.053	$\frac{3}{16}$	.216	.228	.244	.264
$\frac{1}{8}$	.072	.087	.105	$\frac{1}{4}$	.289	.305	.326	.353
$\frac{1}{4}$	.144	.175	.210	$\frac{5}{16}$	.361	.381	.407	.442
$\frac{3}{8}$	.216	.262	.314	$\frac{3}{8}$	.433	.457	.489	.530
$\frac{1}{2}$	.288	.350	.420	$\frac{1}{2}$	.577	.610	.652	.707
$\frac{5}{8}$	.360	.437	.525	$\frac{5}{8}$	.721	.762	.815	.883
$\frac{3}{4}$	.433	.525	.629	$\frac{3}{4}$	.866	.915	.979	1.060
$\frac{7}{8}$	.505	.612	.734	$\frac{7}{8}$	1.010	1.067	1.142	1.237
1	.577	.700	.839	1	1.154	1.220	1.305	1.414
$1\frac{1}{8}$	.649	.787	.944					
$1\frac{1}{4}$	.721	.875	1.049					
$1\frac{3}{8}$	.794	.962	1.153					
$1\frac{1}{2}$	.866	1.050	1.259					
$1\frac{3}{4}$	1.010	1.225	1.469					
2	1.154	1.400	1.677					
$2\frac{1}{4}$	1.298	1.575	1.888					
$2\frac{1}{2}$	1.442	1.750	2.097					
$2\frac{3}{4}$	1.588	1.925	2.307					
3	1.732	2.100	2.517					
$3\frac{1}{2}$	2.020	2.450	2.937					
4	2.308	2.800	3.356					
$4\frac{1}{2}$	2.598	3.150	3.776					
5	2.885	3.501	4.195					

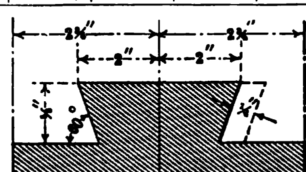


FIG. 11

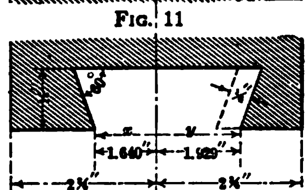


FIG. 12.

Standard T slots for machine tools are greatly to be desired. Table 11 and the accompanying sections give the well-considered proportions of Wm. SELLERS & Co., which, providing as they do for all conditions, deserve general adoption.

The proportions are based on the use of square-head bolts where the size of the head is $1\frac{1}{2}$ times the diameter of the body plus $\frac{1}{8}$ in. They represent three conditions: First, both parts of the slot, that

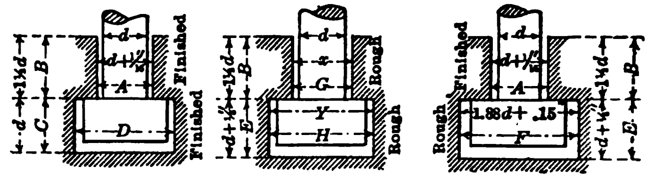


TABLE 11.—WM. SELLERS & CO.'S STANDARD T SLOTS
Dimensions in inches

d	A	B	C	D	E	F	G	H	X	Y
$\frac{1}{2}$	$\frac{9}{16}$	$\frac{3}{4}$	$\frac{3}{8}$	1	$\frac{5}{8}$	$1\frac{5}{16}$	$\frac{9}{16}$	1	$d+\frac{1}{16}$ in.	$1.5d+\frac{1}{8}$ in.
$\frac{5}{8}$	$\frac{11}{16}$	$\frac{11}{16}$	$\frac{3}{8}$	$1\frac{3}{16}$	$\frac{3}{4}$	$1\frac{5}{16}$	$\frac{11}{16}$	$1\frac{3}{16}$	$d+\frac{1}{16}$ in.	$1.5d+\frac{1}{8}$ in.
$\frac{3}{4}$	$\frac{13}{16}$	$1\frac{1}{8}$	$\frac{3}{4}$	$1\frac{3}{8}$	$\frac{7}{8}$	$1\frac{5}{16}$	$\frac{7}{8}$	$1\frac{1}{16}$	$d+\frac{1}{8}$ in.	$1.5d+\frac{3}{16}$ in.
$\frac{7}{8}$	$\frac{15}{16}$	$1\frac{5}{16}$	$\frac{7}{8}$	$1\frac{9}{16}$	1	$1\frac{13}{16}$	1	$1\frac{5}{8}$	$d+\frac{1}{8}$ in.	$1.5d+\frac{3}{16}$ in.
1	$1\frac{1}{16}$	$1\frac{1}{2}$	1	$1\frac{3}{4}$	$1\frac{1}{8}$	2	$1\frac{1}{8}$	$1\frac{13}{16}$	$d+\frac{1}{8}$ in.	$1.5d+\frac{3}{16}$ in.
$1\frac{1}{8}$	$1\frac{3}{16}$	$1\frac{11}{16}$	$1\frac{1}{8}$	$1\frac{15}{16}$	$1\frac{1}{4}$	$2\frac{1}{4}$	$1\frac{1}{4}$	2	$d+\frac{1}{8}$ in.	$1.5d+\frac{3}{16}$ in.
$1\frac{1}{4}$	$1\frac{5}{16}$	$1\frac{7}{8}$	$1\frac{1}{4}$	$2\frac{1}{8}$	$1\frac{3}{8}$	$2\frac{1}{2}$	$1\frac{3}{8}$	$2\frac{3}{16}$	$d+\frac{1}{8}$ in.	$1.5d+\frac{3}{16}$ in.
$1\frac{3}{8}$	$1\frac{7}{16}$	$2\frac{1}{16}$	$1\frac{3}{8}$	$2\frac{5}{16}$	$1\frac{1}{2}$	$2\frac{3}{4}$	$1\frac{1}{2}$	$2\frac{5}{8}$	$d+\frac{1}{8}$ in.	$1.5d+\frac{3}{16}$ in.

for the head and for the body of the bolt, are finished; second, both are rough, that is, unfinished cores; third, the slot for the head is cored and that for the body of the bolt finished.

In the first two cases the two parts of the slot are, of necessity, concentric, and only a moderate amount of clearance is required. In the last case the two parts are almost certain to be out of parallel, and it is obvious that more side clearance must be allowed for the head. As much width as possible without danger of allowing the head to turn is therefore provided.

The consequences of the prevailing confusion of T slots may be greatly mitigated by the use of adapters between fixtures and machine-tool platens as shown in Fig. 13. Instead of making the fixtures with integral tongues, they are made with slots *of standard*

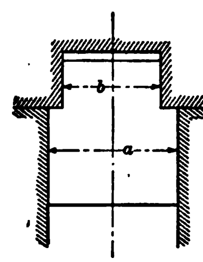


FIG. 13.—Adapter for miscellaneous T-slots and fixtures.

width. A pair of adapters for each machine tool having the dimension a to suit the slot in the platen and the dimension b to suit the tandard fixture slot will enable any fixture to be used on any tool.

Face plates of lathes, boring mills, etc., should never have 8 or 16 slots, but 12, which permits the use of either 3 or 4 straps, the former insuring against distortion, while the latter is most convenient in adjustment.

Shaft Couplings

The accompanying tables, 15 and 16, of flexible shaft couplings represent the practice of the General Electric Co. (*Amer. Mach., Sept.* 29, 1910). The form shown in Table 15 uses flat leather links and is self-explanatory. That shown in Table 16 uses two endless belts placed side by side on the forms or arms of spiders. The belting used is a specially prepared leather which is designed to be used with a tension of 400 lbs. per sq. in. of cross-section, the rating being given both in kilowatts and horse-power, per revolution. The work performed by couplings of this kind is greater than would be expected of the same cross-section of belting, owing to the absence of slippage and the fact that the leather is firmly supported by the other parts of the coupling.

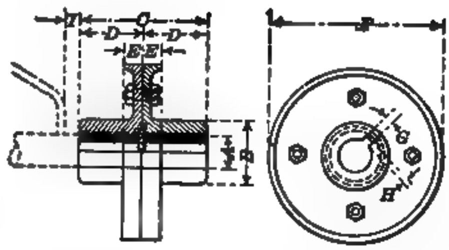


TABLE 13.—CAST-STEEL FLANGE COUPLINGS

Dimensions in inches

Size of shaft A	B	C	D	E	F	G	H	I	Horse-power rating per revolution, with 33,000 lbs. torsion in shaft	Approximate weight
1	2	4	2	1	5¼	⅜	⅜	½	.0102	10
1¼	2½	6	3	1 3/16	5⅝	½	¼	½	.0200	13
1½	2¾	6	3	1½	6¼	½	5/16	½	.0346	20
1¾	3	8	4	1 9/16	7½	½	5/16	½	.0550	27
2	3½	8	4	1 9/16	8½	½	⅜	½	0821	40
2½	4½	10	5	1⅞	9¼	½	⅜	½	.1604	53
3	5	12	6	1 15/16	10¾	⅝	½	½	.2770	80
3½	6	12	6	2	12¼	⅝	½	½	.4400	130
4	6¾	14	7	2 3/16	13	1	½	¾	.6670	160
4½	7½	14	7	2 5/16	14	1	½	¾	9355	200
5	8½	16	8	2¼	15¼	1¼	½	¾	1.2832	280
5½	9	16	8	2½	16¾	1¼	½	¾	1 7081	320
6	9½	18	9	2½	17¾	1¼	¾	¾	2.2176	410
6½	10½	18	9	2½	19	1¼	¾	1	2.8195	465
7	11¾	20	10	3¼	20¼	1½	¾	1	3 5210	595
7½	12	20	10	3¼	21¼	1½	¾	1	4.3310	[illegible]
8	12¾	24	12	3¼	22¼	1½	¾	1	5 2570	840
8½	13¾	24	12	3½	23¾	1½	¾	1	6.3050	965
9	14¾	28	14	3 13/16	25½	2	⅞	1	7.4840	1315
10	16	32	16	4	27	2	⅞	1	10.2670	1650
11	17½	32	16	4 5/16	28¾	2½	1	1	13.6650	1950
12	19	36	18	4½	31¼	2½	1	1	17.7410	2580
13	20½	36	18	4 13/16	33	2½	1	1	22.5560	2970
14	22	40	20	5	35¼	2½	1	1	28.1730	3775
15	23¾	40	20	5½	37½	2½	1	1	34.6510	4320
16	25¼	48	24	5¾	39½	3½	1½	1	42.0530	5730

TABLE 12.—CAST-IRON FLANGED SHAFT COUPLINGS

Dimensions in inches

A	B	C	D	E	F	G	H	J	L	M	Q	R	S	T	Key W	Key X
1	2	5¼	1¼	7/16	1¼	1½	3½	7/16	½	¼	1¾	13/16	⅜	4	¼	⅛
1¼	2½	6¼	1⅝	13/16	1¼	1¾	4¼	½	[illegible]	¼	2	15/16	⅜	4	5/16	5/32
1½	3	7	2⅛	⅞	1 1/16	2¼	4 11/16	½	5/16	¼	2¼	15/16	⅜	4	⅜	3/16
1¾	3½	7¾	2⅝	⅞	1 11/16	2⅝	5¼	11/16	⅝	¼	2½	⅞	½	5	7/16	7/32
2	4	8½	3	⅞	1 13/16	3	5½	13/16	⅝	¼	2¾	⅞	½	5	½	¼
2¼	4½	9¼	3⅜	1	1 13/16	3⅜	6¼	¾	7/16	¼	2⅞	⅞	½	5	⅝	5/16
2½	5	9¾	3⅞	1	1 15/16	3⅞	6¾	¾	7/16	¼	2¾	⅞	½	5	⅝	5/16
2¾	5½	10¾	4¼	1⅛	1 15/16	4¼	7¼	¾	7/16	¼	3	⅞	½	5	¾	⅜
3	6	11¾	4½	1⅛	2⅛	4½	8½	13/16	7/16	¼	3¼	1	½	6	¾	⅜
3¼	6½	12	4¾	1⅛	2¼	4¾	8¾	13/16	½	¼	3½	1	½	6	⅞	7/16
3½	7	12¾	5¼	1 3/16	2 3/16	5¼	9¾	1 13/16	½	¼	3½	1	½	6	⅞	7/16
3¾	7½	13½	5⅝	1¼	2⅜	5⅝	10¼	½	½	¼	3¾	1	½	6	1	½
4	8	14½	6	1¼	2⅜	6	10¾	13/16	[illegible]	¼	4	1⅛	½	6	1	½
4¼	8½	15¼	6⅜	1¼	2½	6⅜	11¼	13/16	9/16	¼	4¼	1⅛	½	6	1⅛	9/16
4½	9	16¼	6¾	1¼	2½	6¾	12¼	1	⅝	¼	4½	1⅛	½	8	1⅛	9/16
5	10	18	7½	1½	3 1/16	7½	13½	1 1/16	13/16	¼	4¾	1¼	1	8	1¼	⅝
5½	11	19½	8¼	2 1/16	3⅜	8¼	15	1⅛	⅞	¼	5¼	1¼	1	8	1⅜	11/16
6	12	21	9	2¼	3 11/16	9	16½	1 3/16	15/16	¼	5¾	1⅜	1⅛	8	1½	¾
6½	13	23	9¾	2¼	3 13/16	9¾	17½	1¼	15/16	⅜	6¼	1⅜	1⅛	8	1⅝	⅞
7	14	24½	10½	2⅜	4¼	10½	19	1 5/16	⅞	⅜	[illegible]	1½	1¼	8	1⅝	⅞
7½	15	26	11¼	2½	4⅝	11¼	20¼	1⅜	15/16	⅜	6¾	1½	1¼	8	1¾	⅞
8	16	27½	12	3	4¾	12	21¾	1 7/16	1 1/16	⅜	7¼	1⅝	1⅜	8	1¾	⅞
8½	17	29	12¾	3¼	5	12¾	22¾	1 7/16	1⅛	⅜	8	1⅝	1⅜	10	1⅞	15/16
9	18	31	13½	3¼	5½	13½	24¼	1 9/16	1 3/16	⅜	8½	1⅝	1⅜	10	1⅞	15/16
9½	19	32½	14¼	3⅜	5⅝	14¼	25¾	1½	1 3/16	⅜	8¾	1⅝	1⅜	10	2	1
10	20	34	15	3½	5 13/16	15	26¾	1⅝	1¼	⅜	9¼	1¾	1½	10	2	1
10½	21	36	15¾	4	5 15/16	15¾	28¼	1⅝	1¼	⅜	9¾	1¾	1½	10	2⅛	1
11	22	37½	16½	4¼	6 1/16	16½	29¾	1⅝	1⅜	⅜	10¼	1¾	1½	12	2⅛	1
11½	23	39	17¼	4½	6 13/16	17¼	30¾	1¾	1⅜	⅜	10¾	1⅞	1⅝	12	2¼	1
12	24	41	18	[illegible]	6 13/16	18	32¼	1¾	1½	⅜	11	1⅞	1½	12	2¼	1

Silent pawls of different constructions are shown in Figs. 18–21. Fig. 18 illustrates the principle of a device used with the brake mechanism on heavy cranes and hoists in steel mills. The driving member *A* is shown rotating in the direction *R*, thus driving the ratchet *B* by means of the pawl *P*. When *A* reverses and moves in the direction *L*, the pawl is raised until it meets the stop pin *F*, which motion is caused by the change in position of the links *D* and *E*. The locked linkage now causes the spring clamp *G* to move with it, while the resistance due to its spring tension keeps the pawl raised above the ratchet teeth as long as rotation in that direction is continued. When *A* again begins to move in the direction *R*, the pawl instantly drops, as clamp *G* remains stationary during the change in position of links *D* and *E*.

In Fig. 19, the driver *A* rotates in the direction *L*, the pawl *P* is driven by *A*, through the contact of *E* at *X*, the pin *B* having merely moved the pawl about its axis *D* at the beginning of motion. The pin *D*, which carries the pawl, is itself carried by the sliding block *E* and the extended end of *D* also passes through an arm of the casting *G*, which latter is an easy fit on the shaft. When *A* begins to rotate in the direction *R*, the block *E* remains stationary, while the pin *C* advances against the projecting finger on the pawl *P*, causing it to rotate about *D*, thus bringing the point above the ratchet teeth. At this instant face *H* of sliding block *E* comes against the face *V*, which rotates all the parts except the ratchet as far as the stroke is set, the reversal at the end of the stroke again bringing the pawl into engagement, and driving the ratchet.

In Fig. 20, the arm *A* carrying the pawls *PP* is made to oscillate about the shaft *S* by a variable throw crank used for changing the length of feed. The ratchet disk *R*, which is keyed to the shaft, carries on its hub a split hub *H* to which is riveted a piece of sheet steel *D*, in which are cut slots *YY* at an angle of about 45 deg. with a radial line; into the pawls are driven pins *XX*.

In feeding, the arm *A* moving in a counter-clockwise direction causes the pins *XX* in the pawls *PP* to ride down in the slot *YY*, thus forcing the pawls into engagement with the ratchet disks, thereby turning the device as a whole. Upon reversal the pins ride outward until they reach the end of the slots and then drag the sheet-steel piece, which has an adjusting screw in the split hub, back with them.

In Fig. 21, a wheel, part of which is shown at *A*, is given a variable back and forth motion, as indicated by the arrows. The friction piece *B* runs in a channel turned through the ratchet teeth, and is

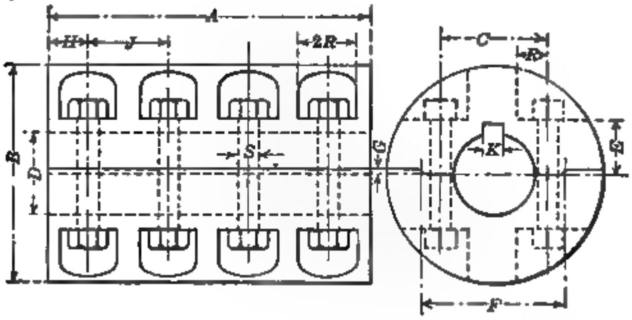


TABLE 14.—CLAMP SHAFT COUPLINGS

Dimensions in inches

D	A	B	C	E	F	G	H	J	K	R	S	Weight, lbs.
1 7/16	6	4 1/4	2 1/2	1	3 3/4	1/8	1 1/2		5/16	15/16	1/2	18.7
1 11/16	7	4 3/4	3	1 3/16	4	3/16	1 3/4		5/16	1 1/16	3/4	29.0
1 15/16	8	5 1/2	3 1/4	1 3/8	4 1/4	1/4	2		5/16	1 5/16	3/4	42 7
1 7/16	9	6 1/4	3 1/2	1 1/2	4 1/2	1/4	2 1/4		5/16	1 5/16	3/4	57 9
2 7/16	10	6 3/4	3 3/4	1 5/8	5 1/4	1/4	2 1/2		5/16	1 5/16	3/4	78.9
2 11/16	11	7 3/4	4	1 7/8	5	1/4	1 3/4	2 3/4	5/16	1 5/16	3/4	94.6
2 15/16	12	8	4 1/4	2	5 3/4	1/4	1 1/2	3	11/16	1 5/16	3/4	125.2
3 3/16	13	8 1/4	4 1/2	2 1/4	5 1/2	1/4	1 1/4	3 1/4	11/16	1 5/16	3/4	149 5
3 7/16	14	9	4 3/4	2 1/4	6	1/4	1 3/4	3 1/2	11/16	1 5/16	7/8	181.7
3 11/16	15	9 1/2	5	2 1/2	6 1/2	1/4	1 1/2	3 3/4	11/16	1 5/16	7/8	228.6
3 15/16	16	10 1/4	5 1/4	2 3/4	7	1/4	2	4	11/16	1 5/16	1	277.9
4 7/16	18	11 1/4	6	3	7 3/4	1/4	2 1/2	4 1/2	11/16	1 5/16	1	389.8
4 15/16	20	12 1/4	6 3/4	3 3/4	8 1/2	1/4	2 1/2	5	11/16	1 5/16	1 1/4	406.7

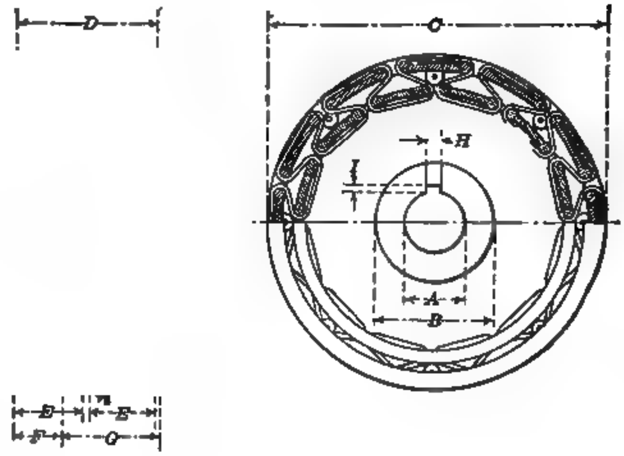


TABLE 16.—LACED LEATHER FLEXIBLE COUPLINGS

Dimensions in inches

Bore	Dimensions								Rating per revolution with 400 lbs. per sq. in. tensile stress in belt		Max. r.p.m.	Weight of complete in lbs.
A	[illegible]	C	D	E	F	G	H	I	Kw.	h.p.		
1	2	5	3	1 1/16	15/16	2 1/16	1/2	1/4	0032	.0043	1800	8
1 1/2	3	8 1/4	4	1 15/16	[illegible]	3 3/16	3/4	5/16	.01492	.02	1500	27
2	3 1/2	9 1/2	5	2 3/16	1 3/16	3 15/16	1/2	1/4	.0258	.0346	1500	39
3	5	15 1/2	[illegible]	2 15/16	1 5/8	5 7/16	3/4	3/8	.1196	.1604	1200	115
4	6 3/4	18 1/2	8	3 15/16	2 1/16	5 1/2	1	1/2	.2066	.277	900	189
5	8 1/4	24 1/2	10	4 15/16	3 1/16	6 15/16	1 1/4	1/2	.4901	.657	750	367
6	9 3/4	30 1/2	12	5 15/16	4 1/16	7 7/16	1 1/2	3/4	.9573	1 2832	600	611
7	11 3/4	37	14	6 15/16	5 3/16	8 1/2	1 1/2	3/4	1.654	2.2176	450	1033
8	12 3/4	43	16	7 15/16	5 15/16	9 1/2	1 3/4	3/4	2 627	3 5215	350	1527
9	15 1/2	49	18	8 15/16	6 15/16	9 3/4	2	7/8	3 9214	5 2566	300	2201
10	16 3/4	[illegible]	20	9 15/16	7 15/16	9 3/4	2	7/8	3.9214	5.2566	300	2376
11	18 1/2	55	22	10 15/16	8 15/16	10 7/16	2 1/4	1	5.5833	7.4844	250	3171
12	19 3/4	55	24	11 15/16	9 15/16	10 7/16	2 1/4	1	5.5833	7.4844	250	3439
13	21 1/2	61	26	12 15/16	10 15/16	11 7/16	2 1/4	1	7.6591	10.2669	200	4485
14	23	61	28	13 15/16	11 15/16	11 7/16	2 1/4	1	7 6591	10 2669	200	4831

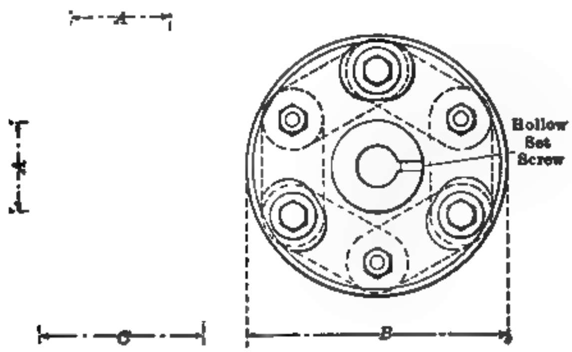


TABLE 15.—LEATHER LINK FLEXIBLE COUPLINGS

Dimensions in inches

Bore	Dimensions						Rating per revolution with 400 lbs. per sq in. tensile stress in belt		Max. r.p.m.	Weight in lbs.
	A	[illegible]	C	D	E	F	Kw	H p.		
3/4	1 1/4	3 3/4	2 3/4	1 1/2	15/16	3/8	.0012	.0016	1800	2 3/4
1	2	5	4	1 15/16	3/4	3/8	0032	.0043	1800	7 1/2
1 1/2	2 3/4	6 3/4	6	2 15/16	1 1/16	5/8	.0076	.0102	1800	15
2	3 1/4	8 1/4	8	3 15/16	1 1/4	3/4	0149	.0200	1800	27
2 1/2	4 1/4	9 1/2	10	4 15/16	1 7/16	3/4	.0258	.0346	1800	43
3	5	11 1/2	12	5 15/16	1 3/4	7/8	.0410	.0550	1800	75
3 1/2	6	12 1/2	12	5 15/16	1 15/16	1 1/8	0612	0821	1800	103

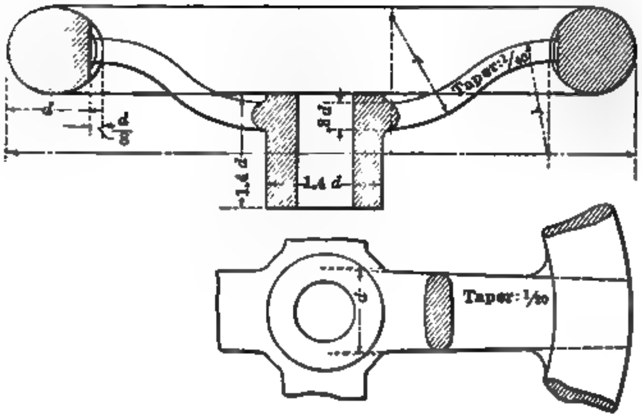
TABLE 17.—FORGED WRENCHES

Dimensions in inches

D	L	T^1	T^2	T^3	B^1	B^2
3/8	5	3/8	1/4	1/4	3/4	5/8
1/2	6 1/2	1/2	1/4	1/4	15/16	15/16
[illegible]	8	1/2	5/16	5/16	1 1/8	3/4
3/4	9 1/2	1/2	5/16	5/16	1 1/4	15/16
1	11	5/8	3/8	1/4	1 3/8	7/8
1 1/8	12 1/2	5/8	3/8	1/4	1 1/2	15/16
1 1/4	14	3/4	7/16	5/16	1 5/8	1
1 3/8	15 1/2	3/4	7/16	5/16	1 3/4	1 1/16
1 1/2	17	3/4	1/2	3/8	1 7/8	1 1/8
1 5/8	18	7/8	1/2	3/8	2	1 3/16
1 3/4	19	7/8	9/16	7/16	2 1/8	1 1/4
1 7/8	20	7/8	9/16	7/16	2 1/4	1 5/16
2	21	1	5/8	1/2	2 3/8	1 3/8

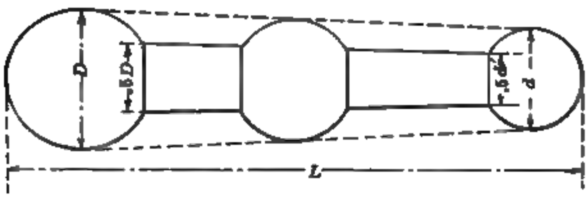
held by the spring and pin, and operates the pawl by alternately coming in contact first on one side and then the other of the conical hole at *a* and *b*.

The dimensions of wrenches may be obtained from Figs. 22, 23 and 24 and Table 17. In Fig. 22 one-half the bolt diameter is divided into four equal parts by lines *abcd*. The point where the circle *e*, representing the inside nut diameter, crosses line *a* locates the first center; the point where line *fg*, from the middle of one side to the corners, crosses line *c* locates the second center; the point where the 30-deg. line crosses line *c* locates the third center, the center of the bolt being the fourth center. *R* may be made equal to the bolt diameter. Angle wrenches, Fig. 24, are better than the straight pattern as they may be used in more confined places.



$d = .1D + .3$ in. $r = .2D - \frac{1}{8}$ in.

FIG. 14.—Hand wheels.



$D = .135L + \frac{1}{2}$ in. $d = .1L + \frac{1}{2}$ in.

FIG. 15.—Ball cranks.

FIG. 16.—Machine tool handles.

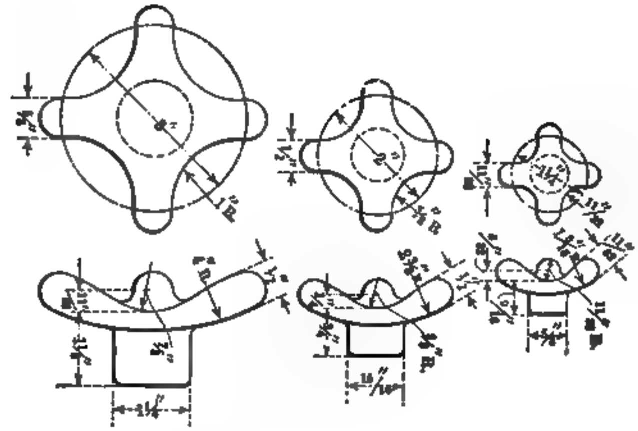


FIG. 17.—Machine tool knobs.

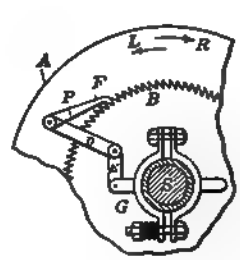


FIG. 18.

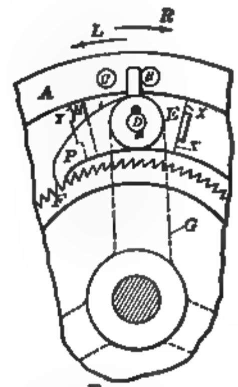


FIG. 19.

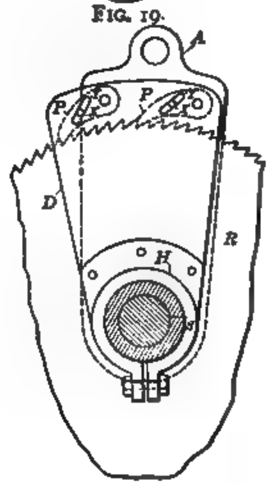


FIG. 21. FIG. 20.

FIGS. 18 to 21.—Silent pawls.

Jig and Fixture Details

Fixture cams may be made self-locking by keeping the rise of the cam within the angle of repose. Using a mean value for this angle the rise of the cam, Fig. 26, for each 9 deg. of arc is given by the formula

$$x = .005\ d$$

in which x = rise of cam, ins., for each 9 deg. of arc
d = diameter of cam, ins.

When fixture cams are placed on horizontal axes the handles should be so placed that their weight will tend to tighten the cams as in Fig. 27, and not as in Fig. 28.

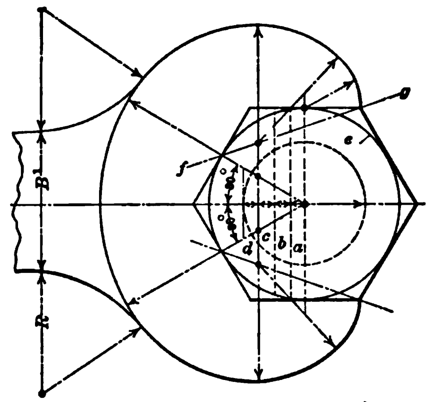


FIG. 22.

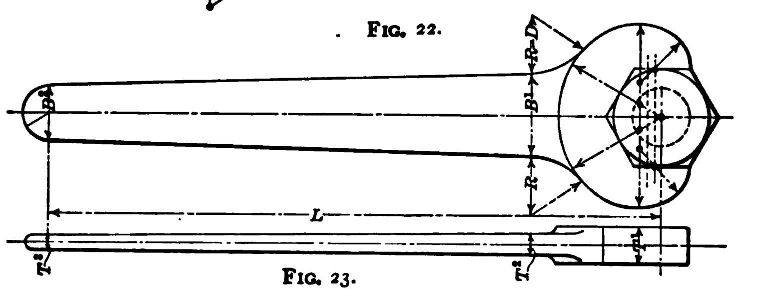


FIG. 23.

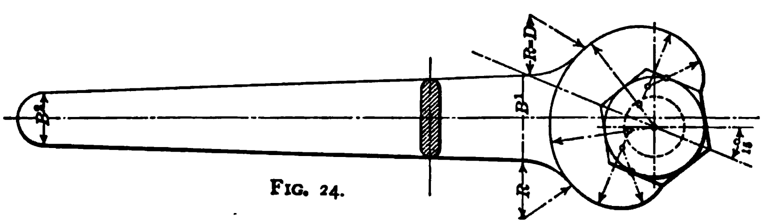


FIG. 24.

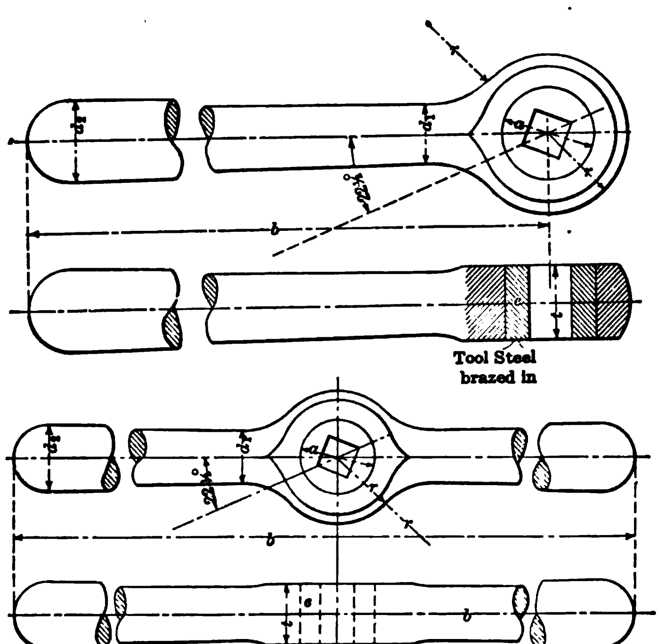
FIGS. 22 to 24.—Dimensions of forged wrenches.

Suitable clearances between punches and dies for accurate work are given in Table 34 by E. DEAN (*Amer. Mach., May* 4, 1905). The table relates to the blanking, perforating and forming of flat stock in the power press for parts of adding machines, cash registers, typewriters, etc.

In this class of work it is generally desired to make two different kinds of cuts with the dies used. First, to leave the outside of the blank of a semi-smooth finish, with sharp corners, free from burrs and with the least amount of rounding on the cutting side. Second, to leave the holes and slots that are perforated in the parts as smooth and straight as possible, and true to size. The table is the result of three years' experimenting on this class of work, and has stood the test of three years of use since it was compiled and it has worked out to the entire satisfaction of those who have used it.

The die always governs the size of the work passing through it. The punch governs the size of the work that it passes through. In blanking work the die is made to the size of the work wanted and the punch smaller. In perforating work the punch is made to size of work wanted and the die larger than the punch. The clearance between the die and punch governs the results obtained.

Fig. 29 shows the application of the table in determining the clearance for blanking or perforating hard rolled steel .060 in. thick. The clearance given in the table for this thickness of metal is .0042, and the sketch shows that for blanking to exactly 1 in. diameter this amount is deducted from the diameter of the punch, while for perforating the same amount is added to the diameter of the die. For a sliding fit make punch and die .00025 to .0005 in. larger; and for a driving fit make punch and die .0005 to .0015 in. smaller.



Diameter of tap $= D$
$b = 24D$ for double ended wrench
$b = 16D$ for single ended wrench
$t = D$ $\quad d_1 = D - \frac{1}{16}$in. $\quad d_2 = D + \frac{1}{16}$in.
$r = D + \frac{1}{16}$ in. $\quad a = D + \frac{1}{8}$ in.

FIG. 25.—Tap wrenches.

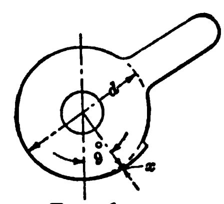


FIG. 26.

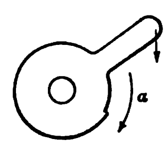


FIG. 27.
Correct Way

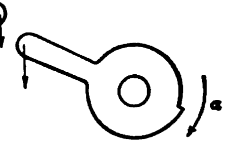


FIG. 28.
Wrong Way

FIGS. 26 to 28.—Self-locking fixture cams.

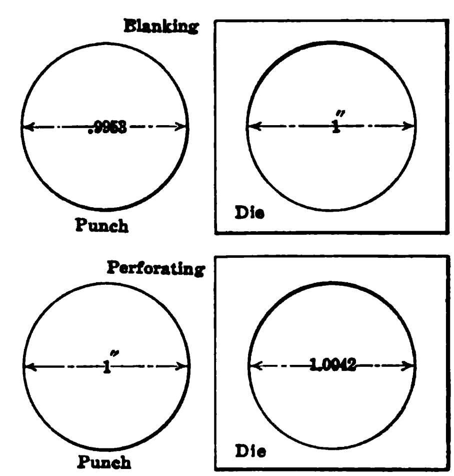
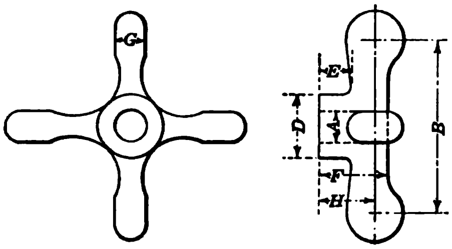


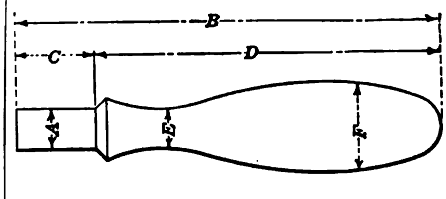
FIG. 29.—Location of allowance in blanking and punching.

Table 18.—Malleable Iron Cross Knobs



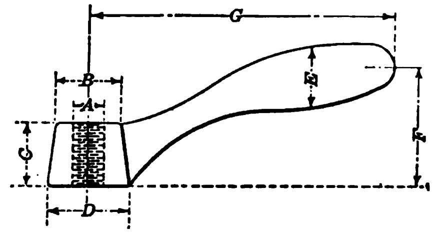
A	B	C	D	E	F	G	H
1/4″	1 5/32″	5/32″	1/2″	5/16″	9/16″	1/4″	7/16″
5/16″	1 11/16″	7/32″	9/16″	5/16″	9/16″	5/16″	1/2″
3/8″	2″	5/16″	11/16″	3/8″	3/4″	3/8″	5/8″
7/16″	2 5/16″	11/32″	3/4″	5/16″	3/4″	3/8″	9/16″
1/2″	2 7/16″	3/8″	7/8″	3/8″	7/8″	3/8″	5/8″

Table 19.—Handles—Straight



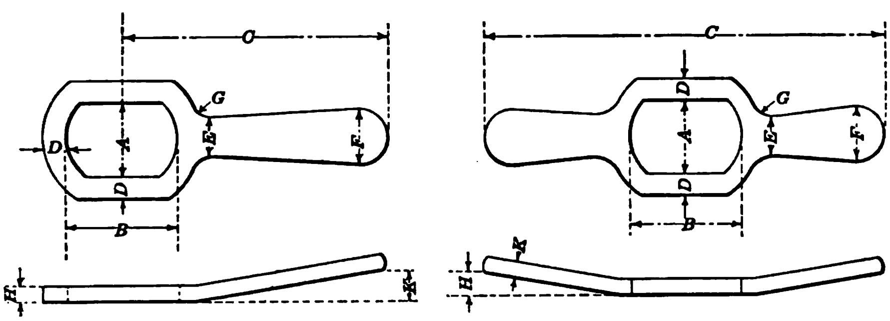
A		B	C	D	E	F
Rough diam.	Finish diam.					
7/16″	3/8″	2 9/16″	19/32″	1 21/32″	3/8″	7/8″
1/2″	7/16″	3 1/2″	3/4″	2 3/4″	15/32″	1″
1/2″	7/16″	3 3/4″	3/4″	3″	15/32″	1 1/8″
1/2″	7/16″	4 1/2″	7/8″	3 5/8″	17/32″	1 1/4″
9/16″	1/2″	5″	1″	4″	17/32″	1 5/16″
11/16″	5/8″	5 3/4″	1″	4 3/4″	21/32″	1 1/2″

Table 20.—Handles—Bent



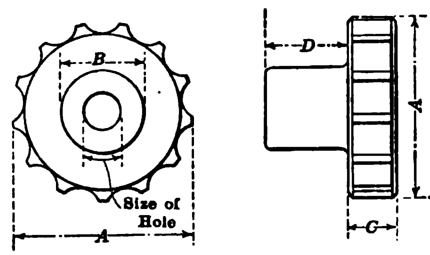
A	B	C	D	E	F	G
1/4″ –24	1/2″	1/2″	5/8″	1/2″	15/16″	2 3/8″
5/16″–18	1/2″	1/2″	5/8″	1/2″	15/16″	2 3/8″
3/8″ –16	1/2″	1/2″	5/8″	1/2″	15/16″	2 3/8″
7/16″–14	3/4″	3/4″	1″	11/16″	1 1/16″	3 3/4″
1/2″ –13	3/4″	3/4″	1″	11/16″	1 1/16″	3 3/4″
9/16″–12	1″	7/8″	1 1/4″	3/4″	1 1/4″	4 1/2″
5/8″ –11	1″	7/8″	1 1/4″	3/4″	1 1/4″	4 1/2″
3/4″ –10	1″	7/8″	1 1/4″	3/4″	1 1/4″	4 1/2″

Table 21.—Single and Double Handled Wrenches



Single handle									Double handle								
A	B	C	D	E	F	G	H	K	A	B	C	D	E	F	G	H	K
7/8″	1 3/8″	3 1/2″	5/16″	1/2″	3/4″	1/2″	1/4″	1/2″	2″	2 1/2″	9″	7/16″	7/8″	1″	1/2″	1/2″	3/8″
1″	1 1/2″	3 1/2″	5/16″	1/2″	3/4″	1/2″	1/4″	1/2″	2 1/4″	2 3/4″	9″	7/16″	7/8″	1″	1/2″	1/2″	3/8″
1 1/4″	1 3/4″	4″	5/16″	5/8″	7/8″	1/2″	1/4″	1/2″	2 3/4″	3 1/4″	10″	1/2″	7/8″	1″	1/2″	1/2″	3/8″
1 5/8″	2 1/8″	4 1/2″	3/8″	3/4″	1″	1/2″	1/4″	1/2″	3″	3 1/2″	10″	5/8″	7/8″	1″	1/2″	1/2″	3/8″

Table 22.—Malleable Iron Knobs



Size of hole	A	B	C	D
1/4″	1 1/2″	3/4″	3/8″	1/2″
5/16″	1 1/2″	3/4″	3/8″	1/2″
3/8″	1 1/2″	3/4″	3/8″	1/2″
7/16″	1 1/2″	3/4″	3/8″	1/2″
1/2″	2″	1 1/16″	1/2″	5/8″
9/16″	2″	1 1/16″	1/2″	1/2″
5/8″	2″	1 1/16″	1/2″	1/2″
3/4″	2 1/2″	1 1/4″	5/8″	5/8″
7/8″	2 1/2″	1 1/4″	5/8″	5/8″

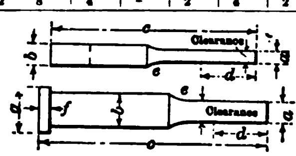


Table 23.—Two Standard Punches

Dimensions in inches

a				b	c	d	e	f	g	Clearance
3/8	5/16	9/32	1/4	1/2	2 1/2	5/8	1 1/2			.005
1/4	7/32	3/16	5/32	3/8	2 1/2	5/8	1 1/2	1/8	1/2	.003
9/32	7/32	1/4	3/16	3/8	2 1/2	5/8	1 1/2			.003
5/32				3/8	2 1/2	5/8	1 1/2			.003
1/8				1/4	2 1/4	5/8	1 1/2			.002
5/8				11/16	3 3/4	1	1 1/2			.005
3/8				11/16	3 3/4	1	1 1/2			.005
13/32				11/16	3 1/2	1	1 1/2			.005
3/8				11/16	3 1/2	1	1 1/2			.005
5/16	11/32	3/8	13/32	1/2	2 1/2	5/8	1 1/2	1/8	5/8	.003
7/16				1/2	2 1/2	5/8	1 1/2	1/8	5/8	.003
3/16 to 7/16				1/2	3	5/8	1 1/2			.003

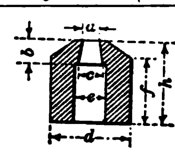
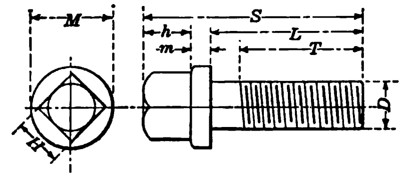


Table 24.—Round Dies

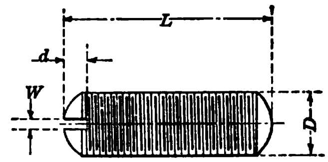
Dimensions in inches

a				h	d	f	e	b	c
1/8	5/32	3/16	7/32	3/4	3/4	5/8	a+1/32	1/4	3°
1/4	9/32			3/4	3/4	5/8	a+1/32	1/4	3°
3/32	1/8	5/32	3/16	7/8	7/8	3/4	a+1/32	1/4	3°
7/32	1/4	9/32	5/16	7/8	7/8	3/4	a+1/32	1/4	3°
11/32	3/8	13/32	7/16	1	1	7/8	a+1/16	3/8	3°
15/32	1/2			1	1	7/8	a+1/16	3/8	3°
17/32	9/16	19/32	5/8	1	1 1/4	7/8	a+3/32	1/2	3°
21/32	11/16			1	1 1/4	7/8	a+1/8	1/2	3°
23/32	3/4	25/32	13/16	1	1 1/2	7/8	a+1/8	1/2	3°
27/32	7/8			1	1 1/2	7/8	a+1/8	1/2	3°



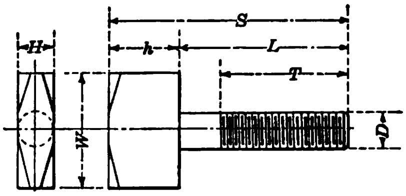
D	Thrd.	L	T	h	m	H	M	S
$\frac{1}{4}$	Shop Standard	1	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{3}{32}$	$\frac{1}{4}$	$\frac{7}{16}$	$1\frac{11}{32}$
$\frac{5}{16}$		1	$\frac{3}{4}$	$\frac{5}{16}$	$\frac{1}{8}$	$\frac{5}{16}$	$\frac{1}{2}$	$1\frac{7}{16}$
$\frac{3}{8}$		$1\frac{1}{2}$	1	$\frac{3}{8}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{5}{8}$	2
$\frac{7}{16}$		2	$1\frac{1}{2}$	$\frac{7}{16}$	$\frac{3}{16}$	$\frac{7}{16}$	$\frac{11}{16}$	$2\frac{5}{8}$
$\frac{1}{2}$		2	$1\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{16}$	$\frac{1}{2}$	$\frac{13}{16}$	$2\frac{11}{16}$

Table 25. Collar Head Jig Screws



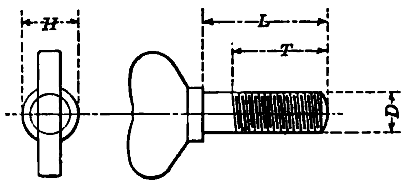
D	Thrd.	L	W	d
$\frac{3}{16}$	Shop Standard	$\frac{1}{2}$	.032	$\frac{1}{16}$
$\frac{1}{4}$		$\frac{3}{4}$	.040	$\frac{3}{32}$
$\frac{5}{16}$		1	.057	$\frac{3}{32}$
$\frac{3}{8}$		1	$\frac{1}{16}$	$\frac{1}{8}$
$\frac{7}{16}$		$1\frac{1}{2}$	$\frac{5}{64}$	$\frac{1}{8}$
$\frac{1}{2}$		$1\frac{1}{2}$	$\frac{5}{64}$	$\frac{1}{8}$

Table 26. Headless Jig Screws



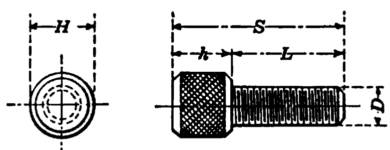
D	Thrd.	H	h	L	S	T	W
$\frac{5}{16}$	Shop Standard	$\frac{5}{16}$	$\frac{5}{8}$	$1\frac{1}{2}$	$2\frac{1}{8}$	$1\frac{1}{8}$	$1\frac{1}{16}$
$\frac{3}{8}$		$\frac{3}{8}$	$\frac{11}{16}$	$1\frac{5}{8}$	$2\frac{5}{16}$	$1\frac{1}{8}$	$1\frac{3}{16}$
$\frac{7}{16}$		$\frac{7}{16}$	$\frac{3}{4}$	$1\frac{7}{8}$	$2\frac{5}{8}$	$1\frac{1}{4}$	$1\frac{5}{16}$
$\frac{1}{2}$		$\frac{1}{2}$	$\frac{3}{4}$	$1\frac{7}{8}$	$2\frac{5}{8}$	$1\frac{1}{4}$	$1\frac{5}{16}$

Table 27. Locking Jig Screws



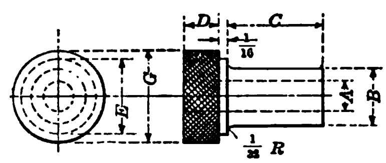
D	Thrd.	H	L	T
$\frac{1}{4}$	Shop Standard	$\frac{3}{8}$	1	$\frac{3}{4}$
$\frac{5}{16}$		$\frac{7}{16}$	1	$\frac{3}{4}$
$\frac{3}{8}$		$\frac{1}{2}$	$1\frac{1}{2}$	1

Table 28. Winged Jig Screws



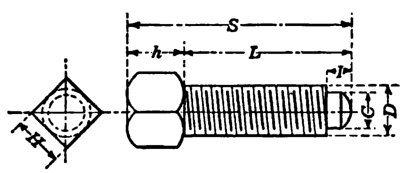
D	Thrd.	L	h	S	H
$\frac{1}{4}$	Shop Standard	$\frac{3}{4}$	$\frac{3}{8}$	$1\frac{1}{8}$	$\frac{1}{2}$
$\frac{5}{16}$		1	$\frac{1}{2}$	$1\frac{1}{2}$	$\frac{9}{16}$
$\frac{3}{8}$		1	$\frac{9}{16}$	$1\frac{9}{16}$	$\frac{5}{8}$

Table 29. Nurled Head Jig Screws



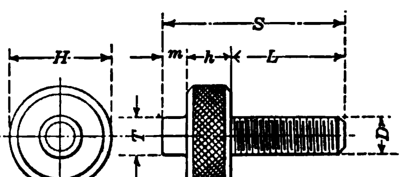
A	B	C	D	E	G
No. 52	$\frac{1}{4}$	$\frac{9}{16}$	$\frac{1}{4}$	$\frac{1}{16}$	$\frac{9}{16}$
No. 30	$\frac{5}{16}$	$\frac{5}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{5}{8}$
No. 12	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{5}{16}$	$\frac{9}{16}$	$\frac{11}{16}$
$\frac{1}{4}$	$\frac{1}{2}$	$\frac{11}{16}$	$\frac{5}{16}$	$\frac{11}{16}$	$\frac{13}{16}$
$\frac{5}{16}$	$\frac{9}{16}$	$\frac{3}{4}$	$\frac{5}{16}$	$\frac{3}{4}$	$\frac{7}{8}$
$\frac{3}{8}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{3}{8}$	$\frac{13}{16}$	$\frac{15}{16}$
$\frac{7}{16}$	$\frac{11}{16}$	$\frac{13}{16}$	$\frac{3}{8}$	$\frac{7}{8}$	1
$\frac{1}{2}$	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{7}{16}$	$\frac{13}{16}$	$1\frac{1}{16}$
$\frac{9}{16}$	$\frac{13}{16}$	$\frac{7}{8}$	$\frac{7}{16}$	1	$1\frac{1}{8}$
$\frac{5}{8}$	$\frac{7}{8}$	$\frac{15}{16}$	$\frac{1}{2}$	$1\frac{1}{16}$	$1\frac{3}{16}$
$\frac{11}{16}$	$\frac{15}{16}$	1	$\frac{1}{2}$	$1\frac{1}{8}$	$1\frac{1}{4}$
$\frac{3}{4}$	$1\frac{1}{16}$	1	$\frac{9}{16}$	$1\frac{1}{4}$	$1\frac{7}{16}$
$\frac{13}{16}$	$1\frac{1}{8}$	$1\frac{1}{16}$	$\frac{9}{16}$	$1\frac{5}{16}$	$1\frac{1}{2}$
$\frac{7}{8}$	$1\frac{1}{4}$	$1\frac{1}{8}$	$\frac{5}{8}$	$1\frac{7}{16}$	$1\frac{5}{8}$
$\frac{15}{16}$	$1\frac{5}{16}$	$1\frac{3}{16}$	$\frac{5}{8}$	$1\frac{1}{2}$	$1\frac{11}{16}$
1	$1\frac{3}{8}$	$1\frac{1}{4}$	$\frac{11}{16}$	$1\frac{9}{16}$	$1\frac{3}{4}$

Table 32. Loose Bushings for Jigs



D	Thrd.	L	h	I	C	H	S
$\frac{1}{4}$	Shop Standard	1	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{5}{32}$	$\frac{1}{4}$	$1\frac{1}{4}$
$\frac{5}{16}$		1	$\frac{5}{16}$	$\frac{1}{8}$	$\frac{13}{64}$	$\frac{5}{16}$	$1\frac{5}{16}$
$\frac{3}{8}$		$1\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{8}$	$\frac{17}{64}$	$\frac{3}{8}$	$1\frac{7}{8}$
$\frac{7}{16}$		$1\frac{1}{2}$	$\frac{7}{16}$	$\frac{3}{16}$	$\frac{5}{16}$	$\frac{7}{16}$	$1\frac{15}{16}$
$\frac{1}{2}$		$1\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{16}$	$\frac{11}{32}$	$\frac{1}{2}$	2

Table 30. Square Head Jig Screws



D	Thrd.	L	h	m	S	H	T
$\frac{1}{4}$	Shop Standard	$\frac{3}{4}$	$\frac{5}{16}$	$\frac{3}{16}$	$1\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{4}$
$\frac{5}{16}$		1	$\frac{3}{8}$	$\frac{7}{32}$	$1\frac{19}{32}$	$\frac{7}{8}$	$\frac{5}{16}$
$\frac{3}{8}$		1	$\frac{7}{16}$	$\frac{1}{4}$	$1\frac{11}{16}$	1	$\frac{3}{8}$

Table 31. Nurled Head Jig Screws

Standard Punches, Dies and Punch Holders

BY A. C. CLAIRE (*Amer. Mach., July 4, 1912*)

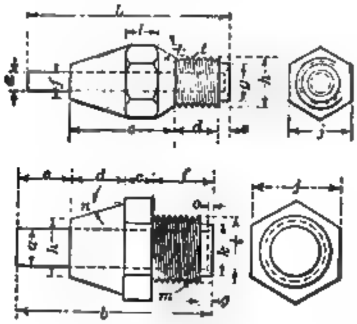


TABLE 33.—PUNCH HOLDERS

Dimensions in inches

a	b	c	d	e	f	g	h	j	k	l	+
3/32 to 3/16	2 1/2	1 3/16	1/2	1/4	1/2	1/2	5/8		60°	1/4	16
	2 1/2	1 5/16	1/2	3/8	3/8	1/2	5/8	3/4	60°	1/4	16

a	b	c	d	e	f	g	h	j	k	l	m	n	o
3/32 to 3/16	2 1/4		11/16	3/8	13/32	1/4	3/8	1/4		1/2	20		
3/16 to 3/8	2 1/2		11/16	3/8	19/32	1/4	5/16	1/4		3/8	20		
1/8 to 3/16	2 1/2		11/16	3/8	3/4	1/4	3/8	1 1/8		3/8	10		
3/32 to 3/16	3		5/8	3/8	13/16	1/4	3/8			1/2	20		
3/4	3	11/16	15/16	1/2	1 1/8	1/4	1 1/4	1 1/2		1 1/4	10		
1	3	11/16	15/16	3/8	1 1/8	1/4	1 1/2	1 3/4		1 1/2	10		

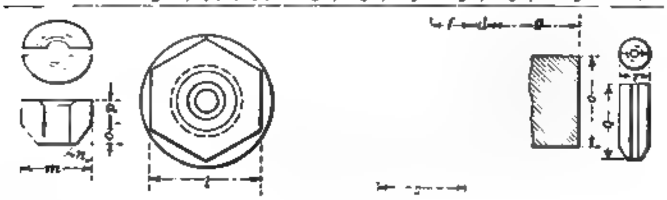


TABLE 35.—HOLDER FOR LARGE PUNCHESS

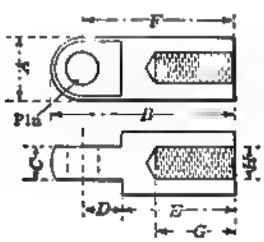
Dimensions in inches

a	b	c	d	e	f	g	h	j	k	l	m	n	o	p	q	r	s	+	u
	1 1/2	1 3/16	3/4	1	3/8	2	3/4	1/2	3/16	1 1/4	1/2	30°	3/32	3/32	1/2	3/16	3/4″ 12 thr'd	3/4	1/2

TABLE 34.—CLEARANCES FOR PUNCH AND DIE FOR DIFFERENT THICKNESSES AND MATERIALS

Thickness of stock, in.	Clearance for brass and soft steel, in.	Clearance for medium rolled steel, in.	Clearance for hard rolled steel, in.
010	.0005	.0006	.0007
.020	.001	.0012	.0014
.030	.0015	.0018	.0021
.040	.002	.0024	.0028
.050	.0025	.003	.0035
.060	003	.0036	0042
.070	0035	.0042	.0049
.080	004	.0048	.0056
.090	0045	.0054	.0063
.100	.005	.006	.007
.110	.0055	.0066	.0077
.120	.006	.0072	.0084
.130	.0065	.0078	.0091
.140	007	.0084	0098
.150	0075	.009	.0105
.160	.008	.0096	.0112
.170	.0085	.0102	.0119
.180	.009	.0108	.0126
.190	0095	.0114	.0133
.200	010	.012	.014

Miscellaneous Small Parts



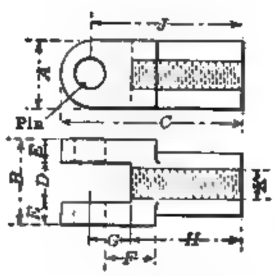


TABLE 36.—SMALL KNUCKLE JOINTS

Dimensions in inches

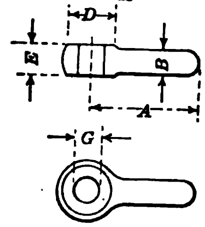
Pin	A	B	C	D	E	F	G	H
1/8	[illegible]	3/8	5/32	3/16	7/16	5/8	3/8	5-40
5/32	3/16	13/32	3/16	7/32	15/32	11/16	1/2	8-32
3/16	3/8	1 3/16	[illegible]	1/4	3/4	1	5/8	10-32
1/4	1/2	1 7/16	5/16	5/16	7/8	1 3/16	3/4	1/4-20
5/16	5/8	1 11/16	3/8	3/8	1	1 3/8	7/8	5/16
3/8	3/4	2 1/8	7/16	1/2	1 1/4	1 5/8	1 1/16	3/8
7/16	7/8	2 7/16	1/2	9/16	1 7/16	2	1 1/4	7/16
1/2	1	2 3/4	9/16	5/8	1 5/8	2 1/4	1 3/8	1/2
5/8	1 1/4		3/4	13/16	1 13/16	2 5/8	1 5/8	5/8
3/4	1 1/2		7/8	15/16	2 1/16	3	1 3/4	3/4
7/8	1 3/4		1	1 1/16	2 7/16	3 1/2	2 1/8	7/8
1	2 1/8		1 1/8	1 5/16	2 13/16	4	2 1/2	1

Pin	A	B	C	D	E	F	G	H	J	K
[illegible]	1/4	3/16	3/4	3/32	5/64	9/32	3/16	7/16	5/8	5-40
5/32	5/16	3/8	29/32	[illegible]	3/32	3/8	7/32	17/32	13/16	8-32
3/16	3/8	1/2	1 3/16	[illegible]	1/8	15/32	1/4	5/8	1	10-32
[illegible]	1/2	5/8	1 7/16	3/16	5/32	1/2	5/16	3/4	1 3/16	1/4-20
5/16	5/8	3/4	1 11/16	3/8	3/16	5/8	3/8	1	1 3/8	5/16
3/8	3/4	7/8	2 1/8	7/16	7/32	3/4	1/2	1 1/4	1 1/2	3/8
7/16	7/8	1	2 7/16	1/2	1/4	7/8	9/16	1 7/16	2	7/16
1/2	1	1 1/8	2 3/4	9/16	9/32	15/16	5/8	1 5/8	2 1/4	1/2
5/8	1 3/8	1 3/8		3/4	7/16	1 3/16	13/16	1 13/16	2 3/4	5/8
[illegible]	1 5/8	1 5/8		7/8	1/2	1 1/2	15/16	2 1/16	3	[illegible]
7/8	1 7/8	2 1/8		1	9/16	1 13/16	1 1/16	2 7/16	3 1/2	7/8
1	2 1/8	2 3/8		1 1/8	[illegible]	1 7/8	1 3/16	2 13/16	4	

Table 37.—S. A. E. Yoke and Eye Rod End Standards

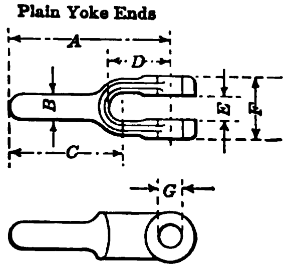
Dimensions in inches

Rod Ends



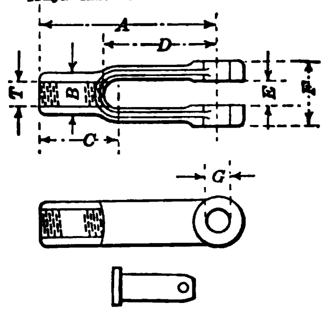
A	B	D	E	G
1 1/4	3/16	3/8	3/16	3/16
1 1/4	1/4	1/2	9/32	1/4
1 3/8	5/16	19/32	11/32	5/16
1 1/2	3/8	11/16	7/16	3/8
1 1/2	7/16	13/16	1/2	7/16
1 3/4	1/2	15/16	9/16	1/2

Plain Yoke Ends



A	B	C	D	E	F	G
1 1/4	3/16	7/8	7/16	3/16	7/16	3/16
1 3/4	1/4	1 1/4	5/8	9/32	5/8	1/4
2	5/16	1 3/8	3/4	11/32	3/4	5/16
2 1/8	3/8	1 7/16	27/32	7/16	7/8	3/8
2 1/4	7/16	1 1/2	1	1/2	1	7/16
2 1/2	1/2	1 5/8	1 1/8	5/16	1 1/8	1/2

Adjustable Yoke Ends

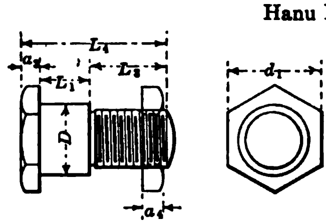


A	B	C	D	E	F	G	T
1 9/16	5/16	11/16	1	3/16	7/16	3/16	3/16 32 U. S.
2	7/16	7/8	1 1/4	9/32	5/8	1/4	1/4 28 ALAM
2 1/4	1/2	1	1 7/16	11/32	3/4	5/16	5/16 24 ALAM
2 1/2	5/8	1 1/8	1 5/8	7/16	7/8	3/8	3/8 24 ALAM
2 7/8	23/32	1 1/4	1 7/8	1/2	1	7/16	7/16 20 ALAM
3	13/16	1 7/16	1 7/8	9/16	1 1/8	1/2	1/2 20 ALAM

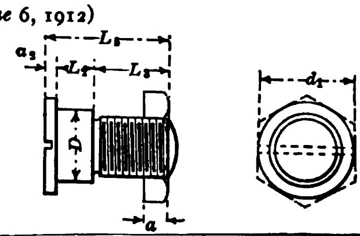
Table 38.—Round and Hexagon Head Studs for Cam Rolls, Levers, Etc.

Dimensions in inches

Hanu Engineering Co., (*Amer. Mach.*, *June* 6, 1912)



Nom. size	L_4	a_3	L_1	D	L_3	d_1	a_4	Thread
1/4	1 1/8	3/16	0.377	0.249	9/16	13/32	3/16	1/4 ×24
5/16	1 3/16	3/16	0.408	0.311	5/8	1/2	3/16	1/4 ×24
3/8	1 15/64	3/16	0.439	0.374	21/32	9/16	3/16	5/16 ×18
7/16	1 21/64	3/16	0.470	0.436	23/32	5/8	7/32	3/8 ×16
1/2	1 7/16	3/16	0.502	0.499	3/4	23/32	7/32	7/16 ×14
5/8	1 5/8	3/16	0.564	0.623	27/32	7/8	1/4	1/2 ×13
3/4	1 25/32	7/32	0.627	0.748	15/16	1	1/4	9/16 ×12
7/8	1 63/64	7/32	0.690	0.873	1 1/16	1 5/32	5/16	5/8 ×12(11)
1	2 1/8	1/4	0.752	0.998	1 1/8	1 5/16	5/16	3/4 ×12(10)
1 1/4	2 15/32	9/32	0.877	1.248	1 5/16	1 5/8	3/8	1 ×12 (8)
1 1/2	2 3/4	5/16	1.002	1.498	1 7/16	1 7/8	3/8	1 1/4 ×12 (7)
1 3/4	3 3/32	11/32	1.127	1.748	1 5/8	2 3/16	7/16	1 1/2 ×12 (6)
2	3 7/16	3/8	1.152	1.998	1 13/16	2 1/2	1/2	1 3/4 ×12 (5)



Nom. size	L_5	a_2	L_2	D	L_3	d_1	a_4	Thread
1/4	61/64	5/64	0.282	0.249	9/16	13/32	3/16	1/4 ×24
5/16	1 1/64	5/64	0.313	0.311	5/8	1/2	3/16	1/4 ×24
3/8	1 5/64	3/32	0.330	0.374	21/32	9/16	3/16	5/16 ×18
7/16	1 11/64	3/32	0.360	0.436	23/32	5/8	7/32	3/8 ×16
1/2	1 15/64	7/64	0.376	0.499	3/4	23/32	7/32	7/16 ×14
5/8	1 25/64	1/8	0.423	0.623	27/32	7/8	1/4	1/2 ×13
3/4	1 35/64	9/64	0.470	0.748	15/16	1	1/4	9/16 ×12
7/8	1 47/64	5/32	0.518	0.873	1 1/16	1 5/32	5/16	5/8 ×12(11)
1	1 55/64	11/64	0.564	0.998	1 1/8	1 5/16	5/16	3/4 ×12(10)
1 1/4	2 11/64	13/64	0.658	1.248	1 5/16	1 5/8	3/8	1 ×12 (8)
1 1/2	2 27/64	15/64	0.752	1.498	1 7/16	1 7/8	3/8	1 1/4 ×12 (7)
1 3/4	2 47/64	17/64	0.845	1.748	1 5/8	2 3/16	7/16	1 1/2 ×12 (6)
2	3 3/64	19/64	1.189	1.998	2 1/16	2 1/2	1/2	1 3/4 ×12 (5)

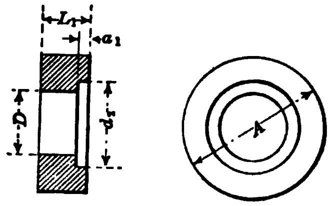


Table 39.—Cam Rollers
Dimensions in inches

Nom. size	D	L_1	a_1	d_2	A
1/4	1/4	3/8	3/32	7/16	7/8
5/16	5/16	13/32	3/32	17/32	1
3/8	3/8	7/16	7/64	19/32	1 1/8
7/16	7/16	15/32	7/64	21/32	1 1/4
1/2	1/2	1/2	1/8	3/4	1 3/8
5/8	5/8	9/16	9/64	29/32	1 5/8
3/4	3/4	5/8	5/32	1 1/32	1 7/8
7/8	7/8	11/16	11/64	1 3/16	2 1/8
1	1	3/4	3/16	1 11/32	2 3/8
1 1/4	1 1/4	7/8	7/32	1 21/32	2 7/8
1 1/2	1 1/2	1	1/4	1 29/32	3 3/8
1 3/4	1 3/4	1 1/8	9/32	2 7/32	3 7/8
2	2	1 1/4	5/16	2 17/32	4 3/8

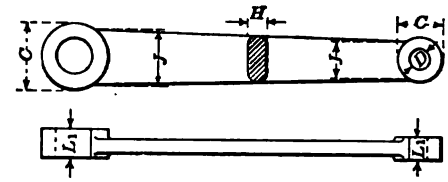


Table 40.—Cast-iron Rock Arms
Dimensions in inches

Nom. size	D	C	L_1	Tap	H	J
1/4	1/4	5/8	3/8	1/4 ×24	5/16	1/2
5/16	5/16	3/4	13/32	1/4 ×24	5/16	5/8
3/8	3/8	7/8	7/16	5/16 ×18	5/16	11/16
7/16	7/16	1	15/32	3/8 ×16	5/16	13/16
1/2	1/2	1 1/16	1/2	7/16 ×14	5/16	7/8
5/8	5/8	1 5/16	9/16	1/2 ×13	11/32	1
3/4	3/4	1 1/2	5/8	9/16 ×12	3/8	1 3/16
7/8	7/8	1 3/4	11/16	5/8 ×12(11)	13/32	1 3/8
1	1	2	3/4	3/4 ×12(10)	7/16	1 9/16
1 1/4	1 1/4	2 3/8	7/8	1 ×12 (8)	1/2	1 7/8
1 1/2	1 1/2	2 7/8	1	1 1/4 ×12 (7)	1/2	2 1/4
1 3/4	1 3/4	3 1/4	1 1/8	1 1/2 ×12 (6)	9/16	2 5/8
2	2	3 3/4	1 1/4	1 3/4 ×12 (5)	5/8	3

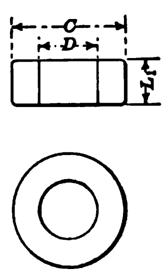


Table 41.—Collars
Dimensions in inches

Nom. size	D	C	L_1	Taper pin No.	Set screw
1/4	1/4	5/8	3/8		
5/16	5/16	3/4	13/32		
3/8	3/8	7/8	7/16		
7/16	7/16	1	15/32		
1/2	1/2	1 1/16	1/2	0–1 1/16	
5/8	5/8	1 5/16	9/16	2–1 5/16	
3/4	3/4	1 1/2	5/8	3–1 1/2	1/4 ×20
7/8	7/8	1 3/4	11/16	4–1 3/4	1/4 ×20
1	1	2	3/4	5–2	5/16 ×18
1 1/4	1 1/4	2 3/8	7/8	6–2 3/8	5/16 ×18 3/8 ×16
1 1/2	1 1/2	2 7/8	1	7–2 7/8	
1 3/4	1 3/4	3 1/4	1 1/8	8–3 1/4	1/2 ×12
2	2	3 3/4	1 1/4	8–3 3/4	1/2 ×12

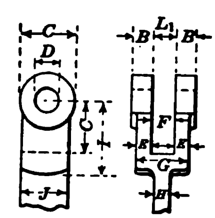


Table 42.—Cast-iron Yoke Ends
Dimensions in inches

Nom. size	L_1	B	C	D	E	F	G	H	I	J
1/4	3/8	5/16	5/8	1/4	1/4	3/8	7/8	5/16	15/16	1/2
5/16	13/32	21/64	3/4	5/16	1/4	13/32	29/32	5/16	1 1/16	5/8
3/8	7/16	11/32	7/8	3/8	1/4	7/16	15/16	5/16	1 3/16	11/16
7/16	15/32	23/64	1	7/16	1/4	15/32	31/32	5/16	1 3/8	13/16
1/2	1/2	3/8	1 1/16	1/2	1/4	1/2	1	5/16	1 7/16	7/8
5/8	9/16	13/32	1 5/16	5/8	1/4	23/32	1 7/32	11/32	1 3/4	1
3/4	5/8	7/16	1 1/2	3/4	9/32	13/16	1 11/32	3/8	1 7/8	1 5/16
7/8	11/16	15/32	1 3/4	7/8	5/16	7/8	1 15/32	13/32	2 3/16	1 3/8
1	3/4	1/2	2	1	5/16	15/16	1 9/16	7/16	2 1/2	1 9/16
1 1/4	7/8	9/16	2 3/8	1 1/4	11/32	1 1/16	1 25/32	1/2	2 7/8	1 7/8
1 1/2	1	5/8	2 7/8	1 1/2	3/8	1 1/4	2	1/2	3 3/8	2 1/4
1 3/4	1 1/8	11/16	3 1/4	1 3/4	13/32	1 3/8	2 7/32	9/16	3 13/16	2 5/8
2	1 1/4	3/4	3 3/4	2	7/16	1 9/16	2 7/16	5/8	4 1/4	3

By the Hann Engineering Co. (*Amer. Mach., June* 6, 1912)

The most satisfactory plug cock known to the author is the Westinghouse construction, Fig. 30, which, almost from the beginning, has been a standard feature of air brake equipment. In view of its entire success there and the universally recognized defects of the common construction, it deserves general adoption. The handle is placed on the small end of the plug, pressure on the large end holding the parts in place and automatically taking up wear. The taper is 2 ins. per ft. measured on the diameter. For frequent service (rock drill throttle valves) the author has used the blunter angle of $3\frac{1}{2}$ ins. per ft. measured on the diameter in order to obtain greater ease of movement. In air brake service security against movement is of course essential. Note that there must be a hole through the end of the plug to admit pressure to the large end, or the plug will not seat itself.

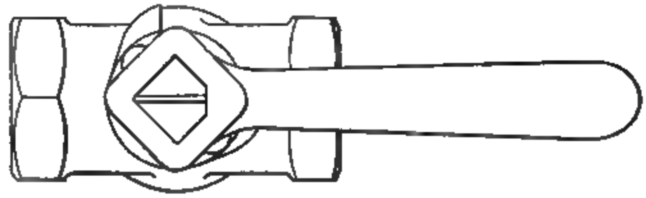


FIG. 30.—The Westinghouse plug cock.

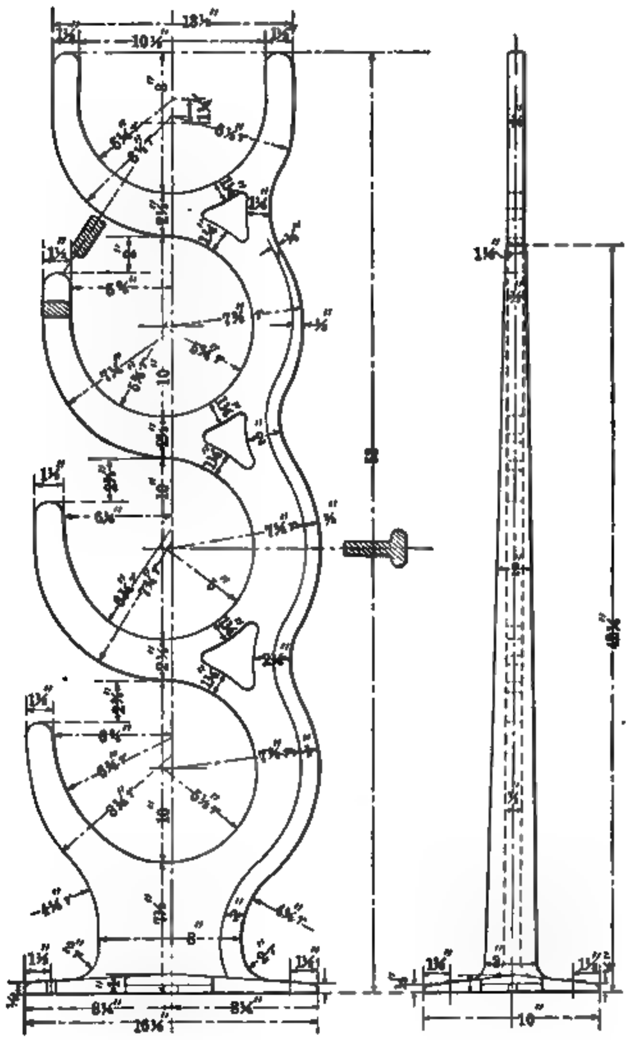
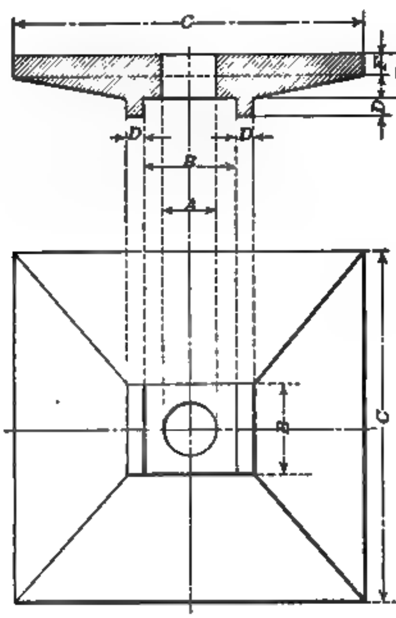


FIG. 31.—Rack for bar stock.



Depth of washer below top of foundation = 50× diameter of bolt. (The washers are usually square.)
A = diam. of bolt + $\frac{1}{8}$ in. (for $\frac{7}{8}$-in. bolts and smaller).
A = diam. of bolt + $\frac{1}{4}$ in. (for 1-in. to $2\frac{1}{4}$-in. bolts).
A = diam. of bolt + $\frac{3}{8}$ in. (for $2\frac{1}{2}$-in. and larger).
B = width of nut or head across flats + $\frac{1}{8}$ in. to $\frac{1}{4}$ in. (square nuts or heads usually used on lower ends of bolts).
C = 8 × diam. of bolt.
D = .375 × diam. of bolt (not over 1 in.).
E = .5 × diam. of bolt.

FIG. 32.—Foundation bolt washers.

PRESS AND RUNNING FITS

The tolerances and allowances suitable for running fits formed the subject of a report by the British Engineering Standards Committee, rendered in 1906. This report was based on an exhaustive investigation, nearly 800 pieces of work from 13 engineering workshops having been measured in order that the final recommendations might fairly represent commercial work. The recommendations of the Committee were presented in both tabular and graphic form, the latter reproduced, by permission, in Fig. 1.

¶ The Committee define *tolerance* as "a difference in dimensions prescribed in order to tolerate unavoidable imperfections of workmanship"; and *allowance* as "a difference in dimensions prescribed in order to allow of various qualities of fit."

The Report also says in part:

"For general engineering practice, the Committee have laid down three classes of workmanship, viz.: First class; second class; third class . . . . . For special cases in which a very high degree of accuracy is required, the Committee have laid down a class of workmanship having 'extra fine tolerances and allowances.' This class is carried up to 3 ins. in diameter and is intended for cases in which extreme accuracy is necessary.

"For running fits the Committee are of the opinion that, wherever possible, the shaft should be the element more nearly approaching the true dimension, and allowance be made on the hole according to the class of fit required. The tolerances on the shaft are negative in order that it may never exceed its true dimensions. . . . . Limit gages adapted to such a system may conveniently be referred to as applying to a 'Shaft Basis.' The reverse system may be termed a 'Hole Basis,' and allowance is then made on the shaft. . . . . .

"In those cases where it is found necessary to adopt the 'Hole Basis,' the tolerances specified for shafts and holes respectively may still be employed, and the standard allowance applied to the shaft instead of to the hole, the minimum diameter of the hole being accurately its nominal diameter."

The allowance suitable for press fits may be obtained from Fig. 2, which was deduced by T. C. Kelly from records of over 800 such fits made at the works of the Lane and Bodley Co. (*Amer. Mach., July* 20, 1899). The chart is for steel or iron shafts and cast-iron cranks. Steel cranks require much smaller allowances for the same pressures but for them no corresponding data are available.

To use the chart select the curve which gives the ratio of the radial thickness of the hub divided by the diameter of the plug. Below the point of intersection of the plug diameter line with the selected curve read pounds. Multiply this reading by the area of the fitted surface in square inches and by the number of thousandths of an inch allowed for the press fit. The result will be the pressure in pounds required to force the plug home.

Following is an example: Diameter of plug 8 ins., length of fit 6 ins., diameter of hub 16 ins., press fit allowance .020 in. Required the pressure to force the parts together.

$$\frac{\text{Radial thickness of hub} = 4 \text{ ins.}}{\text{Diameter of plug} = 8 \text{ ins.}} = .5$$

Finding a point on the 50 per cent. curve opposite 8 ins. and tracing downward, we find 53 lbs. Area of fitted surface $= 8 \times 3.1416 \times 6 = 150$ sq. ins., $53 \times 150 \times 20 = 158{,}000$ lbs.

$= 79$ tons.

The work from which the chart was deduced was of customary workmanship—that is, turned shafts and bored holes. For ground shafts and reamed holes much smaller allowances must be used—not over half those suitable for turned shafts and bored holes. Similarly, turned shafts in reamed holes or ground shafts in bored holes should have three-fourths the allowance suitable for turned shafts and bored holes. For the best results the shaft should be tapered one-half the allowance. Otherwise it scours out the hole most at the entering end which leads to the poorest grip at the shoulder where the best is needed.

The stresses in hubs due to press and shrink fits may be estimated by the use of Fig. 3, which is due to S. H. Weaver, supervisor of mechanical calculation of the General Electric Co., and is plotted from Professor Morley's formula (*Engineering, Aug.* 11, 1911). The use of the chart is explained below it. The shaded portions cover customary dimensions.

The taper press-fit system has obvious advantages over the customary straight-fit system. As used at the works of the Westinghouse Machine Co., a useful simplification has been effected by a slight modification of the taper, due to J. B. Thomas, chief inspector of the works (*Amer. Mach., Aug.* 4, 1904). The change in the taper is from $\frac{1}{16}$ or .0625 to .06 in. per ft., or .005 in. per in., and from $\frac{1}{8}$ or .125 to .120 in. per ft., or .01 in. per in. measured on the diameter. The result of the change in the smaller taper is seen in Table 1 of the diameters at each successive inch of length of a hole of 10 ins. diameter at the large end, the dimensions in the first column being carried to four places, while in the second they are carried to three, where they stop.

TABLE 1.—DIAMETERS AT EACH INCH OF LENGTH OF TAPER PRESS FITS

Taper $\frac{1}{16}$ in. per ft.	Taper .06 in. per ft.
10	10
9.9948	9.995
9.9896	9.990
9.9844	9.985
9.9792	9.980
9.9740	9.975
9.9688	9.970

The slight and otherwise unimportant change in the taper will be seen to lead to round figures for each inch of length, the figures in the second column being much more easily read from the micrometer than those in the first, while, by subtracting from the large diameter five times as many thousandths as the piece has inches of length, the small diameter is obtained directly, a result that, with the taper of $\frac{1}{16}$ in. per ft., can be found only by calculation.

Fig 4 shows a form of gage for large taper holes which Mr. Thomas prefers to full plugs. They are much lighter than full plugs and with them the holes can be gaged independently on different diameters and irregularities in the holes detected. Fig. 5 shows Mr. Thomas's method of gaging the largest holes, the angle of taper being greatly exaggerated. At the left of the hole is a carefully made strip of steel with an upturned end and a row of holes down the center, spaced 1 in. apart. The strip is used in connection with an inside micrometer, as shown. The measurements are not made perpendicular to the axis, but as indicated by the dotted arc. One end of the micrometer being located by a hole in the strip, the other is manipulated precisely as though the hole were straight, the frequent spacing of the holes in the strip permitting the hole to be gaged for uniformity of taper. The dimension thus gaged at the large end is the one given on the

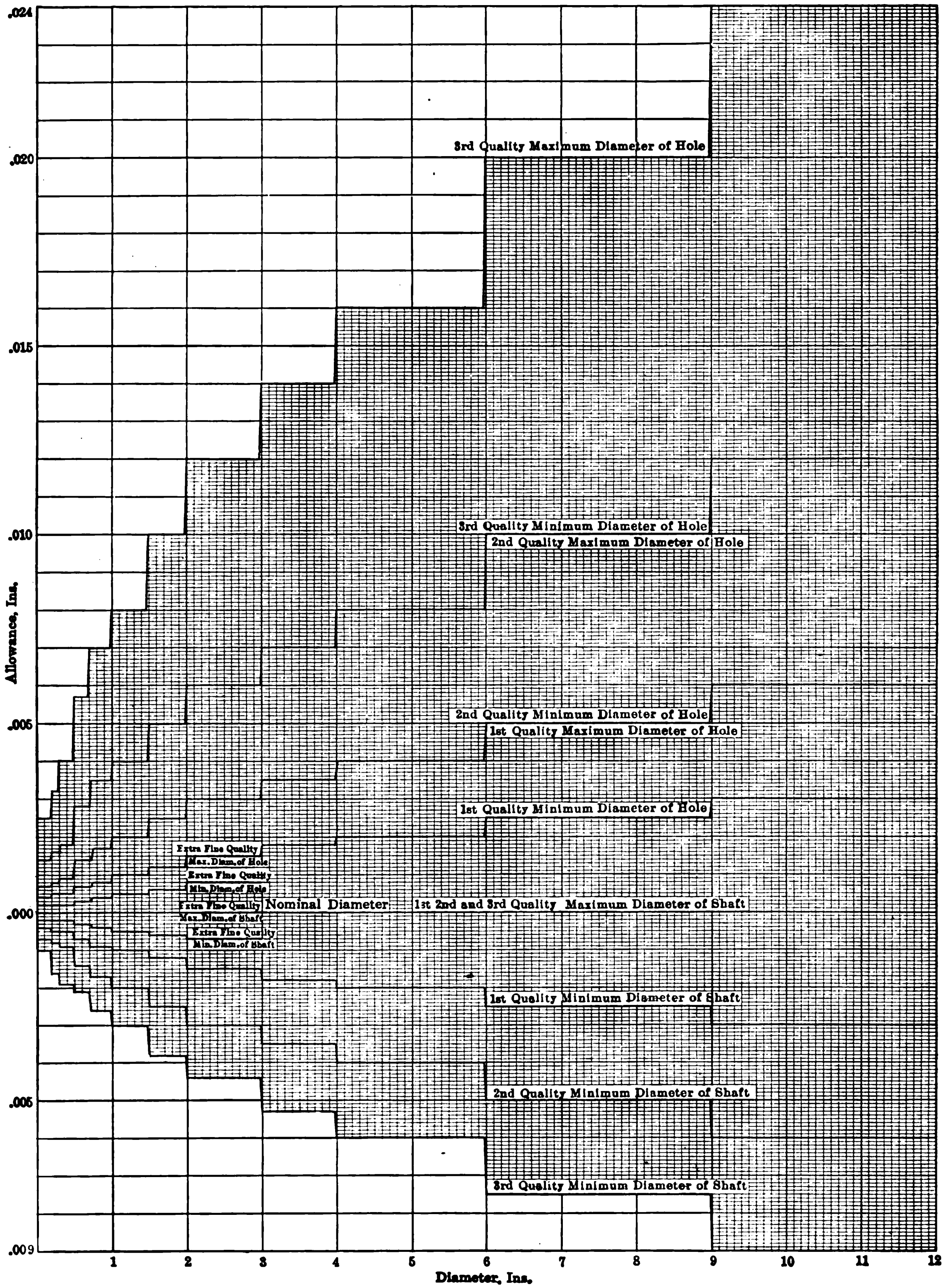


FIG. 1.—British standard tolerances and allowances for running fits.

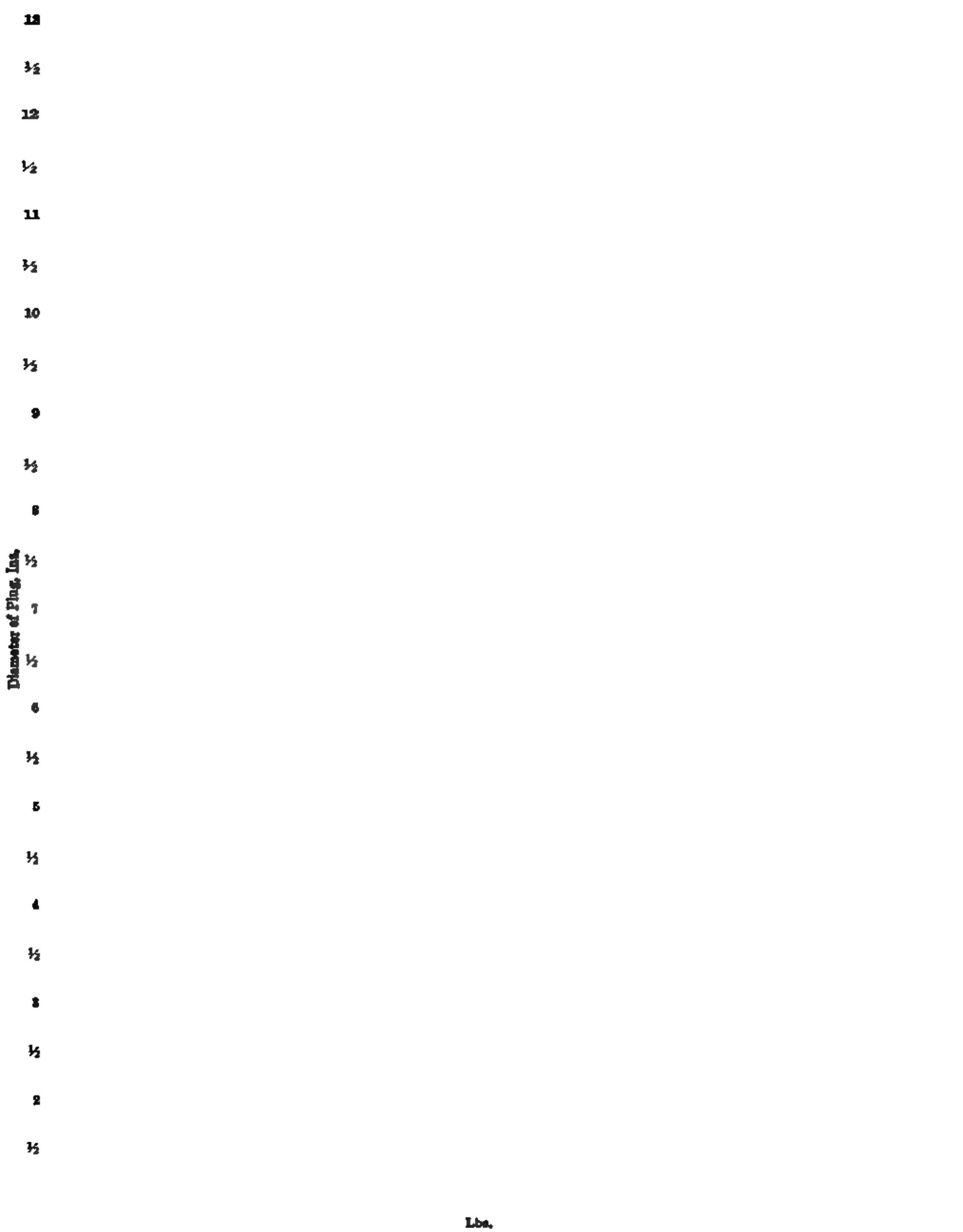


Fig. 2.—Allowances for press fits.

drawing as the true diameter—the microscopic difference between the dimension as called for and as made being of no importance.

The allowance for pressing home is .01 in. on all diameters from 10 to 30 ins.

The pressure required for pressing home is very variable, depending largely on the lubricant used. Thus, with fits of 22 ins. diameter, the pressure required was 139 tons with heavy white lead, 706 with castor oil, and 835 with engine oil, and, with fits of 30 ins. diameter, the tonnage, using heavy white lead, was 100 and, with thinner white lead, 420.

The standard Westinghouse lubricant is 1 lb. of white lead to 1½ pints of linseed oil, this being the mixture which was used with the heavier tonnage of the last case cited.

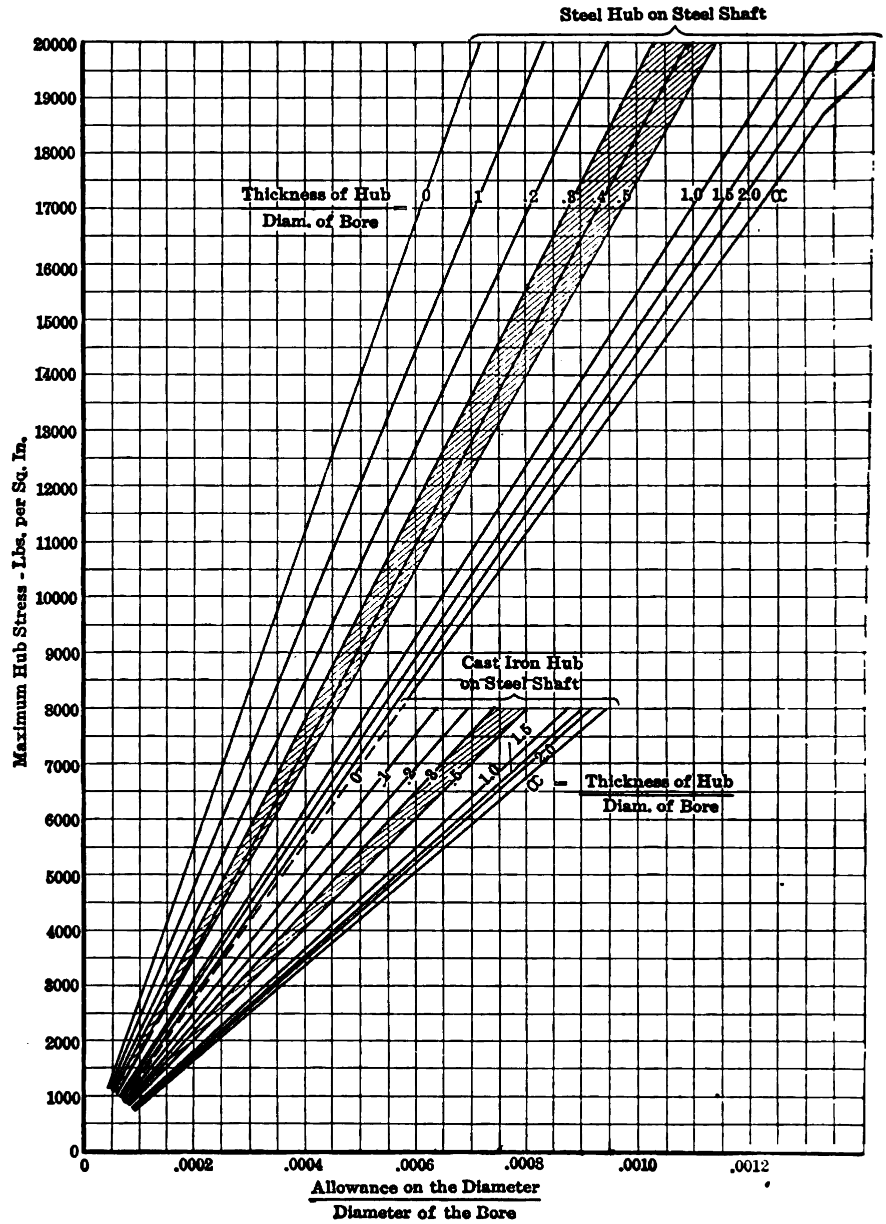
The lubricant is far more effective with taper than with straight fits, due to the surfaces being entirely covered with it when they make contact. It is to this that the great variation when using different lubricants is due.

One advantage of the taper fit is that the plug may be entered in its seat and the two compared directly, whereas, with parallel fits, the comparison can be made with gages only. Thus compared, the distance remaining for pressing home forms the best possible check on both pieces. Thus, with the taper of .005 in. per in. and an allowance of .01 in. for pressing, the plug should enter the hole within 2 ins. of home, or, more generally, the distance by which the parts should not go home, when they are assembled without pressure, should be 1 in. for each five thousandths of pressing allowance.

The slight taper does not in the slightest endanger the security of the work.

The practice of the General Electric Co. in allowances for sliding, press and shrink fits is given in Fig. 6, by JOHN RIDDELL (*Trans. A. S. M. E., Vol.* 24). Mr. Riddell said:

"There are many things to be taken into consideration in laying out these tables and diagrams: First, the relation of length of bore to diameter. In our case the length of hubs



Divide the press or shrink fit allowance by the diameter of the bore and find the ratio on the bottom scale. From it trace up to the line for $\frac{\text{radial thickness of hub}}{\text{diameter of bore}}$ and to the scale of the left where read the fiber stress in the hub.

FIG. 3.—Hub stress for shrink or press fits on solid steel shafts.

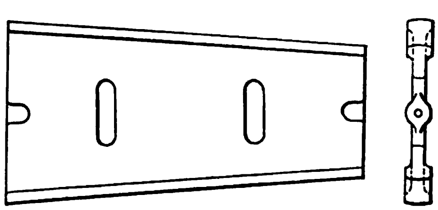


FIG. 4.

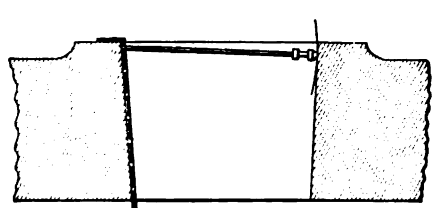


FIG. 5

FIGS. 4 and 5.—Gaging taper holes.

of armature spiders is sometimes several times the diameter; but the actual bearing surface is about equal in length to the diameter, on account of recesses in the hub. Second, the outside diameter of hub. My diagram is laid out on the basis of the hub being twice the diameter of the shaft. Third, the nature of the materials. Fourth, how and where the parts are to be assembled; if they are to be assembled where a suitable hydrostatic press is available, more allowance can be made than if the parts are to be put together by the use of bolts and straps.

"My diagram is based on actual experience extending over a number of years, and is eminently satisfactory.

"There are five curves shown as follows: The left-hand one on the minus side of o line shows allowances for sliding fits. I mean by this such fits as are not loose or free like a running fit, but one that will just slide without any perceptible play. The next curve is on the right or plus side of the o line and shows exactly the same allowances

for tight fits, for parts with light hubs, such as commutator shells, etc. The third curve gives somewhat greater allowances, and is used for steel hubs. The fourth is for our regular armature spiders having solid cast-iron hubs. The fifth shows the amount we have found to be correct for shrinkage fits, and for such heavy articles as couplings."

These allowances for press fits of armature spiders are for assembling in the field where equipment of limited capacity must sometimes be used. They are, therefore, smaller than the allowances that are customary in such work as engine cranks and crank pins.

The practice of the General Electric Co. in allowances and tolerances for journal fits is given in Table 2.

The practice of the Brown and Sharpe Mfg. Co. in allowances and tolerances for ground fits is given in Table 3, by W. A. VIALL (*Trans. A. S. M. E., Vol. 32*).

The practice of the Brown and Sharpe Mfg. Co. in allowances and tolerances for shafts rough turned preparatory for grinding, is given in Table 4 by W. A. VIALL (*Trans. A. S. M. E.*, Vol. 32).

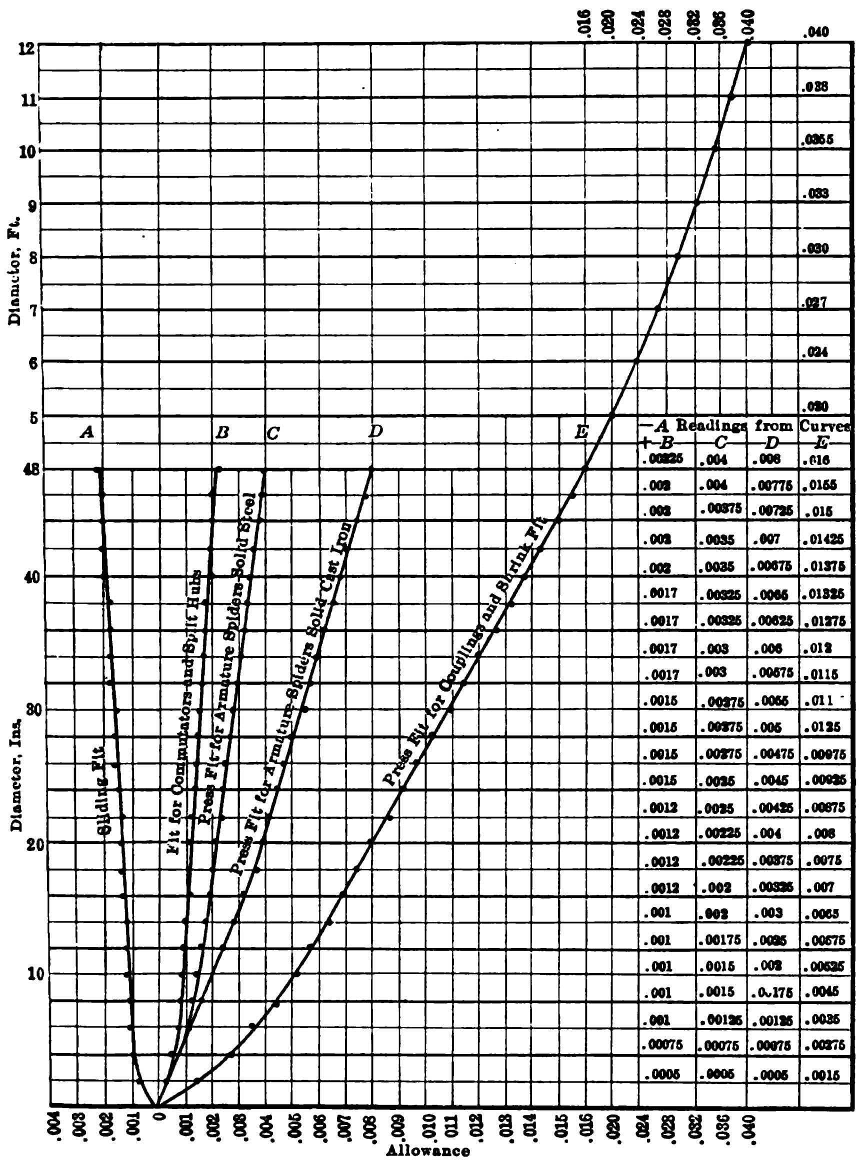
The practice of the C. W. Hunt Co. in allowances and tolerances for fits is given in Tables 5, 6 and 7 (*Amer. Mach., July* 16, 1903).

Table 5 gives all the particulars for press, drive and close or hand fits for parallel shafts ranging between 1 and 10 ins. in diameter. The holes for all parallel fits are made standard, except for the unavoidable variation due to the wear of the reamer, the variation from standard diameter for the various kinds of fits being made in the shaft. This variation is, however, not positive, but is made between limits of accuracy or tolerance. Taking the case of a press fit on a 2-in. shaft, for example, we find that the hole—that is, the reamer—is kept between the correct size and .002 in. below size, while the shaft must be between .002 and .003 in. over size. For a drive or hand fit the limits for the hole are the same as for a press fit, while the shaft in the former case must be between .001 and .002 large and in the latter between .001 and .002 small.

Table 6 gives in the same way the allowances for parallel running fits of three grades of closeness. The variations allowed in the holes are not materially different from those of the preceding table, but the shafts are, of course, below instead of above the nominal size.

Table 7 relates to a feature of the Hunt Company's practice, where the preferred practice with press fits is to make them taper, the taper used being the Hunt standard of $\frac{3}{16}$ in. in diameter for each foot in length. With fits of this character the usual practice is reversed, the variation in diameter being in the hole, while the shaft is kept to standard size. The holes are made with standard reamers, which are maintained at the standard taper, and in each case are sunk into the work to a point determined by Table 7 and defined by an adjustable stop gage, which abuts against a machined face on the work. A plug gage, shown in Fig. 7, is ground to the Hunt taper and to the exact diameter at the zero point A. It is also graduated at intervals of $\frac{1}{16}$ in. of its length as shown. A taper of $\frac{3}{16}$ in. per foot is, very closely, $\frac{1}{1000}$ in. for each $\frac{1}{16}$ in. of length, and each division on the scale thus represents very nearly $\frac{1}{1000}$ in. difference in diameter. One of these intervals is called a "P" and is so entered on the drawings.

All shafts for taper fits are turned to within plus or minus $\frac{1}{1000}$ in. of the nominal size at the large end of the taper. The taper reamer is then sunk in the hole to such a depth that the hole at the large end is small by an amount indicated by the table. Thus for a 2-in. press fit the plug gage must enter the hole to such a depth that its large end registers between the 6 P and 7 P mark, indicating that the hole is between $\frac{6}{1000}$ and $\frac{7}{1000}$ small. The parts are



−A +B	C	D	E
.00225	.004	.008	.016
.002	.004	.00775	.0155
.002	.00375	.00725	.015
.002	.0035	.007	.01425
.002	.0035	.00675	.01375
.0017	.00325	.0065	.01325
.0017	.00325	.00625	.01275
.0017	.003	.006	.012
.0017	.003	.00575	.0115
.0015	.00275	.0055	.011
.0015	.00275	.005	.0125
.0015	.00275	.00475	.00975
.0015	.0025	.0045	.00925
.0012	.0025	.00425	.00875
.0012	.00225	.004	.008
.0012	.00225	.00375	.0075
.0012	.002	.00325	.007
.001	.002	.003	.0065
.001	.00175	.0025	.00575
.001	.0015	.002	.00525
.001	.0015	.0.175	.0045
.001	.00125	.00125	.0035
.00075	.00075	.00075	.00275
.0005	.0005	.0005	.0015

FIG. 6.—The General Electric Co's practice in allowances for sliding, press and shrink fits.

TABLE 4.—BROWN & SHARPE MFG. CO.'S PRACTICE IN TOLERANCES AND ALLOWANCES FOR SHAFTS ROUGH TURNED PREPARATORY FOR GRINDING

Size	Not go on	Go on	Size	Not go on	Go on	Size	Not go on	Go on
	Ins.			Ins.			Ins.	
3/8	.383	.387	15/16	.9455	.9495	1 1/2	1.508	1.512
7/16	.4455	.4495	1	1.008	1.012	1 9/16	1.5705	1.5745
1/2	.508	.512	1 1/16	1.0705	1.0745	1 5/8	1.633	1.637
9/16	.5705	.5745	1 1/8	1.133	1.137	1 11/16	1.6955	1.6995
5/8	.633	.637	1 3/16	1.1955	1.1995	1 3/4	1.758	1.762
11/16	.6955	.6995	1 1/4	1.258	1.262	1 13/16	1.8205	1.8245
3/4	.758	.762	1 5/16	1.3205	1.3245	1 7/8	1.883	1.887
13/16	.8205	.8245	1 3/8	1.383	1.387	1 15/16	1.9455	1.9495
7/8	.883	.887	1 7/16	1.4455	1.4495	2	2.008	2.012

TABLE 2.—GENERAL ELECTRIC CO.'S PRACTICE IN ALLOWANCES AND TOLERANCES FOR JOURNAL FITS

Nominal dimension, ins.	Journal		Bearings: Horizontal		Bearings: Vertical		Bearings: Step		Axle linings for ry. motors	
	Max. diam., ins.	Allowable variation below max diameter	Min. bore, ins.	Allowable variation above min. bore	Min. bore, ins.	Allowable variation above min. bore	Min. bore, ins.	Allowable variation above min. bore	Min. bore, ins.	Allowable variation above min. bore
⅜	.375	.0005	.377	.001	.376	.001	.3755	.0005	.380	.004
½	.500	.0005	.502	.001	.501	.001	.5005	.0005	.505	.004
⅝	.625	.0005	.627	.001	.626	.001	.6255	.0005	.630	.004
¾	.750	.0005	.752	.001	.751	.001	.7505	.0005	.755	.004
⅞	.875	.0005	.877	.001	.876	.001	.8755	.0005	.880	.004
1	1.000	.0005	1.002	.001	1.001	.001	1.0005	.0005	1.005	.004
1⅛	1.125	.0005	1.128	.001	1.127	.001	1.126	.0005	1.130	.004
1¼	1.250	.0005	1.253	.001	1.252	.001	1.251	.0005	1.255	.004
1½	1.500	.0005	1.503	.001	1.502	.001	1.501	.0005	1.505	.004
1¾	1.750	.0005	1.753	.001	1.752	.001	1.751	.0005	1.755	.004
2	2.000	.0005	2.003	.001	2.002	.001	2.001	.0005	2.005	.004
2¼	2.250	.0005	2.253	.001	2.252	.001	2.251	.0005	2.255	.004
2½	2.500	.0005	2.503	.001	2.503	.001	2.501	.0005	2.505	.004
2¾	2.750	.0005	2.754	.002	2.753	.002	2.7515	.0005	2.755	.004
3	3.000	.0005	3.004	.002	3.003	.002	3.0015	.0005	3.005	.004
3½	3.500	.001	3.504	.002	3.504	.002	3.5015	.0005	3.507	.004
4	4.000	.001	4.005	.002	4.004	.002	4.002	.001	4.007	.004
4½	4.500	.001	4.505	.002	4.504	.002	4.502	.001	4.509	.004
5	5.000	.001	5.006	.002	5.005	.002	5.0025	.001	5.009	.004
5½	5.500	.001	5.507	.002	5.505	.002	5.503	.001	5.511	.004
6	6.000	.001	6.009	.002	6.005	.002	6.003	.001	6.011	.004
7	7.000	.001	7.011	.002	7.006	.002	7.0035	.001	7.012	.004
8	8.000	.001	8.012	.003	8.006	.003	8.004	.002	8.013	.004
9	9.000	.001	9.013	.004	9.007	.004	9.0045	.002	.....	.....
10	10.000	.0015	10.014	.005	10.007	.005	10.005	.002	.....	.....
11	11.000	.0015	11.015	.005	11.008	.005	11.0055	.002	.....	.....
12	12.000	.0015	12.016	.005	12.008	.005	12.006	.002	.....	.....
13	13.000	.0015	13.016	.005	13.009	.005	13.0065	.002	.....	.....
14	14.000	.0015	14.016	.005	14.009	.005	14.007	.002	.....	.....
15	15.000	.0015	15.016	.005	15.010	.005	15.0075	.002	.....	.....
16	16.000	.0015	16.016	.005	16.010	.005	16.008	.002	.....	.....
17	17.000	.0015	17.018	.005	17.011	.005	17.008	.002	.....	.....
18	18.000	.0015	18.018	.005	18.011	.005	18.008	.002	.....	.....
19	19.000	.0015	19.018	.005	19.012	.005	19.008	.002	.....	.....
20	20.000	.0015	20.018	.005	20.012	.005	20.008	.002	.....	.....
21	21.000	.002	21.018	.005	21.013	.005	21.008	.002	.....	.....
22	22.000	.002	22.020	.008	22.013	.005	22.008	.002	.....	.....
23	23.000	.002	23.020	.008	23.013	.005	23.008	.002	.....	.....
24	24.000	.002	24.020	.008	24.013	.005	24.008	.002	.....	.....
25	25.000	.003	25.020	.008	.....	.....	.....	.....	.....	.....
26	26.000	.003	26.020	.008	.....	.....	.....	.....	.....	.....
27	27.000	.003	27.022	.008	.....	.....	.....	.....	.....	.....
28	28.000	.003	28.022	.008	.....	.....	.....	.....	.....	.....
29	29.000	.003	29.022	.008	.....	.....	.....	.....	.....	.....
30	30.000	.003	30.022	.008	.....	.....	.....	.....	.....	.....
31	31.000	.003	31.022	.008	.....	.....	.....	.....	.....	.....
32	32.000	.003	32.024	.010	.....	.....	.....	.....	.....	.....
33	33.000	.003	33.024	.010	.....	.....	.....	.....	.....	.....
34	34.000	.003	34.024	.010	.....	.....	.....	.....	.....	.....
35	35.000	.003	35.024	.010	.....	.....	.....	.....	.....	.....
36	36.000	.003	36.024	.010	.....	.....	.....	.....	.....	.....

TABLE 3.—BROWN AND SHARPE MFG. CO.'S PRACTICE IN ALLOWANCES AND TOLERANCES FOR FITS

RUNNING FITS—ORDINARY SPEED

To ½-in. diameter, inc.	.00025 to .00075	Small
To 1 -in. diameter, inc.	.00075 to .0015	Small
To 2 -in. diameter, inc.	.0015 to .0025	Small
To 3½-in. diameter, inc.	.0025 to .0035	Small
To 6 -in. diameter, inc.	.0035 to .005	Small

RUNNING FITS—HIGH SPEED, HEAVY PRESSURE AND ROCKER SHAFTS

To ½-in. diameter, inc.	.0005 to .001	Small
To 1 -in. diameter, inc.	.001 to .002	Small
To 2 -in. diameter, inc.	.002 to .003	Small
To 3½-in. diameter, inc.	.003 to .0045	Small
To 6 -in. diameter, inc.	.0045 to .0065	Small

SLIDING FITS

To ½-in. diameter, inc.	.00025 to .0005	Small
To 1 -in. diameter, inc.	.0005 to .001	Small
To 2 -in. diameter, inc.	.001 to .002	Small
To 3½-in. diameter, inc.	.002 to .0035	Small
To 6 -in. diameter, inc.	.003 to .005	Small

STANDARD FITS

To ½-in. diameter, inc.	Standard to .00025	Small
To 1 -in. diameter, inc.	Standard to .0005	Small
To 2 -in. diameter, inc.	Standard to .001	Small
To 3½-in. diameter, inc.	Standard to .0015	Small
To 6 -in. diameter, inc.	Standard to .002	Small

DRIVING FITS—FOR SUCH PIECES AS ARE REQUIRED TO BE READILY TAKEN APART

To ½-in. diameter, inc.	Standard to .00025	Large
To 1 -in. diameter, inc.	.00025 to .0005	Large
To 2 -in. diameter, inc.	.0005 to .00075	Large
To 3½-in. diameter, inc.	.00075 to .001	Large
To 6 -in. diameter, inc.	.001 to .0015	Large

DRIVING FITS

To ½-in. diameter, inc.	.0005 to .001	Large
To 1 -in. diameter, inc.	.001 to .002	Large
To 2 -in. diameter, inc.	.002 to .003	Large
To 3½-in. diameter, inc.	.003 to .004	Large
To 6 -in. diameter, inc.	.004 to .005	Large

FORCING FITS

To ½-in. diameter, inc.	.00075 to .0015	Large
To 1 -in. diameter, inc.	.0015 to .0025	Large
To 2 -in. diameter, inc.	.0025 to .004	Large
To 3½-in. diameter, inc.	.004 to .006	Large
To 6 -in. diameter, inc.	.006 to .009	Large

SHRINKING FITS—FOR PIECES TO TAKE HARDENED SHELLS ¼ IN. THICK AND LESS

To ½-in. diameter, inc.	.00025 to .0005	Large
To 1 -in. diameter, inc.	.0005 to .001	Large
To 2 -in. diameter, inc.	.001 to .0015	Large
To 3½-in. diameter, inc.	.0015 to .002	Large
To 6 -in. diameter, inc.	.002 to .003	Large

SHRINKING FITS—FOR PIECES TO TAKE SHELLS, ETC., HAVING A THICKNESS OF MORE THAN ¼ IN.

To ½-in. diameter, inc.	.0005 to .001	Large
To 1 -in. diameter, inc.	.001 to .0025	Large
To 2 -in. diameter, inc.	.0025 to .0035	Large
To 3½-in. diameter, inc.	.0035 to .005	Large
To 6 -in. diameter, inc.	.005 to .007	Large

GRINDING LIMITS FOR HOLES

To ½-in. diameter, inc.	Standard to .0005	Large
To 1 -in. diameter, inc.	Standard to .00075	Large
To 2 -in. diameter, inc.	Standard to .001	Large
To 3½-in. diameter, inc.	Standard to .0015	Large
To 6 -in. diameter, inc.	Standard to .002	Large
To 12-in. diameter, inc.	Standard to .0025	Large

The limits given in the table can be recommended for use in the manufacture of machine parts to produce satisfactory commercial work. These limits should be followed under ordinary conditions. Special cases should always be considered, as it may be desirable to vary slightly from the tables.

C. W. Hunt Co.'s Practice in Allowances and Tolerances

Table 5.—Limits to Diameters of Parallel Shafts and Bushings (Shafts Changing)

Diameters		1 in.	2 ins.	3 ins.	4 ins.	5 ins.	6 ins.	7 ins.	8 ins.	9 ins.	10 ins.	Formula
Press fit	Shaft	+.001 +.002	+.002 +.003	+.003 +.004	+.004 +.005	+.005 +.006	+.006 +.007	+.007 +.008	+.008 +.009	+.009 +.010	+.010 +.011	+(.001 d+.000) +(.001 d+.001)
Drive fit	Shaft	+.0005 +.0015	+.001 +.002	+.0015 +.0025	+.002 +.003	+.0025 +.0035	+.003 +.004	+.0035 +.0045	+.004 +.005	+.0045 +.0055	+.005 +.006	+(.0005 d+.000) +(.0005 d+.001)
Hand fit	Shaft	−.001 −.002	−.001 −.002	−.001 −.002	−.002 −.003	−.002 −.003	−.002 −.003	−.003 −.004	−.003 −.004	−.003 −.004	−.003 −.004	....
All fits	Hole	+.000 −.002	+.000 −.002	+.000 −.002	+.000 −.003	+.000 −.003	+.000 −.003	+.000 −.004	+.000 −.004	+.000 −.004	+.000 −.004	....

Table 6.—Limits to Diameters of Parallel Journals and Bearings (Journals Changing)

Diameters		1 in.	2 ins.	3 ins.	4 ins.	5 ins.	6 ins.	7 ins.	8 ins.	9 ins.	10 ins.	Formula
Close fit	Shaft	−.003 −.005	−.004 −.006	−.005 −.007	−.006 −.008	−.007 −.009	−.008 −.010	−.009 −.011	−.010 −.012	−.011 −.013	−.012 −.014	−(.001 d+.002) −(.001 d+.004)
Free fit	Shaft	−.008 −.011	−.009 −.012	−.010 −.013	−.011 −.014	−.012 −.015	−.013 −.016	−.014 −.017	−.015 −.018	−.016 −.019	−.017 −.020	−(.001 d+.007) −(.001 d+.010)
Loose fit	Shaft	−.023 −.028	−.026 −.031	−.029 −.034	−.032 −.037	−.035 −.040	−.038 −.043	−.041 −.044	−.044 −.049	−.047 −.052	−.050 −.055	−(.003 d+.020) −(.003 d+.025)
All fits	Hole	+.000 −.002	+.000 −.002	+.000 −.002	+.000 −.002	+.000 −.003	+.000 −.003	+.000 −.003	+.000 −.004	+.000 −.004	+.000 −.004	....

Table 7.—Limits to Diameters of Taper Shafts and Bushings (Holes Changing)

Diameters		1 in.	2 ins.	3 ins.	4 ins.	5 ins.	6 ins.	7 ins.	8 ins.	9 ins.	10 ins.	Formula
Press fit	Hole	−5 P −6 P	−6 P −7 P	−7 P −8 P	−8 P −9 P	−9 P −0 P	−10 P −11 P	−11 P −12 P	−12 P −13 P	−13 P −14 P	−14 P −15 P	−(Pd+4 P) −(Pd+5 P)
Drive fit	Hole	−½ P −1 P	−1 P −2 P	−1 P −2½ P	−2 P −3 P	−2½ P −3½ P	−3 P −4 P	−3½ P −4½ P	−4 P −5 P	−4½ P −5½ P	−5 P −6 P	−(½ Pd+0) −(½ Pd+P)
Hand fit	Hole	+0 −1 P	+0 −1 P	+0 −1 P	+0 −1 P	+0 −1 P	+0 −1 P	+0 −1 P	+0 −1 P	+0 −1 P	+0 −1 P	+0 −P

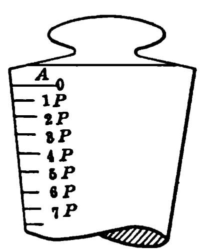


Fig. 7.—C. W. Hunt Co.'s gage for taper press fits.

then pressed together until the true sizes match—that is, in the case of the 2-in. fit, the parts would be pressed between $\frac{6}{16}$ and $\frac{7}{16}$ in.

In case the shafts and wheels thus fitted are not driving members, no key is used, the grip of the press fit being found to be all sufficient. In case they are driving members, the shaft is keyseated for one or more Woodruff keys, the key being placed in position before the parts are pressed together and being entirely hidden work the work is done.

In all cases the tables apply to steel shafts and cast-iron wheels or other members. In the right-hand columns of the tables the formulas from which the allowances are calculated are given, and from which the range of tables may be extended.

BALANCING MACHINE PARTS

Balancing Rotating Parts

Two states of perfect balance must be distinguished—standing or static and running or dynamic balance. Standing balance insures running balance in the case of thin disks but not in the case of long drums, multiple-throw crank shafts or similar pieces.

The method of obtaining standing balance by means of a rolling mandrel supported on a pair of straight-edges is too well known to need description. It is adequate for many cases but is not sufficiently delicate for high speeds.

The importance of accurate balance in high speed machinery is shown by the fact that a weight of 1 oz. rotating at 3600 r.p.m. at 1 ft. radius produces an unbalanced centrifugal force of 276 lbs.

Greater sensitiveness than that of the common parallels is characteristic of the fixture shown in Fig. 1, by the L. A. Goodnow Foundry Co. (*Amer. Mach., June* 16, 1910) by whom it is used for balancing fly-wheels. It consists of two large, turned cast-iron cones, slightly truncated, through which passes an eye bolt having a pivot point projecting downward within the eye, and a large link threaded through the eye, having a bearing for this pivot joint.

The turned fly-wheel is supported in a horizontal position, held by the two cones entering the bore from either side. Because of the point of suspension at the top, the fly-wheel is free and can take any position, depending upon whether it is in balance or out of balance. If it is out of balance, that fact is easily determined by a spirit level on the edge of the rim balanced by an equal weight opposite. Weights are then applied to bring it into a truly horizontal position. After this has been done the weights are weighed and a line is scratched on the inside of the rim indicating the point where weight should be applied and its amount.

Another standing balance apparatus of high sensitiveness is shown in Fig. 2, by P. Fenaux (*Amer. Mach., July* 30, 1908). Although giving standing balance only, it appears to be adequate for *small* drum-shaped pieces revolving at high speed and it was, in fact, designed for the small armatures of electrically driven phonographs.

The apparatus consists of a base *A* with two supports *B* for the axle *C*. The supports are of hardened tool steel and of such shape that the knife-edges of *C* bear on two points only. The balancing part is formed of two flanges *D* connected by *C* and the counterweight *E*, of such weight that it will balance the armature of the smallest weight. The armature is placed in the notches of the flanges. The pin *F* is used for noting the position when balancing. One of the flanges is lengthened by a rod, threaded and ended by a point. This point comes in front of an index fixed on the plate *A*. Toward the end, on each side of the rod *G* comes a small rubber stop *H*. The upper one is fixed on a rather stiff spring *K*; the lower one on a spring *L* supported by a long spring *M*. On the rod are screwed three nuts.

The centers of the armature and of *D* are above the edges of the knives, thus bringing the center of gravity of the system above the points of support, so that the smallest difference on one side or the other will produce a large movement of the point of *G*. The armature to be balanced is put in the notches with a slot in line with the top of the pin *F*. The lower stop is lowered and the point is brought in line with the index by moving *N*, one of the nuts. Then the armature is moved half a turn. If in this new position the point has a tendency to go under the index, that means that the side of the armature next the pin is too light, and the side first placed there is to be drilled. Leaving the nut *N* as it is, one of the two others is moved, so as to bring the point again in line with the index, and this movement indicates, by comparison, to what depth the hole or holes must be drilled. If instead of coming down the point had pushed against the spring *K*, the reverse operation would have been performed.

The principle of Fig. 1 has been developed by the Westinghouse Machine Co. into the highly sensitive and accurate apparatus (patented) shown in Figs. 3–10 (*Amer. Mach., July* 13, 1911).

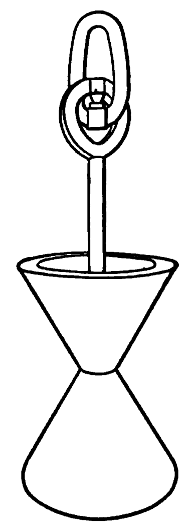


Fig. 1.—A sensitive standing balance fixture.

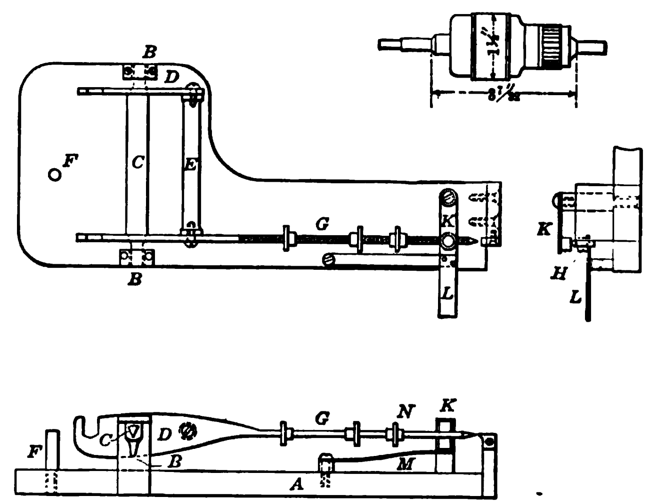


Fig. 2.—A sensitve standing balance apparatus.

This machine is used for giving a running balance to the rotors of Parsons steam turbines, its application for this purpose being due to the fact that if a long drum be divided into elementary disks by planes perpendicular to the axis and each slice be given a standing balance, the drum made up of the assembled disks will be in both standing and running balance. The rotor of the Westinghouse turbine is so divided, the disks being sufficiently thin to give an entirely satisfactory result. Theorectically, the customary balancing

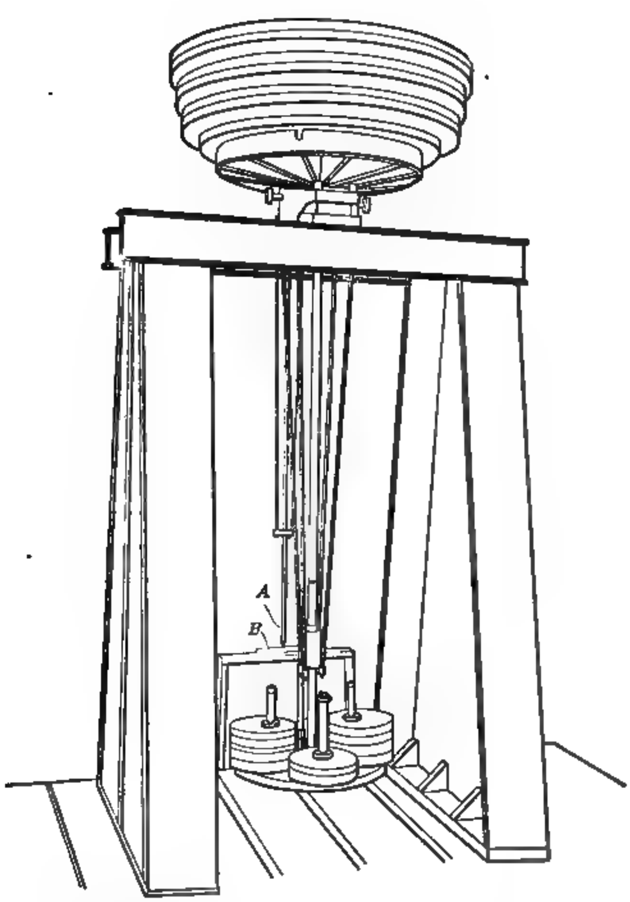


FIG. 3.—The Westinghouse balancing machine.

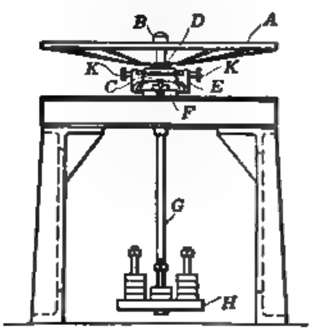


FIG. 4.

FIG. 5.

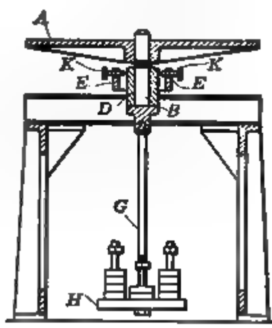


FIG. 6.

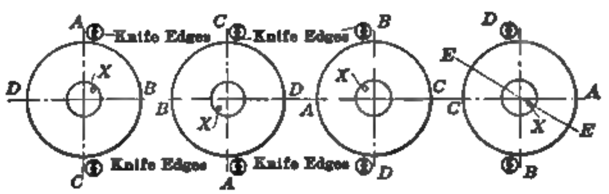


FIG. 7. FIG. 8. FIG. 9. FIG. 10.

FIGS. 4 to 10.—Details of the Westinghouse balancing machine and of the method of balancing

straight-edges would give the same result, but actually they are not sufficiently sensitive.

Fig. 3 shows a disk section in process of testing for standing balance. The balancing machine consists of a turntable A, Figs. 4, 5 and 6, so mounted as to rotate on spindle B in socket D. Socket D is supported by flanges resting upon open beam E and, while it closely fits the opening in E on one axis, it may be slid along the axis at right angles across E, by means of the two adjusting screws K, as most clearly shown in Fig. 6. Beam E rests upon knife-edges C, which in turn rest in self-aligning sockets in blocks F; these latter rest upon the main supports. H is a counterweight on rod G which is rigid with D.

The operation of the machine is shown in Figs. 7 to 10 inclusive. Assuming that the turntable and all the parts connected with it are first properly balanced, so that the upper surface of the turntable will remain accurately in a horizontal plane during a complete rotation, the disk to be balanced is placed upon the turntable, most carefully centered with reference to the spindle and properly clamped in position. Four points, A, B, C and D, are located upon the periphery of the disk at 90 deg. from each other, at the same radius from the center of the spindle, and the hanging counterweight is so adjusted that the combined apparatus located upon the knife-edges will oscillate very slowly, indicating that the center of gravity of the combined mass be just below the plane of the knife-edges. The spindle socket is now moved along the beam by means of the adjusting screws until the beam is balanced in the horizontal position. This will bring the point X, which indicates the position of a vertical line passing through the center of gravity, into the vertical plane in which the knife-edges are located, as in Fig. 7.

Next, the turntable is turned 180 deg., so as to bring the point X into the position represented in Fig. 8, and thus out of alignment with the knife-edges, in which position the beam will be deflected from its previous condition of balance. Sufficient weight should now be added at some point, as at D, to bring the beam again into the position of horizontal balance. The amount of weight added at this point D we may represent by n.

The turntable is now turned 90 deg. and the beam moved by means of the adjusting screws until it is brought into the horizontal position, when the point X will be in the position indicated in Fig. 9. The turntable is now given another movement of 180 deg. to the position indicated in Fig. 10 and weight added at some point, as at C, sufficient to bring the beam again into the horizontal or balanced position. The amount of weight added at the point C may be represented by n'. It remains now to locate the line EF which lies in both the geometric center and the plane of the center of gravity. This may be done by determining the angle θ which is made by the lines EF and AC.

By equating moments about the axes and throwing out small factors which would not materially affect the result, the following expression is obtained:

$$Tan.\ \theta = \frac{n}{n'},$$

in which n' must be the greater weight.

Then, the weight necessary to be added to point E or to be taken away from point F equals.

$$\tfrac{1}{2} cos.\ \theta\ n',$$

n' being the greater of the two weights.

The object of shifting the turntable so as to bring the center of gravity over the knife-edges is to secure just double the effect of the faulty balance when the turntable is turned 180 deg. This is indicated in the above formula by the factor $\frac{1}{2}$.

Final balancing of the turbine disks or sections is obtained by drilling at the points found by this method and in accordance with Table 1 giving the depths of holes of various diameters to remove certain weights of metal. This table is obviously of equal application to any other standing balance apparatus.

The horizontal position of the turntable is determined by the pointer *A* and scale *B*, Fig. 3, the pointer being made to oscillate equally on each side of the zero point in the manner common with delicate chemical balances, thus eliminating even the small friction of the knife-edges.

The complete problem of balancing the rotating parts of high-speed machinery is by no means a simple one. In addition to the fact that revolving parts, if of considerable length, may be, and often are, in standing without being in running balance, is the question of the flexibility of the supporting shaft.

While it is necessary to include this condition in any complete discussion of the balancing problem, it may be omitted in all ordinary cases in which the shaft has the usual degree of stiffness. The only point needing discussion in order to make clear the putting of ordinary machine parts in running balance is that out of which grows the fact that such balance does not necessarily exist although the parts may be in perfect standing balance.

Fig. 11 shows two balls *a* and *b* mounted on arms extended from a revolving shaft, the balls being of equal weight and the arms of equal weight and length, under which circumstances it is obvious that the system as a whole will be in perfect standing balance. If revolved at high speed, however, a centrifugal force *ac* will be generated in the ball *a* and a corresponding centrifugal force *bd* in the ball *b*, and as these forces are not in line with one another, the net result as the piece revolves is a revolving couple which gives rise to vibration of the shaft.

The correction of this condition of affairs is simple enough in principle and requires nothing more than the addition of two counterbalance pieces *e* and *f*, equal in weight and radial distance to *a* and *b*, or of less weight and greater radial distance, or *vice versa*, such that the centrifugal force of the added pieces balances that of the original balls, respectively, and it is, moreover, obvious that the addition of these balancing pieces will not disturb the standing balance of the system.

In actual revolving pieces which have been placed in standing balance we do not have plainly visible weights like the balls *a* and *b*, of Fig. 11, but instead we have the fact that metals are not perfectly homogeneous. For example, two disks upon a shaft as in Fig. 12, although in perfect standing balance, may have heavy spots located at *a* and *b*, which will obviously act precisely like the balls of Fig. 11, the placing of the system in running balance involving the addition of counterweights opposite the heavy spots, precisely like the counterweights of Fig. 11.

The same condition of standing balance with running unbalance may exist in a long drum, Fig. 13, the heavy spots being indicated as before at *ab*.

The difficulty of the problem lies in the fact that no test to which the parts can be subjected when at rest, on parallel strips for example, will give any indication of the position or weight of the heavy spots, nor is it possible to determine them by any known means. The placing of the parts in running balance consists of finding the positions and weights of the counterweights which destroy the vibrations and between the weights and positions of these counterweights there may be an indefinite number of combinations, any one of

TABLE 1.—DEPTH OF DRILLING NECESSARY TO REMOVE GIVEN WEIGHTS WHEN BALANCING MACHINE PARTS

Weight, oz.	Brass: Depth to drill in ins.					Steel: Depth to drill in ins.					Cast-iron: Depth to drill in ins.				
	1-in. drill	¾ drill	½ drill	⅜ drill	¼ drill	1-in. drill	¾ drill	½ drill	⅜ drill	¼ drill	1-in. drill	¾ drill	½ drill	⅜ drill	¼ drill
50	13.25	....	....	....	....	14.19	25.26	....	....	....	15.43	27.50	....	....	....
40	10.60	18.86	....	....	....	11.36	20.20	....	....	....	12.36	22.00	....	....	....
30	7.95	14.16	....	....	....	8.52	15.16	....	....	....	9.27	16.50	....	....	....
20	5.30	9.44	21.22	....	....	5.68	10.06	22.73	....	....	6.19	10.94	24.75	....	....
10	2.65	4.72	10.60	18.86	....	2.83	5.05	11.38	20.21	....	3.08	5.47	12.38	22.02	....
9	2.38	4.24	9.54	16.97	....	2.55	4.55	10.23	18.18	....	2.78	4.95	11.12	19.81	....
8	2.12	3.77	8.48	15.08	....	2.27	4.04	9.08	16.16	36.37	2.47	4.40	9.90	17.60	39.60
7	1.85	3.30	7.42	13.20	29.71	1.98	3.54	7.96	14.14	31.83	2.16	3.86	8.66	15.41	34.65
6	1.59	2.83	6.36	11.31	25.46	1.71	3.04	6.82	12.12	27.28	1.86	3.31	7.43	13.20	29.72
5	1.32	2.36	5.30	9.43	21.22	1.41	2.54	5.68	10.10	22.73	1.54	2.75	6.18	11.00	24.75
4	1.06	1.88	4.24	7.54	16.96	1.14	2.01	4.55	8.08	18.18	1.24	2.20	4.96	8.80	19.80
3	.79	1.41	3.18	5.65	12.72	.85	1.51	3.41	6.06	13.63	.93	1.65	3.71	6.60	14.85
2	.53	.94	2.12	3.77	8.48	.57	1.01	2.28	4.04	9.08	.62	1.09	2.48	4.40	9.90
1	.26	.47	1.06	1.88	4.24	.28	.50	1.14	2.02	4.55	.31	.55	1.24	2.20	4.96
.9	.24	.42	.95	1.69	3.81	.26	.45	1.02	1.83	4.08	.28	.49	1.11	1.98	4.34
.8	.21	.37	.85	1.52	3.39	.22	.40	.91	1.62	3.64	.25	.44	.99	1.76	3.96
.7	.18	.33	.74	1.33	2.96	.19	.35	.79	1.4	3.17	.22	.39	.87	1.54	3.46
.6	.16	.28	.63	1.14	2.54	.17	.30	.67	1.22	2.72	.19	.33	.74	1.32	2.97
.5	.13	.23	.53	.95	2.12	.14	.25	.57	1.01	2.28	.15	.28	.62	1.10	2.48
.4	.11	.19	.43	.76	1.69	.11	.20	.45	.81	1.83	.12	.22	.50	.88	1.98
.3	.08	.14	.31	.57	1.27	.09	.15	.33	.61	1.37	.09	.17	.37	.66	1.49
.2	.05	.09	.21	.38	.85	.06	.10	.22	.41	.91	.06	.11	.25	.44	.99
.1	.02	.05	.11	.19	.42	.02	.05	.11	.20	.45	.03	.06	.12	.22	.50
Deduct for point of drill.	.10	.075	.05	.04	.03	.10	.075	.05	.04	.03	.10	.075	.05	.04	.03

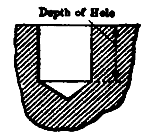
Intermediate weights may be found by adding together the depths of hole corresponding to the different weights that go to make up the whole.

Example:

Suppose the piece is out of balance—27.4 oz. and it is convenient to use a 1-in. drill.
The depth corresponding to 20 oz. = 5.30 ins.
The depth corresponding to 7 oz. = 1.85 ins.
The depth corresponding to 0.4 oz. = 0.10 ins.

Total is 7.25
Deduct for point of drill............. 0.10 in.

Required depth of hole.............. 7.15 ins.



which gives little indication of the positions and weights of the disturbing heavy spots.

The method of putting revolving parts in running balance is thus explained by E. R. Douglas (*Amer. Mach., Feb. 22, 1906*).

Theory helps at the start in one important particular. It shows, simply, that *any unbalanced system of rotating bodies, mounted on a stiff shaft, can be balanced by two weights, located at proper radii in two planes perpendicular to the shaft.*

Moreover, of the eight factors involved, viz.: location of the two planes, amount of the two weights, angular locations of the weights and the radii at which they are placed, any four may be assigned at the start and the other four determined. This simplifies things greatly.

It is generally most convenient to fix the position of the planes and the radii of the weights by providing in the designs for two circumferential sets of balancing pockets, one near each end, and then determine the proper weights to be put in them and their angular locations.

The first step is to find the high side of the rotating body, as by holding a piece of chalk against it. To be practicable, there should be a finished cylindrical surface somewhere near each end of the body preferably over the balancing pockets, and their trueness should be tested by turning the body over slowly in its own bearings while a lathe indicator or a surface gage is brought up to them. If not perfectly true a light cut should be taken off while still turning slowly, to make them so. These will then be the reference surfaces. A good material for marking is the chalk-like substance called kiel, either red or blue. It will show on iron, brass, or copper. A red or blue pencil will do. Ordinary chalk does not stick long enough to be of much use.

Bring the body up to full speed, or as near it as the vibration will allow, and hold the kiel lightly against each of the finished surfaces. Shut down, and after coming to rest turn over slowly by hand, examining the kiel-marks carefully. It will generally be seen that one side of each finished ring is marked while the other is clean, showing in which way each end of the body was displaced in running. Usually the positions of the high spots on the two ends will not be the same. Each high spot should have its middle point carefully located and marked on the body near it by white paint or otherwise. It may be more convenient to locate the middle of the clean or low spot and take that of the high spot as directly opposite.

At first thought it might appear that the high spot would indicate directly the position of the heavy spot, and that the balance weight should be directly opposite. This is far from the truth. Generally speaking, for any position of the heavy spot, the high spot may come anywhere around the circumference. Fundamental facts are these: The three things which determine the position of the high spot with reference to that of the heavy spot are momentum, elasticity, and friction of the parts which are in vibration.

Consider the first alone. Suppose the body to rotate on a stiff shaft in bearings which are absolutely unconstrained and free to move in any direction, without friction of any sort. Momentum alone limits the motion of the body due to the unbalanced forces. Under this condition the high spot will come *directly opposite the heavy spot.*

Consider the second limitation. Suppose the body to be without appreciable momentum (as by consisting entirely of a very light mass at a very long radius) and to rotate in bearings whose motion is restrained in every direction only by equal elasticity. The same conditions would result if it rotated on an elastic and very light shaft in bearings of great rigidity. Then would the high spot come *directly over the heavy spot.*

Consider the third limitation. Suppose the body to be without appreciable momentum and to rotate on a stiff shaft in bearings the motion of which in every direction is limited entirely by friction. Then will the high spot come *ninety degrees back of the heavy spot,* in a direction that is opposite to that of rotation. The results of these three conditions are illustrated in Fig. 14.

Every body in rotation is affected more or less by each of these three factors. The location of the high spot will depend entirely on the relations between them. Since we do not know these we must once more resort to experiment.

If it be practicable, rotate the body in the opposite direction and, having first removed the kiel marks with sandpaper, apply the kiel again. Slowing down, observe these new marks. It will generally be found that they are in a different position than the first ones. Since for opposite rotation the high spot should be displaced from the heavy spot by an equal amount in the opposite direction, the heavy spot must be half way between the two marks; on which side can be told only by trial.

To determine this, first mark the new high spots with paint or otherwise in such a manner as to distinguish them from the first. Put lead balancing weights temporarily in the balancing pockets on both ends half way between the first and second marks. These weights should be of considerable size so as, if possible, to more than balance the heavy spots. Then come slowly up to or toward speed and, first having cleaned off the previous marks with sandpaper, apply the kiel again. If the weights are heavy enough and the position is right the new high spots should be opposite to those first determined for the same direction of rotation. If they are not, either the weights are on the wrong side of the body and should be shifted just 180 deg., or the weights are not heavy enough. In either case a few trials and some common sense should reverse the marks.

If it be impracticable to run in the opposite direction the case is not quite so easy. The process then is to apply weights of considerable size on each end in any position, making them heavy enough to outweigh many times the original heavy spots. Then, coming slowly as far as practicable up toward speed, mark the new high spots. Their position with reference to the weights applied will indicate, if those weights be heavy enough, the general relation which holds in the body between a heavy spot and its high spot. It may need a few trials with the weights in different positions completely to verify this. When known, the positions of the original heavy spots, and of the weights required to balance them, are at once determinable. It will be advisable to prove the correctness of this determination by moving the heavy weights to the positions thus indicated and observing whether the marks are reversed, as they should be.

Whichever method may have been pursued, the rest is comparatively simple after the original marks have been reversed; for the correct position of the balance weights has then been found and it remains only to reduce their amounts, a little at a time first one and then the other, until the correct balance, as shown by true running and absence of vibration, is obtained. The temporary weights may then be made permanent.

It must not be supposed that the above process of balancing is rapid or easy. Many more trials than are described will have to be made, for purposes of verification and for other reasons, and bodies running as fast as steam turbines take a good deal of time in coming up to speed and slowing down. But any attempt to balance them without some rational line of procedure is time wasted, and that here described has given good results.

It will be apparent that a rotating body in balance at one speed is also in balance at all other speeds. The prevailing impression to the contrary is due to the fact that unbalanced bodies have a speed of maximum vibration, at which the vibration may be violent, although above or below it the vibration may be comparatively mild. This speed is nothing more than that at which the rotations of the body synchronize with the natural period of vibration of the supports. Parts that are but slightly out of balance may require a high speed for effective balancing but this is due to the necessity for magnifying the effect of a slight unbalance.

Everything so far with one exception has referred to bodies rotating on very stiff shafts. If the shafts be long and slender, and so somewhat flexible, new conditions may enter. Under such conditions a number of heavy spots, located at different places along the body, may each cause a bend or kink in the shaft, somewhat as indicated in Fig. 15. Of course, where the body can be made up entirely of rings, separately balanced, the possibility of this is almost completely removed, but this cannot always be done. In such cases it will not be sufficient to have a ring of balancing pockets at each end of the body, but other such rings must be provided at intermediate points. Unless the shaft be very flexible indeed, one or two intermediate rings should be sufficient; of course the shaft always should be designed to have as much stiffness as possible. Finished reference surfaces should then be provided as nearly as possible over each row of balancing pockets, and kiel marks put on each to determine their high spots. The general methods of procedure are the same as with stiff shafts, but many more trials will usually have to be made before the balance is correct.

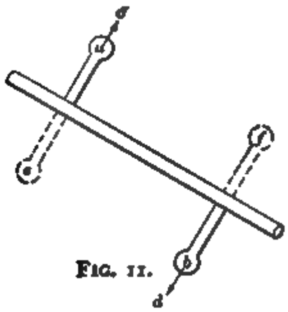


FIG. 11.

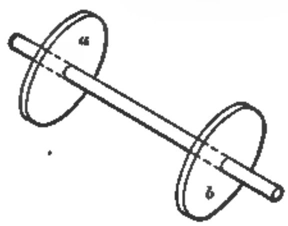


FIG. 12.

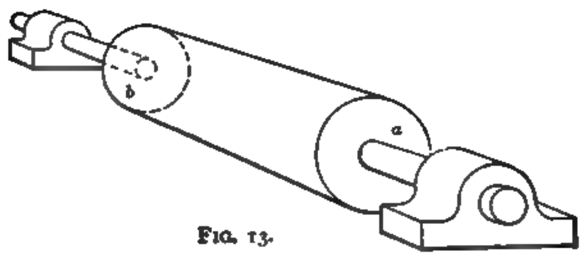


FIG. 13.

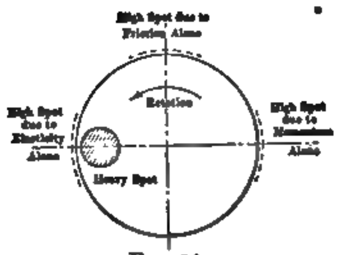


FIG. 14.

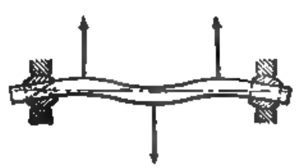


FIG. 15.

FIGS. 11 to 15.—Theory and practice of running balance.

The phenomena attending the rotation of loaded elastic shafts are of considerable interest. When such bodies are run, any slight unbalance, will, of course, set up a centrifugal force tending to bend the center of the shaft slightly out from the axis of rotation in the direction of the unbalance. As the speed and the centrifugal force increase the amount of this deflection also increases, but at a faster rate than the former quantities. A speed is finally reached at which the deflection would be indefinitely great, except that, of course, the shaft would break before this was reached if the constraint of the hub and bearings did not prevent it, or if the speed were not increased so rapidly that the shaft had no time to deflect to the breaking point. This speed is called the *critical speed* of the shaft. Its value depends on the flexibility of the shaft and on the amount and distribution of the weights on it.

If the speed be raised so rapidly as safely to pass the critical value, a curious thing occurs; the deflection, which, as already explained, for light bodies rotating on flexible shafts in rigid bearings is in the direction of the unbalance, changes to the opposite side and, as the speed is increased, decreases in amount. If the speed be made very great indeed, the body finally reaches a condition of rotation about its center of gravity, the shaft being slightly bent.

This is the state of affairs which obtains in the De Laval turbine, where the wheel is thin and comparatively light and the shaft long and flexible, so that the resulting stresses on the bearings are small. It is designed to run above its critical speed even if somewhat out of balance.

Turbines of the Parsons and other types and turbo-electric generators cannot, however, be made on this plan. Even when they can be made up of separately balanced rings it will sometimes happen that a slight unbalance will show after assembling, and with turbo-generators the case is worse on account of the wire windings which have to be put on them after the other parts are assembled. It is therefore essential to be able to give tham a final correct running balance.

FIG. 16.—The Norton running balance machine.

In order to eliminate, as far as possible, in this work, all unknown quantities, it is advisable to mount the body to be balanced, if possible, in bearings which are unconstrained in at least one direction, by carrying the bearings separately on rollers, or suspending them from supports above, or otherwise, so that their sidewise motion may be as free as possible, and drive by a vertical belt. The kiel should then be held in contact on the horizontal diameter. At the start the speed should be only high enough to develop perceptible vibration and allow of marking the high spots. As the balance approaches perfection, the speed may be raised toward its full value

without difficulty. Means should be provided to hold the bearings at their proper distance from each other and prevent endwise vibration.

Our own work is quite large and runs at from 1900 to 3600 r.p.m. We support it in babbitt-lined bearings which are cooled by water jackets on the outside and lubricated by a good grade of oil fed in at several points on each side of the bearing through small flexible tubes. These bearings we support elastically by resting them on cushions of rubber.

Owing to the flexibility of the shafts and the weight which would be required in supporting frames for a perfectly free horizontal mounting, we are not able to work with negligible elasticity; hence we find it as convenient to include its effects in our tests, especially as we are easily able, by running first in one direction and then in the other, to make allowance for it.

We drive by a light belt from an electric motor on the floor above. Having convenient means of varying the speed of this motor through a wide range and of using it as an electric brake when slowing down, we can run off tests pretty rapidly.

We apply the keil on the vertical axis, holding it very lightly in the fingers and resting the hand on a light support over the rotor.

We find it necessary, in order to get the most accurate results, to *grind* the reference rings over the balancing pockets by a small motor-driven emery wheel, while the work is turned slowly in its own bearings, which are blocked rigidly during the job. If the lathe work has been carefully done, this is not a long operation, as the reference rings do not need to be over $\frac{3}{4}$ or 1 in. wide. The surface left by the emery wheel is also excellently adapted to retain the lightest touches of the keil. To preserve this surface, we clean the marks off with benzine rather than with sandpaper.

The methods described by Mr Douglas may be carried out most expeditiously by the running balance machine of the Norton Grinding Co., Fig. 16 (*Amer. Mach., Dec.* 16, 1909), and by it the theoretical principles of running balance have been experimentally proven (*Amer. Mach., Aug.* 11, 1910).

The piece to be balanced is carried at each end on four rollers which are mounted in suitable cradles and carried on the upper ends of inverted pendulums. The lateral motion thus provided is limited by rubber disks through which the pendulum rods pass. Multiplying vertical pointers, plainly shown, are so connected with the rods as to vibrate with them and to magnify the vibration to the eye. Adjustable scriber points are provided, the markings being made more distinct by coating the shaft with red paint. The machine includes an electric motor together with a friction disk drive by which the speed may be varied and the direction of motion be reversed for reasons that have been explained by Mr. Douglas.

Among other things the machine has demonstrated that high-speed rotating parts should be so designed that they will not distort from centrifugal action, if rotated free at any speed at which they may run in use, and thus destroy a state of running balance.

Mr. Norton has found cases in which the ordinary four-throw automobile crank shaft bends something like $\frac{1}{16}$ in. at the low speed of 1200 r.p.m.

Balancing Reciprocating Parts

For the position of the center of gravity of counterweights of usual forms see Center of Gravity.

Reciprocating parts driven by a crank and connecting rod may be balanced *in the direction of the reciprocation* at the expense of unbalance in a direction at right angles thereto. To do this, consider the mass of the reciprocating parts, including all of the connecting rod, as concentrated at the center of the crank pin and calculate its centrifugal force. Then a mass, added on a radial line opposite the crank pin or subtracted on the side of the crank pin, which will generate an equal centrifugal force will balance the reciprocating parts in the direction of reciprocation. At the same radius as the crank-pin center, the weight should obviously equal that of the reciprocating parts. At any other radius the weight is inversely proportional to the radius—the radius being understood to be that of the center of gravity of the mass

This mass will give perfect balance in the direction of the reciprocation with a Scotch yoke or slotted-cross-head. With a connecting rod it gives a slight overbalance at one center and a slight underbalance at the other, the result being the best that can be obtained.

The center of gravity of the counterweight, for perfect results, must be in line with the center of the piston rod which, in center-crank engines, can be secured by dividing the weight equally between the two cranks. In side-crank engines there must be a slight offset with a resulting negligible horizontal twisting moment. In slow-speed engines it is frequently impracticable to place sufficient counterweight in the crank disk because of lack of room. Such engines do not commonly require balancing but, when necessary, satisfactory results have been obtained by placing as much of the counterweight as possible in the crank disk and the remainder in the fly-wheel. For a applicationn of this plan to large Corlies engines see a paper, *Counterweights for Large Engines*, by Dr. D. S. Jacobus in *Trans. A. S. M. E., Vol.* 26. The final result was a considerable horizontal twisting moment but a nearly complete stoppage of serious vibrations.

In horizontal engines the provision of a suitable counterweight in the crank disc balances the parts in a horizontal direction but it introduces a tendency toward vertical vibration equal to the tendency toward horizontal vibration with no counterweight whatever, and this tendency must be resisted by the foundation.

In vertical engines, on a proper foundation, the perfect balance with this arrangement is vertical, where it is not needed, while the unbalance is horizontal where it does harm. Such engines are best when entirely unbalanced, as that arrangement leaves the unbalance vertical where it is resisted by the foundation.

Reciprocating parts driven by more complex mechanism than a crank and connecting rod, may be balanced in the direction of the reciprocation, at the expense of unbalance in a direction at right angles thereto. The application of the method to a rock crusher is thus explained by Prof. O. P. Hood (*Amer. Mach., Nov.* 26, 1908): The crusher balanced was located in a rock house of a Lake Superior copper mine, where the elevation of the crusher above the ground led to an actual horizontal vibration of the rock house of .22 in., which was reduced to a negligible amount by the method described.

It is now feasible to run crushers of the type described at any reasonable speed without the danger of racking the building or requiring unusually heavy construction to resist useless forces.

The problem was as follows:

A mass of approximately 8 tons vibrated 175 times a minute through a short path, the nature of the vibration being determined by an eccentric revolving at a uniform speed, a pitman, toggle joint and swinging jaw. Is the nature of this movement such that a rotating weight, properly placed, can balance the inertia of the swinging jaw and parts moving with it? The forces which move the jaw must react against the frame and building to which it is attached and move the mass of these to produce the vibration. To find the amount of force it is necessary to find the velocity of the moving jaw, from which we can find the acceleration of the mass.

Fig. 17 shows diagrammatically the arrangements of parts of a crusher although very much distorted. Let BB' represent the path of the eccentric crank, AB the pitman articulated at A to the two links AC and AD. Link AC abuts against the frame at C and the point D is constrained to move about H as a center, or practically in the direction ED. As the eccentric revolves, the point D (a part of the jaw DH) has a small motion which crushes the rock against the frame. The character of this motion is readily found by finding the position of D for several positions of B, as shown in

dotted lines at $B'A'D'$. An inspection of this diagram shows that A moves about C as a center and also relatively about D, so that if the eccentric circle be drawn AB'' FG and arcs AF from C and AG from D as a center, the distance, as $A'K$ between the two arcs opposite any point in the circle as B'' measures the distance the point D has moved from its inner position. Thus $A'K$ is equal to $D'D$, and this corresponds to the position of the crank shown at B'.

In the actual crusher under consideration the eccentricity OB is $\frac{1}{2}$ in., AB 37 ins., AC 13 ins., AD 23 ins., JA $3\frac{1}{2}$ ins., JE 3 ins.

The jaw movement and the eccentric circle are so small that it is well to magnify the movement image, as in Fig. 18, where the arcs, AF and AG are drawn properly in proportion to the eccentric circle, which is here marked into 10-deg. spaces. Since the eccentric revolves uniformly these 10-deg. spaces are passed in equal times and, when plotted in Fig. 19, as horizontal distances, represent time. Laying off vertically in Fig. 19 the several distances similar to $A'K$ for each angle of the eccentric, we find the curve OJ, whose distance from the axis OX represents the jaw movement magnified. The actual maximum jaw movement was found to be .484 in. and this is represented in the diagram by XJ, which measures 1.95 ins.,[1] therefore, 1 in. in hight of the diagram represents .248 in. of jaw movement. From the curve OJ we can find the velocity at any point, for the velocity is the rate at which the jaw is changing its position, and by drawing tangents as LM at the 40-deg. point, then the vertical distance MN represents the distance the jaw would have traveled while the eccentric was moving the time LN, provided the rate had been uniform. The distance LN is taken as any convenient distance as 150 deg., but is taken the same for each point on the curve OJ. Plotting the MN distances with time as the base gives the curve OVX, which is the velocity curve of the jaw. The scale of this curve can be found from any of the triangles LMN, for the time is given by the constant base LN and the space by MN, the velocity being equal to the space divided by the time. The distance MN measures 1.74 ins., which equals $1.74 \times .248 = .431$ in., movement of the jaw. It is convenient to assume one revolution per second for the speed so that since the base $LN = 150$ deg., the time will be $150 \div 360 = .417$ sec.

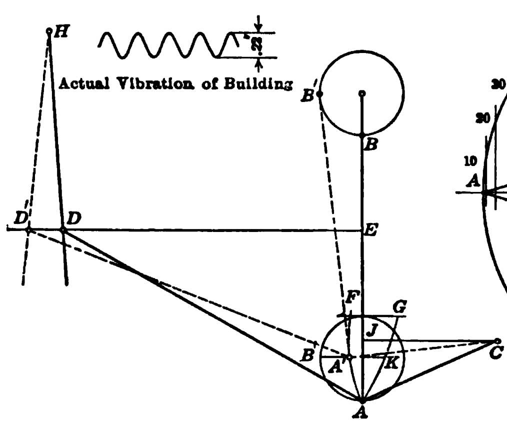


FIG. 17.—Diagrammatic arrangement of crusher parts.

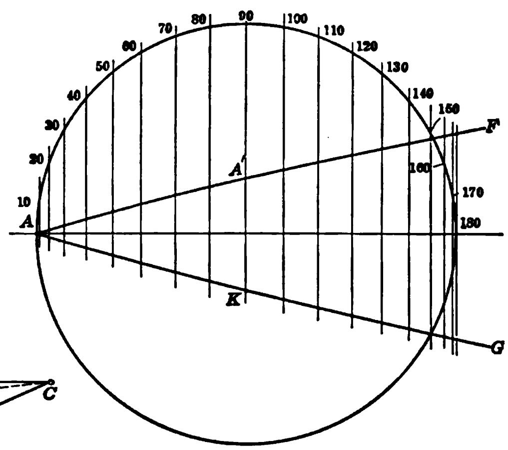


FIG. 18.—Diagram of magnified jaw movement.

The velocity when running at 60 r.p.m. equals $.431 \div .417 = 1.034$ ins. per sec. at the 40-deg. point, and this is represented by the hight MN of 1.74 ins. One inch in height of the velocity curve, therefore, represents $1.034 \div 1.74 \times 12 = .05$ ft. per sec.

To change the velocity of a mass requires a force proportional to the amount of change of velocity made in a second, that is, in proportion to the acceleration which is the rate of change of velocity. From the velocity curve we can find this acceleration by drawing tangents as before. To illustrate, at the 60-deg. point the velocity changes at the rate shown by the tangent $L'M'$ and in the time $L'N'$ would change the amount $M'N'$. Plotting these values of $M'N'$ for each of the points of the velocity curve we have the acceleration curve $RK'S$. When the velocity is the greatest, and before it begins to decrease, the velocity is unchanging for an instant, therefore there is no acceleration as shown by the curve $RK'S$ having a zero value at K'. Up to this point the jaw has been increasing in velocity and, therefore, requiring a force to make the change, but after passing K' the jaw velocity is decreasing and it now requires a force to stop it. This must evidently be in the opposite direction to the first force and is, therefore, shown below the line OX. Since the forces required are proportional to the acceleration, the curve $RK'S$ must also represent to some other scale the forces tending to shake the machine in the line of direction of the movement of the jaw.

That particular point on the jaw, whose movement has the same effect as if all the moving mass was concentrated at that point, is called the center of gyration which is measured from the point H about which the jaw swings. By computation from the drawings of the section of the casting this was found to be 44 ins. from the center, and here the velocities would be greater than at D in the proportion of 44 to 37, or 19 per cent. more.

The scale of the curve $RK'S$ can be found from the triangle $L'M'N'$, where $M'N' = 1.92$ ins. This represents $1.92 \times .05 = .096$ ft. per sec. as the velocity gained in the time $L'N'$. One-half second is represented by OX, which is 6.28 ins. long. Therefore, $L'N'$ represents .254 sec. The acceleration is $.096 \div .254 = .384$ ft. per sec. per sec. This is represented by a line $M'N'$ 1.92 ins. long, therefore, each inch in height represents $.384 \div 1.92 = .2$ ft. per sec. per sec. acceleration.

The maximum force required is evidently at the beginning of the jaw movement when the force is represented by OR. Here at 60 r.p.m. the acceleration is 47 ins.$\times .2 = .94$ ft. per sec. per sec., which at the radius of gyration of the jaw will be 19 per cent. more, or 1.12 ft. per sec. per sec. The mass moved weighs 16,000 lbs. The force required to move this will be $16{,}000 \div 32$, equals 500, multiplied by the acceleration 1.12 or 560 lbs. when running at 60 r.p.m. The force of acceleration will vary as the square of the velocity, so that it is no wonder that several such crushers will shake a building when run at 175 r.p.m., as many do, when the shaking force for each crusher amounts to nearly two and a half tons.

From the curve of acceleration it is noted that this force is applied as a push to the building at the beginning of the jaw movement, growing less in amount until it reverses just before the quarter revolution. The force then becomes a pull on the building, reaching an amount about 72 per cent. of the maximum push, but keeping at it longer. From 180 to 360 deg. the acceleration curve would be symmetrical about the line JS so that there would be a push on the building for about 46 per cent. of the time, and a pull for 54 per cent. An inspection of the curve shows that it does not depart widely from that form which would be made by an unbalanced weight revolving with the eccentric. Such a curve would be a sine curve.

Laying off OS' equal to XS we will take a point T half way between S' and R and assume this as the average force to be balanced, for

[1] The dimensions refer to the original drawing of which the engraving is a reduced copy.

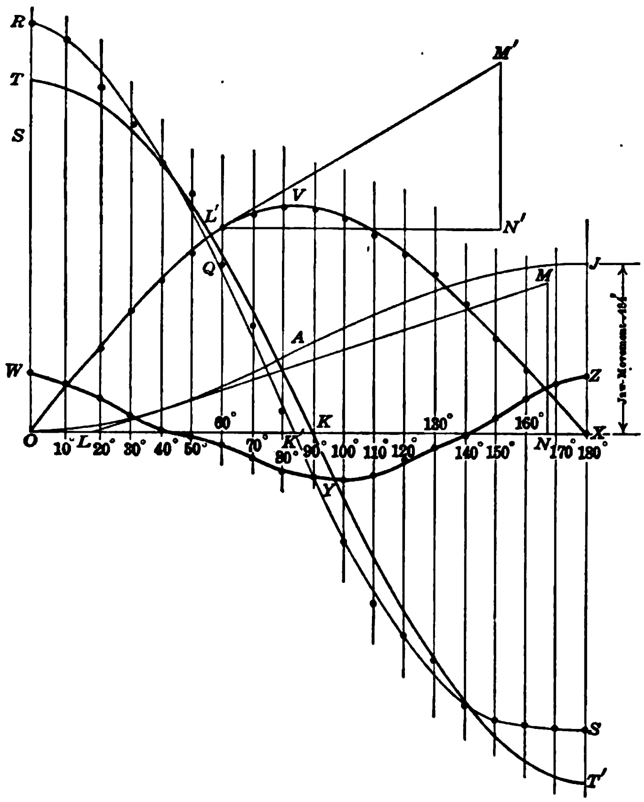


FIG. 19.—Graphical solution of the crusher balancing problem.

if underbalanced at R it will be equally overbalanced at S. The curve TT' is readily laid in as a sine curve. To furnish this balancing force we can place a weight in the fly-wheel that can give the forces TT', but this must be in such phase with the moving jaw that OT shall oppose OR instead of being in phase with it as shown.

We find that there is a convenient place in the fly-wheel of the crusher where the counterweight mass will be about 2 ft. from the center of the shaft. Here the velocity at 1 r.p.m. will be 12.50 ft. per sec., and the force will be equal to the weight times the velocity squared divided by 32 times the radius in ft. The force OT scales 480 lbs., if OR represents 560, so that the weight we seek figures 196 lbs. Half of this can be put in each fly-wheel in such place that, as the jaw begins to move forward the counterweight will begin to move back.

In the diagram Fig. 19, if we combine the curve $RK'S$ and TKT' we have the resultant curve WYZ. This shows that the shaking forces have been greatly reduced and their alternations have been doubled so that the smaller force also has a shorter time to produce movement before reversal.

This resultant XYZ shows that a nearly complete balance could be had by adding to the single rotating weight here described a second weight rotating at twice the revolutions per minute of the main eccentric. If the inertia of this secondary weight was made equal and opposed to OW the final resultant would be nearly a straight line. The secondary weight would have to be geared to the main shaft with a gear ratio of 2 to 1, making a complication of parts not warranted or needed in the usual crusher installation, but still of possible value in extreme cases.

The Hooke Universal Coupling

The Hooke universal shaft coupling, Figs. 1 and 2, does not, when used singly, transmit a uniform motion and, when two couplings are used to connect offset shafts, they are often so assembled as to double the irregularity of a single coupling, although, when correctly assembled, the irregularities neutralize each other and give as a final result a true, uniform motion. Fig. 1 shows the correct and Fig. 2 the incorrect arrangement. In the former the yokes of the intermediate shaft are in the same plane while in the latter they are at right angles to one another.

It is also necessary that the angles between the intermediate and the end axes be equal. This follows as a matter of course if the end axes be parallel but otherwise it must be provided for.

Actual constructions are shown in Figs. 3, 4, and 5. In Fig. 3 the offsetting of the pivot pins to permit their passing each other introduces a small additional error. In Fig. 4, the bearing bushings are clamped in position and also locked by detent pins. In Fig. 5 the cross takes the form of an external split ring, which holds the bearing bushings. The pivots are integral with the forks. The exterior of the forks and the interior of the rings are spherical to retain the grease.

In comparing the movements of the two shafts two methods are possible (*a*) The angular velocities or (*b*) the angular positions of the shafts may be compared. The angular velocities of the two shafts are given by the equation:

$$\frac{v}{V} = \frac{\cos A}{1 - \sin^2 A \sin^2 B}$$

in which V = speed of driving shaft,
v = speed of driven shaft,
A = angle between shafts,
B = angle of rotation of driving shaft from position of shaft A, Fig. 1.

Relative speeds of the driven shaft for various angles between the shafts have been calculated from this equation by EARL BUCKINGHAM (*Amer. Mach., Jan.* 16, 1913). The results are given in Table 1 and graphically in Fig. 6.

The relative positions of the two shafts are given by the equation:

$$\tan C = \tan B \cos A$$

in which A = angle between shafts,
B = angle of rotation of driving shaft from position of shaft A, Fig. 1,
C = angle of rotation of driven shaft from corresponding position.

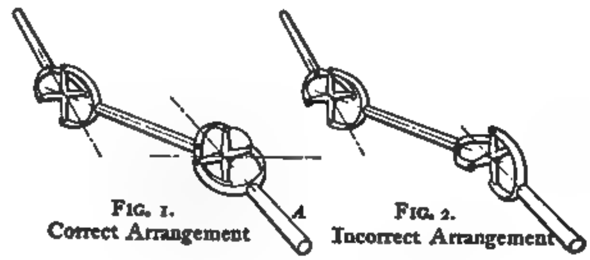


FIG. 1. Correct Arrangement

FIG. 2. Incorrect Arrangement

FIG. 3. FIG. 4. FIG. 5.

FIGS. 1 to 5.—The Hooke universal shaft coupling.

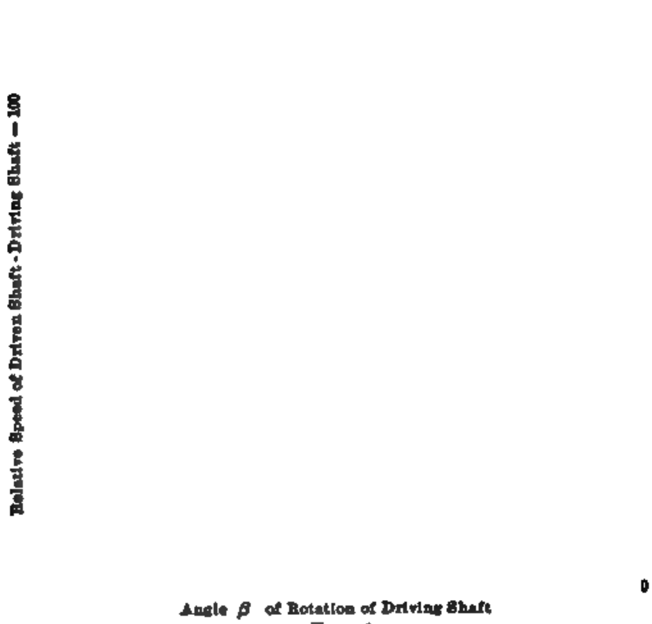


FIG. 6.

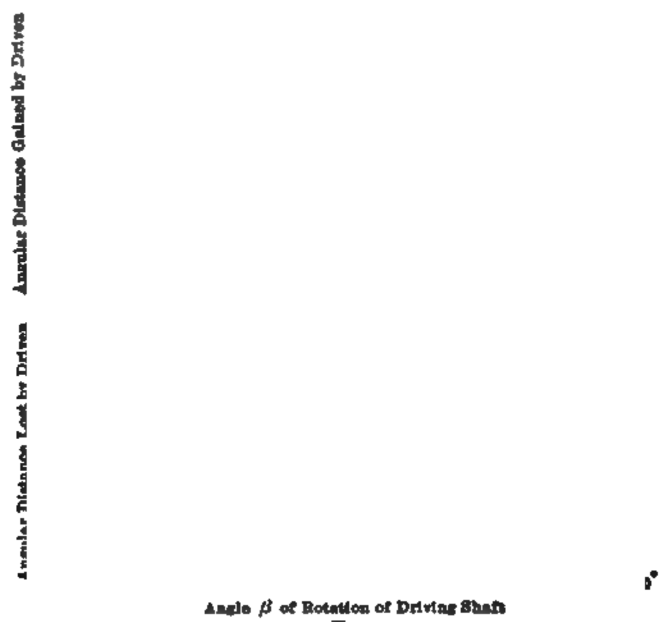


FIG. 7.

FIGS. 6 and 7.—Errors of velocity and position of the Hooke universal shaft coupling.

Mr Buckingham has also calculated the relative positions of the two shafts for various angles between the shafts by this equation. The results are given in Table 2 and graphically in Fig. 7.

TABLE 1.—RELATIVE SPEED OF DRIVEN SHAFT—SPEED OF DRIVING SHAFT = 100

Angle B of rotation of driving shaft	Angle A between shafts									
	0°	10°	20°	30°	40°	50°	60°	70°	80°	89° 59′
0	100	98.5	93.97	86.6	76.6	64.3	50.0	34.2	17.4	.029
5	100	98.5	94.1	86.8	76.8	64.6	50.3	34.4	17.5	.029
10	100	98.6	94.5	87.3	77.6	65.4	51.2	35.1	17.9	.029
15	100	98.7	94.8	88.1	78.8	66.9	52.6	36.4	18.6	.030
20	100	98.8	95.3	89.2	80.5	69.0	54.8	38.1	19.6	.032
25	100	99.0	96.0	90.7	82.7	71.8	57.7	40.6	21.0	.035
30	100	99.2	96.8	92.4	85.7	75.3	61.5	43.9	22.9	.039
35	100	99.46	97.5	94.9	88.7	79.7	66.4	48.2	25.5	.043
40	100	99.72	98.7	96.6	92.4	84.9	72.5	53.8	29.0	.049
45	100	99.98	99.8	98.97	96.6	91.0	80.0	61.2	33.7	.058
50	100	100.22	100.9	101.5	101.1	98.0	89.3	71.0	40.6	.070
55	100	100.5	101.9	104.0	105.9	106.0	100.6	83.9	49.7	.088
60	100	100.7	103.0	106.6	111.0	114.8	114.2	101.2	63.7	.116
65	100	100.9	103.9	108.9	115.9	124.1	130.2	124.5	85.4	.162
70	100	101.1	104.8	111.1	120.6	133.4	148.0	155.2	120.9	.249
75	100	101.3	105.4	113.4	124.6	142.0	162.7	194.1	219.4	.434
80	100	101.4	105.9	114.3	127.7	149.1	183.4	238.1	292.3	.965
85	100	101.5	106.3	115.1	129.8	154.5	195.5	276.6	462.8	3.83
90	100	101.6	106.4	115.4	130.5	155.5	200.0	292.3	575.8	3428.0

leaves at G, depending upon the direction of rotation. Part of the circle HKQ is cut away at the left for clearing the arms of the star wheel.

In using this movement, the designer may either determine the number of slots he wants in the star wheel, which will limit the relative time of operation and the dwell of shaft A during one revolution of shaft B; or he may settle ***approximately*** the relation between operating time and locking time on shaft B, which will limit the number of slots in the star wheel.

Let N = number of slots in star wheel. Examining the drawing it will be seen that angle Δ must always be 90 deg. in order that pin E may enter the slot properly, and

$$\text{Angle } \alpha = \frac{360}{N} \qquad (a)$$

To anyone familiar with geometry, it also is plain that

$$\alpha + \beta = 180 \text{ deg.}$$

from which

$$\beta = 180 - \alpha \qquad (b)$$

Again consulting the drawing, it may be seen that the smallest number of slots with which it would be possible to operate the star wheel would be three. The greatest number of slots possible depends upon the diameter of the star wheel and the size of the slots. If the slots could be considered infinitely narrow, their number might be infinite. Thus the theoretical limits for number of slots lie between 3 and infinity. Of course, the largest number possible in practice will not be very great, but the probabilities are that this limit will not often have to be reached.

TABLE 2.—DIFFERENCE IN ANGULAR MOTIONS OF SHAFTS

Angle B of rotation of driving shaft	Angle A between shafts									
	0	10°	20°	30°	40°	50°	60°	70°	80°	89° 59′
	Angle C of rotation of driven shaft									
	0	0	0	0	0	0	0	0	0	0
	5	4-55-28	4-42- 0	4-19-58	3-50- 3	3-10-15	2-30-18	1-42-50	0-52-14	
	10	9-51- 4	9-24-29	8-40-56	7-41-33	6-27-59	5- 2-18	3-27- 4	1-45-14	0- 0-10
	15	14-46-56	14- 7-58	13- 3-52	11-35-58	9-46-20	7-37- 0	5-14-10	2-39-50	
	20	19-43-11	18-52-54	17-29-43	15-34-47	13-10- 4	10-17-51	7- 5-46	3-37- 0	0- 0-23
	25	24-39-57	23-39-45	21-59-26	19-39-27	16-41- 8	13- 7-27	9- 3-41	4-37-46	
	30	29-37-18	28-28-52	26-33-54	23-51-31	20-21-38	16- 6- 8	11-10-13	5-43-30	0- 0-35
	35	34-35-20	33-20-39	31-13-57	28-12-31	24-13-54	19-17-43	13-28- 3	6-55- 0	
	40	39-34- 7	38-15-20	36- 0-19	32-43-57	28-20-27	22-45-33	16- 0-47	8-17-24	0- 0-50
	45	44-33-41	43-13- 9	40-53-37	37-27-13	32-43-56	26-33-54	18-52-53	9-51- 4	
	50	49-34- 3	48-14-12	45-54-17	42-23-39	37-27-13	30-47-23	22-10-33	11-41-31	0- 1-12
	55	54-35-12	53-18-31	51- 2-36	47-34-15	42-33- 6	35-31-47	26- 1-58	13-55-41	
	60	59-37- 7	58-26- 0	56-18-36	52-59-44	48- 4-12	40-53-36	30-38-33	16-44-22	0- 1-45
	65	64-39-44	63-36-28	61-42- 0	58-40-13	54- 2-28	46-59-49	36-15-27	20-25-29	
	70	69-42-59	68-49-38	67-12-15	64-35-10	60-28-47	53-56-51	43-13- 9	25-30-20	0- 2-45
	75	74-46-45	74- 5- 5	72-46-14	70-43-16	67-22-16	61-48-47	51-55-24	32-56-45	
	80	79-50-56	79-22-36	78-29-30	77- 2-34	74-39-37	70-34-29	62-43-36	44-33-41	0- 5-40
	85	84-55-24	84-40-51	84-13-53	83-29- 5	82-14-57	80- 4-30	75-39- 4	63-15-35	0-11-25
	90	90	90	90	90	90	90	90	90	

The Geneva Stop

The designing of the Geneva stop is shown in Fig. 8 and explained as follows by E. KWARTZ (*Amer. Mach., June* 8, 1911):

Referring to the illustration, the driving roller E is shown leaving the star wheel, after having turned the latter through part of one revolution; or in the position of entering the star wheel for driving, if the direction of rotation is reversed. The round part HKQ, which may be cast in one with the crank disk O, is in position to lock the star wheel until the roller enters at G; or releasing it until the roller

Suppose that the number of slots is determined, and one desires to find the relation between operating time and locking time during one revolution of shaft B. By formula (a) angle β expresses the operating time in degrees. If it is desired as a fraction of one revolution of shaft B, call this fraction B_t.

Then, $$B_t = \frac{\beta}{360}, \text{ but from } (b)$$

$$\beta = 180 - \alpha$$

and from (a) $$\alpha = \frac{360}{N}$$

Therefore, $$B_t = \frac{180 - \frac{360}{N}}{360}$$

or $$B_t = \frac{1}{2} - \frac{1}{N} \qquad (c)$$

From formula (c) we may obtain

$$N = \frac{1}{\frac{1}{2} - B_t} \qquad (d)$$

This formula will give N only approximately, unless the answer should be a whole number. As N is the number of slots, it can, of course, be only a whole number, and B_t eventually must be made to correspond.

Examples.—Assume, first, that number of slots N is determined; say $N=6$. Then from formula (a), $\alpha = \frac{360}{6} = 60$ deg. From formula (b), $\beta = 180 - 60 = 120$ deg.; and from formula (c), $B_t = \frac{1}{2} - \frac{1}{6} = \frac{1}{3}$ revolution of shaft B, for operating time.

Changing the conditions, assume $B_t = \frac{1}{4}$; that is, $\frac{1}{4}$ turn of shaft B is desired to operate the star wheel. Then, from formula (d), $N = \frac{1}{\frac{1}{2} - \frac{1}{4}} = 4$ slots required in the star wheel. It would seem logical that four slots would give $\frac{1}{4}$ operating time, but this does not hold for all fractions, as the next example will show.

Assume $B_t = \frac{1}{5}$. Then from formula (d), $N = \frac{1}{\frac{1}{2} - \frac{1}{5}} = \frac{10}{3} = 3\frac{1}{3}$; but, as N can be only a whole number, we will have to be satisfied with either three or four slots in star wheel and take B what it comes for this number.

In most cases where this device would be used, one probably would start out by deciding upon a certain center distance, if this is not already fixed; then construct a semicircle $DEFC$ upon this center distance and lay out an angle $DCE = \frac{\alpha}{2}$. Connect DE and draw line GD, extending it toward H, which will make an angle $QDH = \beta$, limiting the ends of the clearance cut QH of the locking circle.

Radius r of the locking circle is somewhat a matter of choice. As a standard, the nearest sixteenth of an inch to the result obtained from expression: $r = DE - 1\frac{1}{8}d$, d being the diameter of driving roller may be taken. The shape of the clearance cut is found by tracing one arm of the star wheel on one piece of tracing cloth and the crank-pin roller, center of crank disk and circle HKQ on another piece of tracing cloth. Fasten these pieces with pins to the drawing board with their proper center distance, placing the crank-pin tracing on top, and rotate together, tracing the arm of the star wheel in different positions on the crank-disk tracing while turning over center D. Draw a curve QH that will clear these marks. The curve may be cast with a considerable clearance, but the end points H and Q should be located properly on the legs of angle β, as described.

Distance DE = radius of crank circle $= DC \sin \frac{\alpha}{2}$. To find the extreme radius of star wheel, make an accurate layout, and scale distance CP; or calculate thus: $CE = DC \cos \frac{\alpha}{2}$. Assume diameter of driving roller $= d$. Then,

$$CP = \sqrt{\left(\frac{d}{2}\right)^2 + (CE)^2}$$

If an accurate layout is made, the calculating of CP is not necessary. It will be seen from the illustration that the number of slots may not be very much less than 6 before the crank disk O will interfere with the hub of the star wheel. When such becomes the case, the crank disk will have to be placed on the opposite side of the star wheel.

Rock Arms and Link Work

The length of rocker arms to divide equally the side vibration of the connecting link may be obtained from the formulas

$$a = \frac{b^2 + 4c^2}{4c}$$

$$c = \frac{a + \sqrt{a^2 - b^2}}{2}$$

the notation being as in Fig. 9.

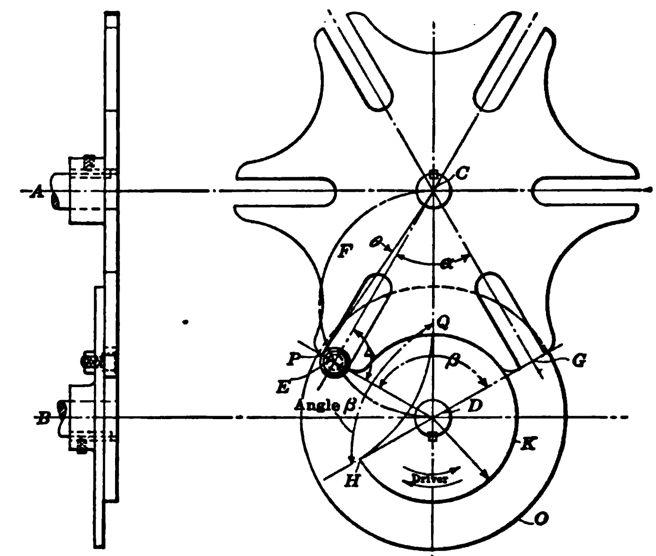


FIG. 8.—Designing the Geneva stop.

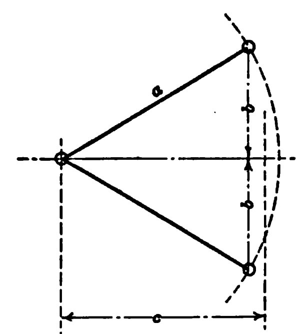


FIG. 9.—The length of rocker arms.

Vibrating levers are frequently required to transmit a reversed motion along two parallel lines, with a given stroke along each line and a given distance apart. Let AB, Fig. 10, represent the stroke and center line of one motion, CD the stroke and center line of the other, and EF the vertical distance between them. To find the position of the central stud and length of the levers: Lay off $EH = \frac{1}{2}$ stroke AB and $FN = \frac{1}{2}$ stroke CD on opposite sides of EF, and draw HN. The intersection of HN with FE is the center of oscillation. Draw GK at right angles with HN and G and K are the middle and extreme positions of the upper pin. Draw MR at right angles with HN and M and R are the extreme and middle positions for the lower pin. This gives the length GO for the upper arm and RO for the lower.

Solving the problem mathematically:

$$\frac{s}{l} = \tan \alpha, \text{ and } \frac{b}{s} = \tan \alpha^1,$$

$$b = \frac{s^2}{l}$$

and $$L = l + \frac{s^2}{l}$$

Also, $$AB : EO :: CD : FO.$$

This gives a simple formula for computing the length of arms which will give an equal vibration on each side of the central line of motion.

A problem in link work, which occurs in the layout of Corliss valve gears, together with its solution, by E. H. BERRY (*Amer. Mach., Aug.* 13, 1908), is shown in Figs. 11 and 12.

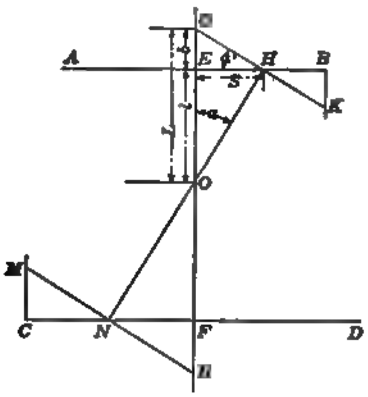


FIG. 10.—Laying out vibrating levers.

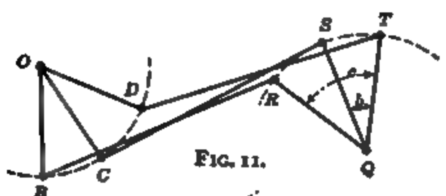


FIG. 11.

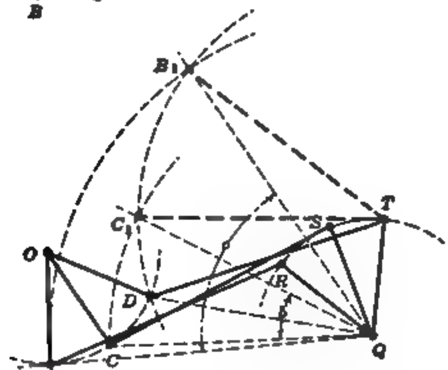


FIG. 12

FIGS. 11 and 12.—A problem in link work and its solution.

Given the point O and the three positions OB, OC, and OD of an arm of known length swinging about O; given the point Q and the angles b and c; required the length of the arm QR and the length of the link BR.

Solution: Draw QB and QC as in Fig. 12. With center Q and with radius QB draw the indefinite arc BB_1 with the same center, and with radius QC draw the indefinite arc CC_1. Lay off the angle BQB_1 equal to the given angle c, and lay off the angle CQC_1 equal to the given angle b. Find the center T of a circle passing through the three points B_1, C_1 and D. Then QT is the required length of arm, and DT is the required length of link.

When obstructions interfere with the rise and fall of the end of a vibrating lever, it may be made to travel in an approximately straight line by the construction shown in Figs. 13 and 14, by A. E. GUY (*Amer. Mach., Apr.* 21, 1898). The lever slides over a guide block swiveled to a fixed point and is driven by an oscillating crank arm connected to its lower end. Mr. Guy has found that, assuming the upper end B, Fig. 13, to be guided in a straight line, when the angle 2α, Fig. 13, is as large as 75 deg., and OE = about one-third of OB, the path of the point E is almost an arc of circle, and for $2\alpha = 90$ deg., which value may be considered as extreme for an ordinary lever, the curve coincides with the arc of a circle until near the ends E and E', when it bends inward.

Consequently if the point E, Fig. 13, is made to travel along the arc of a circle EAE', the path of point B will be very nearly a straight line. It is easy to find the radius of the arc since it passes through three points whose positions are known.

In Fig. 13 the lever is shown at its two extreme positions, BE, and $B'E'$. Draw AE, and at its middle draw the perpendicular DF, then DA is the radius of the arc. For many purposes this graphical method is sufficiently accurate, but for other cases the following equations may be used:

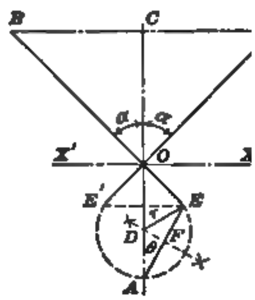


FIG. 13. FIG. 14.

FIGS. 13 and 14.—A straight line lever mechanism.

In Fig. 13 let
$$BE = a$$
$$OE = b$$
$$DA = r$$

and we have
$$\tan\theta = \frac{b \sin\alpha}{a(1 - \cos\alpha)} \qquad (a)$$
$$\text{and } r = \frac{b \sin\alpha}{\sin 2\theta} \qquad (b)$$

Since angle α is known, the value of angle θ will be easily found by formula (a) when the sine of 2θ will be taken from trigonometrical tables and introduced in equation (b).

The Ball Expansion Drive Stud

The ball expansion drive stud, Figs. 15 and 16, invented, patented and largely used by the Link Belt Co., was by that company presented to the mechanical public, without fee or royalty, through the *American Machinist* of Dec. 9, 1909.

The illustrations show sections before and after driving. The rivet or stud is a plain piece of stock having a hole drilled in one end, with a chamfer surrounding the hole and then cut off from a bar of cold-rolled stock.

A hard-steel ball, bought at a very low price from the culls taken from the balls selected by makers of bearings, and slightly larger than the hole in the end of the stud, is dropped into the hole ahead of the stud, which is then driven into place over the steel ball, as shown in Fig. 16. The chamfered end of the stud aids in closing it around the ball.

The amount of expansion on the lower end of the stud depends upon the difference between the diameter of the hole and that of the ball. The diameter of the hole in the end of the stud should be about three-quarters of the outside diameter, and the depth of the hole in the stud should equal the diameter of the ball. Excessive driving weakens rather than increases the hold. The depth of the hole in the casting should be about twice the diameter of the stud for small sizes and 1½ times for large sizes, while the difference in diameter between the ball and the hole in the stud should be about $\frac{1}{32}$ in.

Tests have shown that $\frac{3}{16}$- and ¼-in. studs are about 20 per cent. stronger than bolts of the same diameter, and the average grip of a ⅜-in. stud is nearly equal to the breaking strength of a bolt of this size.

These studs are greatly superior to screwed-in studs. They have been used by the Link Belt Co. with complete success in sizes up to

FIG. 15. FIG. 16.

FIG. 17. FIG. 18

FIGS. 15 to 18.—The ball expansion drive stud.

⅝ in. Table 3 gives the dimensions for small sizes. While the experience of the Link Belt Co. has shown complete security of the construction, the larger sizes may, if desired, be given a security which no one can question by the method shown in Figs. 17 and 18, by PROFESSOR SWEET (*Amer. Mach., Jan.* 19, 1905). A simple wabble drill, Fig. 17, chambers the bottom of the hole while the ball expands the steel into the chamber, Fig. 18.

TABLE 3.—DIMENSIONS OF EXPANSION DRIVE STUDS

	In.	In.	In.
Diameter of stud	$\frac{3}{16}$	¼	⅜
Depth of hole	⅜	½	⅝
Size of ball	⅛	$\frac{3}{16}$	$\frac{9}{32}$
Diameter of center bore	$\frac{1}{16}$	$\frac{3}{32}$	⅛
Depth of center bore	$\frac{3}{32}$	$\frac{5}{32}$	$\frac{7}{32}$

Balance Diaphragms

The effective balancing area of a diaphragm, when opposed to a poppet valve with a knife-edge seat, has been worked out by JAMES CLARK (*Amer. Mach., Oct.* 27, 1904). Referring to Fig. 19, he finds the effective area of the diaphragm to be expressed by the formula:

$$\text{effective area } = \frac{\pi}{3}(R^2+Ra-2a^2)$$

in which R = outer radius of diaphragm, ins.,

a = radius of stem connecting valve and diaphragm, ins.

To make use of the formula, calculate the area to be balanced, that is, the annular area between valve and stem. Assuming, for example, the radius of the valve to be ⅝ in. and of the stem ¼ in., this area is:

$$\pi\left[\left(\frac{5}{8}\right)^2-\left(\frac{1}{4}\right)^2\right]=\pi\,\frac{21}{64}$$

which is to be equated with the formula for the effective area of the diaphragm giving:

$$\frac{\pi}{3}(R^2+Ra-2a^2)=\pi\,\frac{21}{64}$$

or,

$$R^2+Ra-2a^2=\frac{63}{64}$$

which, as $a=\frac{1}{4}$ becomes:

$$R^2+\frac{R}{4}=\frac{71}{64}$$

which, solved for R, gives

$$R=1.185 \text{ or } -.935 \text{ in.}$$

of which the positive value is the one available.

FIG. 19.—Effective balancing area of diaphragms.

The formula of the preceding paragraph may be applied to the balancing of poppet valves with other than knife-edge seats by the consideration of the following paragraph.

The effective pressure or equilibrium area of poppet valves, when closed, formed the subject of an experimental investigation by PROF. S. W. ROBINSON (*Trans. A. S. M. E., Vol.* 4). These experiments showed the presence of a creeping film of steam in the valve seat, the pressure of which, beginning at the pressure of the high-pressure edge, decreases to that of the low-pressure edge and acts to partially balance the applied pressure. The result of the experiments was to develop a formula for the area of the surface which, multiplied by the applied pressure, eqauls the actual closing pressure, this area being the effective pressure or equilibrium area of the valve. For flat-seated poppet valves the formula is:

$$d=2r\sqrt{\frac{R}{r}-\frac{p_2}{p_1}\left(\frac{R}{r}-1\right)} \qquad (a)$$

in which d = diameter of the equilibrium area, ins.,

r = inside radius of valve, ins.,

R = outside radius of valve, ins.,

p_1 = pressure on inner area, lbs. per sq. in. abs.,

p_2 = pressure on outer area, lbs. per sq. in. abs.

Then

$$P=\frac{1}{4}\,\pi d^2(p_1-p_2) \qquad (b)$$

in which P = total effective pressure of the valve against its seating surface.

For conical seated poppet valves the slant height of the inner and outer cones are to be treated as r and R. These are to be substituted

in formula (*a*) and the value of *d* found. The cone is then to be developed and such part of $\frac{1}{4}\pi d^2$ used as the developed cone is of 360 deg.

Formula (*a*) applies strictly to non-elastic fluids only; but the differences between calculated results from the formula and experimental results using steam pressure were small.

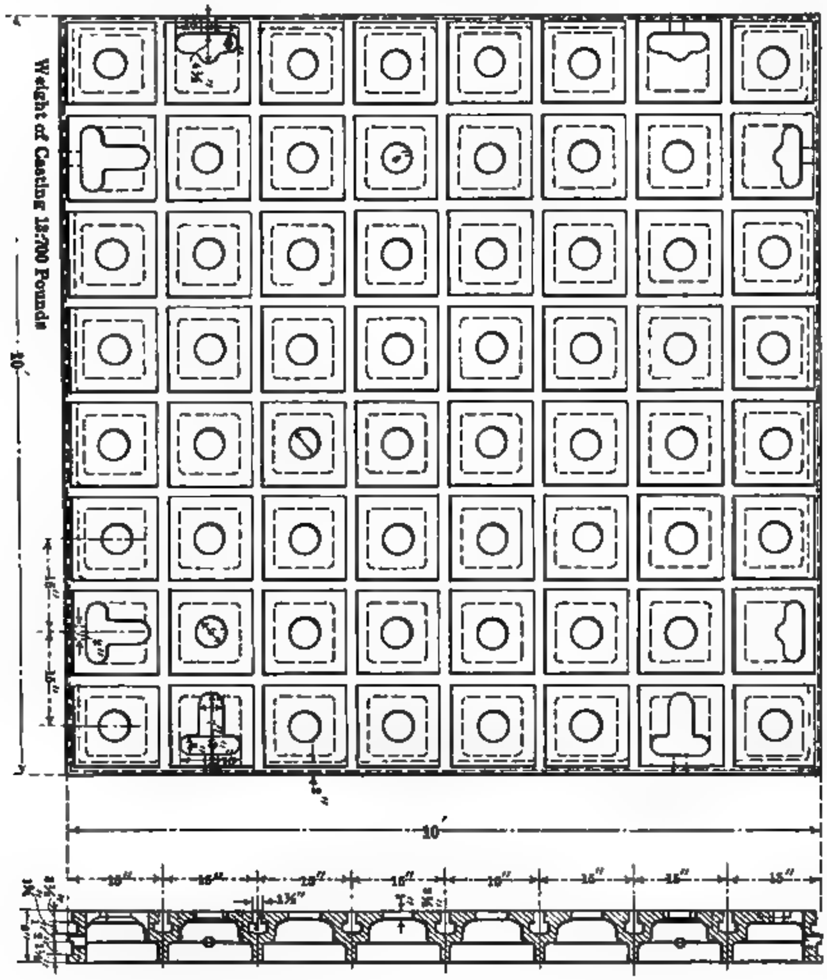


FIG. 20.—Dimensions of a section of cast-iron floor plate.

Surface Plates

FIG. 21.—Section of a floor plate foundation.

Cast-iron Floor Plates

The construction of cast-iron floor plates for use with portable machine tools, as practiced at the works of the General Electric Co., is shown in Figs. 20–22, by John Riddell, who originated this system of doing heavy machine work (*Amer. Mach., Nov.* 28, 1907). Fig. 20 gives the dimensions of one section, which is planed and grooved on the edges, surfaced on top and provided with regularly spaced holes for pouring the grouting.

The holes should be made of a size such that, with rags for packing, a piece of pipe about 3 ft. long may be inserted. Pouring the pipe full of grout produces a hydrostatic head which forces the grout under the plate and forms a solid bed for it.

Fig. 21 relates to an earlier pattern, when the plates were made heavier and less dependance placed on the foundation. It, however, shows the character and dimensions of the foundation. Fig. 22 gives a section of a floor at the same works used for erecting and testing but not for machining operations. Various details are shown, including the method of locating and supporting the rails while putting in the foundation. If the rails and beams are reasonably straight, no machine work, other than drilling, is necessary.

Laying out Approximate Ellipses

The layout of approximate ellipses by four circular arcs may be facilitated by the use of Fig. 23, by S. J. TELLER (*Amer. Mach., Feb.* 6, 1908). To use the chart, find the length of the major axis on one side of the sheet and the length of the minor axis on the top or bottom. Follow the corresponding horizontal and vertical lines to their intersection and read on the curved lines or by interpolation the radius of the larger arcs. Then find the length of the minor axis on one side of the sheet, and the length of the major axis on the top or bottom. Follow the corresponding horizontal and vertical lines to their intersection and read on the curved lines the radius of the small arcs. In some cases it may be found more convenient to put in the small arcs by the cut and try method, as it is a simple matter to draw in arcs which will connect the large arcs and pass through the ends of the major axis.

The layout of approximate ellipses by eight circular arcs may be facilitated by the method shown in Figs. 24 and 25 (*Amer. Mach., Mar.* 18, 1909).

Lay out the long diameter *AB* and the short diameter *CD*, Fig. 24, crossing each other centrally at *F*. Construct the parallelogram *AECF*, and draw the diagonal *AC*. From *E* draw a line at right angles to *AC*, crossing the long diameter at *H* and meeting the short diameter, extended, if necessary, at *G*; *H* is the center, and *AH* the radius for the end of the ellipse; *G* is the center, and *CG* is the radius for the side.

To get the third radius lay off a base line *AB*, Fig. 25, of any convenient length, and divide it into five equal parts by the points 1, 2, 3 and 4. At one end of this line erect the perpendicular *AC*, equal to the radius *AH*, and at the other end erect the perpendicular *BD*, equal to the radius *CG*. Connect the upper ends of these perpendiculars by the line *CD*. From point 2 erect the perpendicular 2 *E*. The length 2 *E* will be the desired third radius. With the compasses set to this radius find, by trial, a center *I*, Fig. 24, from which a curve can be struck which will be just tangent to the curves struck from the centers *H* and *G*. Lines drawn from *I* through *H* and from *G* through *I* will determine the meeting points of the different curves. From other centers similarly located the remainder of the ellipse is formed. In many cases one-half the major axis is a satisfactory value for the third radius.

For narrow ellipses the radius *AH* with which the ends are formed should be lengthened as follows: When the breadth of the ellipse is one-half of its length lengthen *AH* one-eighth; if the breadth of the ellipse is one-third of its length, make *AH* one-quarter longer; if the breadth is one-quarter of the length make *AH* one-half longer.

Arcs of Circles

The radius of a circular arc of which the span and rise are given, may be found as follows: In Fig. 26, g being half the span, h the rise and r the radius,

$$r=\frac{\frac{g^2}{h}+h}{2}$$

To lay off the length of a circular arc on a straight line; Draw the chord ab, Fig. 27, and extend it. Make $bc=\frac{1}{2}ab$. With c as a center strike the arc ad. The length bd of the tangent at b equals the length of the arc ab very nearly. If the arc is of 60 deg. the error is a little less than $\frac{1}{1000}$ of the length of the arc, the error varying as the fourth power of the angle subtended.

To lay off the length of a straight line on a circular arc: Draw the arc tangent to the given line ab, Fig. 28, at a. Make $ac=\frac{1}{4}\,ab$. With c as a center strike the arc bd. The length ad of the arc equals the length ab very nearly. The error of this construction and the law of its variation are the same as those of the one above. Both the above rules are due to PROFESSOR RANKINE (*Rules and Tables*).

Circular arcs of large radius may be drawn by the instrument shown in Fig. 29. The pencil a is located at the extremity of the rise of the arc and knife-edged weights bb are placed at the extremities of the chord. The parts being then clamped in position, the pencil will trace a true circular arc as shown.

A compass for circular arcs of large radius, which is not a makeshift, is shown in Figs. 30 and 31 by U. PETERS (*Amer. Mach., Oct.* 12, 1890). The entire instrument is 25 ins. long and it will fit arcs of any radius up to infinity. It consists of the rod A, an assortment of metal disks D and D_1, drawing pen holder P and weight g, Fig. 30.

By placing on the rod A one disk D_1 of somewhat smaller diameter at a certain distance from the other D, it is clear that by rolling the instrument over a plane (by means of handle H), on the principle of rolling cones, every point of the rod will describe an arc of a certain radius.

The relations between the desired radius R, Fig. 31, the distance a between the disk edges and the diameters d and d_1 of the disks are given by the equation:

$$R=\frac{ad}{d-d_1}$$

If the larger disk be of 4 ins. diameter, this becomes:

$$R=\frac{4a}{4-d_1}$$

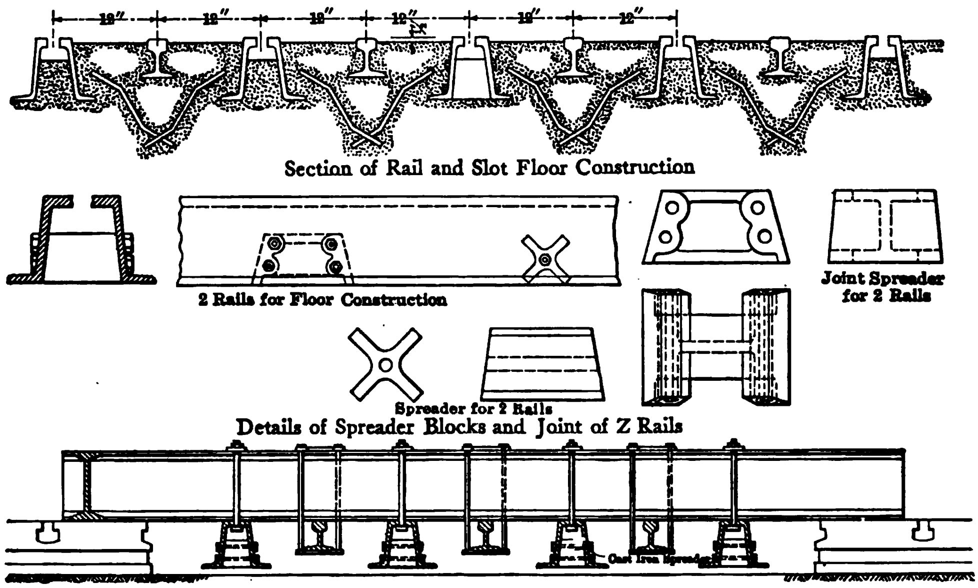


FIG. 22.—Section and details of an erecting and testing floor.

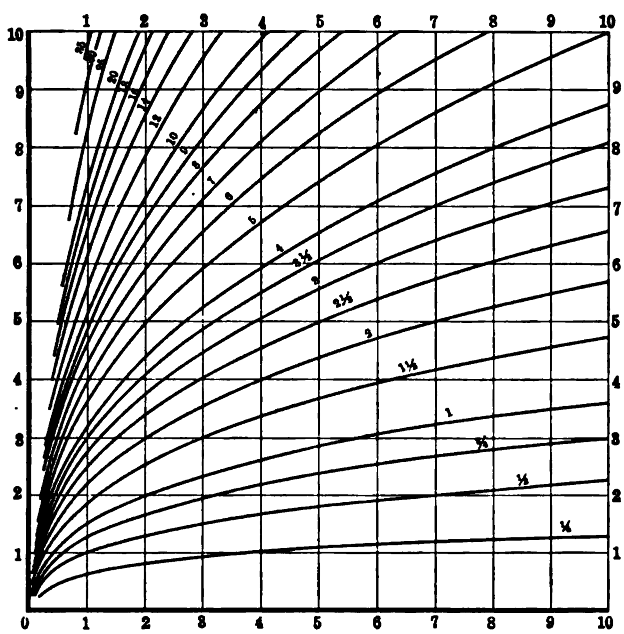


FIG. 23.—Laying out approximate ellipses by four circular arcs.

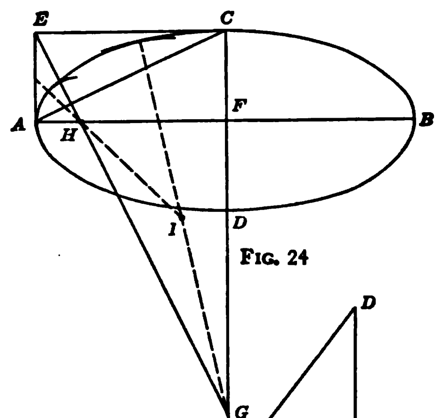


FIG. 24

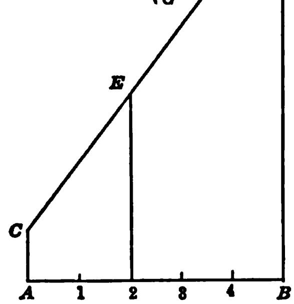


FIG. 25.

FIGS. 24 and 25.—Laying out approximate ellipses by eight circular arcs.

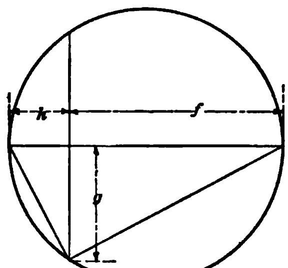


FIG. 26.—Calculating the radius of an arc for a given span and rise.

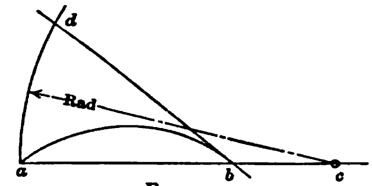


FIG. 27.

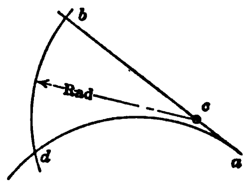


FIG. 28.

FIGS. 27 and 28.—Lengths of arcs and straight lines.

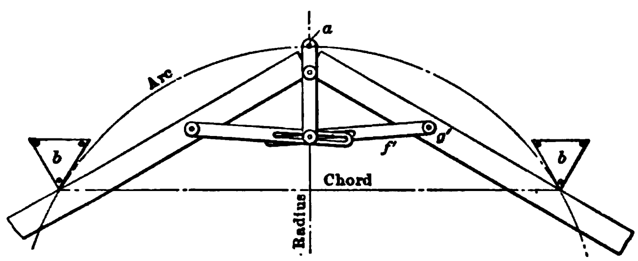


FIG. 29.—Instrument for drawing circular arcs of large radius.

Addition of Binary Fractions

The addition of binary fractions is usually made an unnecessarily laborious operation. They should be added in essentially the same manner as decimals, adding first those with the largest denominator, dividing the result by 2, setting down the remainder of 1, if there is any, and carrying the quotient to the fractions having the next smaller denominator. The following illustration will show the analogy between the processes of adding these and decimal fractions. Let it be required to add the quantities:

$$4\tfrac{3}{8}+3\tfrac{1}{4}+4\tfrac{7}{16}+\tfrac{3}{32}+2\tfrac{5}{32}+1\tfrac{9}{16}+\tfrac{9}{32}+2\tfrac{3}{4}=?$$

Beginning with the fractions having the largest denominator—32—we add them and obtain:

$$3+5+9=\tfrac{17}{32}=\tfrac{8}{16}+\tfrac{1}{32}$$

the $\tfrac{1}{32}$ forming part of the answer and the $\tfrac{8}{16}$ being carried to the sixteenths, which are next added, thus:

$$8+7+9=\tfrac{24}{16}=\tfrac{12}{8}+\tfrac{0}{16}$$

the $\tfrac{0}{16}$ forming part of the answer and the $\tfrac{12}{8}$ being carried to the eighths, which are then added, thus:

$$12+3=\tfrac{15}{8}=\tfrac{7}{4}+\tfrac{1}{8}$$

the $\tfrac{1}{8}$ forming again part of the answer and the $\tfrac{7}{4}$ being again carried to the fourths thus:

$$7+1+3=\tfrac{11}{4}=2+\tfrac{3}{4}$$

Proceeding as before with the whole numbers we have

$$2+4+3+4+2+1+2=18$$

and, annexing the several remainders to the final 18, we have the answer

$$18+\tfrac{3}{4}+\tfrac{1}{8}+\tfrac{0}{16}+\tfrac{1}{32}=18\tfrac{29}{32} \text{ or } 18\tfrac{7}{8}+\tfrac{1}{32}$$

the latter being the preferable method of expression on drawings.

Standard Cross-sections

Standard cross-sections for drawings in accordance with the recommendations of a committee of the *A. S. M. E.*, 1912, are given

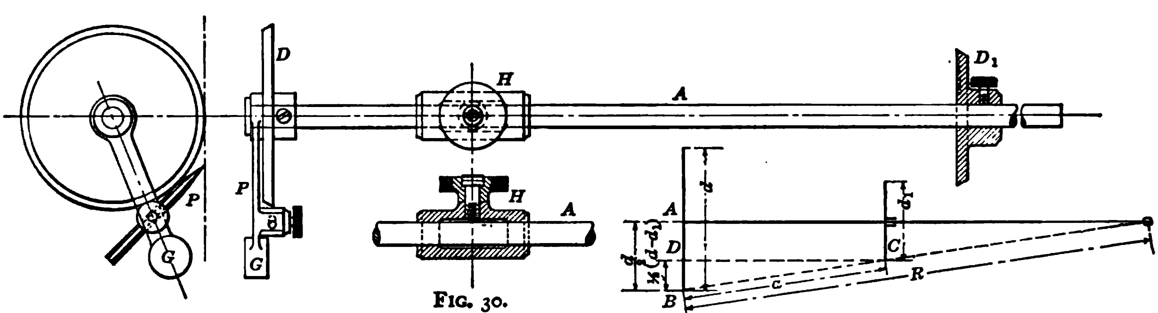


FIG. 30.

FIG. 31.

FIGS. 30 and 31.—Compass for large circles.

or

$$a=\frac{(4-d_1)R}{4}$$

from which Table 4 is obtained.

Example.—Required the settings for an arc of 52 ft. radius. Consulting the table, the third change disk of diameter $3\tfrac{15}{16}$ ins. should be placed on the rod at a distance $a=\frac{R}{64}=\tfrac{13}{16}$ ft. $=9\tfrac{3}{4}$ ins.

TABLE 4.—DISK DIAMETERS AND SETTINGS FOR LARGE CIRCLE COMPASS

Radius		Diameter of change disk, ins.	Distance a between steady and change disk edges
R in ins.	R in ft.		
24 to 96	2 to 8	3	$a=\tfrac{1}{4}\ R$
96 to 384	8 to 32	$3\tfrac{3}{4}$	$a=\tfrac{1}{16}\ R$
384 to 1536	32 to 128	$3\tfrac{15}{16}$	$a=\tfrac{1}{64}\ R$
1536 to 6144	128 to 512	$3\tfrac{63}{64}$	$a=\tfrac{1}{256}R$

in Figs. 32 and 33. The author gives them, not because he believes in such conventions but because many others do, and if they are to be used at all uniformity is obviously desirable. The committee recommend that subdivisions of any of the materials shown generically in Fig. 1, should be made by taking one of these standard cross-sections as a basis and making minor changes, but maintaining the general characteristics; or by writing on the standard section the name of the material. To illustrate, the committee has subdivided concrete into concrete blocks, cyclopean concrete and reinforced concrete, as shown in Fig. 2; and also wrought steel into nickel, chrome and vanadium steels.

In the author's opinion the method shown in the lower right-hand corner of Fig. 32 is the only one to be encouraged. The others are nothing but hieroglyphs which require memorization by all concerned or the constant consultation of a key. The hieroglyphs are memorized by few and why they, with the necessary key, should be pre-

ferred to self-explanatory English has never been explained. It is impossible to make such a schedule complete, as Fig. 33 will show, and resort must be made to simple English in the end. Why not use it in all cases? The whole plan is a case of system gone to seed.

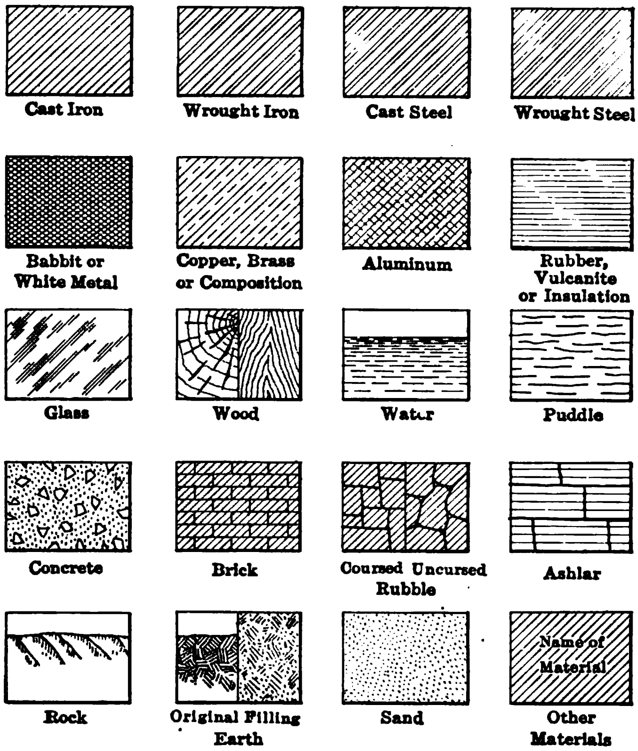


FIG. 32 - Recommended Standard Cross section

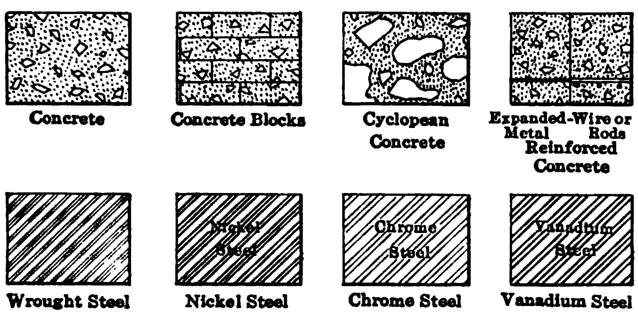


FIG. 33. - Typical Subdivisions
A. S. M. E. standard cross sections.

Filing Notes and Clippings

Every engineer finds a systematic plan of filing and indexing notes of experience and clippings from technical papers a necessity. No technical paper is worth preserving entire, such preservation, in fact, soon defeating its own purpose by the bulk of unclassified information to which it leads.

A satisfactory plan should embody the following features: (1) A minimum of pasting and indexing; (2) indefinite expansibility; (3) notes, clippings and references to books should be kept together; (4) all related information should be grouped together.

Repeated publication in technical papers of methods of filing and indexing leads the author to include here his own method (*Amer. Mach., March* 11, 1909). It is an adaptation of the vertical filing system, a file box (containing one-half the alphabet) being shown in Fig. 34.

The articles which it is desired to preserve are simply clipped out and folded to uniform size. A letter is written at the title of the article to indicate its place in the box, the clipping is dropped into place, and that is all there is of it. The index letters on the clippings are necessary to insure their replacement where they belong after consultation. Many articles may be indexed under several heads, and unless the index letter is used an article is sure to be dropped into the wrong place at some time in the future.

Notes are written on sheets of stiff squared paper, cut to the size to which the clippings are folded, which are dropped in place among the clippings. These sheets also serve for references to books and

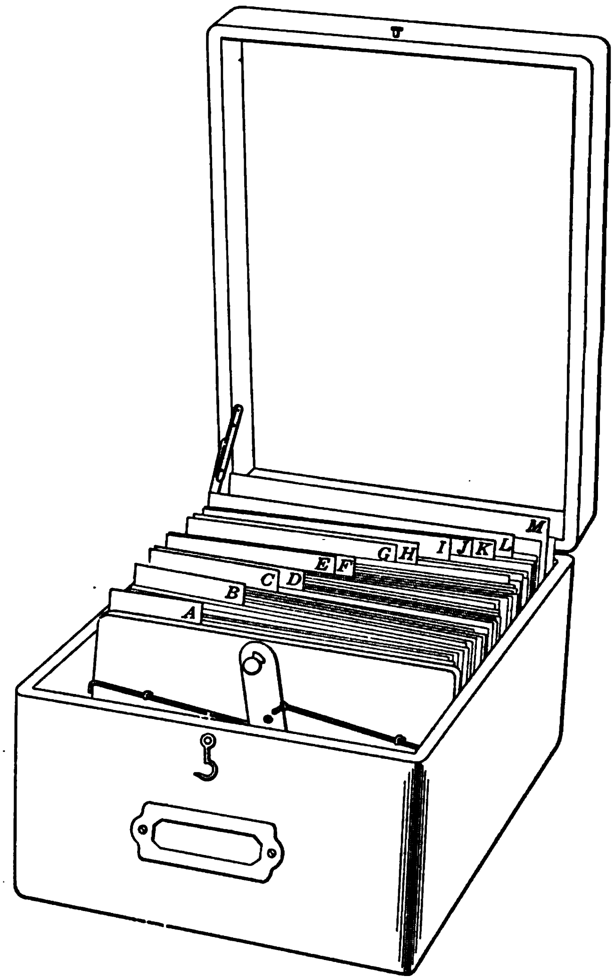


FIG. 34.—Index file for notes and clippings.

for cross references to articles printed on the backs of others. As the collection grows, folders are used to segregate matter on various subjects and when the box is full its contents are divided between two boxes as in the illustration.

The size of the box should be such as to take the standard 6×9 in. page. Pages from standard 9×12 periodicals require a single fold.

Blue Print Solution

1 oz. red prussiate of potash,
$3\frac{1}{2}$ oz. water,
$\frac{3}{4}$ oz. citrate of iron and ammonia,
$3\frac{1}{2}$ oz. water.

Metallic Indicator Paper

The paper should be sized with glue or paste, and zinc oxide, in powder, should be sprinkled on before the size is dry. When dry the paper must be pressed or rolled smooth. Running it through a photographic burnisher gives the best surface.

For a pencil, use a brass wire with end rounded and buffed smooth.

Fig. 35.—American railroad clearances. Official composite cross-section by the American Railway Association.

For the carbon content of steel suitable for various cutting tools, see Index.

Power Constants for Lathe Tools

The pressure on cutting tools formed the subject of an exhaustive investigation by F. W. Taylor and his associates (*Trans. A. S. M. E., Vol.* 28). Following are Mr. Taylor's general conclusions regarding the tangential pressure of the chip on lathe tools when cutting cast-iron.

(*A*) The total tangential pressure of the chip on the tool in cutting cast-iron of the different qualities experimented upon varies between the low limit of 35 tons (of 2000 lbs.) per sq. in. sectional area of chip for soft cast-iron, when a coarse feed is used, and 99 tons per sq. in. sectional area of chip for hard cast-iron, when a fine feed is used.

(*B*) In cutting the same piece of cast-iron, the pressure of the chip on the tool per sq. in. sectional area of chip grows considerably greater as the chip becomes thinner, and slightly greater as the cut becomes more shallow in depth. The following are the high and low limits of pressure per sq. in. of sectional area of the chip when light and heavy cuts are taken on the same piece of cast-iron:

Depth of cut $\frac{1}{8}$ in.×feed .0328 in.: Total pressure per sq. in sectional area of chip, 128,000 lbs. Depth of cut $\frac{27}{64}$ in.×feed .1292 in.: Total pressure per sq. in. sectional area of chip, 75,000 lbs.

(*C*) The same fact mathematically expressed is that in cutting the same piece of cast-iron, the pressure of the chip on the tool per sq. in. sectional area of chip grows greater as the thickness of the chip grows less in proportion to (thickness of feed) $^{\frac{1}{4}}$.

The pressure of the chip per sq. in. of section also grows greater as the depth of the cut grows less in proportion to (depth of cut) $^{\frac{1}{15}}$.

(*D*) The effect upon the pressure of the chip on the tool of a change in the thickness of the feed and the depth of the cut is the same for hard and soft cast-iron, and is represented by the same general formula, with a change merely of the constant.

(*E*) In taking cuts having the same depth and the same feed, the pressure of the chip on the tool becomes slightly greater the larger the cutting tool that is used. This increase in the pressure follows from the fact that the larger the curve of the cutting edge of the tool the thinner the shaving becomes.

Following are the corresponding conclusions regarding the tangential pressure on the chip when cutting steel:

(*A*) The total pressure of the chip on the tool in cutting steel of the different qualities experimented upon varies between the low limit of 92 tons (of 2000 lbs.) per sq. in., and the high limit of 168 tons per sq. in. sectional area of the chip.

(*B*) In cutting the same piece of steel, the pressure of the chip on the tool per sq. in. of sectional area of the chip grows very slightly greater as the chip becomes thinner, and is practically the same whether the cut is deep or shallow. The following are typical cases illustrating the relative pressures of a thin feed on the one hand and a coarse feed on the other:

Depth of cut $\frac{3}{16}$ in.×feed .0156 in.: Total pressure per sq. in. sectional area of chip, 295,000 lbs. Depth of cut $\frac{3}{16}$ in.×feed .125 in.: Total pressure per sq. in. sectional area of chip 257,000 lbs.

(*C*) The same fact mathematically expressed is that in cutting the same piece of steel, the pressure of the chip on the tool per sq. in. of sectional area of the chip grows faster as the thickness of the chip grows less in proportion to (thickness of feed) $^{\frac{1}{15}}$.

The pressure of the chip is in direct proportion to the depth of the cut.

(*D*) Within the limits of cutting speed in common use, the pressure of the chip upon the tool is the same whether fast or slow cutting speeds are used.

(*E*) The pressure of the chip upon the tool depends but little upon the hardness or softness of the steel being cut, but increases as the quality of the seel grows finer. In other words, high grades of steel, whether soft or hard, give greater pressures on the tool than are given by inferior qualities of steel.

(*F*) The pressure of the chip on the tool per sq. in. of sectional area of the chip depends both upon the tensile strength of the steel and its percentage of stretch, and increases both as the tensile strength and stretch increase; although a higher tensile strength has more effect than a large percentage of stretch in increasing the pressure.

Mr. Taylor considers the most important conclusion resulting from his experiments on the pressure of the chip on the tool to be that tho gearing designed in lathes, boring mills, etc., for feeding the tool should be sufficiently strong to deliver at the nose of the tool a feeding pressure equal to the entire driving pressure of the chip upon the lip surface of the tool.

The pressures on cutting tools may be determined from Figs. 1 and 2, by H. L. Seward (*Amer. Mach., Nov.* 16, 1911), which represent the formulas developed by Mr. Taylor as follows:

For steel, $P = 230{,}000\ DF^{\frac{14}{15}}$

For cast-iron, $P = CD^{\frac{14}{15}}F^{\frac{3}{4}}$

in which P = tangential pressure, lbs.,

D = depth of cut, ins.,

F = feed, ins.,

C = a constant which varies from 45,000 for soft cast-iron to 69,000 for hard cast-iron.

Note that in Mr. Taylor's experiments the pressures were measured *at the tool* and do not include the effort necessary to drive the machine.

Below the charts will be found directions for their use.

Power Constants for Twist Drills

The torque and thrust of twist drills formed the subject of extensive tests by Dempster Smith and P. Poliakoff (*Proc. I. M. E.*, 1909). The results have been plotted by W. T. Sears, Mech. Engr., Niles-Bement-Pond Co. (*Amer. Mach., Sept.* 5, 1912) whose charts are repeated in Figs. 3 and 4 from which the torgue and thrust of twist drills within their range may be obtained. The use of the charts is explained below them.

The steel experimented upon was of medium hardness—.29 per cent. carbon, .625 per cent. manganese.

Note that in the experiments the torque was measured at the drill and does not include the friction of the driving mechanism.

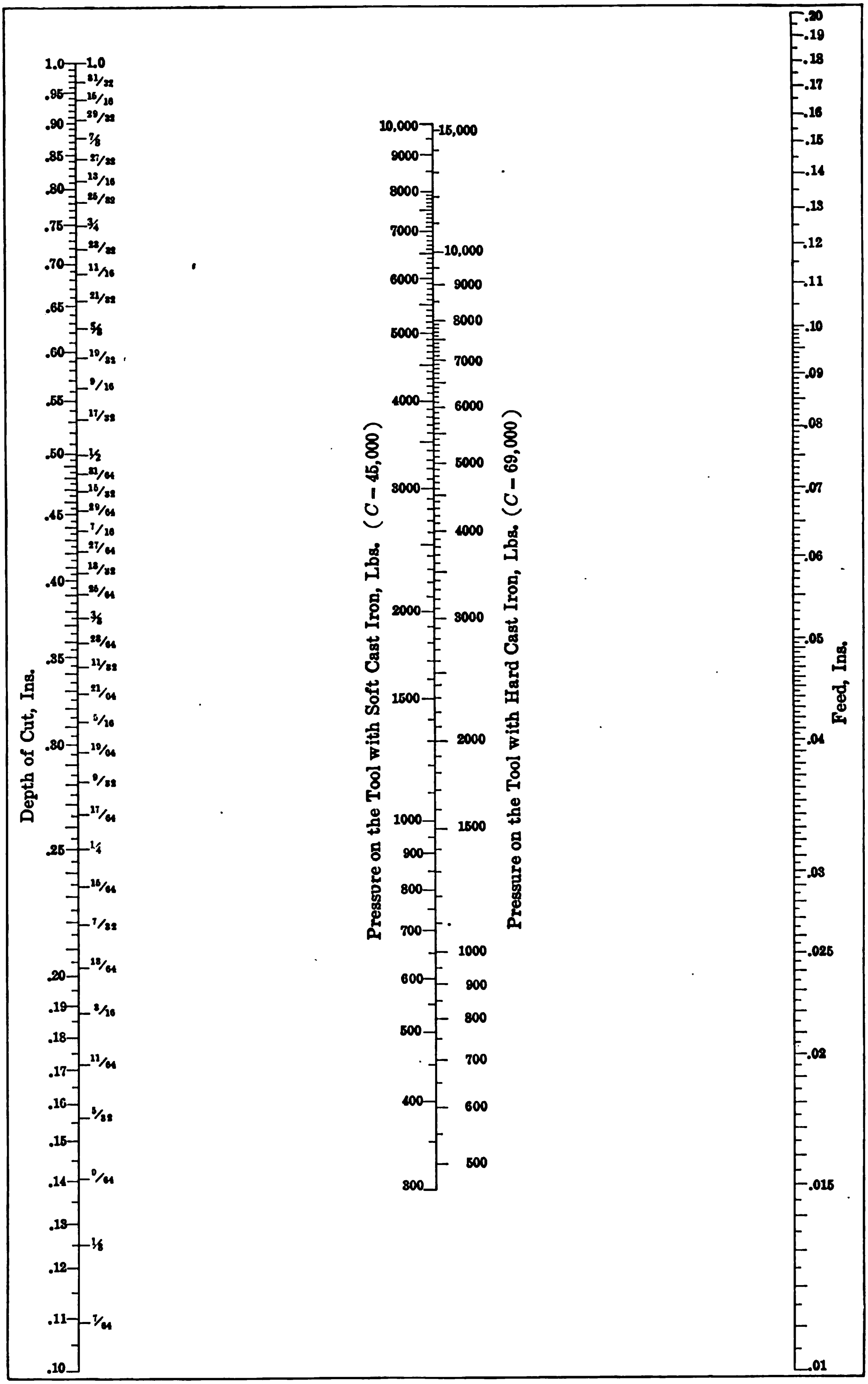
Experiments on smaller drills in cast-iron only, were made by C. S. Farey and E. A. Adams (*Journal Worcester Polytechnic Institute*, 1906). The results for torque, after averaging, are presented in Fig. 5 (*Amer. Mach., Feb.* 14, 1907) together with those for thrust (Henry Hess, *Amer. Mach., Apr.* 25, 1907). The use of the torque chart is best shown by an example:

For a $\frac{3}{4}$-in. drill at a feed of 15 thousandths per revolution, raise a perpendicular from the intersection *a* of the size of drill, and the 15 thousandths feed lines to the observation line. To find the height of the intersection, *ab* may be taken in the dividers and compared with the vertical scale of torques, or we may follow one set of diagonals to *c*, and the other to *d*, and there read 195 lb.-ins.

Size of Drill	Drill Speed in R.P.M. High Speed Steel Drill, Cast Iron	High Speed Steel Drill, Steel	Carbon Steel Drill, Cast Iron	Carbon Steel Drill, Steel
3/8	1019	1019	357	271
25/64				
13/32				
27/64				
7/16	873	873	306	226
29/64				
15/32				
31/64				
1/2	754	764	267	188
33/64				
17/32				
35/64				
9/16	679	679	238	170
37/64				
19/32				
39/64				
5/8	612	612	214	150
41/64				
21/32				
43/64				
11/16	556	556	195	135
45/64				
23/32				
47/64				
3/4	510	510	178	121
49/64				
25/32				
51/64				
13/16	471	471	165	110
53/64				
27/32				
55/64				
7/8	437	437	153	100
57/64				
29/32				
59/64				
15/16	408	408	143	92
61/64				
31/32				
63/64				
1	382	382	134	85
1 1/64				
1 1/32				
1 3/64				
1 1/16	359	359	126	79
1 5/64				
1 3/32				
1 7/64				
1 1/8	340	336	119	73
1 9/64				
1 5/32				
1 11/64				
1 3/16	322	317	113	68
1 13/64				
1 7/32				
1 15/64				
1 1/4	306	298	107	63
1 17/64				
1 9/32				
1 19/64				

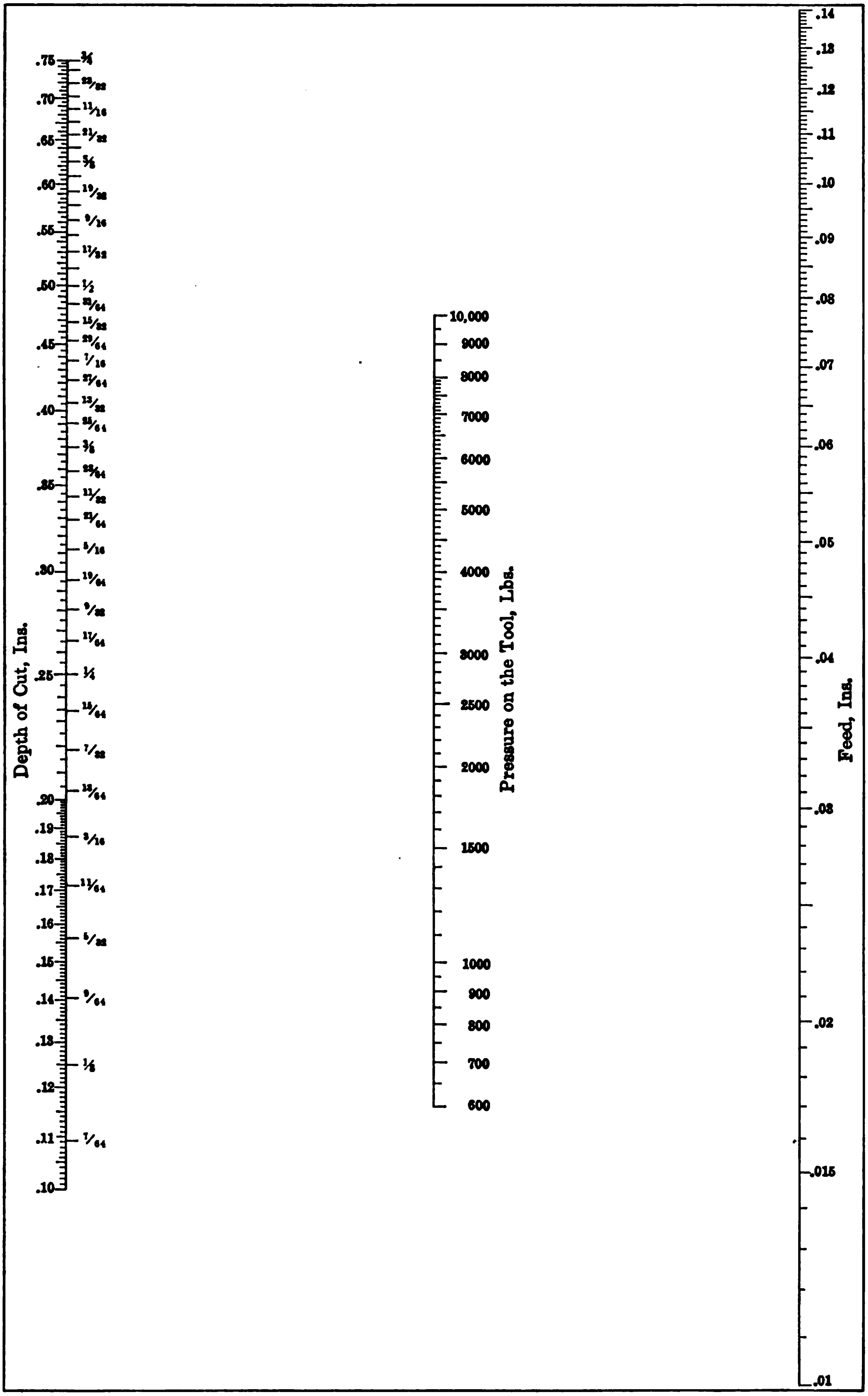
Size of Drill	Drill Speed in R.P.M. High Speed Steel Drill, Cast Iron	High Speed Steel Drill, Steel	Carbon Steel Drill, Cast Iron	Carbon Steel Drill, Steel
1 5/16	291	282	102	59
1 21/64				
1 11/32				
1 23/64				
1 3/8	278	267	97	56
1 25/64				
1 13/32				
1 27/64				
1 7/16	266	254	93	52
1 29/64				
1 15/32				
1 31/64				
1 1/2	255	241	88	49
1 17/32				
1 9/16	244	230	86	46
1 19/32				
1 5/8	235	220	82	43
1 21/32				
1 11/16	226	211	79	41
1 23/32				
1 3/4	218	202	76	39
1 25/32				
1 13/16	211	194	73	37
1 27/32				
1 7/8	204	187	71	35
1 29/32				
1 15/16	197	179	69	33
1 31/32				
2	191	172	67	31
2 1/32				
2 1/16	185	165	65	29
2 3/32				
2 1/8	180	159	63	28
2 5/32				
2 3/16	175	154	61	26
2 7/32				
2 1/4	170	149	59	25
2 9/32				
2 5/16	165	144	57	24
2 11/32				
2 3/8	161	139	56	23
2 13/32				
2 7/16	157	134	54	21
2 15/32				
2 1/2	153	130	53	20
2 17/32				
2 9/16	149	126	52	19
2 19/32				
2 5/8	145	122	51	18
2 21/32				
2 11/16	142	118	50	17
2 23/32				
2 3/4	139	115	49	16
2 25/32				
2 13/16	136	110	47	15
2 27/32				
2 7/8	133	107	46	14
2 29/32				
2 15/16	130	104	45	13
2 31/32				
3	127	102	44	13

Table 1.—Speed of twist drills. By H. M. Norris (*Amer. Mach.*, *July* 20, 1911).



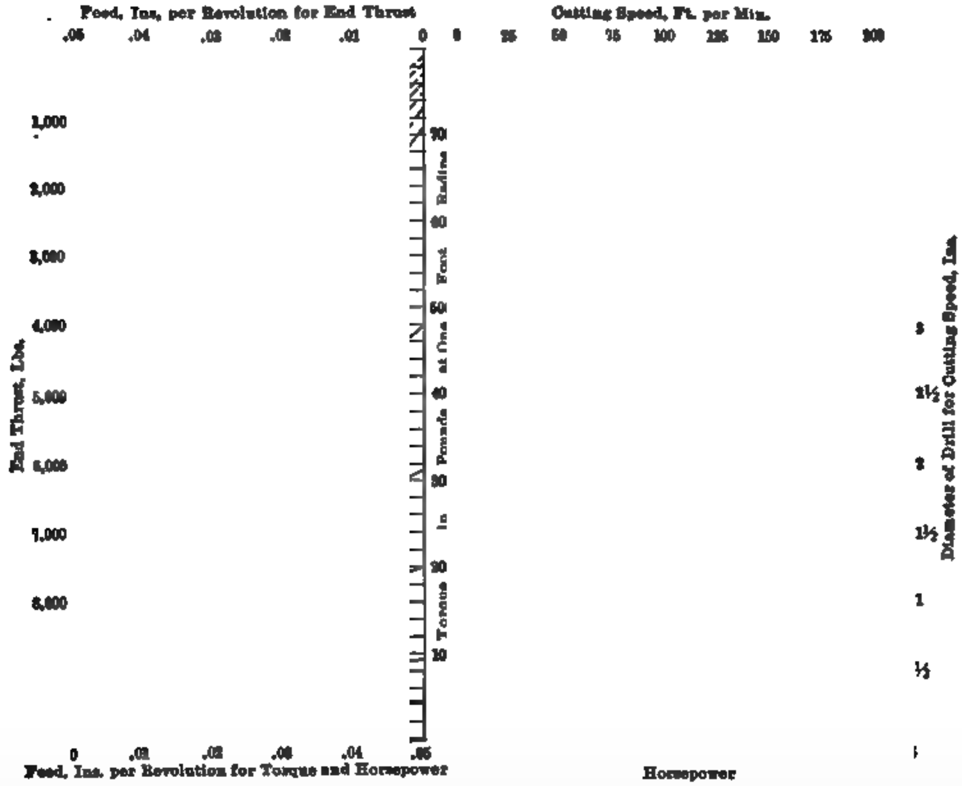
Connect the depth of cut and the rate of feed and read the pressure on the tool from the middle scale.

FIG. 1.—Pressure on lathe tools when cutting cast-iron.



Connect the depth of cut and the rate of feed and read the pressure on the tool from the middle scale.

Fig. 2.—Pressure on lathe tools when cutting steel.



Enter the left, lower scale of *feed per resolution of spindle for tongue and horse-power* at the point representing the given feed; trace vertically upward to the *full* curve of the given drill diameter; then horizontally from this intersection to the right, crossing the scale *torque in lbs. at 1 ft. radius*, from which the torque can be read; then continuing to the inclined line of the given speed. From this intersection trace vertically downward to the *horse-power* scale and read the horse-power required. To find the end thrust: Enter the left upper scale of *feed in ins. per resolution for end thrust* with the given feed and trace vertically downward to the *broken* line representing the size of drill; then horizontally from this intersection to the left to the scale *end thrust in lbs.* from which the thrust can be read. To find the cutting speed: Enter the right vertical scale *diameter of drill in ins. for cutting speed* with the given size of drill; trace horizontally to the left to an intersection with the line representing the given speed in rev. per min.; then vertically upward to the scale *cutting speed in ft. per min.* and read the cutting speed required.

Fig. 3.—Torque, end thrust and horse-power of twist drills drilling cast-iron.

As it is always permissible to use the results of a set of experiments somewhat beyond their actual range, the diagram has been extended in dotted lines to include drills up to 1½-ins. and feeds up to 30 thousandths—the full line portion of the diagram representing the field of the actual experiments, and the dotted portions the extensions. The length of *ef*, obtained by the dividers or by following the diagonals as before, shows the torque for a 1-in. drill under a feed of 25 thousandths to be 476 lb.-ins.

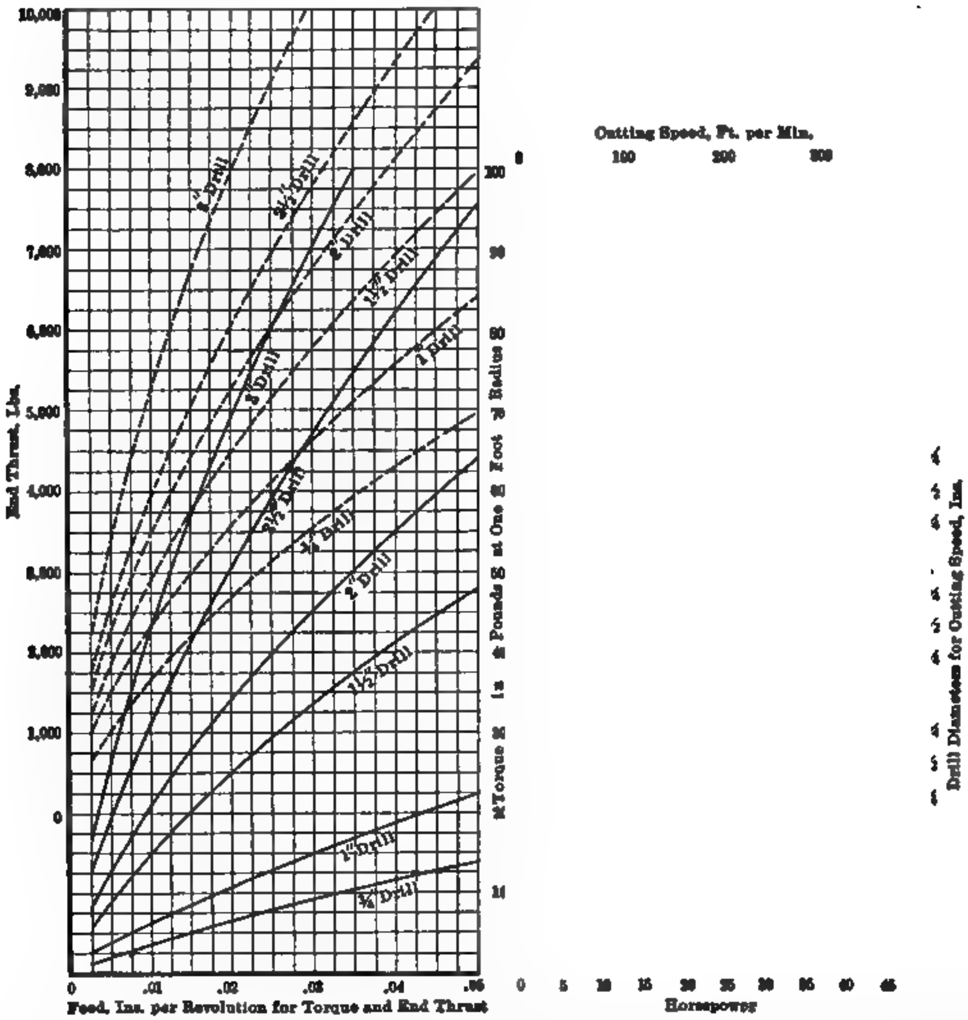
In these experiments, also, the torque was measured at the drill. The chart for thrust is self-explanatory.

Power Constants for Drilling Machines

A very complete series of tests of the power consumed in drilling cast-iron from the solid was made by the Bickford Drill and Tool Co., and reported by H. M. Norris, mechanical engineer of the company (*Amer. Mach., Sept.* 18, 1902) and given here in Fig. 6. The tests were made on a No. 1 Bickford New Radial machine, the power being obtained from an electric motor. The horse powers given are the equivalents of the current readings and include the motor losses. Consultation of the chart will show a sudden increase of power consumption with the 1½-in. drill, due, at least in large measure, to a change in the driving gear train at this point by which the load was carried through an additional gear which causes the power consumed by the idle machine to increase from .24 to .8 h.p.

The chart gives the relation between the feed and the total amount of power consumed for all the observations. The vertical scale gives the horse-power and the horizontal scale the feed in thousandths of an inch per revolution. The smaller figures just within the base line give the feeds actually used in the tests. The upper left-hand part of the diagram gives the data for the larger sizes of drills and for this a second set of horse-power figures is given. The figures for the feed are common to both parts of the chart. The dots which represent the data for the diffrent drills are distinguished from one another as indicated in the small table which appears on the chart. The data for the ½-, ¾-, 1-, and 1¼-in. drills form an extremely satisfactory and progressive set, but there is an abrupt change at the 1½-in. drill for which we have no explanation. The line for this drill begins at the left about where it should to continue the series, but it runs at a much steeper angle and this change in the angle appears to represent a change in the law, as the lines for the larger drills are much more nearly parallel to the 1½-in. line than to the lines for the smaller sizes.

There is less appearance of a law connecting the different sizes in the upper part of the diagram than in the lower. The line for the 2-in. drill is below the one for the 1¾-in., but the 2½-in. is above the 2-in., while the 2½-in. repeats the erratic angle of the 1½-in. Differences of grinding, of sharpness and of quality in the drills themselves naturally enter into the data between different drills, while those factors are, or may be made, constant in all tests on any



Enter the left lower scale of *feed in ins. per revolution for torque and end thrust* at the point representing the given feed; trace vertically upward to an intersection with the *full* curve representing the given drill diameter; then horizontally to the right, crossing the scale of *torque in lbs. at 1 ft. radius*, from which the required torque may be read; then to the line representing the given drill speed and from this intersection vertically downward to the horse-power scale, from which the power may be read. To find the end thrust: Enter the scale of feed per revolution; trace vertically upward to the *broken* line representing the size of drill. From this intersection trace horizontally to the left to the scale *end thrust in lbs.* from which the desired thrust may be read. The cutting speed is found in the same manner as on the preceding chart for cast-iron.

FIG. 4.—Torque, end thrust and horse-power of twist drills drilling steel.

one drill. It is hence to be expected that tests between different drills will show more erratic results than tests of the same drill under varying conditions.

It may fairly be accepted that these tests establish the fact that the law connecting the power with the rate of feed is a straight-line law.

Power Constants for Milling Machines

Two very complete sets of tests of the power required for slab milling were made by Alfred Herbert, Ltd., and reported by P. V. VERNON (*The Engineer*, 1909), the machine used being of the Herbert knee type. The following data are extracted from Mr. Vernon's report. The horse-powers are the equivalents of the current readings and include the motor losses and also a constant loss of 1.8 h.p. consumed in driving a jack shaft and countershaft through which the power was transmitted.

Slabbing mild steel, average of 44,	2.52 h.p. per cu. in. per min.
Slabbing mild steel, minimum,	1.95 h.p. per cu. in. per min.
Slabbing mild steel, maximum,	3.02 h.p. per cu. in. per min.
Slabbing cast-iron, average of 38,	1.10 h.p. per cu. in. per min.
Slabbing cast-iron, minimum,	.89 h.p. per cu. in. per min.
Slabbing cast-iron, maximum,	1.25 h.p. per cu. in. per min.

In these tests, in cast-iron, the feeds ranged between $1\frac{11}{16}$ and $10\frac{7}{16}$ ins. per min., the depth of cut between .14 and 1.10 ins. and the material removed between 7.39 and 15.23 cu. ins. per min. In steel, the feeds ranged between $\frac{5}{8}$ and $10\frac{3}{4}$ ins. per min., the depth of cut between .10 and 1.10 ins. and the material removed between 2.88 and 6.27 cu. ins. per min.

More recent tests by Mr. VERNON (*Proc. Manchester Asso. of Engrs.*, 1912) have led to the following conclusions:

1. A 5-in. double belt driving a 16-in. pulley at a speed of 400 r.p.m. (100,531 sq. ins. of belt surface per min.) geared to drive a

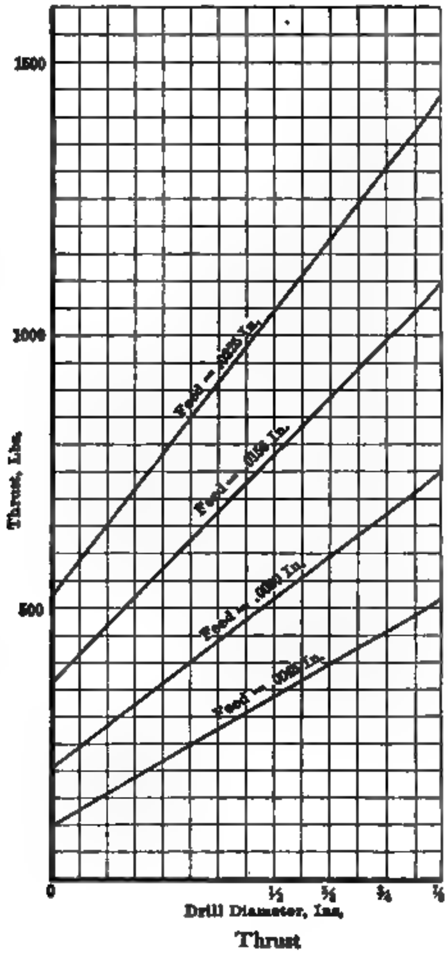


Fig. 5.—Torque and end thrust of twist drills drilling cast-iron.

4½-in. cutter at 70 ft. per min., is able to remove as much as 48.1 cu. ins. of cast-iron and 24.31 cu. ins. of mild steel in a minute.

2. 2090 sq. ins. of double belt passing over a pulley in 1 min. will remove 1 cu. in. of cast-iron on a miller.

3. 4135 sq. ins. of double belt passing over a pulley in 1 min. will remove 1 cu. in. of mild steel on a miller.

4. A 4½-in. cutter on a 2-in. arbor running at 70 ft. per min. is capable of removing at least 3.63 cu. ins. and possibly as much as 6.01 cu. ins. of cast-iron, and at least 2.125 cu. ins., and possibly as much as 3.03 cu. ins. of mild steel per min. for each inch of width up to 8 ins.

Tests by A. L. DeLeeuw for the Cincinnati Milling Machine Co. on knee type machines, gave the results shown in Fig. 7 (*Trans. A. S. M. E.*, 1911). The horse-powers given are the net outputs of the motors after the motor losses have been deducted from the current readings.

Additional tests by Mr. DeLeeuw (*Amer. Mach.*, *Aug.* 8, 1912) give the results of Tables 1–6. As a rule the material is specified by reference to a particular block whose physical properties are given in Table 7. Where there is no reference of this nature the material was machinery steel of a tensile strength of 55,000 lbs. per sq. in., .26 per cent. carbon and .5 per cent. manganese.

Table 1.—Results of Cutting Tests with an 8-in., 12-bladed, High-power Face Mill in Machinery Steel, Cut 5 Ins. Wide, Block *B*

Depth of cut, ins.	Spindle speed, r.p.m.	Feed in ins. per min.	H.p. delivered to machine	Cu. ins. of metal removed per min.	Cu. ins. of metal removed per h.p. min.
¼	20½	12.31	10.96	7.69	.702
	20½	12.26	11 52	7.66	664
	20½	12.26	11 52	7.66	664
	20½	12.24	11.52	7.65	.664
5/16	20½	7.51	10.96	7.04	.642
	20	7.34	10 70	6.88	.643
	20	7.38	11 25	6.92	.615
	20½	7.61	11.52	7.13	.618
⅜	20	5 9	12 62	7 375	.584
	20½	5.97	13 20	7.45	.564
	20	5.9	13.20	7.375	.558
	20½	5 97	13 46	7.45	554
7/16	20	4.54	13.20	7.09	.537
	20½	4.66	13.46	7.28	542
	20½	4 68	13.20	7.31	.554
	19½	4 46	13 46	6.97	.518
½	20	3.48	10.96	6.53	506
	20	3 49	11 81	6 54	.554
	20	3 54	12 62	6.64	.526

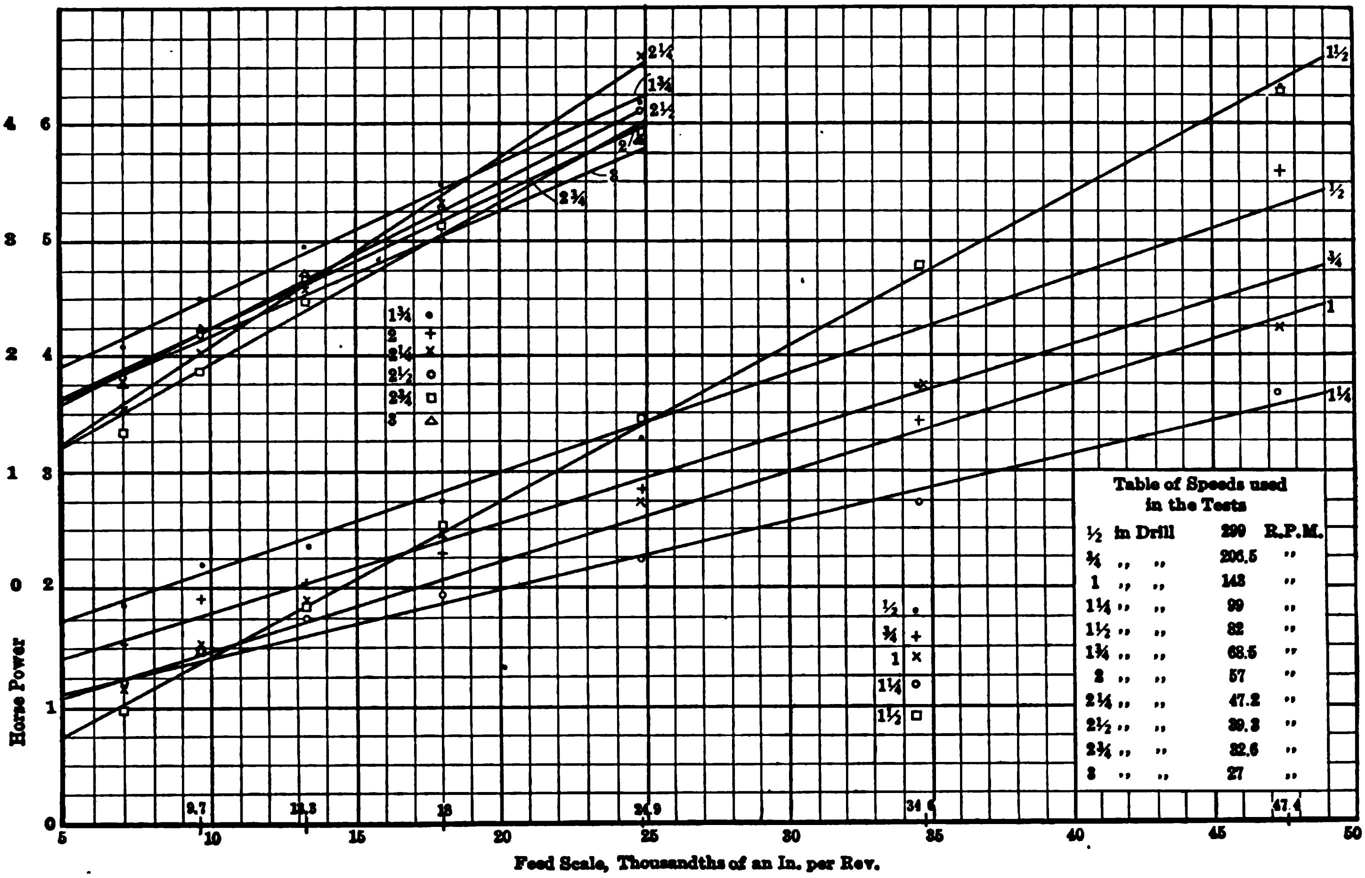


FIG. 6.—Horse-power required to drive drilling machines in cast-iron.

TABLE 2.—RESULTS OF CUTTING TESTS WITH AN 8-IN., 12-BLADED, HIGH-POWER FACE MILL IN MACHINERY STEEL, CUT 5 INS. WIDE, BLOCK *A*

Depth of cut, ins.	Spindle speed, r.p.m.	Feed in ins. per min.	H.p. delivered to machine	Cu. ins. of metal removed per min.	Cu. ins. of metal removed per h.p. min.
1/8	19	11.34	8.113	7.088	.873
	19½	11.72	7.818	7.325	.937
	19½	11.53	7.555	7.21	.954
	20	11.81	7.555	7.381	.977
1/8	26	16.0	14.843	10.000	.674
	25	15.4	13.473	9.625	.714
	25	15.4	13.473	9.625	.714
	25	15.4	13.473	9.625	.714
3/16	20	7.44	10.319	6.975	.686
	20	7.29	9.749	6.834	.701
	20	7.32	10.319	6.863	.665
	20	7.32	10.319	6.863	.665
1/4	19½	5.75	11.41	7.188	.63
	20	5.81	11.959	7.263	.608
	19½	5.69	11.138	7.113	.638
	20	5.84	11.41	7.30	.64
5/16	20½	4.6	12.52	8.568	.684
	20	4.47	11.959	8.325	.696
	20	4.57	11.959	8.512	.712

In all these tests the efficiency of the motor was taken into consideration, and the horse-power values given have the motor losses eliminated.

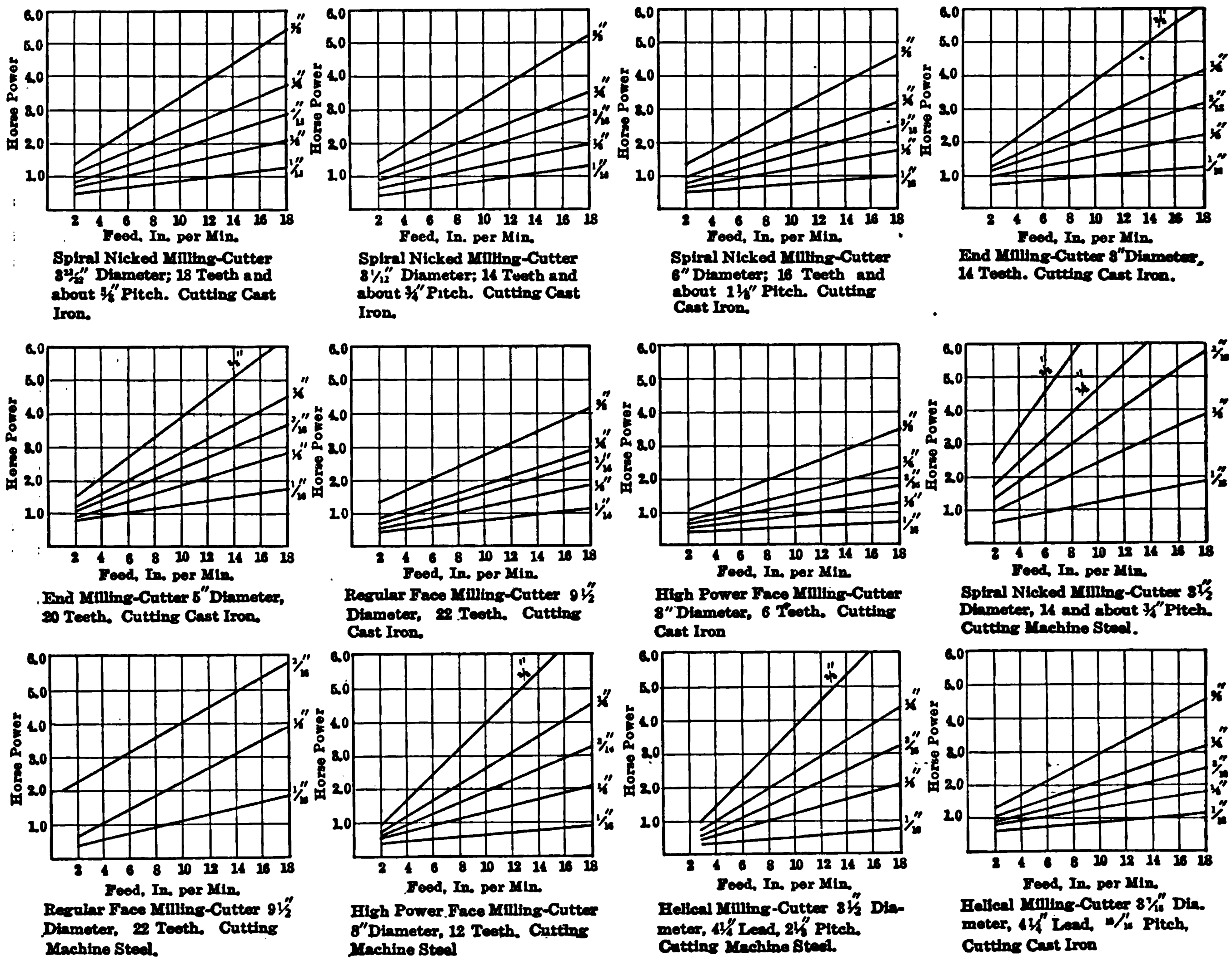
Table 8 gives the results of cuts by Mr. DeLeeuw on German irons and steels. The tests of speigeleisen are noteworthy, showing but little more power consumed than for cast-iron.

In a report of some extremely (record) heavy slab milling on Niles-Bement-Pond planer type machines, W. H. TAYLOR (*Amer. Mach., Jan.* 14, 1909) gives data from which the following are extracted. The horse-powers are the equivalents of the current readings, and include the motor losses.

Slabbing mild steel, average of 15,	2.07 h.p. per cu. in. per min.
Slabbing mild steel, minimum,	1.52 h.p. per cu. in. per min.
Slabbing mild steel, maximum,	3.71 h.p. per cu. in. per min.
Slabbing cast-iron, average of 8,	1.06 h.p. per cu. in. per min.

TABLE 3.—RESULTS OF CUTTING TESTS WITH AN 8-IN., 12-BLADED, HIGH-POWER FACE MILL IN MACHINERY STEEL, CUT 5 INS. WIDE, BLOCK *A*

Depth of cut, ins.	Spindle speed, r.p.m.	Feed in ins. per min.	H.p. delivered to machine	Cu. ins. of metal removed per min.	Cu. ins. of metal removed per h.p. min.
1/8	20	11.92	7.92	7.45	.94
	20	11.83	7.34	7.39	1.007
	19½	11.73	7.62	7.33	.962
	19½	11.73	7.62	7.33	.962
3/16	19½	7.25	7.34	6.80	.927
	20	7.29	7.07	6.83	.967
	20	7.29	7.07	6.83	.967
	19½	7.25	7.07	6.80	.964
1/4	19½	5.7	8.17	7.12	.872
	19	5.58	7.62	6.97	.915
	19	5.54	8.17	6.92	.847
	19	5.58	8.17	6.97	.854
5/16	20	4.53	8.17	7.08	.867
	20	4.56	7.34	7.125	.972
	20	4.56	7.92	7.125	.90
	20	4.53	8.17	7.08	.867
3/8	20	3.47	7.07	6.51	.921
	20	3.54	7.62	6.64	.873
	20	3.49	7.07	6.54	.925
	20	3.52	7.62	6.60	.868



Spiral Nicked Milling-Cutter 3 23/32" Diameter; 18 Teeth and about 5/8" Pitch. Cutting Cast Iron.

Spiral Nicked Milling-Cutter 3 1/32" Diameter; 14 Teeth and about 3/4" Pitch. Cutting Cast Iron.

Spiral Nicked Milling-Cutter 6" Diameter; 16 Teeth and about 1 1/8" Pitch. Cutting Cast Iron.

End Milling-Cutter 8" Diameter, 14 Teeth. Cutting Cast Iron.

End Milling-Cutter 5" Diameter, 20 Teeth. Cutting Cast Iron.

Regular Face Milling-Cutter 9 1/2" Diameter, 22 Teeth. Cutting Cast Iron.

High Power Face Milling-Cutter 8" Diameter, 6 Teeth. Cutting Cast Iron

Spiral Nicked Milling-Cutter 3 1/2" Diameter, 14 and about 3/4" Pitch. Cutting Machine Steel.

Regular Face Milling-Cutter 9 1/2" Diameter, 22 Teeth. Cutting Machine Steel.

High Power Face Milling-Cutter 8" Diameter, 12 Teeth. Cutting Machine Steel

Helical Milling-Cutter 3 1/2" Diameter, 4 1/4" Lead, 2 1/8" Pitch. Cutting Machine Steel.

Helical Milling-Cutter 3 7/16" Diameter, 4 1/4" Lead, 15/16 Pitch, Cutting Cast Iron

Radiating lines give depth of cut. The observations are reduced to 1 in. width of cut. Composition of steel: carbon, .20; manganese, .50.

Fig. 7.—Power required to drive milling machines in machinery steel.

Table 4.—Results of Cutting Tests with an 8-in., 12-bladed, High-power Face Mill in Machinery Steel, Cut 5 Ins. Wide, Block C

Depth of cut, ins.	Spindle speed, r.p.m.	Feed in ins. per min.	H.p. delivered to machine	Cu. ins. of metal removed per min.	Cu. ins. of metal removed per h.p. min.
1/8	22	12.96	6.85	8.10	1.183
	21 1/2	12.89	7.14	8.056	1.128
	21 1/2	12.89	6.85	8.056	1.177
	22	13.00	7.42	8.125	1.096
3/16	21 1/2	7.96	6.85	7.462	1.09
	21 1/2	7.96	6.85	7.462	1.09
	21 1/2	7.92	6.85	7.425	1.084
	21 1/2	7.87	6.85	7.378	1.078
1/4	21	6.16	7.42	7.70	1.039
	21 1/2	6.22	7.14	7.775	1.09
	21 1/2	6.26	7.14	7.825	1.096
5/16	21 1/2	4.89	7.14	7.64	1.07
	21 1/2	4.85	7.14	7.578	1.062
	21 1/2	4.88	7.71	7.625	.99
	22	4.92	7.42	7.687	1.037
3/8	21 1/2	3.80	7.14	7.13	1.000
	22	3.86	6.85	7.237	1.056
	21 1/2	3.78	6.85	7.08	1.034
	21 1/2	3.80	6.85	7.13	1.042

Table 5.—Results of Cutting Tests with 4 1/2-in., 10-toothed, Spiral-nicked Cutter on Machinery Steel, Cut 5 Ins. Wide

Depth of cut, ins.	Spindle speed, r.p.m.	Feed in ins. per min.	H.p. delivered to machine	Cu. ins. of metal removed per min.	Cu. ins. of metal removed per h.p. min.
3/8	40 1/2	6.22	12.35	11.66	.944
	41	6.25	12.60	11.72	.93
	41	6.27	12.60	11.76	.933
3/8	40	7.8	15.12	14.625	.968
	40	7.8	14.55	14.625	1.001
	40	7.8	14.82	14.625	.987
1/2	40	6.09	15.93	15.225	.955
	39 1/2	6.06	15.93	15.15	.95
	39 1/2	6.05	15.93	15.125	.948
	40	6.11	16.20	15.275	.943
5/8	40	4.71	17.07	14.72	.863
	39 1/2	4.68	17.90	14.62	.816
	39 1/2	4.68	18.75	14.62	.779
	39	4.66	17.07	14.56	.853
3/4	39	3.62	18.20	13.575	.745
	39	3.62	18.20	13.575	.745
	39 1/2	3.63	17.63	13.61	.772
	39 1/2	3.63	17.38	13.61	.782

TABLE 6.—RESULTS OF CUTTING TESTS WITH A 10-IN., 16-BLADED, HIGH-POWER FACE MILL IN MACHINERY STEEL, CUT 5 INS. WIDE

Depth of cut, ins.	Spindle speed, r.p.m.	Feed in ins. per min.	H.p. delivered to machine	Cu. ins. of metal removed per min.	Cu. ins. of metal removed per h.p. min.
½	15½	4.61	12.64	11.525	.912
	15½	4.61	12.94	11.525	.89
	15½	4.65	12.94	11.625	.898
	15	5.81	17.05	14.525	.852
	15	5.81	16.24	14.525	.893
	15	5.81	16.50	14.525	.881
⅜	15½	7.56	13.20	14.175	1.074
	15¾	7.63	13.20	14.30	1.083
	15½	7.56	12.64	14.175	1.121
	15¾	7.66	12.64	14.37	1.137
	15¾	7.63	12.94	14.30	1.106
5/16	16	7.70	11.00	12.02	1.092
	16	7.68	11.00	12.00	1.09
	15¾	7.63	11.27	11.92	1.058

TABLE 7.—PHYSICAL PROPERTIES OF MACHINERY STEEL BLOCKS USED IN THE TESTS

Block	Limit of elasticity, lbs. per sq. in.	Elongation per cent. of length	Reduction per cent. of section
A	36,400	36	66
B	36,200	36.5	59.6
C	37,400	36.5	60

Slabbing cast-iron, minimum,	.79 h.p. per cu. in. per min.
Slabbing cast-iron, maximum,	1.53 h.p. per cu. in. per min.
Channelling mild steel, average of 6,	2.33 h.p. per cu. in. per min.
Channelling mild steel, minimum,	1.69 h.p. per cu. in. per min.
Channelling mild steel, maximum,	3.33 h.p. per cu. in. per min.

These tests were of extraordinary severity, a feed in steel as high as 9½ ins. per min. under a cut of ¾ in. depth, consuming 162 h.p. and removing 82 cu. ins. per min., having been included. In cast-iron the extreme duty was the removal of 105 cu. ins. per min. under a feed of 7 ins. per min. and a depth of cut of 1 in., the power consump-

TABLE 8.—POWER CONSUMPTION BY MILLING MACHINES OPERATING ON GERMAN IRONS AND STEELS

Material	Cutter	Speed of cutter, r.p.m.	Feed in ins. per min.	Depth of cut, in.	Cu. ins. of metal removed per min.	Cu. ins. of metal removed per h.p. min.
Bar No. 1—Cast steel, 56,000–70,000 lbs. tensile strength; width of material 6 ins.	Cutter: high-power face mill, 10 ins. diameter, high-speed steel.	14	4.82	⅛	3.52	.837
		14	7.83	⅛	5.87	1.005
		14	10.3	⅛	7.74	1.105
		14	6.52	¼	9.80	8.08
	Cutter: spiral mill with nicked teeth, 4 ins. diameter, high-speed steel.	32	12.96	⅛	9.73	.640
		32	6.18	¼	9.27	.608
		32	3.92	⅜	8.80	.620
Bar No. 2—Spiegeleisen, 17,028 lbs. tensile strength; width of material, 4 ins	Cutter: spiral mill with nicked teeth, 4 ins. diameter, carbon steel.	20	12.5	⅛	6.25	1.03
		20	12.5	¼	12.5	1.02
		20	7.5	⅜	11.25	.990
Bar No. 3—Gray iron, German medium hard machine cast-iron, generally used; width of material, 4 ins.	Cutter: high-speed steel, high-power face mill, 16 blades, 10 ins. diameter.	21	16.1	⅛	8.05	2.12
		21	15.7	3/16	11.76	2.18
		21	15.5	¼	15.5	2.31
		21	14.6	⅜	21.9	1.63
	Cutter: high-speed steel, spiral mill, 4½ ins. diameter.	49	20.82	⅛	10.41	1.50
		49	20.70	¼	20.7	2.67
		49	16.24	⅜	24.4	2.17
		49	12.96	½	25.82	1.65
Bar No. 3A—Gray iron, German medium hard machine cast-iron, generally used; width of material, 4 ins.	Cutter: high-speed steel, face mill, 18 blades, 8 ins. diameter.	25	16.4	⅛	8.20	1.412
		25	15.5	¼	15.5	1.84
		25	14.6	⅜	21.9	1.370
		25	9.7	½	19.4	1.64
	Cutter: high-speed steel, spiral mill with nicked teeth, 4½ ins. diameter.	40	20.82	⅛	10.41	1.14
		40	20.70	¼	20.70	1.44
		40	20.52	⅜	30.8	1.97
		40	7.62	⅝	19.05	1.29
Bar No. 4—Siemens-Martin steel, 85,000–99,000 lbs. tensile strength; width of material, 3½ ins.	Cutter: high-speed steel, face mill, 8 ins. diameter.	17	0.9	¼	0.786	.378
		17	2.28	¼	1.99	.538
		17	3.04	¼	2.56	.540
		17	4.05	¼	3.52	.567
	Cutter: high-speed steel, cutter with nicked teeth, 4½ ins. diameter.	16	20.82	⅛	9.10	.587
		16	12.96	3/16	8.49	.575
		16	10.1	¼	8.85	.548
		16	5.31	⅜	6.97	.605
Bar No. 5—Chrome-nickel steel, 122,000–141,000 lbs. tensile strength; width of material, 3⅝ ins.	Cutter: high-speed steel, high-power face mill, 10 ins. diameter.	14	4.82	⅛	2.18	.592
		14	16.4	⅛	7.42	.747
		14	6.53	3/16	4.43	.720
		14	13.3	3/16	9.02	.628
	Cutter: high-speed steel, spiral mill with nicked teeth, 4 ins. diameter.	13	20.86	1/16	4.78	.467
		13	20.86	⅛	9.45	.680
		13	8.44	3/16	5.70	.692
		13	5.31	¼	4.64	.493

tion being 47 h.p. The h.p. per cu. in. decreased as the amount of metal removed increased, as follows, for steel:

Max. duty, 162 h.p., 82. cu. ins. per min. removed, consumption 1.99 h.p. per cu. in.

Min. duty, 29 h.p., 7.82 cu. ins. per min. removed, consumption 3.71 h.p. per cu. in.

and as follows for cast-iron:

Max. duty, 89. h.p., 105 cu. ins. removed per min., consumption .85 h.p. per cu. in.

Min. duty, 40 h.p., 26 cu. ins. removed per min., consumption 1.53 h.p. per cu. in.

Sizes of Motors for Machine Tools

The sizes of motors for machine tools, according to the practice of the Westinghouse Electric and Mfg. Co., are given in Table 9 in which the horse-power recommended is based on average practice; it may be decreased for very light work and must often be increased for heavy work. The class of motor is indicated by the symbols *A*, *B*, *C*, explained below. The meaning of these symbols is sometimes modified by notes under the tables.

(*A*) Adjustable-speed shunt-wound direct-current motors wherever a number of speeds are essential.

(*B*) Constant-speed shunt-wound direct-current motors where the speeds are obtainable by a gear-box or cone-pulley arrangement or where only one speed is required.

(*C*) Squirrel-cage induction motor where direct current is not available. A gear-box or cone-pulley arrangement must be used to obtain different speeds.

TABLE 9.—SIZES OF MOTORS FOR MACHINE TOOLS

BOLT AND NUT MACHINERY

BOLT CUTTERS

Motor—*A*, *B*, or *C*

	Size, ins.	H.p.
Single	1, 1¼, 1½	1 to 2
	1¾, 2	2 to 3
	2¼, 3½	3 to 5
	4, 6	5 to 7½
Double	1, 1½	2 to 3
	2, 2½	3 to 5
Triple	1, 1½, 2	3 to 7½

BOLT POINTERS

Motor—*B* or *C*

1½, 2½	1 to 2

NUT TAPPERS

Motor *A*, *B*, or *C*

Four-spindle	1, 2	3
Six-spindle	2	3
Ten-spindle	2	5

NUT FACING

Motor—*B* or *C*

1, 2	2 to 3

BOLT HEADING, UPSETTING AND FORGING

Motor—*A*,[1] *B*,[2] or *C*[3]

Size, ins.	H.p.
¾ to 1½	5 to 7½
1½ to 2	10 to 15
2½ to 3	20 to 25
4 to 6	30 to 40

[1] Speed variation is sometimes desired when different sizes of bolts are headed on the same machine.

[2] Compound-wound direct-current motor.

[3] Wound secondary or squirrel-cage motor with approximately 10 per cent. slip.

BORING AND TURNING MILLS

Motor—*A*, *B*, or *C*

Size	Horse-power Average	Heavy
37 to 42 ins.	5 to 7½	7½ to 10
50 ins.	7½	7½ to 10
60 to 84 ins.	7½ to 10	10 to 15
7 to 9 ft.	10 to 15	
10 to 12 ft.	10 to 15	30 to 40
14 to 16 ft.	15 to 20	
16 to 25 ft.	20 to 25	

BULLDOZERS OR FORMING OR BENDING MACHINES

Motor—*B*[1] or *C*[2]

With, ins.	Head movement, ins.	H. p.
29	14	5
34	16	7½
39	16	10
45	18	15
63	20	20

[1] Compound-wound motor.

[2] Wound secondary or squirrel-cage motor with approximately 10 per cent. slip.

BUFFING LATHES

Motor—*B* or *C*

Wheels No.	Diam., ins.	H.p.
2	6	¼ to ½
2	10	1 to 2
2	12	2 to 3
2	14	3 to 5

For brass tubing and other special work use about double the above horse-power.

DRILLING AND BORING MACHINES

Motor—*A*, *B*, or *C*

	H.p.
Sensitive drills up to ½ in.	¼ to ¾
Upright drills, 12 to 20 ins.	1
Upright drills, 24 to 28 ins.	2
Upright drills, 30 to 32 ins.	3
Upright drills, 36 to 40 ins.	5
Upright drills, 50 to 60 ins.	5 to 7½

	Horse-power Heavy	Average
Radial drills, 3-ft. arm.	3	1 to 2
Radial drills, 4-ft. arm.	5 to 7½	2 to 3
Radial drills, 5 to 6 and 7-ft. arm.	5 to 7½	3 to 5
Radial drills, 8 to 9 and 10-ft. arm.	7½ to 10	5 to 7½

CYLINDER BORING MACHINES

Motor—*A*, *B*, or *C*

Diam. of spindle, ins.	Max. boring diam., ins.	H.p
4	10	7½
6	30	10
8	40	15

TABLE 9.—SIZES OF MOTORS FOR MACHINE TOOLS—(*Continued*)

PIPE THREADING AND CUTTING-OFF MACHINES

Motor—*A*, *B*, or *C*

Size pipe, ins.	H.p.
1/4 to 2	2
1/2 to 3	3
1 to 4	3
1 1/4 to 6	3 to 5
2 to 8	3 to 5
3 to 10	5
4 to 12	5
8 to 18	7 1/2
24	10

PLANERS

Motor—*A*,[1] *B*,[1] or *C*

Width, ins.	Distance under rail, ins.	H.p.
22	22	3
24	24	3 to 5
27	27	3 to 5
30	30	5 to 7 1/2
36	36	10 to 15
42	42	15 to 20
48	48	15 to 20
54	54	20 to 25
60	60	20 to 25
72	72	25 to 30
84	84	30
100	100	40

Normal length of bed in feet is about 1/4 the width in inches.
See also second table below.
[1] Compound-wound motor.

ROTARY PLANERS

Motor—*A*, *B*, or *C*

Dia. of cutter, ins.	H.p.
24	5
30	7 1/2
36 to 42	10
48 to 54	15
60	20
72	25
84	30
96 to 100	40

HYDROSTATIC WHEEL PRESSES

Motor—*B* or *C*

Size, tons	H.p.
100	5
200	7 1/2
300	7 1/2
400	10
600	15

ROLLS—BENDING AND STRAIGHTENING

Motor—*B*[1] or *C*[2]

Width, ft.	Thickness, ins.	H.p.
4	3/8	5
6	5/16	5
6	7/16	7 1/2
6	3/4	15
8	7/8	25
10	1 1/8	35
10	1 1/2	50
24	1	50

[1] Standard bending roll motor.
[2] Wound secondary induction motor.

PUNCHING AND SHEARING MACHINES

Presses for notching sheet iron, motor—*A*, *B*, or *C*, 1/2 to 3 h.p.

PUNCHES

Motor—*B*[1] or *C*[2]

Dia., ins.	Thickness, ins.	H.p.
3/8	1/4	1
1/2	1/2	2 to 3
5/8	5/8	2 to 3
3/4	3/4	3 to 5
7/8	3/4	5
1	1/2	5
1	1	7 1/2
1 1/4	1	7 1/2 to 10
1 3/4	1	10 to 15
2	1	10 to 15
2 1/2	1 1/2	15 to 25

[1] Compound-wound motor.
[2] Wound secondary or squirrel-cage motor with approximately 10 per cent. slip on the larger sizes.

SHEARS

Motor—*B*[1] or *C*[2]

Width, ins.	Horse-power Cut 3/8-in. iron	Horse-power Cut 3/4 in. iron
30 to 42	3	5
50 to 60	4	7 1/2
72 to 96	5	10

Bolt shears.......................... 7 1/2 h.p.
Double-angle shears.................... 10 h.p.

[1] Compound-wound motor.
[2] Wound secondary or squirrel-cage induction motor with 10 per cent. slip.

LEVER SHEARS

Motor—*B*[1] or *C*[2]

Size, ins.	H.p.
1 × 1	5
1 1/2 × 1 1/2	7 1/2
2 × 2	10
6 × 1	
2 1/2 × 2 1/2	15
1 × 7	
2 3/4 × 2 3/4	20
1 1/2 × 8	
3 1/2 × 3 1/2	30
4 1/2 round	

[1] Compound-wound motor.
[2] Wound secondary or squirrel-cage motor with approximately 10 per cent. slip.

PLATE SHEARS

Motor—*B*[1] or *C*[2]

Size of metal cut, ins.	Cut per min.	Length of stroke, ins.	H.p.
3/8 × 24	35	3	10
1 × 24	20	3	15
2 × 14	15	4 1/4	30
1 × 42	20	4	20
1 1/2 × 42	15	4 1/2	60
1 1/4 × 54	18	6	75
1 1/2 × 72	20	5 1/2	10
1 1/4 × 100	10 to 12	7 1/2	75

[1] Compound-wound motor.
[2] Wound secondary or squirrel-cage motor with approximately 10 per cent. slip.

TIN PLATE SQUARING SHEARS

Motor—*B* or *C*

Size of plates, ins.	Cuts per min.	H.p.
54 × 54 3/16 packs	30	7 1/2
72 × 72 3/16 packs	30	7 1/2

TABLE 9.—SIZES OF MOTORS FOR MACHINE TOOLS—(*Continued*)

SHAPERS

Motor—*A*, *B*, or *C*

Stroke, ins.	H.p. single head
12 to 16	2
18	2 to 3
20 to 24	3 to 5
30	5 to 7½
TRAVERSE HEAD SHAPER	
20	7½
24	10

SAWS—COLD AND CUT OFF

Motor—*A*, *B*, or *C*

Size of saw, ins.	H.p.
20	3
26	5
32	7½
36	10 to 15
42	20
48	25

SLOTTING AND KEY SEATING

Motor—*A*, *B*, or *C*

Stroke, ins.	H.p.
6	3
8	3 to 5
10	5
12	5
14	5 to 7½
16	7½
18	7½ to 10
20	10 to 15
24	10 to 15
30	10 to 15

HORIZONTAL BORING, DRILLING AND MILLING MACHINES

Motor—*A*, *B*, or *C*

Size of spindle, ins.	H.p. for single spindle
3½ to 4½	5 to 7½
4½ to 5½	7½ to 10
5½ to 6½	10 to 15

For machines with double spindles use motors of double the horse-power given.

MULTIPLE SPINDLE DRILL

Motor—*A*, *B*, or *C*

Size of drill, ins.	Up to	H.p.
1/32 to ¼	6 to 10 spindle	3
1/16 to ⅜	10	5
3/16 to ½	10	7½
¼ to ¾	10	10
⅜ to 1	10	10 to 15
2	4	7½
2	6	10
2	8	15

EMERY WHEELS, GRINDERS, ETC.

Motor—*B* or *C*

Wheels No.	Size, ins.	H.p.
2	6	½ to 1
2	10	2
2	12	3
2	18	5 to 7½
2	24	7½ to 10
2	26	7½ to 10

MISCELLANEOUS GRINDERS

Motor—*B* or *C*

	H.p.
Wet tool grinder	2 to 3
Flexible swinging, grinding, and polishing machine	3
Angle cock grinder	3
Piston rod grinder	3
Twist-drill grinder	2
Automatic tool grinder	3 to 5

GRINDING MACHINES (GRINDING SHAFTS, ETC.)

Motor—*A*, *B*, or *C*

Dia. wheel, ins.	Length work, ins.	Horse-power Average work	Horse-power Heavy work
10	50	5	7½
10	72	5	7½
10	96	5	7½
10	120	5	7½
14	72	10	15
18	120	10	15
18	144	10	15
18	168	10	15

GEAR CUTTERS

Motor—*A*, *B*, or *C*

Size, ins.	H.p.
36×9	2 to 3
48×10	3 to 5
30×12	5 to 7½
60×12	5 to 7½
72×14	7½ to 10
64×20	10 to 15

HAMMERS

Motor—B[1] or *C*[2]

Size, lbs.	H.p.
15 to 75	½ to 5
100 to 200	5 to 7½

Bliss drop hammers require approximately 1 h.p. for every 100-lb. weight of hammer head.

[1] Compound-wound motor.

[2] Wound secondary squirrel-cage motor with approximately 10 per cent. slip.

LATHES

Motor—*A*, *B*, or *C*

ENGINE LATHES

Swing, ins.	Horse-power Average	Horse-power Heavy
12	½	2
14	¾ to 1	2 to 3
16	1 to 2	2 to 3
18	2 to 3	3 to 5
20 to 22	3	7½ to 10
24 to 27	5	7½ to 10
30	5 to 7½	7½ to 10
32 to 36	7½ to 10	10 to 15
38 to 42	10 to 15	15 to 20
48 to 54	15 to 20	20 to 25
60 to 84	20 to 25	25 to 30

WHEEL LATHES

Size, ins.	H.p.	Tail stock motor[1] h.p.
48	15 to 20	5
51 to 60	15 to 20	5
79 to 84	25 to 30	5
90	30 to 40	5 to 7½
100	40 to 50	5 to 7½

[1] Standard machine-tool traverse motor.

TABLE 9.—SIZES OF MOTORS FOR MACHINE TOOLS—(*Continued*)

AXLE LATHES

	H.p.
Single	5, 7½, 10
Double	10, 15, 20

MILLING MACHINES

Motor—*A*, *B*, or *C*

VERTICAL SLABBING MACHINES

Width of work, ins.	H.p.
24	7½
32 to 36	10
42	15

HORIZONTAL SLAB MILLERS

Width between housings, ins.	Horse-power Average	Horse-power Heavy
24	7½ to 10	10 to 15
30	7½ to 10	10 to 15
36	10 to 15	20 to 25
60	25	50 to 60
72	25	75

VERTICAL MILLING MACHINES

Height under work, ins.	H.p.
12	5
14	7½
18	10
20	15
24	20

PLAIN MILLING MACHINES

Table feed, ins.	Cross feed, ins.	Vertical feed, ins.	H.p.
34	10	20	7½
42	12	20	10
50	12	21	15

UNIVERSAL MILLING MACHINES

Machine No.	H.p.
1	1 to 2
1½	1 to 2
2	3 to 5
3	5 to 7½
4	7½ to 10
5	10 to 15

Sizes of motors for various metal and wood-working machines, according to L. R. POMEROY (*General Electric Review*, 1907), are given in Table 11.

The sizes of motors suitable for various commercial presses forms the subject of an article by F. C. FLADD (*Amer. Mach., May* 28, 1903). The list is made up of presses and motors in satisfactory use, includes direct-belt and chain-driven presses (belt-driven preferred by Mr. Fladd) and is given in Table 12.

Methods of making more accurate determinations of motor capacity for machine tools, especially under heavy and fluctuating loads, have been explained by A. G. POPCKE, Industrial Elect. Engr., Westinghouse Electric and Mfg. Co. (*Amer. Mach., Sept.* 26, 1912) as follows:

The preliminary data required are: On direct current: Horse-power, speed and voltage. On alternating current: Horse-power, speed, voltage, frequency and phase.

The voltage, frequency and phase are determined by the electric circuit in the shop. The horse-power depends upon the work done. Whether to use a high- or a low-speed motor depends on the gears that must be used. A comparatively low speed is usually necessary.

The power required to drive a machine tool depends upon the following: The tools used. Amount of metal removed in a given time. The metal cut.

The tools used are of three general types: Lathe type tools, used on lathes, boring mills, planers and shapers. Drills. Milling cutters.

The amount of metal is usually expressed in cubic inches removed per minute. The rate of removing metal for a given job depends upon the tools used, the strength of the machine tool, the strength of the work, the accuracy desired, and the nature of the metal cut.

In roughing work, the question of horse-power must be carefully considered so that the most economical motor is applied. The tendency is to select a motor to suit the maximum capacity of the machine tool. This is very rarely reached for any length of time. Modern motors will operate successfully, at loads 100-125 per cent. above the rated loads. The following information must be obtained to determine the horse-power of the motor to be used for any tool-cutting metal:

Type of tool. Average cut taken: Depth (all tools considered) in inches. Feed per revolution in inches. Cutting speed in feet per minute. Duration. Maximum cut taken: Depth in inches. Feed per revolution. Cutting speed in feet. Duration of peak maximum load. Number of peaks per hour.

With this information it is possible to estimate the average and maximum horse-power required from the cubic inches of metal removed per minute for the cuts taken for average and maximum.

In all cases the area of the cut is taken as equal to the depth multiplied by the feed.

For revolving work or tools, the cutting speed may be quickly determined from Fig. 8, the use of which is explained below it. The chart may obviously be used in the reverse direction with equal facility. The cubic inches of metal removed per minute may be quickly determined from Fig. 9, the use of which is explained below it. To determine the horse-power required the cubic inches of metal removed per minute are multiplied by a constant.

For estimating purposes the constants of Table 10 based on average shop conditions are useful in figuring horse-power *at the cutting tool* when round-nose tools are used:

TABLE 10.—RELATION BETWEEN POWER AND VOLUME OF METAL REMOVED

Cast-iron	.3 to .5	h.p. per cu. in. per min.
Wrought iron, Machinery steel	.6	h.p. per cu. in. per min.
Steel, 50 carbon and harder	1.00 to 1.25	h.p. per cu. in. per min.
Brass and similar alloys	.2 to .25	h.p. per cu. in. per min.

For twist drills the consumption of power per cu. in. of metal removed is approximately double the above.

The constants will vary with the angle and sharpness of the tool, but are close enough to determine the size of motors in the majority of cases. A few tests in any shop using motor-driven tools can easily be made to determine the constants for their particular tools. The tendency is to use tools in accordance with the conditions arrived at from tests made by F. W. Taylor, and others, which see. The above constants hold under these conditions.

The friction load of the machine tool should be added to obtain the total horse-power. However, where the horse-power to remove metal is large, the friction is a small percentage and can be neglected.

In selecting the size of a motor it must be remembered that the load is intermittent in the majority of cases. The heating of a motor in supplying power for work of an intermittent nature is de-

TABLE 11.—SIZES OF MOTORS FOR VARIOUS METAL AND WOOD-WORKING MACHINES

Bolt and Nut Machinery, Helve Hammers

	Motor h.p. required to drive
One and one-half inch single-head bolt cutter	1½
Pratt & Whitney No. 4 turret bolt cutter	2
Two-spindle stay bolt cutter	2
One and one-half inch Acme double-head bolt cutter	2½
One and one-half to two and one-half Acme nut facer	2½
Six-spindle nut tapper	3
One and one-half inch triple-head bolt cutter	3
Three-fourths to two and a half inch double-head bolt cutter	3
Two-inch triple-head bolt cutter	5
Four-spindle stay bolt cutter	5
Bradley hammer	7½
Grinders	
Air-cock grinder	1
Link grinder	3
Sellers universal grinder for tools	5
Norton 18×96-in. piston-rod grinder	5
Saws for Wood	
Band saw, 36-in. wheel	3
Band saw, 42-in. wheel	5
Swing cut-off saw	5
Band saw, 48-in. wheel	7½
Greenlee No. 1½ self-feed rip saw	10
Greenlee vertical automatic cut-off saw	15
Forty to forty-six inch saws	15
Automatic band resaw	20
Greenlee No. 6 automatic cut-off saw	20
Greenlee No. 3 rip saw	20
Woods No. 4 rip saw	20
Extra heavy automatic rip saw	25
Wood-working Tools	
Fay-Egan single-spindle vertical boring machine	3
Fay-Egan three-spindle vertical boring machine	4
Fay-Egan No. 6 vertical mortiser and borer	6
Fay-Egan No. 7 tenoner or gainer	7½
Fay-Egan universal wood worker	7½
Fay-Egan four-spindle vertical borer	7½
Fay-Egan five-spindle vertical borer	10
Fourteen-inch inside molder	12
Fay-Egan universal tenoner and gainer	12
Fay-Egan vertical tenoner	12
Greenlee automatic vertical tenoner	15
Fay-Egan No. 3 gainer, also Greenlee	15½
Greenlee extra-range five-spindle borer and mortiser	15
Greenlee vertical mortiser	15
Fay-Egan automatic gainer, also combination gainer and mortiser	20
Fay-Egan No. 8 vertical saw and gainer	20½
Vertical hollow chisel mortiser and borer	20
Fay-Egan 14½-in. double-cylinder surfacer	20½
Heavy outside molder	20
Six-roll direct-connected planer and matcher	25
Double-cylinder fast flooring machine	30
Double-cylinder planer and matcher	30
Fay-Egan No. 8 automatic tenoner	30½
Woods No. 27 matcher	35
Four-side timber planer, heavy	60

TABLE 12.—HORSE-POWER OF MOTORS FOR VARIOUS PRESSES

Name and number of press.	H.p. of motor required.
Bliss, No. 21	2
Bliss, No. 20	1
Bliss, No. 19	1
Bliss, No. 18	1
Bliss, No. 52	2
Bliss, No. 30A	3
Bliss-Stiles punch, fly-wheel pattern, No. 0	½
Bliss-Stiles punch, fly-wheel pattern, No. 1	1
Bliss-Stiles punch, fly-wheel pattern, No. 2	1
Bliss-Stiles punch, fly-wheel pattern, No. 3	2
Bliss-Stiles punch, fly-wheel pattern, No. 4	3
Bliss-Stiles punch, fly-wheel pattern, No. 5	3
Bliss-Stiles punch, geared pattern, No. 5 doing heavy work	5
Bliss, geared for heavy work, No. 36	5
Bliss, double crank geared, No. 3½	5
Bliss, double crank geared, No. 5	7½
Bliss circular shear, No. 105	½
Bliss double action, fitted with double roll feeds, No. 68N	2
Stiles special five-slide gang press, geared, No. 102	3
Bliss automatic feed armature disk press, No. 16A	1
Bliss toggle, No. 3½	10
Bliss-Stiles 200-lb. automatic board lift drop, improved	3
Bliss-Stiles 400-lb. automatic board lift drop, improved	4
Bliss-Stiles 800-lb. automatic board lift drop, improved	7½
Hilles & Jones combined punch and shear, No. 2	5
Ferracute, No. SG86	5
Ferracute, direct geared, No. C5	3
Ferracute, 18-inch throat, No. P21	2
Bliss, geared with side cut-off attachment, No. 74½	3
Bliss, deep throat for light punching, No. 47½	1
Hilles & Jones, combined punch and shear, No. 3, 36-in. throat, capable of punching 1¼-in. hole through 1-in. stock	10
Hilles & Jones, combined punch and shear, No. 3, 12-in. gap, punching 1-in. hole through 1-in. stock	7½
Hilles & Jones, single punch, 36-in. throat, punching 1½-in. holes through 1½-in. stock	10
Hilles & Jones, horizontal punch, No. 3, 20-in. throat punching 1-in. holes through ¾-in. stock	7½
Hilles & Jones angle shear, No. 3 cutting 1-in. stock	10
Williams & White, bulldozer, No. 6, 20-in. stock, 38-in. die face.	7½
Bliss geared power shear, 36-in. cut, cutting sheet steel ¼-in. thick.	5
Heavy alligator geared cut-off shear, capable of shearing 5-in. by 1-in. bar iron	5
Small armature disk notching press	1
Large coining presses at U. S. Mint, striking up silver dollars, r.p.m., 80; pressure, 160 tons	7½
Smaller coining presses, striking up quarter dollars, r.p.m., 100; pressure, 60 tons	3
Planchet presses at Mint, double roll feed	3
Double cut-off shear at Mint	3

termined by the square root of the mean square of the power required to perform the various operations taking place in a complete cycle. This value will be termed the root mean square value, or r.m.s. value. The method of figuring the r.m.s. value of any intermittent load is best explained by working out an example. Suppose the power fluctuates as follows during a cycle of operations:

Power required	Duration
10 h.p.	10 seconds
5 h.p.	30 seconds
3.5 h.p.	25 seconds
1 h.p.	20 seconds
0 h.p.	20 seconds

The r.m.s. value is figured as follows:

1. Multiply the square of each value of power by its duration.
2. Add the products thus obtained.
3. Divide the sum by the time to complete the cycle (the sum of the times of the various components).
4. Take the square root of the quotient.

The result will be the r.m.s. value. The time can be expressed in seconds or minutes and the power in horse-powers, kilowatts or—the voltage being constant—in amperes, the same units being used throughout a given problem. In the above example the r.m.s. value is determined as follows:

(1)
$$10^2 \times 10 = 1000$$
$$5^2 \times 30 = 750$$
$$3.5^2 \times 25 = 306.25$$
$$1^2 \times 20 = 20$$
$$0 \times 20 = 0$$

(2)
$$2076.25$$

(3) Total time of cycle $= 10+30+25+20+20 = 105$ sec.

$$\frac{2076.25}{105} = 19.8$$

(4) $\sqrt{19.8} = 4.45$ h.p. = r.m.s. value

Thus 4.45 is the r.m.s. value of this cycle and the heating of the motor will be the same as if it were run at a constant load of 4.45 h.p. The maximum load on a 5-h.p. motor would be twice the full load, or 100 per cent. overload for 10 sec. An up-to-date alternating-current induction motor or commutating-pole, direct-current motor will carry this successfully under the given conditions. A 5-h.p. motor would, therefore, be used in this case.

The limits above rated loads to be considered when selecting alternating-current motors to carry widely fluctuating intermittent loads are:

1. Pull at the starting torque.
2. Speed regulation.

For direct-current motors:

1. Commutation.
2. Speed regulation.
3. Stability.

The pull at the starting torque of an induction motor is from 2.5 to 3.5 times the full-load torque.

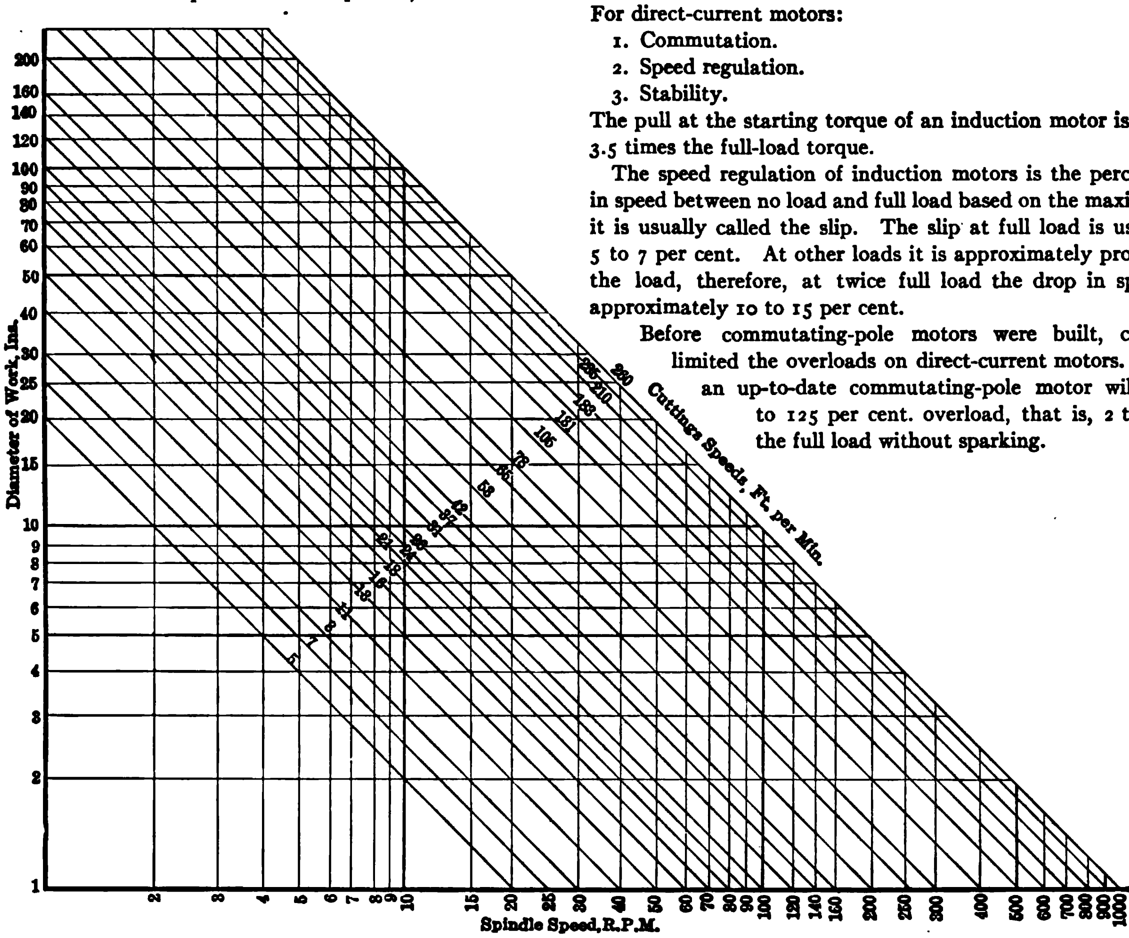
The speed regulation of induction motors is the percentage drop in speed between no load and full load based on the maximum speed; it is usually called the slip. The slip at full load is usually about 5 to 7 per cent. At other loads it is approximately proportional to the load, therefore, at twice full load the drop in speed will be approximately 10 to 15 per cent.

Before commutating-pole motors were built, commutation limited the overloads on direct-current motors. At present an up-to-date commutating-pole motor will carry 100 to 125 per cent. overload, that is, 2 to 2.25 times the full load without sparking.

It is customary to express the speed regulation of direct-current motors in terms of the full-load speed, because the full-load speed is the rated speed of the motor. At full load the speed regulation is 10 to 15 per cent., depending on the rating of the motor. At overloads the effect on noncommutating-pole motors is a decrease in speed proportional to the load; but on commutating-pole motors the speed in many cases tends to increase between full load and 100 per cent. overload.

This type of motor will, therefore, have approximately the same speed at twice full load as it has at full load. If the effect of the interpoles is too strong the tendency is to make a commutating-pole motor oscillate in speed. This speed oscillation will cause a similar variation on armature current of gradually increasing intensity, until something gives way; a fuse will blow, a circuit breaker open or the motor will be injured by "bucking over," that is, flashing across brushes, or burning out the armature.

There is a relation existing between speed regulation and stability. A commutating-pole motor can be designed to be stable at overloads. This will increase the drop in speed. Better speed regula-

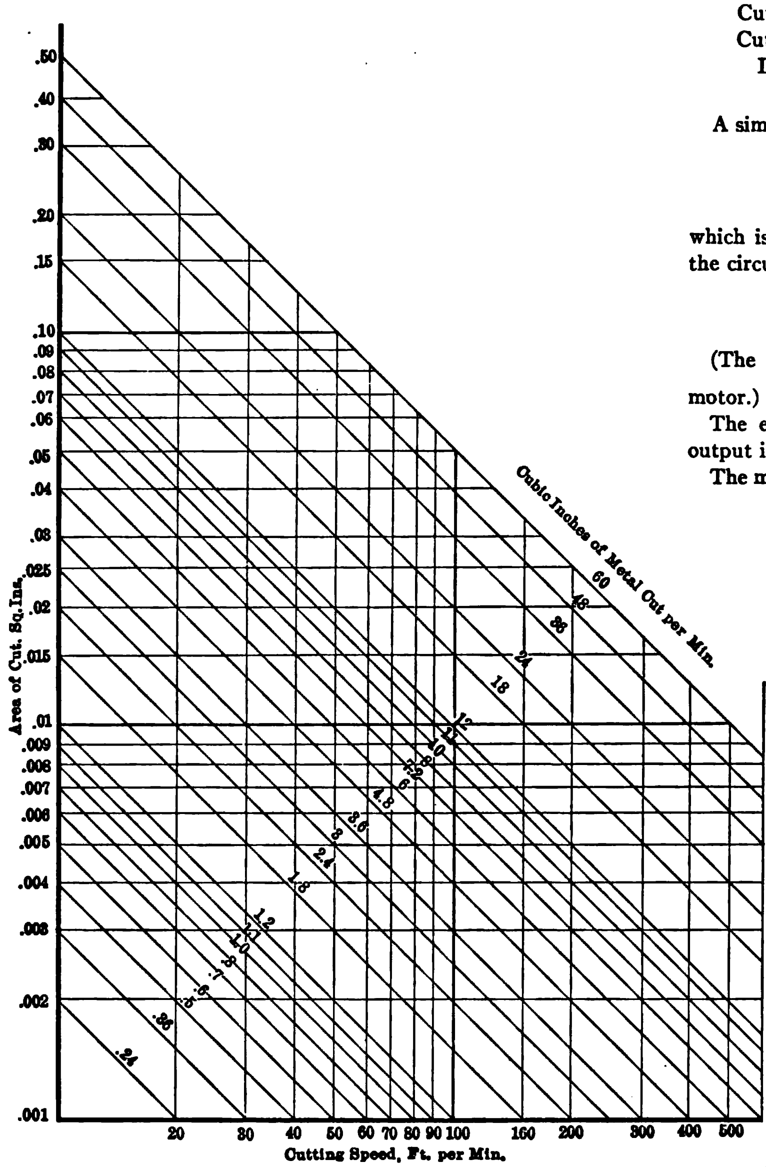


Trace vertically from the r. p. m. to the intersection with the horizonal through the diameter where read cutting speed. Thus a piece of work of 3 in. diameter at 60 r. p m has 47 ft. per min. surface speed.

FIG. 8.—Relation of spindle speed, cutting speed and diameter of work.

tion makes stability less certain. Reliable designers of this type of motor strike a happy medium between these two, and the commercial result is that in most cases these motors can be safely operated on intermittent loads where the maximum load is twice the rated load.

A large reduction in speed giving a stable motor, is an advantage in machine-tool work. It often occurs when long, continuous cuts are taken, that on one part of a casting the depth of cut is greater than on another, due to irregularities in casting. When cutting through the heavy part the speed should be reduced, thus protecting the cutting tools and machine tool as well as the work. For this reason adjustable-speed motors with a speed reduction as high as 25 per cent., can be used to advantage.



Trace vertically from the cutting speed to the intersection with the horizontal through the area of cut (produced by feed and depth of cut) where read cu. ins. of metal removed per min. Thus for $\frac{1}{16}$ in. feed and $\frac{1}{4}$ in. depth of cut (.015 sq. in. area of cut) and 60 ft. per min. cutting speed, 11+cu. in. per min. are removed.

FIG. 9—Relation beween area of cut, cutting speed and volume of metal removed.

Let us apply these principles in determining the horse-power of a motor in actual machine-tool service. A record, Fig. 10, taken with a graphic recording ammeter will be used for this purpose. The record was taken on a lathe driven with a direct-current, adjustable-speed motor. The cycle of operations when turning the shaft, Fig. 11, is as follows:

	Amp.	Time	Calculated r.m.s. (Amp.)$^2\times$time
Cut *ab*	33	180 sec.	$1089\times180=196{,}020$
Cut *bc*	30	140 sec.	$900\times140=126{,}000$
Idle	0	112 sec.	$0\times112=$ 0
Cut *de*	30	171 sec.	$900\times171=153{,}900$
Cut *ec*	9	260 sec.	$81\times260=$ 21,060
Idle	0	190 sec.	$0\times190=$ 0
		1053	496,980

A similar cycle is then repeated.

$$\frac{496{,}980}{1053}=470$$

$$\sqrt{470}=21.7 \text{ amperes}$$

which is the r.m.s. value of current. At 220 volts, the voltage of the circuit, the r.m.s. h.p. input to the motor is

$$\frac{21.7\times220}{1000\times746}=6.4 \text{ h.p. input}$$

(The h.p. input to motor $=\dfrac{\text{amperes}\times\text{volts}}{1000\times746}$ for any direct-current motor.)

The efficiency of the motor being 86 per cent., the r.m.s. h.p. output is 5.5 h.p.

The maximum load occurs when the cut *ab* is taken, which requires

$$\frac{33\times220}{1000\times746}=9.75 \text{ h.p.}$$

input or, at 85 per cent. efficiency, an output of 8.3 h.p.

If a 5-h.p. motor is used, 8.3 h.p. will be 66 per cent. overload, and considering what has been said above, a modern 5-h.p. motor will pull this satisfactorily. The r.m.s. value, or that upon which heating depends, is 10 per cent. above the rated load. All 5-h.p. motors made by reliable manufacturers will carry this load without overheating.

The cycle just discussed represents maximum work done in this lathe, the average load being less severe. Hence a 5-h.p. motor is the proper motor to drive it. Usually a test cannot be conveniently made. In these cases the power cycles can be figured from the rate of removing metal and the time required for each cut. The method of estimating the power has already been explained. The time of a cut is estimated as follows when knowing the length of the cut, the feed per revolution and the spindle speed while taking the cut:

The product obtained by multiplying the feed per revolution and the r.p.m. of the spindle will give the advance of the cutting tool per minute. Dividing this into the length of the cut will give the time to complete the cut in minutes. With this information and the time to make adjustments when the motor is shut down or running idle, the r.m.s. value can be figured with the rules already given.

The practice in the past, still largely used, is to select the motor with reference to the size of a machine, as the swing of a lathe. The strength of a lathe and, therefore, the horse-power it can transmit naturally increases with the size of the lathe; but the quantities which determine the horse-power are those just discussed and their application will avoid misapplications. In many cases, heavier cuts are taken on 18- than on 24-in. lathes, the smaller-swing lathe, therefore, requiring the larger motor.

On machine tools where light cuts are taken, it is not necessary to figure the horse-power for cutting because 2-h.p. is required to

start the tool and run it idle and will do the light cutting successfully.

The rule for figuring horse-power just described is applicable in determining the power to cut metal wherever the round-nose type of tool is used, as in vertical boring mills, shapers, slotters, and planers. On planers the peak load for reversing must be considered in determining the size of motor.

On planers the general tendency is to use motors that are too large. This tendency originated when non-interpole, direct-current motors, only, were available, and a peak load caused considerable

FIG. 10.

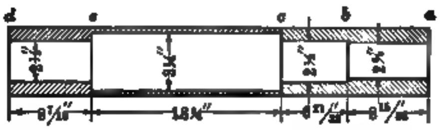


FIG. 11.

FIGS. 10 AND 11.—A piece of lathe work and a graphic record of current readings.

sparking when the planer was reversed. A large motor was, therefore, necessary on account of the reversal. When using alternating-current induction motors, or direct-current, commutating-pole motors this precaution need not be taken.

Table 13 shows the results of tests made on various sizes of planers with a graphic meter. Note the difference between the motors usually specified and those recommended. The recommended motors are alternating current and are operating their planers successfully.

Fly-wheels can be used to advantage on the countershaft from which the forward and reverse belts are driven. The fly-wheel will reduce the peak load on the motor occurring when the planer is reversed. In this way the horse-power of the driving motor can often be reduced.

It is evident that making an investigation as outlined results in the selection of the most economical size of motor for the work done, and in a smaller motor than that usually specified, because the tendency is to select motors to suit the maximum capacity of machine tools and no advantage is taken of the fact that motors will stand heavy overloads for short intervals.

The selection of electric motors for machine driving includes other questions than that of horse-power. These have been explained by A. G. POPCKE, Indust. Elect. Engr. Westinghouse Electric & Mfg. Co., (*Amer. Mach., Oct.* 3, 1912) as follows:

The speed of the shaft on the machine where power is applied is the principal factor which determines the speed of the motor to be connected. On forging machines using large fly-wheels these speeds are as low as 50 to 60 r.p.m.; on machine tools, such as lathes, drills, millers, etc., they average between 200 and 300 r.p.m. Speeds as high as 1000 to 2000 r.p.m. occur on grinders and wood-working machines.

Modern practice is to standardize the speeds of motors This practice has been brought about by the extensive use of alternating current. Since 60 cycles are used in the majority of alternating-current systems, the standard speeds of direct-current motors are approximately the same as the speeds of 60-cycle, alternating-current motors.

The speeds obtainable with the 60-cycle motors mostly used are 1700 to 1800; 1100 to 1200; 850 to 900; 650 to 720, and 550 to 600 r.p.m. The higher speed given in each case is the synchronous speed at which the motor runs when not loaded. The speed decreases from 5 to 7 per cent. as the motor is loaded.

On 25-cycle circuits the speeds of motors most frequently used are 700 to 750; 550 to 600, and 350 to 375 r.p.m. The speeds of direct-current motors are given in the second column of Table 14. A reference thereto will show the relation to the speeds of the alternating-current motors just given.

When a belt drive is to be used the quantities to be considered are: Speed reduction; pulley sizes; belt speeds; motor speed; distance between pulley centers; arc of contact; size of belt; use of idle pulleys; mounting of the motor.

Obtaining the required speed reduction involves the size of the motor pulley, machine pulley and belt speed. The sizes of the pulleys used on motors have been standardized according to ratings,

TABLE 13.—MOTORS FOR PLANERS

Manufacturer	Size	Motor used for test	Kw. cut stroke	Kw. return stroke	Kw. reversal to cut	Kw. reversal to return	Remarks	Motor recommended based on test, h.p.	Motor usually specified, h.p.
	ins. ft.								
Gray	56×15	3	1.3	3.5	4.0	5.3	Average work,		
Gray	56×15	5	1.8	2.8	3.5	5.3	5 tons on tab.	5	15
Gray	56×15	5	2.5	....	6	6	Short stroke		
Gray	54×16	30	4	6	8	10.5	Aver. stroke		
		30	4	7	10	12	Short stroke	5	15
Gray	54×16	5	1.8	2.3	3.5	5.5	Aver. stroke		
Bement-Miles	48×12	5	2	7	8	9	Aver. work	7½	15
Chandler	24×10	7½	2	4.5	4.3	5.5	Motor geared balance wheel	5	7½
Detrick & Harvey	42×12 open side	5	1.5	2.5	5	7	Aver. work	7½	15
Bement-Miles	48×12	30	5	10	14	19	No. bal. wheel	7½	15
Bement-Miles	37× 8	5	1.8	3	4	6	Aver. work	5	10
Pratt & Whitney	36× 8	5	1.5	2	2.5	4	Aver. work	3	5
Gray	36× 8	5	1.8	2	3	5	Aver. work	3	5

TABLE 14.—STANDARD MOTOR RATINGS STANDARD AND MINIMUM PULLEYS AND BELT SPEED WITH STANDARD PULLEY

1	2	3	4	5	6	7	8
H.p.	R.p.m.	Standard pulley		Minimum pulley		Belt speed standard pulley, ft. per min.	Leather belt
		Dia.	Face	Dia.	Face		
1	1700	3½	2½	3	1¾	1560	Single
2	1700	3½	3	3	3	1560	Single
	1200	4	3	3	3	1250	Single
	850	4	4	3½	4	890	Single
3	1800	4	3	3	3	1890	Single
	1150	4	3	3½	4	1200	Single
	850	5	4½	4	4½	1110	Single
5	1800	4	4	3½	4	1890	Single
	1200	5	4½	4	4½	1570	Single
	850	6	5	4½	5	1340	Single
7½	1700	5	4½	4	4½	2220	Single
	1150	6	5	4½	5	1800	Single
	975	7	6	5	6	1790	Single
	850	7	6	5	6	1560	Single
	650	8	7	6	7	1360	Single
10	1700	6	5	4½	5	2670	Single
	1300	7	6	5	6	2380	Single
	1150	7	6	5	6	2100	Single
	850	8	7	6	7	1780	Single
	730	8	7	6	7½	1530	Single
	600	9	8	6½	9	1410	Single
15	1700	7	6	5	6	3100	Single
	1250	8	7	6	7	2620	Single
	1100	8	7	6	7½	2300	Single
	825	9	8	6½	9	1940	Single
	675	10	9	7	8	1770	Single
	600	11	10	7½	9½	1730	Single
20	1700	8	7	6	7	3560	Single
	1100	9	8	6½	9	2600	Single
	900	10	9	7	8	2360	Single
	750	11	10	7½	9½	2160	Single
	650	11	10	8	9½	1870	Single
25	1400	9	8	6½	9	3300	Single
	1100	10	9	7	8	2880	Single
	950	11	10	7½	9½	2730	Single
	825	11	10	8	9½	2370	Single
	600	12	12	9	10½	1880	Double
30	1700	9	8	6½	9	4000	Single
	1150	11	10	7½	9½	3300	Single
	975	11	10	8	9½	2800	Single
	725	12	12	9	10½	2280	Double
	600	13	12	10	11	2040	Double
35	1700	10	9	7	8	4450	Single
	1150	11	10	8	9½	3330	Single
	850	12	12	9	10½	2670	Double
	675	13	12	10	11	2300	Double
40	1700	11	10	7½	9½	4900	Double
	950	12	12	9	10½	3000	Single
	775	13	12	10	11	2640	Double
	600	14	12	12	13	2200	Double
50	1700	11	10	8	9½	4900	Double
	975	13	12	10	11	3320	Double
	750	14	12	12	13	2750	Double
	565	16	13	12½	15	2360	Double

i.e., horse-power, and speed of motor. These are given in Table 14, column 3. This fixes standard practice for belt speeds (see Table 14, column 7).

As the size of motor pulley is reduced on any motor, the strains on the motor bearings and shaft are increased. A minimum pulley is, therefore, specified by motor manufacturers for each motor rating (see Table 14, column 5). The maximum size of the pulley on a motor is required only where speeds higher than the motor speed are required. This is, in nearly all cases, limited by the belt speed, which should not exceed 5000 ft. per min.

In some cases, with small motors especially, the size and location of the motor are such that the diameter of the motor limits the largest pulley.

The success of a belted motor application depends largely upon the arc of contact. The distance between centers of motor pulley and machine pulley, as well as the speed reduction, determines the arc of contact on the smallest pulley, usually the motor pulley.

Motors can be furnished with idler pulley attachments, and these are applied to advantage where it is necessary to overcome a small arc of contact. When necessary to obtain extremely low speeds back-geared motors should be used.

A good standard for the back-geared type of motor is one having a speed reduction of 6 to 1 between its armature and countershaft speeds. Usually, if the required reduction in speed exceeds 6 to 1, a back-geared motor should be used. For instance, if the reduction is 12 to 1 between the motor speed and the machine speed, a back-geared motor with a 6 to 1 speed reduction should be used, and the further reduction 2 to 1 obtained by means of a pulley on the countershaft of the back-geared motor.

The pulleys furnished with motors make provision for the proper width of the belt. Table 14 shows whether a single or double belt should be used. The width of the belt should be 1 in. narrower than the pulley face on pulleys up to 12-in. face; above that it should be 2 ins. narrower than the pulley face.

The cost of a motor of given horse-power increases as the rated speed decreases. For instance, the cost of a 10-h.p. motor at 1200 r.p.m. is approximately the same as a 5-h.p. motor at 600 r.p.m. The cost increases in the same proportion as the square root of the torque figured at 1-ft. radius. From a cost point of view, therefore, as high a speed motor as possible should be used, but the diameter of minimum pulley specified should not be gone below.

When the machine pulley is fixed, as when belting to a fly-wheel, the motor pulley must suit the requirements of the machine. Care must be taken not to go below the minimum motor pulley and the arc of contact must also be carefully considered, for in these cases the reduction is usually large.

When the machine pulley can be chosen to suit, the standard motor pulley, Tables 14 and 15, will assist in selecting the proper speed of motor and size of pulleys. Table 15 gives the machine speed at the left column and the motor speeds at the top of the table. The figures in the body of the table are the speed reductions for any combination of machine and motor speed indicated.

The letter *B* indicates that the motor is to be belted directly, and the symbol *Bbg* indicates that a back-geared motor be belted. The figure after *Bbg* indicates the reduction between the motor countershaft and the driven machine, if a back-geared motor with a 6 to 1 reduction is used.

The heavy-faced type indicate the method recommended in the majority of cases for the combination where it occurs. Thus, for machine speeds between 600 and 1500, use 1800-r.p.m. motors; between 350 and 600, use 1200-r.p.m. motors; between 250 and 350 use 900-r.p.m. motors; between 150 and 250, use 720-r.p.m. motors. For the smaller power requirements, and between 150 to 200 for the large power requirements, use 600-r.p.m. motors. Below 100 and 150 r.p.m. it is best to use back-geared motors.

It is poor practice to use back-geared motors whose initial speed is 1700-1800 r.p.m. in the majority of cases. In applications requiring from 10 to 20 h.p., 1200-r.p.m. back-geared motors should be used; above this 900-r.p.m. or 720-r.p.m. back-geared motor should be used.

Before deciding upon any belt drive the arc of contact should be carefully checked. In machine-tool work, for applications where belts are used, the distance between centers is usually between 3 and 5 ft. Motor pulleys range from 3 to 12 ins., and the arc of contact is usually considered when the ratio of reduction is between 3 and 6.

Table 16 shows the arc of contact, knowing the size of the motor pulley, ratio of reduction and the distance between pulley centers. Table 17 shows the effect of the arc of contact on the transmitting power of the belt. The decrease with decreased arc of contact is expressed by a percentage which the power transmitted at a given arc of contact is of the power transmitted at 180 deg.

To transmit the required power the pulley and belt width must be increased or an idler pulley must be used to increase the arc of contact. An example will best illustrate the application of Tables 14, 15 and 16. The speed of the machine is 185 r.p.m.; the h.p. required is $7\frac{1}{2}$; the distance between centers is 5 ft. What motor speed and what pulleys should be used for the belt drive?

Refer to Table 15. This shows that for 150 to 200 r.p.m. a 720-r.p.m. motor should be used.

Refer to Table 14. A $7\frac{1}{2}$-h.p. 650-r.p.m. motor has an 8×7-in. standard pulley and a 6×7-in. minimum pulley.

The speed reduction with this motor is

$$\frac{650}{185}=3.5$$

Refer to Table 16. The arc of contact for a ratio of reduction of 3.5 (average of 3 to 4), the distance between centers of 5-ft. and 8-in. motor pulley is 160 deg. (average of 164 and 157), and with a 6-in. motor pulley is 165 deg. (average of 162 and 168). Either will give successful service. The machine pulley would be with an 8-in motor pulley $3.5\times8=28$ ins. and with a 6-in. motor pulley, $3.5\times6=21$ ins.

The face in either case will be 7 ins. and a single 6-in. leather belt should be used. The combination of 8-in. motor pulley and 28-in. machine pulley is preferred because the motor pulley is standard.

The above example covers a case where the machine pulley can be seclected at will. In cases where a motor is to be belted to a fly-wheel or to a pulley on a machine which cannot be easily changed, the procedure is as explained in the following example: The size of the machine pulley (fly-wheel) is 72 ins.; the speed of the pulley is 100 r.p.m.; the h.p. required is 15, and the distance between centers is 6 ft. What motor speed and motor pulley should be used?

Consider a reduction of 6 : 1 belted directly. The motor speed must be 600. The size of the motor pulley

$$\frac{\text{machine pulley}}{\text{ratio of reduction}}=\frac{72}{6}=12\text{ ins.}$$

Table 14 shows that a 12-in. pulley can be used with a 15-h.p., 600-r.p.m. motor. It is 1-in. above the standard pulley diameter.

Table 16 shows that for a 12-in. motor pulley, a ratio of reduction of 6, and 6 ft. distance between centers, the arc of contact is outside the limits of the table and the arc of contact very small (below 120 deg.)

A successful drive can be obtained by using a 12×10-in pulley on the motor and employing an idler pulley. It is not customary for motor manufacturers to supply idler attachments on such large motors.

When a geared drive is to be used the points to be considered are the following: Speed reduction; pitch of the gears, number of teeth on the gears (pinion and gear); face of the gear; pitch-line speed; distance between centers; use of idler gears and mounting of the motor.

TABLE 15.—RELATION OF MACHINE AND MOTOR SPEEDS. RECOMMENDATIONS FOR BELT DRIVE

		Approximate motor speed				
		1800	1200	900	720	600
Speed of driven machine	1500	1.2 B				
	1000	1.8 B	1.5 B			
	800	2.2 B	1.5 B	1.12 B		
	600	3 B	2 B	1.5 B	1.2 B	1.2 B
	500	3.6 B	2.4 B	1.8 B	1.44 B	1.2 B
	400	4.5 B	3 B	2.25 B	1.8 B	1.5 B
	350	5.13 B	3.4 B	2.52 B	2.06 B	1.7 B
	300	6 B	4 B	3 B	2.4 B	2 B
	250	7.2	4.8 B	3.6 B	2.9 B	2.4 B
	200	9	6 B	4.5 B	3.6 B	3 B
	150	12	8 Bbg 1.33	6 B	4.8 B	4 B
	100	18	12 Bbg 2	9 Bbg 1.5	7.2 Bbg 1.2	6 B
	90	20	13.4 Bbg 2.23	10 Bbg 1.67	8 Bbg 1.33	6.7 Bbg 1.11
	80	22.5	15 Bbg 2.5	11.3 Bbg 1.88	9 Bbg 1.5	7.5 Bbg 1.25
	70	25.3	17.1 Bbg 2.85	12.9 Bbg 2.15	10.2 Bbg 1.7	8.6 Bbg 1.43
	60	30	20 Bbg 3.33	15 Bbg 2.5	12 Bbg 2	10 Bbg 1.67
	50	36	24 Bbg 4	18 Bbg 3	14.4 Bbg 2.4	12 Bbg 2

B = Motor belted direct. **Bbg.** = Back-geared motor belted. Bbg = 1.33, etc., the number indicates reduction from countershaft speed. The heavy-faced type indicates the motor speed recommended in most cases.

TABLE 16.—RELATION BETWEEN MOTOR PULLEY, DISTANCE BETWEEN CENTERS OF PULLEYS, RATIO OF REDUCTION AND ARC OF BELT CONTACT.

Ratio of reduction	Distance between centers, ft.	Diameter of motor pulley, ins.									
		3	4	5	6	7	8	9	10	11	12
	3	170	166	163	160	157	153	150	147	145	141
3	4	173	170	167	165	163	161	158	156	155	151
	5	175	172	170	168	167	164	162	161	160	157
	3	165	160	155	150	145	142	156	132	126	122
4	4	168	165	162	158	154	152	148	144	140	137
	5	172	168	166	162	159	157	155	151	148	146
	3	160	153	148	142	134	128	122			
5	4	165	161	157	152	146	142	138			
	5	168	164	162	157	153	150	146			
	3	153	147	139	131	122					
6	4	161	156	150	144	138					
	5	164	161	156	152	146					

TABLE 17.—RELATION OF ARC OF CONTACT TO POWER TRANSMITTED

Arc of contact	Per cent. of power transmitted
180	100
170	94
160	89
150	83
140	78
130	72
120	67

TABLE 18.—STANDARD MOTOR RATINGS AND DATA FOR GEARED CONNECTIONS

H.p.	R.p.m.	Diam. pitch	Number of teeth: Standard pinion	Number of teeth: Min. rawhide pinion	Number of teeth: Min. pinion steel	Face: Steel	Face: Rawhide and cloth	Std. pitch line speed	Min. dia.	Max. No. of teeth for p.l. speed of 1000 ft. per min.	Max. No. of teeth for p.l. speed of 2000 ft. per min.
1	1700	8	17	15	13	1	1½	940	1.63	18	36
1	1200	8	17	15	13	1½	2	665	1.63	25	50
2	1700	8	17	15	13	1½	2	940	1.63	18	36
2	1200	8	22	20	19	1¾	2¼	870	2.38	25	50
2	850	6	18	21	19	1¾	2¼	615	2.38	36	72
3	1800	8	22	20	19	1¾	2¼	1300	2.38	....	34
	1150	8	22	21	19	1¾	2¼	830	2.38	26	52
	850	6	18	18	18	2¾	3⅜	670	3	27	54
5	1800	8	22	21	19	1¾	2¼	1300	2.38	....	34
	1200	6	18	18	18	2¾	3⅜	940	3	19	38
	850	6	21	19	18	2¾	3⅜	990	3	27	54
7½	1700	6	18	18	18	2¾	3⅜	1400	3	....	26
	1150	6	21	19	18	2¾	3⅜	1050	3	20	40
	975	5	19	18	18	3	3¾	970	3.6	19	38
	850	5	19	18	18	3	3¾	850	3.6	22	44
	650	5	20	18	18	3	3¾	685	3.6	29	58
10	1700	6	21	19	18	2¾	3⅜	1420	3	....	27
	1300	6	22	19	18	2¾	3⅜	1250	3	....	35
	1150	5	19	18	18	3	3¾	1150	3.6	....	33
	850	5	20	18	18	3	3¾	890	3.6	22	44
	730	5	21	18	18	3½	4¼	805	3.6	26	52
	600	5	21	19	19	3½	4¼	665	3.8	31	62
15	1700	5	19	18	18	3	3¾	1700	3.6	....	22
	1250	5	20	18	18	3	3¾	1300	3.6	....	30
	1150	5	21	18	18	3½	4¼	1210	3.6	....	35
	825	5	21	19	19	3½	4¼	910	3.8	23	46
	675	4½	22	18	18	4	5	870	4	25	50
	600	4½	22	19	19	4	5	770	4.22	29	58
20	1700	5	20	18	18	3	3¾	1780	3.6	....	22
	1100	5	21	19	19	3½	4¼	1220	3.8	....	35
	900	4½	22	18	18	4	5	1150	4	19	38
	750	4½	22	19	19	4	5	960	4.22	23	46
	650	4	21	18	18	4½	5¼	890	4.5	23	46
25	1400	5	21	19	19	3½	4¼	1550	3.8	....	27
	1100	4½	22	18	18	4	5	1400	4	....	31
	950	4½	22	19	19	4	5	1220	4.22	....	36
	825	4	21	18	18	4½	5¼	1130	4.5	18	36
	600	4	22	19	18	4½	5¼	860	4.5	25	50
30	1700	5	21	19	19	3½	4¼	1880	3.8	....	22
	1150	4½	22	19	19	4	5	1470	4.22	....	30
	975	4	21	18	18	4½	5¼	1330	4.5	....	31
	725	4	22	19	18	4½	5¼	1050	4.5	21	42
	600	3½	20	.....	18	4½	.....	970	5.53	20	40
35	1700	4½	22	18	18	4	5	2180	4	....	20
	1150	4	21	18	18	4½	5¼	1580	4.5	....	27
	850	4	22	19	18	4½	5¼	1220	4.5	18	36
	675	3½	20	.....	18	4½	.....	1080	5.53	18	36
40	1700	4½	22	19	19	4	5	2180	4.22	....	20
	950	4	22	19	18	4½	5¼	1370	4.5	....	32
	775	3½	20	.....	18	4½	.....	1250	5.53	....	32
	600	3	18	.....	15	4½	.....	940	5	19	38
50	1700	4	21	18	18	4½	5¼	2340	4.5	....	18
	975	3½	20	.....	18	4½	.....	1580	5.53	....	25
	750	3	18	.....	15	4½	.....	1170	5	15	30
	565	3	20	.....	18	4½	.....	990	6	20	40

Here also, each motor rating has a minimum pinion for the same reason, limiting stresses. The pitch, number of teeth and face for motor pinions have been standardized for back-gear motors and the best practice when gearing a motor directly to machines is to use these motor pinions as far as possible.

The pitch-line speed is limited by noise when steel pinions are used. A speed of 1000 ft. per min. should not be exceeded if quiet operation is desired. Between 1000 and 2000 r.p.m., rawhide or cloth pinions should be used; 2000 ft. per min. should not be exceeded if it can possibly be avoided.

Table 18 gives the standard motor ratings and additional data for geared drives all of which are useful when working out geared motor applications.

TABLE 19.—ADJUSTABLE SPEED MOTOR RATINGS AND DATA FOR GEARED CONNECTIONS

H.p.	R.p.m.: Min.	R.p.m.: Max.	Smallest pulley: Dia.	Smallest pulley: Face	Gear data: Diam. pitch	Gear data: Face: Steel	Gear data: Face: Rawhide	Gear data: Min. teeth	Gear data: Min. diam.	Pitch-line speed, at min. diam.: Min. speed	Pitch-line speed, at min. diam.: Max. speed	Max. teeth not to exceed 2000 ft. per min. at max. speed
1	740	2200	3	3	8	1¾	2¼	19	2.38	460	1380	27
	600	1800	3	3	8	1¾	2¼	19	2.38	375	825	46
	450	1800	3	4	8	1¾	2¼	19	2.38	280	1120	34
2	1100	2200	3	3	8	1¾	2¼	19	2.38	690	1380	27
	740	2200	3	4	8	1¾	2¼	19	2.38	460	1380	27
	450	1800	4	4½	6	2¾	3⅜	18	3.0	355	1420	25
3	1000	2000	3	4	8	1¾	2¼	19	2.38	625	1250	30
	660	2000	4	4½	6	2¾	3⅜	18	3.0	520	1560	23
	450	1800	4½	5	6	2¾	3⅜	18	3.0	355	1422	25
	375	1500	5.	6	6	2¾	3⅜	18	3.0	294	1176	30
5	1000	2000	4	4½	6	2¾	3⅜	18	3.0	790	1580	23
	750	1500	4½	5	6	2¾	3⅜	18	3.0	590	1180	30
	600	1800	5	6	6	2¾	3⅜	18	3.0	470	1410	25
	450	1800	6	7	5	3	3¾	18	3.6	425	1700	21
	375	1500	6	7½	5	3½	4¼	18	3.6	355	1420	25
7½	900	1800	5	6	6	2¾	3⅜	18	3.0	705	1410	25
	800	1600	5	6	5	3	3¾	18	3.6	755	1510	24
	600	1800	6	7	5	3	3¾	18	3.6	570	1710	21
	500	1500	6	7½	5	3½	4¼	18	3.6	475	1425	25
	450	1800	6½	9	5	3½	4¼	19	3.8	450	1800	21
	350	1400	6½	9	5	3½	4¼	19	3.8	350	1400	27
10	850	1700	6	7	5	3	3¾	18	3.6	800	1600	22
	750	1500	6	7	5	3	3¾	18	3.6	710	1420	25
	600	1800	6	7½	5	3½	4¼	18	3.6	570	1710	21
	500	1500	6½	9	5	3½	4¼	19	3.8	500	1500	25
	450	1800	6½	9	5	3½	4¼	19	3.8	450	1800	21
	375	1500	7	8	4½	4	5	18	4.0	390	1560	23
15	780	1560	6½	9	5	3½	4¼	19	3.8	780	1560	24
	600	1200	7	8	4½	4	5	18	4.0	630	1260	28
	500	1500	7½	9½	4½	4	5	19	4.22	555	1665	23
	400	1200	8	9½	4	4½	5¼	18	4.5	470	1410	25
	375	1500	9	10½	4	4½	5¼	18	4.5	440	1760	20
20	650	1300	7½	9½	4½	4	5	19	4.22	720	1440	26
	550	1100	8	9½	4	4½	5¼	18	4.5	645	1290	28
	500	1500	9	10½	4	4½	5¼	18	4.5	590	1770	20
	400	1200	10	11	3½	4½	....	18	5.53	580	1740	21
	300	1200	12	13	3	4½	....	15	5.0	390	1560	19
25	550	1100	9	10½	4	4½	5¼	18	4.5	645	1290	28
	400	1200	12	13	3	4½	....	15	5.0	525	1575	19
	300	1200	12½	15	3	4½	....	18	6.0	470	1880	19
30	550	1100	10	11	3½	4½	....	18	5.53	800	1600	23
	350	1050	12½	15	3	4½	....	18	6.0	550	1650	22
	250	1000	14	18	3	4½	....	18	6.0	390	1560	23
40	550	1100	12	13	3	4½	....	15	5.0	720	1440	21
	350	1050	12½	15	3	4½	....	18	6.0	550	1650	22
	250	1000	16.	21	3	4½	....	19	6.33	415	1660	23
50	500	1000	12½	15	3	4½	....	18	6.0	790	1580	23
	325	975	16	21	3	4½	....	19	6.33	540	1620	23

If reductions greater than 7 to 1 are required, it is usually necessary to obtain the reduction by the use of two sets of gears. Back-geared motors can be used to furnish one set of gears in these cases. Thus if a reduction of 10 to 1 is desired, a back-geared motor with a standard 6 to 1 reduction, with a further reduction from the counter-shaft of the motor to the machine of $\frac{10}{6}$ to 1 or 1.66 to 1 will fulfill the requirements.

An example will explain how to proceed in a motor application where gears are to be used: The speed of the driven shaft of the machine is 210 r.p.m.; the h.p. is 10; the motor is mounted on the machine and the limiting distance between centers is 12 ins. What are the sizes of gear and pinion to be used? The machine is a punch and shear.

In this case a pitch-line speed of approximately 1000 ft. per min. will be employed. Table 18 shows that a 10-h.p. motor at 850 r.p.m. is the highest speed motor that can be used for this pitch-line speed. The ratio of reduction is then

$$\frac{850}{210} = 4.05 \text{ (use 4 to 1)}$$

TABLE 20.—POWER REQUIREMENTS OF MACHINE TOOLS IN GROUPS

Kind	Size	Observed horse-power, maximum	Observed horse-power, average	Remarks
BORING MACHINES				
Bullard, single head	36 ins.	.78	.52	
Bullard, double head	42 ins.	1.72	1.08	
CAM CUTTERS				
Brainard	No. 2		.67	
Brainard	No. 4	.48	.32	
Brainard	No. 5	.48	.32	
Lathe type, single head			.32	
Lathe type, double head			.50	
CUTTING-OFF MACHINES.				
Hurlbut-Rogers	1 11/16 ins.		.12	
Hurlbut-Rogers	2 ins.	.28	.14 to .18	
Hurlbut-Rogers	3 ins.	.34	.20 to .22	
DRILLING MACHINES.				
Prentice Bros. radial	No. 0		.72	
Prentice Bros. radial	No. 1	3.18	1.12	
Woodward & Rogers	Sensitive single-spindle		.31	
Dwight-Slate	2-spindle		.32	
Woodward & Rogers	Sensitive 3-spindle		.35	
Woodward & Rogers	4-spindle		.48	
Woodward & Rogers	6-spindle		.71	
Prentice upright	16 ins.		.25	
Prentice upright	18 ins.		.35	
Prentice upright	20 ins.		.42	
Prentice upright	22 ins.		.59	
Blaisdell upright	24 ins.		.47	
Blaisdell upright	26 ins.		.22	
Blaisdell upright	28 ins.		.25	
Blaisdell upright	30 ins.		.30	
Blaisdell upright	34 ins.		.45	
Blaisdell upright	36 ins.		.53	
Blaisdell upright	46 ins.		.63	
Blaisdell upright	50 ins.		.83	
GEAR CUTTERS				
Brainard	No. 4½		.15 to .32	
Gould & Eberhardt	No. 3		.20	
Brown & Sharpe	No. 3		.20	
GRINDERS				
Brown & Sharpe cutter and reamer grinder	No. 3		.32	
C. H. Besly & Co. gardner grinder.	No. 4	1.42	.53	
Brown & Sharpe plain	No. 11		.80	
Brown & Sharpe surface	No. 2		.40	
Brown & Sharpe surface	No. 3		.50	
Brown & Sharpe universal.	No. 1		.60	
Brown & Sharpe universal.	No. 2		.76	
Diamond wet tool grinder.		3.29	.97	Carrying one 20-in. wheel
Leland & Faulconer wet grinder.			.41 to .82	Carrying two 24-in. wheels
DROP HAMMERS				
Blondell	40 lbs.		.10	
Pratt & Whitney	250 lbs.		2.00	
Pratt & Whitney	400 lbs.		2.50	
Pratt & Whitney	600 lbs.		3.00	
Pratt & Whitney	800 lbs.		3.50	
Pratt & Whitney	1000 lbs.		4.00	
Billings & Spencer	1500 lbs.		5.00	
POWER HAMMERS				
Bradley	100 lbs.		1.50	
Bradley	150 lbs.		1.75	
KEYSEATER				
Baker Bros	No. 4	.64	.28 to .32	
LATHES				
Reed boring lathe	20 ins.		.35	
Reed boring lathe	30 ins.		.41	
Reed engine lathe	12 ins.		.24	
Reed lathe	14 ins.	.48	.26	
Prentice	16 ins.		.34	
LATHES—(*Continued*)				
Reed	16 ins.	.48	.36	
Blaisdell	18 ins.		.39	
Blaisdell	20 ins.		.44	
Reed	22 ins.	.37	.32	
Reed	24 ins.		.25	
Blaisdell	24 ins.		.31	
Prentice	28 ins.		.31	
Draper	38 ins.		.58	
Reed speed lathe	10 ins.		.10	
Reed speed lathe	14 ins.		.12	
Putnam squaring-up lathe.	15 ins.		.25	
Gisholt turret lathe	Size H		.70	
Potter & Johnston semi-automatic.	No. 1	1.63	.33 to .63	
Jones & Lamson flat turret.	2×24 ins.	1.97	1.20 to 1.80	
Wood turning lathe	14 ins.		.31	
Wood turning lathe	16 ins.		.36	
Wood turning lathe (Putnam gap).	36 ins.	1.50	1.30	
MILLING MACHINES				
Brainard	No. 1	.47	.30	
Brainard	No. 3	.64	.26	
Brainard	No. 4		.19 to .29	
Brainard	No. 4½		.13 to .19	
Brainard	No. 6		.26	
Brainard	No. 7		.83	
Brainard	No. 14		.25	
Brainard	No. 15		.25	
Becker vertical	No. 3		.26	
Becker vertical	No. 5		.55	
Becker-Brainard	No. 3		17 to .25	
Brown & Sharpe	No. 1		.15	
Brown & Sharpe	No. 2		.25	
Brown & Sharpe	No. 5		.30	
Reed	No. 7		.83	
Pratt & Whitney hand	No. 1½		.20	
PLANERS				
Whitcomb	17 ins.	2.01	1.00 to .43	
Whitcomb	22 ins.×5 ft.	2.34	1.16 to .53	
Putnam	22 ins.×5 ft.	1.44	.70	
Putnam	24 ins.×6 ft.		.84	
Putnam	26 ins.×5 ft.	1.59	.81	
Putnam	30 ins.×6 ft.	4.91	1.31	
Putnam	30 ins.×8 ft.	5.46	1.56	
Powell	36 ins.×10 ft.	4.00	1.60	
Pond	50 ins.×9 ft.	2.94	1.14	
Wood panel planer	34 ins.	7.75	3.70	
Wood surface	24 ins.	3.40	2.00	
POLISHING STANDS				
Brown & Sharpe	No. 3		1.00	
Diamond	No. 5		1.19	
PUNCH PRESSES				
Bliss	No. 3	2.59	1.26	
PROFILING MACHINES				
Garvin	No. 1		.50	
Pratt & Whitney	No. 6		.40	
BAND SAW				
Fay & Co.	36-in. wheels	3.00	.87	Used for pattern work
CIRCULAR SAWS				
Kimball Bros.	9-in. b.ade	3.77	1.05	
Whitney	9-in. blade	3.75	1.04	
White	13-in. blade	5.82	1.21	
HACK SAW				
	12 to 14 ins.		.06	
SCREW MACHINES				
Brown & Sharpe automatic.	No. 1		.60	
Pratt & Whitney automatic.	No. 2		.37	
Pratt & Whitney	No. 2		.72	

Table 20.—Power Requirements of Machine Tools in Groups—(*Continued*)

Kind	Size	Observed horse-power, maximum	Observed horse-power, average	Remarks	Kind	Size	Observed horse-power, maximum	Observed horse-power, average	Remarks
Screw Machines—(*Continued*)					Screw Machines—(*Continued.*)				
Pratt & Whitney	No. 3	....	.80		Pratt & Whitney hand.	No. 2	....	.43	
Brown & Sharpe automatic.	No. 3	....	.80		Pratt & Whitney hand.	No. 2½	....	.47	
					Pratt & Whitney hand.	No. 3	....	.50	
Pratt & Whitney automatic.	No. 3–O	1.04	.90		Shapers				
					Lodge & Davis	14 ins.	....	.35	
Pratt & Whitney	No. 3–B	1.04	.90		Hendey	20 ins.	....	.50	
Brown & Sharpe	No. 00	....	.36		Hendey	24 ins.	....	.52 to .70	
Cleveland	¾ in.	....	.40		Hendey	28 ins.	....	.52 to .70	
Cleveland	2 ins.	....	.87		Tapping Machine				
Cleveland	2¾ ins.	1.04	.90		Pratt & Whitney	No. 2	....	.10	

The distance between centers for any set of gears is determined by the formula:

$$a = \frac{b}{2P}$$

where a is the distance between centers in ins., b is the sum of the number of teeth in both gears and P is the diametral pitch. In this case

$$a = 12 = \frac{b}{2 \times 5} \text{ or } b = 120$$

The number of teeth in the pinion is,

$$\frac{b}{\text{Ratio of reduction plus 1}} = \frac{120}{5} = 24 = \text{number of teeth}$$

in the motor pinion. The number of teeth in the gear is $4 \times 24 = 96$.

Table 18 shows that the pitch-line speed for this motor with 20 teeth is 890 ft. per min. The pitch-line speed with 24 teeth is

$$\frac{24}{20} \times 890 = 1070 \text{ ft. per min.}$$

If quiet operation is desired a cloth or rawhide pinion should be used with a 3½-in. face. Thus the gears are specified as follows: Motor pinion (rawhide) $P = 5$, face 3½ ins., 24 teeth. Machine gear (steel) $P = 5$, face 3 ins., 96 teeth.

Applications of adjustable speed-motors are dealt with in a similar way. The belt speeds and pitch-line speeds must be carefully considered on the maximum speeds of these motors. The minimum pulleys and pinions are determined by the minimum speeds of the motors.

Table 19 contains the ratings mostly used and pulley and gear nformation for this type of motor.

Power Requirements of Machine Tools in Groups

Data relating to the power required to drive machine tools in groups are much more difficult to obtain and are correspondingly

Table 21.—Friction Load Due to Line and Countershafts

Department	Per cent. of friction load to the total load
Cam-cutting department	26
Cutting-off department	43
Cuttermaking department	27
Chucking department	26
Light drilling department	23
Heavy drilling department	34
Grinding department	21
Lathe department	25
Milling department	25
Planing department	26
Patternmaking department	17
Jig and fixture making department	37

Table 22.—Power Requirements of Machine Tools in Groups

Kind of machine	Kind of work	Per cent. of machines running	Floor area for machine and operator in sq. ft.	Total average power per machine in watts[1]	Total average power per machine in watts[2]	Friction and line-shaft load per machine in watts[2]	Average power used in doing actual work, in watts[1]	Total power per sq. ft. of floor area, in watts
No. 2 horizontal Rockford boring mills	Boring bearings in aluminum cases	85	150	1620	1320	1100	300	8.8
No. 4 Cincinnati millers	Light milling on aluminum	100	120	995	995	830	500	8.3
16-in. Lodge & Shipley lathes	Turning small forgings	60	55	900	555	500	87	10.1
Double disk grinders; double buffers; two-wheel emery stands.	Grinding and polishing	55	55	1800	1000	300	830	18.2
24-in. Bullard vertical lathes	Heavy cuts on cast-iron fly-wheels	100	100	1350	1350	350	1000	13.5
24-in. Gould & Eberhardt gear cutters	Cutting small cast-iron gears	100	65	333	333	250	83	5.1
Four-head Ingersoll milling machines	Making four cuts on cast-iron cylinders.	100	300	3550	3550	2300	1250	11.8
Baker single and Bausch multi-spindle drills.	Drilling and tapping cast-iron	40	70	1530	637	550	217	9.1
Heald grinders, No. 60 internal grinders.[3]	Cylinder grinding	85	70	2830	2430	1860	500	34.7
No. 6 Whitney hand millers	Keyseating small cast-iron gears	60	40	365	220	120	165	5.5
Landis No. 2 grinders	Grinding cam shafts	80	90	1875	1500	1000	625	16.7
Norton 10 by 50-in. grinders	Grinding pistons and small forgings	70	100	2000	1400	1100	450	14
Jones & Lamson flat turret lathes	Machining small forgings	85	65	675	560	200	375	8.6
Eight spindle Cincinnati gang drills	Drilling and reaming connecting rods (8 holes).	100	110	2840	2840	2000	840	25.8
Potter & Johnston automatics	Turning small cast-iron gears	100	75	690	690	440	250	9.2
1¼-in. Gridley automatics	Machining cast-iron pistons	100	200	1520	1520	1250	270	7.6
No. 4 Warner & Swasey turret lathe	Machining small forgings	65	55	560	360	310	70	6.5
24-in. Cincinnati drill presses	Small drilling on forgings	90	40	520	474	345	100	11.8

[1] Deducting idle machines. [2] Including idle machines. [3] Exhaust fan not considered.

less numerous than those relating to tools fitted with individual motors. The individual motor must be proportional to the maximum requirements of the tool, while the group motor has to meet only the average requirement, this average taking into account the fact that some of the tools are normally idle at any one moment. Group driving therefore calls for much smaller motor capacity than individual driving.

An excellent determination of the requirements for group driving, by L. P. Alford (*Amer. Mach., Oct.* 31, 1907), is given in Table 20, which includes the results of many thousand observations extending over a period of about six months in a plant comprising over 2000 machine tools. The experiments were made prior to the introduction of high-speed steel and on machine tools, generally speaking, of the light or medium class used in making light automatic machinery. The rough parts were made with a small surplus of material to be removed, making the work of the cutting tools light. Since the chief use of high-speed steel is in removing large quantities of stock, the tests have permanent value for the conditions under which they were made.

The tests were made possible by the arrangement of the works in departments, each department being devoted to tools of a given kind, not usually differing much in size.

In the case of departments containing a variety of sizes, the results were arrived at by a process of elimination. The tests were made under strictly working conditions.

The motor capacity for a department is subject to correction from the total obtained from Table 20 to cover the factor of departmental slip due to that lessening of the average horse-power values of machine tools due to working conditions in the department. During the progress of the tests the horse-power actually used by all of the machine tools in the plant was checked against the value obtained by computation after applying the individual horse-power values to the entire machine-tool equipment. Certain classes of machine tools were eliminated, such as speed lathes, grindstones, tool grinders, snagging grinders and others which are intermittent in their use. A comparison of the gross horse-power value so obtained showed that the sum of the individual power values was 20 per cent. higher than the actual horse-power used in the factories. Therefore in using these data for the purpose for which they were collected, the values obtained for the various departments by using the individual machine-tool values as given in Table 20 were reduced 20 per cent. before being used to determine the size of motor required. Two other exceptions were also made in its use. The power values were reduced one-half when applied to the machine tools of the jig- and fixture-making department and to the experimental department. The reason is obvious, as the tools are there used intermittently and with light cuts and fine feeds.

To the machine tool load as thus determined, the load due to the line- and countershafts is to be added. Table 21 gives these friction loads for the plant at which the tests were made. For additional information on the friction of shafting see Index.

Additional data of the same character are given in Table 22 by H. C. Spillman (*Mchy., June,* 1913). The tests embodied in this table were made in an automobile engine factory after it had been in operation about nine months.

The transmission equipment consists of a 20-h.p. motor for each department, driving two or three lengths of line-shafting, each about 70 ft. long. The shafting is $2\frac{7}{16}$ ins. diameter, running at 240 r.p.m., and supported on Hyatt roller bearings at intervals of 10 ft. (For additional information on the friction of shafting with plain and Hyatt bearings, see Friction of Shafting.) The shafting was carefully lined up when installed and re-checked with a surveyor's level. The equipment was in excellent condition.

In making the power tests, every precaution was taken to avoid errors. The motors were all tested for their efficiency at different loads and the electrical instruments were carefully calibrated. Readings were taken when the motor was driving the line-and countershafts only, no machines being in operation. The electrical losses were deducted, which gave the net shafting and countershafting loss, and this was carefully proportioned among the different machines. The second reading was taken with the machines running light, which gave the total friction loss. After these tests were completed, the machines were put into operation and readings were taken every fifteen minutes. A record was also made of the number of machines in operation and the kind of work the machines were doing. Each motor in this factory drives only one kind of machine tools, which greatly assisted in obtaining accurate results for the amount of power consumed by each size and kind of machine. The data were carefully tabulated and the floor area occupied by each machine, and space allotted for the operator were noted; also the total length of line-shafting. The trucking aisles and all other space not used for manufacturing was deducted, so that the unit values per sq. ft. of floor area include the machine and sufficient space for the operator and material. The results show that the line-shafting and countershafting consume 30 per cent. of the total power, and the total friction losses absorb 72 per cent. of the total power. This makes a 42 per cent. loss of power from the countershafting to the machine tools, and only 20 per cent. of the total power is utilized in doing work. The electrical loss shows 8 per cent. of the total power. In the table there are two items mentioned as follows: Total average power per machine, deducting idle machines; total average power per machine, including idle machines. These items include all the mechanical power of that department, such as line-shafting, countershafting, machine friction and power consumed in doing work on the machines. In the first case this total power is equally divided among all the machines which are in operation. In the second case it is divided equally among all the machines, both running and idle. The electrical losses are omitted in all cases.

Power Constants for Punching and Shearing

The power required for punching and shearing formed the subject of experiments by Prof. G. C. Anthony (*Amer. Mach., May* 22, 1913). The apparatus employed consisted of a hydraulic bolster below the die and connected to an ordnance indicator by which indicator diagrams of the pressures were obtained (*Trans. A. S. M. E., Vol.*, 33). Examples of these diagrams to a reduced scale are given in Fig. 12. The steel plate tested was from the Lukins Iron and Steel Co., and was of $\frac{1}{4}$, $\frac{5}{16}$, $\frac{3}{8}$, $\frac{7}{16}$, $\frac{1}{2}$ and $\frac{5}{8}$ in. thickness, having an average tensile strength of 59,000 lbs. per sq. in. with elongation of 27 per cent. and reduction of area of 55 per cent. Flat, bevel and spiral punches of 3 deg. of clearance were included in the tests. The cards were interpreted for both maximum pressure and ft.-lbs. of work required.

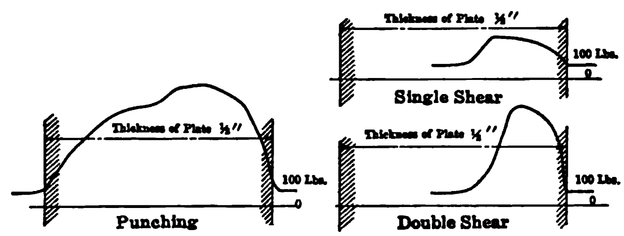


Fig. 12.—Indicator diagrams from punching and shearing experiments.

Fig. 13 gives the work and maximum pressures developed when using flat punches having .06 in. clearance. Figs. 14 and 15 give the effects of clearance and shape of the punch on the work and maximum pressure required for punching. The character of the punch and amount of clearance are given at the top of the charts; the ft.-lbs. of work and maximum pressure are at the left, and the

thickness of the plate is indicated on the charts. Save in two or three cases the minimum values for work and pressure were obtained by the use of the flat punch, and in one of these cases, the spiral punch in ⅜-in. plate, the value is questionable by reason of insufficient data.

While efficiency in the use of bevel and spiral punches in thick plates has been frequently questioned, it has been believed that a decrease in pressure was general when they were used in punching thin plates, but the results of these experiments do not confirm this. The bevel and spiral punches crowd the metal to the walls of the die, thus producing unnecessary friction, while the real cutting edge, which is on the die, does not produce the effect of a bevel shear.

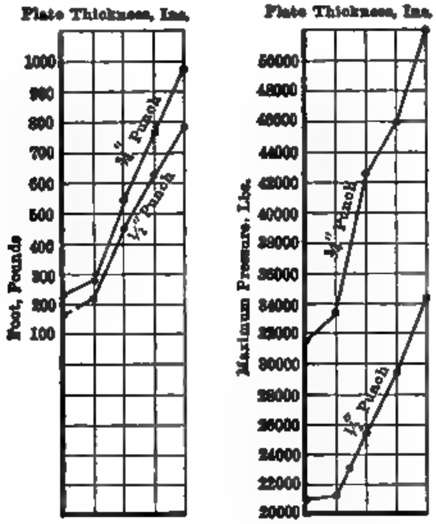


FIG. 13.—Work and pressure of punching steel plates with .06 in. clearance.

Fig. 16 gives the work and maximum pressures required per sq. in. for punching and for single and double shear of plate and rivet. The tension value has been added for purpose of comparison.

It will be observed that the work required for punching is approximately double that for shearing; that the ultimate shearing strength of the plate is about 75 per cent. of the tensile strength; that the ultimate strength of the rivet in double shear is 1.82 greater than in single shear; that the ultimate strength of the plate in double shear is 1.95 greater than in single shear.

FIG. 14.—Effects of clearance and of form of punch on work of punching.

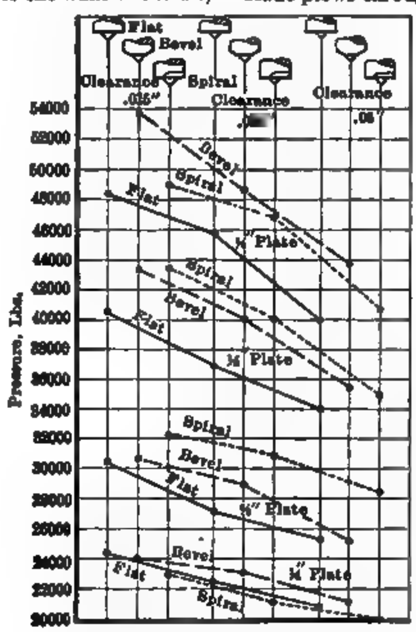


FIG. 15.—Effects of clearance and of form of punch on maximum pressure of punching.

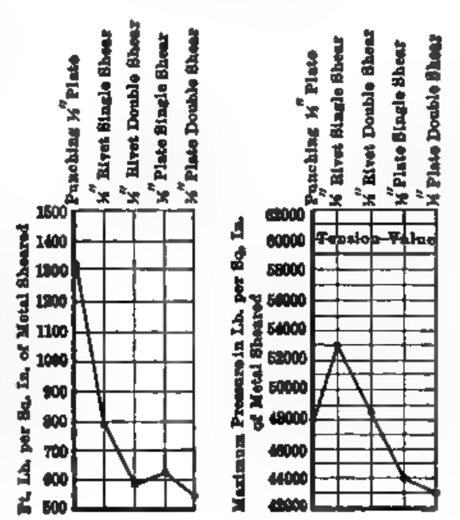


FIG. 16.—Punching and shearing values of steel plate.

TABLE 23.—SHEARING VALUES OF HOT STEEL BLOOMS OF .20 CARBON AND 70,000 LBS. ULTIMATE STRENGTH

Size of bloom, ins.	Temperature Fahr. about	Max. pressure, lbs. per sq. in.	Energy, ft.-lbs. per sq. in.
9×9	2500	5,000	540
6×6	2500	9,000	
4×6½	2500	11,000	800

TABLE 24.—SHEARING VALUES OF COLD STEEL BARS OF 70,000 LBS. ULTIMATE STRENGTH

Thickness of bars, ins.	Angle of knives, deg.	Max. pressure, lbs. per sq. in.	Energy, in.-lbs. per inch of width
1	Flat	48,000	1200
1	4	36,000	1000
1	8	22,000	700
1½	Flat	48,000	2500
1½	4	45,000	2000
1½	8	32,000	1600

Experiments with similar apparatus were made and reported by H. V. Loss (*Journal of the Franklin Institute, Dec.* 1899). Mr. Loss's experiments covered the shearing of hot blooms from 4×4 ins. to 10×10 ins., and of cold bars from ¾ to 2½ ins. thick and 4 to 8 ins. wide. His results are summarized in Tables 23 and 24. The apparent anomaly of greater energy consumed when cutting hot metal is apparent only. With cold metal the bar breaks after a comparatively small depth of penetration, while, with hot metal, the shear blade plows through the entire thickness before the parts separate.

At a temperature of about 1800 deg. Fahr. the maximum pressure increases about 50 per cent. for the larger and 100 per cent. for the smaller sizes. At the same temperature the energy increases about 40 per cent. for the larger and 75 to 80 per cent. for the smaller sizes.

The pressure required to drive rivets may be obtained from Fig. 17 (*Amer. Mach., July* 13, 1911), which is based on formulas by Wilfred Lewis.

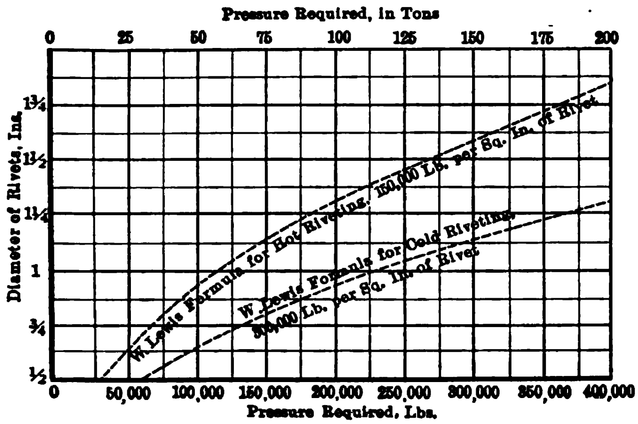


FIG. 17.—Pressure required to drive rivets.

Power Constants for Centrifugal Fans

The power required to drive Sturtevant centrifugal fans is given in Tables 25 and 26 from tests by the Interior Conduit and Insulation Co. (*Amer. Mach., Dec.* 31, 1896).

Centrifugal fans consume an amount of power which is dependent upon the opening of the outlet and the amount of air which the fan is allowed to pass—an obstruction in the outlet operating to decrease the power consumed, which is at a maximum when the outlet is entirely free. As fans are actually used for blowing fires and many other purposes, the resistance of the fire operates as an obstruction and the power consumed is, during normal conditions, reduced from the maximum. Nevertheless, at various times this resistance is reduced or may be absent, when the power consumed at once mounts up to the maximum.

With fan driven by a belt or special engine, this increase is a matter of little moment so long as the belt or engine is able to drive the fan, and on this account the figures for power given in the catalog of fan makers show what is supposed to be the average or normal requirements. When fans are driven by electric motors the conditions are changed, since an electric motor has no limit of capacity beyond which it slows down or stalls, but, on the other hand, takes more and more current in the endeavor to drive the load, until a burn-out results. Consequently electric motors for fans should be proportioned with reference to the maximum requirements, and not, as with steam engines, to the mean.

The figures of the tables are no doubt the equivalents of the current readings which necessarily exceed the actual power consumed by the fans.

A pressure of 4 oz. is amply sufficient for ordinary forge fires. There is a tendency toward specifications for higher pressures than this, even up to 8 oz., but it is doubtful if such pressures ever reach the fire, the convenient blast gate cutting the pressure down to lower figures.

The horse-power required to drive centrifugal fans has been investigated by A. E. Guy, and the results are given below (*Amer. Mach., June* 29, 1911).

When the fan takes the air from the atmosphere and delivers into a duct, and particularly when that duct or pipe is circular, it is comparatively easy to measure the approximate capacity of the apparatus when the air handled is at a moderate temperature. The instrument needed for the operation is very simple and can be easily made. Fig. 18 represents a conbination of Pitot and pressure tubes connected to a glass U-tube containing water. The end of the assembled tubes should be inserted into the delivery pipe as shown. A straight part of the pipe should be selected where the flow is not likely to be disturbed by the influence of bends, valves, etc. The gage should be inserted into the pipe for about one-sixth the diameter and turned so that the open end of the Pitot tube is against the current. If the tube is not so placed the readings will not be correct.

TABLE 25.—POWER REQUIRED TO DRIVE STURTEVANT STEEL PRESSURE BLOWERS

The Upper Figures of Each Set Give the Power Consumed with the Outlet One-third Open; the Middle Figure, Two-thirds Open, and the Lower Figure Fully Open

Pressure of blast	4 oz.		5 oz.		6 oz.		7 oz.		8 oz.	
Size No. of blower	Rev.	H.p.	Rev.	H.p.	Rev.	H.p.	Rev.	H.p.	Rev.	H.p.
		.7		1.0		1.3				
2	3103	1.4	3445	2.0	3756	2.6				
		2.1		3.0		3.9				
		1.0		1.4		1.8				
3	2456	2.0	2753	2.8	3006	3.6				
		3.0		4.2		5.4				
		1.4		1.9		2.5				
4	2224	2.8	2470	3.8	2692	5.0				
		4.2		5.7		7.5				
		2.0		2.8		3.6		4.6		
5	1814	4.0	2026	5.6	2215	7.2	2387	9.2		
		6.0		8.4		10.8		13.8		
		2.6		3.6		4.7		6.0		7.3
6	1619	5.2	1797	7.2	1960	9.4	2009	12.0	2258	14.6
		7.8		10.8		14.1		18.0		21.9
		3.6		5.0		6.5		8.3		10.4
7	1344	7.2	1507	10.0	1641	13.0	1768	16.6	1898	20.8
		10.8		15.0		19.5		24.9		31.2
		4.5		6.4		8.4		10.6		13.0
8	1200	9.0	1330	12.8	1445	16.8	1565	21.2	1675	26.0
		13.5		19.2		25.2		31.8		39.0
		5.9		8.3		10.9		13.8		16.9
9	1035	11.8	1145	16.6	1250	21.8	1350	27.6	1446	33.8
		17.7		24.9		32.7		41.4		50.7
		7.9		11.2		14.5		18.4		22.5
10	902	15.8	995	22.4	1085	29.0	1168	36.8	1253	45.0
		23.7		33.6		43.5		55.2		67.5

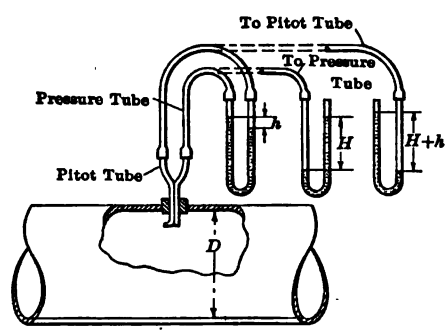


FIG. 18.—Apparatus for finding the pressure and flow of air in blast pipes.

With the two rubber tubes in place the difference in the heights of the columns of water in the U-tube shows the velocity head causing the flow in the duct. Disconnecting the Pitot tube from the glass gage and measuring the height between the two levels, will indicate the pressure head against which the air is delivered. Again connecting the Pitot tube and disconnecting the pressure tube, will show, by the difference in the hights of the water columns, the total head produced by the fan. This total head is composed of the static head measured by the pressure tube, plus the velocity head shown when the two tubes are used together.

TABLE 26.—POWER REQUIRED TO DRIVE STURTEVANT MONOGRAM BLOWERS

The Upper Figures of Each Pair Give the Power Consumed with the Outlet One-half Open, and the Lower Figure with the Outlet Fully Open

Pressure of blast	1 oz.		1½ oz.		2 oz.		2½ oz		3 oz.		3½ oz.		4 oz.		5 oz.	
Size No. of blower	Rev.	H.p.	Rev.	H.p.	Rev.	H.p.	Rev.	H.p.	Rev.	H.p.	Rev.	H.p.	Rev.	H.p.	Rev.	H.p.
0	1863	.15 .30	2274	.28 .56	2615	.44 .88	2912	.6 1.2	3177	.8 1.6						
1	1632	.21 .42	1992	.38 .76	2291	.60 1.20	2550	.8 1.7	2782	1.1 2.2	2992	1.4 2.8				
2	1373	.28 .56	1677	.5 1.0	1928	.79 1.6	2147	1.1 2.2	2343	1.5 2.9	2520	1.8 3.7				
3	1167	.40 .80	1425	.7 1.4	1638	1.1 2.2	1824	1.6 3.1	1900	2.0 4.1	2140	2.6 5.2	2279	3.2 6.3	2527	4.4 8.8
4	1050	.5 1.1	1227	1.0 2.0	1410	1.5 3.0	1570	2.1 4.2	1713	2.8 5.6	1842	3.5 7.0	1961	4.3 8.6	2176	6.0 12.0
5	852	.7 1.5	1038	1.3 2.7	1194	2.1 4.2	1330	2.9 5.9	1450	3.9 7.8	1560	4.9 9.8	1662	6.0 11.9	1843	8.4 16.7
6	726	1.0 2.1	886	1.8 3.7	1018	2.9 5.7	1134	4.0 8.0	1237	5.3 10.5	1331	6.7 13.4	1417	8.1 16.2	1571	11.4 22.7
7	632	1.3 2.7	772	2.4 4.9	878	3.8 7.5	988	5.3 10.6	1078	6.9 13.9	1159	8.7 17.3	1234	10.7 21.4	1368	15.0 30.0
8	545	1.7 3.4	665	3.1 6.2	766	4.8 9.5	852	6.7 13.4	930	8.8 17.6	1000	11.1 22.3	1065	13.5 27.0	1180	18.2 36.5
9	477	2.2 4.5	583	4.1 8.2	671	6.3 12.7	748	8.9 17.8	815	11.7 23.4	876	14.8 29.7	932	18.0 36.1	1034	25.3 50.6
10	426	2.9 5.9	519	5.4 10.8	598	8.3 16.7	667	11.7 23.3	726	15.4 30.7	781	18.0 36.1	831	23.7 47.3	922	33.2 66.4
36	362	4.0 8.1	443	7.4 14.8	511	11.4 22.9	567	16.0 32.0	611	21.1 42.1	665	26.7 53.4	712	32.5 64.9	785	45.5 91.0
37	316	5.3 10.7	345	9.8 19.7	413	15.2 30.3	493	21.2 42.5	538	27.9 55.9	579	35.4 70.9	615	43.0 86.1	683	60.3 120.7

Calling the velocity of flow v ft. per sec., the velocity head h ins. of water, and the static pressure head H ins. of water,

$$v=\sqrt{2g\frac{p}{d}}=\sqrt{\frac{1,746,700\times h}{406.7+H}}=1321\sqrt{\frac{h}{(406.7+H)}}$$

in which,

p = pressure in lbs. per sq. ft.,

d = weight in lbs. of 1 cu. ft. of free air at 50 deg. Fahr. = .077884,

406.7 = ins. of water, corresponding to atmospheric pressure.

Knowing the inside diameter D, in ins., of the delivery pipe, the volume discharged in cu. ft. per sec. is

$$\frac{\pi D^2}{4\times 144}\times v$$

But this air is at a pressure H and the corresponding volume of free air per min. would be

$$\frac{\pi D^2\times v\times 60\times(406.7+H)}{4\times 144\times 406.7}=\frac{D^2\times v\times(406.7+H)}{1242}\text{ cu. ft. per min.}$$

The horse-power in air delivered would be

$$\frac{\text{volume per min.}\times\text{pressure per sq. ft.}}{33,000}$$

One cu. ft. of water weighs 62.35 lbs.; 1 in. of water equals

$$\frac{62.35}{12}=5.196\text{ lbs. per sq. ft.}$$

Hence,

$$\frac{\text{vol. per min.}\times 5.196\times H}{33,000}=\text{air h.p.}$$

or

$$\text{air h.p.}=\frac{\text{cu. ft. per min.}\times H}{6350}$$

Substituting for the volume and velocity their repsective values:

$$\text{air h.p.}=\frac{D^2\times v\times H\times(406.7+H)}{6350\times 1242}=$$

$$D^2\times H\times\left[1321\sqrt{\frac{h}{406.7+H}}\right]\times(406.7+H)=\frac{D^2H\sqrt{h}\times(406.7+H)}{5970}$$

As the efficiency of ordinary blowers is about 50 per cent., multiplying the air horse-power as just obtained by 2 gives approximately the shaft horse-power necessary to run the blower. While reading the gages the speed should be kept constant, and the time selected when the flow of air is uniform.

The gage readings and particularly that of the velocity head should be very close, for which reason it is preferable to use a U-tube of rather small diameter.

The formulas given are intended for approximate work only. The density of the air depends so much upon the temperature that the method would not apply to hot-blast work, for instance. Corrections should also be made for altitude and humidity.

Power Constants for Moving Heavy Loads

The power required to move heavy loads on wheels may be obtained from Fig. 19, by A. D. HARRISON (*Amer. Mach., June* 18, 1908). The chart was originally designed for hoists and cranes but is applicable to analogous conditions. It represents the formula:

$$\text{Brake h.p.}=.0097\left[\frac{WS}{D}(d+.7)\right]$$

in which W = total weight of structure, tons,

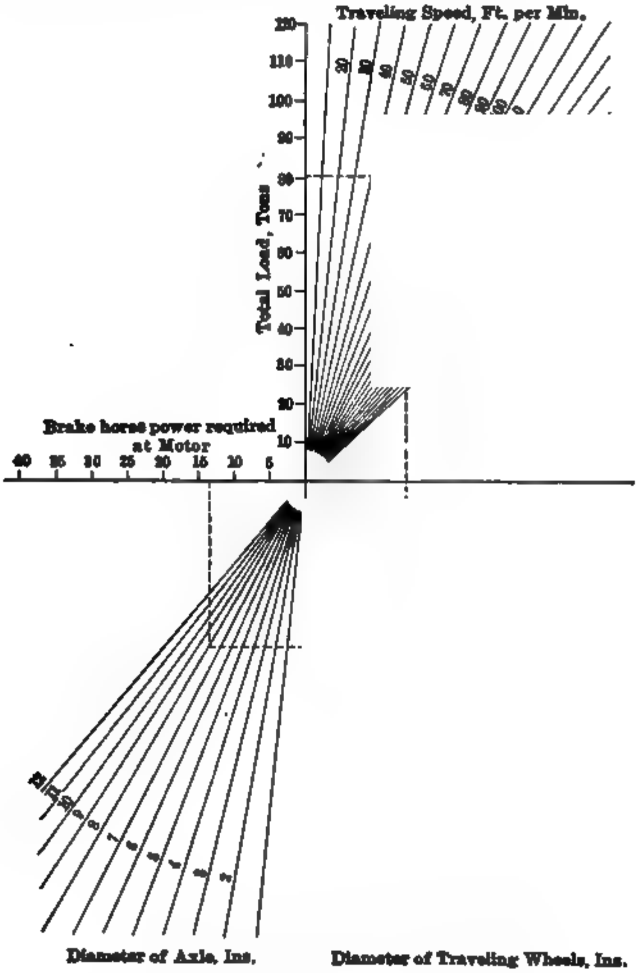
S = speed, ft. per min.,

D = diameter of wheels, ins.,

d = diameter of axles, ins.

The coefficient of rolling friction is taken at .035, the coefficient of sliding friction at .1 and the efficiency of the gearing at .70.

The use of the chart is shown by an example below it.



Starting with a load of 80 tons, trace to the right to the speed —60 ft. per min.—then down to the wheel diameter—27 ins.—then to the left to the axle diameter—8 ins.—and then up to the horse-power—13.

Fig. 19.—Power required to move heavy loads on wheels.

Measuring the Energy of Hammer Blows

The measurement of the energy of hammer blows by the compression of lead plugs, formed the subject of experiments by the Niles-Bement-Pond Co., which were reported by W. T. Sears, Mech. Engr. of the company (*Amer. Mach., Mar.* 10, 1910).

Previous experiments of this kind have, usually, been made by comparing the compressions obtained under hammers with the measured compressions obtained under testing machines. The speed of the compression is, however, known to affect the results and, hence, these tests were made under falling weights at speeds equal to those obtained in actual practice in a steam hammer, in order to get final results which could be depended upon in steam-hammer work. These results are, finally, compared with those obtained from slow-speed or static tests.

The velocity of a hammer ram at the instant before impact with the anvil, depends on friction, the total mean effective pressure on the piston and the distance it has fallen through.

For a Niles-Bement-Pond 1100-lb. steam hammer of 28-in. stroke, the maximum velocity, assuming a constant pressure of 100 lbs. per sq. in. on the piston on its downward stroke, and neglecting friction, would be in the neighborhood of 35 ft. per sec., and this corresponds to the speed due to gravity alone, acting through a distance of over 30 ft.

Table 27.—The Compression of $1\frac{1}{2} \times 1\frac{1}{2}$-in. Lead Plugs under Falling Weights

Note.—Where division lines include two or more plug numbers, the weight was dropped upon as many plugs at one time as there indicated.

Plug No.	Falling weight, lbs.	Drop, ins.	In.-lbs. of energy	Plug dimensions, ins. Long	Upset to	Upset	Average upset	Striking velocity, ft. per sec.
13	20	240	4,800	1 489	.979	.51	. . .	35 9
14	20	120	2,400	1.494	1.165	329	. . . . .	25 4
15	20	240	4,800	1.492	.983	.[illegible]	.	35 9
16	20	360	7,200	1.492	.848	.644	. . . . . .	43.9
17	50	120	6,000	1.492	911	581	. . . . .	25 4
18	50	240	12,000	1.492	.656	.836	. .	35.9
19	50	360	18,000	1 492	505	987	. . . .	43 9
20	100	360	36,000	1.491	307	1.184	. . . .	43.9
21	100	240	24,000	1.491	.409	1 082	. . . . .	35.9
22	100	120	12,000	1.489	652	.837	. . . . . .	25.4
23	150	360	54,000	1.491	.194	1 297	. . . .	43.9
24	150	240	36,000	1.495	290	1.205	. .	35 9
25	200	120	24,000	1.492	.401	1 091	. . . .	25 4
26	200	240	48,000	1.492	219	1 274	. . .	35.9
27	200	120	24,000	1.489	.405	1 084	. . . .	25.4
28	150	120	18,000	1 492	501	.991	. . . . . .	25.4
29	150	240	36,000	1.502	275	1 227	. . . .	35 9
30	150	240	36,000	1.503	275	1 228		35.9
31 32	150	240	36,000	1.498 1 498	502 507	.996 .991	.993	35.9
33 34 35	150	240	36,000	1 5 1.498 1 495	.670 .681 677	.830 .817 .818	.822	35.9
36 37 38 39	150	240	36,000	1.5 1 5 1 498 1 5	769 .778 .778 .771	.731 .722 720 .729	.725	35 9
40	150	240	36,000	1 498	.279	1 219	. . . .	35 9
41 42 43 44	200	240	48,000	1 497 1.502 1 502 1 502	.663 .660 660 .650	.834 .842 .842 .852	.842	35.9
45 46 47 48	[illegible]	356.5	71,300	1.5 1 501 1 5 1 502	.515 .525 .525 .515	.985 976 .975 987	.981	43 7

Weight of anvil—8660 pounds.

The plugs, which were $1\frac{1}{2}$ ins. diameter by $1\frac{1}{2}$ ins. long, were tested, in most cases, one at a time by placing them on an anvil, having a weight of over 8000 lbs. and striking them with different size cylindrical weights, weighing from 20 to 200 lbs. dropping from different heights up to 360 ins. In addition to a drop on a single plug, the

150-lb. weights were tried with two, three and four plugs, and the 200-lb. weight with four plugs.

The falling weights were guided by two lengths of piano wire stretched tight vertically. The weights were tripped without giving any initial velocity, and there is not much question but that the actual and theoretical velocities at instant of impact were, very closely, the same, the friction loss due to the guides and air being, undoubtedly, very slight.

There was certainly some loss, even if small, and therefore the compressions obtained were perhaps a trifle less than they should have been.

Table 27 gives the results of these tests.

In plotting the energy curves, Fig. 20, which are the values that were wanted, it was found that there was not so much difference in the higher speeds, as was perhaps to be expected from the considerable difference that occurred at the low speeds.

In other words, the higher the velocity, up to the maximum of speeds tested, the less the energy curves varied.

Somewhat similar, though less complete, tests using copper cylinders, which have been often referred to, were made by Prof. R. H. Thurston (*Amer. Mach.*, *Dec.* 24, 1903). The original object of these tests was to determine the comparative efficiencies of crank and friction roll (board drop) presses. Two hammers of each type were tested, the falling weights being about 300 and 900 lbs. respectively. They were adjusted to fall 28 ins., that being the maximum lift of the crank drop hammer. The effect attainable by utilizing the full 60-in. lift of the friction roll hammer was not determined experimentally, but it is easily calculable from the data obtained.

The gages used in measuring the work done by the hammers were cylinders of pure merchant copper, prepared for the purpose. They measured: Size No. 1, 2½ ins. long, 1¼ ins. diameter; size No. 2, 2 ins. long, 1 in. diameter; size No. 3, 1¼ ins. long, ⅝ in. diameter.

Of these, a considerable number were prepared and divided into three sets: one for use with each kind of hammer, and one for testing and standardizing in testing machines. The work done by crushing

Table 28.—Work Done by Drop Hammers as Measured by the Compression of Copper Cylinders

	Friction roll drop hammer				Crank lift drop hammer			
Weight of drop	903 lbs.		319 lbs.		925 lbs.		290 lbs.	
Size of copper cylinders	1¼×2½" No. 1	1×2" No. 2	1×2" No. 2	⅝×1¼" No. 3	1¼×2½" No. 1	1×2" No. 2	1×2" No. 2	⅝×1¼" No. 3
Area in sq. ins. under compression curves (see chart).	ADE	AHI	ANO	ARS	ABC	AFG	ALM	APQ
	45.22	45.26	13.75	13.76	35.10	36.25	10.75	10.50
	Average 45.34		Average 13.75½		Average 35.67		Average 10.67½	
Reduced to work done or in.-lbs.	22,715	22,630	6,875	6,880	17,550	18,075	5,875	5,250
	Average 22,672		Average 6,877		Average 17,812		Average 5,312	
Reduced to work done or ft.-lbs.	Average 1,884		Average 576		Average 1,484		Average 443	
Work done per lb. of drop in ft.-lbs.	Average 25.10		Average 21.56		Average 19.14		Average 18.30	
Work done per lb. of drop in ft.-lbs.	Average 2.09		Average 1.8		Average 1.6		Average 1.52	

This is quite clearly illustrated in the chart, Fig. 20, which gives the energy curves worked up from the Niles-Bement-Pond tests and from tests made at Purdue University. It would seem as if, after a speed of say 10 ft. per sec. was obtained, that a further increase in compression speed makes very little change.

The speed of 10 ft. is simply a guess, and it may be 5 ft. or 1, or even less.

This was a point which was not important to the company, but it would seem to be vitally important in measuring energy of blows where the speed is low, for all tests so far made show that energy calculations of a slow-moving blow cannot be even closely estimated unless the speed is known.

Curve *A* is worked up from slow-moving or static-pressure tests at Purdue University.

Curves *B* and *C* are the energy curves, resulting from Niles-Bement-Pond slow-moving or static tests of which the speeds are given.

Curve *D* is the result of the low velocity drop tests made by the Niles-Bement-Pond Co., which are not shown in the table, but which were made roughly in a hammer, having a falling weight of 1330 lbs., an anvil weight of 16,400 lbs. and a maximum drop of 38 ins.

Curve *E* is plotted from the tabulated results given in Table 27, and is the curve that is used for hammer calculations.

Curve *F* is plotted from the published results of Purdue drop tests, in which the maximum velocity is 19$\frac{7}{10}$ ft. per sec.

In order to check up new lots of plugs from time to time, static or slow-moving tests are obtained, and if these agree with previous ones, it is assumed that the action at the high speeds will also be practically the same, thus giving fairly dependable results.

No appreciable difference has been noted in new lead obtained from time to time, or in lead that has been used for tests and remelted.

The lead should be reasonably pure, though small amounts of mpurities do not appear to affect the accuracy of the results.

the standards in the testing machine to the same extent that companion specimens were crushed under the hammers, gave a measure of the action of the latter, and permitted a fair comparison to be made. The amount of work done in the slowly acting testing machine in producing a given compression is somewhat less than where the same effect is suddenly produced, as by a falling weight; but this difference effects the two hammers nearly alike, and, if the difference were measurable, it would be found to tell against the drop which falls most rapidly—the friction roll hammer, in this case.

The results of the experiments thus made are exhibited in Table 28 and Fig. 21. The final results of the table are given in ft.-lbs. of work per lb. of hammer, and the unavoidable differences in size are thus eliminated.

The chart, Fig. 21, was made thus: The compression of each set of gage cylinders was averaged for each of the two styles of hammer. These average compressions were laid off, on a convenient scale, horizontally from the left toward the right. Erecting ordinates at the extremities of the abscissas thus measured off, proportional to the loads required to produce the same compression as determined by the testing machine, and shown on the chart by the curve laid down by plotting the loads and compressions obtained by test, a measure of the work done by the hammer is obtained.

This was done for each hammer, and a set of measures is thus given of the work done by each machine, and the effects produced by the hammers are rendered easily comparable.

Comparing the tabulated figures, it is seen that the friction roll drop hammers performed, respectively, 25.1 and 21.56 in.-lbs., or 2.1 and 1.8 ft.-lbs. of work per lb. of weight of drop or hammer, while the crank lift hammer gives 19.14 and 18.3 in.-lbs., or 1.5 ft.-lbs. per lb. of hammer falling 27½ ins. The theoretical effect would be 27½ in.-lbs, or 2.25 ft.-lbs. The "efficiencies" of the two are, therefore, 90 per cent. for the friction roll hammer, and less than 70 per cent. for the crank lift hammer.

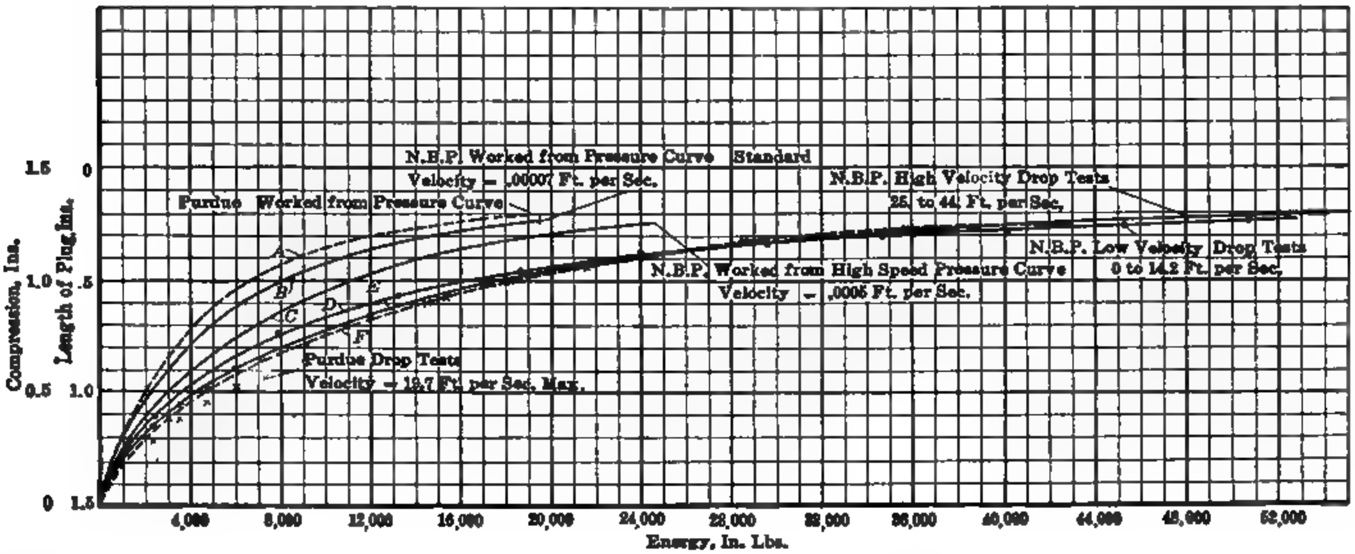


FIG. 20.—Energy curves from compression tests of lead plugs.

FIG. 21.—Work done and pressures obtained by drop hammers as measured by the compression of copper cylinders.

Cutting Capacity of Power Presses

The cutting capacity of power presses has been analyzed by E. W. ZEH (*Amer. Mach., Oct.* 12, 1905), the result being the chart, Fig 22. The chart is based on the principle that the cutting length increases inversely as the square of the thickness of the material. That is,

$$l = \frac{A}{t^2 s} \qquad (a)$$

in which l = cutting length, ins.

A = energy required to shear a flat bar using parallel cutting edges, in.-lbs.,

t = thickness of material, ins.,

s = ultimate resistance to shearing.

The fact that the material is severed before the upper knife has descended the full thickness of the bar brings in another influence which has to be taken into account. This depth of penetration as it may be called, varies greatly. It is influenced by the ductility and thickness of the material and it increases as the thickness decreases, but not in simple proportion. Table 29 gives the results of some experiments with soft steel, in which t stands again for the thickness of the material, and p for the depth of penetration.

TABLE 29.—THE DEPTH OF PENETRATION IN SOFT STEEL

t =	1	$\frac{3}{4}$	$\frac{5}{8}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{5}{16}$
p =	.25	.31	34	.37	.44	.47
t =	$\frac{1}{4}$	$\frac{3}{16}$	$\frac{1}{8}$	$\frac{3}{32}$	$\frac{1}{16}$	$\frac{1}{32}$
p =	.5	.56	.62	67	.75	.87

In Fig. 21 the curve C shows in a graphical manner how the depth of penetration varies.

Taking into cansideration the depth of penetration, formula (*a*), for the cutting length will now have the following form:

$$l = \frac{A}{t^2 ps} \qquad (b)$$

When the cutting length of a power press for a certain thickness t and resistance s is given, and it is desired to know the cutting ledgth l_1 for another thiekness t_1 of the resistance s_1 the following formula, which is derived from (*b*), will give the answer:

$$l_1 = \frac{t^2 lps}{t_1^2 p_1 s_1} \qquad (c)$$

in which p_1 stands for the depth of penetration for the new thickness introduced.

To find the angle which the upper knife must have to keep th maximum pressure within the limit of the press, formula (*d*) may b inverted thus:

$$\cot \alpha = \frac{P}{.5t^2 s}$$

When closed cutting dies are used, for instance, a round blankin die, it is customary for practical reasons to give the die (or punch several high points, and the question arises: How does the numbe of cutting points affect the pressure which is required to penetrat the material?

In answer to this question refer to Fig. 24, in which c is the devel oped circumference or cutting length of a round die. According t

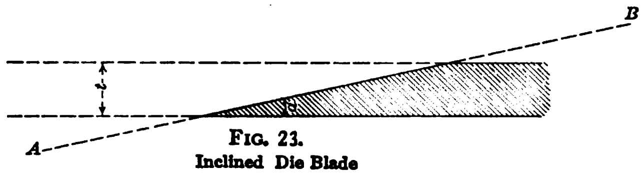


FIG. 23.
Inclined Die Blade

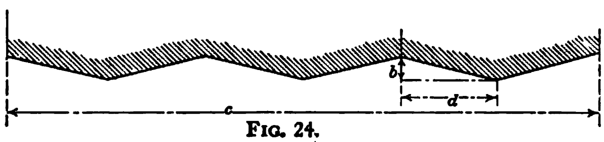


FIG. 24.
Die Blade with Multiple Points

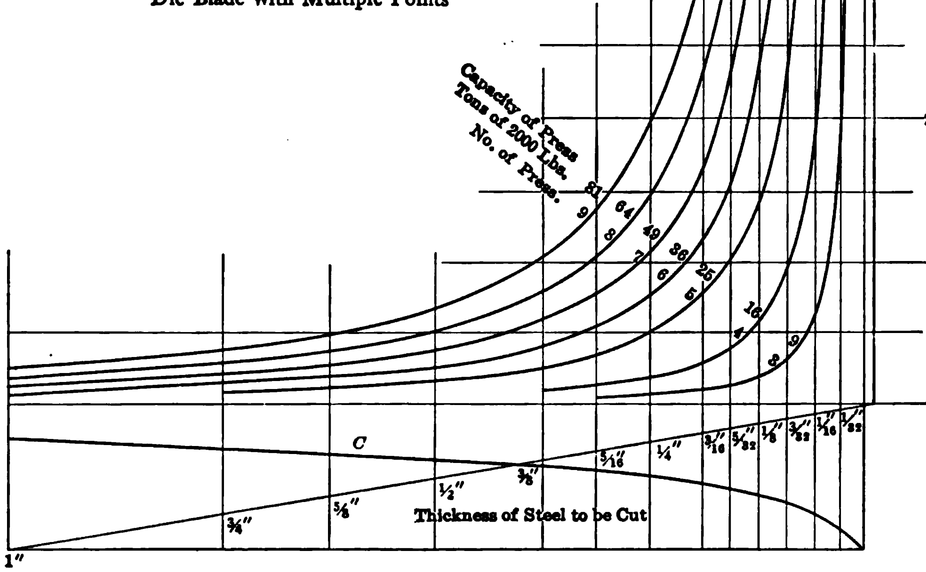


FIG. 22.—Cutting capacity of power presses.

While the cutting length of which a press is capable may be obtained from these formulas, another factor has also to be taken into consideration. The pressure required to force the knife or die through the material must not exceed a certain maximum which is fixed for each press. To keep the pressure within this limit, it is often necessary to incline the edge of one of the dies.

When the cutting edge AB, Fig. 23, descends, it finds a resistance

$$P = .5\, t^2 \cot. \alpha s, \qquad (d)$$

in which t = thickness of the material,
s = ultimate resistance to shearing,
α = angle of the knife,

This formula is limited in one direction to which attention should be called, viz., the width of the bar divided by its thickness must be greater than cot α.

formula (*d*) the pressure necessary for one inclined side of the punch is equal to $.5t^2$ cot. αs, consequently for both sides of one cutting point the pressure is twice this amount or t^2 cot. αs. If n be the number of cutting points, the total pressure necessary is

$$P = t^2 \cot. \alpha sn \qquad (f)$$

$$\cot \alpha = \frac{d}{b} \text{ and } d = \frac{c}{2n}$$

consequently $$\alpha = \frac{c}{2n} \div b$$

This value, substituted in formula (*f*), gives:

$$P = \frac{t^2 cs}{2b} \qquad (g)$$

In this formula the number of high points does not appear at all,

owing that the pressure is not influenced by the number of such oints and merely depends upon the amount of shearing b. To scertain the amount of shearing b which the die must have to keep e pressure within the limit P, formula (h) may be used:

$$b=\frac{t^2cs}{2P} \qquad (h)$$

Although developed from a plain round cutting die, these formulas ill hold good for irregular dies. In this case it is only necessary to bsesve that all sections of the cutting edge have the same inclination.

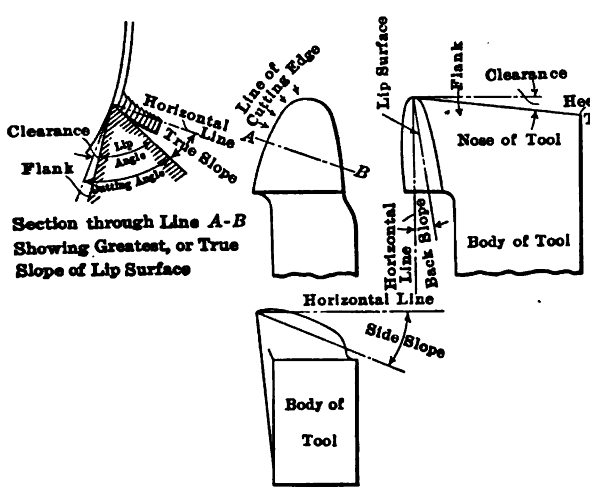


FIG. 25.—Definitions of tool elements.

Following are two practical applications of the formulas:

(1) A die has a cutting length of 14 ins. in $\frac{1}{8}$-in. soft steel. Which size press is required?

The chart shows that the No. 5 press cuts $\frac{1}{8}$-in. steel about 14 ins. long. Consequently this press will do the work. This press, exerting a maximum pressure of 25 tons, we have to shear the die sufficiently to keep the pressure within this limit. According to formula (h) this shear must be at least

$$\frac{1\times14\times60{,}000}{64\times2\times50{,}000}=.14 \text{ in.}$$

This being the extreme limit, it would be well to increase the shear somewhat, say to .2 or .25 in.

Taylor's Tool Forms

The shape and duty of roughing tools formed the subject of exhaustive experimental investigation by F. W. TAYLOR and his associates (*Trans. A. S. M. E., Vol.* 28). Mr. Taylor defines the various elements of cutting tools by means of the outline sketches, Fig. 25. Regarding the values of the various angles for roughing tools he makes the following recommendations:

Contrary to the opinion of almost all novices in the art of cutting metals, the clearance angle and the back-slope and side-slope angles of a tool are by no means among the most important elements in the design of cutting tools, their effect for good or evil upon the cutting speed and even upon the pressure required to remove the chip being much less than is ordinarily attributed to them.

The clearance angle should have the following values:

(A) For standard shop tools to be ground by a trained grinder or on an automatic grinding machine, a clearance angle of 6 deg. should be used for all classes of roughing work.

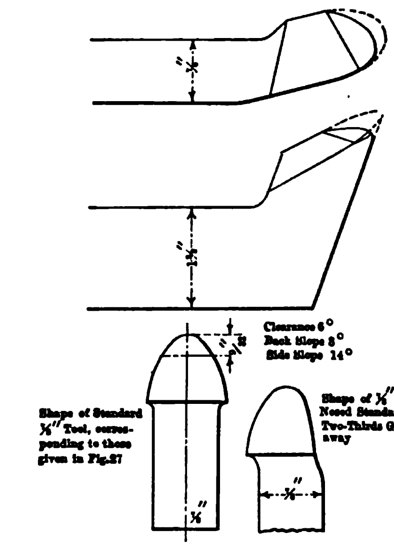


Fig. 26.—Taylor's standard $\frac{7}{8}$-in. roughing tool.

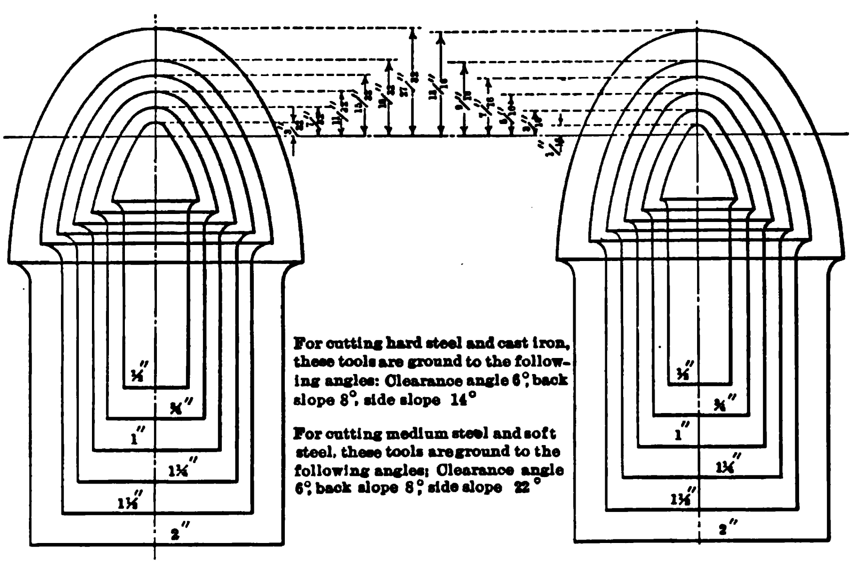


FIG. 27.—Blunt tool for cutting hard steel and cast-iron.

FIG. 28.—Sharp tool for cutting medium and soft steel.

The cutting capacity of a given power press for a certain thickness of a certain material being known, it is possible to draw a characteristic curve which shows the cutting lengths for any other thickness of the same material. Such curves have been drawn in Fig. 22 for sizes Nos. 3 to 9 of the Zeh & Hahnemann power presses. They are based upon steel of an ultimate strength of 60,000 lbs. per sq. in.

These presses are graded in such a manner that they exert a maximum pressure in tons equal to the square of their number. The No. 5 press, for instance, exerts a maximum pressure of 5×5 or 25 tons; the No. 6 of 6×6, or 36 tons, etc. These pressures must be known to a die-maker in order to enable him to determine the proper amount of shear by the formulas given to obtain a safe result.

(*B*) In shops in which each machinist grinds his own tools a clearance angle of from 9 deg. to 12 deg. should be used.

The latter recommendation is based on the fact that when the workmen grind their own tools they usually grind the clearance and lip angles without gages, merely by looking at the tool and guessing at the proper angles; and much less harm will be done by grinding clearance angles considerably larger than 6 deg. than by getting them considerably smaller.

(*C*) For standard tools to be used in a machine shop for cutting metals of average quality: Tools for cutting cast-iron and the harder

steels, beginning with a low limit of hardness, of about carbon .45 per cent., say, with 100,000 lbs. tensile strength and 18 per cent. stretch, should be ground with a clearance angle of 6 deg., back slope 8 deg., and side slope 14 deg., giving a lip angle of 68 deg.

(*D*) For cutting steels softer than, say, carbon .45 per cent. having about 100,000 lbs. tensile strength and 18 per cent. stretch, tools should be ground with a clearance angle of 6 deg., back slope of 8 deg., side slope of 22 deg., giving a lip angle of 61 deg.

(*E*) For shops in which chilled iron is cut a lip angle of iron 86 deg. to 90 deg. should be used.

(*F*) In shops where work is mainly upon steel as hard or harder than tire steel, tools should be ground with a clearance angle of 6 deg., back slope 5 deg., side slope 9 deg., giving a lip angle of 74 deg.

(*G*) In shops working mainly upon extremely soft steels, say, carbon .10 per cent. to .15 per cent., it is probably economical to use tools with lip angles keener than 61 deg.

(*H*) The most important consideration in choosing the lip angle is to make it sufficiently blunt to avoid the danger of crumbling or spalling at the cutting edge.

(*I*) Tools ground with a lip angle of about 54 deg. cut softer

TABLE 30.—TAYLOR'S CUTTING SPEEDS IN STEEL

Standard ½-in. Tool

Depth of cut in ins.	Feed in ins.	Cutting speed in ft. per min. for a tool which is to last 1 hr. and 30 min. before regrinding		
		Soft steel	Medium steel	Hard steel
1/16	1/64	510	255	116
	1/32	322	161	73.2
	1/16	203	102	46.2
3/32	1/64	445	223	101
	1/32	281	141	63.9
	1/16	177	88.7	40.2
	3/32	135	67.4	30.7
1/8	1/64	404	202	91.8
	1/32	255	128	57.9
	1/16	161	81	36.6
3/16	1/64	359	179	81.6
	1/32	226	113	51.4
1/4	1/64	330	165	25

Standard ⅝-in. Tool

Depth of cut in ins.	Feed in ins.	Cutting speed in ft. per min. for a tool which is to last 1 hr. and 30 min. before regrinding		
		Soft steel	Medium steel	Hard steel
1/16	1/64	548	274	125
	1/32	358	179	81.6
	1/16	235	117	53.3
3/32	1/64	467	234	106
	1/32	306	153	69.5
	1/16	200	100	45.5
	3/32	156	78	35.5
1/8	1/64	417	209	94.8
	1/32	273	136	62
	1/16	179	89.3	40.6
	3/32	140	69.8	31.7
3/16	1/64	362	181	82.2
	1/32	236	118	53.8
	1/16	155	77.4	35.2
1/4	1/64	328	164	74.5
	1/32	215	107	48.8
3/8	1/64	286	143	65

Standard ¾-in. Tool

Depth of cut in ins.	Feed ins.	Cutting speed in ft. per min. for a tool which is to last 1 hr. and 30 min. before regrinding.		
		Soft steel	Medium steel	Hard steel
3/32	1/64	482	241	110
	1/32	323	161	73.4
	1/16	217	108	49.3
	3/32	172	85.8	39
1/8	1/64	423	212	96.1
	1/32	284	142	64.5
	1/16	190	95.2	43.2
	3/32	151	75.3	34.2
	1/8	128	63.8	29
3/16	1/64	358	179	81.4
	1/32	240	120	54.5
	1/16	161	80.5	36.6
	3/32	127	63.7	28.7
1/4	1/64	320	160	72.7
	1/32	215	107	48.8
	1/16	144	72	32.7
3/8	1/64	276	138	62.7
	1/32	185	92.4	42

Standard ⅞-in. Tool

Depth of cut in ins.	Feed in ins.	Cutting speed in ft. per min. for a tool which is to last 1 hr. and 30 min. before regrinding		
		Soft steel	Medium steel	Hard steel
3/32	1/64	476	238	108
	1/32	325	162	73.8
	1/16	222	111	50.4
	3/32	177	88.4	40.2
1/8	1/64	420	210	95.5
	1/32	286	143	65
	1/16	195	97.6	44.4
	3/32	156	77.9	35.4
	1/8	133	66.4	30.2
3/16	1/64	352	176	80.0
	1/32	240	120	54.5
	1/16	164	82	37.3
	3/32	131	65.5	29.8
	1/8	112	56	25.5
1/4	1/64	312	156	70.9
	1/32	213	107	48.4
	1/16	145	72.6	33
	3/32	116	58.1	26.4
3/8	1/64	264	132	60
	1/32	180	90.2	41
	1/16	122	61.1	27.8
1/2	1/64	237	118	53.8
	1/32	162	80.8	36.7

TABLE 30.—TAYLOR'S CUTTING SPEEDS IN STEEL—(*Continued*)

Standard 1-in. Tool

Depth of cut in ins.	Feed in ins.	Cutting speed in ft. per min. for a tool which is to last 1 hr. and 30 min. before regrinding		
		Soft steel	Medium steel	Hard steel
3/32	1/64	490	245	111
	1/32	339	169	77
	1/16	235	117	53.4
	3/32	189	94.5	43
1/8	1/64	427	214	97
	1/32	296	148	67.2
	1/16	205	102	46.6
	3/32	165	83	37.5
	1/8	142	71	32.3
3/16	1/64	358	179	81.3
	1/32	247	124	56.1
	1/16	171	85.5	38.8
	3/32	138	69	31.3
	1/8	118	59	26.8
	3/16	95	47.5	21.6
1/4	1/64	315	157	71.6
	1/32	218	109	49.5
	1/16	150	75	34.1
	3/32	121	60.5	27.5
	1/8	104	52	23.6
3/8	1/64	263	132	59.8
	1/32	182	91	41.4
	1/16	126	62.8	28.5
	3/32	101	50.6	23
1/2	1/64	232	116	52.7
	1/32	161	80.5	36.6
	1/16	111	55.7	25.3

Standard $1\frac{1}{4}$-in. Tool

Depth of cut in ins.	Feed in ins.	Cutting speed in ft. per min. for a tool which is to last 1 hr. and 30 min. before regrinding		
		Soft steel	Medium steel	Hard steel
3/32	1/64	518	259	118
	1/32	366	183	83.2
	1/16	257	129	58.4
	3/32	209	105	47.5
1/8	1/64	450	225	102
	1/32	317	158	72
	1/16	223	112	50.7
	3/32	182	90.8	41.4
	1/8	157	78.5	35.7
3/16	1/64	370	185	84.1
	1/32	260	130	59.1
	1/16	183	91.7	41.6
	3/32	149	74.6	33.8
	1/8	129	64.5	29.3
	3/16	105	52.6	23.8
1/4	1/64	322	161	73.2
	1/32	227	113	51.6
	1/16	159	79.7	36.1
	3/32	130	65	29.5
	1/8	112	56.1	25.5
	3/16	91.4	45.7	20.8
3/8	1/64	264	132	60
	1/32	186	93.1	42.3
	1/16	131	65.5	29.8
	3/32	107	53.4	24.1
	1/8	92.2	46.1	20.8
1/2	1/64	230	115	52.3
	1/32	162	80.9	36.8
	1/16	114	56.9	25.9
	3/32	92.6	46.3	21

qualities of steel, and also cast-iron, with the least pressure of the chip upon the tool. The pressure upon the tool, however, is not the most important consideration in selecting the lip angle.

(*J*) In choosing between side slope and back slope in order to grind a sufficiently acute lip angle, the following considerations, given in the order of their importance, call for a steep side slope and are opposed to a steep back slope: (*a* With side slope the tool can be ground many more times without weakening it; (*b*) the chip runs off sideways and does not strike the tool posts or clamps; (*c*) the pressure of the chip tends to deflect the tool to one side, and a steep side slope tends to correct this by bringing the resutant line of pressure within the base of the tool; (*d*) easier to feed.

(*K*) The following consideration calls for at least a certain amount of back slope: An absence of back slope tends to push the tool and the work apart, and therefore to cause a slightly irregular finish and a slight variation in the size of the work.

Fig. 26 shows Mr. Taylor's standard $\frac{3}{4}$-in. tool and Figs. 27 and 28 show the dimensions of other sizes.

Feed and Depth of Cut

Following are Mr. Taylor's conclusions regarding the relation between feed and depth of cut:

(*A*) With any given depth of cut metal can be removed faster, *i.e.*, more work can be done, by using the combination of a coarse feed with its accompanying slower speed than by using a fine feed with its accompanying higher speed. In most cases it is not practicable for the operator to take the coarsest feeds, owing either to the ack of pulling power of the machine or the elasticity of the work. Therefore, the above rule is only, of course, a broad general statement.

(*B*) The cutting speed is affected more by the thickness of the shaving than by the depth of the cut. A change in the thickness of the shaving has about three times as much effect on the cutting speed as a similar or proportional change in the depth of the cut has upon the cutting speed. Dividing the thickness of the shaving by 3 increases the cutting speed 1.8 times, while dividing the length that the shaving bears on the cutting edge by 3 increases the cutting speed 1.27 times.

(*C*) Expressed in mathematical terms, the cutting speed varies with the standard round-nosed tool approximately in inverse proportion to the square root of the thickness of the shaving or of the feed.

(*D*) With the best modern high-speed tools, varying the feed and the depth of the cut causes the cutting speed to vary in practically the same ratio whether soft or hard metals are being cut.

(*E*) The same general formula expresses the laws for the effect of depth of cut and feed upon the speed, the constants only requiring to be changed.

(*F*) The same general type of formula expresses the laws governing the effect of the feed and depth of cut upon the cutting speed when using the different sized standard tools.

Tables 30 and 31 give Mr. Taylor's determinations regarding depth of cut, feed and speed for high-speed steel tools:

A study of Mr. Taylor's and Prof. J. T. Nicholson's experiments,

TABLE 31.—TAYLOR'S CUTTING SPEEDS IN CAST-IRON

Standard ⅝-in. Tool

Depth of cut in ins.	Feed in ins.	Cutting speed in ft. per min. for a tool which is to last 1 hr. and 30 min. before regrinding		
		Soft cast-iron	Medium cast-iron	Hard cast-iron
3/32	1/64	206	103	60
	1/32	147	73.3	42.8
	1/16	97.5	48.8	28.5
	3/32	76	38	22.2
	1/8	64.1	32.1	18.7
1/8	1/64	194	97	56.7
	1/32	138	69.3	40.4
	1/16	93.1	46.5	27.2
	3/32	72.1	36.1	21.3
	1/8	41.8	20.9	12.2
3/16	1/64	182	91	53
	1/32	128	64	37.7
	1/16	86.1	43.1	25.1
	3/32	67.4	33.7	19.6
1/4	1/64	173	86.3	50.4
	1/32	122	61	35.7
	1/16	81.9	41	23.9

Standard ¾-in. Tool

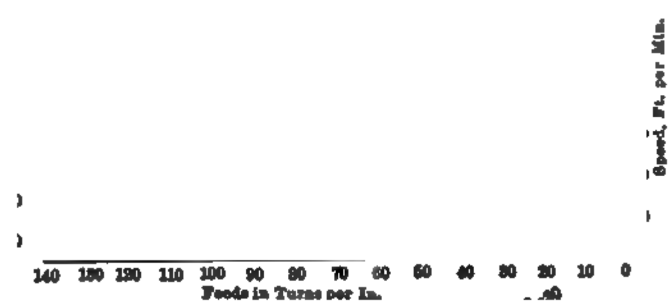
Depth of cut in ins.	Feed in ins.	Cutting speed in ft. per min. for a tool which is to last 1 hr. and 30 min. before regrinding		
		Soft cast-iron	Medium cast-iron	Hard cast-iron
3/32	1/64	216	108	63
	1/32	160	80	46.6
	1/16	110	55	32.2
	3/32	88.4	44.2	25.8
	1/8	75.4	37.7	22
1/8	1/64	200	100	58.6
	1/32	148	74	43.3
	1/16	104	51.8	30.2
	3/32	82.6	41.3	24.1
	1/8	69.6	[illegible].8	20.3
3/16	1/64	183	91.6	68
	1/32	135	67.5	39.4
	1/16	[illegible]	47	27.4
	3/32	75.4	37.7	22
	[illegible]	64.3	32.2	18.8
1/4	1/64	171	85.7	50.1
	1/32	126	63.2	36.9
	1/16	87.8	43.9	25.6
	3/32	70.4	35.2	20.6
3/8	1/64	156	77.8	45.4
	1/32	116	57.8	33.8
	1/16	79.7	39.9	23.3

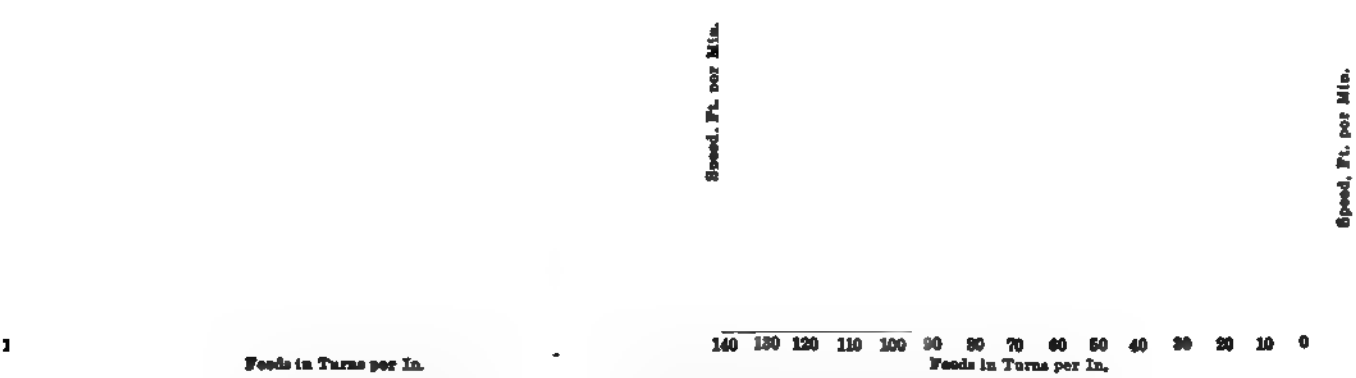
the latter made at the Manchester School of Technology, has led E. C. HERBERT (*Amer. Mach., June* 24, 1909) to the discovery of a law by which apparent discrepancies between the experiments are reconciled. Mr. Herbert expresses his law, which he calls the cube law, thus:

Since the maximum thickness of the chip is *generally* proportional to the feed or traverse, we will call

t = traverse,
c = depth of cut,
a = area of the cut, and
s = cutting speed.

From what has been said above it follows that if t_1, c_1, a_1, s_1 represent the values of these factors for any given working conditions, and t_2, c_2, a_2, s_2 represent their value for another set of working conditions,





A piece of cast-iron 2 ins. diameter is to be turned down to 1½ ins. diameter. Follow the line marked turning of the chart for cast-iron to its intersection with the ¼-in. depth of cut line whence trace vertically to the bottom where read the feed 34.2 turns per in. and from the same point trace horizontally to the right where read the speed 52 ft. per min.

FIG. 29.—Feeds and speeds in average practice.

Table 31.—Taylor's Cutting Speeds in Cast-iron—(*Continued*)

Standard ¾-in. Tool

Depth of cut in ins.	Feed in ins.	Cutting speed in ft. per min. for a tool which is to last 1 hr. and 30 min. before regrinding		
		Soft cast-iron	Medium cast-iron	Hard cast-iron
3/32	1/64	222	111	65
	1/32	169	84.3	49.2
	1/16	120	59.8	34.9
	3/32	97	48.5	28.3
	1/8	83.4	41.7	24.4
	3/16	66.4	33.2	19.4
1/8	1/64	203	102	59.3
	1/32	156	78.2	45.6
	1/16	110	55	32
	3/32	88.8	44.4	25.9
	1/8	76.2	38.1	22.3
	3/16	60.9	30.4	17.8
3/16	1/64	181	90.6	52.9
	1/32	137	68.5	40
	1/16	97.7	48.9	28.5
	3/32	78	39	22.8
	1/8	67.5	33.7	19.7
	3/16	54.2	27.1	15.8
1/4	1/64	167	83.6	48.8
	1/32	126	63.2	36.9
	1/16	90.8	45.4	26.3
	3/32	72.7	36.3	21.2
	1/8	62.7	31.3	18.3
3/8	1/64	150	75	43.8
	1/32	113	56.7	33.1
	1/16	81	40.5	23.6
	3/32	65.5	32.7	19.1

Standard ⅞-in. Tool

Depth of cut in ins.	Feed in ins.	Cutting speed in ft. per min. for a tool which is to last 1 hr. and 30 min. before regrinding		
		Soft cast-iron	Medium cast-iron	Hard cast-iron
3/32	1/64	220	110	64.2
	1/32	169	84.6	49.4
	1/16	122	61.2	35.7
	3/32	99.8	49.9	29.1
	1/8	86.4	43.2	25.2
	3/16	70.1	35.1	20.5
1/8	1/64	202	101	58.9
	1/32	156	77.8	45.4
	1/16	112	56.2	32.8
	3/32	91.8	45.9	26.8
	1/8	79.3	39.7	23.2
	3/16	64.3	32.2	18.8
3/16	1/64	178	89	52
	1/32	137	68.6	40.1
	1/16	99.4	49.7	29
	3/32	81	40.5	23.7
	1/8	70.1	35	20.5
	3/16	56.8	28.4	16.6
1/4	1/64	163	81.5	47.7
	1/32	126	62.9	36.7
	1/16	90.8	45.4	26.5
	3/32	74.1	37	21.6
	1/8	64.1	32	18.7
	3/16	52	26	15.2
3/8	1/64	144	71.8	41.9
	1/32	111	55.4	32.3
	1/16	80	40	23.4
	3/32	65.3	32.6	19.1
	1/8	56.4	28.2	16.5
	3/16	45.8	22.9	13.4
1/2	1/64	135	67.5	39.4
	1/32	104	52.1	30.4
	1/16	75.2	37.6	22
	3/32	61.4	30.7	17.9
	1/8	43.1	21.6	12.6

then the heating of the cutting edge and, by assumption, the durability of the tool will remain unaltered so long as the relation

$$t_1a_1s_1^3 = t_2a_2s_2^3$$

holds good. From which it follows that for constant durability of the cutting tool

$$s_2 = s_1\sqrt[3]{\frac{t_1a_1}{t_2a_2}}$$

or, since $a = tc$,

$$s_2 = s_1\sqrt[3]{\frac{t_1^2c_1}{t_2^2c_2}}.$$

The cube law may be most conveniently stated thus:

The cutting speed varies inversely as the cube root of the product of traverse by area of cut; or alternatively, *the cutting speed varies inversely as the cube root of the product of depth of cut by traverse squared.*

In cutting cast-iron, the cube law as stated above is only applicable in the case of coarse feeds. When the feed is less than 1/16 in. the thickness of the chip has very little influence on the speed, which varies inversely as the cube root of the area of cut approximately.

An examination of a large collection of data led Stanley H. Moore to construct Fig. 29 (*Amer. Mach., Dec.* 25, 1902) for the best feed and speed values for various depths of cut, the term "best feed and speed" being understood to mean that combination that will remove a maximum amount of material when due consideration is given to economy and the time required for changing and grinding the tools.

The use of the charts is explained by an example below them.

Speeds for Tapping and Threading

Cutting speeds for tapping and threading, as followed in the shops named, are as follows (*Amer. Mach., Aug.* 3, 1911):

By the F. E. Wells Co., for tapping cast-iron:

¼	⅜	½	⅝	¾	inch holes
382	255	191	153	127	r. p. m.

using an oil or soda compound.

For soft steel and iron:

¼	⅜	½	⅝	¾	inch holes
299	153	115	91	76	r. p. m.

using oil as a lubricant.

The National Machine Company uses 233 r.p.m. up to ¼ in. diameter and 140 r.p.m. for sizes between ¼ and ½ in., using a screw-cutting oil as a lubricant.

They tap holes as deep as four tap diameters by power.

TABLE 31.—TAYLOR'S CUTTING SPEEDS IN CAST-IRON—(*Continued*)

Standard 1-in. Tool

Depth of cut in ins.	Feed in ins.	Cutting speed in ft. per min. for a tool which is to last 1 hr. and 30 min. before regrinding		
		Soft cast-iron	Medium cast-iron	Hard cast-iron
3/32	1/64	226	113	66
	1/32	177	88.4	51.6
	1/16	130	64.8	37.8
	3/32	107	53.5	31.2
	1/8	92.8	46.4	27.1
	3/16	75.7	37.8	22.1
1/8	1/64	205	102	59.8
	1/32	160	85.1	46.8
	1/16	118	58.8	34.3
	3/32	97	48.5	23.3
	1/8	84.2	42.1	24.6
	3/16	68.6	34.3	20
3/16	1/64	181	90.6	52.9
	1/32	142	70.8	41.3
	1/16	104	51.9	30.3
	3/32	85.8	42.9	25
	1/8	74.3	37.2	21.7
	3/16	60.6	30.3	17.7
1/4	1/64	165	82.3	48.1
	1/32	129	64.4	37.5
	1/16	94.3	47.1	27.5
	3/32	77.8	38.9	22.7
	1/8	67.5	33.7	19.7
	3/16	55	27.5	16.1
3/8	1/64	143	71.5	41.8
	1/32	112	56	32.6
	1/16	81.9	41	23.9
	3/32	67.6	33.8	19.7
	1/8	58.6	29.3	17.1
	3/16	57.5	28.7	16.8
1/2	1/64	132	66.2	38.6
	1/32	104	51.6	30.2
	1/16	75.8	37.9	22.1
	3/32	62.6	31.3	18.3
	1/8	54.2	27.1	15.8
	3/16	44.2	22.1	12.9

Standard 1½-in. Tool

Depth of cut in ins.	Feed in ins.	Cutting speed in ft. per min. for a tool which is to last 1 hr. and 30 min. before regrinding		
		Soft cast-iron	Medium cast-iron	Hard cast-iron
3/32	1/64	239	119.6	69.8
	1/32	191	95.3	55.6
	1/16	142	70.8	41.3
	3/32	118	59.1	34.4
	1/8	103	51.7	30.2
	3/16	85	42.5	24.8
1/8	1/64	216	108	63.1
	1/32	172	86.2	50.3
	1/16	128	64	37.3
	3/32	107	53.4	31.2
	1/8	93.4	46.7	27.3
	3/16	76.8	38.4	22.4
3/16	1/64	187	93.5	54.6
	1/32	149	74.6	43.6
	1/16	111	55.5	32.7
	3/32	92.5	46.3	27
	1/8	73.1	36.5	21.3
	3/16	66.4	33.2	19.4
1/4	1/64	168	84.1	49.1
	1/32	134	67.2	39.2
	1/16	99.8	49.9	29.1
	3/32	83.2	41.6	24.3
	1/8	72.6	36.3	21.2
	3/16	59.7	29.8	17.4
3/8	1/64	144	71.8	41.9
	1/32	115	57.3	33.4
	1/16	85.1	42.6	24.8
	3/32	70.9	35.5	20.7
	1/8	62	31	18.1
	3/16	51	25.5	14.9
1/2	1/64	131	55.6	38.3
	1/32	105	52.3	30.5
	1/16	77.6	38.8	22.7
	3/32	64.7	32.4	18.9
	1/8	56.6	28.3	16.5
	3/16	46.5	23.3	13.6
3/4	1/64	112	56	32.7
	1/32	89.2	44.6	26
	1/16	66.2	33.1	19.3
	3/32	55.2	27.6	16.1
	1/8	48.3	24.2	14.1
	3/16	39.7	19.8	11.6

By the Landis Machine Co., for threading cast-iron in machines of the bolt-cutter type:

1/4	3/8	1/2	3/4	1	1½	2	ins.
200	150	125	100	85	55	45	r. p. m.

with petroleum as a lubricant.

For soft steel and iron:

1/4	3/8	1/2	3/4	1	1½	2	ins.
280	220	175	140	115	75	6	r. p. m.

with compound or screw-cutting oil.

The speeds are for high-speed steel dies. Some users of the machines run at a much higher rate, the figures given being conservative and easily attained.

The Bignall & Keeler Mfg. Co., aims to have its pipe-threading machines run at a cutting speed of 15 ft. per min. They advise nothing but lard oil on the dies.

The Standard Engineering Co. also recommends a cutting speed of 15 ft. per min.

The number of teeth in milling cutters may be determined from Fig. 30, by W. G. Groocock, which gives the practice of the Woolwich arsenal (*Amer. Mach., Aug.* 17, 1911). The chart contains also lines for the lead of the spiral. Mr. Groocock's practice is to use a 14-deg. spiral on end and finishing mills and 20 to 25 deg. on roughing end and slab mills, with an occasional slab mill of 30 deg. spiral and fewer teeth.

Milling machine cutters of greatly increased pitch of teeth formed the subject of extended tests by the Cincinnati Milling Machine Co., which were reported on by A. L. DELEEUW (*Trans. A. S. M. E. Vol.* 33). The dimensions found most advantageous, as regards capacity and power consumption, are shown in Fig. 31. For the power consumption obtained in these tests, see Power Requirements of Milling Machines.

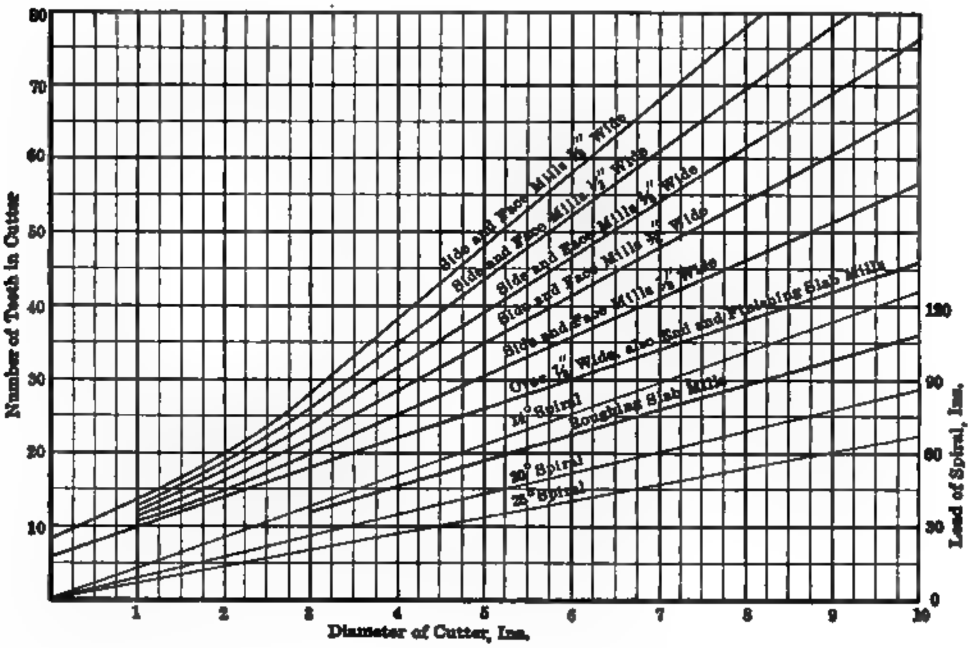


FIG. 30.—Number of teeth in milling cutters.

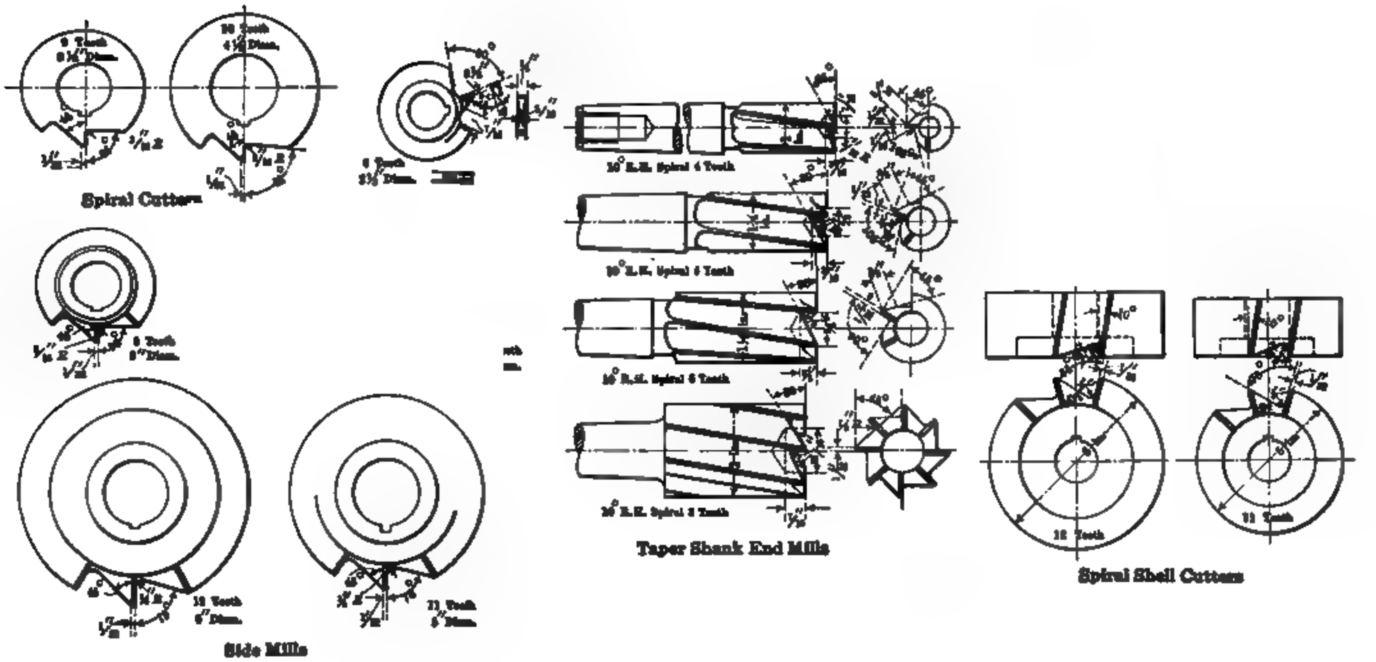


FIG. 31.—Coarse pitch milling cutters.

CAST-IRON

The following particulars and tables relating to the properties and uses of cast-iron are extracted from the report of Dr. J. J. Porter, chairman of a committee of the American Foundrymen's Association (*Trans. A. F. A., Vol.* 19).

Cast-iron is a complex alloy of six or more elements. The common elements are: Iron, carbon, silicon, sulphur, phosphorus, manganese; and the other elements sometimes present are: Copper, nickel, oxygen, nitrogen, aluminum, titanium and vanadium.

Carbon is the most important element in cast-iron. It exists in many forms, all of which are included under the two heads of graphite and combined carbon. The total carbon is dependent upon the temperature in the blast-furnace, the conditions of melting and the percentage of other metalloids. Graphite weakens iron. The amount depends upon the per cent. of total carbon, the rate of cooling, the per cent. of silicon, the per cent. of sulphur, and the per cent. of manganese. Combined carbon hardens iron and may increase or decrease the strength. The amount depends upon the per cent. silicon, the rate of cooling, the per cent. sulphur and the per cent. manganese.

Silicon exists in cast-iron in the form of silicides. Its chief effects are through its action on the carbon. Increasing the silicon decreases the total carbon because it replaces carbon in the molten solution. Increasing the silicon increases the graphite because it replaces carbon in the solid solution, the displaced carbon being precipitated as graphite.

Phosphorus exists in cast-iron as the phosphide Fe_3P which is insoluble in the solid iron-carbon solution. Phosphorus decreases the total carbon. According to Upton, the effect of phosphorus on carbon is to slightly increase graphite and decrease total carbon.

Sulphur exists in cast-iron as iron sulphide and manganese sulphide. Iron sulphide forms a eutectic with iron melting at 1780 deg. Fahr. and insoluble in the solid iron-carbon solution. It therefore forms films between the iron crystals and causes brittleness.

Manganese sulphide does not form these films and is less detrimental. Manganese has a greater affinity than iron for sulphur and with enough manganese all the sulphur will be in combination with it.

Sulphur has a greater tendency to segregate than any other constituent of cast-iron. This tendency is greatest with manganese sulphide. Sulphur tends to decrease graphite and increase combined carbon.

The presence of silicon decreases the amount of sulphur which cast-iron can take up. Much sulphur reduces the total carbon, and *vice versa.*

Manganese may exist in cast-iron as manganese sulphide or as manganese carbide. It tends to harden iron. It can neutralize sulphur and will also remove dissolved oxide at high temperatures, as in the blast-furnace.

Traces of copper are common in pig iron. Its effects on cast-iron are poorly understood. Cast-iron will take up only about 5 per cent. copper and this does not affect the casting properties. Copper accentuates the red-shortness due to sulphur. Copper prevents a complete evolution of sulphur in iron analysis.

Small amounts of nickel occur in many pig irons. Its effects on the strength and ductility of cast-iron are relatively unimportant.

The strength of cast-iron is dependent upon nine factors: 1, per cent. of graphite; 2, size of graphite flakes; 3, per cent. of combined carbon; 4, size of primary crystals of solid solution, Fe-C-Si; 5, amount of dissolved oxide; 6, per cent. of phosphorus; 7, per cent. of sulphur; 8, per cent. of silicon; 9, per cent. of manganese.

The size of graphite flakes accounts for many cases of difference in strength of irons of the same composition. The factors influencing the size are very poorly understood.

The effect of dissolved oxide is probably important. To reduce oxide we may get the best brands of pig iron, avoid oxidizing conditions in the cupola, and use deoxidizing agents.

Phosphorus lessens strength, particularly resistance to shock. One per cent. produces a marked effect.

Sulphur may indirectly strengthen iron through decreasing the graphite, but is more likely to weaken it through causing blowholes and high shrinkage.

Silicon and manganese act chiefly indirectly. Silicon should be kept as low as possible and still have the necessary softness. Manganese should be high, but if too high produces weakness.

Of the elastic properties only toughness and elasticity are important in cast-iron. The sum of these properties is given by the deflection. The factors influencing them are about the same as those influencing strength.

Maximum rigidity with the least sacrifice of strength and toughness is obtained through the use of manganese and combined carbon.

Hardness is due both to combined carbon and *gamma* solid solution. The latter explains the cases of hard cast-iron which are yet low in combined carbon.

Phosphorus has only a slight hardening effect. Manganese may soften iron through its action on the sulphur, but in larger amounts will harden it. Sulphur is an energetic hardening agent. Silicon softens iron due to its action in decreasing combined carbon up to a certain point. Beyond this point it hardens, due to its direct action. Combined carbon is the chief hardening agent in cast-iron.

In chilled iron the factors influencing the depth and quality of the chill are, pouring temperature, and percentage of silicon, sulphur, phosphorus and total carbon. The higher the pouring temperature the deeper the chill. Sulphur causes a brittle chill and is undesirable. Phosphorus injures the strength of chill and causes a sharp line between the white and gray portions. Manganese increases the hardness of the chill and its resistance to heat strains.

The grain structure and porosity depend on the size and percentage of the graphite. The fusibility of cast-iron depends primarily on combined carbon, and to a less extent on the phosphorus. Graphite affects the melting-point only in so far as it dissolves in the iron at temperatures below the melting-point.

Fluidity is determined by per cent. silicon, per cent. phosphorus, freedom from dissolved oxide and temperature above the freezing-point.

The following Table 1 of classified castings is taken by Dr. Porter partly from published results but chiefly from replies to inquiries. Thickness is taken into consideration since this largely determines the percentage of silicon necessary, and it has been the aim to subdivide the various classes according to section wherever possible. In this respect the endeavor has been to follow the definitions of the American Society for Testing Materials, who have grouped castings according to thickness as follows:

"Castings having any section less than ½ in. thick shall be known as light castings.

"Castings in which no section is less than 2 ins. thick shall be known as heavy castings.

"Medium castings are those not included in the above definitions."

Table 1.—Chemical Composition of Iron Castings for Various Purposes

The last analysis under each head is preceded by the word "Sug." (abbreviated from suggested) and is the tentative standard or probable best analysis *suggested* by the committee. Under is abbreviated by "und."

	Silicon	Sulphur	Phos.	Mang.	Comb. carb.	Total carb.
Acid Resisting Castings:						
	1.00%	.050%	.50%			3.00%
	2.30	low	.20	.41%		3.60
	.80–2.00	.02–.03	.40–.60	1.00–2.00		3.00–3.50
Sug.	1.00–2.00	und. .05	und. .40	1.00–1.50		3.00–3.50
Agricultural Machinery, Ordinary:						
	2.20–2.80%	und. .085%	und. .70%	und. .70%		
	2.65	.050	.81	.70	.15%	3.50%
	2.25	.070	.70	.80	.30	3.50
	2.10	.068	.73	.45	.47	3.42
	2.00	.089	.89	.46	.50	3.39
Sug.	2.00–2.50	.06–.08	.60–.80	.60–.80		
Agricultural Machinery, Very Thin:						
	2.90%	.050%	.85%	.70%	.10%	3.50%
	2.50	.080	.65	.60	.30	3.50.
Sug.	2.25–2.75	.06–.08	.70–.90.	.50–.70		
Air Cylinders:						
	1.20–1.50%	und. .09%	.35–.60%	.50–.80%		
	1.90	.074	.50	.65		
	1.12	.085	.40	.70	.70%	3.50%
	.95	.100	.30	.90	.80	3.40
	2.00	.070	.30	.60	.40	
Sug.	1.00–1.75	und. .09	.30–.50	.70–.90		3.00–3.30
Ammonia Cylinders:						
	1.20–1.90%	und. .095%	und. .70%	.60–.80%		
Sug.	1.00–1.75	und. .09	.30–.50	.70–.90		3.00–3.30%
Annealing Boxes, Pots and Pans:						
	1.20%	.060%	.10%	.40%		
	1.80	.03	.70.	.60		2.90%
	1.53	.04	.33	1.08	.58	3.68
Sug.	1.40–1.60	und. .06	und. .20	.60–1.00		low
Automobile Castings:						
	1.80%	.030%	.50%	.70%	.60%	3.50%
	2.65	.076	.45	.65	.55	
	2.35	.072	.60	.70	.40	
Sug.	1.75–2.25	und. .08	.40–.50	.60–.80		
Automobile Cylinders:						
	1.65%	.076%	.45%	.65%	.55%	
	2.31	.094	.50	.43	.51	3.35%
	2.70	.053	.46	.23	.44.	3.02
	2.45	.102	.72	.41	.41	3.47
	2.59	.083	.57	.47	.11	3.35
	2.55	.104	.82	.32	.09	3.04
	2.98	.047	.89	.27	.14	3.19
	2.67	.111	.73	.38	.10	3.24
	2.30	.084	.81	.52	.59	3.35
	1.60	.083	.54	.42	.66	3.75
	3.26	.159	.93	.44	.03	2.87
	1.72	.091	.58	.48	.62	2.52
	1.67	.068	.44	.82	.62	3.91
	1.38	.093	.62	.52	.76	3.61
	1.47	.075	.13	.60		
	1.50	.103	.86	.43		
	1.99	.130	.65	.39	.45	3.17
	1.89	.090	.70	.39	.77	3.34
	2.29	.090	.83	.60	.90	4.16
Sug.	1.75–2.00	und. .08	.40–.50	.60–.80	.55–.65	3.00–3.25
Automobile Fly-wheels:						
	2.35%	.072%	.60%	.70%	.40%	
	3.10	.045	.35	.55	.27	
Sug.	2.25–2.50	und. .07	.40–.50	.50–.70		
Balls for Ball Mills:						
	1.00%	.100%	.30%	.50%		low
Sug.	1.00–1.25	und. .08	und. .20	.60–1.00		low

	Silicon	Sulphur	Phos.	Mang.	Comb. carb.	Total carb.
Bed Plates:						
	2.20%	.090%	.55%	.50%		
	1.32	.090	.40	.60		
	1.65		.28	.92	.72%	
	1.85	.080	.60	.55	.50	3.25–3.50%
	1.80–2.20	.04–.06	.45–.55	.40–.50	.40–.50	3.40–3.60
	1.65–1.85	.070	.65–.80	.60–.75		3.85
Sug.	1.25–1.75	und. .10	.30–.50	.60–.80		
Boiler Castings:						
	2.50%	und. .07%	und. .20%	.80–1.0%		
	2.25	.060	.62	.59		
Sug.	2.00–2.50	und. .06	und. .20	.60–1.0		
Brake Shoes:						
	1.50%		low			low
	2.00–2.50%	und. .15%	und. .70%	und. .70%		
	2.00–2.50	und. .15	und. .70	und. .70		
	1.40–1.80	.06–.08	.50–.80	.45–.60	.40–.65%	3.50%
	1.86	.183	1.93	.33	1.22	3.01
Sug.	1.40–1.60	.08–.10	.30	.50–.70		low
Car Castings, Gray Iron. *See also Brake Shoe and Car Wheels:*						
	2.20–2.80%	und. .085%	und. .70%	und. .70%		
	1.40–1.80	.06–.08	.50–.80	.45–.60	.40–.65%	3.50%
	2.25	.050	.60	.75		3.50
	1.75	.070	.85	.60		
Sug.	1.50–2.25	und. .08	.40–.60	.60–.80		
Car Wheels, Chilled:						
	.50–.70%	.05–.07%	.35–.45%	.30–.50%	.50–.75%	3.50%
	.58–.68	.05–.08	.25–.45	.15–.27	.63–1.0	
	.73	.080	.43	.44	1.25	4.31
	.86	.127	.35	.49	.92	3.47
	.70	.08	.50	.40	.60	3.50
	.58	.141	.38	.48	.90	3.63
	.57	.101	.41	.42		
	.68	.188	.36	.53		
	.67	.170	.38	.81	.74	3.66
	.50–.60	.08–.10	.30–.40	.45–.55	.70–.80	3.50
Sug.	.60–.70	.08–.10	.30–.40	.50–.60	.60–.80	3.50–3.70
Car Wheels, Unchilled. *See Wheels.*						
Chilled Castings:						
	.80–1.00%	.09–.11%	.50%	.50%		
	1.20–1.40		low			low
	1.00	.08	.40	.75		3.25%
	1.35	.117	.60	.54	.65%	3.00
	.50	.200	.45	1.50	3.00	3.00
	1.20	.090	.30	.50	1.20	3.20
	1.20	.080	.30	1.25		3.50
	.75	.090	.30	.30	3.00	3.20
Sug.	.75–1.25	.08–1.0	.20–.40	.80–1.2		
Chills:						
	2.07%	.073%	.31%	.48%	.23%	2.64%
Sug.	1.75–2.25	und. .07	.20–.40	.60–1.0		
Collars and Couplings for Shafting:						
	1.60%	.040%	.55%	.55%	.30%	3.57%
Sug.	1.75–2.00	und. .08	.40–.50	.60–.80		
Cotton Machinery. *See also Machinery Castings:*						
	2.20–2.30%	und. .09%	.70%	.60%	.45%	3.45%
Sug.	2.00–2.25	und. .08	.60–.80	.60–.80		
Crusher Jaws:						
	.80–1.00%	.09–.11%	.50%	.50%		
	1.00	.080	.40	.75		3.25%
	.50	.20	.45	1.50	3.00%	3.00
Sug.	.80–1.00	.08–.10	.20–.40	.80–1.2		

TABLE 1.—CHEMICAL COMPOSITION OF IRON CASTINGS FOR VARIOUS PURPOSES—(*Continued*)

	Silicon	Sulphur	Phos.	Mang.	Comb. carb.	Total carb.
CUTTING TOOLS, CHILLED CAST-IRON:						
	1.35%	.117%	.60%	.54%	.65%	3.00%
Sug.	1.00–1.25	und. .08	.20–.40	.60–.80		
DIES FOR DROP HAMMERS:						
	1.40%	.060%	.10%	.40%		
	1.40	.090	.40	.70	1.00%	3.20%
Sug.	1.25–1.50	und. .07	und. .20	.60–.80		low
DIAMOND POLISHING WHEELS:						
	2.70%	.063%	.30%	.44%	1.60%	2.97%
DYNAMO AND MOTOR FRAMES, BASES AND SPIDERS, LARGE:						
	1.95%	.042%	.40%	.39%	.59%	3.82%
	1.90	.08	.47	.60	.64	3.79
	2.15	.070	.75	.60	.55	3.80
	2.10	.070	.55	.40		3.50
Sug.	2.00–2.50	und. .08	.50–.80	.30–.40	.20–.30	low
DYNAMO AND MOTOR FRAMES, BASES AND SPIDERS, SMALL:						
	3.19%	.075%	.89%	.35%	.06%	2.95%
	2.30	.070	.55	.40		3.50
	2.50	.070	.75	.60	.55	3.95
Sug.	2.50–3.00	und. .08	.50–.80	.30–.40	.20–.30	low
ELECTRICAL CASTINGS:						
	3.19%	.075%	.89%	.35%	.06%	2.95%
	1.95	.042	.40	.39	.59	3.82
	1.90	.080	.47	.60	.64	3.79
	2.15	.070	.75	.60	.55	3.80
	2.50	.070	.75	.60	.55	3.95
	2.10	.070	.55	.40		3.50
	2.30	.070	.55	.40		3.50
Sug.	2.00–3.00	und. .08	.50–.80	.30–.40	.20–.30	low
ECCENTRIC STRAPS. *See Locomotive Castings and Machinery Castings:*						
ENGINE FRAMES. *See also Machinery Castings:*						
	2.25%	.080%	.55%	.60%		
	1.60	.090	.50	.60		
	1.32	.100	.40	.60		
Sug.	1.25–2.00	und. .09	.30–.50	.60–1.0		
FARM IMPLEMENTS:						
	2.00%	.089%	.89%	.46%	.50%	3.39%
	2.10	.068	.68	.45	.47	3.32
Sug.	2.00–2.50	.06–.08	.50–.80	.60–.80		
FIRE POTS:						
	2.50%	und. .07%	und. .20%	.80–1.0%		
Sug.	2.00–2.50	und. .06	und. .20	.60–1.0		low
FLY-WHEELS. *See also Automobile Fly-wheels and Machinery Castings:*						
	2.20%	.090%	.55%	.50%		
	1.50	.090	.50	.60		
Sug.	1.50–2.25	und. .08	.40–.60	.50–.70		
FRICTION CLUTCHES:						
	2.00–2.50%	und. .15%	und. .70%	und. .70%		
Sug.	1.75–2.00	.08–.10	und. .30	.50–.70		low
FURNACE CASTINGS:						
	2.50%	und. .07%	und. .20%	.80–1.0%		
	2.00	.085	.35	.53		
	1.85	.090	.70	.60		
Sug.	2.00–2.50	und. .06	und. .20	.60–1.00		low
GAS ENGINE CYLINDERS:						
	1.45%			.65%		
	1.98	.090%	.84%	.63		
	1.21	.117	.40	.35	1.40%	3.74%
	1.00–1.25	.04–.08	.20–.40	.70–.80	.60–.80	3.00–3.10
Sug.	1.00–1.75	und. .08	.20–.40	.70–.90		3.00–3.30
GEARS, HEAVY:						
	1.40%	.060%	.10%	.40%		
	.94	.150	.43	.31	1.47%	
	1.60	.080	.40	.60		3.50%
	1.50–1.75	.080	.40–.60	.50–.70		
	1.00–1.25	.075	.40	.80–1.0		very low
	1.40–1.60	.04–.08	.30–.50	.40–.60	.50–.80	3.20–3.40
Sug.	1.00–1.50	.08–.10	.30–.50	.80–1.0		low
GEARS, MEDIUM:						
	1.50–2.00%	und. .08%	.35–.60%	.50–.80%		
	1.90	.060	.10	.40		
	2.30	.060	.60	.60		3.75%
	1.90	.100	.69	.58	.55%	3.83
Sug.	1.50–2.00	und. .09	.40–.60	.70–.90		
GEARS, SMALL:						
	3.43%		1.42%	.90%		
	2.00	.100%	.50	.70		3.50%
Sug.	2.00–2.50	und. .08	.50–.70	.60–.80		
GRATE BARS:						
	2.75%	low	low			
	2.00	.085%	.35%	.53%		
Sug.	2.00–2.50	und. .06	und. .20	.60–1.0	und. .30	low
GRINDING MACHINERY, CHILLED CASTINGS FOR:						
	.50%	.200%	.45%	1.50%	3.00%	3.00%
Sug.	.50–.75	.15–.20	.20–.40	1.5–2.0		
GUN CARRIAGES:						
	.94%	.050%	.44%	.31%	.63%	3.03%
	1.00	.050	.30	.60	1.10	2.50
Sug.	1.00–1.25	und. .06	.20–.30	.80–1.0		low
GUN IRON:						
	1.34%	.003%	.08%	1.00%	.93%	3.12%
	1.19	.055	.41	.42	1.13	3.18
	1.53	.050	.29	.45	.42	3.43
	.98	.06	.43	.43	.75	1.74
	.30		.44	3.55	1.70	3.90
	1.20	.100	.30	.80	1.00	3.00
Sug.	1.00–1.25	und. .06	.20–.30		.80–1.0	low
HANGERS FOR SHAFTING:						
	1.60%	.040%	.55%	.55%	.30%	3.57%
Sug.	1.50–2.00	und. .08	.40–.50	.60–.80		
HARDWARE, LIGHT:						
	1.84%		.58%	1.04%		
	2.20		.74	1.10		
	2.50		1.21	1.16		
	2.51	.110%	.62	.41	.24%	3.18%
	2.70	.030	.60	.50	.40	3.60
	2.50	und. .050	.60	.70		
	2.00–2.25	.050	.85	.40		3.85–4.00
Sug.	2.25–2.75	und. .08	.50–.80	.50–.70		
HEAT RESISTANT IRON:						
	1.20%	.060%	.10%	.40%		
	1.67	.032	.09	.29	.43%	3.87%
	2.15	.086	1.26	.41	.13	3.30
	2.02	.070	.89	.29	.84	3.60
	1.53	.040	.33	1.08	.58	3.68
	2.07	.073	.31	.48	.23	2.64
	1.80	.030	.70	.60		
	2.75	low	low			
	2.50	und. .07	und. .20	.80–1.0		
	1.76	.075	.63	.79	.56	3.68
	2.00	.030	.70			
Sug.	1.25–2.50	und. .06	und. .20	.60–1.00	und. .30	low
HOLLOW WARE:						
	2.51%	.110%	.62%	.41%	.24%	3.18%
Sug.	2.25–2.75	und. .08	.50–.70	.50–.70		
HOUSINGS FOR ROLLING MILLS:						
	1.00–1.25%	.085%	.65%	.75%		low
Sug.	1.00–1.25	und. .08	.20–.30	.80–1.0		low
HYDRAULIC CYLINDERS, HEAVY:						
	1.00%	.050%	.30%	.60%	1.10%	2.50%
	.90	.136	.39	.25	1.44	3.34
	.80–1.50	.07–.11	.35–.50			
	1.12	.085	.40	.70	.70	3.50
	.95	.100	.30	.90	.80	3.40
	1.15	und. .08	.50	.60	1.15	
	.90–1.20	.06–.08	.30–.50	.80–1.0	.80–1.0	2.90–3.10
Sug.	.80–1.20	und. .10	.20–.40	.80–1.0		low

TABLE 1.—CHEMICAL COMPOSITION OF IRON CASTINGS FOR VARIOUS PURPOSES—(*Continued*)

	Silicon	Sulphur	Phos.	Mang.	Comb. carb.	Total carb.
Hydraulic Cylinders, Medium:						
	1.40%	.060%	.10%	.40%		
	1.90	.074	.50	.65		
	1.62	.08	.50	.60		
	1.75	.070	.40	.55	.50%	
Sug.	1.20–1.60	und. .09	.30–.50	.70–.90		low
Ingot Molds and Stools:						
	1.20%	.060%	.10%	.40%		
	1.67	.032	.09	.29	.43%	3.87%
Sug.	1.25–1.50	und. .06	und. .20	.60–1.0		
Locomotive Castings, Heavy:						
	1.40–2.00%	und. .085%	und. .60%	und. .70%		
	1.25–1.50	.06–.08	.40–.60	.45–.60	.50–.70%	3.50%
	1.62	.098	.40	.49		
Sug.	1.25–1.50	und. .08	.30–.50	.70–.90		
Locomotive Castings, Light:						
	1.40–2.00%	und. .085%	und. .60%	und. .70%		
	1.50–2.00	.06–.08	.40–.60	.45–.60	.45–.55%	3.50%
Sug.	1.50–2.00	und. .08	.40–.60	.60–.80		
Locomotive Cylinders:						
	1.25–1.75%	und. .10%	und. .90%			
	1.40–2.00	und. .085	und. .60	und. .70%		
	1.25–1.50	.06–.08	.40–.60	.45–.60	.50–.70%	3.50%
	1.00–1.40	und. .11	.40–.90	.40–.90		
	1.41	.092	.38	.39		
	1.56	.061	.45	.78		
Sug.	1.00–1.50	.08–.10	.30–.50	.80–1.0		
Machinery Castings, Heavy:						
	1.05%	.110%	.54%	.35%	.33%	2.98%
	.85	.030	.35	.92		
	.80–1.50	.030–.050	.35–.50			
	.90–1.50	.09–1.2*	.15–.40	.20–.80	.10–.30	2.50–2.90
	1.85	.100	.50	.60		3.50
	1.30	.090	.40	.60		
	1.85	.120	.60	.45		3.40–3.55
	1.75	.100	.50	.70	.80	3.65
Sug.	1.00–1.50	und. .10	.30–.50	.80–1.0		low
Machinery Castings, Medium:						
	1.83%	.078%	.50%	.31%	.43%	2.93%
	2.25	.080	.55	.60		
	1.60	.060	.66			
	2.29	.071	.66	.49		
	1.60	.090	.50	.60		
	2.10	.110	.67	.50		3.40–3.55
	2.25	.060	.75	.55		
	2.00	.100	.75	.50	.75	3.50
	1.76	.075	.63	.79	.56	3.68
	2.00	.100	.50	.50	.50	3.60
	2.35	.075	.45	.65	.30	
	1.80	.060	.80	.50	.70	
	2.06	.075	.78	.47		3.45
	1.40	low	.20	.40		
	2.00	.030	.70			
	1.85	.08	.60	.50–.60	.50	3.25–3.50
	1.50–2.10	.08–.09	.40–.80	.20–.60	.10–.40	2.60–3.20
	1.80–2.10	und. .09	.40–.90	.40–.90		
Sug.	1.50–2.00	und. .09	.40–.60	.60–.80		
Machinery Castings, Light:						
	2.04%	.044%	.58%	.39%	.32%	3.84%
	2.25	.080	.70	.50	.20	3.55
	2.76	.037	1.19		.13	3.66
	2.49	.097	.90	.42		3.40
	2.51	.084	.62	.61		3.46
	2.50	.100	.60	.70		3.50
	3.00	.060	.65	.50		3.50
	2.40	.050	.47	.59		
	2.85	.064	.67	.65		
	2.52	.062	.66	.68		
	3.15	.050				
	2.50	.100	.70	.60		3.40–3.55
	2.20–2.80	.06–.08	.60–1.3	.20–.40	.10–.60	3.00–3.60
Sug.	2.00–2.50	und. .08	.50–.70	.50–.70		
Ornamental Work:						
	4.19%	.080%	1.24%	.67%	.03%	2.88%
	2.51	.110	.62	.41	.24	3.18
	2.25		.60–.90			
Sug.	2.25–2.75	und. .08	.60–1.0	.50–.70		
Permanent Molds:						
	2.15%	.086%	1.26%	.41%	.13%	3.30%
	2.02	.070	.89	.29	.84	3.60
Sug.	2.00–2.75	und. .07	.20–.40	.60–1.0		
Permanent Mold Castings:						
	2.00–3.00%					3.00–4.00%
Sug.	1.50–3.00	und. .06%		und. .40%		
Piano Plates:						
	2.00%	low	.40%	.60%		
Sug.	2.00–2.25	und. .07	.40–.60	.60–.80		
Pillow Blocks:						
	1.60%	.040%	.55%	.55%	.30%	3.50%
Sug.	1.50–1.75	und. .08	.40–.50	.60–.80		
Pipe:						
	2.00%	.060%	.60%	.60%		
	2.00	.060	1.00	.60		
Sug.	1.50–2.00	und. .10	.50–.80	.60–.80		
Pipe Fittings:						
	2.88%		.41%	1.10%		
	1.70	.058	.50	.73	1.16	4.18
	2.51	.110	.62	.41	.24	3.18
Sug.	1.75–2.50	und. .08	.50–.80	.60–.80		
Pipe Fittings for Superheated Steam Lines:						
	1.72%	.085%	.89%	.48%	.17%	2.45%
	1.40–1.60	.06–.09	.20–.40	.45–.75		3.00–3.25
Sug.	1.50–1.75	und. .08	.20–.40	.70–.90		low
Piston Rings:						
	1.35%			.40%		
	1.60	.08%	1.15%	.35	.60%	
	1.50–2.00	.06–.08	.40–.60	.45–.60	.45–.55	3.50
Sug.	1.50–2.00	und. .08	.30–.50	.40–.60		low
Plow Points, Chilled:						
	1.20–1.40%		low			low
	1.20	.090%	.30%	.50%	1.20	3.20%
	.75	.090	.30	.30	3.00	3.20
	1.20	.080	.30	1.25		3.50
Sug.	.75–1.25	und. .08	.20–.30	.80–1.0		
Propeller Wheels:						
	1.15%		.32%	.51%	.60%	
	1.40	low	.20	.40		
Sug.	1.00–1.75	und. .10%	.20–.40	.60–1.0		low
Pulleys, Heavy:						
	1.75%	.040%	.55%	.55%	.30%	3.57%
	2.40	.060	.60	.60		3.75
Sug.	1.75–2.25	und. .09	.50–.70	.60–.80		
Pulleys, Light:						
	2.20–2.80%	und. .08%	und. .70%	und. .70%		
	2.40	und. .08	.95	.70		
	2.72	.040	.50	.66		
	2.52	.075	.77	.68		3.37%
	3.35	.089	.70	.47		3.42
	2.25	.040	.55	.55	.30	3.57
	2.15	.080	.70	.60	.40	3.55
Sug.	2.25–2.75	und. .08	.60–.80	.50–.70		
Pumps, Hand:						
	2.30–2.75%	und. .08%	.60–1.0%	.30–.50%		
Sug.	2.00–2.25	und. .08	.60–.80	.50–.70		
Radiators:						
	2.15%	low	.80%	.45%	.50%	3.50%
	2.45	.104%	.44	.40	.35	3.40
Sug.	2.00–2.25	und. .08	.60–.80	.50–.70	.50–.60	
Railroad Castings:						
	2.20–2.80%	und. .08%	und. .70%	und. .70%		
	1.40–1.80	.06–.08	.50–.80	.45–.60	.40–.65%	3.50%
	2.25	.050	.60	.75		
	1.75	.070	.85	.60		
Sug.	1.50–2.25	und. .08	.40–.60	.60–.80		

TABLE 1.—CHEMICAL COMPOSITION OF IRON CASTINGS FOR VARIOUS PURPOSES—(*Continued*)

	Silicon	Sulphur	Phos.	Mang.	Comb. carb.	Total carb.
ROLLS, CHILLED:						
	.50–1.00%	.01–.06%	.20–.80%	.15–1.5%		2.60–3.25%
	.80	.100	.88	.16	.91	2.84%
	.71	.058	.54	.39	1.38	3.00
	.65	.050	.25	1.50	.63	3.50
Sug.	.60–.80	.06–.08	.20–.40	1.0–1.2		3.00–3.25
ROLLS, UNCHILLED (SAND CAST):						
	.75%	.030%	.25%	.66%	1.20%	4.10%
SCALES:						
	1.67%		1.92%	1.90%		
	2.12		.61	.80		
	1.70		.63	1.60		
Sug.	2.00–2.30	und. .08	.60–1.0	.50–.70		
SLAG CAR CASTINGS:						
	1.76%	.075%	.63%	.79%	.56%	3.68%
	2.00	.030	.70			
Sug.	1.75–2.00	und. .07	und. .30	.70–.90		
SOIL, PIPE AND FITTINGS:						
	2.00%	.060%	1.00%	.60%		
Sug.	1.75–2.25	und. .09	.50–.80	.60–.80		
STEAM CYLINDERS, HEAVY:						
	1.41%	.092%	.38%	.39%		
	.95	.100	.30	.90	.80%	3.40%
	1.10	.136	.43	.33	.99	3.30
	1.00	.080	.20–.30	1.00	.75	3.00
	1.35–1.50	.080	.50	.75		3.65
	1.30–1.40	.04–.08	.40–.50	.70–.80	.70–.80	3.00–3.20
	.90–1.20	.09–.12	.20–.40	.70–.90		und. 3.50
Sug.	1.00–1.25	und. .10	.20–.40	.80–1.0		low
STEAM CYLINDERS, MEDIUM:						
	1.66%	.065%	.70%	.90%		
	1.60	.063	.72	.85		
	1.70	.070	.70	.75		
	1.70	.075	.60	.92		3.50%
	1.40–2.00	.085	.70	.30–.70		
	1.50–2.00	und. .08	.35–.60	.50–.80		
	1.40–1.60	und. .09	.40–.90	.40–.90		
	1.50–1.65	.080	.60	.60–.70		
	1.50–1.80	.070	.43	.76		
	1.85	.080	.60	.50–.60	.50%	3.25–3.50
	1.75	.100	.65	.55		3.40–3.55
	1.32	.136	.43	.33	.99	3.30
	1.12	.085	.40	.70	.70	3.50
	2.00	.100	.50	.70	.40	3.50
STEAM CYLINDERS, MEDIUM—(*Continued*):						
	2.00	.070	.30	.60		
	1.50	.070	.75	.70		3.50
	1.59	.109	.60	.38		3.34
	1.86		.29	.55	.52	
	1.90	.074	.50	.65		
	1.56	.061	.45	.78		
Sug.	1.25–1.75	und. .09	.30–.50	.70–.90		
STOVE PLATE:						
	2.90%		.73%	1.40%		
	2.59	.072%	.62	.37	.35%	3.30%
	3.19	.084	1.16	.38	.33	3.41
	2.75	.050	1.00	.80	.18	3.38
	2.79	.077	1.40	.32	.20	3.22
	2.51	.110	.62	.41	.24	3.18
	2.76	.071	.63	.63	.37	3.50
	2.76	.084	.65	.54		
	2.50	.060	1.00	.60		
	2.60	.050	.60	.60		
	2.50–3.00	und. .10	.60–.80	.40–.60		3.00–4.00
Sug.	2.25–2.75	und. .08	.60–.90	.60–.80		
VALVES, LARGE:						
	1.20–1.50%	und. .09%	.35–.60%	.50–.80%		
	1.00	.100	.50	.90		
	1.67		.26	.45	.69%	
Sug.	1.25–1.75	und. .09	.20–.40	.80–1.0		
VALVES, SMALL:						
	1.70%	.058%	.50%	.74%	1.16%	4.18%
	2.23	.075	.67	.67		
Sug.	1.75–2.25	und. .08	.30–.50	.60–.80		low
WATER HEATERS:						
	2.15%	.050%	.40%	.50%		
Sug.	2.00–2.25	und. .08	.30–.50	.60–.80		
WHEELS, LARGE:						
	2.10%	.040%	.40%	.70%		
Sug.	1.50–2.00	und. .09	.30–.40	.60–.80		
WHEELS, SMALL:						
	2.10%	.050%	.40%	.50%		
	1.60	.083	.60	.39		
Sug.	1.75–2.00	und. .08	.40–.50	.50–.70		
WHITE IRON CASTINGS:						
	.50%	.150%	.20%	.17%	2.90%	
	.93	.250	.70	.50		2.50

TABLE 2.—TESTS OF MALLEABLE CASTINGS

Tension Tests

Section	Area	Tensile strength, lbs. per sq. in.	Elongation in 8 ins., per cent.	Reduction area, per cent.
Round.........	.793	43100	8.70	3.75
Round.........	.817	43000	5.87	4.76
Round.........	.801	43400	6.21	3.98
Round.........	.219	41130	7.70	3.40
Round.........	.202	44700	13.00	3.63
Round.........	.210	43050	5.80	3.52
Swuare........	.277	36700	4.70	2.00
Square........	.277	38100	3.72	3.00
Square........	.283	37520	4.21	2.71
Square........	1.040	38460	4.10	3.30
Square........	1.030	38000	1.95	2.88
Square........	1.050	37860	2.38	2.94
Rect..........	.244	37600	3.87	3.80
Rect..........	.218	37250	3.22	4.70
Star..........	.584	34600	4.20	3.10
Star..........	.523	36500	7.20	2.50
Star..........	.575	37200	4.80	3.50

Compression Tests

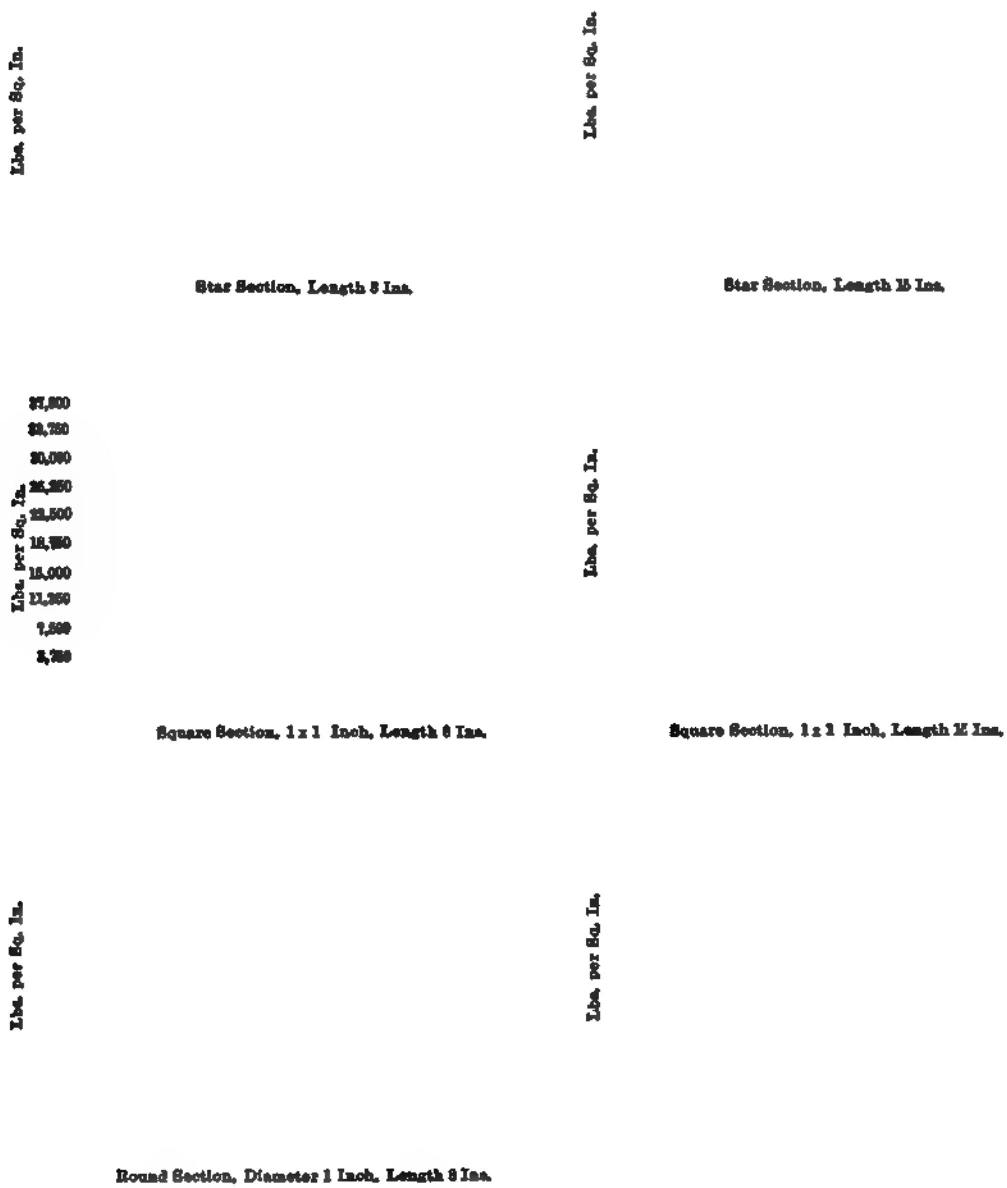
Section	Area	Length, ins.	Compressive strength, lbs. per sq. in.	Final area
Round.........	.835	15	32950	.883
Round.........	.847	15	31700	.901
Round.........	.801	15	33240	.886
Round.........	.213	7.5	33300	.222
Round.........	.209	7.5	32600	.221
Round.........	.204	7.5	34600	.215
Square........	.282	7.5	32580	.291
Square........	.263	7.5	33200	.272
Square........	.254	7.5	31870	.278
Square........	1.051	15	29650	1.070
Square........	1.040	15	30450	1.066
Square........	1.048	15	29700	1.070
Star..........	.453	15	31900	.465
Star..........	.436	15	32200	.448
Star..........	.457	15	30400	.467

Malleable Castings

Malleable castings, made by the Buhl Malleable Co., were tested and reported on by C. M. DAY (*Amer. Mach., Apr.* 5, 1906) and from the report the following facts are taken. Tensile and compressive tests were made on round, square, rectangular, and cruciform sections. The rounds were of ½ and 1 in. diameters, the squares of ½ and 1 in. sides, the rectangles ½×¾ in., while the crosses were 1 in. wide with 4 ribs ½ in. thick. The results are given in Table 2.

Mr Day makes the following comments on the tests: It was found that the round section gave the best results both in the 1-in. and the ½-in. sizes, as well as in both tension and compression tests. The round section, besides having a greater tensile strength, also had a greater elongation and reduction in area. This may be due to the fact that the iron casts better in round sections or that the outer skin of a malleable casting is not its strongest part. Every one of the round bars showed a perfect fracture, with a good skin and a smooth velvety interior. It is generally thought that malleable iron does not cast well in round sections and that a section with narrow ribs is the strongest possible section. This is why the star-shaped section was made to test. In only one of the star pieces did the fracture show up well. In the others there were signs of shrinkage. It seems rather strange that these results were quite the reverse from what were expected; the round section being the strongest, next the square, then the rectangular section, and lastly the star; varying inversely as the perimeter exposed.

Fig. 1 gives representative stress strain diagrams from the tests.



Tension tests. Compression tests.

FIG. 1.—Stress-strain diagrams of malleable castings.

Following are the specifications of the Bureau of Steam Engineering of the U. S. Navy Department for malleable castings:

Malleable-iron castings for which physical requirements are specified may be made by either the open-hearth or the air-furnace process.

Sulphur must not exceed .06 per cent., and phosphorus must not exceed .225 per cent.

The transverse breaking load for a bar 1 in. square, loaded at the middle and resting on supports 1 ft. apart, shall be not less than 3,000 lbs., deflection being at least ½ in.

The minimum tensile strength of the material must be 40,000 lbs. per sq. in., and the elongation at least 2½ per cent. in 2 ins.

Castings must be true to pattern, free from blemishes, scale, and shrinkage cracks.

Malleable castings must be neither "over" nor "under" annealed. They must have received their full heat in the oven at least 60 hours after reaching that temperature, and shall not be dumped until they are at least "black hot."

STEEL

For steel for springs, see also Springs.

For steel for boilers, see Steam Boilers.

A list of heat treatments will be found at the end of the section.

The composition and physical properties of steel for a large variety of purposes are given in Table 1 of representative specifications by C. A. TUPPER (*Amer. Mach., Mar.* 24, 1910). The table is the result of an inquiry extending through two years of time into the practice of many of the largest machinery building and structural work companies and has been submitted to a number of large users and manufacturers of steel, including engineers and chemists. It represents advanced practice.

With the table Mr. Tupper makes some observations on the trend of practice in these matters from which the following extracts are taken:

For carbon-steel masses of considerable weight rotating at great speed, such as the body of the spindle of a horizontal steam turbine running up to, say, 3600 r.p.m., metal of high endurance, having a tensile strength of 85,000 to 100,000 lbs. and an elastic limit of 55,000 to 70,000 lbs. is now required, with elongation in 2 ins. of 25 to 30 per cent., contraction 40 to 48 per cent. The same is also true of other rotating elements turning at much slower speed but subjected, at the same time, to heavy pressure or resistance, such as the runner of a hydraulic turbine or the impeller of a centrifugal pump.

For pumping engines the conclusions of manufacturers and engineers vary, some being of the judgment that an ultimate tensile strength of 55,000 to 60,000 lbs. is ample, though an elastic limit of at least 30,000 to 40,000 lbs. is required. Elongation in 2 ins. (the present tendency apparently being to adhere to that standard) of 20 to 28 per cent. and contraction of 35 to 45 per cent. between 25 and 35 per cent., being considered satisfactory, are allowed for under these conditions. Sulphur and phosphorus should not exceed .05 per cent. each, and some users place .02 to .03 per cent. as the limit.

In the purchase of steel billets, manufacturers find, as a matter of shop economy, that it is advisable to carry, as far as possible, stocks of generally suitable characteristics, rather than divide specifications.

As to the carbon content, the grade of billet best adapted to engines or other machinery having a reciprocating motion, where no excessive strains or stresses are likely to be set up, should run from 25 to 33 points. These can be machined to better advantage than the 50-point carbon steel often recommended. In annealing, such steel is customarily heated to from 1650 to 1800 deg. Fahr. for 10 to 12 hours, and allowed to cool very slowly. A higher temperature of heating is permissible, but not above the extreme limit reached in forging and most manufacturers fear to go above 1850 deg. Fahr. For the greater tensile strengths required in the operation of such machinery, up to, say, 70,000 lbs., the use of 33- to 35-point carbon steel is to be recommended.

It should be recognized that overheating is more injurious to high-carbon steel than too low; also, that if the reduction in hammering or pressing has been great enough so that the coarsening of the grain at the high temperature to which the steel has been heated prior to such hammering or pressing has been well effaced, the harm of overheating increases not only with the distance above the final point of recalescence to which the temperature is raised, but also in direct ratio to the percentage of carbon. Steel that has become dangerously crystalline may—in many cases, at least—be restored to specification conditions, or better, by reheating and manipulation; but the necessity for this ought to be avoided just as far as possible.

For less particular service, such as ordinary shop machinery, the usual stock billets of 15- to 25-point carbon, having a tensile strength of about 55,000 lbs., elastic limit practically negligible, and purchased with no more than ordinary physical or chemical requirements, are regarded as sufficient for all purposes.

For special machinery parts subject to excessive wear, such as crank pins, valves, compression rods, table clutches, return cranks, etc., a carbon content of at least 50 to 55 points is needed. Such forgings, or even castings, should also show an ultimate tensile strength of at least 75,000 lbs., if not annealed, and a minimum of 65,000 to 75,000 lbs. if thoroughly annealed, with elastic limit, elongation and contraction correspondingly high.

For carbon-steel shafts and axles subjected to heavy strains (the tendency now being, however, to use some special alloy such as vanadium) the requirements of machinery and truck builders vary considerably, but a content of .35 to .45 per cent. carbon, manganese not above .45 to .50 per cent., silicon .05 per cent., sulphur not to exceed .03 per cent. and phosphorus within .04 per cent. are considered safe. In actual manufacturing, where such steel is used, these standards have not heretofore been very generally realized, but the increasing frequency of accidents and breakdowns, under the severe requirements of modern power, mill and traction service, is compelling builders to draw the lines tighter and tighter.

It should be remembered, however, that the greater the percentage of carbon in billets, the higher is the temperature required in forging—not less than 1650 deg. Fahr. for high-carbon steel; hence costs may be kept down by using billets as low in carbon as the requirements of the finished product will permit.

The selection of the proper steel for crank pins has always been a vexed question, particularly where such parts are subjected to heavy stresses and the force of sudden shocks—as in the case of a reversing engine for rolling-mill service, when the rolls bite the ingot. 45- to 55-point carbon steel, with tensile strength of 75,000 lbs. will meet ordinary requirements of heavy duty, but for continuously severe operating conditions, as in the instance above cited, a special alloy steel such as chrome vanadium of high tensile strength and great toughness is desirable. An example of this will be found in the accompanying table.

For large traveling cranes a leading builder states that the grade of castings best suited to his requirements is of open-hearth steel, having an ultimate tensile strength between 66,000 and 68,000 lbs., elastic limit of 33,000 to 35,000 lbs., and chemical content of .24 per cent. carbon, .48 per cent. manganese, .20 per cent. silicon, .06 per cent. phosphorus, and .04 per cent. sulphur. Steel for truck wheels carries .03 per cent. carbon, .59 per cent. manganese, .63 per cent. silicon, .454 per cent. phosphorus and .152 per cent. sulphur; truck-wheel tires .44 per cent. carbon, .78 per cent. manganese, .28 per cent. silicon, .38 per cent. phosphorus and .048 per cent. sulphur. The figures given are taken from actual tests of steel that, all things considered, has proved most satisfactory. For pinions, armature shafts, truck axles, etc., a rolled open-hearth steel, 30 to 35 points carbon is used, and, for the cross shafts on the crane bridges, turned and ground shafting that runs high in carbon. The steel and other metal used in the construction of the crane-motors is such as electrical manufacturers employ in building high-grade motors for heavy duty.

Some years ago attention was directed, by tests of the new armor plate made for naval vessels, to the great tensile strength, toughness and ductility of the then little-known nickel steel. Experiments with shafts, axles, spindles and other parts of vehicles or machinery rotating at considerable speeds also showed that it possessed the

extremely valuable quality of resistance to fatigue, that it readily withstood shocks of all kinds as well as those of shell impact and that steel high in nickel was practically not subject to cracking or similar rupture.

Forgings made from nickel steel have, in general, the same requirements as those of ordinary carbon steel of the same class, with the addition of 2.5 to 3.5 per cent. nickel. This raises the ultimate strength anywhere from 10,000 to 80,000 lbs. per. sq. in. (depending upon the other characteristics of the metal), and the elastic limit in proportion, without sacrificing the ductility. In fact, the last-named is usually increased.

The proportions of nickle commercially usable in large forgings intended for machinery parts are limited by a curious property of the metal, viz., that its mixture in high-carbon steel up to and beyond a certain percentage increases the hardness of the steel. Between these two points there is a small range, the working limits of which are about as stated above, in which nickel steel can be machined to the best advantage. For low-carbon nickel steel the range is somewhat greater, and it can be easily worked cold with nickel under 3 to 5.5 per cent.

Specifications for small forgings, which are subsequently to be reduced by grinding, may call for as much nickel as is desirable. Steel can be forged readily, without regard to its nickel content.

Nickel-steel forgings, also, do not ordinarily need annealing, except where the latter is intended to be carried to the tempering stage, as there is already sufficeint homogeneity in the structure.

Allowing, however, for the truth of all that has been said above, it is a fact indicative of the rate of modern progress that for many purposes nickel steel has already had its day, and the addition or substituiton of other alloys, to form such combinations as chrome nickel, nickel vanadium and chrome vanadium steels, has become general. Vanadium has an even more favorable effect than nickel alone and increases the ductility and toughness of the steel containing it. This is now used for engine, locomotive and automobile parts, as well as in bridges, viaducts or other structures where there is much vibration. Chrome-nickel enters similarly into the composition of machinery steel. Titanium appears to give results even better than those mentioned (although this is not generally conceded) and at the same time does not appear in the finished product. It apparently acts as a scavenger to remove impurities.

Advocacy of vanadium steel is particularly strong at present among the expert metallurgists employed by machinery builders.

A chrome-nickel, chrome-vanadium, chrome-nickel-vanadium or other special alloy steel used for the rotating parts of extremely compact high-speed machinery is ordinarily required to have an ultimate tensile strength of about 120,000 lbs., with elongation of 16 to 30 per cent. in 2 ins. and the extremely high elastic limit of 100,000 lbs. Where such machinery is subjected to extraordinary stresses, however, the tensile strength may run as high on test as 165,000 to 175,000 lbs., with elastic limit very little under these figures and elongation up to 32 per cent. or beyond.

In the forging of these alloyed steels, much more than in their machining, special skill is usually required, particularly when they are high in silicon; and the temperature must be maintained at above 200 to 250 deg. Fahr. under the melting-point, thus necessitating, with a large piece, several reheatings, it being a well-recognized fact that forgings made at a temperature just above the final recalescence point are always strongest. A small variation in the proportioning of the alloys makes considerable difference in forging conditions; hence machinery builders find it necessary to check their specifications very closely with the results of actual tests in the shop, before arbitrarily demanding this or that from the steel manufacturers. Failure to proceed very cautiously on that basis has, not infrequently, led to heavy losses.

Heat treatment of steel in the shop—and there is nothing which will be more likely to influence specifications in future—comes under two heads: annealing and tempering. The former, only, will be considered here, and without further reference to tool steel, which is a subject quite by itself.

Until very recently, annealing has not been given by builders of high-speed or heavy machinery the attention it deserves, and, even to-day, the art is practised to a far less extent than the average reader probably supposes.

By the annealing of steel before it leaves the mills, a uniformity of structure is given to billets which does much to relieve or prevent subsequent internal strains; and re-annealing of ordinary carbon steels in the shop, after forging or machining, is, as a rule highly desirable, for the reason that, at each of the three periods of recalescence or "absorption" in heating and cooling (*i.e.*, six periods both ways) an actual re-arrangement of the molecules takes place while an increase in temperature is temporarily arrested, and such disturbance as there may have been of the physical structure of the piece, tending to crystallization, gives way to a restoration of the desired conditions.

In annealing, furnaces especially designed for the purpose, and preferably gas-fired, should be provided. The too prevalent tendency among machinery builders who do their own forging, to use an ordiary forge fire in annealing, leads to some pernicious results—results which, nevertheless, have to be taken into account in the preparation of specifications.

A slow raising of the temperature to the final point of recalescence, with careful observation by means of a pyrometer, and even slower cooling, are essential to good practice.

Chemical analysis, which originally met with so much opposition when introduced in metal-working plants, has been swung to the other extreme, so much so that undue reliance has, of late, been placed upon it in many quarters. Chemistry, property applied, is, of course, essential in determining the characteristics of steel; but it does not take the place of physical tests, and microscopic, or photo-microscopic apparatus will determine things that are altogether outside the range of chemistry.

In testing, the appearance of a crystalline fracture, or any trace of crystallization, should be sufficient to at once cause the rejection of a forging, if annealed steel is required; but for unannealed forgings crystalline fractures may be regarded as normal if the steel is high in carbon. A circumstance to be here observed, and one often overlooked by machinery builders, is the fact that forgings made in custom shops will not be annealed in advance unless annealing is specified; also that they will not be physically tested unless such tests are asked for or they are purchased under definite physical specifications. In testing, a bar is ordinarily severed from an end having the full diameter of the forging, the cut being made in an axial direction about half way of the radius, according to the U. S. Naval standard.

Since the development of the so-called high-speed press, the forging of steel by means of continuous hydraulic pressure, rather than by the use of a steam hammer, has been coming more and more into favor among builders of machinery subjected to severe stresses, for the reason that it results in a more uniform, homogeneous and reliable product.

No matter how powerful a steam hammer may be, the force of its blow is not ordinarily felt very far below the surface of a forging, and, even with the exercise of the greatest skill, the depth of compression forms a very irregular line around the center of the piece, leaving the interior far from solidified; whereas, with a press, the molecular structure of the forging is almost uniformly condensed, the compression being felt through its diameter. The press will also work very close to a shoulder, or even forge a squared up shoulder, thereby saving metal and time in machining.

These facts are becoming so well recognized that the day of the large steam hammer is drawing to a close; henceforth, for heavy work, presses will be quite generally installed as new equipment is needed. They are mentioned here particularly on account of the influence which they are beginning to exert on specifications.

TABLE 1.—REPRESENTATIVE SPECIFICATIONS FOR STEEL

Class of steel and where used. Customary heat treatment understood	Physical properties: Tensile strength, lbs per sq in	Elastic limit, lbs. per sq. in.	Elongation per cent. in 2 ins.	Reduction area in per cent.	Percentage of ingredients: Carbon	Manganese	Phosphorus	Sulphur	Nickel	Chromium	Vanadium	Remarks
compressors	55-60,000	30-40,000	22-28	35-45	.25-.33	.48-.50	.03-.07	.03-.05				General service.
r	55-60,000	30-40,000	20-27	30-40	.15-.25	.30-.40	.04-.08	.05-.06				Where service varies but little with no unusual strains.
Heavy crank forging	79,000	58,000	32	62	.29	.53	.05	.05	3.2			Test of one crank.
Gears, continual mesh	50-60,000	34-42,000	40-44	58-62	.30-.35	.45-.49	.03-.05	.03-.04				Requirements vary widely with uses.
Gears, sliding	55-60,000	42-45,000	43-48	65-72	.20-.23	.24-.32	.04-.05	.02-.03		.30	.12	Some automobile gears show over three times the strength here given.
Cranks, shafts, axles, pins, webs, rods, etc.	60-70,000	35-50,000	23-38	32-47	.28-.35	.45-.58	[2].05	.04-.05				Ordinary practice for moderate service. Tendency toward alloy steels.
Shaft and axles for heavy duty	80-85,000	55-65,000	25-30	34-40	.32-.36	.45-.60	.04-.06	.04-.05				
Heavy shafts and high-speed rotating parts	65-70,000	35-45,000	25-28	40-43	.30-.38	.47-.60	.04-.05	.03-.04				
Propeller shafts and rotating parts of hydraulic machinery subject to vibratory strains.	85-100,000	60-70,000	31-39	43-48	.31-.37	.44-.52	.03-.04	.01-.03	3.25	1.00		For heaviest and most severe service. Tendency is toward use of special steels of extreme toughness and resistance to fatigue.
	90-105,000	65-80,000	36-41	66-72	.43-.48	.49-.56	[2].04	[2].04				
	121,000	114,000	23	64	.28	.56	trace	trace		1.04	.13	Test of one crank.
	66,000	41,000	42	61	.32	.49	.04	.04				Test of one axle.
	135-160,000	100-120,000	15-18	37-44	.45-.48	.89-.90	[1].03	[1].03		1.10	.18	
	85-110,000	55-75,000	25-34	40-48	.26-.30	.28-.36	[2].05	[2].04				
	75,000	37,500	22	32	.27-.29	.49-.51	[1].05	[1].05				One recent specification.
., for light	50-60,000	30-35,000	28-30	38-45	.25-.36	.38-.50	.04-.05	.03-.05				Tendency toward higher grade steel; not found adequate expression.
t pressure	80-85,000	45-50,000	22-30	38-42	.26-.30	.33-.46	[1].05	.03-.04	3.25			Tendency toward special alloy steels.
:e	70-75,000	40-52,000	24-27	32-39	.45-.53	.58-.62	.04-.05	.03-.04				Tendency toward special alloy steels.
	110-125,000	90-100,000	25-33	42-48	.27-.35	.33 .30	[2].03	.01-.03		1.10	.16	
	60-70,000	35-40,000	21-34	28-37	.35-.56	.62-.70	.04-.05	.04-.05				
	105-120,000	80-100,000	18-22	33-37	.34-.36	.50-.55	[2].04	[2].03		.95	.15	
	70-75,000	38-46,000	in 8" 12-38		.27-.30	.50-.70	.05-.07	.05-.06				
Eye bars of nickel steel	85-100,000	[1]48,000	in 10' [1]10		.45	.70			3.25			Specifications for St. Louis municipal bridge over Mississippi River.
Rivets for boilers and tanks	45-50,000	23-27,000	in 8" 30-34		.10-.18	.20-.25	.05-.08	.05-.07				Rivets such as are carried in stock.
Rivets for bridge and structural work	55-60,000	28-33,000	32-36		.11-.16	.22-.28	.04-.08	.03-.07				
Structural steel for buildings	55-65,000	30-35,000	in 8" 21-25		.28-.45	.50-.60	.05-.06	.04-.05				Common range.
Structural steel for ships and bridges	55-60,000	28-36,000	in 8" 18-26		.35-.40	.40-.60	.04-.07	.03-.06				Not used much in future for important work.
Bridge steel	62-70,000	37-40,000	in 8" 22-27		.25-.43	.48-.55	[1].04	[1].04				Recent specifications representing good modern practice.
Crane steel	65-70,000	33-36,000	in 8" 21-23		.24-.26	.45-.48	[1].06	[1].04				Average specifications of a leading crane builder.
	55-60,000	29-33,000	in 8" 17-19	52-60	.18-.22	.34-.42	.06-.08	.05-.07				
	65-70,000	45-55,000	in 8" 25-30	48-55	.22-.26	.30-.50	.04-.06	.03-.04				
ine pump	90-110,000	70-85,000	24-28	56-66	.25-.31	.42-.51	[2].05	[2].04	3.50	.80	.17	
	187,000	183,000	14	50	.40	.77				1.22		Figures given by American Vanadium Co.
	237,500	227,100	10	35	.48	.90				1.25	.18	Extreme test noted
	227,000	224,000	in 8" 12	42	.34	.40				1.00	.17	Extreme test noted
	60-70,000	27-35,000	20-22	30-34	.20-.38	.30-.45	.05-.08	.05-.06				Concerning basic and acid open-hearth steel castings, see the explanation in the text of article accompanying this chart.
	[1]60,000	[1]30,000	[1]20	[1]25	.20-.30	.50-.65	[1].06	[1].05				
	70-85,000	33-38,000	18-21	28-32	.40-.45	.45-.55	.04-.06	.03-.06				
	[1]70,000	[1]35,000	[1]15	[1]20	.30-.40	.55-.70	[1].06	[1].05				
	80-110,000	45-50,000	12-15	18-30	.40-.55	.60-.78	.05-.07	.04-.06				
	[1]80,000	[1]40,000	[1]10	[1]15	.40-.50	.60-.75	[1].06	[1].05				
	[1]70,000	[1]40,000	[1]20	[1]30	.20-.25	.45-.50	[1].06	[1].05			.20	Figures given by custom foundry.
l stresses	69-75,000	34-35,000	17-23	35-47	.27-.28	.63-.69	[1].06	[1].05				Range of a series of tests.
Steel castings for variety of heavy service	62-78,000	33-41,000	16-24	33-49	.31-.34	.62-.71	[1].06	[1].05				Range of a series of 12 tests.
Steel castings for pumps	65,000	34,000	18	24	.26	.48	.05	.04				One specification
Steel rails, heavy O. H. carbon	90-100,000	55-60,000	in 8" 20-30	30-40	.75-1.15	.60-.75	.03-.08	.04-.06				Averages of figures available; practice varies greatly.

[1] Not less than. [2] Not over. NOTE 1.—Elongation s in 2 in except where given differently. Percentage of elongation generally adopted is 1,400,000, divided by the ultimate strength. This, however, varies with the chemical content and heat treatment. NOTE 2.—The figures here given relate particularly to basic open-hearth steel, except where specified. NOTE 3.—The silicon is not given owing to the rarity with which it is specified. NOTE 4.—The figures are intended to represent average practice, except where otherwise mentioned, and no rigid limits are intended.

One of the effects, which does not come under any of the other headings here treated, is a reduction (for pressed forgings) of 40 to 50 per cent. in the area of the initial section as compared with that which must be specified for hammered forgings.

Steel that has been forged by pressure and subsequently annealed shows greater homogeneity than hammered forgings and greater elongation. It also has greater toughness, offers more resistance to all manner of strains and has a higher elastic limit.

It may be stated here also that, in all cases, permissible variations in the chemical analysis of steel, no matter for what purpose intended, should be given—rather than insisting upon strict adherence to the figures of a single analysis that is considered favorable; as it is practically necessary for the steel manufacturer to have a certain amount of leeway in either direction. "We have learned," says the superintendent of one prominent crucible-steel plant, "that a chemical specification that permits of no leeway has usually been prepared by a novice. If we had no range to work on we would have to go out of the business." At the same time the exercise of too great latitude in such matters should be carefully guarded against; and the proper relation both of chemical conditions and physical characteristics ought to be rigidly insisted upon where the character of the service demands it.

TABLE 2.—PHYSICAL AND CHEMICAL CHARACTERISTICS OF STEEL FORGINGS FOR ENGINES OF THE U. S. NAVY

NOTE.—Class C forgings will not be tested unless there is reason to doubt that they are of a quality suitable for the purpose for which intended. Tests, if required, shall be made at the expense of the contractor, and may be made at the point of delivery.

Class	Material	Treatment	Minimum tensile strength, lbs. per sq. in.	Minimum elastic limit, lbs. per sq. in.	Minimum elongation, per cent. in 2 ins.	Maximum percentage of— P.	Maximum percentage of— S.	Without showing cracks or flaws must cold bend about an inner diameter of—	Suitable uses
H. G	Open-hearth nickel steel,	Annealed and oil-tempered.	95,000	65,000	21	.05	.05	1 in. through 180°.	Bolts and studs for all moving parts of main engines, shaft couplings, main bearing caps, thrust bearing side rods, main engine framing and moving parts of circulating pumps; connecting rods, caps and bolts; eccentric rods; main circulating pump engine working parts; piston rods; suspension links and link blocks; valve stems.
A	Open-hearth, either nickel or carbon steel.	Annealed. Oil-oil-tempering optional.	80,000	50,000	25	.05	.05	1 in. through 180°.	Coupling bolts; crossheads and slippers; crank, thrust, line, stern tube, tail, and propeller shafts; main bearing cap bolts; outboard coupling; reverse arms and blocks; rotor shaft; thrust bearing side rods; turning engine worm; working parts, reversing gear; working parts, pumps.
B	Open-hearth carbon steel.	Annealed	60,000	30,000	30	.05	.05	½ in. through 180°.	Bearer bars; Curtis turbine shaft; engine columns and tie-rods; H.p. relief valve stems; main steam valve stems; main bearing cap bolts; piston rod nuts; piston valve followers; pipe flanges; rotor drum and wheel; stole-plate wedges; swivel pins for crosshead; working levers and gears.
C	Open-hearth or Bessemer steel.	Annealed	52,000	....	28	.....	.....	1 in. through 180°.	Gland for cylinder liners; small parts of eccentrics; uptake and smoke-pipe forgings.

TABLE 3.—PHYSICAL AND CHEMICAL CHARACTERISTICS OF STEEL CASTINGS FOR THE U. S. NAVY

NOTE.—Class C castings will not be tested unless there are reasons to doubt that they are of a quality suitable for the purpose for which they are intended. Tests, if required, may be made at the building yard. The inspector will select a sufficient number of castings and have them crushed, bent, or broken, and note their behavior and the appearance of the fracture.

Class symbol	Chemical composition, Not over— P.	Chemical composition, Not over— S.	Physical requirements: Minimum tensile strength	Minimum yield point	Minimum elongation	Minimum reduction of area	Bending test; cold bend (not less than)	Suitable uses
			Lbs. per sq. in.	Lbs. per sq. in.	Per cent. in 2 ins.	Per cent.		
Special	.04	.04	90,000	57,000	20	30	90 deg. about an inner diameter of 1 in.	
A	.05	.05	80,000	35,000	17	20	90 deg. about an inner diameter of 1 in.	Engine frame strongbacks; I.P. and L.P. pistons; pistons and followers for piston valves; reverse arms; separators; valve stem crosshead.
B	.06	.05	Maximum 80,000 Minimum 60,000	30,000	22	25	120 deg. about an inner diameter of 1 in.	Boiler fittings; crosshead backing guide; cylinder and valve chest covers; engine bedplates; H.p. cylinder relief valves; I.P. and L.P. piston followers; main bearing caps and shoes; main steam valves; outboard coupling casing; pipe flanges; pipe flanges, slip joints, and safety and relief valves for superheated steam; piston rod and valve stem stuffing boxes; reducing valves; reverse shaft bearings; thrust horseshoes; valve stem guide and crosshead cap.
C	.06	.05	....	....	....	....	....	Unimportant castings.

Table 2 gives the physical and chemical characteristics specified by the Bureau of Steam Engineering of the U. S. Navy Department for engine forgings. The specified treatment is as follows:

All forgings shall be annealed as a final process, unless otherwise directed. All tempered forgings, if forged solid, and if more than 5 ins. in diameter in any part of their lengths, not including collars, palms, or flanges, shall be bored through axially before tempering, and the bore shall be of sufficient size to enable the manufacturer to get the requisite tempering effect. Forgings, such as crank shafts, thrust shafts, etc., may, previous to tempering, be machined in a manner best calculated to insure that the tempering effect reaches the desired portions. In this case, the inspector will decide upon the location of the test pieces if they cannot be taken in the manner hereinafter described. All forgings shall be free from slag, cracks, blowholes, hard spots, sand, foreign substances, and all other defects affecting their value.

Table 3 gives the physical and chemical characteristics of steel castings specified by the U. S. Navy Deparment.

Steel for resisting shock should be of high carbon content. In the past the accepted dictum was that for this purpose a low carbon steel should be used, the idea being that low carbon steel is tough and able to stand punishment and that high carbon steel is brittle. The fallacy of this reasoning was first shown by experience with steam-hammer piston rods at the Crescent steel works about 1880 and the demonstration was made complete by the experience of rock-drill manufacturers. In rock drills low carbon steel was a complete failure, high carbon steel being found, early in the history of the industry, to be the only suitable material.

Looking back, with the superior wisdom that comes after the event, the traditional view now seems absurd. It is now clear that what is wanted is a material that will absorb and give back again the greatest number of ft.-lbs. of energy without change of form; that is, without passing its elastic limit. That is to say, the property wanted is resilience and not toughness. In other words, we should aim at the properties of a spring and not at those of a piece of lead, which is equivalent to saying that we want high and not low carbon steel, and, by the same token, the steel should be in the tempered and not the annealed condition.

The properties of steels of various carbon percentages, as regards the elastic resilience, have not been studied with sufficient care to enable the exact composition most suitable for resisting shock to be stated. Analogy would indicate that the composition most suitable for springs is the correct one for shock resistence, and the same remark applies to the heat treatment. At the same time, the author in his own experience with rock drills, made use of steel with carbon percentage as high as 1.25 and with conspicuous success. Such steels were hardened with great care to avoid overheating and then drawn to a straw color.

Steel for Cutting Tools

The percentage of carbon suitable for carbon steel tools may be obtained from Table 4 (*Amer. Mach., Nov.* 21, 1912). To some the carbon percentages will seem high but the table is the result of much investigation.

Table 4.—Carbon Percentage in Carbon Steel Tools

Anvil facing	.70 to .80
Arbor, saw	.35 to .50
Auger bit	.50 to .65
Axes of various shapes for cutting wood	1.00 to 1.10
Ball bearing races	1.10 to 1.20
Ball-peen hammer	.80 to .90
Band saw	.70 to .80
Barrel, gun	.30 to .70
Barrel, gun, drill for boring	1.10 to 1.20
Bar, digging	.80 to .90
Bar, pinch	.70 to .80
Bit, auger	.50 to .65
Bit, ax	1.00 to 1.10
Bit, stone, channeling machine	1.00 to 1.20
Bit, for stone drilling	.70 to .90
Blacksmith's cold chisel	.70 to .80
Blacksmith's hammer	.70 to .80
Blacksmith's hot chisel	.80 to .90
Blade, knife	1.10 to 1.20
Blade, pocket knife	.90 to 1.00
Blade, reamer	1.10 to 1.20
Blade, skate	.80 to .90
Blade, table cutlery	.70 to 1.10
Blanking punch for files	1.20 to 1.30
Boilermaker's snap	.60 to .70
Boilermaker's beading tool	.70 to .85
Bolt dies, cold heading	.60 to .70
Bolt machine plunger	.60 to .70
Brick chisel	.60 to .70
Broad axe	1.00 to 1.10
Bucket teeth for dredges	.70 to .80
Button set	.60 to .70
Cabinet file	1.20 to 1.30
Cant dog	.90 to 1.00
Cant hook	.80 to .90
Cant-saw file	1.20 to 1.30
Cape chisel	.80 to .90
Car and locomotive spring	.90 to 1.10
Cartridge shell die	1.20 to 1.30
Cartridge shell punch	1.20 to 1.30
Carving knife	1.00 to 1.10
Carving fork	.55 to .65
Calking chisel	.80 to .90
Center, lathe	1.00 to 1.10
Channeling machine bit, stone	1.00 to 1.20
Chisel, blacksmith's cold	.70 to .80
Chisel, chipping	.80 to .90
Chisel, brick	.60 to .70
Chisel, carpenter's	1.00 to 1.30
Chisel, file cutting	1.10 to 1.20
Chisel, hot	.80 to .90
Chisel, machinist's	.80 to .90
Chisel, railroad track	.70 to .80
Chisel, stone cutter's	1.10 to 1.20
Chisel, wood-working	1.00 to 1.30
Chuck jaw	.80 to .90
Circular saw	.80 to .90
Cleaver, butcher's	.80 to .90
Cold-heading bolt die	.60 to .70
Cold chisel, blacksmith's	.70 to .80
Cold cutting die for metal	1.10 to 1.20
Cold-punching horseshoe die	1.00 to 1.10
Cone, bicycle	1.00 to 1.10
Crosscut saw	.90 to 1.00
Crowbar	.70 to .80
Crucible machinery steel	.35 to .50
Cruciform drill steel	.80 to .90
Cutter blank, milling	1.10 to 1.20
Cutter, flue	1.20 to 1.30

TABLE 4.—CARBON PERCENTAGE IN CARBON STEEL TOOLS—(*Continued*)

Cutter, glass	1.20 to 1.30
Cutter, nail	1.10 to 1.20
Cutter, pipe	1.10 to 1.20
Cutter, horse hoof	.70 to .85
Cutter, clinch (farrier's)	.80 to .90
Cutting die, cold, for metal	1.10 to 1.20
Cutting die, paper	1.10 to 1.20
Cylinder, pneumatic hammer	.80 to .90
Die, cold-heading for bolts	.60 to .70
Die, cartridge shell	1.20 to 1.30
Die, cold cutting for metal	1.10 to 1.20
Die, drop forging	.60 to .75
Die, leather cutting	.80 to .90
Die, drop hammer	.60 to .80
Die, horseshoe cold-punching	1.00 to 1.10
Die, nail	1.10 to 1.20
Die, paper cutting	1.10 to 1.20
Die, pipe	1.10 to 1.20
Die, rivet	.60 to .75
Die, shoe-upper cutting	.70 to .80
Die, silversmith's, stamping	1.10 to 1.20
Die, silver spoon, drop	.80 to .90
Die, threading	1.00 to 1.10
Die, wire drawing	1.30 to 1.50
Dog, cant	.90 to 1.00
Drift pin	.80 to .90
Digging bar	.80 to .90
Drag saw	.90 to 1.00
Dredge bucket teeth	.70 to .80
Drill, cruciform	.80 to .90
Drill for drilling tool steel	1.00 to 1.20
Drill for shotgun barrels	1.10 to 1.20
Drill, quarry	.75 to .90
Drill, twist	1.10 to 1.20
Driver, screw	.60 to .70
Drop-forging die	.60 to .75
Edge, scythe	1.00 to 1.10
Expander roll for tubes	1.00 to 1.10
Eyepin for tie rods	.70 to .80
Facing, anvil	.70 to .80
File, blanking punch for	1.20 to 1.30
File, cutting chisel for	1.10 to 1.20
Files in general	1.20 to 1.30
Flat chisel	.80 to .90
Flatter, blacksmith's	.80 to .90
Flue cutter, boiler	1.20 to 1.30
Forging die, drop	.60 to .75
Fork, pitch	.90 to 1.10
Gang saw	.90 to 1.00
Glass cutter	1.20 to 1.30
Glove die, leather	.80 to .90
Glut or stone wedge	.60 to .70
Grab	.70 to .90
Granite point	1.22 to 1.35
Grass hook	.60 to .70
Gun barrel	.30 to .70
Gun-barrel reamer	1.10 to 1.20
Hammer, blacksmith's	.70 to .80
Hammer, ball-peen	.80 to .90
Hammer, bush for stone	1.20 to 1.30
Hammer, machinst's	.90 to 1.00
Hammer, nail machine	1.00 to 1.10
Hammer, peen	1.15 to 1.20
Hammer, pneumatic cylinder	.80 to .90
Hardie	.70 to .80
Hatchet	1.10 to 1.20
Hoe	.80 to .90
Hook, cant	.80 to .90
Hook, grass	.60 to .70
Horseshoe die, cold punching	1.00 to 1.10
Hot chisel	.80 to .90
Hot punch	.80 to .90
Ice plow	1.10 to 1.20
Jaw, chuck	.80 to .90
Knife blade	1.10 to 1.20
Knife, butcher's	.75 to .90
Knife, carving	1.00 to 1.10
Knife, cobbler's	.85 to 1.00
Knife, farrier's	.80 to .90
Knife, machine	1.10 to 1.20
Knife, paper	1.10 to 1.20
Knife, pen	.80 to .95
Knife, pruning	.75 to .85
Knife, putty	.90 to 1.00
Knife, drop-forging die for table	.60 to .75
Knife, shear, for paper	.80 to 1.00
Knife, wood-working	1.10 to 1.20
Lathe tool	1.00 to 1.20
Lathe center	1.00 to 1.10
Lawn-mower blade	.90 to 1.00
Locomotive and car spring	.90 to 1.10
Machine knife	1.10 to 1.20
Machinery steel, crucible	.35 to .50
Machinist's hammer	.90 to 1.00
Magnet, permanent	1.20 to 1.30
Magnet for telephone call bell	.50 to .60
Magnet for telephone	.90 to 1.10
Mandrel	1.00 to 1.10
Mattock	.60 to .80
Maul, railroad	.70 to .80
Maul, woodchopper's	.70 to .80
Mill pick	1.20 to 1.30
Mill saw	1.20 to 1.30
Milling cutter blank	1.10 to 1.20
Mining tools, drills, picks, etc	.65 to 1.10
Molder's hand tools	1.00 to 1.10
Mower blade, lawn	.90 to 1.00
Nail cutter	1.10 to 1.20
Nail die	1.10 to 1.20
Nail-machine hammer	1.00 to 1.10
Nail puller	.90 to 1.00
Paper-cutting die	1.10 to 1.20
Paper knife	1.10 to 1.20
Paving and plug drill	1.10 to 1.20
Peen hammer	1.15 to 1.20
Pick ax	.70 to .80
Pick mill,	1.20 to 1.30

TABLE 4.—CARBON PERCENTAGE IN CARBON STEEL TOOLS—(*Continued*)

Pin, drift	.80 to .90
Pincers (farrier's)	.80 to .90
Pinch bar	.70 to .80
Pipe cutter	1.10 to 1.20
Pipe die	1.10 to 1.20
Pit saw	.90 to 1.00
Pitchfork	.90 to 1.10
Pitching chisel for stone	1.00 to 1.15
Planer tools for metal	1.00 to 1.20
Planer tools for stone	.80 to 1.30
Planer tools for wood	1.10 to 1.20
Pliers	.60 to .95
Plow blade for ice	1.10 to 1.20
Plug and paving drill	1.10 to 1.20
Plunger for bolt machine	.60 to .70
Pneumatic-hammer cylinder	.80 to .90
Point, granite	1.22 to 1.35
Point, clay pick	.70 to .80
Pruning shears	.90 to 1.00
Puller, nail	.90 to 1.00
Punch, boilermaker's	.85 to .95
Punch, cartridge shell	1.20 to 1.30
Punch, for file blanks	1.20 to 1.30
Punch, hot	.80 to .90
Punch, railroad track	.70 to .80
Punch, washer	.80 to 1.00
Putty knife	.90 to 1.00
Quarry drill	.75 to .90
Railroad car and locomotive spring	.90 to 1.10
Railraod-spike maul	.80 to .90
Railroad-track chisel	.70 to .80
Ramrod	.60 to .70
Razor	1.00 to 1.15
Razor blade, safety	1.00 to 1.20
Reamer blade	1.00 to 1.20
Reamer, hand	1.00 to 1.10
Rivet, die	.60 to .75
Rivet set	.65 to .75
Road-scraper blade	.60 to ..70
Roll, expander	1.00 to 1.10
Saws for wood in general	.80 to .90
Saw, arbor	.35 to .50
Saw file	1.20 to 1.30
Saw blade, band	.70 to .80
Saw, circular	.80 to .90
Saw, crosscut	.90 to 1.00
Saw, drag	.90 to 1.00
Saw for steel	1.60
Saw, gang	.90 to 1.00
Saw, mill	1.20 to 1.30
Saw, pit	.90 to 1.00
Saw swage	.80 to .90
Saw teeth, inserted for wood	.90 to 1.00
Scraper blade for roads	.60 to .70
Scraper tube	1.20 to 1.30
Scraper, wood-working	.90 to 1.00
Screw-driver	.60 to .70
Scythe edge	1.00 to 1.10
Setscrew	.60 to .70
Set, button	.60 to .70
Set, mason's	.75 to 1.00
Set, rivet	.65 to .75
Shaper tools	1.00 to 1.20
Shear knife	.80 to 1.00
Shears, pruning	.90 to 1.00
Shell, drawing punch for cartridge	1.20 to 1.30
Shell, drawing die for cartridge	1.20 to 1.30
Shoe die, for cutting leather	.70 to .80
Shotgun barrel drill	1.10 to 1.20
Shovel teeth, dredge	.60 to .70
Silversmith's die for stamping	1.10 to 1.20
Silver-spoon die	.80 to .90
Skate-blade steel	.80 to .90
Sledge	.70 to .80
Spade	.40 to .50
Spindle for cotton or wool	.50 to .60
Star drill	.80 to .90
Steel for files	1.20 to 1.30
Steel for welding	.70 to .85
Stonecutter's chisels	1.10 to 1.20
Stone drilling bit	.70 to .90
Stone planer tools	.80 to 1.30
Stone wedge or glut	.70 to .80
Swage, saw	.80 to .90
Table-knife blade	.70 to 1.10
Tap	1.00 to 1.20
Tooth, dredge bucket	.70 to .80
Teeth, inserted for wood saw	.90 to 1.00
Tools, mason's	.60 to 1.35
Tools, molder's	1.00 to 1.10
Tools, pitching	1.00 to 1.15
Track chisel, railroad	.70 to .80
Track punch	.70 to .80
Trowel, mason's	.40 to .50
Tube scraper	1.20 to 1.30
Twist drill	1.10 to 1.20
Vise, jaw	.70 to .80
Washer, punch	.70 to .80
Washer, punch	.80 to 1.00
Wedge, stone	.70 to .80
Well bit for stone drilling	.70 to .90
Wire-drawing die	1.30 to 1.50
Woodchopper's maul	.70 to .80
Wood-planer blades	1.10 to 1.20
Wood-saw inserted teeth	.90 to 1.00
Wood-working chisel	1.00 to 1.30
Wood-working knife	1.00 to 1.20
Wrench	.70 to .80

Composition and Properties of Carbon Steels

The following data are from the report of the iron and steel division of the standards committee of the Society of Automobile Engineers, Janurary 1912.

A numeral index system has been adopted in the numbering of the metal specifications contained in this report. This system renders it possible to employ specification numerals on shop drawings and blue prints, that are partially descriptive of the quality of material covered by such number; for example, to the carbon steels have been assigned the whole number 10, the numbers following the dash indicating the content of carbon; wherefore 10—10 is the specification numeral for .10 carbon steel.

In the case of steels containing nickel, the approximate percentage

of nickel is indicated in each case by the first figure to the left of the dash in the various specifications. In the case of the chromium steels and of the chromium-vanadium steels, the first figure to the left of the dash indicates approximately the chromium content. In the case of the silico-manganese steel, the corresponding figure indicates the silicon content.

The basic numerals for the various qualities of steels herein specified are as follows:

Carbon steels	10—
Carbon steel, screw stock	11—
Carbon steel, castings	12—
Nickel steels (3.50 per cent. nickel)	23—
Nickel-chromium steels, low nickel	31—
Nickel-chromium steels, medium nickel	32—
Nickel-chromium steels, high nickel	33—
Nickel-chromium-vanadium steels, low nickel	41—
Nickel-chromium-vanadium steels, medium nickel	42—
Chromium steels, 1.00 per cent. chromium	51—
Chromium steels, 1.20 per cent. chromium	52—
Chromium-vanadium steels	61—
Silico-manganese steel	92—
Valve metal, 96 per cent. nickel	96—
Valve metal, 30 per cent. nickel	230—

These steels may be of open-hearth, crucible or electric manufacture, and must be homogeneous, sound and free from physical defects, such as pipes, seams, heavy scale or scabs and surface and internal defects visible to naked eye.

These steels will be purchased on the basis of chemical analysis. The specifications indicate the desired chemical composition. Any shipments not conforming to these specifications after careful check analysis may be rejected.

The notes and instructions following the chemical specifications are not to be considered in any way a part of these specifications. They are adeed solely for the information of the user of the steels and the guidance of the purchaser in the selection of proper steels for his different purposes. They should not be incorporated in the specification when ordering steel.

Materials to be sampled shall be considered under three classes, namely:

1. Wire, tubing, sheet and rod metal less than 1½ in. in size shall be sampled across or through the entire section.

2. Forgings or pieces of irregular shape shall be sampled by drilling or cutting at thickest and thinnest sections, or through or across entire section. In drop forgings changes of carbon may be looked for in the outer ⅛ in. of metal.

3. Bars and billets or other shapes above 1½ in. thick shall be drilled at half radius, or half-way between center and exterior surfaces.

Recognizing the wide variance in methods used for the determination of sulphur, the final reference method shall be the gravimetric (aqua regia) method, by oxidation.

The figures for physical characteristics refer to sections common to automobile use, that is, bars from 1-in. round to 1½ in. round. The high elastic limits can be obtained only on small sections or with severe heat treatments and the low elastic limits can be expected on heavy sections or with less severe treatments.

Carbon Steels

Specification No. 10—10

.10 Carbon Steel

The following composition is desired:

Carbon	.05 to .15% (.10% desired)
Manganese	.30 to .60% (.45% desired)
Phosphorus, not to exceed	.04%
Sulphur, not to exceed	.04%

This is usually known in the trade as soft, basic open-hearth steel. It is a material commonly used for seamless tubing, pressed steel frames, pressed steel brake-drums, sheet steel brakebands and pressed steel parts of many varieties. It is soft and ductile and will stand much deformation without cracking.

This steel in a natural or annealed condition is of low strength, and must not be used where much strength is required. This quality of material is considerably stronger after cold drawing or rolling; that is, its elastic limit is raised by such working. This is important in view of the fact that many wire and sheet metal parts above mentioned are used in the cold-rolled or cold-drawn form.

It must not be forgotten that when this steel (so cold worked) is heated, as for bending, brazing, welding, or the like, the elastic limit returns to that characteristic of the annealed material. *This remark also applies to all materials that have an increased elastic limit produced by cold working.*

This material in a natural or annealed state does not machine freely. It will tear badly in the turning, threading and broaching operations. Heat treatment produces but little benefit, and that not in strength but in toughness. It is possible to quench this grade of steel and put it in a condition to machine better than the annealed state.

The heat treatment which will produce a little stiffness is to quench at 1500 deg. Fahr. in oil or water. No drawing is required.

This steel will case-harden but is not as suitable for this purpose as specification 10—20.

Physical Characteristics

	Annealed	Cold rolled or cold drawn
Elastic limit, lbs., per sq. in.	28,000 to 36,000	40,000 to 60,000
Reduction of area	55–65 per cent.	45–55 per cent.
Elongation in 2 ins.	30–40 per cent.	Unimportant

Specification No. 10—20

.20 Carbon Steel

The following composition is desired:

Carbon	.15 to .25% (.20% desired)
Manganese	.50 to .80% (.65% desired)
Phosphorus, not to exceed	.04%
Sulphur, not to exceed	.04%

This steel is known to the trade as .20 carbon, open-hearth steel, and often as machine steel.

This quality is intended primarily for case-hardening. It forges well and machines well, but should not be considered as screw-machine stock. It may therefore be used for a very large variety of forged, machined and case-hardened parts of an automobile where strength is not paramount.

Steel of this quality may also be drawn into tubes and rolled into cold-rolled forms, and, as a matter of fact, makes a better frame than specification 10—10, because of the slightly higher carbon and resulting strength. The increased carbon content has no detrimental effect as far as usage is concerned, and it is only the most difficult of cold forming operations that cause it to crack during the forming. For automobile parts it may be safely used interchangeably with specification 10—10, as far as cold pressed shapes are concerned.

Heat treatment of this steel produces but little change as far as strength is concerned, but does cause a desirable refinement of grain after forging, and the toughness is materially increased. A simple quenching operation from about 1500 deg. Fahr. in oil, is all that is necessary. Treatment will often help the machining qualities.

Case-hardening is the most important treatment for this quality of steel. The character of the operation must depend upon the im-

portance of the part to be treated and upon the shape and size. There is a certain group of parts in an automobile which are not called upon to carry much load or withstand any shock. The only requirement is hardness. Such parts are fairly illustrated by screws and by rod-end pins. The simplest form of case-hardening will suffice, viz., heat treatment *A* below.

Another class of parts demands the best treatment (heat treatment *B*), such as gears, steering-wheel pivot-pins, cam-rollers, push-rods and many similar details of an automobile which the manufacturer learns by experience must be not only hard on the exterior surface but possess strength as well. The desired treatment is one which first refines and strengthens the interior and uncarbonized metal. This is then followed by a treatment which refines the exterior, carbonized, or high carbon, metal.

In the case of very important parts, the last drawing operation should be continued from one to three hours, to insure the full benefit of the operation.

The objects of drawing are two-fold: First, and not least important, is the relieving of all internal strains produced by quenching; second, a decrease in hardness, which is sometimes desirable. The hardness begins to decrease very materially from 350 deg. Fahr. up, and the operation must be controlled as dictated by experience with any given part.

There are certain very important pieces that demand all of these operations, but the last drawing operation may be omitted with a large number. Experience teaches what degree of hardness and toughness combined is necessary for any given part. It is impossible to lay down a general rule covering all different uses. If the fundamental principle is well understood, there should be no trouble in developing the treatment to a proper degree.

Following the foregoing treatment, a fractured part should show a fine-grained exterior, without any appearance of shiny crystals. The smaller the crystals the better. The interior may show a silky, fibrous condition or a fine crystalline; but it must not show a coarse, shiny, crystalline condition.

Physical Characteristics

	Annealed	Cold rolled or cold drawn	Heat treatment *C* or *D*
Elastic limit, lbs. per sq. in.	30,000 to 40,000	40,000 to 75,000[2]	40,000[1] to 75,000
Reduction of area......	45–60%	30–35%	15–60%
Elongation in 2 ins.....	25–35%	Unimportant	15–35%

There is little use in giving the physical characteristics of a carbonized steel, inasmuch as any test must be deceptive because of the very high carbon exterior case which cracks and fails long before the soft and tough interior does. This means that the rupture is fragmental and progressive, and misleading.

Specification No. 10—30

.30 Carbon Steel

The following composition is desired:

Carbon...........................	.25 to .35% (.30% desired)
Manganese........................	.50 to .80% (.65% desired)
Phosphorus, not to exceed..........	.04%
Sulphur, not to exceed.............	.04%

[1] The elastic limit is under control in two ways—by choice of quenching medium (oil, water or brine) and by varying the final drawing temperature. In the interpretation of the physical characteristic figures, it must be remembered that only the minimum figures as to toughness (*i.e.*,) reduction and elongation) may be expected with the highest degree of strength (*i.e.*, elastic limit); and, conversely, that the highest degree of toughness may be expected with the lowest elastic limit. This remark applies to all heat treated steels. It would be manifestly impossible to obtain the highest percentage of elongation and the highest elastic limit on the same specimen.

[2] In sections not over ½ in. round or ½ in. sheets or flats.

This material is sometimes referred to in the trade as .30 carbon machine steel.

It is primarily for use as a structural steel. It forges well, machines well and responds to heat treatment in the matter of strength as well as toughness; that is to say, intelligent heat treatment will produce marked increase in the elastic limit. It may be used for all forgings such as axles, driving-shafts, steering pivots and other structural parts. It is the best all-around structural steel for such use as its strength warrants.

Heat treatment for toughening and strength is of importance with this steel. The heat treatment must be modified in accordance with the experience of the individual user, to suit the size of the part treated and the combination of strength and toughness desired. The steel should be heat treated in all cases where reliability is important.

Machining may precede the heat treatment, depending somewhat upon convenience and the character of the treatment. If the highest strength is demanded, a strong quenching medium must be employed; for example, brine. In such case, the elastic limit will be correspondingly high and the steel correspondingly hard and difficult to machine. On the other hand, if a moderately high elastic limit is all that is desired, an oil quench will suffice and machining may follow without any difficulty whatever.

Heat treatment *C* below is the simplest form of heat treatment. The drawing operation (No. 3) must be varied to suit each individual case. If great toughness and little increased strength are desired, the higher drawing temperatures may be used, that is in the neighborhood of 1100 deg. Fahr. to 1200 deg. Fahr. If much strength is desired and little toughness, the lower temperatures are available. Even the lowest of the temperatures given will produce a quality of steel, after oil quenching, that is very tough—sufficiently tough for many important parts. In fact, with some parts the drawing operation (No. 3) may be entirely omitted.

Results better than obtainable with the sequence of operations of heat treatment *C* may be obtained by a so-called double treatment *D*, which produces a refinement of grain not possible with one treatment and is resorted to in parts where extremely good qualities are desired.

This quality of steel is not intended for case-hardening, but by careful treatment it may be safely case-hardened. This should be in emergencies only, rather than as a regular practice and, if at all, only with the double treatment followed by the drawing operation; that is, the most painstaking form of case-hardening.

Physical Characteristics

	Annealed	Cold rolled or cold drawn	Heat treatment *C* or *D*
Elastic limit, lbs. per sq. in.	35,000 to 45,000	Not usually worked in this manner, except for wire.	40,000 to 80,000
Reduction of area......	40–55%	....	30–60%
Elongation in 2 ins.....	20–30%	....	10–30%

Specification No. 10—40

.40 Carbon Steel

The following composition is desired:

Carbon...........................	.35 to .45% (.40% desired)
Manganese........................	.50 to .80% (.65% desired)
Phosphorus, not to exceed..........	.04%
Sulphur, not to exceed.............	.04%

This material is ordinarily known to the trade as .40 carbon machine steel. It represents a structural steel of greater strength

than specification 10—30. Its uses are more limited and are confined in a general way to such parts as demand a high degree of strength and a considerable degree of toughness. At the same time, with proper heat treatment the fatigue-resisting (endurance) qualities are very high—higher than with any of the foregoing specifications.

This steel is commonly used for crankshafts, driving shafts and propeller-shafts. It has also been used for transmission gears, but it is not quite hard enough without case-hardening and is not tough enough with case-hardening to make safe transmission gears. It should not be used for case-hardened parts, except in an emergency. Other specifications are decidedly better for this purpose. In a properly annealed condition it machines well, but not well enough for screw-machine work; but certainly well enough for all-around machine-shop practice. The best heat treatment for this quality of steel for crankshafts and similar parts is heat treatment *E* below.

Physical Characteristics

	Annealed	Heat treatment *E*
Elastic limit, lbs. per sq. in.	40,000 to 50,000	45,000 to 100,000
Reduction of area	40–50%	25–55%
Elongation in 2 ins.	20–25%	5–25%

SPECIFICATION No. 10—50

.50 Carbon Steel

The following composition is desired:

Carbon	.45 to .55%	(.50% desired)
Manganese	.50 to .80%	(.65% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	

This specification differs very little from specification 10—40. Owing to its higher carbon content it is somewhat harder to machine, but not seriously. It is also somewhat stronger. It can be used for gears with a little better results than the preceding specification. The same form of heat treatment may be used, with suitable modifications to fit individual cases.

Physical Characteristics

	Annealed	Heat treatment *E*
Elastic limit, lbs. per sq. in.	45,000 to 60,000	50,000 to 110,000
Reduction of area	30–40 per cent.	15–50 per cent.
Elongation in 2 ins.	15–20 per cent.	5–20 per cent.

SPECIFICATION No. 10—80

.80 Carbon Steel

The following composition is desired:

Carbon	.75 to .90%	(.80% desired)
Manganese	.25 to .50%	(.35% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	

This quality is ordinarily known to the trade as spring steel. Its use generally is for springs of light section. The hardening and drawing of springs, that is, the heat treatment of them, is, as a rule, in the hands of the springmaker, but in case it is desired to treat, as for small springs, heat treatment *F* below is recommened. It must be understood that the higher the drawing temperature (operation 3), the lower will be the elastic limit of the material. On the other hand, if the material be drawn at too low a temperature, it will be brittle. A few practical trials will locate the best temper for any given shape or size.

The physical characteristics of heat-treated spring steel are best determined by transverse test. This is because steel as hard as tempered spring steel is very difficult to hold firmly in the jaws of a tensile testing machine. There is more or less slip, and side strains are bound to occur, all of which tend to produce misleading results.

The physical characteristics in the annealed condition may be omitted, inasmuch as this grade of steel is not ordinarily used for structural parts in such condition. Careful examination of the fracture of the treated material is desirable. After tempering no suitable spring steel should be coarsely crystalline. It should be finely crystalline, and in some cases should show a partly fibrous fracture.

Physical Characteristics

(Transverse Test)

	Heat treatment *F*
Elastic limits, lbs. per sq. in.	90,000 to 160,000
Reduction of area	Not determined in transverse test.
Elongation	Not determined in transverse test.

SPECIFICATION No. 10—95

.95 Carbon Steel

The following composition is desired:

Carbon	.90 to 1.05%	(.95% desired)
Manganese	.25 to .50%	(.35% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	

This is a grade of steel used generally for springs. Properly heat treated, extremely good results are possible. Substantially the same remarks apply to this quality of steel as to specification 10—80. It is possible that the quenching temperature (operation 1, heat treatment *F*) of the heat treatment may be lowered slightly because of the increase in carbon, and it is also probable that the drawing temperature (operation 3) will not be the same.

Physical Characteristics

The physical characteristics of a tempered spring of this quality are substantially those of specification 10—80, possibly a little higher. The thought will naturally arise as to what is to be gained by the use of this material. The answer is that this steel will possess a finer grain and endure longer, providing the treatment is suitable; also, that with thicker and heavier metal the treatment will penetrate deeper because of the increased carbon content. It is for this reason that this quality of steel should be used for the heavier types of springs.

The same remarks as made in regard to tests and inspection in connection with specification 10—80 apply to this steel.

Screw Stock

SPECIFICATION No. 11—14

The following composition is desired:

Carbon	.08 to .20%
Manganese	.30 to .80%
Phosphorus, not to exceed	.12%
Sulphur	.06 to .12%

This steel may be made by any process. It is intended for use where high screw-machine production is the important factor, strength and toughness being a secondary consideration.

Steel Castings

SPECIFICATION No. 12—35

The following composition is desired:

Carbon	.30 to .40%	(.35% desired)
Manganese	.50 to .80%	(.65% desired)
Phosphorus, not to exceed	.05%	
Sulphur, not to exceed	.05%	
Silicon	.10 to .30%	

Steel castings may be made by any approved process. Genuine steel castings, and not malleable iron or complex mixtures often found in the market masquerading under the name of steel, are referred to. Genuine steel castings should be annealed and may be heat treated to great advantage. A steel casting of the composition given in the specification should be so tough as to bend to a considerable angle before breaking. The elastic limit of such a casting in an annealed condition is in the neighborhood of 35,000 lbs. per sq. in.

Like other castings, steel castings are subject to blow-holes. Consequently, they should not be used in the vital parts of an automobile. It is impossible to inspect against blow-holes, and steel castings for axles, crank-shafts and steering-spindles are used only at great risk. The specification has been prepared with the idea of furnishing a fair commercial analysis. Freedom from blow-holes and proper physical condition are of more importance than the absolute analysis.

Nickel Steels

In connection with the purchase and use of alloy steels it should be borne in mind that such steels should be used in the heat-treated condition only, that is, not in an annealed or natural condition. In the latter condition there is a slight benefit, perhaps, as compared with plain carbon steels, but as a rule nothing commensurate with the increased cost. In the heat-treated condition, however, there is a very marked improvement in physical characteristics.

SPECIFICATION No. 23—15

.15 Carbon, 3½ Per cent. Nickel Steel

The following composition is desired:

Carbon	.10 to .20%	(.15% desired)
Manganese	.50 to .80%	(.65% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Nickel	3.25 to 3.75%	(3.50% desired)

This quality of steel is embraced in these specifications to furnish a nickel steel that is suitable for carbonizing purposes. Steel of this character, properly carbonized and heat treated, will produce a part with an exceedingly tough and strong core, coupled with the desired high carbon exterior.

This steel is also available for structural purposes, but is not one to be selected for such purpose when ordering materials. Much better results will be obtained with one of the other nickel steels of higher carbon. It is intended for case-hardening gears, for both the bevel driving and transmission systems, and for such other case-hardened parts as demand a very tough, strong steel with a hardened exterior.

The case-hardening sequence may be varied considerably, as with specification 10—20, those parts of relatively small importance requiring a simpler form of treatment. As a rule, however, those parts which require the use of nickel steel require the best type of case-hardening, viz: heat treatment *G* below.

The second quench (operation 6) must be conducted at the lowest possible temperature at which the material will harden. It will be found that sometimes this is lower than 1300 deg. Fahr. In connection with certain uses it will be found possible to omit the final drawing (operation 7) entirely, but for parts of the highest importance this operation should be followed as a safeguard. Parts of intricate shape, with sudden changes of thickness, sharp corners and the like, should always be drawn, in order to relieve the internal strains.

Physical Characteristics

Much is to be learned from the character of the fracture. The center should be fibrous in appearance, and the exterior, high carbon metal closely crystalline, or even silky. When used for structural purposes, the physical characteristics will range about as follows:

	Annealed	Heat treatment *H* or *K*
Elastic limit, lbs. per sq. in.	35,000 to 45,000	40,000 to 80,000
Reduction of area	45–65 per cent.	40–65 per cent.
Elongation in 2 ins.	25–35 per cent.	15–35 per cent.

SPECIFICATION No. 23—20

.20 Carbon, 3½ Per cent. Nickel Steel

The following composition is desired:

Carbon	.15 to .25%	(.20% desired)
Manganese	.50 to .80%	(.65% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Nickel	3.25 to 3.75%	(3.50% desired)

This quality may be used interchangeably with specification No. 23—15. Although intended primarily for case-hardening, it may be properly used for structural parts, with suitable heat treatment, and will give elastic limits somewhat higher than material provided by the preceding specification. For case-hardening heat treatment *G* should be followed, and for structural purposes the treatment should be in accordance with heat treatment *H* or *K*; the quenching temperatures, as with other specifications, being modified to meet individual cases.

Physical Characteristics

	Annealed	Heat treatment *H* or *K*
Elastic limit, lbs. per sq. in.	40,000 to 50,000	50,000 to 125,000
Reduction of area	40–65 per cent.	40–65 per cent.
Elongation in 2 ins.	20–30 per cent.	10–25 per cent.

SPECIFICATION No. 23—25

.25 Carbon, 3½ Per cent. Nickel Steel

The following composition is desired:

Carbon	.20 to .30%	(.25% desired)
Manganese	.50 to .80%	(.65% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Nickel	3.25 to 3.75%	(3.50% desired)

This is a quality of steel that may be case-hardened successfully. Suitable treatment (*G*) gives a product that is satisfactory for gears, whether of the transmission or rear axle bevel type. The treatment after carbonizing must be slightly modified to meet the increase in carbon content. This is also a useful quality of steel for many structural parts, its response to heat treatment (either *H* or *K*) being most satisfactory.

Physical Characteristics

The physical characteristics of this steel may be considered as practically those obtained with specification 23—20, slight modifications in the treatment much more than offsetting the slight difference in the carbon content.

	Annealed	Heat treatment *H* or *K*
Elastic limit, lbs. per sq. in.	40,000 to 50,000	60,000 to 130,000
Reduction of area	40–60 per cent.	30–60 per cent.
Elongation in 2 ins.	20–30 per cent.	10–25 per cent.

SPECIFICATION No. 23—30

.30 Carbon, 3½ Per cent. Nickel Steel

The following composition is desired:

Carbon	.25 to .35%	(.30% desired)
Manganese	.50 to .80%	(.65% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Nickel	3.25 to 3.75%	(3.50% desired)

This quality of steel is primarily for heat-treated structural parts where strength and toughness are sought; such parts as axles, front-wheel spindles, crank-shafts, driving-shafts and transmission shafts. Wide variations as to elastic limit are possible by the use of different quenching mediums—oil, water or brine—and variation in drawing temperatures, from 500 deg. Fahr. up to 1200 deg. Fahr. The form of treatment is heat treatment *H*, a higher refinement of which is heat treatment *K*.

This material may be case-hardened, but is rather high carbon for the practice of the average case-hardening department. The lower ranges of carbon—in the neighborhood of .25—are satisfactory, but the upper ranges—in the neighborhood of .35—approach the danger point, and steel of the latter carbon content must be correspondingly carefully handled.

Physical Characteristics

	Annealed	Heat treatment *H* or *K*
Elastic limit, lbs. per sq. in.	45,000 to 55,000	65,000 to 150,000
Reduction of area	35–55 per cent.	25–55 per cent.
Elongation in 2 ins.	15–25 per cent.	10–25 per cent.

SPECIFICATION No. 23—35

.35 Carbon, 3½ Per cent. Nickel Steel

The following composition is desired:

Carbon	.30 to .40%	(.35% desired)
Manganese	.50 to .80%	(.65% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Nickel	3.25 to 3.75%	(3.50% desired)

This quality of steel is subject to precisely the same remarks as specification 23—30. It will respond a little more sharply to heat treatment and can be forced to higher elastic limits. The difference will be small except in extreme cases.

Physical Characteristics

	Annealed	Heat treatment *H* or *K*
Elastic limit, lbs. per sq. in.	45,000 to 55,000	65,000 to 160,000
Reduction of area	35–55 per cent.	25–55 per cent.
Elongation in 2 ins.	15–25 per cent.	10–25 per cent.

SPECIFICATION No. 23—40

.40 Carbon 3½ Per cent. Nickel Steel

The following composition is desired:

Carbon	.35 to .45%	(.40% desired)
Manganese	.50 to .80%	(.65% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Nickel	3.25 to 3.75%	(3.50% desired)

SPECIFICATION No. 23—45

.45 Carbon, 3½ Per cent. Nickel Steel

The following composition is desired:

Carbon	.40 to .50%	(.45% desired)
Manganese	.50 to .80%	(.65% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Nickel	3.25 to 3.75%	(3.50% desired)

Physical Characteristics

	Annealed	Heat treatment *H* or *K*
Elastic limit, lbs. per sq. in.	55,000 to 70,000	70,000 to 200,000
Reduction of area	30–50 per cent.	15–55 per cent.
Elongation in 2 ins.	15–25 per cent.	5–20 per cent.

SPECIFICATION No. 23—50

.50 Carbon, 3½ Per cent. Nickel Steel

The following composition is desired:

Carbon	.45 to .55%	(.50% desired)
Manganese	.50 to .80%	(.65% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Nickel	3.25 to 3.75%	(3.50% desired)

The above nickel steels, specifications 23—40, 23—45 and 23—50, are qualities not in wide use but available for certain purposes. The carbon contents being higher than generally used, greater hardness is obtainable by quenching; and as increased brittleness accompanies the greater hardness, the treatments given must be modified to meet such condition. For example, the final quench may be at a considerably lower temperature, and the final drawing temperature, or partial annealing, must be carefully chosen, in order to produce the desired toughness and other physical characteristics.

The strength of these steels, as with specifications 23—25, 23—30 and 23—35, depends upon the treatment and may be controlled closely over a wide range. The degree of brittleness must be carefully watched and guarded against. Proper treatment will yield very strong and tough steel; not as tough as specifications 23—25, 23—30 and 23—25, but nevertheless tough enough to fill a number of needs.

Nickel Chromium Steels

These classes of alloy steel are important. There are three types in common use. The difference between these types consists in the amount of alloying elements present. The types may be classified as low nickel-, medium nickel-, and high nickel-chromium steels, viz., class 31—, 32— and 33—, as given in the foregoing specifications. In general it may be said that the heat treatments and the properties induced thereby are much the same as in the case of plain nickel steels, except that the effects of the heat treatments are somewhat augmented by the presence of the chromium, and further that these effects increase in these three types with increasing amounts of nickel and chromium.

SPECIFICATION No. 31—15

.15 Carbon, Low Nickel Chromium Steel

The following composition is desired:

Carbon	.10 to .20%	(.15% desired)
Manganese	.50 to .80%	(.65% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Nickel	1.00 to 1.50%	
Chromium	.30 to .75%	

SPECIFICATION No. 31—20

.20 Carbon, Low Nickel Chromium Steel

The following composition is desired:

Carbon	.15 to .25%	(.20% desired)
Manganese	.50 to .80%	(.65% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Nickel	1.00 to 1.50%	
Chromium	.30 to .75%	

SPECIFICATION No. 31—25

.25 Carbon, Low Nickel Chromium Steel

The following composition is desired:

Carbon	.20 to .30%	(.25% desired)
Manganese	.50 to .80%	(.65% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Nickel	1.00 to 1.50%	
Chromium	.30 to .75%	

SPECIFICATION No. 31—30

.30 Carbon, Low Nickel Chromium Steel

The following composition is desired:

Carbon	.25 to .35%	(.30% desired)
Manganese	.50 to .80%	(.65% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Nickel	1.00 to 1.50%	
Chromium	.30 to .75%	

SPECIFICATION No. 31—35

.35 Carbon, Low Nickel Chromium Steel

The following composition is desired:

Carbon	.30 to .40%	(.35% desired)
Manganese	.50 to .80%	(.65% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Nickel	1.00 to 1.50%	
Chromium	.30 to .75%	

SPECIFICATION No. 31—40

.40 Carbon, Low Nickel Chromium Steel

The following composition is desired:

Carbon	.35 to .45%	(.40% desired)
Manganese	.50 to .80%	(.65% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Nickel	1.00 to 1.50%	
Chromium	.30 to .75%	

SPECIFICATION No. 31—45

.45 Carbon, Low Nickel Chromium Steel

The following composition is desired:

Carbon	.40 to .50%	(.45% desired)
Manganese	.50 to .80%	(.65% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Nickel	1.00 to 1.50%	
Chromium	.30 to .75%	

SPECIFICATION No. 31—50

.50 Carbon, Low Nickel Chromium Steel

The following composition is desired:

Carbon	.45 to .55%	(.50% desired)
Manganese	.50 to .80%	(.65% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Nickel	1.00 to 1.50%	
Chromium	.30 to .75%	

Substantially the same remarks apply to these various types as apply to nickel steels. In other words, the carbon content may be varied from the lowest to the highest, depending upon the physical qualities sought. The physical characteristics obtainable will vary with the carbon, and the heat treatment must be chosen accordingly.

Specifications 31—15 and 31—20 (.15 and .20 carbon) are intended primarily for case-hardening (heat treatment *G*). These steels ought not be used in the natural condition, but if desired may be used for structural purposes, in which case heat treatments *H* and *K* are recommended.

Specifications 31—25, 31—30, 31—35, 31—40 (.25 to .40 carbon) are intended primarily for structural purposes in a heat-treated condition (heat treatments *H* and *K*). Specification 31—25 may be used for case-hardening, as also may specification 31—30 if necessary.

Specifications 31—45, 31—50 (.45 and .50 carbon) may be used for gears and other structural parts where a high degree of strength and hardness are demanded and where toughness is not of first importance. Heat treatment *K* is recommended for such parts, the final drawing (operation 5) being carried out at 250-550 deg. Fahr., or at such temperature as will give the desired physical characteristics.

Physical Characteristics

Specifications 31—15, 31—20

	Annealed	Heat treatment *H* or *K*
Elastic limit, lbs. per sq. in.	30,000 to 40,000	40,000 to 100,000
Reduction of area	40–55 per cent.	40–65 per cent.
Elongation in 2 ins.	25–35 per cent.	15–25 per cent.

Specifications 31—25, 31—30

	Annealed	Heat treatment *H* or *K*
Elastic limit, lbs. per sq. in.	40,000 to 55,000	50,000 to 125,000
Reduction of area	35–50 per cent.	25–55 per cent.
Elongation in 2 ins.	20–30 per cent.	10–25 per cent.

Specifications 31—35, 31—40

	Annealed	Heat treatment *H* or *K*
Elastic limit, lbs. per sq. in.	45,000 to 60,000	55,000 to 150,000
Reduction of area	30–45 per cent.	25–50 per cent.
Elongation in 2 ins.	15–25 per cent.	5–20 per cent.

Specifications 31—45, 31—50

	Annealed	Heat treatment *H* or *K*
Elastic limit, lbs. per sq. in.	55,000 to 70,000	60,000 to 175,000
Reduction of area	30–50 per cent.	20–45 per cent.
Elongation in 2 ins.	15–25 per cent.	5–15 per cent.

Medium Nickel Chromium Steels

It will be noted that this type of nickel chromium steel is of the same composition as the preceding type, except that it contains more nickel and more chromium. The physical characteristics are omitted for the reason that results will be obtained that are intermediate the low nickel chromium alloys and the high nickel chromium alloys.

Specification No. 32—15

.15 Carbon, Medium Nickel Chromium Steel

The following composition is desired:

Carbon	.10 to .20%	(.15% desired)
Manganese	.30 to .60%	(.45% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Nickel	1.50 to 2.00%	(1.75% desired)
Chromium	.75 to 1.25%	(.75% desired)

Specification No. 32—20

.20 Carbon, Medium Nickel Chromium Steel

The following composition is desired:

Carbon	.15 to .25%	(.20% desired)
Manganese	.30 to .60%	(.45% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Nickel	1.50 to 2.00%	(1.75% desired)
Chromium	.75 to 1.25%	(1.00% desired)

Specification No. 32—25

.25 Carbon, Medium Nickel Chromium Steel

The following composition is desired:

Carbon	.20 to .30%	(.25% desired)
Manganese	.30 to .60%	(.45% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Nickel	1.50 to 2.00%	(1.75% desired)
Chromium	.75 to 1.25%	(1.00% desired)

Specification No. 32—30

.30 Carbon, Medium Nickel Chromium Steel

The following composition is desired:

Carbon	.25 to .35%	(.30% desired)
Manganese	.30 to .60%	(.45% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Nickel	1.50 to 2.00%	(1.75% desired)
Chromium	.75 to 1.25%	(1.00% desired)

Specification No. 32—35

.35 Carbon, Medium Nickel Chromium Steel

The following composition is desired:

Carbon	.30 to .40%	(.35% desired)
Manganese	.30 to .60%	(.45% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Nickel	1.50 to 2.00%	(1.75% desired)
Chromium	.75 to 1.25%	(1.00% desired)

Specification No. 32—40

.40 Carbon, Medium Nickel Chromium Steel

The following composition is desired:

Carbon	.35 to .45%	(.40% desired)
Manganese	.30 to .60%	(.45% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Nickel	1.50 to 2.00%	(1.75% desired)
Chromium	.75 to 1.25%	(1.00% desired)

Specification No. 32—45

.45 Carbon, Medium Nickel Chromium Steel

The following composition is desired:

Carbon	.40 to .50%	(.45% desired)
Manganese	.30 to .60%	(.45% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Nickel	1.50 to 2.00%	(1.75% desired)
Chromium	.75 to 1.25%	(1.00% desired)

Specification No. 32—50

.50 Carbon, Medium Nickel Chromium Steel

The following composition is desired:

Carbon	.45 to .55%	(.50% desired)
Manganese	.30 to .60%	(.45% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Nickel	1.50 to 2.00%	(1.75% desired)
Chromium	.75 to 1.25%	(1.00% desired)

High Nickel Chromium Steels

These steels differ from the two preceding types (31— and 32—) in the fact that they contain still more nickel and chromium. Attention is called to the fact that there will be some difference noted between the high nickel chromium and the medium nickel chromium heat treatments. It will be found that the possible ranges of treatment will differ slightly; also that the resulting physical characteristics will vary correspondingly. Annealing before machining will be found necessary for all carbons. The higher percentages of alloy elements make machining in a natural condition difficult.

Specification No. 33—15

.15 Carbon, High Nickel Chromium Steel

The following composition is desired:

Carbon	.10 to .20%	(.15% desired)
Manganese	.30 to .60%	(.45% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Nickel	3.25 to 3.75%	(3.50% desired)
Chromium	1.25 to 1.75%	(1.50% desired)

This quality of steel is intended primarily for case-hardening (heat treatment *L*). It may also be used for structural purposes, but is not first choice for such purposes.

Specification No. 33—20

.20 Carbon, High Nickel Chromium Steel

The following composition is desired:

Carbon	.15% to .25%	(.20% desired)
Manganese	.30% to .60%	(.45% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Nickel	3.25% to 3.75%	(3.50% desired)
Chromium	1.25% to 1.75%	(1.50% desired)

This quality of steel is also intended primarily for case-hardened parts, and when so used demands the most careful treatment (heat treatment *L*). It may also be used for structural purposes, but, as with other alloys, there is little gain over carbon steel unless it be used in a properly heat-treated condition. The heat treatment is substantially the same sequence of operations as apply to other chrome nickel steels dealt with herein, with such modifications as may be determined by practical experiment.

Physical Characteristics

	Annealed	Heat treatment *M* or *P*
Elastic limits, lbs. per sq. in.	40,000 to 50,000	50,000 to 125,000
Reduction of area	45–60 per cent.	30–65 per cent.
Elongation in 2 ins.	20–25 per cent.	5–20 per cent.

SPECIFICATION No. 33—25

.25 Carbon, High Nickel Chromium Steel

The following composition is desired:

Carbon	.20 to .30%	(.25% desired)
Manganese	.30 to .60%	(.45% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Nickel	3.25 to 3.75%	(3.50% desired)
Chromium	1.25 to 1.75%	(1.50% desired)

This quality of chrome nickel steel is intermediate between that preferred for case-hardening (specification 33—20) and the next higher quality (specification 33—30) for heat-treated structural parts. With the case-hardening treatment (heat treatment *L*) there may be slight modifications necessary. When properly heat treated this steel will answer for many structural parts. Heat treatments *M* and *P* are recommended, *P* being a higher refinement of *M*.

Physical Characteristics

	Annealed	Heat treatment *M* or *P*
Elastic limit, lbs. per sq. in.	40,000 to 50,000	60,000 to 140,000
Reduction of area	45–60 per cent.	30–65 per cent.
Elongation in 2 ins.	20–25 per cent.	5–20 per cent.

SPECIFICATION No. 33—30

.30 Carbon, High Nickel Chromium Steel

The following composition is desired:

Carbon	.25 to .35%	(.30% desired)
Manganese	.30 to .60%	(.45% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Nickel	3.25 to 3.75%	(3.50% desired)
Chromium	1.25 to 1.75%	(1.50% desired)

This grade of nickel chromium steel is intended primarily for structural parts of the most important character; and should always be heat treated. It is suitable for crank-shafts, axles, spindles, drive-shafts, transmission shafts and, in fact, the most important structural parts of cars. The heat treatment recommended is the same as in the case of specification 33—25. This steel is not intended for case-hardening. If case-hardening be attempted, the highest degree of care must be exercised.

Physical Characteristics

	Annealed	Heat treatment *L* or *M*
Elastic limit, lbs. per sq. in.	45,000 to 55,000	60,000 to 175,000
Reduction of area	40–55 per cent.	30–60 per cent.
Elongation in 2 ins.	15–25 per cent.	5–30 per cent.

SPECIFICATION No. 33—35

.35 Carbon, High Nickel Chromium Steel

The following composition is desired:

Carbon	.30 to .40%	(.35% desired)
Manganese	.30 to .60%	(.45% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Nickel	3.25 to 3.75%	(3.50% desired)
Chromium	1.25 to 1.75%	(1.50% desired)

Substantially the same remarks apply to this specification as apply to specification 33—30. The carbon content is but five points higher, and the physical characteristics, either annealed or otherwise heat treated, will not differ materially from those given for specification 33—30, and are repeated below. This quality should not be used for case-hardening.

Physical Characterisiics

	Annealed	Heat treatment *M* or *P*
Elastic limit, lbs. per sq. in.	45,000 to 55,000	60,000 to 175,000
Reduction of area	40–55 per cent.	30–60 per cent.
Elongation in 2 ins.	15–25 per cent.	5–20 per cent.

SPECIFICATION No. 33—40

.40 Carbon, High Nickel Chromium Steel

The following composition is desired:

Carbon	.35 to .45%	(.40% desired)
Manganese	.30 to .60%	(.45% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Nickel	3.25 to 3.75%	(3.50% desired)
Chromium	1.25 to 1.75%	(1.50% desired)

This quality of steel is suitable for structural parts where unusual strength is demanded. Higher elastic limit is possible under a given treatment than with material like specifications 33—30 or 33—35. The toughness will not be quite as great, but this does not bar the material from uses where toughness is not the controlling factor and where strength is. Heat treatment *P* is recommended. This steel should be thoroughly annealed for machining. This quality of steel should not be case-hardened.

Physical Characteristics

	Annealed	Heat treatment *P*
Elastic limit, lbs. per sq. in.	50,000 to 60,000	65,000 to 200,000
Reduction of area	40–50 per cent.	20–50 per cent.
Elongation in 2 ins.	15–25 per cent.	2–15 per cent.

SPECIFICATION No. 33—45

.45 Carbon, High Nickel Chromium Steel

The following composition is desired:

Carbon	.40 to .50%	(.45% desired)
Manganese	.30 to .60%	(.45% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Nickel	3.25 to 3.75%	(3.50% desired)
Chromium	1.25 to 1.75%	(1.50% desired)

The use of this steel is largely for gears, where extreme strength and hardness are necessary. The carbon is sufficiently high to cause the material, in the presence of chromium and nickel, to become hard

enough to make a good gear when quenched, without case-hardening (carbonizing).

This steel is difficult to forge. During the forging operation it should be kept at a thoroughly plastic heat and not hammered or worked after dropping to ordinary forging temperatures as cracking is liable to follow. The steel also becomes so very hard as to forge with great difficulty. On the other hand, too high a temperature is not advisable, as the steel becomes red-short and breaks. In brief, the forging temperature limits are narrow and this steel must be reheated more frequently than any of the other steels dealt with in this report. To heat treat for gears, use heat treatment Q. This steel should be thoroughly annealed for machining—operations 1 and 2.

The final drawing operation must be conducted at a heat which will produce the proper degree of hardness. The desired Brinell hardness for a gear is between 430 and 470, the corresponding Shore hardness being from 75 to 85. This quality of steel should not be case-hardened.

Physical Characteristics

	Annealed	Heat treatment Q
Elastic limit, lbs. per sq. in.	50,000 to 60,000	150,000 to 250,000
Reduction of area	40–50 per cent.	15–25 per cent.
Elongation in 2 ins.	15–25 per cent.	2–15 per cent.

Nickel Chromium Vanadium Steels

Attention is called to the fact that there is already in use a new series of steels corresponding to the class of specifications 31— and 32—, but with the addition of vanadium (the proportions of the other elements remaining unchanged). The amount of vanadium to be specified should be "not less than .12 per cent. (.18 per cent. desired)." As in the case of the chromium vanadium steels of class 60—, herein specified, the effect of this vanadium content is to increase the elastic limit without appreciably reducing the ductility. The vanadium also increases the fatigue-resisting (endurance) qualities of the steels. The heat treatments and remarks as to application for this class of steels are essentially the same as for classes 31— and 32—. The physical characteristics obtained are very similar.

Chromium Steels

The use of this type of steel is restricted almost entirely to ball and roller bearings. The physical characteristic most desired is extreme hardness. As the automobile manufacturer rarely works this quality of steel, no farther remarks are given here.

Specification No. 51—95

.95 Carbon, 1 Per cent. Chromium Steel

The following composition is desired:

Carbon	.90 to 1.05%	(.95% desired)
Manganese	.20 to .45%	
Phosphorus, not to exceed	.03%	
Sulphur, not to exceed	.03%	
Chromium	.90 to 1.10%	(1.00% desired)

Specification No. 51—120

1.20 Carbon, 1 Per cent. Chromium Steel

The following composition is desired:

Carbon	1.10 to 1.30%	(1.20% desired)
Manganese	.20 to .45%	
Phosphorus, not to exceed	.03%	
Sulphur, not to exceed	.03%	
Chromium	.90 to 1.10%	(1.00% desired)

Specification No. 52—95

.95 Carbon, 1.20 Chromium Steel

The following composition is desired:

Carbon	.90 to 1.05%	(.95% desired)
Manganese	.20 to .45%	
Phosphorus, not to exceed	.03%	
Sulphur, not to exceed	.03%	
Chromium	1.10 to 1.30%	(1.20% desired)

Specification No. 52—120

1.20 Carbon, 1.20 Chromium Steel

The following composition is desired:

Carbon	1.10 to 1.30%	(1.20% desired)
Manganese	.20 to .45%	
Phosphorus, not to exceed	.03%	
Sulphur, not to exceed	.03%	
Chromium	1.10 to 1.30%	(1.20% desired)

Chromium Vanadium Steels

Specification No. 61—15

.15 Carbon, Chromium Vanadium Steel

The following composition is desired:

Carbon	.10 to .20%	(.15% desired)
Manganese	.50 to .80%	(.65% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Chromium	.70 to 1.10%	(.90% desired)
Vanadium, not less than	.12%	(.18% desired)

Chromium carbon steel containing small amounts of vanadium has found much usage in automobile parts, particularly springs, axles, driving-shafts and gears. It is used interchangeably with carbon steel, nickel steel and nickel chromium steel.

Specification 61—15 is provided in these specifications to furnish a quality that is highly suitable for carbonizing purposes, such as gears and case-hardened parts of importance. Properly treated parts of this quality will be found to possess an extremely high degree of strength and toughness. This steel is also available for structural purposes, but should not be selected for such purpose when ordering materials. Better results are obtainable from some of the other qualities to be mentioned. The treatment recommended for case-hardening is heat treatment S.

The high initial quenching temperature of this steel is noteworthy, that is, something a little over 1600 deg. Fahr. This feature is different from other steels referred to in this report and characteristic of chromium vanadium steel. The heat for second quench (operation 5) should be conducted at the lowest possible temperature that will harden the exterior carbonized surface. Practical experiment will develop the best temperature for local conditions in any hardening room.

Physical Characteristics

		Heat treatment T
Elastic limit, lbs. per sq. in.	35,000 to 45,000	50,000 to 90,000
Reduction of area	50–70 per cent.	40–70 per cent.
Elongation in 2 ins.	25–30 per cent.	10–25 per cent.

Specification No. 61—20

.20 Carbon, Chromium Vanadium Steel

The following composition is desired:

Carbon	.15 to .25%	(.20% desired)
Manganese	.50 to .80%	(.65% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Chromium	.70 to 1.10%	(.90% desired)
Vanadium, not less than	.12%	(.18% desired)

This quality is also primarily for case-hardening. It is used for the most important case-hardened parts; that is, case-hardened shafts, gears and the like. It may also be used in a heat-treated condition for structural purposes, but for such work some of the specifications following are to be preferred, particularly where higher strength is desired. The case-hardening treatment recommended is that covered by heat treatment *S*. For structural purpose heat treatment *T* is recommended.

Physical Characteristics

	Annealed	Heat treatment *T*
Elastic limit, lbs. per sq. in.	40,000 to 50,000	55,000 to 100,000
Reduction of area	50–65 per cent.	45–65 per cent.
Elongation in 2 ins.	20–30 per cent.	10–25 per cent.

Specification No. 61—25

.25 Carbon, Chromium Vanadium Steel

The following composition is desired:

Carbon	.20 to .30%	(.25% desired)
Manganese	.50 to .80%	(.65% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Chromium	.70 to 1.10%	(.90% desired)
Vanadium, not less than	.12%	(.18% desired)

The difference between this and the preceding specification is very slight and they may be used interchangeably for structural purposes. This steel may be case-hardened but it should not be first choice for such parts. The physical characteristics may be considered as practically the same as given for specification 61—20.

Physical Characteristics

	Annealed	Heat treatment *T*
Elastic limit, lbs. per sq. in.	40,000 to 50,000	55,000 to 100,000
Reduction of area	50–65 per cent.	45–65 per cent.
Elongation in 2 ins.	20–30 per cent.	10–25 per cent.

Specification No. 61—30

.30 Carbon, Chromium Vanadium Steel

The following composition is desired:

Carbon	.25 to .35%	(.30% desired)
Manganese	.50 to .80%	(.65% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Chromium	.70 to 1.10%	(.90% desired)
Vanadium, not less than	.12%	(.18% desired)

This quality of steel is intermediate in the carbon range and may be used interchangeably with specification 61—25 for structural purposes. It should not be used for case-hardening. When treated as recommended by heat treatment *T* it possesses a high degree of combined strength and toughness.

Physical Characteristics

	Annealed	Heat treatment *T*
Elastic limit, lbs. per sq. in.	45,000 to 55,000	60,000 to 150,000
Reduction of area	50–60 per cent.	25–55 per cent.
Elongation in 2 ins.	20–25 per cent.	5–15 per cent.

Specification No. 61—35

.35 Carbon, Chromium Vanadium Steel

The following composition is desired:

Carbon	.30 to .40%	(.35% desired)
Manganese	.50 to .80%	(.65% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Chromium	.70 to 1.10%	(.90% desired)
Vanadium, not less than	.12%	(.18% desired)

This specification provides a first-rate quality of steel for structural parts that are to be heat treated. The fatigue-resisting (endurance) qualities of this material are excellent, which, of course, is to be expected in view of the high degree of refinement of grain resulting from heat treatment.

Physical Characteristics

	Annealed	Heat treatment *T*
Elastic limit, lbs. per sq. in.	45,000 to 55,000	60,000 to 150,000
Reduction of area	50–60 per cent.	25–55 per cent.
Elongation in 2 ins.	20–25 per cent.	5–15 per cent.

Specification No. 61—40

.40 Carbon, Chromium Vanadium Steel

The following composition is desired:

Carbon	.35 to .45%	(.40% desired)
Manganese	.50 to .80%	(.65% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Chromium	.70 to 1.10%	(.90% desired)
Vanadium, not less than	.12%	(.18% desired)

This is a very good quality of steel to be selected where a high degree of strength is desired, coupled with a good measure of toughness. Its fatigue-resisting qualities are very high, and it is a first-class material for high-duty shafts. Heat treatment *T* is recommended.

Physical Characteristics

	Annealed	Heat treatment *T*
Elastic limit, lbs. per sq. in.	50,000 to 60,000	65,000 to 175,000
Reduction of area	45–55 per cent.	15–50 per cent.
Elongation in 2 ins.	15–25 per cent.	2–15 per cent.

Specification No. 61—45

.45 Carbon, Chromium Vanadium Steel

The following composition is desired:

Carbon	.40 to .50%	(.45% desired)
Manganese	.50 to .80%	(.65% desired)
Phosphorus, not to exceed	.04%	
Sulphur, not to exceed	.04%	
Chromium	.70 to 1.10%	(.90% desired)
Vanadium, not less then	.12%	(.18% desired)

This quality of steel contains sufficient carbon in combination with chromium and vanadium to harden to a considerable degree when quenched at a proper temperature, and may be used for gears and springs. For structural parts where exceedingly high strength is desirable, heat treatment *T* should be followed. For gears this steel should be annealed after forging, this anneal to consist of operations 1 and 2 of heat treatment *U*.

The last drawing operation may be modified to obtain any desired hardness.

Physical Characterictics

	Annealed	Heat treatment U
Elastic limit, lbs. per sq. in.	55,000 to 65,000	150,000 to 200,000
Reduction of area	40–55 per cent.	10–25 per cent.
Elongation in 2 ins.	15–25 per cent.	2–10 per cent.

SPECIFICATION No. 61—50

.50 Carbon, Chromium Vanadium Steel

The following composition is desired:

Carbon	.45 to .55% (.50% desired)
Manganese	.50 to .80% (.65% desired)
Phosphorus, not to exceed	.04%
Sulphur, not to exceed	.04%
Chromium	.70 to 1.10% (.90% desired)
Vanadium, not less than	.12% (.18% desired)

Substantially the same remarks as made in regard to specification 61—45 apply to this quality. In this grade, however, we also find a material that is suitable for springs. With a proper sequence of heating, quenching and drawing, very high elastic limits are obtained. For spring material heat treatment *U* is recommended, except that the last drawing (operation 5) will be carried farther—probably from 500–900 deg. Fahr. This final drawing temperature will have to vary with the section of material being handled, whether light spiral springs or heavy flat springs.

Physical Characteristics

	Annealed	Heat treatment U
Elastic limit, lbs. per sq. in.	60,000 to 70,000	150,000 to 225,000
Rduction of area	35–50 per cent.	15–35 per cent.
Elongation in 2 ins.	15–20 per cent.	2–10 per cent.

Silico-Manganese Steel

This steel stands in a class by itself. It has been more or less standardized by usage as a spring steel. It is also used somewhat for gears. All parts made of this steel should be heat treated.

SPECIFICATION No. 92—50

Silico-Manganese Steel

The following composition is desired:

Carbon	.45 to .55% (.50% desired)
Manganese	.50 to .80% (.65% desired)
Silicon	1.50 to 2.00% (1.75% desired)
Phosphorus, not to exceed	.04%
Sulphur, not to exceed	.04%

For structural parts the treatment should be heat treatment *V*.

Suitable temperatures for any given thickness of piece, and the character of the quenching medium must be determined experimentally. When used for springs this material may be treated as above, with proper modification as to drawing temperature, with the probability that 800 deg. Fahr. as a drawing temperature will give about the proper characteristics.

Physical Characteristics

	Annealed	Heat treatment V
Elastic limit, lbs. per sq. in.	55,000 to 65,000	60,000 to 180,000
Reduction of area	30–45 per cent.	10–40 per cent.
Elongation in 2 ins.	20–25 per cent.	5–20 per cent.

Valve Metals

These materials are high nickel valve metals. They do not respond to heat treatment. The best that can be done with them is to treat for the purpose of securing uniformity of condition, by annealing or quenching at ordinary temperatures (1500 deg. Fahr. or thereabouts). Change of strength or ductility cannot be expected to any commercial degree.

SPECIFICATION No. 296—

Valve Metal No. 1

This metal shall contain not less than 96 per cent. of nickel.

This material shall be malleable.

SPECIFICATION No. 230—

Valve Metal No. 2

This metal shall contain:

Carbon, not over	.50 per cent.
Manganese, not over	1.50 per cent.
Phosphorus, not over	.04 per cent.
Sulphur, not over	.04 per cent.
Nickel	28.00 to 35.00 per cent.

The remainder to be iron.

List of Heat Treatments

Heat Treatment A

After forging or machining—

1. Carbonize at a temperature between 1600° F. and 1750° F. (1650°–1700° F. desired).
2. Cool slowly or quench.
3. Reheat to 1450°–1500° F. and quench.

Heat Treatment B

After forging or machining—

1. Carbonize at a temperature between 1600° F. and 1750° F. (1650°—1700° F. desired).
2. Cool slowly in the carbonizing mixture.
3. Reheat to 1500°–1550° F.
4. Quench.
5. Reheat to 1400°–1450° F.
6. Quench.
7. Draw in hot oil at a temperature which may vary from 300°–450° F., depending upon the degree of hardness desired.

Heat Treatment C

After forging or machining—

1. Heat to 1475°–1525° F.
2. Quench.
3. Reheat to 600°–1200° F. and cool slowly.

Heat Treatment D

After forging or machining—

1. Heat to 1500°–1550° F.
2. Quench.
3. Reheat to 1400°–1450° F.
4. Quench.
5. Reheat to 600°–1200° F. and cool slowly.

Heat Treatment E

After forging or machining—

1. Heat to 1500°–1550° F.
2. Cool slowly.
3. Reheat to 1400°–1450° F.
4. Quench.
5. Reheat to 600°–1200° F. and cool slowly.

Heat Treatment F

After shaping or coiling—

1. Heat to 1425°–1475° F.
2. Quench in oil.
3. Reheat to 400°–800° F., in accordance with degree of temper desired, and cool slowly.

Heat Treatment G

After forging or machining—

1. Carbonize at a temperature between 1600° F. and 1750° F. (1650°–1700° F. desired).
2. Cool slowly in the carbonizing material.
3. Reheat to 1450°–1525° F.
4. Quench.
5. Reheat to 1300°–1400° F.
6. Quench.
7. Reheat to a temperature from 250°–500° F. (in accordance with the necessities of the case) and cool slowly.

Heat Treatment H

After forging or machining—

1. Heat to 1500°–1550° F.
2. Quench.
3. Reheat to 600°–1200° F. and cool slowly.

Heat Treatment K

After forging or machining—

1. Heat to 1500°–1550° F.
2. Quench.
3. Reheat to 1300°–1400° F.
4. Quench.
5. Heat to 600°–1200° F. and cool slowly.

Heat Treatment L

After forging or machining—

1. Carbonize at a temperature between 1600° F. and 1750° F. (1650–1700° F. desired).
2. Cool slowly in the carbonizing mixture.
3. Reheat to 1400°–1500° F.
4. Quench.
5. Reheat to 1300°–1400° F.
6. Quench.
7. Heat to 250°–500° F. and cool slowly.

Heat Treatment M

After forging or machining—

1. Heat to 1450°–1500° F.
2. Quench.
3. Reheat to a temperature between 500° F. and 1250° F. and cool slowly.

Heat Treatment P

After forging or machining—

1. Heat to 1450°–1500° F.
2. Quench.
3. Reheat to 1375°–1425° F.
4. Quench.
5. Reheat to a temperature between 500° F. and 1250° F. and cool slowly.

Heat Treatment O

After forging—

1. Reheat to 1475°–1525°. (Hold at this temperature one-half hour, to insure thorough heating.)
2. Cool slowly.
3. Reheat to 1450°–1500° F.
4. Quench.
5. Reheat to 250°–550° F. and cool slightly.

Heat Treatment S

After forging or machining—

1. Carbonize at a temperature between 1600° F. and 1750° F. (1650°–1700° F. desired).
2. Cool slowly in the carbonizing mixture.
3. Reheat to 1600°–1700° F.
4. Quench.
5. Reheat to 1475°–1550° F.
6. Quench.
7. Reheat to 250°–550° F. and cool slowly.

Heat Treatment T

After forging or machining—

1. Heat to 1600°–1700° F.
2. Quench.
3. Reheat to some temperature between 500° F. and 1300° F. cool slowly.

Heat Treatment U

After forging—

1. Heat to 1525°–1600° F. (Hold for about one-half hour.)
2. Cool slowly.
3. Reheat to 1650°–1700° F.
4. Quench.
5. Reheat to 350°–550° F. and cool slowly.

Heat Treatment V

After forging or machining—

1. Heat to 1650°–1750° F.
2. Quench.
3. Reheat to a temperature between 600° F. and 1400° F. and cool slowly.

For alloys for bearings see Index.

Copper-Tin-Zinc Alloys

The tensile strengths of copper-tin-zinc alloys are given in Fig. 1 from The Materials of Construction by Prof. J. B. Johnson. The location of any point within the triangle indicates the composition. Thus, point *a* stands for 40 per cent. copper, 20 per cent. zinc, and 40 per cent. tin. Again, the contour lines give the tensile strengths for the useful alloys. The composition and strength of copper-tin or copper-zinc alloys may in like manner be read from the sides of the triangle. As put by Professor Johnson, "So much depends on the purity of the ingredients and on the manipulation of the process of melting and casting, that this chart, or any similar record, must be taken as showing what *may* be obtained rather than what *will* be obtained from the use of these particular mixtures."

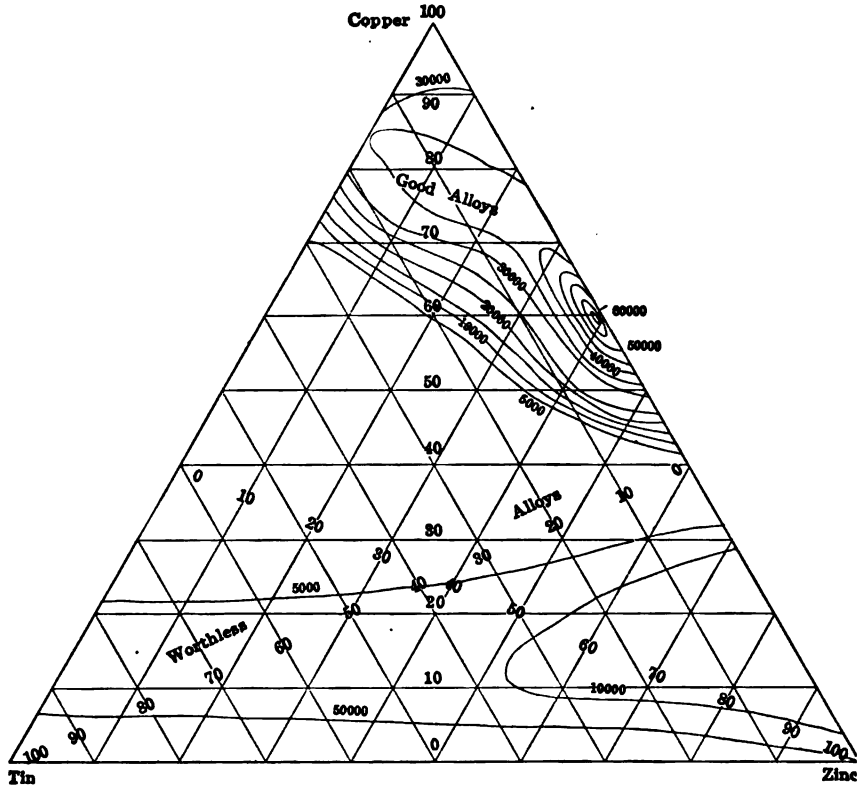


FIG. 1.—Composition and strength of copper-tin-zinc alloys.

The following data are from the report of the sheet metals division of the Standards Committee of the Society of Automobile Engineers, January, 1912. They are approximate and should be used as a guide only. If the figures are of particular interest to an engineer, a special inquiry should be sent to the mill manufacturing, giving size, temper, etc., with a request for tensile strength and elongation figures covering the particular requirements.

Standard Sheet Brass

SPECIFICATION No. 33

The following composition is desired:

Copper	64.00 to 67.00 per cent.
Zinc	33.00 to 36.00 per cent.
Lead not to exceed	.50 per cent.
Iron not to exceed	.10 per cent.

Sheet brass shall be furnished annealed or hard rolled. Annealed brass is to be designated as light annealed, or soft. Hard rolled brass shall be furnished in the following tempers, and the amount of reduction in thickness from the annealed sheet shall be as follows, expressed in Brown & Sharpe gages:

Temper.	B. & S. numbers
Quarter hard	1
Half hard	2
Hard	4
Extra hard	6
Spring	8

Thickness, (B. & S. gage)	Limits, ins.	Up to 5 ins. wide inclusive, ins.	Over 5 ins. wide to 8 ins. inclusive, ins.	Over 8 ins. wide to 11 ins. inclusive, ins.	Over 11 ins. wide to 14 ins. inclusive, ins.
No. 0000 to No. 0 inc.	(.4600–.3248)	±.0044	±.0048	±.0051	±.0055
Below 0 to No. 4 inc.	(.3248–.2043)	±.0039	±.0043	±.0046	±.0050
Below 4 to No. 8 inc.	(.2043–.1284)	±.0034	±.0038	±.0041	±.0045
Below 8 to No. 14 inc.	(.1284–.0640)	±.0029	±.0033	±.0036	±.0040
Below 14 to No. 18 inc.	(.0640–.0403)	±.0025	±.0029	±.0033	±.0037
Below 18 to No. 24 inc.	(.0403–.0201)	±.0020	±.0024	±.0028	±.0032
Below 24 to No. 28 inc.	(.0201–.0126)	±.0016	±.0020	±.0024	±.0028
Below 28 to No. 32 inc.	(.0126–.0079)	±.0013	±.0017	±.0020	±.0024
Below 32 to No. 35 inc.	(.0079–.0056)	±.0010	±.0014	±.0017	±.0022
Below 35 to No. 38 inc.	(.0056–.0039)	±.0008	±.0012	±.0015	±.0019

Standard sheet brass is for use in the manufacture of lamps, horns, flexible tubes, and ornamental work in general.—Tensile strength, hard, about 60,000 lbs. per sq. in.; elongation, about 5 per cent. in 2 ins. Tensile strength, soft, about 48,000 lbs. per sq. in.;

elongation, about 50 per cent. in 2 ins. Drawing brass and spinning brass are special qualities of brass for the operations indicated by the name.

Low Brass

Used on account of color, resistance to corrosion and atmospheric changes, and on account of superior ductility.

SPECIFICATION No. 34

The following composition is desired:

Copper	78.00 to 81.00	per cent.
Zinc	19.00 to 22.00	per cent.
Lead not to exceed	.20	per cent.
Iron not to exceed	.10	per cent.

Specifications for temper, gage variation, etc., shall be the same as for sheet brass. Tensile strength, hard, about 75,000 lbs. per sq. in.; elongation, about 5 per cent. in 2 ins. Tensile strength, soft, about 42,000 lbs. per sq. in.; elongation, about 50 per cent. in 2 ins.

Brazing Brass

SPECIFICATION No. 35

The following composition is desired:

Copper	74.00 to 76.00	per cent.
Zinc	24.00 to 26.00	per cent.
Lead not to exceed	.25	per cent.
Iron not to exceed	.10	per cent.

Specifications for temper, gauge variation, etc., shall be the same as for sheet brass. This material is used for parts where brazing or silver soldering is required. This material has about the same physical properties as low brass.

Free Cutting Brass

SPECIFICATION No. 36

The following composition is desired:

Copper	61.00 to 64.00	per cent.
Zinc	33.00 to 38.00	per cent.
Lead	1.25 to 2.00	per cent.
Iron not to exceed	.10	per cent.

This grade of material contains lead, which makes it free cutting and suitable for work on which machining is to be done. It does not bend or form readily, because of its "shortness." Specifications for temper, gage variation, etc., shall be the same as for sheet brass. It has a tensile strength when hard of about 75,000 lbs. per sq. in., with an elongation of about 3 per cent. in 2 ins. When soft, its tensile strength is about 50,000 lbs. per sq. in., with an elongation of about 35 per cent. in 2 ins.

Red Metal or Commercial Bronze

SPECIFICATION No. 37

The following composition is desired:

Copper	88.00 to 91.00	per cent.
Zinc	9.00 to 12.00	per cent.
Lead not to exceed	.20	per cent.
Iron not to exceed	.10	per cent.

Specifications for temper, gage, variation, etc., shall be the same as for sheet brass. This material has a rich gold color and is used for screen wires, radiators and in other places subject to corrosion. It is also used for ornamental parts where its color is desired. Its tensile strength, hard, is about 55,000 lbs per sq. in., with an elongation of about 5 per cent. in 2 ins. Soft, it has a tensile strength of about 37,000 lbs. per sq. in., and an elongation of about 40 per cent. in 2 ins.

Gilding Metal

SPECIFICATION No. 38

The following composition is desired:

Copper	94.00 to 96.00	per cent.
Zinc	4.00 to 6.00	per cent.
Lead not to exceed	.15	per cent.
Iron not to exceed	.06	per cent.

Specifications for temper, gage variation, etc., shall be the same as for sheet brass. This material is used for radiators. It has a tensile strength of about 45,000 to 55,000 lbs. per sq. in., with an elongation of about 5 per cent. in 2 ins. when hard. Annealed soft its tensile strength is about 35,000 lbs. per sq. in., with an elongation of about 35 per cent. in 2 ins.

Phosphor Bronze

Phosphor bronze is composed of copper, tin and phosphorus in proportions varied to suit the requirements of the trade. Specifications for temper, gage variation, etc., shall be the same as for sheet brass.

Copper Sheets and Strips

Copper sheets and strips shall be at least 99.50 per cent. pure, and shall be either soft or furnished with such roller temper as may be specified.

For Copper in Rolls.—Less than .060 in. thick, variation .002 in. under and .001 in. over gage; .060 in. and thicker, variation .003 in. under and .003 in. over gage.

For Copper in Sheets.—Up to and including 48 ins. wide the variation in thickness may be 5 per cent. under or over gage. Over 48 ins. in width, up to and including 60 ins. wide, the variation in thickness may be 7 per cent. under or over. Test specimens cut from soft copper sheet shall have a minimum tensile strength of 30,000 lbs. per sq. in., with an elongation of at least 25 per cent. in two (2) inches for gages not less than .030 in. thick.

German Silver

German silver in rolls and sheets is to be specified according to color and service required in the following standard grades: 5 per cent., 15 per cent., 18 per cent., 20 per cent., 25 per cent., 30 per cent. nickel, the balance being copper and zinc. It will be supplied soft or with such roller temper as may be required.

Brass Rods

For Cold Heading.—The material shall be suitable for cold working, such as the heading of rivets and the rolling of threads for screws.

SPECIFICATION No. 39

The following composition is desired:

Copper	61.50 to 64.50	per cent.
Zinc	35.50 to 38.50	per cent.
Lead	Not to exceed .50	per cent.
Iron	Not to exceed .10	per cent.

The temper shall be produced by annealing sufficiently to give the metal the softness required for heading. The material should be ordered for heading, and the order accompanied by a sample or drawing to show the mechanical operations required. This material has a tensile strength of about 35,000 to 40,000 lbs. per sq. in., with an elongation of about 50 per cent. in 2 ins.

Free-cutting Brass Rod.

Material suitable for automatic screw-machine work.

SPECIFICATION No. 40

The following composition is desired:

Copper	61.50 to 64.50	per cent.
Zinc	31.50 to 35.50	per cent.
Lead	2.25 to 3.50	per cent.
Iron	Not to exceed .10	per cent.

All free cutting brass rods shall be furnished hard drawn, unless otherwise specified for when ordered.

Rods shall not vary in diameter more than the amount specified in the following table:

Up to and including ½ in.,	.0015 over or under required diameter.
From ½ in. to and including 1 in.,	.002 over or under required diameter.
From 1 in. to and including 3 ins.,	.0025 over or under required diameter.

This material is suitable for automatic screw machine work. Its tensile strength is about 65,000 lbs. per sq. in., with about 15 per cent. elongation in 2 ins.

Tobin Bronze.

Turned and straightened rods for various purposes where strength and resistance to corrosion are required; also for hot forging. Rods up to and including 1 in. in diameter shall have a tensile strength of not less than 62,000 lbs. per sq. in. Rods larger than 1 in. and up to and including 7 ins. in diameter shall have a tensile strength of 60,000 lbs. per sq. in.

All rods not larger than 1 in. in diameter shall have an elongation of at least 25 per cent. in 2 ins. All rods larger than 1 in. in diameter shall have an elongation of at least 28 per cent. in 2 ins. The elastic limit, or the point at which rapid elongation begins, shall be at least 30,000 lbs. per sq. in. for all sizes.

Tubing

Tubing can be furnished in copper and the commercial alloys of copper and zinc, such as high brass, bronze, phosphor bronze, and Tobin bronze. The composition shall be as specified to meet the requirements of use. The temper of the tubing shall be as specified in the order, and may be hard, half hard or annealed. If annealed, the tubing may be soft, or light annealed.

The following variation on inside and outside diameter and the thickness of the walls shall be allowed on all commercial tubing:

Outside and Inside Dimensions

Up to ½ in. inclusive	.002 in. over or under
Over ½ in. to and including ¾ in.	.0025 in. over or under
Over ¾ in. to and including 1 in.	.003 in. over or under
Over 1 in. to and including 1¼ ins.	.0035 in. over or under
Over 1¼ ins. to and including 1½ ins.	.004 in. over or under
Over 1½ ins. to and including 1¾ ins.	.0045 in. over or under
Over 1¾ ins. to and including 2 ins.	.005 in. over or under
Over 2 ins.	¼ of 1 per cent. over or under

No combination of variations on the same tube shall make the thickness of the wall vary from the nominal by more than the following amounts:

Thickness of Wall

Up to and including $\frac{1}{32}$ in.	.001 in. over or under
Over $\frac{1}{32}$ in. to and including $\frac{1}{16}$ in.	.002 in. over or under
Over $\frac{1}{16}$ in. to and including $\frac{3}{32}$ in.	.003 in. over or under
Over $\frac{3}{32}$ in. to and including ⅛ in.	.005 in. over or under
Over ⅛ in. to and including ¼ in.	.008 in. over or under
Over ¼ in. to and including $\frac{5}{16}$ in.	.0125 in. over or under
Over $\frac{5}{16}$ in. to and including ⅜ in.	.015 in. over or under

On all stock where the above commercial variations are not permissible limits shall be specified in the order.

Brass Casting Metals

RED BRASS

Specification No. 27

Copper	85.00 per cent.
Tin	5.00 per cent.
Lead	5.00 per cent.
Zinc	5.00 per cent.

A tolerance of 1 per cent. plus or minus will be allowed in the above. Impurities of over .25 per cent. will not be permitted.

NOTE.—A high grade of composition metal, and an excellent bearing where speed and pressure are not excessive. Largely used for light castings, and possesses good machining qualities.

YELLOW BRASS

Specification No. 28

Copper	62.00 to 65.00 per cent.
Lead	2.00 to 4.00 per cent.
Zinc	36.00 to 31.00 per cent.

Total impurities in excess of .50 per cent. will not be permitted.

NOTE.—This alloy represents a high grade of yellow brass; is tough and possesses good machining qualities. Its use is suggested in preference to ordinary commercial yellow brass castings, which are, generally speaking, a miscellaneous assortment of mixtures, some of them containing considerable amounts of iron (from 1 to 3 per cent.). This is very undesirable, as it renders the castings liable to blow-holes, hard spots and, in some cases, small particles of metallic iron.

Cast Manganese Bronze

SPECIFICATION No. 29

Manganese bronze is understood to mean a metal constituted principally of copper and zinc in the approximate proportion of 60 to 40, iron being present in small and manganese in variable quantities. Main dependence will be placed upon physical specifications.

Tensile strength	60,000 lbs. per sq. in.
Yield point	30,000 lbs. per sq. in.
Elongation in 2 ins.	20 per cent.

NOTE.—Manganese bronze is of value for castings where strength and toughness are required. Specifications are not severe, being easily met by all makers of quality castings. Test coupons should be attached to castings made in the sand, the use of chills, special sand or artificial methods of cooling being prohibited. This precaution prevents the use of inferior metals.

Aluminum Alloys

No. 1

Specification No. 30

Aluminum, not less than	90.00 per cent.
Copper	8.50 to 7.00 per cent.

Total impurities shall not exceed 1.7 per cent. of which not over .2 shall be zinc. No other impurities than carbon, silicon, iron, manganese and zinc shall be allowed.

NOTE.—This is one of the lightest of the aluminum alloys, possessing a high degree of strength, and can be used where a tough, light alloy of these characteristics is required in automobile construction.

No. 2

Specifictaon No. 31

Aluminum, not less then	80.00 per cent.
Zinc, not over	15.00 per cent.
Copper, between	2.00 and 3.00 per cent.
Manganese, not to exceed	.40 per cent.

Total impurities shall not exceed 1.65 per cent., of which not more than .50 per cent. should be silicon, not more than 1.00 per cent. iron, and not more than .15 per cent. lead.

NOTE.—This mixture possesses strength, closeness of grain, and can be cast solid and free from blowholes. It is a light metal, its specific gravity being in the neighborhood of 3.00.

No. 3

Specification No. 32

Aluminum	65.00 per cent.
Zinc	35.00 per cent.

Total impurities in excess of 1.65 per cent. will not be permitted.

NOTE.—This is a mixture that can be used where cheap castings not to be subjected to any great strains are desired. It is a desirable mixture for flat plates, foot-boards, running boards, etc. It is quite brittle and will not equal in toughness or strength specifications 30 and 31.

On aluminum alloys the standard specimen of reference shall be the same as indicated for standard steel tensile test-specimen. Test piece shall be tested with the skin on. We recommend a test bar $\frac{1}{2}$ in. in diameter at the breaking section and filleted to a $\frac{3}{4}$ in. diameter threaded end. Fillet should extend for at least $\frac{3}{8}$ in. Test bar to be attached to casting, use of chills or artificial means of cooling being prohibited.

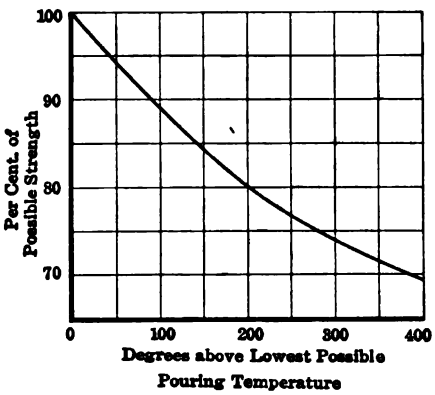


FIG. 2.—Effect of pouring temperature on the strength of aluminum castings.

Aluminum Alloys

The design of parts to be made of aluminum castings is subject to restrictions which are thus explained by H. W. GILLETT (*Society of Automobile Engineers* 1911):

Owing to certain physical properties of aluminum, such as its high contraction on cooling and its weakness when just solidified—that is, its hot shortness—aluminum castings require more careful design than almost any other casting metal.

In passing from the molten to the solid state, aluminum contracts a good deal; when a heavy and a thin section come next to each other, the thin place will freeze first. If the thin section is so situated as to lie between a heavy section and a gate or riser the supply of metal is thereby cut off from the molten mass in what is to be the heavy part of the casting. The contraction of freezing has to take place, and instead of taking place uniformly over this heavy part and maintaining the exact shape of the mold, it will often draw away from a corner and produce a shrink. We can induce the heavy portion to freeze more quickly by placing the chill in the mold at that point, but it is difficult to accomplish the end completely by this method; it greatly increases the time required to put up the mold and produces unsightly chill marks on the casting.

The ideal casting, therefore, is one of as nearly uniform section throughout as is practical, since that means that the whole casting solidifies at the same time, so that contraction is uniform.

On account of the hot shortness of aluminum the shrinkage strains set up when a heavy section joins a thin one often cause the metal to give away entirely at that point, and a crack appears. If it is inevitable that light and heavy sections come together, the cooling strain should be distributed by joining the sections by a smooth curve, that is, a liberal fillet.

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There is no one factor in foundary practice that more gravely affects the strength of the casting than the pouring temperature. The reason for this again, is, the speed of crystallization. The cooler the metal can be poured into the mold the more quickly it solidifies and the less time the crystals have to grow or arrange themselves, and the result is a mass of closely interlocking crystals forming a strong fine-grained material.

The effect of pouring temperature was well shown by a set of test bars, all of which were cast from the same pot of metal with exactly similar molds, the only variable being the pouring temperature. The average results obtained in this series of tests are given in Fig. 2, which shows that the lower the pouring temperature the stronger the casting.

This has a distinct bearing on design, since the lowest temperature at which a casting can be poured is that to which the thinnest section will just escape a misrun. If the casting is so designed that this crucial section compels hot pouring, all of the thicker parts will freeze too slowly and will be weaker than they should be. By slightly increasing the section of the thinnest parts, a casting can often be poured 100 deg. colder and the strength of the whole casting be increased at least 10 per cent. If the bulk of a casting is from $\frac{1}{4}$ to $\frac{1}{2}$ in. thick, one little part $\frac{1}{8}$ in. thick will give a resultant casting, on account of the high pouring temperature required, whose average strength is about 16,000 lbs. per sq. in. instead of 18,000 lbs. or over. The call for lightness has led many designers to overlook this vital point.

The great influence of the pouring temperature is the reason why separately cast test bars show only the quality of the ingot metal and nothing at all as to the strength of the corresponding casting, even though the test bar and casting may be poured from the same pot of metal. Aluminum test bars should be made on the castings. Were this stipulation not made the foundryman who wishes can pour the casting as hot as he pleases, allow his metal to cool way down and then pour separate test bars which will then show an utterly fictitious strength in comparison with the casting.

The general lack of attention to pouring temperatures, not only in commercial practice, but in most of the investigations on aluminum, vitiates many of the published data on aluminum alloys and accounts for a great many irregulatities and seeming contradictions in the results. In comparing the different aluminum alloys, really comparable results can only be obtained by pouring at the same number of degrees above the melting-point of the particular alloy in question in all cases, thus allowing the same time for crystallization and producing an analogous condition.

Core work always means trouble. It takes time to set cores in the mold correctly, and if a lot of small cores are used the danger of shifts is greatly increased. If, on the other hand, large cores are used, they must be made hard enough to allow handling them and setting them in the mold, which requires not only a solid core, but one reinforced by iron rods and wires. This makes them hard to crush, and on large cores inside of thin walls of metal, introduces danger of cracking. When we have a core completely surrounded by walls of metal it is a question whether the tensile strength of the metal as it solidifies is greater than the compressive strength of the core. Let the core be ever so slightly too hard and the casting is inevitably ruined.

If cores must be used, the core prints should be large and deep, so as to anchor the cores firmly without the use of chaplets to hold the cores in place, since it is impossible for the molten metal to fuse a chaplet into the body of the casting, without pouring at a temperature far above that necessary to give the greatest strength. When a job requires cores, the first question that should be asked by the patternmaker is if that pattern cannot be made so as to allow the use of green-sand core, or at least a green-sand half. Green sand will crush and give away when the casting contracts on cooling, where a hard, dry sand core will not crush and will crack the casting.

Casting Alloys Specified by the Bureau of Steam Engineering, U. S. Navy

Name	Composition by percentage						Purposes for which suitable	Tensile strength, minimum	Yield point, minimum	Elongation in 2 ins. (minimum) per cent.
	Copper	Tin	Zinc	Iron, maximum	Lead, maximum	Miscellaneous				
Commercial brass.....	64–68	......	32–34	2.0	3.0	....	Name and number plates; cases for instruments; oil cups; distributing boxes.			
Muntz metal..........	59–62	......	39–41	....	.6					
Brazing metal.........	84–86	......	Rem.	.06	.3	....	Brazing metal, and all flanges and fittings that are to be brazed.			
Gun bronze	87–89	9–11	1–3	.06	.2	....	All composition valves 4 ins. in diameter and above; expansion joints, flanged pipe fittings, gear wheels, bolts and nuts, miscellaneous brass castings, all parts where strength is required of brass castings or where subjected to salt water, and for all purposes where no other alloy is specified. Composition valves: Safety and relief, feed check and stop, surface blow, drain, air, and water cocks, main stop, throttle, reducing, sea, safety sluice, and manifolds at pumps. Condenser, Distiller, Feed-water heater, Oil cooler: Heads, shapes, and water chests. Pumps: Air-pump casing, valve seats, buckets, main circulating, water cylinders, valve boxes, water pistons stuffing boxes, followers, glands, in general the water end of pumps complete except as specified. Stuffiing boxes: Glands, bushings for iron or steel boxes. Blowers: Bearing boxes Journal boxes: Distance pieces Miscellaneous: Grease extractors; steam strainers, separators, casing for stern tube and propeller shafts, propeller hub caps. Bearings: Main, stern tube, strut and spring. Spring bearings: Glands and baffles Reciprocating engine: Intermediate and low pressure relief valves and casings, crosshead brasses, crank pin brasses, eccentric straps and distance pieces.	30,000	15,000	15
Journal bronze........	82–84	12.5–14.5	2.5–4.5	.06	1.0	....	Journal boxes, guide gibs, bushings, sleeves, slippers, etc. Reciprocating engine: Valve stem crosshead bottom brass; link block gibs, suspension link brasses.			
Manganese bronze.....	57–60	.75	37–40	1.0	....	Aluminum, 0.5; manganese 0.3.	Propeller hubs, blades, engine framing, and composition castings requiring great strength.	60,000	30,000	20
Cast naval brass.......	59–63	.5–1.5	Rem.	.06	.6	....	Valve handwheels, hand-rail fittings, ornamental and miscellaneous castings, and valves in water chests of condensers.			
Phosphor-bronze	80–90	6–8	Rem.	.06	.2	Phosphorus, .3.	Castings where strength and incorrodibility are required.	40,000	20,000	20
Screw pipe fittings, brass	77–80	4	13–19	.1	3.0	....	For composition screwed fittings.......			

Fusible Alloys

Alloys	Bismuth	Lead	Tin	Cadmium	Melting-point
					Deg. Fahr.
Newton's....	50.0	31.25	18.75	....	204
Rose's......	50.0	28.10	24.64	....	212
Darcet's....	50.0	25.00	25.00	....	200
Wood's.....	50.0	24.00	14.00	12.00	160
Lupowitz's..	50.0	27.00	13.00	10.00	140

WEIGHT OF MATERIALS

Table 1.—Specific Gravity and Weight of Metals

Material	Specific gravity	Weight in lbs. of one: Cu. ft.	Weight in lbs. of one: Cu. in.	Cu. ins. in one lb.
luminum—cast	2.569	160	.093	10.80
luminum—wrought	2.681	167	.097	10.35
luminum—bronze	7.787	485	.281	3.56
ntimony	6.712	418	.242	4.13
rsenic	5.748	358	.207	4.83
ismuth	9.827	612	.354	2.82
rass—cast: from	7.868	490	.284	3.53
to	8.430	525	.304	3.29
average	8.109	505	.292	3.42
rass—Muntz-metal	8.221	512	.296	3.37
rass—naval (rolled)	8.510	530	.307	3.26
rass—sheet	8.462	527	.305	3.28
rass—wire	8.558	533	.308	3.24
Bronze (gun-metal): from	8.478	528	.306	3.27
to	8.863	552	.319	3.13
average	8.735	544	.315	3.18
opper—cast	8.622	537	.311	3.22
opper—hammered	8.927	556	.322	3.11
opper—sheet	8.815	549	.318	3.15
opper—wire	8.895	554	.321	3.12
Gold (pure)	19.316	1203	.696	1.44
Gold standard 22 carat fine (Gold 11—Copper 1)	17.502	1090	.631	1.59
Iron—cast: from	6.904	430	.249	4.02
to	7.386	499	.266	3.76
average	7.209	464	.260	3.85
Iron—wrought: from	7.547	470	.272	3.56
to	7.803	486	.281	3.68
average	7.707	480	.278	3.60
Lead—cast	11.368	708	.410	2.44
Lead—sheet	11.432	712	.412	2.43
Manganese	8.012	499	.289	3.46
Nickel—cast	8.285	516	.299	3.35
Nickel—rolled	8.687	541	.313	3.19
Platinum	21.516	1340	.775	1.29
Silver	10.517	655	.379	2.64
Steel: from	7.820	487	.282	3.55
to	7.916	493	.285	3.51
average	7.868	490	.284	3.53
Tin	7.418	462	.267	3.74
White Metal (Babbitt's)	7.322	456	.264	3.79
Zinc—cast	6.872	428	.248	4.04
Zinc—sheet	7.209	449	.260	3.85

Table 2.—Specific Gravity and Weight of Wood

	Specific gravity	Average	Weight per cu. ft. lbs., average
Alder	.56 to .80	.68	42
Apple	.73 to .79	.76	47
Ash	.60 to .84	.72	45
Bamboo	.31 to .40	.35	22
Beech	.62 to .85	.73	46
Birch	.56 to .74	.65	41
Boe	.91 to 1.33	1.12	70
Cedar	.49 to .75	.62	39
Cherry	.61 to .72	.66	41
Chestnut	.46 to .66	.56	35
Cork	.24	.24	15
Cypress	.41 to .66	.53	33
Dogwood	.76	.76	47
Ebony	1.13 to 1.33	1.23	76
Elm	.55 to .78	.61	38
Fir	.48 to .70	.59	37
Gum	.84 to 1.00	.92	57
Hackmatack	.59	.59	37
Hemlock	.36 to .41	.38	24
Hickory	.69 to .94	.77	48
Holly	.76	.76	47
Hornbeam	.76	.76	47
Juniper	.56	.56	35
Larch	.56	.56	35
Lignum vitae	.65 to 1.33	1.00	62
Linden	.604	....	37
Locust	.728	....	46
Mahogany	.56 to 1.06	.81	51
Maple	.57 to .79	.68	42
Mulberry	.56 to .90	.73	46
Oak, Live	.96 to 1.26	1.11	69
Oak, White	.69 to .86	.77	48
Oak, Red	.73 to .75	.74	46
Pine, White	.35 to .55	.45	28
Pine, Yellow	.46 to .76	.61	38
Poplar	.38 to .58	.48	30
Spruce	.40 to .50	.45	28
Sycamore	.59 to .62	.60	37
Teak	.66 to .98	.82	51
Walnut	.50 to .67	.58	36
Willow	.49 to .59	.54	34

Table 3.—Weights of Iron, Brass, and Copper Wire
Birmingham or Stubbs gage

No. of gage	Dia. in in.	Weight in lbs. per 1000 linear ft.: Iron	Brass	Copper
0000	.454	546.21	589.29	623.2
000	.425	478.65	516.41	546.1
00	.380	382.66	412.84	436.6
0	.340	306.34	330.50	349.5
1	.300	238.50	257.31	272.1
2	.284	213.74	230.60	243.9
3	.259	177.77	191.79	202.8
4	.238	150.11	161.95	171.3
5	.220	128.26	138.37	146.3
6	.203	109.20	117.82	124.6
7	.180	85.86	92.63	97.96
8	.165	72.14	77.83	82.31
9	.148	58.05	62.62	66.23
10	.134	47.58	51.34	54.29
11	.120	38.16	41.17	43.54
12	.109	31.49	33.97	35.92
13	.095	23.92	25.80	27.29
14	.083	18.26	19.70	20.83
15	.072	13.73	14.82	15.67
16	.065	11.19	12.08	12.77
17	.058	8.92	9.62	10.17
18	.049	6.36	6.86	7.259
19	.042	4.67	5.04	5.333
20	.035	3.25	3.52	3.704
21	.032	2.71	2.93	3.096
22	.028	2.08	2.24	2.370
23	.025	1.66	1.79	1.890
24	.022	1.28	1.39	1.463
25	.020	1.06	1.14	1.209
26	.018	.863	.926	.979
27	.016	.680	.732	.774
28	.014	.529	.560	.592
29	.013	.438	.483	.511
30	.012	.382	.412	.435
31	.010	.266	.286	.302
32	.009	.212	.232	.244
33	.008	.167	.183	.193
34	.007	.133	.140	.148
35	.005	.066	.071	.075
36	.004	.052	.046	.048

TABLE 4.—WEIGHTS OF SEAMLESS BRASS TUBING PER LINEAR FOOT, LBS.
1/8 to 2 1/2 Outside Diameter. Nos. 1 to 25 Stubbs Iron Gage

No. of gage	Thickness in ins.	1/8	3/16	1/4	5/16	3/8	7/16	1/2	9/16	5/8	3/4	7/8	1	1 1/4	1 1/2	1 3/4	2	2 1/4	2 1/2
1	.300												2.42	3.28	4.10	5.03	5.88	6.75	7.62
2	.284												2.35	3.16	4.03	4.80	5.57	6.45	7.26
3	.259											1.85	2.22	2.98	3.72	4.48	5.23	5.96	6.72
4	.238											1.76	2.10	2.79	3.49	4.18	4.82	5.51	6.25
5	.220											1.68	1.99	2.60	3.26	3.89	4.53	5.14	5.80
6	.203									1.00	1.29	1.58	1.88	2.46	3.04	3.60	4.20	4.80	5.39
7	.180									.90	1.19	1.44	1.71	2.23	2.77	3.28	3.78	4.33	4.83
8	.165					.40	.52	.64	.77	.87	1.11	1.35	1.59	2.07	2.54	3.02	3.50	3.98	4.46
9	.148					.39	.49	.61	.71	.82	1.04	1.25	1.46	1.89	2.32	2.75	3.17	3.60	4.03
10	.134					.38	.46	.58	.65	.77	.96	1.16	1.35	1.74	2.12	2.51	2.90	3.28	3.67
11	.120			.18	.26	.36	.43	.53	.61	.70	.87	1.05	1.22	1.57	1.91	2.27	2.16	2.96	3.31
12	.109			.177	.256	.334	.413	.491	.570	.650	.808	.965	1.12	1.43	1.76	2.07	2.39	2.70	3.01
13	.095			.170	.237	.306	.377	.445	.514	.580	.717	.855	1.00	1.27	1.55	1.82	2.09	2.37	2.65
14	.083			.160	.220	.280	.340	.400	.460	.520	.640	.760	.88	1.12	1.36	1.61	1.84	2.08	2.32
15	.072		.096	.144	.201	.251	.303	.355	.409	.461	.564	.667	.77	.99	1.19	1.40	1.61	1.81	2.02
16	.065	.045	.092	.138	.186	.232	.279	.326	.372	.420	.515	.609	.70	.89	1.07	1.26	1.45	1.64	1.82
17	.058	.044	.087	.128	.169	.212	.255	.295	.338	.380	.463	.548	.64	.80	.97	1.14	1.30	1.48	1.64
18	.049	.043	.078	.113	.150	.183	.220	.255	.291	.325	.397	.467	.54	.67	.82	.96	1.10	1.24	1.39
19	.042	.040	.070	.101	.130	.161	.193	.221	.252	.282	.343	.404	.46	.58	.71	.83	.95	1.07	1.19
20	.035	.036	.062	.086	.113	.136	.163	.188	.214	.238	.289	.339	.39	.49	.59	.69	.80	.893	1.00
21	.032	.034	.057	.081	.104	.128	.151	.173	.196	.219	.266	.312	.357	.450	.542	.635	.727	.820	.913
22	.028	.031	.051	.072	.093	.112	.133	.152	.174	.192	.233	.275	.315	.396	.477	.556	.638	.718	.799
23	.025	.029	.047	.066	.082	.102	.119	.136	.155	.173	.209	.245	.281	.354	.426	.497	.571	.641	.714
24	.022	.026	.042	.058	.074	.090	.107	.121	.137	.154	.186	.218	.249	.312	.376	.438	.502	.566	.629
25	.020	.024	.039	.052	.068	.081	.096	.110	.126	.140	.169	.197	.226	.285	.342	.399	.457	.516	.573

For weights of seamless copper tubing, add 5 per cent. to the weights above.

TABLE 5.—WEIGHTS OF STEEL HEXAGON AND OCTAGON BARS

Dia. or distance across flats	Weight per ft., lbs. Hexagon	Octagon	Dia. or distance across flats	Weight per ft., lbs. Hexagon	Octagon
1/16	.012	.011	1 9/16	7.195	6.905
1/8	.046	.044	1 5/8	7.776	7.446
3/16	.103	.099	1 11/16	8.392	8.027
1/4	.185	.177	1 3/4	9.025	8.635
5/16	.288	.277	1 13/16	9.682	9.264
3/8	.414	.398	1 7/8	10.36	9.918
7/16	.564	.542	1 15/16	11.06	10.58
1/2	.737	.708	2	11.79	11.28
9/16	.932	.896	2 1/8	13.31	12.71
5/8	1.151	1.107	2 1/4	14.92	14.24
11/16	1.393	1.331	2 3/8	16.62	15.88
3/4	1.658	1.584	2 1/2	18.42	17.65
13/16	1.944	1.860	2 5/8	20.31	19.45
7/8	2.256	2.156	2 3/4	22.29	21.28
15/16	2.591	2.482	2 7/8	24.36	23.28
1	2.947	2.817	3	26.53	25.36
1 1/16	3.327	3.182	3 1/8	28.78	27.50
1 1/8	3.730	3.568	3 1/4	31.10	29.28
1 3/16	4.156	3.977	3 3/8	33.57	32.10
1 1/4	4.605	4.407	3 1/2	36.10	34.56
1 5/16	5.077	4.858	3 5/8	38.73	37.05
1 3/8	5.571	5.331	3 3/4	41.45	39.68
1 7/16	6.091	5.827	3 7/8	44.26	42.35
1 1/2	6.631	6.344	4	47.16	45.12

TABLE 6.—WEIGHT OF SPHERES OF VARIOUS METALS

Diameter in ins.	Weight in pounds Steel	Wrought iron	Cast-iron	Copper	Brass	Lead
1	.146	.142	.134	.166	.155	.213
1 1/2	.495	.481	.564	.563	.526	.723
2	1.13	1.1	1.07	1.33	1.25	1.71
2 1/2	2.26	2.2	2.1	2.6	2.4	3.3
3	3.9	3.8	3.6	4.5	4.2	5.8
3 1/2	6.2	6.1	5.8	7.146	6.7	9.2
4	9.27	9	8.6	10.6	9.9	13.6
4 1/2	13.3	13	12.3	15.2	14.2	19.5
5	18.5	18	17	21	19.5	27
5 1/2	24.4	23.7	22.6	27.7	25.9	35.5
6	31.9	31	29.1	36	33.6	46
6 1/2	40	39	36	45.7	42.7	58.7
7	50.5	49	46.2	57	53.3	73
7 1/2	62	60.2	57	70.3	65.7	90.3
8	75.2	73	69	85	79.4	109
8 1/2	90	87.5	83	102.3	95.6	131.4
9	107	104	98.1	121	113	155
9 1/2	126	122.4	116	143	133.6	183.7
10	146	142	134.5	166	155	213
10 1/2	169	165	156.5	193	180	284
11	195	190	180	222	207.5	286
11 1/2	222	216.5	205	253	236.4	325
12	254	247	233.5	288	270	370

Table 7.—Weights of Sheet Iron, Steel, Copper and Brass

Birmingham Gage

No of gage	Thickness in ins.	Weight per sq. ft. Steel	Iron	Copper	Brass
0000	.454	18.5232	18.16	20.5662	19.4312
000	.425	17.3400	17.00	19.2525	18.1900
00	.380	15.5040	15.20	17.2140	16.2640
0	.340	13.8720	13.60	15.4020	14.5520
1	.300	12.2400	12.00	13.5900	12.8400
2	.284	11.5872	11.36	12.8652	12.1552
3	.259	10.5672	10.36	11.7327	11.0852
4	.238	9.7104	9.52	10.7814	10.1864
5	.220	8.9760	8.80	9.966	9.4160
6	.203	8.2824	8.12	9.1959	8.6884
7	.180	7.3440	7.20	8.1540	7.7040
8	.165	6.7320	6.60	7.4745	7.0620
9	.148	6.0384	5.92	6.7044	6.3344
10	.134	5.4672	5.36	6.0702	5.7352
11	.120	4.8960	4.80	5.4360	5.1360
12	.109	4.4472	4.36	4.9377	4.6652
13	.095	3.8760	3.80	4.3035	4.0660
14	.083	3.3864	3.32	3.7599	3.5524
15	.072	2.9376	2.88	3.2616	3.0816
16	.065	2.6520	2.60	2.9445	2.7820
17	.058	2.3664	2.32	2.6274	2.4824
18	.049	1.9992	1.96	2.2197	2.0972
19	.042	1.7136	1.68	1.9026	1.7976
20	.035	1.4280	1.40	1.5855	1.4980
21	.032	1.3056	1.28	1.4496	1.3696
22	.028	1.1424	1.12	1.2684	1.1984
23	.025	1.0200	1.00	1.1325	1.0700
24	.022	.8976	.88	.9966	.9416
25	.020	.8160	.80	.9060	.8560
26	.018	.7344	.72	.8154	.7704
27	.016	.6528	.64	.7248	.6848
28	.014	.5712	.56	.6342	.5992
29	.013	.5304	.52	.5889	.5564
30	.012	.4896	.48	.5436	.5136
31	.010	.4080	.40	.4530	.4280
32	.009	.3672	.36	.4077	.3852
33	.008	.3264	.32	.3624	.3424
34	.007	.2856	.28	.3171	.2996
35	.005	.2040	.20	.2265	.2140
36	.004	.1632	.16	.1812	.1712
Specific gravities		7.85	7.70	8.72	8.24
Weight of a cubic foot		489.6	480.0	543.6	513.6
Weight of a cubic inch		.2833	.2778	.3146	.2972

American or Brown & Sharpe gage

No of gage	Thickness in ins.	Weight per sq. ft. Steel	Iron	Copper	Brass
0000	.460000	18.7680	18.4000	20.8380	19.6880
000	.409642	16.7134	16.3857	18.5568	17.5327
00	.364796	14.8837	14.5918	16.5253	15.6133
0	.324861	13.2543	12.9944	14.7162	13.9041
1	.289297	11.8033	11.5719	13.1052	12.3819
2	.257627	10.5112	10.3051	11.6705	11.0264
3	.229423	9.3605	9.1769	10.3929	9.8193
4	.204307	8.3357	8.1723	9.2551	8.7443
5	.181940	7.4232	7.2776	8.2419	7.7870
6	.162023	6.6105	6.4809	7.3396	6.9346
7	.144285	5.8868	5.7714	6.5361	6.1754
8	.128490	5.2424	5.1396	5.8206	5.4994
9	.114423	4.6685	4.5769	5.1834	4.8973
10	.101897	4.1574	4.0759	4.6159	4.3612
11	.090742	3.7023	3.6297	4.1106	3.8838
12	.080808	3.2970	3.2323	3.6606	3.4586
13	.071962	2.9360	2.8785	3.2599	3.0800
14	.064084	2.6146	2.5634	2.9030	2.7428
15	.057068	2.3284	2.2827	2.5852	2.4425
16	.050821	2.0735	2.0328	2.3022	2.1751
17	.045257	1.8465	1.8103	2.0501	1.9370
18	.040303	1.6444	1.6121	1.8257	1.7250
19	.035890	1.4643	1.4356	1.6258	1.5361
20	.031961	1.3040	1.2784	1.4478	1.3679
21	.028462	1.1612	1.1385	1.2893	1.2182
22	.025346	1.0341	1.0138	1.1482	1.0848
23	.022572	.92094	.90288	1.0225	.96608
24	.020101	.82012	.80404	.91058	.86032
25	.017900	.73032	.71600	.81087	.76612
26	.015941	.65039	.63764	.72213	.68227
27	.014195	.57916	.56780	.64303	.60755
28	.012641	.51575	.50564	.57264	.54103
29	.011257	.45929	.45028	.50994	.48180
30	.010025	.40902	.40100	.45413	.42907
31	.008928	.36426	.35712	.40444	.38212
32	.007950	.32436	.31800	.36014	.34026
33	.007080	.28886	.28320	.32072	.30302
34	.006305	.25724	.25220	.28562	.26985
35	.005615	.22909	.22460	.25436	.24032
36	.005000	.20400	.20000	.22650	.21400
37	.004453	.18168	.17812	.20172	.19059
38	.003965	.16177	.15860	.17961	.16970
39	.003531	.14406	.14124	.15995	.15113
40	.003144	.12828	.12576	.14242	.13456

Table 8.—Weights of Flat Sizes of Steel in Pounds per Linear Foot

	1/2	5/8	3/4	7/8	1	1 1/8	1 1/4	1 3/8	1 1/2	1 3/4	2	2 1/4	2 1/2	2 3/4	3	3 1/2	4	5	6
1/8	.213	.266	.320	.372	.426	.479	.530	.585	.640	.745	.850	.955	1.07	1.18	1.28	1.49	1.70	2.13	2.56
3/16	.319	.399	.480	.558	.639	.718	.790	.878	.960	1.12	1.28	1.43	1.60	1.76	1.92	2.24	2.55	3.20	3.83
1/4	.425	.533	.640	.743	.852	.958	1.06	1.17	1.28	1.49	1.70	1.91	2.13	2.34	2.56	2.98	3.40	4.26	5.11
5/16	.531	.665	.800	.929	1.06	1.20	1.33	1.46	1.60	1.86	2.13	2.39	2.66	2.92	3.19	3.72	4.25	5.32	6.38
3/8	.638	.798	.960	1.12	1.28	1.43	1.59	1.75	1.91	2.23	2.55	2.87	3.20	3.51	3.83	4.46	5.10	6.40	7.66
7/16	.744	.931	1.12	1.30	1.49	1.67	1.86	2.05	2.23	2.60	2.98	3.35	3.72	4.09	4.46	5.21	5.95	7.44	8.92
1/2	....	1.07	1.28	1.49	1.70	1.91	2.13	2.34	2.55	2.98	3.40	3.83	4.26	4.68	5.10	5.96	6.80	8.52	10.20
9/16	....	1.20	1.44	1.67	1.91	2.15	2.39	2.63	2.87	3.35	3.83	4.30	4.78	5.26	5.74	6.69	7.65	9.56	11.50
5/8	....	.....	1.60	1.86	2.12	2.39	2.66	2.92	3.19	3.72	4.26	4.79	5.32	5.86	6.39	7.44	8.52	10.64	12.78
11/16	....	.....	1.76	2.04	2.34	2.63	2.92	3.22	3.51	4.09	4.68	5.26	5.84	6.43	7.01	8.18	9.35	11.70	14.00
3/4	....	.....	.....	2.23	2.55	2.86	3.19	3.50	3.83	4.46	5.10	5.74	6.40	7.02	7.65	8.92	10.20	12.80	15.30
13/16	....	.....	.....	2.41	2.76	3.11	3.45	3.80	4.14	4.83	5.53	6.22	6.91	7.60	8.29	9.67	11.10	13.80	16.60
7/8	....	.....	.....	.....	2.98	3.34	3.72	4.09	4.46	5.21	5.96	6.70	7.46	8.19	8.94	10.42	11.92	14.92	17.88
15/16	....	.....	.....	.....	3.19	3.59	3.98	4.38	4.78	5.58	6.38	7.17	7.97	8.77	9.56	11.20	12.80	15.90	19.10
1	....	.....	.....	.....	.....	3.82	4.25	4.68	5.10	5.96	6.80	7.66	8.52	9.36	10.20	11.92	13.60	17.04	20.40
1 1/8	....	.....	.....	.....	.....	.....	4.78	5.27	5.74	6.71	7.65	8.61	9.59	10.54	11.48	13.41	15.30	19.17	22.95
1 1/4	....	.....	.....	.....	.....	.....	.....	5.85	6.38	7.45	8.50	9.57	10.65	11.71	12.76	14.90	17.00	21.30	25.61
1 1/2	....	.....	.....	.....	.....	.....	.....	7.02	7.67	8.94	10.20	11.49	12.78	14.04	15.30	17.88	20.40	25.56	30.

TABLE 9.—WEIGHTS AND SECTIONAL AREAS OF SQUARE AND ROUND STEEL BARS. BY THE CARNEGIE STEEL CO.

Thickness or diameter in ins.	Weight of □ bar 1 ft. long	Weight of ○ bar 1 ft. long	Area of □ bar in sq. ins.	Area of ○ Bar in sq. ins.
0				
1/16	.013	.010	.0039	.0031
1/8	.053	.042	.0156	.0123
3/16	.119	.094	.0352	.0276
1/4	.212	.167	.0625	.0491
5/16	.333	.261	.0977	.0767
3/8	.478	.375	.1406	.1104
7/16	.651	.511	.1914	.1503
1/2	.850	.667	.2500	.1963
9/16	1.076	.845	.3164	.2485
5/8	1.328	1.043	.3906	.3068
11/16	1.608	1.262	.4727	.3712
3/4	1.913	1.502	.5625	.4418
13/16	2.245	1.763	.6602	.5185
7/8	2.603	2.044	.7656	.6013
15/16	2.989	2.347	.8789	.6903
1	3.400	2.670	1.0000	.7854
1/16	3.838	3.014	1.1289	.8866
1/8	4.303	3.379	1.2656	.9940
3/16	4.795	3.766	1.4102	1.1075
1/4	5.312	4.173	1.5625	1.2272
5/16	5.857	4.600	1.7227	1.3530
3/8	6.428	5.049	1.8906	1.4849
7/16	7.026	5.518	2.0664	1.6230
1/2	7.650	6.008	2.2500	1.7671
9/16	8.301	6.520	2.4414	1.9175
5/8	8.978	7.051	2.6406	2.0739
11/16	9.682	7.604	2.8477	2.2365
3/4	10.41	8.178	3.0625	2.4053
13/16	11.17	8.773	3.2852	2.5802
7/8	11.95	9.388	3.5156	2.7612
15/16	12.76	10.02	3.7539	2.9483
2	13.60	10.68	4.0000	3.1416
1/16	14.46	11.36	4.2539	3.3410
1/8	15.35	12.06	4.5156	3.5466
3/16	16.27	12.78	4.7852	3.7583
1/4	17.22	13.52	5.0625	3.9761
5/16	18.19	14.28	5.3477	4.2000
3/8	19.18	15.07	5.6406	4.4301
7/16	20.20	15.86	5.9414	4.6664
1/2	21.25	16.69	6.2500	4.9087
9/16	22.33	17.53	6.5664	5.1572
5/8	23.43	18.40	6.8906	5.4119
11/16	24.56	19.29	7.2227	5.6727
3/4	25.71	20.20	7.5625	5.9396
13/16	26.90	21.12	7.9102	6.2126
7/8	28.10	22.07	8.2656	6.4918
15/16	29.34	23.04	8.6289	6.7771
3	30.60	24.03	9.0000	7.0686
1/16	31.89	25.04	9.3789	7.3662
1/8	33.20	26.08	9.7656	7.6699
3/16	34.55	27.13	10.160	7.9798
1/4	35.92	28.20	10.563	8.2958
5/16	37.31	29.30	10.973	8.6179
3/8	38.73	30.42	11.391	8.9462
7/16	40.18	31.56	11.816	9.2806
1/2	41.65	32.71	12.250	9.6211
9/16	43.14	33.90	12.691	9.9678
5/8	44.68	35.09	13.141	10.321
11/16	46.24	36.31	13.598	10.680
3/4	47.82	37.56	14.063	11.045
13/16	49.42	38.81	14.535	11.416
7/8	51.05	40.10	15.016	11.793
15/16	52.71	41.40	15.504	12.177
4	54.40	42.73	16.000	12.566
1/16	56.11	44.07	16.504	12.962
1/8	57.85	45.44	17.016	13.364
3/16	59.62	46.83	17.535	13.772
1/4	61.41	48.24	18.063	14.186
5/16	63.23	49.66	18.598	14.607
3/8	65.08	51.11	19.141	15.033
7/16	66.95	52.58	19.691	15.466
1/2	68.85	54.07	20.250	15.904
9/16	70.78	55.59	20.816	16.349
5/8	72.73	57.12	21.391	16.800
11/16	74.70	58.67	21.973	17.257
3/4	76.71	60.25	22.563	17.721
13/16	78.74	61.84	23.160	18.190
7/8	80.81	63.46	23.766	18.665
15/16	82.89	65.10	24.379	19.147
5	85.00	66.76	25.000	19.635
1/16	87.14	68.44	25.629	20.129
1/8	89.30	70.14	26.266	20.629
3/16	91.49	71.86	26.910	21.135
1/4	93.72	73.60	27.563	21.648
5/16	95.96	75.37	28.223	22.166
3/8	98.23	77.15	28.891	22.691
7/16	100.5	78.95	29.566	23.221
1/2	102.8	80.77	30.250	23.758
9/16	105.2	82.62	30.941	24.301
5/8	107.6	84.49	31.641	24.850
11/16	110	86.38	32.348	25.406
3/4	112.4	88.29	33.063	25.967
13/16	114.9	90.22	33.785	26.535
7/8	117.4	92.17	34.516	27.109
15/16	119.9	94.14	35.254	27.688
6	122.4	96.14	36.000	28.274
1/16	125	98.14	36.754	28.866
1/8	127.6	100.2	37.516	29.465
3/16	130.2	102.2	38.285	30.069
1/4	132.8	104.3	39.063	30.680
5/16	135.5	106.4	39.848	31.296

Table 9.—Weights and Sectional Areas of Square and Round Steel Bars. By the Carnegie Steel Co.—(*Continued*)

Thickness or diameter in ins.	Weight of □ bar 1 ft. long	Weight of ○ bar 1 ft. long	Area of □ bar in sq. ins.	Area of ○ bar in sq. ins.
3/8	138.2	108.5	40.641	31.919
7/16	140.9	110.7	41.441	32.548
1/2	143.6	112.8	42.250	33.183
9/16	146.5	114.9	43.066	33.824
5/8	149.2	117.2	43.891	34.472
11/16	152.1	119.4	44.723	35.125
3/4	154.9	121.7	45.563	35.785
13/16	157.8	123.9	46.410	36.450
7/8	160.8	126.2	47.266	37.122
15/16	163.6	128.5	48.129	37.800
7	166.6	130.9	49.000	38.485
1/16	169.6	133.2	49.879	39.175
1/8	172.6	135.6	50.766	39.871
3/16	175.6	137.9	51.660	40.574
1/4	178.7	140.4	52.563	41.282
5/16	181.8	142.8	53.473	41.997
3/8	184.9	145.3	54.391	42.718
7/16	188.1	147.7	55.316	43.445
1/2	191.3	150.2	56.250	44.179
9/16	194.4	152.7	57.191	44.918
5/8	197.7	155.2	58.141	45.664
11/16	200.9	157.8	59.098	46.415
3/4	204.2	160.3	60.063	47.173
13/16	207.6	163	61.035	47.937
7/8	210.8	165.6	62.016	48.707
15/16	214.2	168.2	63.004	49.483
8	217.6	171.0	64.000	50.265
1/16	221.0	173.6	65.004	51.054
1/8	224.5	176.3	66.016	51.849
3/16	228.0	179.0	67.035	52.649
1/4	231.4	181.8	68.063	53.456
5/16	234.9	184.5	69.098	54.269
3/8	238.5	187.3	70.141	55.088
7/16	242.0	190.1	71.191	55.914
1/2	245.6	193.0	72.250	56.745
9/16	249.3	195.7	73.316	57.583
5/8	252.9	198.7	74.391	58.426
11/16	256.6	201.6	75.473	59.276
3/4	260.3	204.4	76.563	60.132
13/16	264.1	207.4	77.660	60.994
7/8	267.9	210.3	78.766	61.862
15/16	271.6	213.3	79.879	62.737
9	275.4	216.3	81.000	63.617
1/16	279.3	219.3	82.129	64.505
1/8	283.2	222.4	83.266	65.397
3/16	287.0	225.4	84.410	66.296
1/4	290.9	228.5	85.563	67.201
5/16	294.9	231.5	86.723	68.112
3/8	298.9	234.7	87.891	69.029
7/16	302.8	237.9	89.066	69.953
1/2	306.8	241.0	90.250	70.882
9/16	310.9	244.2	91.441	71.818
5/8	315.0	247.4	92.641	72.760
11/16	319.1	250.6	93.848	73.708
3/4	323.2	253.9	95.063	74.662
13/16	327.4	257.1	96.285	75.622
7/8	331.6	260.4	97.516	76.589
15/16	335.8	263.7	98.754	77.561
10	340.0	267.0	100.00	78.540
1/16	344.3	270.4	101.25	79.525
1/8	348.5	273.8	102.52	80.516
3/16	352.9	277.1	103.79	81.513
1/4	357.2	280.6	105.06	82.516
5/16	361.6	284.0	106.35	83.525
3/8	366.0	287.4	107.64	84.541
7/16	370.4	290.9	108.94	85.562
1/2	374.9	294.4	110.25	86.590
9/16	379.4	297.9	111.57	87.624
5/8	383.8	301.4	112.89	88.664
11/16	388.3	305.0	114.22	89.710
3/4	392.9	308.6	115.56	90.763
13/16	397.5	312.2	116.91	91.821
7/8	402.1	315.8	118.27	92.886
15/16	406.8	319.5	119.63	93.956
11	411.4	323.1	121.00	95.033
1/16	416.1	326.8	122.38	96.116
1/8	420.9	330.5	123.77	97.205
3/16	425.5	334.3	125.16	98.301
1/4	430.3	337.9	126.56	99.402
5/16	435.1	341.7	127.97	100.51
3/8	439.9	345.5	129.39	101.62
7/16	444.8	349.4	130.82	102.74
1/2	449.6	353.1	132.25	103.87
9/16	454.5	357.0	133.69	105.00
5/8	459.5	360.9	135.14	106.14
11/16	464.4	364.8	136.60	107.28
3/4	469.4	368.6	138.06	108.43
13/16	474.4	372.6	139.54	109.59
7/8	479.5	376.6	141.02	110.75
15/16	484.5	380.6	142.50	111.92

TABLE 10.—WEIGHTS OF BRASS, COPPER AND ALUMINUM BARS

Dia. or distance across flats	Brass			Copper		Aluminum	
	Weight per ft., lbs.			Weight per ft. lbs.,		Weight per ft. lbs.,	
	Round	Square	Hexagon	Round	Square	Round	Square
1/16	.011	.014	.013	.012	.015	.003	.004
1/8	.045	.055	.048	.047	.060	.014	.018
3/16	.100	.125	.108	.106	.135	.032	.041
1/4	.175	.225	.194	.189	.241	.057	.072
5/16	.275	.350	.301	.296	.377	.089	.114
3/8	.395	.510	.436	.426	.542	.128	.163
7/16	.540	.690	.592	.579	.737	.174	.222
1/2	.710	.905	.773	.757	.964	.227	.290
9/16	.900	1.15	.978	.958	1.22	.288	.367
5/8	1.10	1.40	1.24	1.18	1.51	.356	.453
11/16	1.35	1.72	1.45	1.43	1.82	.430	.548
3/4	1.66	2.05	1.73	1.70	2.17	.516	.652
13/16	1.85	2.40	2.03	2.00	2.54	.601	.766
7/8	2.15	2.75	2.36	2.32	2.95	.697	.888
15/16	2.48	3.15	2.71	2.66	3.39	.800	1.02
1	2.85	3.65	3.10	3.03	3.86	.911	1.16
1 1/16	3.20	4.08	3.49	3.42	4.35	1.03	1.31
1 1/8	3.57	4.55	3.91	3.81	4.88	1.15	1.47
1 3/16	3.97	5.08	4.38	4.27	5.44	1.28	1.64
1 1/4	4.41	5.65	4.82	4.72	6.01	1.42	1.81
1 5/16	4.86	6.22	5.33	5.21	6.63	1.57	2.00
1 3/8	5.35	6.81	5.76	5.72	7.24	1.72	2.19
1 7/16	5.86	7.45	6.38	6.26	7.97	1.88	2.40
1 1/2	6.37	8.13	6.92	6.81	8.67	2.05	2.61
1 9/16	6.92	8.83	7.54	7.39	9.41	2.22	2.83
1 5/8	7.48	9.55	8.15	7.99	10.18	2.41	3.06
1 11/16	8.05	10.27	8.80	8.45	10.73	2.59	3.30
1 3/4	8.65	11.00	9.47	9.27	11.80	2.79	3.55
1 13/16	9.29	11.82	10.15	9.76	12.43	2.99	3.81
1 7/8	9.95	12.68	10.86	10.64	13.55	3.20	4.08
1 15/16	10.58	13.50	11.68	11.11	14.15	3.41	4.35
2	11.25	14.35	12.36	12.11	15.42	3.64	4.64
2 1/8	12.78	16.27	13.92	13.67	17.42	4.11	5.24
2 1/4	14.32	18.24	15.72	15.33	19.51	4.61	5.87
2 3/8	15.96	20.32	17.52	17.08	21.74	5.14	6.54
2 1/2	17.68	22.53	19.44	18.92	24.09	5.69	7.25
2 5/8	19.50	24.83	21.24	20.86	26.56	6.27	7.99
2 3/4	21.40	27.25	23.40	22.89	29.05	6.89	8.53
2 7/8	23.39	29.78	25.82	25.02	31.86	7.52	9.58
3	25.47	32.43	27.84	27.24	34.69	8.20	10.44
3 1/4	30.45	38.77	32.76	31.97	40.71	9.62	12.25
3 1/2	35.31	44.96	37.80	37.08	47.22	11.16	14.21
3 3/4	40.07	51.01	43.56	42.11	53.61	12.81	16.31
4	46.12	58.73	49.44	48.43	61.67	14.56	18.56

TABLE 11.—WEIGHT OF STEEL PLATES PER SQ. FT. BOTH THEORETICAL AND WITH COMMERCIAL OVERWEIGHT ALLOWANCE

Thickness ins.	Theoretical weight, lbs.	Allowance for overweight, plates 50 to 75 ins. wide	Adjusted weight, lbs.
		Per cent.	
3/16	7.65	10	8.42
1/4	10.207	10	11.23
5/16	12.75	8	13.78
3/8	15.31	7	16.38
7/16	17.86	6	18.93
1/2	20.41	5	21.44
9/16	22.96	4 1/2	24.00
5/8	25.51	4	26.5
11/16	28.07	3 1/2	29.20
3/4	30.62	3 1/2	30.80
13/16	33.17	3 1/2	34.50

TABLE 12.—WEIGHTS OF FLAT ROLLED STRIPS, HOOP OR BAND STEEL

Pounds per Lineal Foot
Thicknesses by Birmingham Wire Gage
One cu. ft. of steel weighs 489.6 lbs.
For widths from 1/4 in. to 3/4 in. and thicknesses from No. 19 to No. 11 B. W. G.

Width in ins.	No. 19. .042 in.	No. 18. .049 in.	No. 17. .058 in.	No. 16. .065 in.	No. 15. .072 in.	No. 14. .083 in.	No. 13. .095 in.	No. 12. .109 in.	No. 11. .120 in.
1/4	.036	.042	.049	.055	.061	.071	.081	.093	.102
17/64	.038	.044	.052	.059	.065	.075	.086	.098	.108
9/32	.040	.047	.055	.062	.069	.079	.091	.104	.115
19/64	.042	.049	.059	.066	.073	.084	.096	.110	.121
5/16	.045	.052	.062	.069	.077	.088	.101	.116	.128
21/64	.047	.055	.065	.073	.080	.093	.106	.122	.134
11/32	.049	.057	.068	.076	.084	.097	.111	.127	.140
23/64	.051	.060	.071	.079	.088	.101	.116	.133	.147
3/8	.054	.062	.074	.083	.092	.106	.121	.139	.153
25/64	.056	.065	.077	.086	.096	.110	.126	.145	.159
13/32	.058	.068	.080	.090	.099	.115	.131	.151	.166
27/64	.060	.070	.083	.093	.103	.119	.136	.156	.172
7/16	.062	.073	.086	.097	.107	.123	.141	.162	.179
29/64	.065	.075	.089	.100	.111	.128	.146	.168	.185
15/32	.067	.078	.092	.104	.115	.132	.151	.174	.191
31/64	.069	.081	.096	.107	.119	.137	.156	.180	.198
1/2	.071	.083	.099	.111	.122	.141	.162	.185	.204
33/64	.074	.086	.102	.114	.126	.146	.167	.191	.210
17/32	.076	.089	.105	.117	.130	.150	.172	.197	.217
35/64	.078	.091	.108	.121	.134	.154	.177	.203	.223
9/16	.080	.094	.111	.124	.138	.159	.182	.208	.230
37/64	.083	.096	.114	.128	.142	.163	.187	.214	.236
19/32	.085	.099	.117	.131	.145	.168	.192	.220	.242
39/64	.087	.102	.120	.135	.149	.172	.197	.226	.249
5/8	.089	.104	.123	.138	.153	.176	.202	.232	.255
41/64	.091	.107	.126	.142	.157	.181	.207	.237	.261
21/32	.094	.109	.129	.145	.161	.185	.212	.243	.268
43/64	.096	.112	.132	.148	.164	.190	.217	.249	.274
11/16	.098	.115	.136	.152	.168	.194	.222	.255	.281
45/64	.100	.117	.139	.155	.172	.198	.227	.261	.287
23/32	.103	.120	.142	.159	.176	.203	.232	.266	.293
47/64	.105	.122	.145	.162	.180	.207	.237	.272	.300
3/4	.107	.125	.148	.166	.184	.212	.242	.278	.306

To compute the weight of sheet iron on the basis of 480 lbs. per cu. ft. divide the thickness expressed in thousandths by 25. The result is the weight in lbs. per sq. ft.

HEAT

The centigrade thermometer scale is a case of the blind worship of decimals. It possesses no advantage that can be discovered by any except its devotees, while the confusion due to its existence overbalances a hundredfold all the advantages that its advocates imagine they see in it. It has introduced two sets of temperature observations where there might have been one; it has made necessary countless conversions between observations where there might have been none, and to offset this it has introduced no compensating advantage whatever. Every application of the following conversion formulas and tables is an illustration of the harm done by this fussy and amateurish attempt to improve a thing that did not need improvement as well as of the uniform result when metric and other hobby riders endeavor to change established standards of measurement.

Conversions between the Fahrenheit and Centigrade scales may be made by the following formulas:

$$F=\frac{9}{5}C+32$$

$$C=\frac{5}{9}(F-32)$$

in which F = reading by Fahrenheit scale,
C = reading by Centrigade scale.

TABLE 1.—EQUIVALENT TEMPERATURES—CENTIGRADE TO FAHRENHEIT.

C.°	0	10	20	30	40	50	60	70	80	90
	F.	F.	F.	F.	F.	F.	F.	F.	F.	F.
−200	−328	−346	−364	−382	−400	−418	−436	−454	....	....
−100	−148	−166	−184	−202	−220	−238	−256	−274	−292	−310
−0	+32	+14	−4	−22	−40	−58	−76	−94	−112	−130
0	32	50	68	86	104	122	140	158	176	194
100	212	230	248	266	284	302	320	338	356	374
200	392	410	428	446	464	482	500	518	536	554
300	572	590	608	626	644	662	680	698	716	734
400	752	770	788	806	824	842	860	878	896	914
500	932	950	968	986	1004	1022	1040	1058	1076	1094
600	1112	1130	1148	1166	1184	1202	1220	1238	1256	1274
700	1292	1310	1328	1346	1364	1382	1400	1418	1436	1454
800	1472	1490	1508	1526	1544	1562	1580	1598	1616	1634
900	1652	1670	1688	1706	1724	1742	1760	1778	1796	1814
1000	1832	1850	1868	1886	1904	1922	1940	1958	1976	1994
1100	2012	2030	2048	2066	2084	2102	2120	2138	2156	2174
1200	2192	2210	2228	2246	2264	2282	2300	2318	2336	2354
1300	2372	2390	2408	2426	2444	2462	2480	2498	2516	2534
1400	2552	2570	2588	2606	2624	2642	2660	2678	2696	2714
1500	2732	2750	2768	2786	2804	2822	2840	2858	2876	2894
1600	2912	2930	2948	2966	2984	3002	3020	3038	3056	3074
1700	3092	3110	3128	3146	3164	3182	3200	3218	3236	3254
1800	3272	3290	3308	3326	3344	3362	3380	3398	3416	3434
1900	3452	3470	3488	3506	3524	3542	3560	3578	3596	3614
2000	3632	3650	3668	3686	3704	3722	3740	3758	3776	3794
2100	3812	3830	3848	3866	3884	3902	3920	3938	3956	3974
2200	3992	4010	4028	4046	4064	4082	4100	4118	4136	4154
2300	4172	4190	4208	4226	4244	4262	4280	4298	4316	4334
2400	4352	4370	4388	4406	4424	4442	4460	4478	4496	4514
2500	4532	4550	4568	4586	4604	4622	4640	4658	4676	4694
2600	4712	4730	4748	4766	4784	4802	4820	4838	4856	4874
2700	4892	4910	4928	4946	4964	4982	5000	5018	5036	5054
2800	5072	5090	5108	5126	5144	5162	5180	5198	5216	5234
2900	5252	5270	5288	5306	5324	5342	5360	5378	5396	5414
3000	5432	5450	5468	5486	5504	5522	5540	5558	5576	5594
3100	5612	5630	5648	5666	5684	5702	5720	5738	5756	5774
3200	5792	5810	5828	5846	5864	5882	5900	5918	5936	5954
3200	5972	5990	6008	6026	6044	6062	6080	6098	6116	6134
3400	6152	6170	6188	6206	6224	6242	6260	6278	6296	6314
3500	6332	6350	6368	6386	6404	6422	6440	6458	6476	6494
3600	6512	6530	6548	6566	6584	6602	6620	6638	6656	6674
3700	6692	6710	6728	6746	6764	6782	6800	6818	6836	6854
3800	6872	6890	6908	6926	6944	6962	6980	6998	7016	7034
3900	7052	7070	7088	7106	7124	7142	7160	7178	7196	7214
C.°	0	10	20	30	40	50	60	70	80	90

C.°	F.°
1	1.8
2	3.6
3	5.4
4	7.2
5	9.0
6	10.8
7	12.6
8	14.4
9	16.2
10	18.0

EXAMPLE: 1347° C. = 2444° F. + 12.6° F. = 2456.6° F.

Tables 1 and 2 by DR. LEONARD WALDO (*Trans. A. I. M. E.*, 1911) are the most complete that have been prepared. Their range is from absolute zero to the temperature of the electric arc. The equivalents for 10-deg. intervals are read directly and for 1-deg. intervals by the supplementary tables of proportional parts. The tables are used precisely like tables of logarithms as illustrated by the examples below them.

TABLE 2.—EQUIVALENT TEMPERATURES—FAHRENHEIT TO CENTIGRADE

Heavy Face Figures Indicate Recurring Decimals.

F.°	0	10	20	30	40	50	60	70	80	90
	C.	C.	C.	C.	C.	C.	C.	C.	C.	C.
−400	−240.0	−245.5	−251.1	−256.6	−262.2	−267.7	....	....	....	....
−300	−184.4	−190.0	−195.5	−201.1	−206.6	−212.2	−217.7	−223.3	−228.8	−234.4
−200	−128.8	−134.4	−140.0	−145.5	−151.1	−156.6	−162.2	−167.7	−173.3	−178.8
−100	−73.3	−78.8	−84.4	−90.0	−95.5	−101.1	106.6	−112.2	−117.7	−123.3
−0	−17.7	−23.3	−28.8	−34.4	−40.0	−45.5	−51.1	−56.6	−62.2	−67.7
0	−17.7	−12.2	−6.6	−1.1	+4.4	+10.0	+15.5	+21.1	+26.6	+32.2
100	37.7	43.3	48.8	54.4	60.0	65.5	71.1	76.6	82.2	87.7
200	93.3	98.8	104.4	110.0	115.5	121.1	126.6	132.2	137.7	143.3
300	148.8	154.4	160.0	165.5	171.1	176.6	182.2	187.7	193.3	198.8
400	204.4	210.0	215.5	221.1	226.6	232.2	237.7	243.3	248.8	254.4
500	260.0	265.5	271.1	276.6	282.2	287.7	293.3	298.8	304.4	310.0
600	315.5	321.1	326.6	332.2	337.7	343.3	348.8	354.4	360.0	365.5
700	371.1	376.6	382.2	387.7	393.3	398.8	404.4	410.0	415.5	421.1
800	426.6	432.2	437.7	443.3	448.8	454.4	460.0	465.5	471.1	476.6
900	482.2	487.7	493.3	498.8	504.4	510.0	515.5	521.1	526.6	532.2
1000	537.7	543.3	548.8	554.4	560.0	565.5	571.1	576.6	582.2	587.7
1100	593.3	598.8	604.4	610.0	615.5	621.1	626.6	632.2	637.7	643.3
1200	648.8	654.4	660.0	665.5	671.1	676.6	682.2	687.7	693.3	698.8
1300	704.4	710.0	715.5	721.1	726.6	732.2	737.7	743.3	748.8	754.4
1400	760.0	765.5	771.1	776.6	782.2	787.7	793.3	798.8	804.4	810.0
1500	815.5	821.1	826.6	832.2	837.7	843.3	848.8	854.4	860.0	865.5
1600	871.1	876.6	882.2	887.7	893.3	898.8	904.4	910.0	915.5	921.1
1700	926.6	932.2	937.7	943.3	948.8	954.4	960.0	965.5	971.1	976.6
1800	982.2	987.7	993.3	998.8	1004.4	1010.0	1015.5	1021.1	1026.6	1032.2
1900	1037.7	1043.3	1048.8	1054.4	1060.0	1065.5	1071.1	1076.6	1082.2	1087.7
2000	1093.3	1098.8	1104.4	1110.0	1115.5	1121.1	1126.6	1132.2	1137.7	1143.3
2100	1148.8	1154.4	1160.0	1165.5	1171.1	1176.6	1182.2	1187.7	1193.3	1198.8
2200	1204.4	1210.0	1215.5	1221.1	1226.6	1232.2	1237.7	1243.3	1248.8	1254.4
2300	1260.0	1265.5	1271.1	1276.6	1282.2	1287.7	1293.3	1298.8	1304.4	1310.0
2400	1315.5	1321.1	1326.6	1332.2	1337.7	1343.3	1348.8	1354.4	1360.0	1365.5
2500	1371.1	1376.6	1382.2	1387.7	1393.3	1398.8	1404.4	1410.0	1415.5	1421.1
2600	1426.6	1432.2	1437.7	1443.3	1448.8	1454.4	1460.0	1465.5	1471.1	1476.6
2700	1482.2	1487.7	1493.3	1498.8	1504.4	1510.0	1515.5	1521.1	1526.6	1532.2
2800	1537.7	1543.3	1548.8	1554.4	1560.0	1565.5	1571.1	1576.6	1582.2	1587.7
2900	1593.3	1598.8	1604.4	1610.0	1615.5	1621.1	1626.6	1632.2	1637.7	1643.3
3000	1648.8	1654.4	1660.0	1665.5	1671.1	1676.6	1682.2	1687.7	1693.3	1698.8
3100	1704.4	1710.0	1715.5	1721.1	1726.6	1732.2	1737.7	1743.3	1748.8	1754.4
3200	1760.0	1765.5	1771.1	1776.6	1782.2	1787.7	1793.3	1798.8	1804.4	1810.0
3300	1815.5	1821.1	1826.6	1832.2	1837.7	1843.3	1848.8	1854.4	1860.0	1865.5
3400	1871.1	1876.6	1882.2	1887.7	1893.3	1898.8	1904.4	1910.0	1915.5	1921.1
3500	1926.6	1932.2	1937.7	1943.3	1948.8	1954.4	1960.0	1965.5	1971.1	1976.6
3600	1982.2	1987.7	1993.3	1998.8	2004.4	2010.0	2015.5	2021.1	2026.6	2032.2
F.°	0	10	20	30	40	50	60	70	80	90

F.°	C.°
1	0.5
2	1.1
3	1.6
4	2.2
5	2.7
6	3.3
7	3.8
8	4.4
9	5.0

Examples: −246.0° F. = −151.11° C. − 3.33° C. = −154.44° C. 3762° F. = 2071.11° C. + 1.11° C. = 2072.22° C. 2423.5° F. = 1326.666° C. + 1.666° C. + .277° C. = 1328.609° C.

TABLE 2.—EQUIVALENT TEMPERATURES—FAHRENHEIT TO CENTIGRADE—(*Continued*)

Heavy Faced Figures Indicate Recurring Decimals

F.°	0	10	20	30	40	50	60	70	80	90
	C.	C.	C.	C.	C.	C.	C.	C.	C.	C.
3700	2037.7	2043.3	2048.8	2054.4	2060.0	2065.5	2071.1	2076.6	2082.2	2087.7
3800	2093.3	2098.8	2104.4	2110.0	2115.5	2121.1	2126.6	2132.2	2137.7	2143.3
3900	2148.8	2154.4	2160.0	2165.5	2171.1	2176.6	2182.2	2187.7	2193.3	2198.8
4000	2204.4	2210.0	2215.5	2221.1	2226.6	2232.2	2237.7	2243.3	2248.8	2254.4
4100	2260.0	2265.5	2271.1	2276.6	2282.2	2287.7	2293.3	2298.8	2304.4	2310.0
4200	2315.5	2321.1	2326.6	2332.2	2337.7	2343.3	2348.8	2354.4	2360.0	2365.5
4300	2371.1	2376.6	2382.2	2387.7	2393.3	2398.8	2404.4	2410.0	2415.5	2421.1
4400	2426.6	2432.2	2437.7	2443.3	2448.8	2454.4	2460.0	2465.5	2471.1	2476.6
4500	2482.2	2487.7	2493.3	2498.8	2504.4	2510.0	2515.5	2521.1	2526.6	2532.2
4600	2537.7	2543.3	2548.8	2554.4	2560.0	2565.5	2571.1	2576.6	2582.2	2587.7
4700	2593.3	2598.8	2604.4	2610.0	2615.5	2621.1	2626.6	2632.2	2637.7	2643.3
4800	2648.8	2654.4	2660.0	2665.5	2671.1	2676.6	2682.2	2687.7	2693.3	2698.8
4900	2704.4	2710.0	2715.5	2721.1	2726.6	2732.2	2737.7	2743.3	2748.8	2754.4
5000	2760.0	2765.5	2771.1	2776.6	2782.2	2787.7	2793.3	2798.8	2804.4	2810.0
5100	2815.5	2821.1	2826.6	2832.2	2837.7	2843.3	2848.8	2854.4	2860.0	2865.5
5200	2871.1	2876.6	2882.2	2887.7	2893.3	2898.8	2904.4	2910.0	2915.5	2921.1
5300	2926.6	2932.2	2937.7	2943.3	2948.8	2954.4	2960.0	2965.5	2971.1	2976.6
5400	2982.2	2987.7	2993.3	2998.8	3004.4	3010.0	3015.5	3021.1	3026.6	3032.2
5500	3037.7	3043.3	3048.8	3054.4	3060.0	3065.5	3071.1	3076.6	3082.2	3087.7
5600	3093.3	3098.8	3104.4	3110.0	3115.5	3121.1	3126.6	3132.2	3137.7	3143.3
5700	3148.8	3154.4	3160.0	3165.5	3171.1	3176.6	3182.2	3187.7	3193.3	3198.8
5800	3204.4	3210.0	3215.5	3221.1	3226.6	3232.2	3237.7	3243.3	3248.8	3254.4
5900	3260.0	3265.5	3271.1	3276.6	3282.2	3287.7	3293.3	3298.8	3304.4	3310.0
6000	3315.5	3321.1	3326.6	3332.2	3337.7	3343.3	3348.8	3354.4	3360.0	3365.5
6100	3371.1	3376.6	3382.2	3387.7	3393.3	3398.8	3404.4	3410.0	3415.5	3421.1
6200	3426.6	3432.2	3437.7	3443.3	3448.8	3454.4	3460.0	3465.5	3471.1	3476.6
6300	3482.2	3487.7	3493.3	3498.8	3504.4	3510.0	3515.5	3521.1	3526.6	3532.2
6400	3537.7	3543.3	3548.8	3554.4	3560.0	3565.5	3571.1	3576.6	3582.2	3587.7
6500	3593.3	3598.8	3604.4	3610.0	3615.5	3621.1	3626.6	3632.2	3637.7	3643.3
6600	3648.8	3654.4	3660.0	3665.5	3671.1	3676.6	3682.2	3687.7	3693.3	3698.8
6700	3704.4	3710.0	3715.5	3721.1	3726.6	3732.2	3737.7	3743.3	3748.8	3754.4
6800	3760.0	3765.5	3771.1	3776.6	3782.2	3787.7	3793.3	3798.8	3804.4	3810.0
6900	3815.5	3821.1	3826.6	3832.2	3837.7	3843.3	3848.8	3854.4	3860.0	3865.5
7000	3871.1	3876.6	3882.2	3887.7	3893.3	3898.8	3904.4	3910.0	3915.5	3921.1
7100	3926.6	3932.2	3937.7	3943.3	3948.8	3954.0	3960.0	3965.5	3971.1	3976.6
7200	3982.2	3987.7	3993.3	3998.8	4004.4	4010.0	4015.5	4021.1	4026.6	4032.2
7300	4037.7	4043.3	4048.8	4054.4	4060.0	4065.5	4071.1	4076.6	4082.2	4087.7
7400	4093.3	4098.8	4104.4	4110.0	4115.5	4121.1	4126.6	4132.2	4137.7	4143.3
7500	4148.8	4154.4	4160.0	4165.5	4171.1	4176.6	4182.2	4187.7	4193.3	4198.8
7600	4204.4	4210.0	4215.5	4221.1	4226.6	4232.2	4237.7	4243.3	4248.8	4254.4
7700	4260.0	4265.5	4271.1	4276.6	4282.2	4287.7	4293.3	4298.8	4304.4	4310.0
7800	4315.5	4321.1	4326.6	4332.2	4337.7	4343.3	4348.8	4354.4	4360.0	4365.5
7900	4371.1	4376.6	4382.2	4387.7	4393.3	4398.8	4404.4	4410.0	4415.5	4421.1
F.°	0	10	20	30	40	50	60	70	80	90

F.°	C.°
1	0.5
2	1.1
3	1.6
4	2.2
5	2.7
6	3.3
7	3.8
8	4.4
9	5.0

TABLE 3.—KILOGRAM CALORIES, EQUIVALENT TO BRITISH THERMAL UNITS

British thermal	0	1	2	3	4	5	6	7	8	9	British thermal
0	....	.252	.504	.756	1.008	1.260	1.512	1.764	2.016	2.268	0
10	2.52	2.772	3.024	3.276	3.528	3.780	4.032	4.284	4.536	4.788	10
20	5.04	5.292	5.544	5.796	6.048	6.300	6.552	6.804	7.056	7.308	20
30	7.56	7.812	8.064	8.316	8.568	8.820	9.072	9.324	9.576	9.828	30
40	10.08	10.332	10.584	10.836	11.088	11.340	11.592	11.844	12.096	12.348	40
50	12.60	12.852	13.104	13.356	13.608	13.860	14.112	14.364	14.616	14.868	50
60	15.12	15.372	15.624	15.876	16.128	16.380	16.632	16.884	17.136	17.388	60
70	17.64	17.892	18.144	18.396	18.648	18.900	19.152	19.404	19.656	19.908	70
80	20.16	20.412	20.664	20.916	21.168	21.420	21.672	21.924	22.176	22.428	80
90	22.68	22.932	23.184	23.436	23.688	23.940	24.192	24.444	24.696	24.948	90

1 British thermal unit = 251.996 therms, or gram calories.

Table 4.—Melting-points of Metals and Other Substances

These melting-points were collected by Dr. G. K. Burgess, of the Bureau of Standards, Washington, D. C. Those shown in CAPITALS are accepted by the Bureau as standard at this time (1911).

These melting-points were obtained on the purest metals obtainable. Lower melting-points may be expected with metals of less purity.

	Fahrenheit degrees	Centigrade degrees
ALUMINUM	1216	658
ANTIMONY	1166	630
Arsenic	1472	800
Bismuth	518	270
CADMIUM	610	321
Calcium	1481	805
Chromium	2741	1505
COBALT	2714	1490
COPPER	1981	1083
GOLD	1945	1063
Iridium (?)	4172	2300
IRON	2768	1520
LEAD	621	327
Magnesium	1204	651
Manganese	2237	1225
MERCURY	38	39
Molybdenum (?)	4532	2500
NICKEL	2642	1450
PALLADIUM	2822	1550
Phosphorus	111	44
PLATINUM	3191	1755
POTASSIUM	144	62
Rhodium (?)	3452	1900
Silicon	2588	1420
SILVER	1762	961
SODIUM	207	97
Tantalum (?)	5252	2900
TIN	450	232
Titanium (?)	3362	1850
TUNGSTEN	5432	3000
Uranium	4352	2400
Vanadium (?)	3182	1750
ZINC	786	419

(?) Doubtful

Some Other Melting-points

GLASS	1832	1000
GLASS, LEAD FREE	2192	1200
DELTA METAL	1742	950
BARIUM CHLORIDE	1635	891
POTASSIUM CHLORIDE	1325	718
SODIUM CHLORIDE	1472	800
Sulphur	{ 237 248	{ 114 120
Fusible metals:		
1 tin, 2 lead	361	183
1 tin, 1 lead	304	151
3 tin, 2 lead	275	135
4 tin, 4 lead, 1 bismuth	263	128
3 tin, 5 lead, 8 bismuth	212	100

Table 5.—Lineal Expansion of Solids at Ordinary Temperatures

British Board of Trade—from Clark's Manual of Rules, Tables and Data

	For 1 deg. Fahr.	For 1 deg. Cent.
	(Length—1)	
Aluminum (cast)	.00001234	.00002221
Brass (cast)	.00000957	.00001722
Brass (plate)	.00001052	.00001894
Bronze (copper, tin 2½, zinc 1)	.00000986	.00001774
Bismuth	.00000975	.00001755
Cement, Portland (mixed) pure	.00000594	.00001070
Concrete (cement, mortar and pebbles)	.00000795	.00001430
Copper	.00000887	.00001596
Glass, English flint	.00000451	.00000812
Glass, thermometer	.00000499	.00000897
Glass, hard	.00000397	.00000714
Gold, pure	.00000786	.00001415
Iron, wrought	.00000648	.00001166
Iron, cast	.00000556	.00001001
Lead	.00001571	.00002828
Mercury (cubical expansion)	.00009984	.00017971
Nickel	.00000695	.00001251
Platinum	.00000479	.00000863
Platinum 85%; iridium 15%	.00000453	.00000815
Porcelain	.00000200	.00000360
Silver, pure	.00001079	.00001943
Steel, cast	.00000636	.00001144
Steel, tempered	.00000689	.00001240
Tin	.00001163	.00002094
Wood, pine	.00000276	.00000496
Zinc	.00001407	.00002532
Zinc 8; tin 1	.00001496	.00002692

The transmission of heat through metallic tubes, according to the report of the Research Committee A. S. M. E. (*Trans. A. S. M. E., Vol.* 33) may be determined from the following table of constants:

Conditions	B.t.u. transmitted per sq. ft. per hour, per deg. Fahr. difference of temperature
From flue gas to water under the conditions existing in economizers.	2
From flue gas to boiling water under boiler conditions (feed heated to steam temperature).	10
From flue gas to steam or air under superheater or air-heater conditions.	4
From hot air to colder water under air-cooler conditions	4
From condensing steam to water under surface condensing conditions, no air present.	1500
When an ordinary amount of air is present	500 to 600

The heat transfer capacity of metallic tubes is so largely in excess of the heat that can be brought in contact with the tube surfaces, that the material of the tube has little to do with the amount of heat transferred.

STEAM BOILERS

The horse-power of boilers is, in a sense, a misnomer, as that term is a measure properly applicable only to dynamic effect. But as boilers are necessary to drive steam engines, the same measure applied to steam engines has come to be universally applied to the boiler, and cannot well be discarded.

The standard adopted by the judges at the Centennial Exhibition, of 30 lbs. water per hour, evaporated, at 70 lbs. pressure, from 100 deg., for each horse-power, is a fair one for both boilers and engines, and has been favorably received by engineers and steam users generally. The Centennial standard is practically equivalent to the evaporation of 34½ lbs. of water from and at 212 deg. Fahr. per hour. Expressed in this form it has been endorsed by the American Society of Mechanical Engineers.

Square feet of heating surface is no criterion as between different styles of boilers—a square foot under some circumstances being many times as efficient as in others; but when an average rate of evaporation per sq. ft. for any given boiler has been fixed upon by experiment, there is no more convenient way of rating the power of others of the same style. The following table gives an approximate list of sq. ft. of heating surface per h.p. in different styles of boilers, and various other data for comparison:

HEATING SURFACE PER H.P. OF STEAM BOILERS
From Steam by Permission of the Babcock and Wilcox Co.

Type of boiler	Sq. ft. of heating surface for 1 h.p.	Coal per sq. ft. H.S. per hour	Relative economy	Relative rapidity of steaming	Authority
Water-tube	10 to 12	.3	1.00	1.00	Isherwood
Tubular	14 to 18	.25	.91	.50	Isherwood
Flue	8 to 12	.4	.79	.25	Prof. Trowbridge
Plain cylinder	6 to 10	.5	.69	.20	
Locomotive	12 to 16	.275	.85	.55	
Vertical tubular	15 to 20	.25	.80	.60	

By general consent the best digested official rules for the strength of steam boilers are those of Massachusetts, from which the following abstract has been taken:

Steel shall be made by the open-hearth process, and will be considered as manufactured by the basic method unless the report of test states that the acid method has been used.

All plates and rivets used in the construction of steel shells or drums of boilers shall be as specified by the American Society for Testing Materials, adopted 1901.

There shall be three classes of open-hearth boiler plate and rivet steel, namely, flange or boiler steel, fire-box steel and extra soft steel, which shall conform to the limits in chemical composition given in Table 1.

TABLE 1.—CHEMICAL COMPOSITION OF BOILER MATERIALS

	Flange or boiler steel (per cent.)	Fire-box steel (per cent.)	Extra soft steel (per cent.)
Phosphorus shall not exceed	Acid, .06 Basic, .04	Acid, .04 Basic, .03	Acid, .04 Basic, .04
Sulphur shall not exceed	.05	.04	.04
Manganese	.30 to .60	.30 to .50	.30 to .50

Steel for boiler rivets shall be of the extra soft class.

The three classes of open-hearth boiler plate and rivet steel—namely, flange or boiler steel, fire-box steel and extra soft steel—shall conform to the physical qualities given in Table 2.

TABLE 2.—PHYSICAL PROPERTIES OF BOILER MATERIAL

	Flange or boiler steel	Fire-box steel	Extra soft steel
Tensile strength, lbs. per sq. in.	55,000 to 65,000	52,000 to 62,000	45,000 to 55,000
Yield point, in lbs. per sq. in., shall not be less than,	½ T.S.	½ T.S.	½ T.S.
Elongation per cent. in 8 ins. shall not be less than,	25	26	28

For material less than $\frac{5}{16}$ in. and more than ¾ in. in thickness the following modifications shall be made in the requirements for elongation:

(*a*) For each increase of ⅛ in. in thickness above ¾ in. a deduction of 1 per cent. shall be made from the specified elongation.

(*b*) For each decrease of $\frac{1}{16}$ in. in thickness below $\frac{5}{16}$ in. a deduction of 2½ per cent. shall be made from the specified elongation.

The three classes of open-hearth boiler-plate and rivet steel shall conform to the following bending tests; and for this purpose the test specimen shall be 1½ ins. wide, if possible, and for all material ¾ in. or less in thickness the test specimen shall be of the same thickness as that of the finished material from which it is cut, but for material more than ¾ in. thick the bending test specimen may be ½ in. thick.

Rivet rounds shall be tested of full size as rolled.

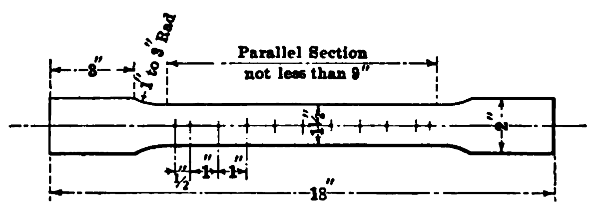


FIG. 1.—Standard test piece of 8 ins. gaged length, piece to be of same thickness as plate.

(*c*) Test specimens cut from the rolled material, as specified above, shall be subjected to a cold bending test and also to a quenched bending test. The cold bending test shall be made on the material in the condition in which it is to be used, and prior to the quenched bending test the specimen shall be heated to a light cherry red, as seen in the dark, and quenched in water, the temperature of which is between 80 deg. and 90 deg. Fahr.

(*d*) Flange or boiler steel, fire-box steel and rivet steel, both before and after quenching, shall bend cold 180 deg. flat on itself without fracture on the outside of the bent portion.

For fire-box steel a sample taken from a broken tensile test specimen shall not show any single seam or cavity more than ¼ in. long in either of the three fractures obtained on the test for homogeneity, as described below.

The standard test specimen of 8 in. gaged length shall be used to determine the physical properties. The standard shape of the test specimen for sheared plates shall be as shown in Fig. 1.

For other material the test specimen may be the same as for sheared plates, or it may be planed or turned parallel throughout its entire length; and in all cases, where possible, two opposite sides of the test specimens shall be the rolled surfaces. Rivet rounds and small rolled bars shall be tested of full size as rolled.

One tensile test specimen will be furnished from each plate as it is rolled, and two tensile test specimens will be furnished from each melt of rivet rounds. In case any of these develops flaws or breaks outside of the middle third of its gaged length, it may be discarded and another test specimen substituted therefor.

For material ¾ in. or less in thickness the bending test specimen shall have the natural rolled surface on two opposite sides. The bending test specimens cut from plates shall be 1½ ins. wide, and for material more than ¾ in. thick the bending test specimen may be ½ in thick. The sheared edges of bending test specimens may be milled or planed. The bending test specimens for rivet rounds shall be of full size as rolled. The bending tests may be made by pressure or by blows.

One cold bending specimen and one quenched bending specimen will be furnished from each plate as it is rolled. Two cold bending specimens and two quenched bending specimens will be furnished from each melt of rivet rounds. The homogeneity test for fire-box steel shall be made on one of the broken tensile test specimens.

The homogeneity test for fire-box steel is made as follows: A portion of the broken tensile test specimen is either nicked with a chisel or grooved on a machine, transversely about $\frac{1}{16}$ in. deep, in three places about 2 ins. apart. The first groove should be made on one side 2 ins. from the square end of the specimen; the second, 2 ins. from it on the opposite side; the third, 2 ins. from the last, and on the opposite side from it. The test specimen is then put in a vise, with the first groove about ¼ in. above the jaws, care being taken to hold it firmly. The projecting end of the test specimen is then broken off by means of a hammer, a number of light blows being used, and the bending being away from the groove. The specimen is broken at the other two grooves in the same way. The object of this treatment is to open and render visible to the eye any seams due to failure to weld up, or to foreign interposed matter or cavities due to gas bubbles in the ingot. After rupture, one side of each fracture is examined, a pocket lens being used, if necessary, and the length of the seams and cavities is determined.

For the purposes of this specification the yield point shall be determined by the careful observation of the drop of the beam or halt in the gage of the testing machine.

In order to determine if the material conforms to the chemical limitations prescribed above, analysis shall be made of drillings taken from a small test ingot. An additional check analysis may be made from a tensile specimen of each melt used on an order, other than in locomotive fire-box steel. In the case of locomotive fire-box steel a check analysis may be made from the tensile specimen from each plate as rolled.

The variation in cross-section of weight of more than 2½ per cent. from that specified will be sufficient cause for rejection, except in the case of sheared plates, which will be covered by the following permissible variations:

(*e*) Plates 12½ lbs. per sq. ft. or heavier, up to 100 ins. wide, when ordered to weight, shall not average more than 2½ per cent. variation above or 2½ per cent. below the theoretical weight; when 100 ins. wide and over, 5 per cent. above or 5 per cent. below the theoretical weight.

(*f*) Plates under 12½ lbs. per sq. ft., when ordered to weight, shall not average a greater variation than the following: Up to 75 ins. wide, 2½ per cent. below the theoretical weight; 75 ins. wide up to 100 ins. wide, 5 per cent. below the theoretical weight; when 100 ins. wide and over, 10 per cent. above or 3 per cent. below the theoretical weight.

(*g*) For all plates ordered to gage there will be permitted an average excess of weight over that corresponding to the dimensions on the order equal in amount to that specified in Table 3.

All finished material shall be free from injurious surface defects and laminations, and must have a workmanlike finish.

Each plate shall be distinctly stamped by the manufacturer with the heat number.

Table 3.—Allowances for Overweight for Rectangular Plates when Ordered to Gage

[Plates will be considered up to gage if measuring not over $\frac{1}{100}$ in. less than the ordered gage. The weight of 1 cu. in. of rolled steel is assumed to be .2833 lb.]

Plates ¼ In. and Over in Thickness

Thickness of plate, in.	Width of plate		
	Up to 75 ins., per cent.	75 to 100 ins., per cent.	Over 100 ins., per cent.
¼	10	14	18
$\frac{5}{16}$	8	12	16
⅜	7	10	13
$\frac{7}{16}$	6	8	10
½	5	7	9
$\frac{9}{16}$	4½	6½	8½
⅝	4	6	8
Over ⅝	3½	5	6½

Each plate shall be distinctly stamped by the manufacturer in at least five places in the following manner: At the four corners, at a distance of about 12 ins. from the edges, and at or near the center of the plate, with the name of the manufacturer, place where manufactured, brand and lowest tensile strength.

Each head shall be distinctly stamped by the manufacturer on each side with the name of the manufacturer, place where manufactured, brand and lowest tensile strength; stamps to be so located as to be plainly visible when the head is finished.

Shells, drums and butt straps shall be of open-hearth fire-box steel, as specified above.

Heads, combustion chambers, furnaces, or any plates that require staying or flanging, shall be of open-hearth flange, fire-box or extra soft steel, as specified above.

Rivets shall be of open-hearth extra soft steel, as specified above.

Cast steel for use in boiler and steam superheater mountings, manhole frames, steam pipe, fittings, side lugs, or any other parts of boilers or superheaters where cast steel is used, shall not have less than 50,000 lbs. tensile strength.

Cast-iron for use in boiler mountings, steam-pipe fittings, side lugs, or any other parts of boilers where cast-iron is permitted to be used, shall not have less than 18,000 lbs. tensile strength.

Cross pipes connecting the steam and water drums of water-tube boilers, and cross boxes, shall be of wrought or cast steel when the working pressure exceeds 160 lbs. per sq. in.

Mud drums of water-tube boilers shall be of wrought or cast steel when the working pressure exceeds 160 lbs. per sq. in.

Pressure parts of superheaters, attached to boilers or separately fired, shall be of wrought or cast steel when the working pressure exceeds 50 lbs. per sq. in.

Boiler and superheater mountings, such as nozzles, cross pipes, steam pipes, fittings, valves and their bonnets shall be of wrought or cast steel when exposed to steam which is superheated over 80 deg. Fahr.

Waterleg and door-frame rings of vertical fire-tube boilers 36 ins. or over in diameter, shall be of wrought or cast steel, or wrought iron.

Waterleg and door-frame rings of locomotive-type boilers shall be wrought or cast steel, or wrought iron.

The resistance to crushing of mild steel shall be taken at 95,000 lbs. per sq. in. of cross-sectional area.

The maximum shearing strength of rivets per sq. in. of cross-sectional area shall be taken as in Table 4.

Table 4.—Shearing Strength of Rivets per Sq. In.

	Lbs.
Iron rivets in single shear	38,000
Iron rivets in double shear	70,000
Steel rivets in single shear	42,000
Steel rivets in double shear	78,000

Table 5 gives the allowable shearing strength of rivets from $\frac{11}{16}$ in. to $1\frac{1}{16}$ ins. in diameter, in pounds.

TABLE 5.—SHEARING STRENGTH OF RIVETS

Diameter of rivet after driving	11/16 in. .6875	3/4 in. .75	13/16 in. .8125	7/8 in. .875	15/16 in. .9375	1 1/16 ins. 1.0625
Cross-sectional area of rivet after driving.	.3712 sq. in.	.4418 sq. in.	.5185 sq. in.	.6013 sq. in.	.6903 sq. in.	.8866 sq. in.
	Allowable shearing strength, in lbs.					
Iron, single shear	14,106	16,788	19,703	22,849	26,231	33,691
Iron, double shear	25,984	30,926	36,295	42,091	48,321	62,062
Steel, single shear	15,590	18,556	21,777	25,255	28,993	37,237
Steel, double shear	28,954	34,460	40,443	46,901	53,843	69,155

The lowest factors of safety used for boilers, the shells or drums of which are exposed to the products of combustion and the longitudinal joints of which are of lap-riveted construction, shall be as follows:

(*a*) Five for boilers not over ten years old.

(*b*) Five and five-tenths for boilers over ten and not over fifteen years old.

(*c*) Five and seventy-five hundredths for boilers over fifteen and not over twenty years old.

(*d*) Six for boilers over twenty years old.

(*e*) Five for boilers, the longitudinal joints of which are of lap-riveted construction and the shells or drums of which are not exposed to the products of combustion.

TABLE 6.—AREAS OF GRATE SURFACES IN SQ. FT. FOR OTHER THAN SPRING-LOADED SAFETY VALVES

Maximum pressure allowed per sq. in. on the boiler		Zero to 25 lbs.	Over 25 to 50 lbs.	Over 50 to 100 lbs.
Diameter of valve in ins.	Area of valve in sq. ins.	Area of grate in sq. ft.		
1	.7854	1.50	1.75	2.00
1¼	1.2272	2.25	2.50	3.00
1½	1.7671	3.00	3.75	4.02
2	3.1416	5.50	6.50	7.00
2½	4.9087	8.25	10.00	11.00
3	7:0686	11.75	14.25	16.00
3½	9.6211	16.00	19.50	21.75
4	12.5660	21.00	25.50	28.25
4½	15.9040	26.75	32.50	36.00
5	19.6350	32.75	40.00	44.00

TABLE 7.—AREAS OF GRATE SURFACES IN SQ. FT. FOR DIRECT SPRING-LOADED SAFETY VALVES

		$W=\frac{75}{3600}$ $P=40$ $A=.401$	$W=\frac{100}{3600}$ $P=65$ $A=.329$	$W=\frac{160}{3600}$ $P=115$ $A=.297$	$W=\frac{160}{3600}$ $P=140$ $A=.244$	$W=\frac{200}{3600}$ $P=190$ $A=.224$	$W=\frac{240}{3600}$ $P=240$ $A=.213$
Maximum pressure allowed per sq. in. on the boiler		Zero to 25 lbs.	Over 25 to 50 lbs.	Over 50 to 100 lbs	Over 100 to 150 lbs	Over 150 to 200 lbs	Over 200 lbs.
Diam'er of valves in in.	Area of valve, in sq. ins.	Area of grate in sq. ft.					
1	.7854	2.00	2.50	2.75	3.25	3.5	3.75
1¼	1.2272	3.25	4.00	4.25	5.00	5.5	5.75
1½	1.7671	4.50	5.50	6.00	7.25	8.0	8.50
2	3.1416	8.00	9.75	10.75	13.00	14.0	15.00
2½	4.9087	12.50	15.00	16.50	20.00	22.0	23.00
3	7.0686	17.75	21.50	24.00	29.00	31.5	33.25
3½	9.6211	24.00	29.50	32.50	39.50	43.0	45.25
4	12.5660	31.50	38.25	42.50	51.50	56.0	59.00
4½	15.9040	40.00	48.50	53.50	65.00	71.0	74.25
5	19.6350	49.00	60.00	66.00	80.00	88.0	92.25

When the conditions exceed those on which Table 7 is based, the following formula shall be used:

$$A=\frac{W70}{P}\times 11$$

In which A = area of direct spring-loaded safety valve in sq. ins. per sq. ft. of grate surface,

W = weight of water in lbs. evaporated per sq. ft. of grate surface per sec.,

P = pressure (absolute) at which the safety valve is set to blow.

Fusible plugs shall be filled with pure tin.

The least diameter of fusible metal shall not be less than ½ in., except for working pressures of over 175 lbs., or when it is necessary to place a fusible plug in a tube, in which cases the least diameter of fusible metal shall not be less than ⅜ in.

The following formulas for the strength of boiler joints are from the Massachusetts rules. In them:

$T.S.$ = tensile strength of plate, lbs. per sq. in.

t = thickness of plate, ins.

b = thickness of butt strap, ins.

P = pitch of rivets, ins., on row having greatest pitch.

d = diameter of rivet after driving, ins.

a = cross-sectional area of rivet after driving, sq. ins.

s = strength of rivet in single shear, as given in Table 4.

S = strength of rivet in double shear, as given in Table 4.

c = crushing strength of mild steel, as given above.

n = number of rivets in single shear in a unit of length of joint.

N = number of rivets in double shear in a unit of length of joint.

Lap joint longitudinal or circumferential, single riveted, Fig. 2.

A = strength of solid plate $= P\times t\times T.S.$

B = strength of plate between rivet holes $=(P-d)t\times T.S.$

C = shearing strength of one rivet in single shear $=n\times s\times a$.

D = crushing strength of plate in front of one rivet $=d\times t\times c$.

Divide B, C or D (whichever is the least) by A, and the quotient will be the efficiency of a single-riveted lap joint.

Lap joint longitudinal or circumferential, double riveted, Fig. 3.

A = strength of solid plate $=P\times t\times T.S.$

B = strength of plate between rivet holes $=(P-d)t\times T.S.$

C = shearing strength of two rivets in single shear $=n\times s\times a$.

D = crushing strength of plate in front of two rivets $=n\times d\times t\times c$.

Divide B, C or D (whichever is the least) by A, and the quotient will be the efficiency of a double-riveted lap joint.

Butt and double strap joint, double riveted, Fig. 4.

A = strength of solid plate $=P\times t\times T.S.$

B = strength of plate between rivet holes in the outer row $=(P-d)t\times T.S.$

C = shearing strength of two rivets in double shear, plus the shearing strength of one rivet in single shear $=N\times S\times a+n\times s\times a$

D = strength of plate between rivet holes in the second row, plus the shearing strength of one rivet in single shear in the outer row $=(P-2d)t\times T.S.+n\times s\times a$.

E = strength of plate between rivet holes in the second row, plus the crushing strength of butt strap in front of one rivet in the outer row $=(P-2d)t\times T.S.+d\times b\times c$.

F = crushing strength of plate in front of two rivets, plus the crushing strength of butt strap in front of one rivet $=N\times d\times t\times c+n\times d\times b\times c$.

G = crushing strength of plate in front of two rivets, plus the shearing strength of one rivet in single shear $=N\times d\times t\times c+n\times s\times a$.

Divide B, C, D, E, F or G (whichever is the least) by A, and the quotient will be the efficiency of a butt and double strap joint, double-riveted.

Butt and double strap joint, triple riveted, Fig. 5.

A = strength of solid plate = $P \times t \times T.S.$

B = strength of plate between rivet holes in the outer row = $(P-d)t \times T.S.$

C = shearing strength of four rivets in double shear, plus the shearing strength of one rivet in single shear = $N \times S \times a + n \times s \times a$.

D = strength of plate between rivet holes in the second row, plus the shearing strength of one rivet in single shear in the outer row = $(P-2d)t \times T.S. + n \times s \times a$.

E = strength of plate between rivet holes in the second row, plus the crushing strength of butt strap in front of one rivet in the outer row = $(P-2d)t \times T.S. + d \times b \times c$.

F = crushing strength of plate in front of four rivets, plus the crushing strength of butt strap in front of one rivet = $N \times d \times t \times c + n \times d \times b \times c$.

G = crushing strength of plate in front of four rivets, plus the shearing strength of one rivet in single shear = $N \times d \times t \times c + n \times s \times a$.

G = strength of plate between rivet holes in the third row, plus the crushing strength of butt strap in front of two rivets in the second row and one (1) rivet in the outer row = $(P-4d)t \times T.S. + n \times d \times b \times c$.

H = crushing strength of plate in front of eight rivets plus the crushing strength of butt strap in front of three rivets = $N \times d \times t \times c + n \times d \times b \times c$.

I = crushing strength of plate in front of eight rivets plus the shearing strength of two rivets in the second row and one rivet in the outer row, in single shear = $N \times d \times t \times c + n \times s \times a$.

Divide B, C, D, E, F, G, H or I (whichever is the least) by A, and the quotient will be the efficiency of a butt and double strap joint quadruple-riveted.

The working pressure of steam boilers is given by the formula:

$$\frac{T.S. \times t \times \%}{R. \times F.S.} = \text{maximum allowable working pressure, lbs. per sq. in}$$

in which $T.S.$ = tensile strength of shell plates, lbs.,

t = minimum thickness of shell plates, ins.,

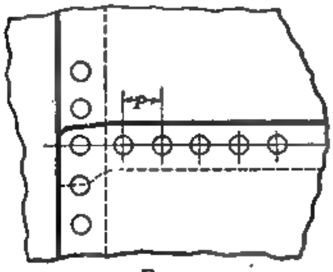


FIG. 2.

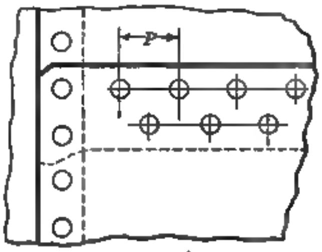


FIG. 3.

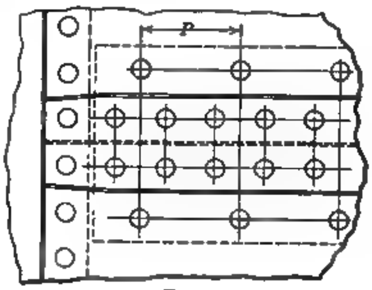


FIG. 4

FIG. 5.

FIGS. 2 to 6.—Boiler joints.

Divide B, C, D, E, F or G (whichever is the least) by A, and the quotient will be the efficiency of a butt and double strap joint, triple-riveted.

Butt and double strap joint, quadruple-riveted, Fig. 6.

A = strength of solid plate = $P \times t \times T.S.$

B = strength of plate between rivet holes in the outer row = $(P-d)t \times T.S.$

C = shearing strength of eight rivets in double shear, plus the shearing strength of three rivets in single shear = $N \times S \times a + n \times s \times a$.

D = strength of plate between rivet holes in the second row, plus the shearing strength of one rivet in single shear in the outer row = $(P-2d)t \times T.S. + n \times s \times a$.

E = strength of plate between rivet holes in the third row, plus the shearing strength of two rivets in the second row in single shear and one rivet in single shear in the outer row = $(P-4d)t \times T.S. + n \times s \times a$.

F = strength of plate between rivet holes in the second row, plus the crushing strength of butt strap in front of one rivet in the outer row = $(P-2d)t \times T.S. + d \times b \times c$.

% = efficiency of longitudinal joint or ligament between tube holes, whichever is the least,

R = radius of the inside diameter of the outside course of the shell or drum,

$F.S.$ = the factor of safety, the lowest value of which in Massachusetts is 5.

The minimum thickness of a convex bumped head shall be determined by the formula:

$$\frac{R \times F.S. \times P}{T.S.} = t$$

The minimum thickness of a concave bumped head shall be determined by the formula:

$$\frac{R \times F.S. \times P}{.6(T.S.)} = t$$

In which R = one-half the radius to which the head is bumped,

$F.S.$ = 5 = factor of safety,

P = working pressure, lbs. per sq. in., for which the boiler is designed,

$T.S.$ = tensile strength, lbs. per sq. in., stamped on the head by the manufacturer,

t = thickness of head, ins.

When a convex or concave head has a manhole opening, the thickness as found by the above formulas shall be increased by not less than $\frac{1}{8}$ in.

When a convex or concave head has a manhole opening, the flange shall be turned inward, and to a depth of not less than three times the thickness of the head.

The minimum thickness of plates in stayed flat surface construction shall be $\frac{5}{16}$ in.

TABLE 8.—MINIMUM THICKNESS OF BOILER PLATES PERMITTED IN MASSACHUSETTS

Shell Plates

When the diameter of shell is—			
36 ins. or under	Over 36 to 54 ins. inclusive	Over 54 to 72 ins. inclusive	Over 72 ins.
$\frac{1}{4}$ in.	$\frac{5}{16}$ in.	$\frac{3}{8}$ in.	$\frac{1}{2}$ in.

Butt Straps

Thickness of shell plates	Minimum thickness of butt straps	Thickness of shell plates	Minimum thickness of butt straps
$\frac{1}{4}$ in.	$\frac{1}{4}$ in.	$\frac{17}{32}$ in.	$\frac{7}{16}$ in.
$\frac{9}{32}$ in.	$\frac{1}{4}$ in.	$\frac{9}{16}$ in.	$\frac{7}{16}$ in.
$\frac{5}{16}$ in.	$\frac{1}{4}$ in.	$\frac{5}{8}$ in.	$\frac{1}{2}$ in.
$\frac{11}{32}$ in.	$\frac{1}{4}$ in.	$\frac{3}{4}$ in.	$\frac{1}{2}$ in.
$\frac{3}{8}$ in.	$\frac{5}{16}$ in.	$\frac{7}{8}$ in.	$\frac{5}{8}$ in.
$\frac{13}{32}$ in.	$\frac{5}{16}$ in.	1 in.	$\frac{3}{4}$ in.
$\frac{7}{16}$ in.	$\frac{3}{8}$ in.	$1\frac{1}{8}$ ins.	$\frac{3}{4}$ in.
$\frac{15}{32}$ in.	$\frac{3}{8}$ in.	$1\frac{1}{4}$ ins.	$\frac{7}{8}$ in.
$\frac{1}{2}$ in.	$\frac{7}{16}$ in.	....	....

Tube Sheets

When the diameter of tube sheet is—			
42 ins. or under	Over 40 to 54 ins. inclusive	Over 54 to 72 ins. inclusive	Over 72 ins.
$\frac{3}{8}$ in.	$\frac{7}{16}$ in.	$\frac{1}{2}$ in.	$\frac{9}{16}$ in.

TABLE 9.—MAXIMUM ALLOWABLE PITCH, INS., OF SCREWED STAY-BOLTS. ENDS RIVETED OVER

Pressure, in lbs. per sq. in.	Thickness of plate						
	$\frac{5}{16}$ in.	$\frac{3}{8}$ in.	$\frac{7}{16}$ in.	$\frac{1}{2}$ in.	$\frac{9}{16}$ in.	$\frac{5}{8}$ in.	$\frac{11}{16}$ in.
	Maximum pitch of stay-bolts, in ins.						
100	$5\frac{1}{2}$	$6\frac{1}{4}$	7	$7\frac{3}{4}$	$8\frac{1}{2}$	......	......
110	$5\frac{1}{4}$	6	$6\frac{3}{4}$	$7\frac{3}{8}$	$8\frac{1}{8}$	......	......
120	$5\frac{1}{8}$	$5\frac{3}{4}$	$6\frac{1}{2}$	$7\frac{1}{8}$	$7\frac{7}{8}$	$8\frac{1}{2}$	......
125	5	$5\frac{5}{8}$	$6\frac{3}{8}$	7	$7\frac{3}{4}$	$8\frac{3}{8}$	......
130	5	$5\frac{5}{8}$	$6\frac{1}{4}$	$6\frac{7}{8}$	$7\frac{5}{8}$	$8\frac{1}{4}$	......
140	$4\frac{3}{4}$	$5\frac{3}{8}$	6	$6\frac{5}{8}$	$7\frac{3}{8}$	[illegible]	......
150	$4\frac{3}{4}$	$5\frac{1}{4}$	$5\frac{7}{8}$	$6\frac{1}{2}$	$7\frac{1}{8}$	$7\frac{3}{4}$	$8\frac{3}{8}$
160	$4\frac{5}{8}$	$5\frac{1}{8}$	$5\frac{3}{4}$	$6\frac{1}{4}$	$6\frac{7}{8}$	$7\frac{1}{2}$	[illegible]
170	$4\frac{1}{2}$	5	$5\frac{5}{8}$	$6\frac{1}{8}$	$6\frac{3}{4}$	$7\frac{1}{4}$	$7\frac{7}{8}$
180	$4\frac{3}{8}$	$4\frac{7}{8}$	$5\frac{1}{2}$	6	$6\frac{1}{2}$	$7\frac{1}{8}$	$7\frac{3}{4}$
190	$4\frac{3}{8}$	$4\frac{7}{8}$	$5\frac{3}{8}$	$5\frac{7}{8}$	$6\frac{3}{8}$	7	$7\frac{1}{2}$
200	$4\frac{1}{4}$	$4\frac{3}{4}$	$5\frac{1}{4}$	$5\frac{3}{4}$	$6\frac{1}{4}$	$6\frac{3}{4}$	$7\frac{3}{8}$
225	$4\frac{1}{8}$	$4\frac{1}{2}$	5	$5\frac{1}{2}$	6	$6\frac{1}{2}$	7
250	4	$4\frac{3}{8}$	$4\frac{3}{4}$	$5\frac{1}{4}$	$5\frac{3}{4}$	$6\frac{1}{4}$	$6\frac{5}{8}$
300	$3\frac{3}{4}$	$4\frac{1}{8}$	$4\frac{1}{2}$	$4\frac{7}{8}$	$5\frac{3}{8}$	$5\frac{3}{4}$	$6\frac{1}{8}$

When the maximum allowable pitch is $5\frac{1}{2}$ ins. or less, the stay-bolts adjacent to a furnace door or other boiler fitting, hand hole or other opening, may have an increased pitch of not over 1 in.

When a pitch not exceeding $8\frac{1}{2}$ ins. is required and is not given in the table, the following formula shall be used:

$$S=\sqrt{\frac{C\times(t+1)^2}{P}+6},\quad P=\frac{C(t+1)^2}{S^2-6},\quad t=\sqrt{\frac{P(S^2-6)}{C}}-1.$$

in which S = maximum pitch of stay-bolts, ins.

C = a constant = 66.

t = thickness of plate, in *sixteenths* of an in.

P = working pressure, lbs. per sq. in.

When hollow stay-bolts are used, having the hole $\frac{3}{8}$ in. in diameter or over, the maximum allwoable pitch given in the above table may be increased by the mean diameter of the stay-bolt:

Mean diameter of stay-bolt =

$$\frac{\text{least outside diameter of stay-bolt} + \text{diameter of hole in stay-bolt}}{2}$$

The area of a segment of a head to be stayed shall be the area enclosed by lines drawn 3 ins. from the shell and 2 ins. from the tubes, as shown in Figs. 7 and 8

FIG. 7.

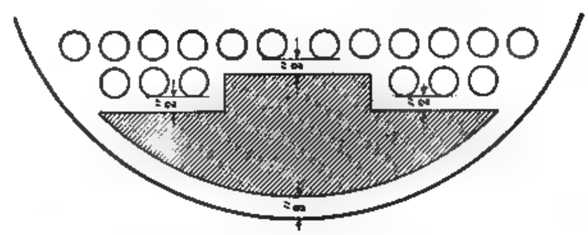


FIG. 8.

FIGS. 7 and 8.—Areas to be braced in steam boiler heads.

FIG. 9.—Allowance for man-hole openings.

When an area is required that is not given in Table 10, the following formula shall be used:

$$\frac{4H^2}{3}\sqrt{\frac{2R}{H}-.608} = \text{area of segment to be stayed, sq. ins.}$$

in which H = distance from tubes to shell, minus 5 ins.

R = radius of boiler, minus 3 ins.

When a flat head has a manhole opening, the flange of which is formed from the solid sheet and turned inward to a depth of not less than twice the thickness of the head, an area 2 ins. wide all around the manhole opening, as shown in Fig. 9, may be deducted from the total area of head, including manhole opening to be stayed.

TABLE 10.—AREAS TO BE BRACED IN ACCORDANCE WITH FIG. 7

Height from tubes to shell.	Diameter of boiler												
	24 ins.	30 ins.	36 ins.	42 ins.	48 ins.	54 ins.	60 ins.	66 ins.	72 ins.	78 ins.	84 ins.	90 ins.	96 ins
	Area to be stayed in sq. ins.												
8 ins.	28	33	37	40	43	47	51	53	55	58	60	63	65
8½ ins.	35	41	46	51	55	59	63	66	70	74	76	80	82
9 ins.	42	49	56	62	67	72	76	82	86	90	92	95	98
9½ ins.	50	58	66	70	80	86	91	96	101	105	111	116	119
10 ins.	57	68	77	85	93	99	106	112	117	123	129	132	137
10½ ins.	66	78	89	98	107	114	123	131	135	142	147	153	160
11 ins.	74	88	100	111	121	130	138	147	155	161	169	174	183
11½ ins.	83	99	112	124	137	146	156	165	173	181	189	196	204
12 ins.	91	109	125	139	151	163	174	184	194	203	213	219	230
12½ ins.		120	138	153	167	180	193	204	216	224	234	243	252
13 ins.	—	132	151	168	183	197	211	224	235	247	256	267	279
13½ ins.		143	164	183	200	216	230	246	258	270	282	293	302
14 ins.	—	155	178	199	217	234	250	266	280	294	305	319	331
14½ ins.		167	192	215	235	254	271	287	303	318	333	345	360
15 ins.	—	178	206	231	252	273	291	309	326	343	357	372	386
15½ ins.			220	247	271	291	312	332	350	368	382	400	417
16 ins.	—	—	235	263	289	312	334	355	374	394	411	423	443
16½ ins.			249	281	308	332	357	380	399	420	436	457	475
17 ins.	—	—	264	297	326	353	378	402	425	447	467	486	502
17½ ins.				314	345	374	400	426	449	471	494	516	536
18 ins.	—	—	—	331	365	396	424	450	476	500	520	543	564
18½ ins.				349	384	417	448	476	501	526	552	577	598
19 ins.	—	—	—	366	404	439	470	500	529	555	580	604	631
19½ ins.				384	424	461	496	528	558	584	613	641	663
20 ins.	—	—	—	401	444	483	519	552	583	613	642	667	699
20½ ins.					464	505	543	578	613	643	675	706	729
21 ins.	—	—	—	—	485	528	568	604	640	673	705	733	766
21½ ins.					505	551	594	632	669	703	739	766	797
22 ins.	—	—	—	—	526	574	618	658	697	734	769	800	835
22½ ins.						597	643	687	726	765	800	835	867
23 ins.	—	—	—	—	—	620	668	713	754	796	830	869	906
23½ ins.						642	695	740	784	827	866	904	945
24 ins.	—	—	—	—	—	667	719	768	814	859	897	939	978
24½ ins.						689	745	797	843	892	934	975	1018
25 ins.	—	—	—	—	—	714	771	825	875	922	966	1010	1051
25½ ins.						737	798	855	907	956	1003	1047	1092
26 ins.	—	—	—	—	—	761	824	882	936	987	1035	1083	1126
26½ ins.							850	909	968	1024	1073	1120	1167
27 ins.	—	—	—	—	—	—	877	939	998	1053	1106	1157	1202
27½ ins.							904	968	1030	1089	1145	1195	1243
28 ins.	—	—	—	—	—	—	930	997	1060	1120	1177	1232	1279
28½ ins.								1028	1092	1157	1211	1270	1321
29 ins.	—	—	—	—	—	—	—	1056	1123	1187	1248	1305	1360
29½ ins.								1084	1155	1221	1284	1347	1400
30 ins.	—	—	—	—	—	—	—	1115	1187	1255	1321	1382	1442
30½ ins.									1218	1290	1358	1424	1480
31 ins.	—	—	—	—	—	—	—	—	1252	1324	1394	1459	1523
31½ ins.									1286	1359	1433	1496	1561
32 ins.	—	—	—	—	—	—	—	—	1317	1394	1467	1538	1605
32½ ins.										1430	1508	1575	1650
33 ins.	—	—	—	—	—	—	—	—	—	1465	1542	1617	1687
33½ ins.										1500	1578	1655	1733
34 ins.	—	—	—	—	—	—	—	—	—	1536	1617	1695	1770
34½ ins.											1654	1735	1816
35 ins.	—	—	—	—	—	—	—	—	—	—	1692	1775	1856
35½ ins.												1810	1900
36 ins.	—	—	—	—	—	—	—	—	—	—	—	1857	1941
36½ ins.													1984
37 ins.	—	—	—	—	—	—	—	—	—	—	—	—	2026

When a greater allowable stress per sq. in. on stays and stay-bolts is required than that allowed in Table 11, the material shall conform to the physical qualities of Table 12, and the maximum allowable stress on such stays or stay-bolts shall be based on a factor of safety of not less than 6½.

TABLE 11.—MAXIMUM STRESS PER SQ. IN. OF NET SECTION OF STAYS AND STAY-BOLTS

Material and type	Size up to and including 1½ ins. diameter or equivalent area	Size over 1½ ins. diameter or equivalent area
Weldless mild steel head to head or through stays.	8000 lbs.	9000 lbs.
Weldless mild steel diagonal or crow-foot stays.	7500 lbs.	8000 lbs.
Weldless wrought-iron head to head or through stays.	7000 lbs.	7500 lbs.
Weldless wrought-iron diagonal or crow-foot stays.	6500 lbs.	7000 lbs.
Welded mild steel or wrought-iron stays	6000 lbs.	6000 lbs.
Mild steel or wrought-iron stay-bolts. . .	6500 lbs.	7000 lbs.

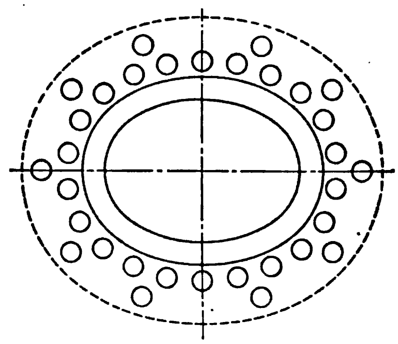


FIG. 10.—Riveting of man-hole frames.

The minimum thickness of cast-iron nozzles shall be determined by the formula:

$$\frac{Pdf}{2S}+.5=t.$$

in which P = working pressure, lbs. per sq. in.,
d = inside diameter of nozzle, ins.,
f = factor of safety = 12,
S = ultimate tensile strength of cast-iron, not less than 18,000 lbs. per sq. in.,
.5 = a constant,
t = thickness of nozzle, ins.,

Manhole frames on shells or drums shall have the proper curvature, and on boilers over 48 ins. in diameter shall be riveted to the shell or drum with two rows of rivets, which may be pitched as shown in Fig. 10. The strength of the rivets in shear shall not be less than the tensile strength of the shell plate removed, on a line parallel to the axis of the shell, through the center of the manhole.

Following is the code of uniform boiler specifications adopted by the American Boiler Manufacturers Association as amended up to 1909:

Materials

Cast-Iron.—Should be of soft, gray texture and high degree of ductility. To be used only for hand-hole plates, crabs, yokes, etc., and manheads. It is a dangerous metal to be used in mud drums, legs, necks, headers, manhole rings or any part of a boiler subject to tensile strains; its use is prohibited for such parts.

Steel.—Homogeneous steel made by the open-hearth or crucible processes, and having the following qualities, is to be used in all boilers.

	Lbs. T. S.
Flange or boiler steel.	55,000 to 65,000

When it is stipulated that the plates are to be flanged, the physical properties shall be the same as required for fire-box steel.

	Lbs. T. S.
Fire-box steel	52,000 to 62,000
Extra soft steel	45,000 to 55,000

	Elongation in 8 ins.
Flange or boiler steel	25 per cent.
Fire-box steel	26 per cent.
Extra soft steel	28 per cent.

Elastic limit to be not less than one-half of the unltimate T. S.

	Chemical requirements	
	Sul.	Phos.
Flange or boiler steel	.03 per cent.	.04 per cent.
Fire-box steel	.03 per cent.	.04 per cent.
Extra soft steel	.03 per cent.	.04 per cent.

For all plates the elastic limit to be at least one-half the ultimate strength; percentage of manganese and carbon left to the judgment of the steel maker.

TABLE 12.—PHYSICAL PROPERTIES OF STAYS AND STAY-BOLTS

Tensile strength, lbs. per sq. in., shall not exceed	62,000
Yield point, in lbs. per sq. in., shall not be less than	½ T.S.
Elongation per cent. in 8 ins. shall not be less than	28

TABLE 13.—ALLOWABLE LOADS ON NET CROSS-SECTION OF STAY-BOLTS, V-THREADS, 12 THREADS PER IN.

Outside diameter of stay-bolts in ins.		Diameter at bottom of thread in ins.	Net-cross sectional area (at bot-bottom of thread) in sq. ins.	Allowable load at 6500 lbs. stress per sq. in.	Allowable load at 7000 lbs. stress per. sq. in.
¾	.7500	.6057	.288	1872	2,016
13/16	.8125	.6682	.351	2282	2,457
⅞	.8750	.7307	.419	2724	2,933
15/16	.9375	.7932	.494	3211	3,458
1	1.0000	.8557	.575	3738	4,025
1 1/16	1.0625	.9182	.662	4303	4,634
1⅛	1.1250	.9807	.755	4908	5,285
1 3/16	1.1875	1.0432	.855	5558	5,985
1¼	1.2500	1.1057	.960	6240	6,720
1 5/16	1.3125	1.1682	1.072	6968	7,504
1⅜	1.3750	1.2307	1.190	7735	8,330
1 7/16	1.4375	1.2932	1.313	8535	9,191
1½	1.5000	1.3557	1.444	9386	10,108

TABLE 14.—ALLOWABLE LOADS ON NET CROSS-SECTION OF STAY-BOLTS, V-THREADS, 10 THREADS PER IN.

Outside diameter of stay-bolts in ins.		Diameter at bottom of thread in ins.	Net-cross sectional area (at bottom of thread) in sq. ins.	Allowable load at 6500 lbs. stress per sq. in.	Allowable load at 7000 lbs. stress per sq. in.
1¼	1.2500	1.0768	.911	5,921	6,377
1 5/16	1.3125	1.1393	1.019	6,623	7,133
1⅜	1.3750	1.2018	1.134	7,371	7,938
1 7/16	1.4375	1.2643	1.255	8,157	8,785
1½	1.5000	1.3268	1.382	8,983	9,674
1 9/16	1.5625	1.3893	1.515	9,847	10,605
1⅝	1.6250	1.4518	1.655	10,757	11,585

Test section to be 8 ins. long, planed or milled edges; its cross-sectional area not less than one-half of 1 sq. in., nor width less than the thickness of the plate.

Bending Test.—Steel up to ½ in. thickness must stand bending double and being hammered down on itself; above that thickness it must bend round a mandrel of diameter of one and one-half times the thickness of plate down to 180 deg. All without showing signs of distress.

Bending test piece to be in length not less than sixteen times thickness of plate, and rough, shear edges milled or filed off. Such pieces to be cut both lengthwise and crosswise of the plate.

All tests to be made at the steel mill. Three pulling tests and three bending tests to be made from each heat. If one fails the manufacturer may furnish and test a fourth piece, but if two fail the entire heat to be rejected.

Certified copies of tests to be furnished each member of A. B. M. A. from heats from which his plates are made.

Rivets to be of good charcoal iron, or of a soft, mild steel, having the same physical and chemical properties as the fire-box plates, and must test hot and cold by driving down on an anvil with the head in a die; by nicking and bending, by bending back on themselves cold, without developing cracks or flaws.

Boiler tubes, of charcoal iron or mild steel especially made for the purpose, and lap welded or drawn; they should be round, straight, free from scales, blisters and mechanical defects, each tested to 500 lbs. internal hydrostatic pressure.

This fact and manufacturers' name to be plainly stenciled on each tube.

TABLE 15.—ALLOWABLE LOADS ON NET CROSS-SECTION OF CIRCULAR STAYS OR RECTANGULAR STAYS OF EQUAL CROSS-SECTIONAL AREA.

Minimum diameter of circular stay in ins.		Net cross-sectional area of stay in sq. ins.	Allowable stress, in lbs. per sq. in. net cros-sectional area					
			6,000	6,500	7,000	7,500	8,000	9,000
			Allowable load, in lbs. on net cross-sectional area					
1	1.0000	.7854	4,712	5,105	5,498	5,891	6,283	......
1 1/16	1.0625	.8866	5,320	5,763	6,206	6,650	7,093	......
1⅛	1.1250	.9940	5,964	6,461	6,958	7,455	7,952	......
1 3/16	1.1875	1.1075	6,645	7,199	7,753	8,306	8,860	......
1¼	1.2500	1.2272	7,363	7,977	8,590	9,204	9,818	......
1 5/16	1.3125	1.3530	8,118	8,795	9,471	10,148	10,824	12,177
1⅜	1.3750	1.4849	8,909	9,652	10,394	11,137	11,879	13,364
1 7/16	1.4375	1.6230	9,738	10,550	11,361	12,173	12,984	14,607
1½	1.5000	1.7671	10,603	11,486	12,370	13,253	14,137	15,904
1 9/16	1.5625	1.9175	11,505	12,464	13,423	14,381	15,340	17,258
1⅝	1.6250	2.0739	12,443	13,480	14,517	15,554	16,591	18,665
1 11/16	1.6875	2.2365	13,419	14,537	15,655	16,744	17,892	20,129
1¾	1.7500	2.4053	14,432	15,634	16,837	18,040	19,242	21,648
1 13/16	1.8125	2.5802	15,481	16,771	18,061	19,352	20,642	23,222
1⅞	1.8750	2.7612	16,567	17,948	19,328	20,709	22,090	24,851
1 15/16	1.9375	2.9483	17,690	19,164	20,638	22,112	23,586	26.535
2	2.0000	3.1416	18,850	20,420	21,991	23,562	25,133	28,274
2⅛	2.1250	3.5466	21,280	23,053	24,826	26,600	28,373	31,919
2¼	2.2500	3.9761	23,857	25,845	27,833	29,821	31,809	35,785
2⅜	2.3750	4.4301	26,580	28,796	31,011	33,226	35,441	39,871
2½	2.5000	4.9087	29,452	31,907	34,361	36,815	39,270	44,178
2⅝	2.6250	5.4119	32,471	35,177	37,883	40,589	43,295	48,707
2¾	2.7500	5.9396	35,638	38,607	41,577	44,547	47,517	53,456
2⅞	2.8750	6.4918	38,951	42,197	45,443	48,689	51,934	58,426
3	3.0000	7.0686	42,412	45,946	49,480	53,015	56,549	63,617

Standard thicknesses by Birmingham wire gage to be

No. 13 for tubes 1 in., 1¼ ins., 1½ ins. and 1¾ ins. diameter.
No. 12 for tubes 2 ins., 2¼ ins., and 2½ ins. diameter
No. 11 for tubes 2¾ ins., 3 ins., 3¼ ins., and 3½ ins. diameter.
No. 10 for tubes 3¾ ins., and 4 ins. diameter.
No. 9 for tubes 4½ ins., and 5 ins. diameter.

Tests.—A section cut from one tube taken at random from a lot of 150 or less must stand hammering down cold vertically without cracking or splitting when down solid.

Length of test pieces:

¾ in. for tubes from 1 in. to 1¾ ins. diameter.
1 in. for tubes from 2 ins. to 2½ ins. diameter.
1¼ ins. for tubes from 2¾ ins. to 3¼ ins. diameter.
1½ ins. for tubes from 3½ ins. to 4 ins. diameter.
1¾ ins. for tubes from 4½ ins. to 5 ins. diameter.

All tubes must stand expanding flange over on tube plate and bending without flaw, crack or opening of the weld.

Stay-bolts to be made of iron or mild steel specially manufactured for the purpose, and must show on:

Test section 8 ins. long; net:

For iron, tensile strength not less than 46,000 lbs.; elastic limit not less than 26,000 lbs.; elongation not less than 22 per cent. for bolts of less than 1 sq. in. area, nor less than 20 per cent. for bolts 1 sq. in. and more in net area.

For steel, tensile strength not less than 55,000 lbs.; elastic limit not less than 33,000 lbs.; elongation not less than 25 per cent. for bolts of less than 1 sq. in. area, nor less than 22 per cent. for bolts 1 sq. in. and more in net area.

Tests.—A bar taken from a lot of 1000 lbs. or less at random, threaded with a sharp die V-thread with rounded edges, must bend cold 180 deg. around a bar of same diameter without showing any crack or flaws.

Another piece, similarly chosen, and threaded, to be screwed into well fitting nuts formed of pieces of the plates to be stayed, and riveted over so as to form an exact counterpart of the bolt in the finished structure; to be pulled in testing machine and breaking stress noted; if it fails by pulling apart the tensile stress per sq. in. of net section is its measure of strength; if it fails by shearing the shear stress per sq. in. of mean section in shear is this measure. The mean section in shear is the product of half the thickness of the plate by the circumference at half height of thread.

Braces and Stays.—Material to be fully equal to stay-bolt stock, and tensile strength to be determined by testing a bar not less than 10 ins. long from each lot of 1000 lbs. or less.

Workmanship and Dimensions

Flanging, bending and forming to be done at a heat suited to the material, but no bending must be done or blow struck on any plate which no longer shows a red by daylight at the working point and at least 4 ins. beyond it

Rolling must be done cold by gradual and regular increments from the straight plate to the exact circle required and the whole circumference including the lap rolled to a true circle.

Bumped heads uniformly dished to a segment of a sphere should have a thickness equal to that of a cylindrical shell of solid plate of same material, whose diameter is equal to the radius of curvature of the dished head.

Rivet holes, manholes, etc., to be allowed for by proportionate increase in the thickness.

Riveting.—Holes made perfectly true and fair by clean cutting punches or drills. Sharp edges and burrs removed by slight counter sinking and burr reaming before and after sheets are joined together.

Under side of original rivet head must be flat, square and smooth. For rivets $\frac{5}{8}$ in. to $\frac{13}{16}$ in. diameter allow $1\frac{1}{2}$ diameters for length of stock to form the head, and less for larger rivets. Allow 5 per cent. more stock for driven head for button set or snap rivets. Use light regulation riveting hammers until rivet is well upset in the hole; after that snap and heavy mauls. For machine riveting more stock is to be left for driven head to make it equal to original head, as fixed by experiment.

Total pressure on the die about 80 tons for $1\frac{1}{8}$ in. to $1\frac{1}{4}$ in. rivets, 65 tons for 1 in., 57 tons for $\frac{15}{16}$ in., 35 tons for $\frac{3}{4}$ in. rivets.

Make heads of rivets equal in strength to shanks by making head at periphery of shank of a height equal to $\frac{1}{3}$ diameter of shank and giving a slight fillet at this point.

Approximately make rivet holes double thickness of thinnest plate; pitch three times rivet hole; pitch lines of staggered rows $\frac{1}{2}$ pitch apart; lap for single riveting equal to pitch, for double riveting $1\frac{1}{2}$ pitch, and $\frac{1}{2}$ pitch more for each additional row of rivets; *exact dimensions determined* by making resistance to shear of aggregate rivet section at least 10 per cent. greater than tensile strength of net or standing metal.

Rivet holes punched with good sharp punches and well fitting dies in A. B M. A. steel up to $\frac{5}{8}$ in. thickness; in thicker plates punch and ream with a fluted reamer, or drill the holes.

Drift pin to be used only with light hammers to pull plates into place and round up the hole, but never to enlarge or gouge holes with heavy hammers.

Calking to be done by hand or pneumatic hammer and Conery or round nosed tool. Avoid excessive calking; the fit must be made in the laying of the plates. The square nosed tool may be used for finishing with great care to avoid nicking lower plate. Calking edges must be prepared by bevel planing, shearing or chipping.

Flat Surfaces.—State the thickness of the plate t in sixteenths of an inch, the pitch p in ins., and use a constant:

$C=112$ for plates $\frac{7}{16}$ in. and under with screw stays with riveted ends.

$C=120$ for plates over $\frac{7}{16}$ in. with screw stays with riveted ends.

$C=140$ for all plates when in addition to screw threads in the plates a nut is used inside and outside on each plate.

When salt, acids or alkali are contained in the feed water, this latter construction is imperative.

Rule.—Multiply this constant C by the square of the thickness of the plate expressed in sixteenths of an inch, and divide by the square of the pitch expressed in ins.; the quotient is the safe working pressure P; that is:

$$P=\frac{C\times t^2}{P^2}$$

Tube holes either punched $\frac{1}{8}$ in. less than required diameter and reamed to full size, or drilled; then slightly countersunk on both sides; should be $\frac{1}{64}$ in. to $\frac{1}{16}$ in. larger than diameter of tube according to size of tube; if copper ferrules are used the hole to be a neat fit for the ferrule. Tube sheet to be annealed after punching and before reaming.

Tube Setting.—Ends of tubes to be annealed (in the tube mill) before setting. The tube to extend through the sheet $\frac{1}{16}$ in. for every inch of diameter. Expand until tight in hole and no more. On end exposed to direct flame, flange the tube partly over on sheet, finishing by beading tool which must not come in contact with the plate; expand slightly after beading.

Copper ferrules Nos. 18 to 14 wire gage should be used in the fire-tube boiler on ends subject to direct heat.

Riveted and lap-welded flues, as prescribed in Rule 11, Sections 8, 9, 10, 11, 12 and 13 of Regulations of Board of Supervising Inspectors of Steam Vessels, approved February, 1895.

Corrugated furnace flues as prescribed in sections 14 and 15 of the same rule.

Stay-bolts to be carefully threaded with sharp clean dies V-thread with round edges; threading machine equipped with a lead screw; holes tapped with tap extending through both sheets to neat smooth fit, so that bolts can be put in by hand lever or wrench with a steady pull; $\frac{1}{8}$ diameter to project for riveting over; with hollow stay-bolts use slender drift pin in the bore while riveting and drive it home to expand the bolt after riveting.

Height of nuts used on screw stays to be at least 50 per cent. of diameter of stay. Largest permissible pitch for screw stays is 10 ins.

Braces and stays shall be subjected to careful inspection and tests as per section 6 and 2. Welding to be avoided where possible, but good clean welds to be allowed a value of 80 per cent. of the solid bar. Rivets by which braces are attached, when the pull on them is other than at right angles to be allowed only half the stress permitted for rivets in the seams.

Manholes should be flanged in, out of the solid plate, on a radius not less than three times the metal thickness to a straight flange; when the plate is $\frac{1}{2}$ in. or less in thickness a reinforce ring to be shrunk around it. Cast-iron reinforce flanges never to be used.

Domes to be avoided when possible; cylindrical portion to be flanged down to the shell of the boiler, and this shell flanged up in-

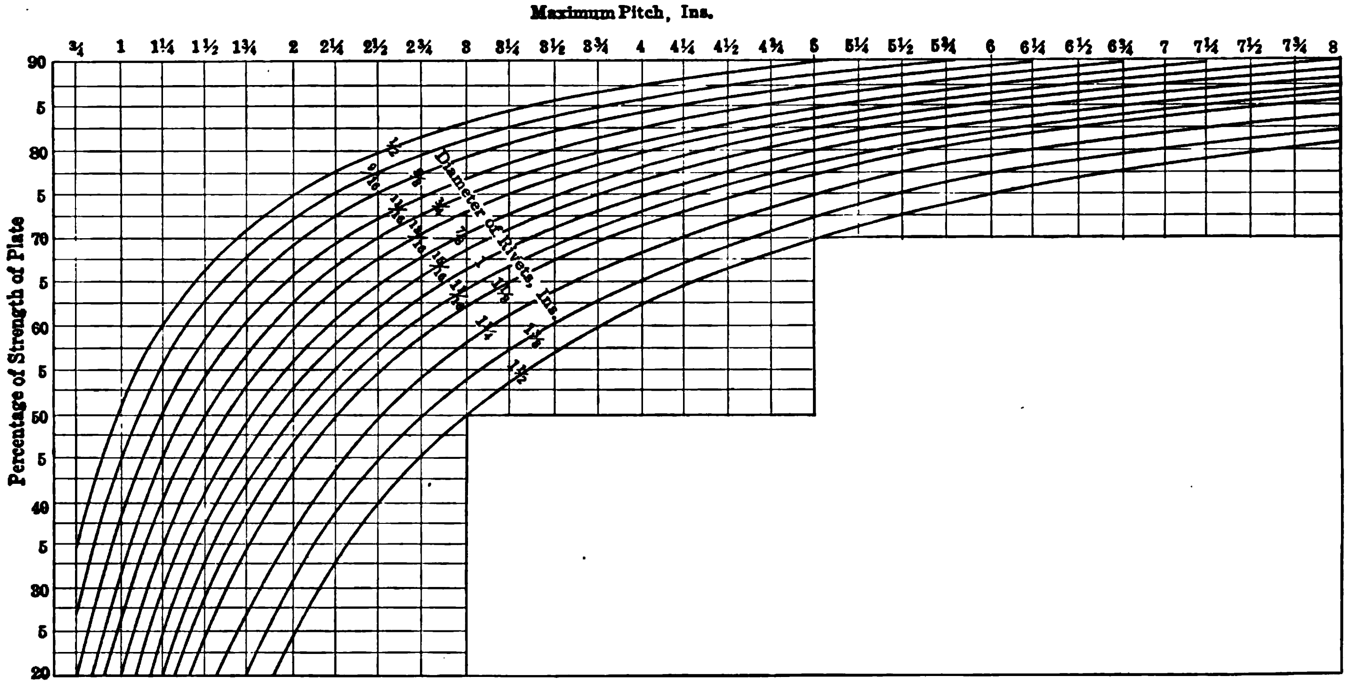
side the dome, or reinforced by a collar flanged at the joint, the flanges double-riveted.

Drums should be put on with collar flanges of A. B. M. A. steel, not less than ⅜ in. thick, double-riveted to shell and drum and single riveted to the neck or leg, or the flanges may be formed on these legs.

assumed for the steel plate and 40,000 lbs. shear strength for the rivets, all figured on the actual net standing metal.

Flat surfaces proportioned as prescribed above have in the constants there given a factor of safety of 5 or a little over.

Bumped heads proportioned as prescribed above to be subject to a factor of safety of 5.



Traced downward from the maximum pitch to the curve for the diameter of the rivets and then to the left where read the percentage of plate strength.

FIG. 11.—Percentage of plate strength of boiler joints.

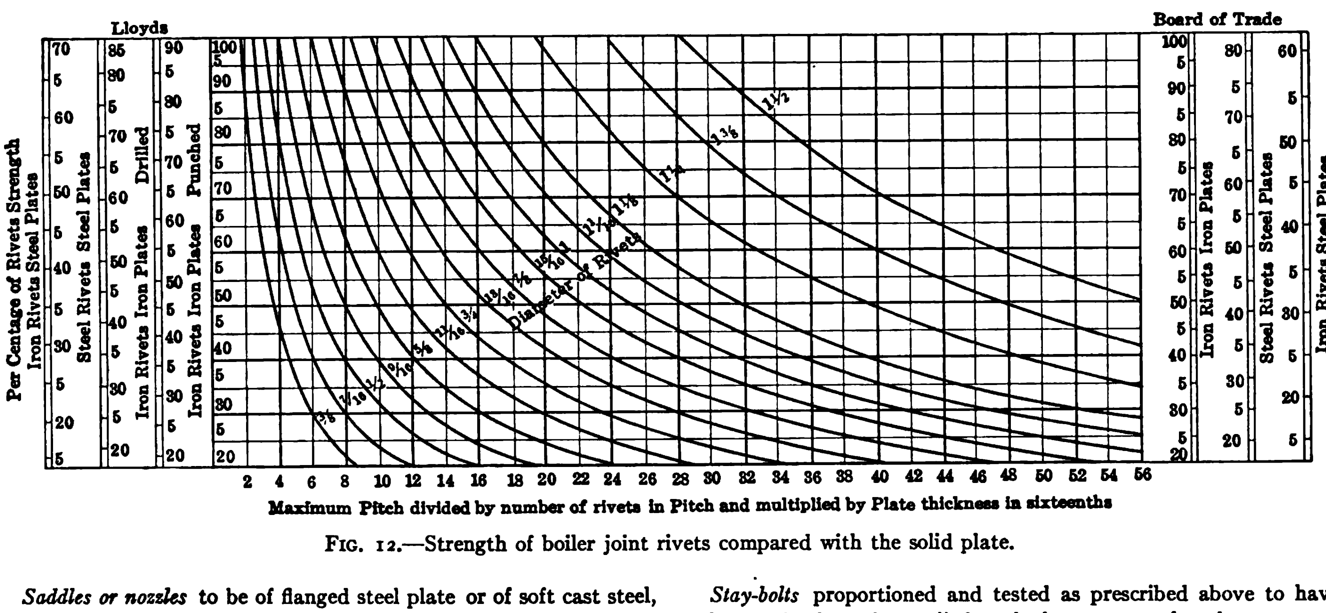


FIG. 12.—Strength of boiler joint rivets compared with the solid plate.

Saddles or nozzles to be of flanged steel plate or of soft cast steel, never of cast-iron.

Factor of Safety

Rivet seams when proportioned and with materials tested as prescribed above shall have 4½ as factor of safety; when not so tested, but inspection of materials indicates good quality, a factor of safety of 5 is to be taken, and at most 55,000 lbs. tensile strength

Stay-bolts proportioned and tested as prescribed above to have a factor of safety of 5 applied to the lowest stress found.

Braces and Stays.—When tested as prescribed above to be allowed a factor of safety of 5; when not so tested but careful inspection shows good stock they may be used up to 6500 lbs. actual direct pull for wrought iron, and 8000 lbs. for mild steel, all per square inch of actual net metal.

TABLE 16.—PROPERTIES OF STANDARD BOILER TUBES AND FLUES. WEIGHTS AND DIMENSIONS ARE NOMINAL

From the National Tube Company's Book of Standards

Diameters, External	Diameters, Internal	Thickness, Ins.	Thickness, B.W.G	Weight per ft.	Length of tube per sq. ft., External surface	Length of tube per sq. ft., Internal surface	Sq. ft. of surface per lineal ft., External surface	Sq. ft. of surface per lineal ft., Internal surface
1¾	1.560	.095	13	1.679	2.182	2 448	.458	.408
2	1 810	.095	13	1.932	1 909	2 110	.523	.473
2¼	2.060	.095	13	2.186	1.697	1.854	.589	.539
2½	2.282	.109	12	2.783	1.527	1.673	654	.597
2¾	2.532	.109	12	3.074	1.388	1.508	719	.662
3	2.782	.109	12	3.365	1.273	1 373	.785	728
3¼	3.010	.120	11	4.011	1.175	1 269	.850	.788
3½	3.260	.120	11	4.331	1.091	1.171	.916	.853
3¾	3.510	.120	11	4.652	1.018	1 088	.981	.918
4	3.732	.134	10	5.532	.954	1.023	1 047	.977
4½	4.232	.134	10	6 248	.848	.902	1.178	1.107
5	4.704	.148	9	7.669	.763	812	1.308	1.231
6	5 670	.165	8	10 282	.636	673	1 570	1.484
7	6.670	.165	8	12 044	.545	.572	1.832	1.746
8	7.670	.165	8	13 807	.477	.498	2.094	2.008
9	8.640	.180	7	16 955	.424	.442	2.356	2.261
10	9 594	.203	6	21.240	.381	.398	2.617	2.511
11	10 560	.220	5	25 329	.347	.361	2.879	2 764
12	11.542	.229	....	28.788	318	.330	3.141	3.021
13	12 524	.238	4	32 439	.293	304	3.403	3.278
14	13.504	.248	....	36.424	.272	.282	3.665	3.535
15	14 482	.259	3	40 775	.254	263	3 926	3.791
16	15 460	.270	......	45.359	.238	.247	4 188	4 047

Hydrostatic Pressure

The hydrostatic test, to be made on completed boilers built strictly to these specifications, is never to exceed working pressure by more than one-third of itself and this excess limited to 100 lbs. per sq. in. The water used for testing to have a temperature of at least 125 deg. Fahr.

Hanging or Supporting the Boiler

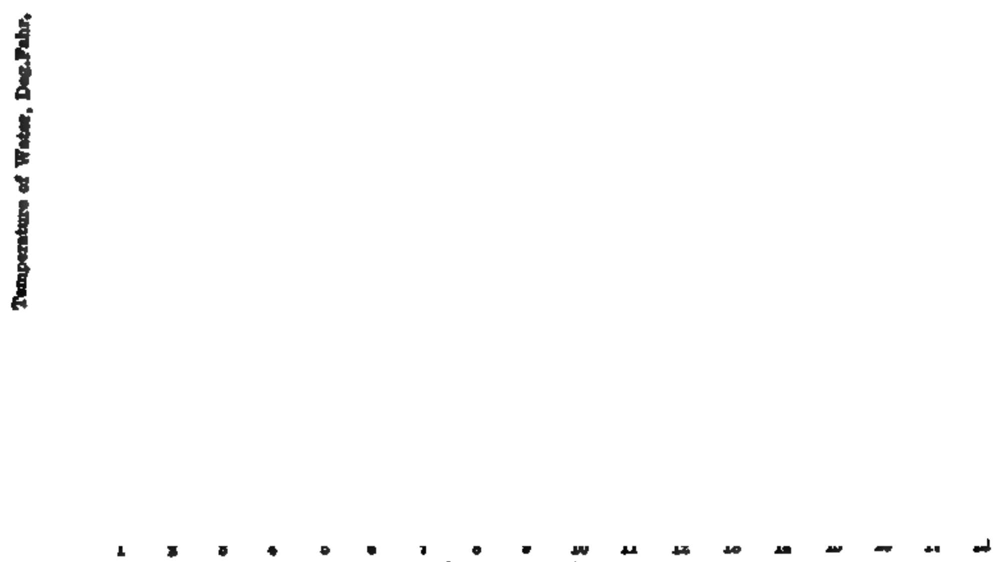
The boiler should be supported on points where there is the greatest excess of strength. Excessive local stresses from weight of boiler and contents must be avoided and distortion of parts prevented by using long lugs or brackets, and only half the stress which they may carry in the seams, to be allowed on rivets.

The supports must permit rebuilding the furnace without disturbing the proper suspension of the boiler. The boiler should be slightly inclined so that a little less water shows at the gage cocks than at the opposite end.

The percentage of plate strength of boiler joints may be obtained from Fig. 11 (*Amer. Mach., June* 16, 1892). The use of the chart is explained below it.

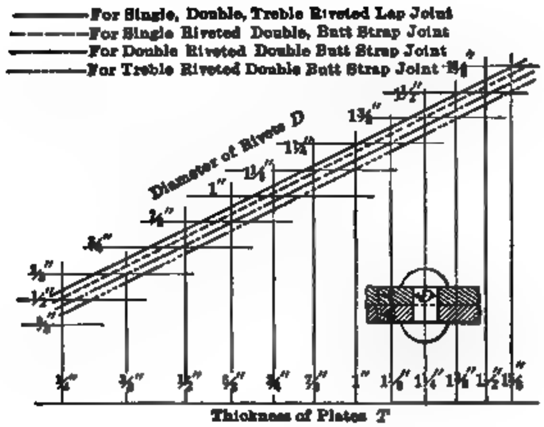
The strength of the rivets of boiler joints compared with the strength of the solid plate may be obtained from Fig. 12 (*Amer. Mach., Apr.* 14, 1892) which is drawn for Lloyd's and the British Board of Trade rules. The use of the chart is best shown by an example:

Find the percentage of strength of rivets in single shear compared with the tensile strength of the solid plate in the case of a double-riveted lap joint; plate thickness ½ in., rivets ¾ in. diameter, pitch 2½ ins., iron plate and rivets. Divide the maximum pitch by the number of rivets in one pitch length and multiply the quotient by the plate thickness in sixteenths of an inch; *i.e.*, $\frac{2.5}{2}\times 8 = 10$. Find



From the final temperature at the left trace horizontally to the diagonal leading from the initial temperature and then down where read the percentage of saving.

FIG. 13.—The saving due to heating feed water for steam boilers.



Rivet Diameters

Double Riveted Double Butt Strap Joint

Single Riveted Lap Joint

Double Riveted Lap Joint

Double Riveted Lap Joint

Double Riveted Double Butt Strap Joint

Double Riveted Double Butt Strap Joint

FIG. 14.—Dimensions of riveted joints.

the intersection of the ordinate 10 on the chart with the curve for ¾-in. rivets and read 71 per cent. for punched plates. If rivets in double shear are considered to be 75 per cent. stronger than in single shear, multiply the result given by the chart by 1.75.

By a reverse reading the rivet diameter can be obtained if the pitch, number of rows, plate thickness and percentage of strength are given.

The dimensions of riveted joints in accordance with Professor Bach's formulas may be determined from Fig. 14, by S. SHIBATA (*Amer. Mach., May* 12, 1904). The diameter of the rivets is to be taken from the upper left-hand chart, after which the other dimensions are to be taken from the chart for the type of joint to be used. For other formulas the charts may be used for the trial layout and thus shorten the process of adjusting the strength of the rivets to that of the

TABLE 17.—PROXIMATE ANALYSIS AND HEATING VALUE OF AMERICAN COALS

From *Steam* By Permission of the Babcock & Wilcox Co.

	Moisture	Volatile matter	Fixed carbon	Ash	Sulphur	Heating value per lb. coal, heat units	Volatile matter per cent. of combustible	Heating value per lb. combustible, heat units	Theoretical evaporation lbs. water from and at 212° per lb. combustible
Anthracite.									
Northern coal field	3.42	4.38	83.27	8.20	.73	13,160	5.00	14,900	15.42
East Middle coal field	3.71	3.08	86.40	6.22	.58	13,420	3.44	14,900	15.42
West Middle coal field	3.16	3.72	81.59	10.65	.50	12,840	4.36	14,900	15.42
Southern coal field	3.09	4.28	83.81	8.18	.64	13,220	4.85	14,900	15.42
Anthracite from one mine.									
Egg, screen 2½–1¾ ins	......	......	88.49	5.66	......	......	......	......	......
Stove, screen 1¾–1¼ ins	......	......	83.67	10.17	......	......	......	......	......
Chestnut, screen 1¼–¾ in	......	......	80.72	12.67	......	......	......	......	......
Pea, screen ¾–½ in	......	......	79.05	14.66	......	......	......	......	......
Buckwheat, screen ½–¼ in	......	......	76.92	16.62	......	......	......	......	......
Semi-Anthracite.									
Loyalsock field	1.30	8.10	83.34	6.23	1.63	13,920	8.86	15,500	16.05
Bernice basin	.65	9.40	83.69	5.34	.91	13,700	10.98	15,500	16.05
Semi-Bituminous.									
Broad Top, Pa	.79	15.61	77.30	5.40	.90	14,820	17.60	15,800	16.36
Clearfield County, Pa	.76	22.52	71.82	3.99	.91	14,950	24.60	15,700	16.25
Cambria County, Pa	.94	19.20	71.12	7.04	1.70	14,450	22.71	15,700	16.25
Somerset County, Pa	1.58	16.42	71.51	8.62	1.87	14,200	20.37	15,800	16.36
Cumberland, Md	1.09	17.30	73.12	7.75	.74	14,400	19.79	15,800	16.36
Pocahontas, Va	1.00	21.00	74.39	3.03	.58	15,070	22.50	15,700	16.25
New River, W. Va	.85	17.88	77.64	3.36	.27	15,220	18.95	15,800	16.36
Bituminous.									
Connellsville, Pa	1.26	30.12	59.61	8.23	.78	14,050	34.03	15,300	15.84
Youghiogheny, Pa	1.03	36.50	59.05	2.61	.81	14,450	38.73	15,000	15.53
Pittsburg, Pa	1.37	35.90	52.21	8.02	1.80	13,410	41.61	14,800	15.32
Jefferson County, Pa	1.21	32.53	60.99	4.27	1.00	14,370	35.47	15,200	15.74
Middle Kittanning seam, Pa	1.81	35.33	53.70	7.18	1.98	13,200	40.27	14,500	15.01
Upper Freeport seam, Pa. and O	1.93	35.90	50.19	9.10	2.89	13,170	43.59	14,800	15.32
Thacker, W. Va	1.38	35.04	56.03	6.27	1.28	14,040	39.33	15,200	15.74
Jackson County, O	3.83	32.07	57.60	6.50	......	13,090	35.76	14,600	15.11
Brier Hill, O	4.80	34.60	56.30	4.30	......	13,010	38.20	14,300	14.80
Hocking Valley, O	6.59	34.97	48.85	8.00	1.59	12,130	42.81	14,200	14.70
Vanderpool, Ky	4.00	34.10	54.60	7.30	......	12,770	38.50	14,400	14.91
Muhlenberg County, Ky	4.33	33.65	55.50	4.95	1.57	13,060	38.86	14,400(?)	14.91
Scott County, Tenn	1.26	35.76	53.14	8.02	1.80	13,700	34.17	15,100(?)	15.63
Jefferson County, Ala	1.55	34.44	59.77	2.62	1.42	13,770	37.63	14,400(?)	14.91
Big Muddy, Ill	7.50	30.70	53.80	8.00	......	12,420	36.30	14,700	15.22
Mt. Olive, Ill	11.00	35.65	37.10	13.00	......	10,490	47.00	13,800	14.29
Streator, Ill	12.00	33.30	40.70	14.00	......	10,580	45.00	14,300	14.80
Missouri	6.44	37.57	47.94	8.05	......	12,230	43.94	14,300(?)	14.80
Lignites and Lignitic Coals.									
Iowa	8.45	37.09	35.60	18.86	......	8,720	51.03	12,000(?)	12.42
Wyoming	8.19	38.72	41.83	11.26	......	10,390	48.07	12,900(?)	13.35
Utah	9.29	41.97	44.37	3.20	1.18	11,030	48.60	12,600(?)	13.04
Oregon lignite	15.25	42.98	33.32	7.11	1.66	8,540	54.95	11,000(?)	11.39

net section of the plate. In all cases the distance from the center of the outermost row of rivets to the edge of the plate is to be taken as one-half the lap of a single riveted joint, as given in the figure for that joint.

The resistance offered by the expanded tubes in tube sheets formed the subject of experiments by PROFS. O. P. HOOD and G. L. CHRISTENSEN (*Trans. A. S. M. E.*, 1908) of which the following are the conclusions:

The slipping point of a 3-in. twelve-gage Shelby cold drawn tube rolled into a straight smooth machined hole in a 1-in. sheet occurs with a pull of about 7000 lbs.

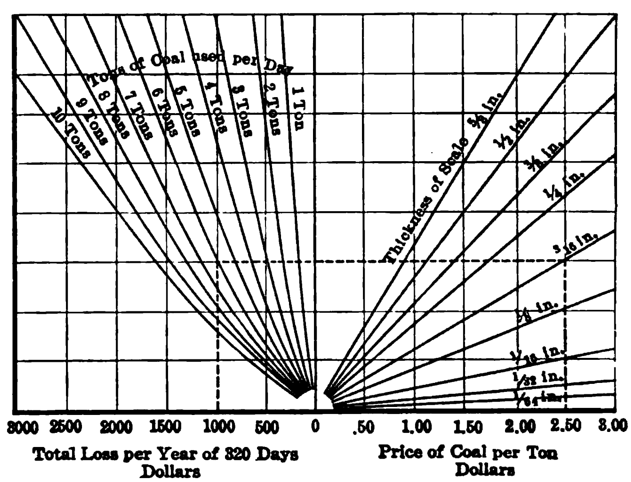
Various degrees of rolling do not greatly affect the point of initial slip.

The frictional resistance of such tubes is about 750 lbs. per sq. in. of tube-bearing area in sheets ⅝ in. and 1 in. thick.

For a higher resistance to initial slip other resistance than friction must be depended upon.

Serrating the tube seat in a straight machined hole by rolling or cutting square edged grooves .01 in. deep and ten pitch will raise the slipping point to three or four times that in a smooth hole.

It is possible to make a rolled joint that will offer a resistance beyond the elastic limit of the tube and remain tight.



From the price of coal per ton trace upward to the line for the thickness of scale, then horizontally to the line for the number of tons of coal consumed per day, then down, and read the loss in dollars per year of 320 days.

FIG. 15.—Loss of coal due to scale in boilers.

TABLE 18.—HORSE-POWER OF CHIMNEYS FOR STEAM BOILERS
From Steam, By Permission of the Babcock & Wilcox Co.

Diameter in ins.	Height of Chimneys and Commercial Horse-power											Side of square, ins.	Effective area, sq. ft.	Actual area, sq ft.
	50 ft.	60 ft.	70 ft.	80 ft.	90 ft.	100 ft.	110 ft.	125 ft.	150 ft.	175 ft.	200 ft.			
18	23	25	27									16	.97	1.77
21	35	38	41									19	1.47	2.41
24	49	54	58	62								22	2.08	3.14
27	65	72	78	83								24	2.78	3.98
30	84	92	100	107	113							27	3.58	4.91
33		115	125	133	141							30	4.48	5.94
36		141	152	163	173	182						32	5.47	7.07
39			183	196	208	219						35	6.57	8.30
42			216	231	245	258	271					38	7.76	9.62
48				311	330	348	365	389				43	10.44	12.57
54				363	427	449	472	503	551			48	13.51	15.90
60				505	536	565	593	632	692	748		54	16.98	19.64
66					658	694	728	776	849	918	981	59	20.83	23.76
72					792	835	876	934	1023	1105	1181	64	25.08	28.27
78						995	1038	1107	1212	1310	1400	70	29.73	33.18
84						1163	1214	1294	1418	1531	1637	75	34.76	38.48
90						1344	1415	1496	1639	1770	1893	80	40.19	44.18
96						1537	1616	1720	1876	2027	2167	86	46.01	50.27
102								1946	2133	2303	2462	90	52.23	56.75
108								2192	2402	2594	2773	96	58.83	63.62
114								2459	2687	2903	3003	101	65.83	70.88
120									2990	3230	3452	106	73.22	78.54
126									3308	3573	3820	112	81.00	86.59
132									3642	3935	4205	117	89.19	95.03
138									3991	4311	4608	122	97.75	103.86
144									4357	4707	5031	127	106.72	113.10

The capacity of safety valves, according to the regulations of the Board of Supervising Inspectors of the Steamboat Inspection Service of the United States, is expressed by the formula:

$$A = .2074 \frac{W}{P}$$

in which A = area of valve disk, sq. ins.,
W = weight of steam discharged per hour, lbs.,
P = absolute pressure, lbs. per sq. in.

The above formula, due to L. D. Lovekin, chief engineer New York Shipbuilding Co., assumes the lift of the valve to be one-thirty-second of its diameter. Experiments by P. G. Darling, mechanical engineer Manning Maxwell and Moore (*Trans. A. S. M. E.* 1909) show that safety valves do not lift in proportion to their diameters; that the lift is practically the same for a large, as for a small valve, smaller for the larger valve if anything, and is around three-thirty-seconds of an inch for all valves in normal condition. From this fact and Napier's formula for the discharge of steam through an orifice, *Power* (*Mar.* 9, 1909) deduces the very simple formula:

$$d = .1 \frac{W}{P}$$

in which d = diameter of disk, ins.
the remaining notation being as before.

Mr. Lovekin's formula, which has proven sufficient, gives the same results as *Power's* formula for valves of 2.64 ins. diameter.

The Massachusetts formula is:

$$A = 770 \frac{W}{P}$$

in which A = total area of safety valve or valves, sq. ins.
W = lbs. of water evaporation per sq. ft. of grate surface per sec.
P = boiler pressure (absolute).

The Philadelphia formula is:

$$A = \frac{22.5\,G}{P \times 8.62}$$

in which A = area of safety valve, sq. ins. per sq. ft. of grate.
G = grate area, sq. ft.
P = boiler pressure (gage).

The saving due to heating feed water for boilers may be determined from Fig. 13 by W. M. WRIGHT (*Power, June* 25, 1912). The use of the chart is explained below it. The chart is calculated for 100 lbs. boiler pressure. For 50 lbs. pressure, percentages are less than .15 higher. For 200 lbs. pressure, percentages are less than .2 lower.

The loss due to scale in boilers may be estimated by the use of Fig. 15 by CHAS. BROSSMANN (*Power, Apr.* 16, 1912). The use of the chart is explained below it.

The horse-power of chimneys is given in Table 18 from Wm. Kent's well known formula, the figures for the horse-power being, however, increased for the larger sizes by unimportant amounts by the Babcock and Wilcox Company. The table is based on the assumption that a commercial horse-power requires an average consumption of 5 lbs. of coal per hour..

TABLE 1.—PROPERTIES OF SATURATED STEAM
From Steam, By Permission of the Babcock & Wilcox Co.

Pressure in lbs. per sq. in. above vacuum	Temperature in deg. Fahr.	Total heat in heat units from water at 32 deg.	Heat in liquid from 32 deg. in units	Heat of vaporization, or latent heat in heat units	Density or weight of cu. ft. in lbs.	Volume of 1 lb. in cu. ft.	Factor of equivalent evaporation at 212 deg.	Total pressure above vacuum
1	101.99	1113.1	70.0	1043.0	.00299	334.5	.9661	1
2	126.27	1120.5	94.4	1026.1	.00576	173.6	.9738	2
3	141.62	1125.1	109.8	1015.3	.00844	118.5	.9786	3
4	153.09	1128.6	121.4	1007.2	.01107	90.33	.9822	4
5	162.34	1131.5	130.7	1000.8	.01366	73.21	.9852	5
6	170.14	1133.8	138.6	995.2	.01622	61.65	.9876	6
7	176.90	1135.9	145.4	990.5	.01874	53.39	.9897	7
8	182.92	1137.7	151.5	986.2	.02125	47.06	.9916	8
9	188.33	1139.4	156.9	982.5	.02374	42.12	.9934	9
10	193.25	1140.9	161.9	979.0	.02621	38.15	.9949	10
15	213.03	1146.9	181.8	965.1	.03826	26.14	1.0003	15
20	227.95	1151.5	196.9	954.6	.05023	19.91	1.0051	20
25	240.04	1155.1	209.1	946.0	.06199	16.13	1.0099	25
30	250.27	1158.3	219.4	938.9	.07360	13.59	1.0129	30
35	259.19	1161.0	228.4	932.6	.08508	11.75	1.0157	35
40	267.13	1163.4	236.4	927.0	.09644	10.37	1.0182	40
45	274.29	1165.6	243.6	922.0	.1077	9.285	1.0205	45
50	280.85	1167.6	250.2	917.4	.1188	8.418	1.0225	50
55	286.89	1169.4	256.3	913.1	.1299	7.698	1.0245	55
60	292.51	1171.2	261.9	909.3	.1409	7.097	1.0263	60
65	297.77	1172.7	267.2	905.5	.1519	6.583	1.0280	65
70	302.71	1174.3	272.2	902.1	.1628	6.143	1.0295	70
75	307.38	1175.7	276.9	898.8	.1736	5.760	1.0309	75
80	311.80	1177.0	281.4	895.6	.1843	5.426	1.0323	80
85	316.02	1178.3	285.8	892.5	.1951	5.126	1.0337	85
90	320.04	1179.6	290.0	889.6	.2058	4.859	1.0350	90
95	323.89	1180.7	294.0	886.7	.2165	4.619	1.0362	95
100	327.58	1181.9	297.9	884.0	.2271	4.403	1.0374	100
105	331.13	1182.9	301.6	881.3	.2378	4.205	1.0385	105
110	334.56	1184.0	305.2	878.8	.2484	4.026	1.0396	110
115	337.86	1185.0	308.7	876.3	.2589	3.862	1.0406	115
120	341.05	1186.0	312.0	874.0	.2695	3.711	1.0416	120
125	344.13	1186.9	315.2	871.7	.2800	3.571	1.0426	125
130	347.12	1187.8	318.4	869.4	.2904	3.444	1.0435	130
140	352.85	1189.5	324.4	865.1	.3113	3.212	1.0453	140
150	358.26	1191.2	330.0	861.2	.3321	3.011	1.0470	150
160	363.40	1192.8	335.4	857.4	.3530	2.833	1.0486	160
170	368.29	1194.3	340.5	853.8	.3737	2.676	1.0502	170
180	372.97	1195.7	345.4	850.3	.3945	2.535	1.0517	180
190	377.44	1197.1	350.1	847.0	.4153	2.408	1.0531	190
200	381.73	1198.4	354.6	843.8	.4359	2.294	1.0545	200
225	391.79	1201.4	365.1	836.3	.4876	2.051	1.0576	225
250	400.99	1204.2	374.7	829.5	.5393	1.854	1.0605	250
275	409.50	1206.8	383.6	823.2	.5913	1.691	1.0632	275
300	417.42	1209.3	391.9	817.4	.644	1.553	1.0657	300
325	424.82	1211.5	399.6	811.9	.696	1.437	1.0680	325
350	431.90	1213.7	406.9	806.8	.748	1.337	1.0703	350
375	438.40	1215.7	414.2	801.5	.800	1.250	1.0724	375
400	445.15	1217.7	421.4	796.3	.853	1.172	1.0745	400
500	466.57	1224.2	444.3	779.9	1.065	.939	1.0812	500

Steam and Coal Consumption

The coal consumption of steam engines varies with the type. Single-cylinder non-condensing engines use 28 to 50 lbs.; ordinary compound condensing engines use 18 to 22 lbs.; while the best types of multi-expansion condensing engines utilizing superheated steam have reduced the steam consumption below 12 lbs. per h.p. hour. Table 2 gives the relative efficiency of various types of pumping engines.

Table 2.—Efficiencies of Various Types of Pumping Engines Using Saturated Steam
From Steam, By Permission of the Babcock & Wilcox Co.

Type		Duty. Million ft.-lbs. work done per 1000 lbs. steam consumed, with varying conditions of service	Lbs. of steam per pump h.p. hour
Condensing.			
Direct acting and crank and fly-wheel,	Triple expansion	125 to 140	16 to 13.5
Direct acting and crank and fly-wheel,	Compound	100 to 120	20 to 16
Direct acting low duty	Triple expansion	75 to 90	26 to 20
Direct acting low duty	Compound	40 to 60	50 to 33
Non-condensing.			
Direct acting low duty	Triple expansion	50 to 70	40 to 28
Direct acting low duty	Compound	30 to 40	66 to 50
Direct acting small sizes	Non-compound	8 to 20	250 to 100
Vacuum pumps, direct acting, independent		8 to 20	250 to 100
Vacuum pumps, fly-wheel, independent		45 to 80	45 to 25
Injectors		2 to 5	1000 to 400

Table 3.—Useful Steam per I.h.p.-hr., S_u in Lbs. with Single-Cylinder Non-condensing Engine Steam Throttled

With new engines S_u may be taken approximately 1.75 lbs. less.

Avg. abs. admission pressure	Cut-off in per cent. of full stroke: 70	60	50	40	33.3	30	25	20	15
30									
35	51.50	51.00							
40	46.00	44.50	44.50						
45	42.50	40.50	39.50	39.00					
50	39.50	37.75	35.75	35.00	35.00				
55	37.85	35.55	33.50	32.25	32.00				
60	36.50	34.00	31.80	30.40	29.95	29.80			
65	35.25	33.00	30.75	28.80	28.20	27.75			
70	34.25	32.00	29.80	27.80	26.85	26.30	25.95		
75	33.50	31.00	29.00	27.00	25.85	25.20	24.75		
80	32.65	30.40	28.20	26.20	25.00	24.30	23.75		
85	32.00	29.80	27.55	25.50	24.45	23.52	23.00	22.10	
90	31.50	29.35	27.15	25.00	23.85	23.05	22.50	21.45	
95	31.00	28.95	26.60	24.60	23.35	22.75	22.00	20.90	
100	30.60	28.50	26.30	24.25	22.90	22.25	21.50	20.50	19.60
105	30.30	28.05	26.00	24.00	22.50	21.95	21.00	20.10	19.45
110	30.00	27.80	25.75	23.65	22.25	21.55	20.75	19.90	19.10
115	29.75	27.50	25.50	23.40	22.00	21.30	20.50	19.50	18.96
120	29.50	27.30	25.25	23.05	21.65	21.10	20.25	19.25	18.52
125	29.15	27.05	24.95	22.80	21.50	20.96	20.00	19.00	18.40
130	28.85	26.80	24.75	22.60	21.35	20.75	19.75	18.85	18.25
135	28.60	26.55	24.50	22.45	21.15	20.50	19.50	18.60	18.00
140	28.45	26.30	24.30	22.30	21.00	20.30	19.30	18.50	17.82
145	28.22	26.15	24.15	22.25	20.80	20.15	19.15	18.25	17.75
150	28.05	26.00	24.05	22.15	20.60	20.00	19.00	17.80	17.55

Table 4.—Useful Steam per I.h.p.-hr., S_u in Lbs. with Single-cylinder, Non-condensing Engines, Automatic Cut-off

With new engines these values may be taken approximately 1.5 lbs. less.

Avg. abs. admission pressure	Cut-off in per cent. of full stroke: 70	60	50	40	33.3	30	25	20	15	12.5	10
35	50.50										
40	44.50	40.50	39.50	38.30	39.50						
45	40.50	36.80	35.50	34.00	33.50	34.00	35.50				
50	38.00	34.80	32.50	30.80	30.25	30.50	31.60				
55	36.00	33.00	30.75	29.00	27.90	27.80	28.55	30.50			
60	34.50	31.75	29.50	27.50	26.40	25.90	26.15	26.50	28.00		
65	33.45	30.75	28.40	26.25	25.00	24.50	24.45	24.40	25.50	27.50	
70	32.50	29.90	27.50	25.20	24.00	23.45	23.10	22.75	23.55	25.00	27.25
75	31.55	29.20	26.75	24.45	23.15	22.60	22.25	21.60	22.10	23.20	25.00
80	31.00	28.50	26.00	23.75	22.50	22.00	21.50	20.80	21.00	21.95	23.00
85	30.50	28.00	25.45	23.20	22.00	21.45	21.00	20.00	20.00	20.80	21.50
90	30.00	27.50	25.00	22.75	21.50	20.95	20.50	19.50	19.50	19.90	20.50
95	29.50	27.00	24.60	22.40	21.10	20.55	20.00	19.00	19.00	19.10	19.50
100	29.10	26.50	24.20	22.00	20.85	20.10	19.50	18.60	18.45	18.50	19.00
105	28.80	26.10	23.90	21.65	20.45	19.80	19.00	18.40	17.85	18.00	18.50
110	28.50	25.75	23.65	21.45	20.10	19.50	18.80	18.15	17.45	17.50	18.00
115	28.25	25.55	23.40	21.20	19.85	19.20	18.55	17.80	17.00	17.10	17.55
120	28.00	25.40	23.15	21.00	19.55	19.00	18.35	17.50	16.60	16.80	17.10
125	27.75	25.25	22.95	20.75	19.40	18.80	18.10	17.20	16.40	16.50	16.65
130	27.50	25.10	22.75	20.50	19.25	18.60	17.90	17.00	16.25	16.25	16.35
135	27.35	24.90	22.55	20.30	19.05	18.45	17.60	16.80	16.10	16.00	16.00
140	27.10	24.65	22.40	20.15	18.85	18.30	17.35	16.55	15.95	15.75	15.55
145	26.90	24.45	22.30	20.00	18.70	18.15	17.10	16.45	15.70	15.55	15.25
150	26.75	24.25	22.15	19.88	18.55	18.00	16.95	16.35	15.50	15.40	15.00

The approximate steam consumption of various types of steam engines may be obtained from Tables 3–8 and the following formulas by J. A. Knesche (*Power, Nov.* 12, 1912). The useful steam is to be taken from Tables 2–6 in accordance with the class of engine under consideration and to the quantity thus obtained additions are to be made as follows:

The greater part of the steam loss within the cylinder is due to condensation and the smaller part to leakage past the piston and valves. The condensation losses S_c are determined from the formula

$$S_c = \frac{K}{\sqrt{P}} \qquad (a)$$

in which S_c = steam losses through condensation,

P = piston speed in ft. per sec.,

K = coefficient as given in Table 7,

when the ratio of stroke to diameter, $\left(\frac{s}{d}\right)$ is approximately 2.

The smaller figures are to be applied to engines that are new or in very good condition.

When $\frac{s}{d}$ differs considerably from 2, the values in Table 7 are to be multiplied by the coefficients which are given in Table 8.

If the admission steam is superheated sufficiently, cylinder condensation may be entirely avoided. With cutoff in the high-pressure cylinder ranging from 40 to 25 per cent. a superheat of from 175 to 250 deg. Fahr. is sufficient to prevent condensation. But even in such a case, S_c must not be taken as zero, because superheated steam, compared with saturated steam, does less work in the engine cylinder on account of the more rapid fall of its expansion curve and also, because heat is required to superheat the steam. If S_c is determined from the formula $S_c = \frac{K}{\sqrt{P}}$ when superheated steam is used, then K will be from $\frac{1}{4}$ to $\frac{1}{3}$ the value for saturated steam, as given in Table 7.

For single-cylinder engines the leakage past the piston S_l may be determined according to the formula

$$S_l = \frac{35.14}{\sqrt{i.hp. \times P}} + \frac{3.62}{P} \qquad (b)$$

in which $i.hp.$ = indicated h.p.,

P = piston speed in ft. per sec.

For compound engines the leakage loss is 80 per cent. and for triple-expansion engines 64 per cent. of the value given by this

TABLE 5.—USEFUL STEAM PER I.H.P.-HR., S_u IN LBS. WITH SINGLE-CYLINDER CONDENSING ENGINES, AUTOMATIC CUT-OFF

With new engines these values may be taken from 1 to 1.5 lbs. less in the smaller cut-offs.

Avg. abs. admission pressure	Cut-off in per cent of full stroke										
	50	40	33.3	30	25	20	15	12.5	10	7	5
35	24.15										
40	23.55	21.50	20.20	19.40	18.60	17.80	16.95	16.95	16.60		
45	23.25	21.10	19.85	19.00	18.35	17.45	16.70	16.50	16.30	16.15	
50	23.00	20.80	19.55	18.75	18.05	17.10	16.45	16.10	16.00	15.75	16.10
55	22.75	20.60	19.30	18.50	17.80	16.90	16.15	15.75	15.75	15.45	15.65
60	22.50	20.45	19.10	18.25	17.55	16.70	15.90	15.50	15.50	15.15	15.45
65	22.30	20.30	18.90	18.05	17.40	16.50	15.70	15.30	15.35	14.95	15.25
70	22.10	20.10	18.70	17.95	17.25	16.35	15.50	15.10	15.20	14.75	15.05
75	21.95	19.97	18.55	17.80	17.10	16.18	15.35	15.00	15.05	14.60	14.85
80	21.80	19.81	18.40	17.72	16.95	16.05	15.20	14.90	14.90	14.50	14.65
85	21.70	19.70	18.25	17.60	16.85	15.97	15.05	14.80	14.75	14.40	14.45
90	21.60	19.60	18.10	17.50	16.75	15.85	14.95	14.70	14.60	14.30	14.25
95	21.50	19.50	18.00	17.42	16.65	15.75	14.85	14.67	14.50	14.20	14.05
100	21.40	19.40	17.93	17.35	16.55	15.65	14.75	14.60	14.40	14.10	13.90
105	21.30	19.30	17.85	17.27	16.45	15.55	14.67	14.53	14.30	14.00	13.80
110	21.20	19.20	17.76	17.15	16.40	15.45	14.60	14.45	14.20	13.95	13.75
115	21.10	19.10	17.67	17.07	16.35	15.37	14.55	14.40	14.10	13.90	13.70
120	21.00	19.00	17.60	17.00	16.30	15.30	14.50	14.35	14.00	13.85	13.65
125	20.90	18.95	17.55	16.95	16.25	15.23	14.45	14.30	13.95	13.80	13.60
130	20.80	18.90	17.50	16.90	16.20	15.15	14.40	14.25	13.90	13.75	13.55
135	20.75	18.85	17.45	16.85	16.15	15.08	14.37	14.20	13.85	13.70	13.50
140	20.70	18.80	17.40	16.80	16.10	15.00	14.35	14.15	13.77	13.65	13.45
145	20.65	18.75	17.35	16.75	16.05	14.95	14.32	14.10	13.75	13.60	13.40
150	20.60	18.70	17.30	16.70	16.00	14.92	14.30	14.05	13.73	13.55	13.35

formula. With engines in very good condition S_l may be only one-half the foregoing values while, with pistons in visibly leaky condition, the leakage loss may be twice this or even more.

The condensation losses in the steam lines plus any water carried over with the steam when the boilers prime may be taken from 4 to 10 per cent., depending upon the size and length of the steam line, its covering and the frequency w th which the boilers prime.

The method of procedure is best shown by an example: Required the steam consumption of a 42×60-in. vertical, single-cylinder, piston-valve throttling engine, the diagrams from which are shown in Fig. 1. Taking first the top end:

Useful steam per indicated h.p.-hr. from Table 3, $S_u = 34.25$ lbs.

Average admission pressure = 67.8 lbs., absolute.

Cut-off = 73.45 per cent.

Ratio of stroke to diameter $\left(\frac{s}{d}\right) = 1.43$.

Piston speed $(P) = 5.36$ ft. per sec.

Then

$$\sqrt{P} = \sqrt{5.36} = 2.315$$

From Tables 7 and 8, $K = 27.93 \times .91 = 25.4$, and from equation (a)

$$S_c = \frac{25.4}{2.315} = 11 \text{ lbs.}$$

The leakage losses S_l from equation (b) are

$$\frac{35.14}{\sqrt{289 \times 5.36}} + \frac{3.62}{5.36} = 1.57 \text{ lbs.}$$

the h.p. being computed from the indicator diagram, Fig. 1. Therefore,

$$S = 34.25 + 11 + 1.57 = 46.82 \text{ lbs.}$$

The steam-line losses are taken at 4 per cent.; hence the total steam consumption is $46.82 \times 1.04 = 48.7$ lbs. per h.p.-hr.

Taking next the bottom end:

Useful steam per indicated h.p.-hr. from Table 3, $S_u = 33.5$ lbs.

Average admission pressure = 57.8 lbs., absolute.

TABLE 6.—USEFUL STEAM PER I.H.P.-HR., S_u IN LBS. WITH COMPOUND CONDENSING ENGINES

These values are for engines in good condition and well defined cut-off and without preheating in the receiver.

With new engines these values may be taken from 1 to 1.5 lbs. less in the smaller cut-offs.

Avg. abs. admission pressure	Cut-off in per cent. of full stroke reduced to low pressure cyl.							
	25	20	15	12.5	10	7	5	4
40								
45	17.50	16.45	15.25	14.80	14.50	14.75	15.30	
50	17.25	16.10	15.00	14.50	14.10	14.25	14.50	
55	16.95	15.85	14.80	14.15	13.65	13.75	14.00	
60	16.70	15.55	14.60	13.85	13.25	13.40	13.50	13.50
65	16.50	15.35	14.40	13.65	13.10	13.05	13.00	13.00
70	16.35	15.15	14.20	13.50	12.95	12.70	12.60	12.75
75	16.25	15.00	14.00	13.35	12.85	12.50	12.25	12.50
80	16.15	14.90	13.85	13.25	12.75	12.30	12.00	12.25
85	16.05	14.80	13.70	13.15	12.65	12.10	11.80	12.00
90	15.95	14.70	13.55	13.05	12.55	11.95	11.60	11.75
95	15.90	14.65	13.45	12.95	12.45	11.85	11.45	11.50
100	15.85	14.60	13.35	12.85	12.35	11.75	11.30	11.35
105	15.82	14.55	13.25	12.80	12.25	11.65	11.20	11.20
110	15.79	14.50	13.15	12.70	12.15	11.55	11.10	11.10
115	15.76	14.45	13.05	12.60	12.05	11.45	11.03	11.00
120	15.73	14.40	13.00	12.50	11.95	11.35	10.94	10.90
125	15.70	14.35	13.00	12.45	11.85	11.25	10.83	10.80
130	15.67	14.30	12.97	12.42	11.75	11.15	10.75	10.70
135	15.64	14.25	12.95	12.39	11.65	11.05	10.67	10.60
140	15.61	14.20	12.93	12.36	11.55	10.95	10.60	10.50
145	15.58	14.17	12.91	12.35	11.45	10.85	10.53	10.40
150	15.55	14.15	12.90	12.32	11.43	10.80	10.47	10.30

TABLE 7.—VALUES OF K IN FORMULA (a)

Engine type	K
Throttling, single-cylinder, non-condensing....	27.938 to 25.952
Automatic cut-off, single-cylinder non-condensing.	23.955 to 19.963
Automatic cut-off, single-cylinder, condensing..	21.959 to 19.963
Compound, condensing....................	19.95 to 17.955
Triple-expansion, condensing...............	16.758 to 15.96

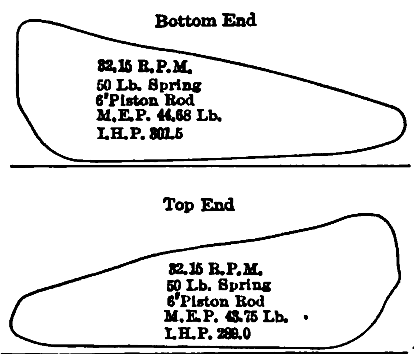


FIG. 1.—Indicator cards for calculated example.

TABLE 8.—CORRECTIONS FOR VALUES OF K

If $\frac{s}{d}$ is approximately	Then K in Table 5 is to be multiplied by
1	.82
1.25	.87
1.50	.91
2.00	1.00
2.50	1.08
3.00	1.15
4.00	1.29
5.00	1.41

TABLE 9.—MEAN FORWARD PRESSURE OF STEAM PER LB. OF INITIAL PRESSURE

Cut-off in fractions of the stroke		Percentage of clearance 0	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5	7
$\frac{1}{10}$	.1	.3303	.3439	.3505	.3568	.3630	.3690	.3750	.3808	.3864	.3919	.3974	.4027	.4076	.4126
$\frac{1}{8}$	.125	.3849	.3966	.4023	.4078	.4132	.4187	.4237	.4287	.4338	.4386	.4433	.4480	.4527	.4571
$\frac{1}{6}$	.167	.4662	.4757	.4802	.4844	.4890	.4933	.4973	.5014	.5056	.5096	.5134	.5173	.5210	.5245
$\frac{3}{16}$	.188	.5013	.5097	.5138	.5181	.5217	.5259	.5295	.5332	.5367	.5405	.5440	.5474	.5511	.5546
$\frac{1}{5}$	.20	.5219	.5298	.5336	.5376	.5414	.5449	.5482	.5517	.5556	.5588	.5623	.5656	.5687	.5716
$\frac{1}{4}$	.25	.5966	.6025	.6059	.6090	.6120	.6148	.6174	.6207	.6229	.6258	.6286	.6312	.6336	.6359
$\frac{3}{10}$	.30	.6609	.6663	.6684	.6712	.6729	.6755	.6779	.6803	.6825	.6845	.6864	.6882	.6911	.6927
$\frac{1}{3}$	.333	.6988	.7029	.7047	.7076	.7092	.7106	.7132	.7144	.7168	.7190	.7212	.7219	.7239	.7257
$\frac{3}{8}$	.375	.7433	.7458	.7476	.7494	.7510	.7525	.7539	.7569	.7582	.7593	.7603	.7630	.7639	.7646
$\frac{2}{5}$	.40	.7665	.7691	.7719	.7729	.7738	.7765	.7772	.7778	.7802	.7806	.7829	.7831	.7853	.7874
$\frac{1}{2}$	.50	.8466	.8484	.8492	.8503	.8513	.8522	.8530	.8539	.8548	.8556	.8565	.8573	.8582	.8590
$\frac{3}{5}$	.60	.9064	.9076	.9081	.9087	.9092	.9097	.9102	.9107	.9112	.9117	.9122	.9127	.9132	.9136
$\frac{5}{8}$	.625	.9188	.9194	.9201	.9206	.9210	.9215	.9220	.9224	.9228	.9233	.9237	.9241	.9245	.9249
$\frac{2}{3}$	.667	.9371	.9378	.9382	.9385	.9389	.9392	.9396	.9399	.9402	.9405	.9408	.9411	.9415	.9418
$\frac{7}{10}$	.70	.9497	.9502	.9505	.9508	.9511	.9513	.9516	.9518	.9521	.9524	.9526	.9528	.9531	.9533
$\frac{3}{4}$	.75	.9657	.9661	.9663	.9665	.9667	.9668	.9670	.9672	.9674	.9675	.9677	.9679	.9680	.9682

Cut-off = 70.5 per cent.

Condensation losses S_c same as for top end or 11 lbs.

Leakage losses $S_l = \frac{35.14}{\sqrt{301.5 \times 5.36}} + \frac{3.62}{5.37} = 1.55$ lbs.

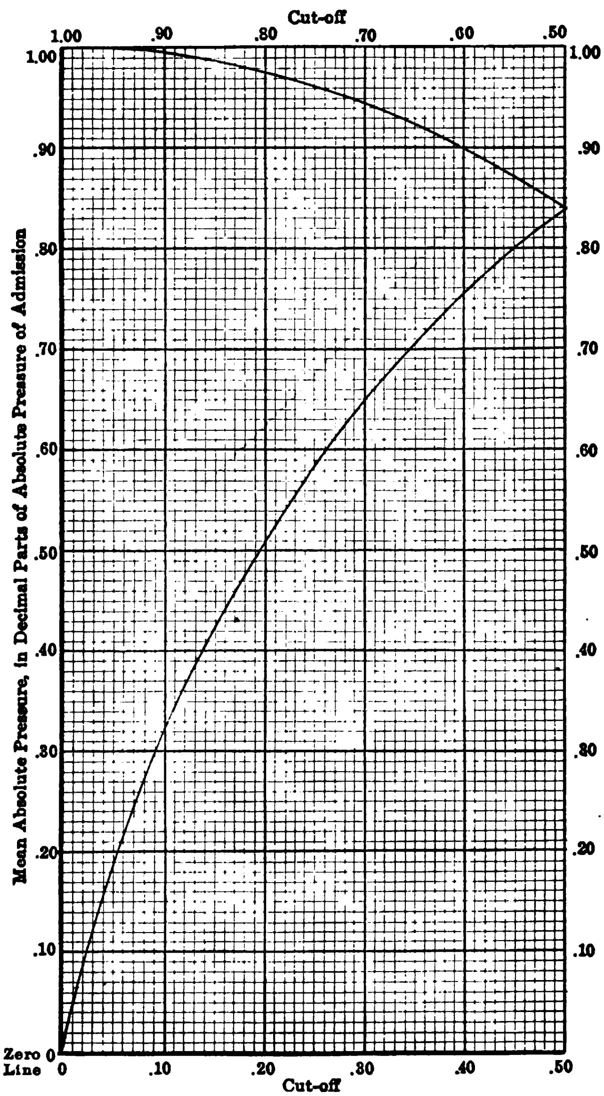


FIG. 2.—Mean forward pressure of steam used expansively.

Therefore,

$$S = 33.5 + 11 + 1.55 = 46.05 \text{ lbs.}$$

The steam-line losses are taken at 4 per cent.; hence the total steam consumption is

$$46.05 \times 1.04 = 47.9 \text{ lbs. per h.p.-hr.}$$

Therefore, the average steam consumption for the engine is 48.3 bs. per i.h.p.-hr.

Power Calculations

The theoretical mean effective pressure of steam used expansively is given by the formula:

$$M.e.p = P\frac{1 + hyp \log r}{r} - p$$

In which P = absolute initial pressure,
p = absolute back pressure,
r = ratio of expansion.

The same results may be more quickly obtained from Table 9, by F. R. Low (*Power*, *Sept.* 26, 1911). The table gives directly the absolute mean forward pressure per lb. of absolute initial pressure. The quantities of the table are to be multiplied by the absolute initial pressure and from the result the absolute back pressure is to be subtracted, the result being the mean effective pressure.

Essentially the same results, neglecting the effect of clearance, may be obtained from Fig. 2 by PROFESSOR RANKINE (*The Steam Engine and Other Prime Movers*) which is self-explanatory.

After obtaining the theoretical m.e.p. it is to be corrected for clearance and compression which may be done, with sufficient accuracy for most purposes, by multiplying the theoretical m.e.p. by .96.

The actual or expected mean effective pressure may then be obtained by multiplying the result by the proper factor from Table 10 from Seaton's *Manual of Marine Engineering*.

TABLE 10.—FACTORS FOR OBTAINING EXPECTED FROM THEORETICAL MEAN EFFECTIVE PRESSURE IN STEAM ENGINES

Type of Engine	Factor
Expansive engine, special valve gear or with separate cut-off valves, cylinders jacketed.	.94
Expansive engine having large ports, etc., and good ordinary valves, cylinders jacketed.	.9 to .92
Expansive engine with ordinary valves and gear as in general practice, unjacketed.	.8 to .85
Compound engines with expansion valve to h.p. cylinder, cylinders unjacketed and with large ports, etc.	.9 to .92
Compound engines with ordinary slide valves, cylinders jacketed and good ports, etc.	.8 to .85
Compound engines as in general practice in merchant marine service with early cut-off in both cylinders, without jackets and expansion valves.	.7 to .8

TABLE 11.—ACTUAL EXPANSION RATIOS

Per cent. of clearance	Points of cut-off															
	.10	.125	.20	.25	.30	.333	.375	.40	.50	.60	.625	.70	.75	.80	.875	.90
.01	9.181	7.481	4.809	3.884	3.258	2.944	2.623	2.463	1.983	1.655	1.590	1.422	1.328	1.246	1.141	1.109
.0125	9	7.363	4.764	3.875	3.24	2.930	2.612	2.454	1.975	1.653	1.588	1.421	1.327	1.246	1.140	1.109
.0150	8.826	7.25	4.720	3.830	3.222	2.916	2.602	2.445	1.970	1.650	1.585	1.419	1.326	1.245	1.140	1.109
.0175	8.659	7.133	4.677	3.803	3.204	2.902	2.592	2.436	1.966	1.647	1.583	1.418	1.325	1.244	1.140	1.108
.02	8.5	7.034	4.635	3.777	3.187	2.889	2.582	2.428	1.961	1.645	1.581	1.416	1.325	1.243	1.138	1.108
.0225	8.346	6.932	4.595	3.752	3.170	2.876	2.574	2.420	1.956	1.642	1.579	1.415	1.324	1.243	1.138	1.108
.0250	8.2	6.833	4.555	3.727	3.153	2.863	2.562	2.411	1.952	1.640	1.576	1.413	1.322	1.242	1.138	1.108
.0275	8.088	6.738	4.516	3.702	3.137	2.850	2.552	2.403	1.947	1.637	1.574	1.412	1.321	1.241	1.138	1.107
.03	7.933	6.645	4.417	3.678	3.121	2.837	2.543	2.395	1.943	1.634	1.572	1.410	1.320	1.240	1.138	1.107
.0325	7.792	6.555	4.440	3.654	3.105	2.824	2.533	2.387	1.938	1.632	1.570	1.409	1.319	1.240	1.138	1.107
.0350	7.666	6.468	4.404	3.631	3.089	2.812	2.524	2.379	1.934	1.629	1.568	1.408	1.318	1.239	1.137	1.106
.0375	7.545	6.390	4.484	3.608	3.074	2.800	2.515	2.371	1.930	1.627	1.566	1.406	1.317	1.238	1.136	1.106
.04	7.428	6.303	4.333	3.58	3.058	2.788	2.506	2.363	1.925	1.625	1.563	1.405	1.316	1.238	1.136	1.106
.0425	7.315	6.229	4.298	3.564	3.043	2.776	2.497	2.355	1.921	1.622	1.561	1.404	1.315	1.237	1.136	1.106
.0450	7.206	6.147	4.256	3.542	3.028	2.764	2.488	2.348	1.917	1.620	1.569	1.402	1.314	1.236	1.135	1.105
.0475	7.102	6.082	4.232	3.521	3.014	2.752	2.479	2.340	1.913	1.617	1.557	1.401	1.313	1.235	1.135	1.105
.05	7	6	4.2	3.5	3	2.741	2.470	2.333	1.907	1.615	1.555	1.400	1.312	1.235	1.135	1.105
.0525	6.901	5.985	4.168	3.478	2.986	2.730	2.461	2.325	1.904	1.613	1.553	1.398	1.311	1.234	1.134	1.104
.0550	6.806	5.861	4.130	3.459	2.971	2.719	2.453	2.318	1.900	1.610	1.551	1.397	1.310	1.233	1.134	1.104
.0575	6.714	5.794	4.106	3.439	2.957	2.708	2.445	2.311	1.896	1.608	1.549	1.396	1.309	1.233	1.134	1.104
.06	6.625	5.729	4.076	3.418	2.944	2.697	2.436	2.304	1.892	1.606	1.547	1.394	1.308	1.232	1.133	1.104
.0625	6.538	5.666	4.047	3.407	2.931	2.686	2.428	2.297	1.888	1.603	1.545	1.393	1.307	1.231	1.133	1.103
.0650	6.454	5.605	4.045	3.380	2.917	2.675	2.420	2.290	1.884	1.601	1.543	1.392	1.306	1.231	1.132	1.103
.0675	6.373	5.545	3.990	3.362	2.904	2.665	2.412	2.283	1.881	1.599	1.541	1.390	1.305	1.230	1.132	1.103
.07	6.294	5.482	3.963	3.342	2.892	2.655	2.404	2.276	1.877	1.597	1.539	1.389	1.304	1.229	1.132	1.103

TABLE 12.—HORSE-POWER OF SINGLE-CYLINDER STEAM ENGINES PER LB. OF MEAN EFFECTIVE PRESSURE

Diameter of cyl., ins.	Diameter of piston-rod, ins.	Speed of piston in ft. per min.				
		1 ft.	400 ft.	500 ft.	600 ft.	700 ft.
10	1¾	.00234	.936	1.17	1.404	1.638
11	1⅞	.00284	1.136	1.42	1.704	1.988
12	2	.00338	1.352	1.69	2.028	2.366
13	2	.00397	1.588	1.985	2.382	2.779
14	2¼	.00460	1.84	2.30	2.76	3.22
15	2⅜	.00529	2.116	2.645	3.174	3.703
16	2½	.00602	2.408	3.01	3.612	4.214
17	2⅝	.0068	2.72	3.40	4.08	4.76
18	2¾	.00762	3.048	3.81	4.572	5.334
19	2⅞	.00849	3.396	4.245	5.094	5.943
20	3	.00941	3.764	4.705	5.646	6.587
21	3⅛	.01038	4.152	5.19	6.228	7.266
22	3¼	.01139	4.556	5.695	6.834	7.973
23	3⅜	.01245	4.98	6.225	7.47	8.715
24	3½	.01356	5.424	6.78	8.136	9.492
25	3⅝	.01472	5.888	7.36	8.832	10.304
26	3¾	.01592	6.368	7.96	9.552	11.144
27	3⅞	.01717	6.868	8.585	10.302	12.019
28	4	.01847	7.388	9.235	11.082	12.929
29	4⅛	.01981	7.924	9.905	11.886	13.867
30	4¼	.02121	8.484	10.605	12.726	14.847
31	4⅜	.02264	9.056	11.32	13.584	15.848
32	4½	.02413	9.652	12.065	14.478	16.891
33	4⅝	.02566	10.264	12.83	15.396	17.962
34	4¾	.02724	10.896	13.62	16.344	19.068
35	4⅞	.02887	11.548	14.435	17.322	20.209
36	5	.03055	12.22	15.275	18.33	21.385
37	5⅛	.03227	12.908	16.135	19.362	22.589
38	5¼	.03404	13.616	17.02	20.424	23.828
39	5⅜	.03585	14.34	17.925	21.51	25.095
40	5½	.03772	15.088	18.86	22.632	26.404
41	5⅝	.03963	15.852	19.815	23.778	27.741
42	5¾	.04159	16.636	20.795	24.954	29.113
43	5⅞	.04360	17.44	21.80	26.16	30.52
44	6	.04565	18.26	22.825	27.39	31.955

Actual expansion ratios at various points of cut-off when the clearance is taken into account are given in Table 11 by ROBERT GRIMSHAW (*Amer. Mach., Jan.* 20, 1883).

The horse power of engines per lb. m.e.p. may be taken from Table 12.

To lay out the hyperbolic or isothermal expansion curve proceed as in Fig. 3. Locate the clearance line *AO* and the line *BO* of absolute vacuum. Through any point, as *C*, draw *CE* parallel and *CD* perpendicular to the atmospheric line. Draw radiating lines *OD*, *OL*, *OM*, etc., and from *D*, *L*, *M*, etc., and *F*, *H*, *J*, etc. draw horizontals and perpendiculars intersecting at *G*, *I*, *K*, etc., which are points of the required curve passing through *C*.

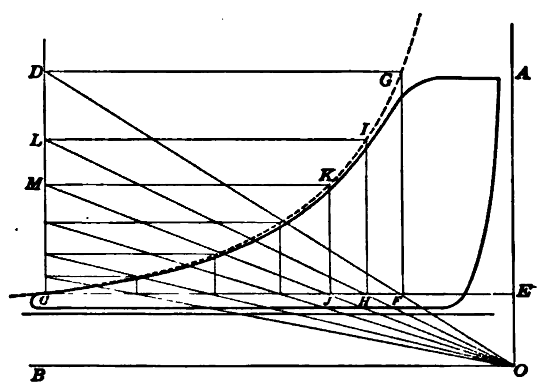


FIG. 3.—Laying out the hyperbolic or isothermal expansion curve.

Construction and Dimensions of Parts

Current practice in the dimensions of steam-engine parts formed the subject of an investigation by O. N. TROOIEN (*Bulletin No.* 252 *of the University of Wisconsin*) from which the following is taken.

Particulars were obtained of a large number of engines ranging between 20 and 400 rated h.p. The data secured were first tabulated and separated into classes and subclasses, the two main classes being high-speed or quick-revolution engines and low-speed or slow-revolution engines (the latter class being principally the

Corliss). Divisions into subclasses were made in the treatment of such parts as the crank pin for center-crank and side-crank engines, while in dealing with such parts as the piston rod or crosshead pin, no such division was thought necessary.

The following symbols of notation are used in the formulas given:

D = diameter of piston, ins.
A = area of piston, sq. ins.
L = length of stroke, ins.
p = unit steam pressure, taken as 125 lbs. per sq. in. above exhaust as a standard pressure.
$H.P.$ = rated horse-power.
N = revolutions per minute.
C = a constant.
K = a constant.
d = diameter of unit under consideration, ins.
l = length of unit under consideration, ins.

The commercial point of cut-off was taken at one-fourth of the stroke.

Other notation than the above is explained as used.

Diameter of piston rod:

$$d = C\sqrt{DL}$$

L being the free length.

Values of C for high-speed engines: Mean .15; maximum .187; minimum .125. For Corliss engines: Mean .114, maximum .156; minimum .1.

Thickness of cylinder wall:

$$t = CD + .28 \text{ in.}$$

in which t = thickness, ins.,
D = diameter of piston, ins.,
C = a constant.

Values of C: Mean .054; maximum .072; minimum .035. No characteristic difference was found between high- and low-speed engines.

Diameter of cylinder-cover stud bolts:

$$d = CD + \tfrac{3}{8} \text{ in.}$$

in which d = diameter of bolts, ins.,
D = diameter of cylinder, ins.,
C = a constant.

Mean value of $C = .04$.

With only one exception the smallest diameter of bolts used in the high-speed engines was $\tfrac{3}{4}$ in., and in the Corliss engines the smallest value was 1 in.

The mean thickness of cylinder flanges for holding cylinder covers, where these were bolted to cylinder flanges, was found to be 1.12 times the thickness of cylinder wall, for both high-speed and Corliss engines.

The thickness of cylinder cover at the center seems to vary a great deal, but for the engines examined it may be taken as 2.75 times the thickness of the cylinder wall for high-speed engines and 1.12 times the thickness of cylinder wall for Corliss engines.

Number of stud bolts for cylinder covers:

$$N = CD$$

in which N = the number of bolts

Mean value of $C = .72$ for high-speed engines, and .65 for Corliss engines.

The least number of bolts used for any engine was found to be six. For additional information on cylinder-cover joints and bolts, including a chart for the diameter and number of bolts, see below.

The clearance volume was found to vary from 5 to 11 per cent. in high-speed engines and from 2 to 5 per cent. in Corliss engines.

Ratio of length of stroke to diameter of cylinder in engines having a speed greater than 200 r.p.m.:

$$L = CD$$

Values of C: Mean 1.07; maximum 1.55; minimum .82.

Ratio of length of stroke to diameter of cylinder in engines having a speed between 110 and 200 r.p.m.:

$$L = CD$$

Values of C: Mean 1.36; maximum 1.88; minimum 1.03.

Ratio of length of stroke to diameter of cylinder in engines having a speed less than 110 r.p.m. (Corliss engines):

$$L = CD + 8 \text{ ins.}$$

Values of C: Mean 1.63; maximum 2.40; minimum 1.15.

Face of piston in terms of diameter:

$$w = CD$$

or

$$w = CD + 1 \text{ in.}$$

in which w = width of piston,
D = diameter of piston,
C = a constant.

Using $w = CD$:

Values of C for high-speed engines: Mean .40; maximum .47; minimum .30. For Corlis engines: Mean .32.

Using the equation $w = CD + 1$ in:

Values of C for high-speed engines: Mean .32; maximum .40; minimum .24. For Corliss engines: Mean .26.

The box type seems to be the prevailing form of piston. The thickness of shell of piston in high-speed engines is about .6 of the thickness of cylinder wall, and for Corliss engines this ratio is about .7.

The prevailing number of rings used for the piston is two, and the rings are usually turned to a diameter $\tfrac{1}{4}$ in. larger than the bore of the cylinder. For additional details of pistons see below.

Piston speed—high-speed engines:

Mean 605, maximum 900, minimum 320 ft. per min.

Piston speed—Corliss engines:

Mean 592, maximum 800, minimum 400 ft. per min.

Area of cross-head shoes:

$$a = CA$$

in which a = area of cross-head shoes:

Values of C: Mean .53; maximum .72; minimum .37.

Pressure on cross-head shoes, steam being assumed to follow as far as half stroke:

$$s = \frac{125}{nC}$$

in which s = pressure on shoes, lbs. per sq. in.,

$$n = \frac{\text{length of connecting rod}}{\text{length of crank}}.$$

For high-speed engines n may be taken as 6 and for Corliss engines as 5.5. Values of s for high-speed engines: Mean 39.5; maximum 57; minimum 28. For Corliss engines: Mean 43; maximum 61; minimum 32.

Under normal conditions of shorter cut-off these values are materially reduced.

Length of bearing part of cross-head pin in terms of its diameter:

$$l = Cd$$

Values of C for high-speed engines: Mean 1.25; maximum 1.5; minimum 1. For Corliss engines: Mean 1.43; maximum 1.9; minimum 1.

Dimensions of cross-head pin:

$$dl = KA$$

Values of K for high-speed engines: Mean .10; maximum .15; minimum .037. For Corliss engines: Mean .115; maximum .19; minimum .037.

Cross-section of connecting rod of high-speed engines at the middle of its length:

$$h = Cb$$

In which h = height,
b = breadth.

Values of C: Mean 2.28; maximum 3; minimum 1.85.

Dimensions of cross-section of connecting rod of high-speed engines at the middle of its length:

$$b = C\sqrt{DL_c}$$

in which L_c = length of rod between centers.

Values of C: Mean .073; maximum .094; minimum .05.

Dimensions of cross-section of connecting rod of Corliss engines (circular section only):

$$d = C\sqrt{DL_c}$$

Values of C: Mean .092; maximum .104; minimum .081.

Length of crank pin in terms of its diameter:

$$l = CD$$

Values of C for high-speed engines: Mean .87; maximum 1.25; minimum .66;. For Corliss engines: Mean 1.14; maximum 1.30; minimum 1.

Diameter of crank pin:

$$d = CD$$

Values of C for high-speed center-crank engines: Mean .40; maximum .526; minimum .28. For side-crank Corliss engines: Mean .27; maximum .32; minimum .21.

Diameter of main journal of high-speed center-crank engines:

$$d = C\sqrt[3]{\frac{H.P.}{N}}$$

Values of C: Mean 6.6; maximum 8.2; minimum 5.4.

For Corliss engines this dimension seems best expressed by the form:

$$d = C\left(\sqrt[3]{\frac{H.P.}{N}} - .3\right)$$

Values of C: Mean 7.2; maximum 8; minimum 6.4.

Length of main journal in terms of its diameter:

$$l = Kd$$

Values of K for high-speed center-crank engines: Mean 2.1; maximum 2.9; minimum 1.6. For Corliss side-crank engines: Mean 1.9; maximum 2.2; minimum 1.62.

Projected area of main journal in terms of piston area:

$$dl = FA$$

Values of F for high-speed center-crank engines: Mean .48; maximum .78; minimum .32. For Corliss side-crank engines: Mean .6; maximum .66; minimum .5.

For additional data on bearings of steam engines see Index and below.

Weight of fly-wheel:

$$W = C \times \frac{H.P.}{D_1^2 N^3}$$

in which W = total weight of wheel, lbs.

This relation gives fairly satisfactory results for high-speed engines up to about 175 horse-power, and for this range the values of C are: Mean 1,300,000,000,000; maximum 2,800,000,000,000; minimum 660,000,000,000.

When high-speed engines of larger size are considered, the relation seems better expressed by:

$$W = C \times \frac{H.P.}{D_1^2 N^3} + 1000$$

Values of C: Mean 720,000,000,000; maximum 1,140,000,000,000; minimum 330,000,000,000.

A somewhat greater uniformity seems to exist among the builders of standard Corliss engines. In these engines the relation seems best expressed by:

$$W = C\frac{H.P.}{D_1^2 N^3} - K$$

Values of C: Mean 890,000,000,000; maximum 1,330,000,000,000; minimum 625,000,000,000. Corresponding values of K: Mean 4000; maximum 6000; minimum 2800.

For additional information on the weight of steam-engine fly-wheels see Fly-wheels.

Diameter of fly-wheel in terms of length of stroke:

$$D_1 = CL$$

in which D_1 = outside diameter of wheel, ins.

Values of C for high-speed engines: Mean 4.4; maximum 5; minimum 3.4. For Corliss engines: Mean 4.4; maximum 5.25; minimum 3.25.

Belt surface per indicated horse-power:

$$S = C \times H.P$$

in which S = velocity of wheel rim, ft. per min., multiplied by the width of belt, ft.

Values for C for high-speed engines; Mean 26.5; maximum 55; minimum 10.

For Corliss engines, this relation seems better expressed by:

$$S = C \times H.P + 1000$$

Values of C: Mean 21; maximum 35; minimum 18.2.

For additional data on main belts for steam engines see Belts.

Velocity of fly-wheel rim in ft. per sec.; for high-speed engines: Mean 70, maximum 82, minimum 48 ft. per sec.

For Corliss engines: Mean 68, maximum 82, minimum 40 ft. per sec.

Weight of reciprocating parts:

$$W = C \times \frac{D^2}{LN}$$

in which W = weight of reciprocating parts (piston, piston rod cross-head and one-half the connecting rod), lbs.

Values of C for high-speed engines (data not obtained for Corliss engines): Mean 2,000,000; maximum 3,400,000; minimum 1,370,000

For the cases where the information was obtainable, the balance weight opposite the crank pin was found to be about 75 per cent. of the weight of the reciprocating parts.

Total weight of engine in terms of horse-power:

$$W = C \times \text{H.P.}$$

in which W = total weight of engine, lbs.

Values of C: Mean 82; maximum 120; minimum 52.

For direct-connected engines, the weight of the engine without the generator was found to be from 10 to 25 per cent. greater than the weight of belt-connected engines of the same capacity.

Values of C for Corliss engines: Mean 132; maximum 164; minimum 102.

The dimensions of main bearings of large engines, to avoid undue heating, according to the practice of the late Edwin Reynolds, should be such that the product of the square root of the speed of rubbing surface in ft. per sec. multiplied by the pressure in lbs. per sq. in. of projected area should never exceed the constant number 375 for an horizontal engine, or 500 for a vertical engine when the shaft was lifted at every revolution.

Locomotive main driving boxes in some cases give a constant as high as 585, but this is accounted for by the cooling action of the air.

Using this principle, Figs. 4 and 5 have been constructed by F. W. SALMON (*Amer. Mach., Sept.* 17, 1903). Fig. 4 gives the velocity of rubbing in ft. per min. and per sec. for shafts from 4 to 17 ins. diameter and for speeds from 60 to 140 r.p.m., and Fig. 5 gives the loads per sq. in. for various velocities and for various constants.

The dimensions of the main frames of steam engines have received less attention in discussion than any other feature. When the Corliss (girder) frame was more popular than now, the author made an examination of a good many such frames (*Amer. Mach., Feb.* 14, 1895). While the resulting data have small application to other types of frames they are given here in the absence of others. The method of comparison was to compute from measurements of the frames the number of sq. ins. in the smallest cross-section, that

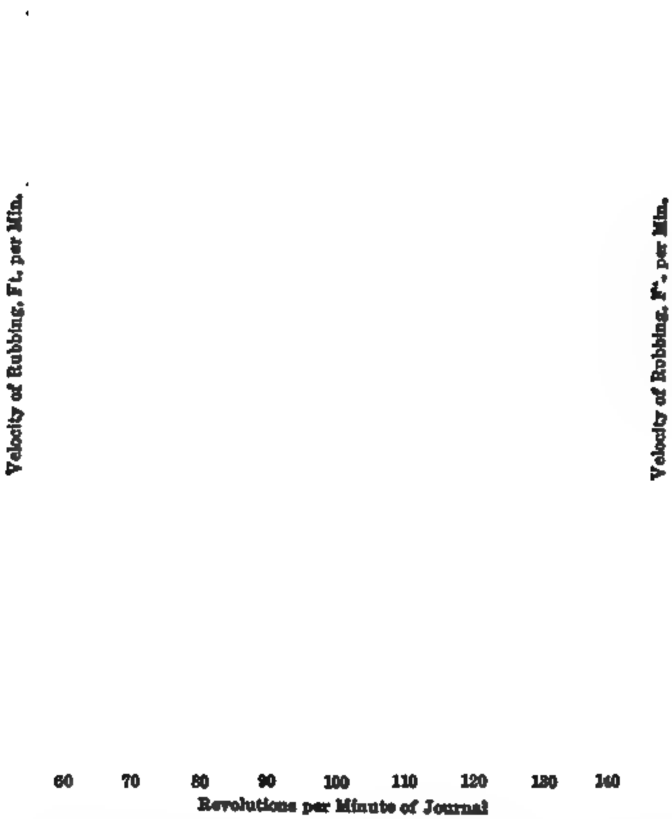


FIG. 4.

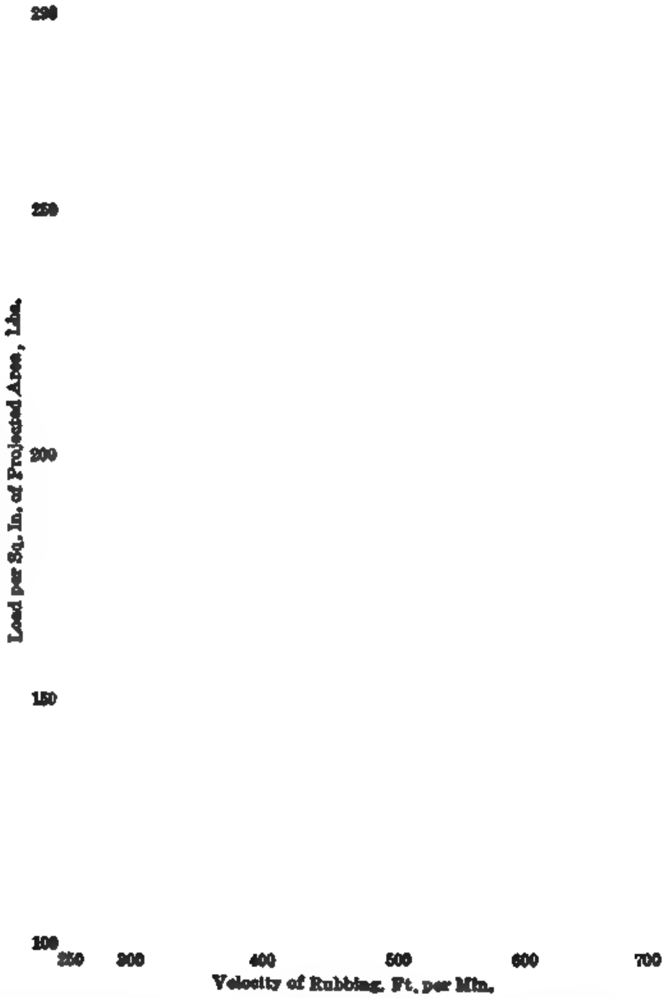


FIG. 5.

From the r.p.m. in the base line of Fig. 4 trace upward to the diagonal for the diameter of the journal, thence horizontally and read velocity of rubbing. Find this velocity in the base line of Fig. 5, trace upward to the curve for the selected constant, thence horizontally and read the appropriate pressure per sq. in. The journal is then to be of a length which will bring the pressure per sq. in. down to this figure.

FIGS. 4 and 5.—Rubbing velocity and safe bearing pressures on main journals of steam engines.

is, immediately behind the pillow block, also the compute the total maximum pressure upon the piston, and to divide the latter quantity by the former. The result gives the number of lbs. pressure upon the piston allowed for each sq. in. of metal in the frame. This, while not the actual strain upon the metal, is strictly comparative and that is all that is required for the purpose.

Representative figures resulting from the examination are given in Table 13.

TABLE 13.—DIMENSIONS OF SMALLEST SECTION OF CORLISS ENGINE FRAMES

Size of engine	Lbs. per sq. in. of smallest section of frame
10×30	217
12×36	248
18×42	278
24×48	360
24×48	395
28×48	575
30×60	350

It will be observed that, speaking generally, the strains increase with the size of the engine and that more cross-section of metal is allowed with relatively long strokes than with short ones, both of which are as we should expect. Other data of a more miscellaneous character show loads of about 300 lbs. on short stroke engines of about 10 ins. diameter of cylinder, while one memorandum of a 32-in. engine which had been running for many years without any indication of weakness gives a strain of 667 lbs.

From the above the author formulated the general rule that in engines of moderate speed, and having strokes up to one and one-half times the diameter of the cylinder, the load per sq. in. of smallest section should be for a 10-in. engine 300 lbs., which figure should be increased for larger bores up to 500 lbs. for a 30-in. cylinder of the same relative stroke. For high speeds or for longer strokes the load per sq. in. should be reduced in accordance with good judgment.

The following additional particulars of steam-engine parts are from Seaton's Manual of Marine Engineering:

Frame bolts: Stress not to exceed 4000 lbs. per sq. in. at bottom of thread or, for a large number of small bolts, 3000 lbs. When possible add 20 per cent. to the cross-section as given by this rule.

Cylinder covers: When above 24 ins. diameter for high- and 40 ins. for low-pressure cylinders, cylinder covers should be made hollow with a depth at the center of about ¼ the diameter of the piston.

Pitch of cylinder-cover bolts should not exceed $\sqrt{\frac{t \times 100}{p}}$ in which t = thickness of cover flange in 16ths in. and p = pressure on cover, lbs. per sq. in.

Flat surfaces of cast-iron sustaining steam pressure should be stiffened by ribs of a pitch not greater than $\sqrt{\frac{t^2 \times 50}{p}}$ in which t = thickness in 16ths in. and p = pressure, lbs. per sq. in. Ribs to be of the same thickness as the flat surface and of a depth = $2\frac{1}{2}$ times the thickness.

Piston of the follower type: Compute

$$x = \frac{D}{50}\sqrt{p} + 1$$

in which D = diameter of piston, ins.
p = effective pressure, lbs. per sq. in.,

Number of ribs in piston	$= \frac{D+20}{12}$
Thickness of ribs in piston	$= .18x$
Thickness of front of piston near hub	$= .2x$
Thickness of front of piston near rim	$= .17x$
Thickness of back of piston	$= .18x$
Thickness of hub around rod	$= .3x$
Depth of piston near center	$= 1.4x$
Diameter of follower bolts	$= .1x \times \frac{1}{4}$ in.
Pitch of follower bolts	$= 10$ diameters

Slide valve rod:

$$\text{Diameter of valve rod} = \sqrt{\frac{L \times B \times p}{F}}$$

in which L = length of valve, ins.,
B = breadth of valve, ins.,
p = maximum pressure, lbs. per sq. in.,
F = 12,000 for long steel rods,
F = 14,500 for short steel rods.

Slot links for link motion:

Let D = diameter of valve rod as above, taking F = 12,000

Diameter of block pin if secured at one end only	$= D$
Diameter of block pin if secured at both ends	$= .75D$
Diameter of eccentric-rod pins	$= .7D$
Diameter of suspension-rod pins if secured at both ends	$= .55D$
Diameter of suspension-rod pins if secured at one end only	$= .75D$
Breadth of link	$= .8D$ to $.9D$
Length of block	$= 1.6D$ to $1.8D$
Thickness of bars of link	$= .7D$
Diameter of suspension rod if but one	$= .7D$
Diameter of suspension rods if two	$= .55D$

The dimensions of snap piston rings may be determined from Figs. 6 and 7 by the author (*Amer. Mach., June* 3, 1909). The ring is cast large with two lugs on the inside as shown in Fig. 6. After being cut off and faced to thickness, the gap is cut out. The ring is then sprung together by a clamp on these lugs, when it is strapped to a face plate, or, better, clamped between two flanges of a special fixture, and is put in the lathe and turned to the true size of the cylinder bore. When the clamp is relieved, the ring expands again, but not to a circular shape; but when put in its place in the cylinder, it resumes its circular form, or very near it, and is a fit all around, as it should be. To secure the best results the ring should be released from the face plate and be reclamped before making the finishing cut.

A reverse procedure is used in making the pattern. If the pattern is turned up round, it will be found when the gap is cut out and closed up, that the casting will have so departed from its circular shape that, unless excessive finish has been allowed on the pattern, the ring will not clean up at all points, and where the cut is heavy, the ring will be thin after turning. To obviate this, make the pattern of pattern size with usual finish, as though the ring were to be solid and of the same size as the cylinder bore. Next saw the pattern apart where the gap of the ring is to be, and insert a piece the size of the gap.

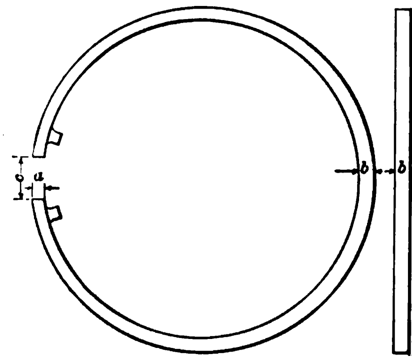


FIG. 6. A Snap Piston Ring

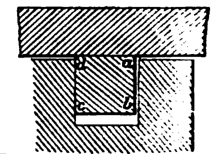


FIG. 8. Action of a Snap Piston Ring under Pressure

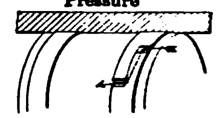
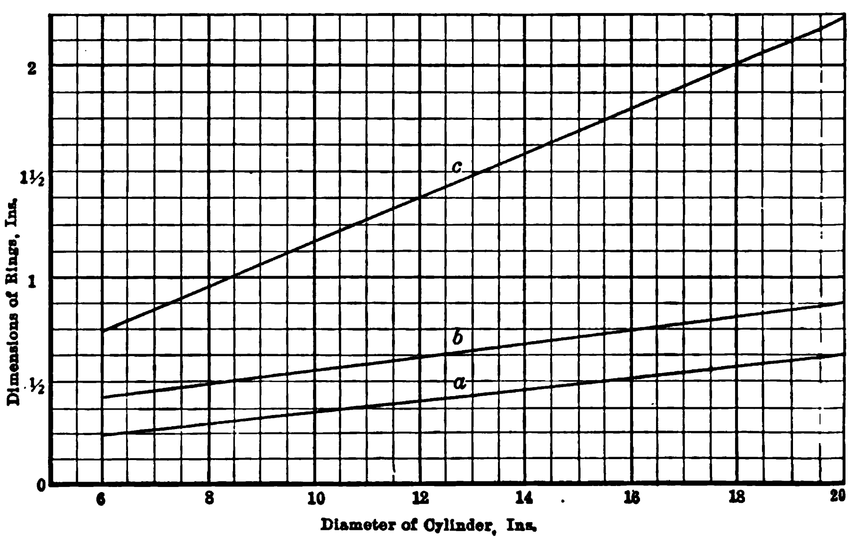


FIG. 9. Steam Blowing Past a Lapped Joint



$a = .0268\ d + .0893$
$b = .0312\ d + .25$
$c = .1071\ d + .1071$

d being the diameter of the cylinder and the remaining notation, as in Fig. 6. All dimensions in inches

FIG. 7.—Dimensions of snap piston rings.

FIGS. 6 to 9.—Snap piston rings and their action.

This will spring the pattern outward to a non-circular form such that when the casting is cut and sprung inward for turning, it will be nearly a true circle in the rough, with a fairly uniform allowance for finish all around and with a gradually tapering thickness, as intended.

Should the rings not come out exactly as intended, the case can be met to a certain extent by changing the width of the gap as there is no nicety about this dimension.

Large numbers of rings have been made to the dimensions of the

chart and with entire success—the extreme size made being of 28 ins. diameter.

Regarding the large increase in the width of gap over the more customary dimensions there is nothing to be said against it, as the strength of such rings is not sufficient to bring about any undue pressure against the bore of the cylinder, while it has the advantage that it reduces the danger of breakage when putting the rings in place—especially with the smaller sizes. The fact is that the rubbing of the rings against the cylinder wall introduces a sort of peening action and no matter what their original strength they soon lose most of it.

TABLE 14.—DIMENSIONS FOR PISTON ENDS OF PISTON RODS

A	B	C	D	E	F	G	H	J	K	L	M	N	P	R	S
ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	
2 3/4	3 1/4	3	2 1/2	4 1/4	3 1/2	3 3/8	2 1/4	5 1/4	3 1/4	3 3/4	3/4	1 1/2	2	4 3/4	8
3	3 1/2	3 1/4	2 3/4	4 1/2	3 3/4	3 1/2	2 1/2	5 1/2	4 1/4	4 1/4	3/4	1 1/2	2	5 1/4	8
3 1/4	3 3/4	3 1/2	3	5 1/4	4 1/4	4 1/4	2 1/2	6 1/4	4 3/4	4 1/2	3/4	1 1/2	2	5 3/4	8
3 1/2	4	3 3/4	3 1/4	5 1/2	4 1/2	3 7/8	2 3/4	7 1/4	5	4 3/4	1	1 1/2	2 1/2	6	6
3 1/2	4	3 3/4	3 1/4	6 1/4	5 1/4	4 3/8	2 3/4	8 1/4	5	4 7/8	1	1 1/2	2 1/2	6	6
3 3/4	4 1/4	4	3 1/2	6 1/2	5 1/4	4 5/8	2 7/8	8 1/4	5 3/8	5 1/4	1	1 1/2	2 1/2	6 1/4	6
4	4 1/2	4 1/4	3 3/4	6 1/2	5 1/2	4 3/4	2 7/8	8 1/2	5 3/4	5 3/4	1	1 1/2	2 1/2	6 3/4	6
4	4 1/2	4 1/4	3 3/4	7	6	5 1/4	2 7/8	8 3/4	5 3/4	5 3/4	1	1 1/2	2 1/2	6 7/8	6
4 1/4	4 3/4	4 1/2	4	7	6	5 3/4	3 1/4	9 1/4	6 1/4	6	1	2	3	7 1/4	6
4 1/2	5	4 3/4	4 1/4	7	6	5 3/4	3 1/4	9 3/4	6 1/2	6 3/4	1	2	3	7 3/4	6

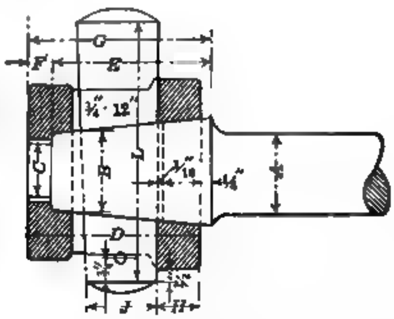


TABLE 15.—DIMENSIONS FOR CROSSHEAD ENDS OF PISTON RODS

A	B	C	D	E	F	G	H	J	K	L
ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.
2 3/4	2 3/4	1 13/16	5 3/4	5 1/4	3/8	6	1 1/2	2 3/4	5/8	9
3	2 3/4	1 13/16	5 3/4	5 1/4	3/8	6	1 1/2	2 3/4	5/8	9
3 1/4	3 1/4	2 1/16	6 1/4	6	3/8	6 1/4	1 3/4	2 3/4	5/8	9
3 1/2	3 3/8	2 1/16	6 1/4	6	3/8	6 1/4	1 3/4	2 3/4	5/8	9
3 3/4	3 5/8	2 13/16	7	6 1/2	3/8	7 1/4	1 3/4	3	7/8	10 1/2
4	3 7/8	2 13/16	7	6 1/2	3/8	7 1/4	1 3/4	3	7/8	10 1/2
4 1/4	4 1/8	3 1/16	7 1/2	7	3/8	7 3/4	2	3 1/4	1	11 1/2
4 1/2	4 3/8	3 1/16	7 1/2	7	3/8	7 3/4	2	3 1/4	1	11 1/2

The force which presses the ring against the cylinder wall is chiefly the steam pressure, compared with which the force exerted by the strength of the ring is insignificant.

Referring to Fig. 8, in which the clearances are exaggerated for clearness, the steam comes down the joint between the piston and the cylinder bore from right to left, and flowing down the joint *ab*, establishes full pressure in that joint and below the ring. As shown experimentally by Prof. S. W. Robinson, the steam also establishes a creeping film in the joint *ad*, beginning at full pressure at *a* and ending at such pressure at *d* as may exist at the left of *d*, the average pressure of the film being about the mean of the initial and the terminal pressures. Under these circumstances the outward pressure prevails and the ring is forced against the cylinder bore. It is doubtful if the eccentric construction has much value beyond satisfying the feeling that it is appropriate for the purpose.

A consideration of the action described in connection with Fig. 8 will show that the practice which some follow, of placing two rings in a groove is wrong. The average pressure per sq. in. of the surfaces in contact with the cylinder bore is substantially the same with two rings as with one, while the two rings, having twice the surface, exert twice the total pressure and double the tendency to wear the cylinder. On the other hand if the rings are placed in separate grooves the second one has only to deal with the pressure due to the leakage past the first, there being a progressive reduction of pressure from ring to ring, the tendency to leak and to wear being measured by the difference of pressure on the two sides of a ring. The fact that two rings in the same groove may be so placed as to break joints has no value, as the action is precisely the same as that of the lapped joint referred to in the next paragraph.

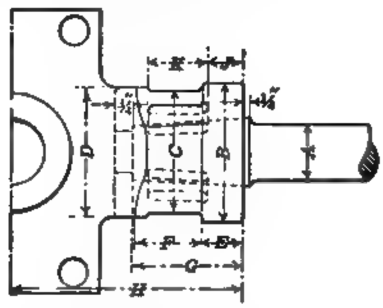


TABLE 16.—DIMENSIONS OF CROSSHEADS

Diameter of rod A	B	C	D	E	F	G	H	J	K
ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.
2 3/4 3	6 1/4	5 1/4	6	1 3/4	3	5	10 1/2	1 1/2	2 13/16
3 1/4 3 1/2	7 1/4	6 1/4	7	2	3 1/4	5 3/4	12	1 3/4	3 1/8
3 3/4 4	8	7	7 1/2	2	3 3/4	6 1/2	13	1 3/4	3 1/2
4 1/4 4 1/2	9	8	8 1/2	2 1/4	6 1/4	6 1/4	14	2	3 1/2

A feature of these rings, as sometimes made, which serves no useful purpose is the lap joint, shown in Fig. 9. With such a joint the steam follows the course indicated by the arrow and escapes as freely as though the lap were absent. The only good effect of this form of joint is to prevent the tendency to streak the cylinder, due to a plain square joint, but this can be obviated just as effectively by cutting the joint at an angle.

Eccentric rings being heaviest opposite the joint, they have a tendency, in horizontal cylinders, to work around to a position with the joint at the top of the piston, where the steam may blow through freely. To prevent this these joints should be placed at or near the bottom of the piston and pins be inserted in the grooves to keep them there.

The practice of scraping the rings into their grooves when followers and junk rings are used, represents wasted effort as a consideration of Fig. 8 will show. Moreover, the accumulation of oil residue tends to stick such rings fast and prevent their expansion. Up to the point where noise results, a slight degree of looseness sideways is advantageous, as it tends to prevent this action.

Locomotive practice in dimensions of pistons, piston rods and steel crossheads, as drawn up by a committee of the Amer. Ry. M.M. Asso. (*Amer. Mach., June* 29, 1911) is given in Tables 14, 15 and 16.

The taper fit of piston rods in pistons and crossheads is almost universal, but the author can see no reason for it. His own practice was to make them a straight sliding fit, bottoming at the end for the crosshead and against a shoulder for the piston. The straight fit is much cheaper, not only as regards the actual fits but because the rod can be made to measure. The taper fit is chiefly a matter of habit and tradition.

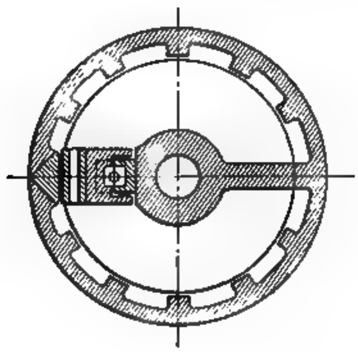


FIG. 10.—Piston valve without packing rings.

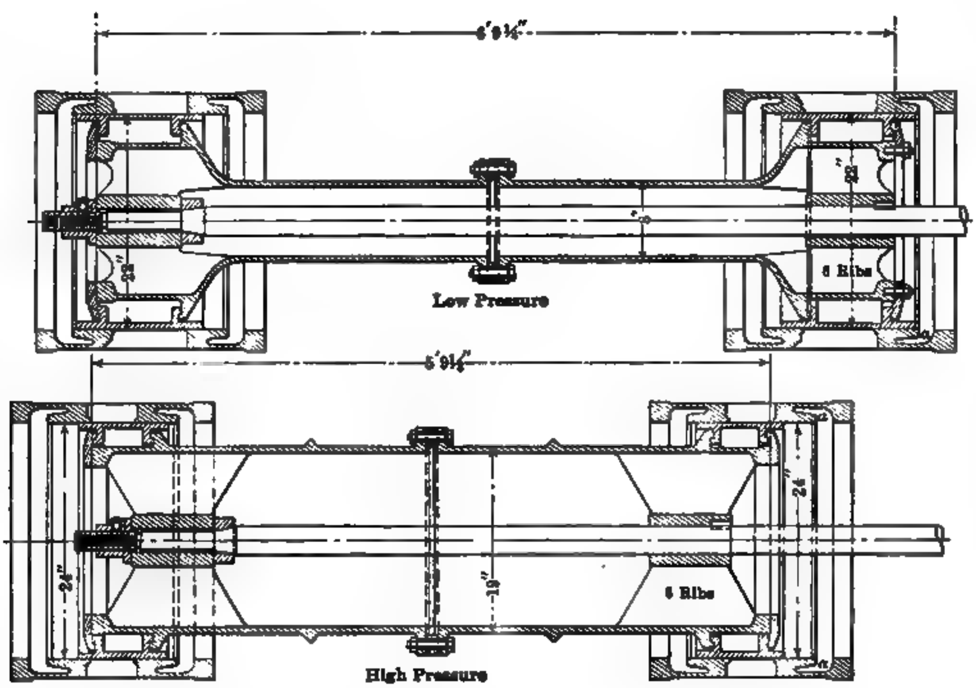


FIG. 11.—Large piston valves for steam engines.

A construction of piston valve which avoids the use of packing rings is shown in Fig. 10 (*W. H. Booth, Amer. Mach., May* 21, 1896). After rough turning, the valve is slotted longitudinally, compressed by an encircling clip, turned at the ends and a V-piece fitted in a V-recess on the inside of the valve, the slot through the body being along the apex of the V. The V-piece is fitted with a spring and wedge combination or other means of setting up. The ends are then bolted on, the encircling clip removed and the valve is finished to its correct diameter. Thus made, it has an initial elasticity and does not require any expanding pressure from the V, the duty of which is merely to close the longitudinal slot.

Large piston valves of this construction as made by the Union Iron Works (*Amer. Mach., Oct.* 12, 1905) are shown with a few leading dimensions in Fig. 11. The object of the deep circumferential recesses *aa* is to facilitate heating of the seats and thus diminish distortion. The admission of steam by the high-pressure valve is by its inside and by the intermediate and low-pressure valves by their outside edges. The rings forming the valves proper are split longitudinally and have tongue pieces—not shown—in the joints as shown in Fig. 10.

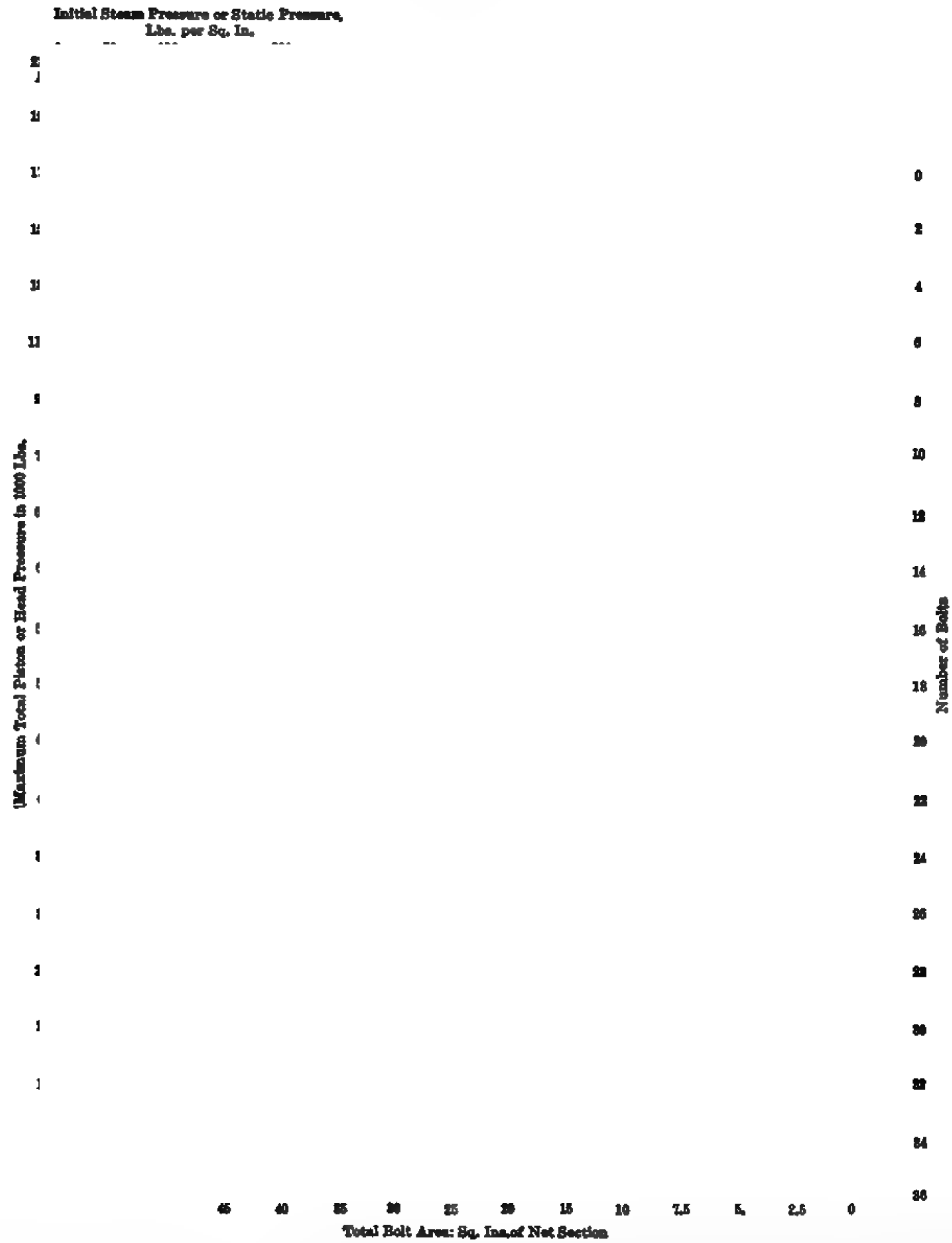
Cylinder Cover Joints

The diameters of cylinder and tank head bolts may be obtained from Fig. 12, by F. K. CASWELL (*Amer. Mach., July* 7, 1898), the use of which is shown by the example below it.

The unit stress for cylinder head bolts in good practice is about 4000 lbs. per sq. in. on the net section, or 3500 lbs. if the bolts are less than ¾ in. diameter, though stresses up to 8000 lbs. are used.

The adjustment of the diameter to the number of bolts to give the required total cross-section is so made that the distance between bolts shall not be so great as to endanger tightness of the joint. For information on this point see above. Provision for tightness with small cylinders gives an excess of strength when customary sizes of bolts are used.

The common gasket joint of cylinder covers is a common nuisance. Fig. 13 shows the joint used on the Straight Line Engine and on the engines of the Ball Engine Co. *The joint is not ground* but simply faced in a lathe without special care or workmanship. The only essential for its success is that it be *narrow*—not over ⅜ in. wide. The distance between stud centers should not exceed about four times the thickness of the cover flange.



To find the number and diameter of bolts for a steam cylinder head of 18 ins. diameter, subjected to a pressure of 175 lbs. per sq. in.: Find 175 on the pressure scale at the top, trace downward to the cylinder diagonal, 18, thence horizontally to the stress diagonal, say 4500 lbs. per sq. in., thence downward to the ⅞-in. diagonal and thence horizontally and read, at the right, 25, the number of bolts, or, from the intersection with the stress diagonal trace downward to the bottom and read 9.75 sq. ins. the total net bolt section. By tracing vertically from the pressure per sq. in., 175 lbs., to the cylinder diagonal, 18, and thence horizontally to the left, the total pressure on the head, 44, may be read in thousands of lbs.

FIG. 12.—Diameter and number of cylinder and tank head bolts.

Stuffing box glands should be made as shown in Fig. 15—not as usual, as shown in Fig. 14. With the form shown in Fig. 14, leakage is apt to take place around the outside of the packing, though the gland be more than tight enough to stop leakage around the rod. The bottom of the box should be flat—not beveled, as in Fig. 14.

Areas of Ports and Pipes

The areas of steam ports and pipes, as determined by an investigation of 165 single-cylinder engines ranging from 20 to 740 h.p. by PROF. JOHN H. BARR (*Trans. A. S. M. E., Vol.* 18), may be expressed by the formula:

$$a = \frac{AV}{C}$$

in which a = area of port or pipe, sq. ins.,
A = area of piston, sq. ins.,
V = velocity of piston, ft. per min.,
C = mean velocity of steam in port or pipe, ft. per min.

FIG. 13.—Cylinder cover joint of the Straight-Line engine.

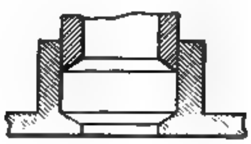


FIG. 14.

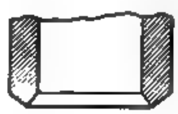


FIG. 15.

FIGS. 14 and 15.—Correct and incorrect construction of stuffing box glands.

For high-speed engines using the same port for both admission and exhaust, the values of C are: Mean 5500; maximum 6500; minimum 4500. For Corliss engine steam ports: Mean 6800; maximum 9000; minimum 5000. For Corliss engine exhaust ports: Mean 5500; maximum 7000; minimum 4000.

For high-speed engine steam pipes the values of C are: Mean 6500; maximum 7000; minimum 4800; For Corliss engine steam pipes: Mean 6000; maximum 8000; minimum 5000.

For high-speed engine exhaust pipes the values of C are: Mean 4400; maximum 5500; minimum 2500. For Corliss engine exhaust pipes: Mean 3800; maximum 4700; minimum 2800.

In the case of plain slide-valve engines it has long been taught that the port should be opened to steam about $\frac{3}{4}$ of its width, but prevailing practice with high-speed single-valve shaft-governor engines has shown that a simple constriction in a steam passage does not obstruct the flow of steam as much as has been supposed and that so large an opening is unnecessary.

Fig. 16 gives indicator cards from such an engine, with 10×10-in. cylinders (*Amer. Mach., Dec.* 6, 1900), taken under the following conditions: The engine was loaded with a friction brake, so that the load could be varied, and cards were taken at various loads. The valve rod had a sharp point attached, which was made to scribe a line on a strip of tin pressed against it at the instant of taking the card. In this way a record of the exact valve travel at the instant was obtained, and by working backward through the known dimensions of the valve and ports, the exact openings which gave the various cards were determined. As it was impracticable to insert the steam gage in the steam chest, it was placed in the steam pipe 20 ft. from the engine. The gage had been recently tested. No doubt the pressure in the chest was somewhat below that shown by the gage, and this loss due to the 20 ft. of pipe is, in the diagrams, added to the loss due to the ports. The cards are, however, fairly comparative, and they show clearly how little effect is produced by the reduced openings at the earlier cut-offs.

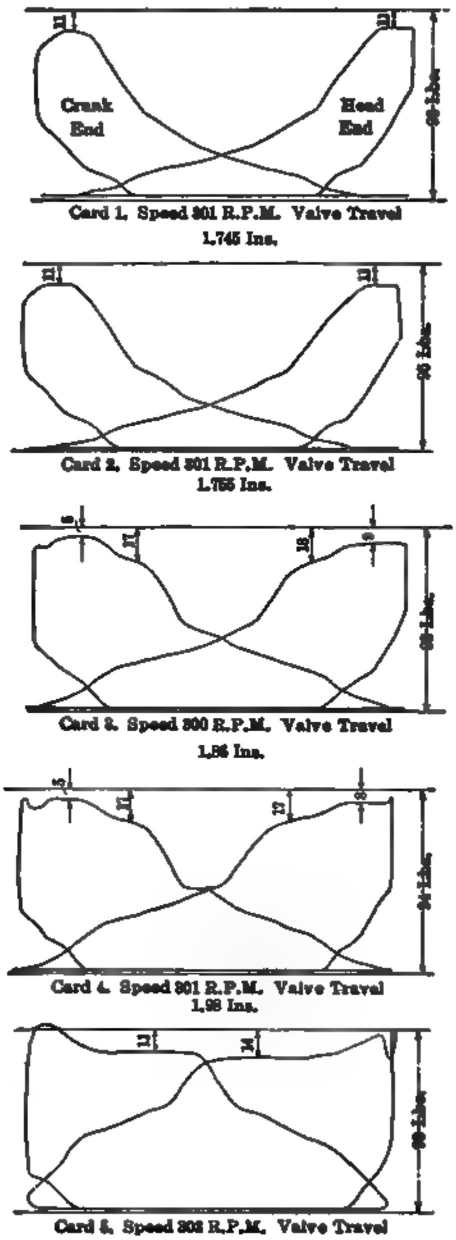


FIG. 16.—Effect of small port openings on indicator cards.

The standard rule for steam ports which calls for an area such that the velocity of the steam in them shall not exceed 6000 ft. per min., would, at the speed of this engine, call for a port area equal to 8.35

per cent. of the piston area, while the actual area of the ports was 12 per cent. of the piston area. Similarly the rule for the port opening would call for an area of opening of 6.25 per cent. of the piston area. The cards are numbered in order, beginning with the shortest cut-off, and Table 17 gives the opening figured as a percentage of the piston area and a comparison of this area with the area called for by the rule.

TABLE 17.—PORT AREAS IN SHAFT GOVERNOR ENGINES

Number of card	Area of port opening as a percentage of piston area	Area of opening divided by area called for by the old rule
1	2.27	.363
2	2.36	.377
3	3.38	.524
4	4.32	.69
5	7.76	1.24

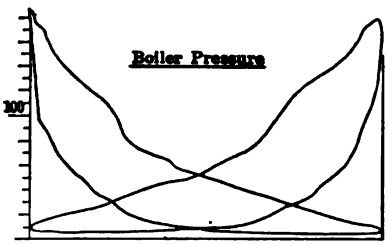


FIG. 17.—Indicator card from a high-speed passenger locomotive.

That is, in cards 1 and 2 the actual area is but little more than one-third of that called for by the rule, while in 3, which represents about an average point of cut-off with economical load, it is but a little over one-half. Comparing the performances, the drop in the steam line of cards 1 and 2 is greater than in cards 3 and 4, measured at the most favorable point of the latter, but less when measured at less favorable points, while it is slightly less than in card 5, although the opening for the latter card has 3.41 times the area of that for the former.

The influence of the steam pipe between the pressure gage and the steam chest vitiates the comparison to a certain extent, as, while its size remains fixed, more steam must be drawn through it with late cut-offs than with early ones, but the inference is unmistakable that a very decided constriction in the steam passage has a very slight effect on the flow of steam.

It is common to explain this action of the small ports by reference to the fact that they go with early cut-offs. The velocity of the piston being less at the early cut-offs, the velocity of the steam through the ports is correspondingly less, and hence it is argued that it should be expected that smaller ports would answer. The author is convinced that the importance of this action is much exaggerated. Were it true to an appreciable extent, the effect of the increased velocity at mid-stroke would appear in the exhaust line. The steam is forced through the exhaust port at various velocities at different positions of the piston, and if the action described had an appreciable influence, the exhaust line would arch upward; but, in point of fact, it is almost invariably straight.

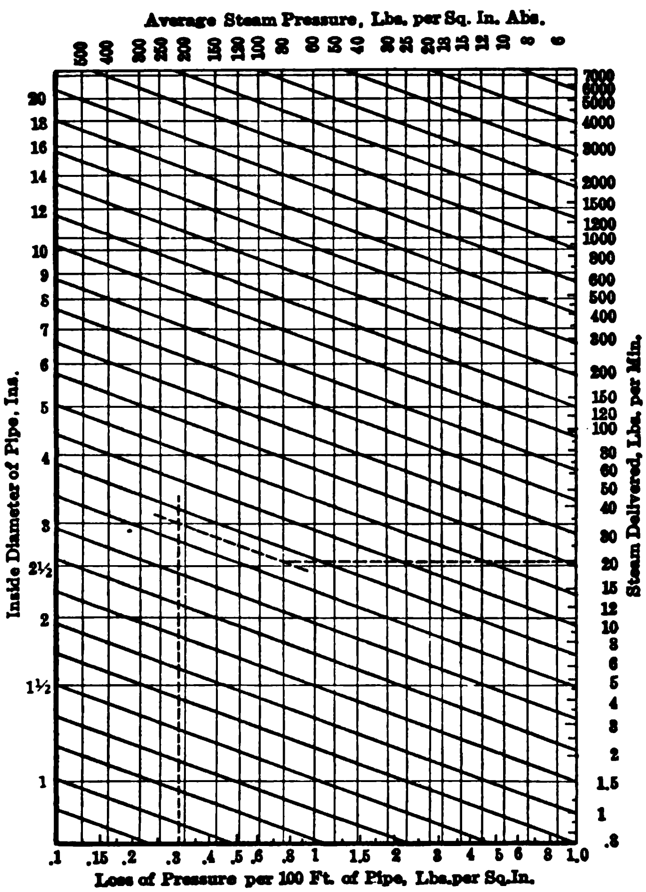


FIG. 18.

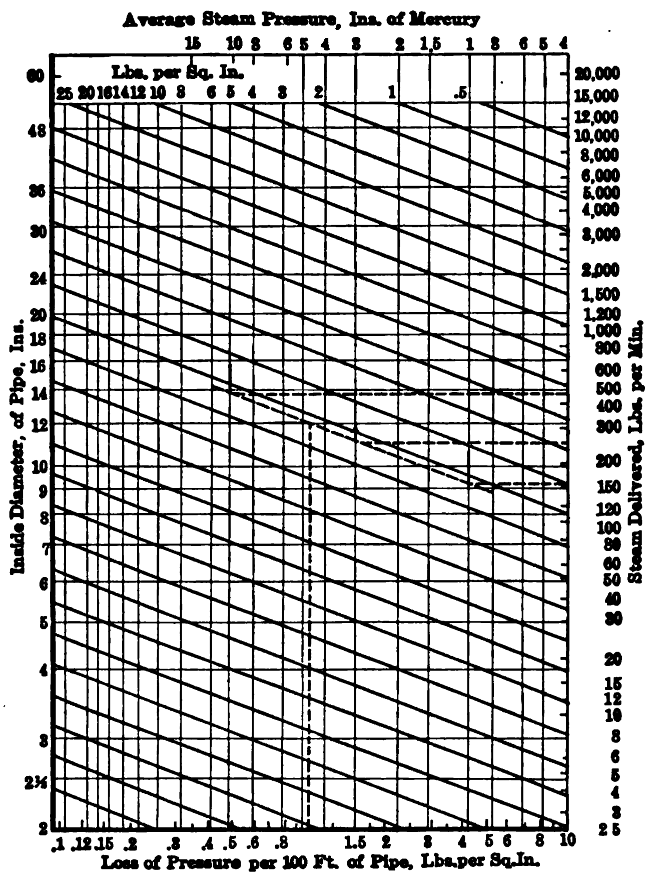


FIG. 19.

From the desired pressure loss say .3 lb. per sq. in. per 100 ft. of pipe length trace vertically to the diameter of the pipe, say 3 ins., thence diagonally to the vertical from the steam pressure, say 80 lbs., thence horizontally to the right where read the quantity of steam delivered 20 lbs. per min.

FIGS. 18 and 19.—Drop of pressure in steam pipe lines.

The ports and port openings of locomotives are invariably smaller than those of stationary engines and yet well-designed locomotive valve gears give surprisingly good results, as shown in Fig. 17, especially after making allowance for the back pressure due to the blast nozzle which is still smaller in area than the ports. The results of an examination of this subject by the author may be found in his *Slide Valve Gears*. The calculations were based on time-card speeds, which are necessarily much less than running speeds, and, on this basis, velocities of steam through the ports (that is, values of C in Professor Barr's formula) were found as high as 11,000 ft. per min., and even this high velocity is still farther increased at the blast nozzle. The use of a single nozzle for both cylinders makes the comparison of steam velocities through ports and nozzle unsatisfactory, a better comparison being that between port and nozzle areas. The area of the nozzles was found to range between 36 and 44 per cent. of the area of two ports.

The drop of pressure in steam-pipe lines is given by the following formula, due to Professor Unwin:

$$W = 87.5\sqrt{\frac{Pyd^5}{L\left(1+\frac{3.6}{d}\right)}}$$

in which W = weight of steam delivered, lbs. per min.,
P = drop in pressure, lbs. per sq. in.,
y = density of steam, lbs. per cu. ft.,
d = diameter of pipe, ins.,
L = length of pipe, ft.

This formula has been accepted with slight and unimportant changes in the coefficient and after extended tests by Prof. R. C. Carpenter and G. H. Babcock. It has been reduced to chart form by Prof. H. V. Carpenter (*Power, Dec.* 17, 1912 and *June* 10, 1913), these charts being given here as Figs. 18 and 19, instructions for use appearing below them. The charts are subject to the caution due to the fact that they are extended far beyond the range of any experiments that have been made. They, however, represent the best existing knowledge of the subject. Fig. 18 is for high and Fig. 19 for low pressures, including those below the atmosphere. The great

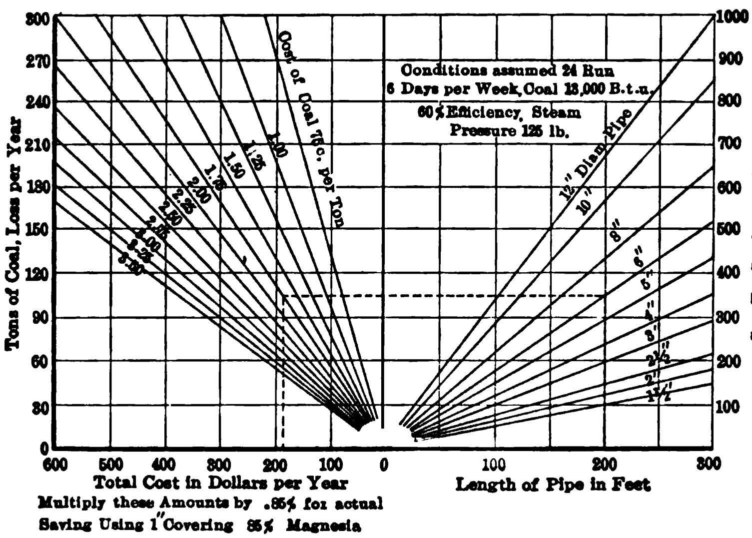


FIG. 20.—Loss of coal due to uncovered pipes.

velocities permissible at low pressures increase the relative importance of elbows.

The charts relate to actual, not nominal, pipe diameters. They apply to saturated steam. For superheated steam instead of the actual pressure use the pressure at which saturated steam has the same weight per cu. ft.

Experiments on the resistance of pipe fittings are few in number a give very discordant results. The formulas by Robert Briggs these losses may be used in the absence of anything better. Th are, for one standard 90-deg. elbow:

$$l = \frac{76d}{1+\frac{3.6}{d}}$$

and for one globe valve:

$$l = \frac{114d}{1+\frac{3.6}{d}}$$

in both of which l = length of pipe, ins., equivalent to one fitting,
d = diameter of pipe, ins.

The resistance of gate valves is negligible.

For the resistance of screwed pipe fittings to the flow of wate see Index.

The loss due to uncovered steam pipes may be determined approxi mately from Fig. 20, by Chas. Brossmann (*Power, Apr.* 16, 1912). To use the chart, trace upward, as shown, from the length of pipe to the diameter, to the left to the cost of coal per ton and then down to the loss in dollars per year. A good covering should prevent about 85 per cent. of this loss. The conditions assumed are a service of 6 days of 24 hours per week, coal of 13,000 B.t.u. heating value, a boiler efficiency of 60 per cent. and a steam pressure of 125 lbs.

TABLE 18.—APPROXIMATE EFFICIENCIES OF VARIOUS STEAM-PIPE COVERINGS REFERRED TO BARE PIPES, BASED ON 1-IN. COVERING From Steam, by Permission of the Babcock and Wilcox Co.

Covering	Efficiency
Asbestocel	76.8
Gast's air cell	74.4
Asbesto-sponge felt	85.0
Magnesia	83.5
Asbestos, navy brand	82.0
Asbesto-sponge hair	86.0
Asbestos fire felt	73.5

A cheap and effective steam-pipe covering may be made of sawdust and lime. The mixture is made up like sand mortar, using one barrel of lime to five of sawdust and allowing several days for it to dry before turning on the steam. A steam line of 197 ft. of 8-in., 219 ft. of 7-in., and 258 ft. of 6-in. pipe was covered with this mixture encased in a wood box 12 ins. square inside and tamped down. A test of 20 days with bare pipe showed a condensation of 1440 lbs. of water per hour or a fraction under 1¼ lbs. per sq. ft. of external surface per hour. The covered pipe showed a condensation of 195 lbs. of water per hour or 2¾ oz. per sq. ft. of external surface of pipe per hour, the loss covered being 14 per cent. of that uncovered. The working steam pressure was 90 lbs. and the air temperature averaged about 64 deg. Fahr. If the wood box is not desired "a little fire clay or flour mixed in makes it possible to wrap it on under a covering of muslin." The mixture is regarded as fireproof (F. A. Nystrom, *Amer. Mach., Mar.* 7 *and Apr.* 4, 1901).

Steam-pipe lines should incline about 1 in. in 10 ft. in the direction of the flow of steam.

Laying out the Slide Valve

Laying out a slide valve may be most conveniently done by the Bilgram diagram, for the demonstration and many additional applications of which see the author's Slide Valve Gears. To lay out a plain slide valve proceed as in Fig. 21. Let AB be the length of

TABLE 19.—LOSS OF HEAT FROM COVERED AND UNCOVERED STEAM PIPES, CALCULATED FOR 160 LBS. PRESSURE AND 60 DEG. FAHR. TEMP. From Steam, by Permission of the Babcock and Wilcox Co.

Pipe	Thickness of covering	½ in.	¾ in.	1 in.	1¼ ins.	1½ ins.	Bare
	B.t.u. per lineal foot per hour	149	118	99	86	79	597
2 ins.	B.t.u. per square foot per hour	240	190	161	138	127	959
	B.t.u. per square foot per hour per 1 deg. difference in temperature	.770	.613	.519	.445	.410	3.198
	B.t.u. per lineal foot per hour	247	193	160	139	123	1085
4 ins.	B.t.u. per square foot per hour	210	164	136	118	104	921
	B.t.u. per square foot per hour per 1 deg. difference in temperature	.677	.592	.439	.381	.335	2.970
	B.t.u. per lineal foot per hour	352	269	221	190	167	1555
6 ins.	B.t.u. per square foot per hour	203	155	127	110	96	897
	B.t.u. per square foot per hour per 1 deg. difference in temperature	.655	.500	.410	.355	.310	289
	B.t.u. per lineal foot per hour	443	337	276	235	207	1994
8 ins.	B.t.u. per square foot per hour	196	149	122	104	92	883
	B.t.u. per square foot per hour per 1 deg. difference in temperature	.632	.481	.394	.335	.297	2.85
	B.t.u. per lineal foot per hour	549	416	337	287	250	2468
10 ins.	B.t.u. per square foot per hour	195	148	120	102	89	877
	B.t.u. per square foot per hour per 1 deg. difference in temperature	.629	.477	.387	.329	.287	2.83

Covering: magnesia, canvas covered.
For calculating radiation for pressure and temperature other than 160 deg. and 60 deg., use B.t.u. figures for 1 deg. difference.

stroke to any convenient scale, *a* being the desired point of cut-off. Draw the crank circle $A'a'B'$ and project point *a* to it, giving Oa', the cut-off position of the crank. Make *de* equal to the desired lead opening and draw *fg* with radius *Og* equal to the desired port opening. Find by trial point *k*, such that a circle struck from it as a center will be tangent to Oa', *fg* and *cd*. The radius, *kl*, of this circle is the lap and the diameter, *mn*, is the travel of the valve. The advance angle is equal to $i'OB'$, the center of the eccentric being at *p*, such that $pq = kn$. If the valve has no inside lap, Oi' is the crank position and *i* the piston position for release and compression. If the valve has inside lap equal to the radius *ko*, compression takes place at *h* and release at *b*. With negative inside lap, release and compression change places.

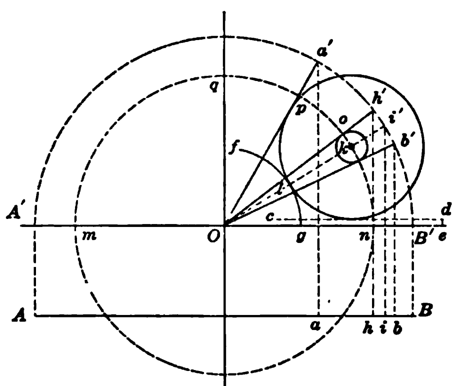


FIG. 21.

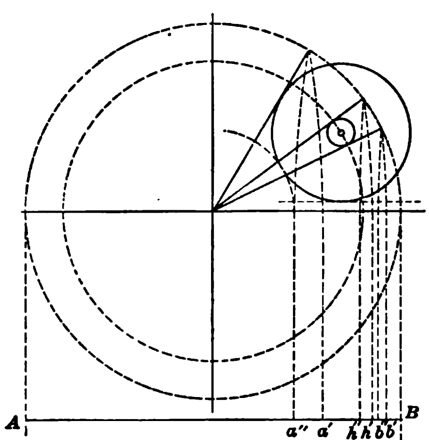


FIG. 22.

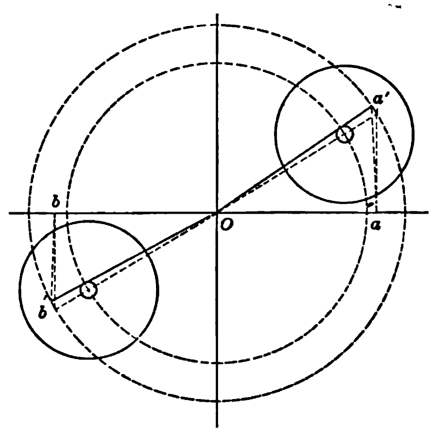


FIG. 23.

FIGS. 21 to 23.—The Bilgram diagram applied to a plain slide valve.

The above construction assumes the slotted cross-head construction or its equivalent, a connecting rod of infinite length, and, when applied to the actual connecting-rod construction, it gives the mean positions of the events of the stroke. To find the actual positions, project, as in Fig. 22, the extremities of the crank positions to the diameter by circular arcs of which the radius equals the length of the connecting rod, giving points a', a'', b', b'', h', h'', of which the single primed letters refer to the outward and the double primed letters to the inward stroke—the cylinder being assumed to lie at the left of the diagram.

To equalize compression and release, lay down the desired compression and release points as in Fig. 23 at *a* and *b*. Project these points to the crank circle by circular arcs with radius equal to the length of the connecting rod, giving points a' and b'. Draw the corresponding crank positions and give the *a* end of the valve an inside positive lap and the *b* end an inside negative lap equal to the radii of the small circles.

The action of a shifting eccentric upon a slide valve may be determined as in Figs. 24 and 25. In Fig. 24 the eccentric swings from a center located on the center line of the crank and on the same side as the crank pin, that is, the center of the arc $dd°$. With the radius of $dd°$ and with a center in the vertical center line, strike the arc $QQ°$. With the eccentric center at the full throw position, *d*, the action of the eccentric on the valve is given by the lap circles struck from *Q* as a center and, similarly, with the eccentric at any other point, d', the action on the valve is given by the lap circles struck from Q' as a center, the three cut-off positions of the crank being shown by tangents to the lap circles. The increasing distance of the lap circles above the horizontal center line as the cut-off is shortened shows the increase of the lead with shortened cut-off, and the actual lead for any point of cut-off may be measured from the diagram.

In Fig. 25 the eccentric is swung from a center on the center line of the crank but opposite the crank pin. The centers of the lap circles are now located on an arc which is convex downward instead of

upward, the general effect being the same but with the important exception that the lead *decreases* as the cut-off is shortened, as shown by the lap circles approaching the horizontal center line as the cut-off is shortened.

The action of a Stephenson link motion on a slide valve is essentially the same as that of a shifting eccentric. If the eccentric rods are "open," that is, if they are not crossed when the eccentrics are placed as in Fig. 26, the lead increases as the cut-off is shortened and the Bilgram diagram is similar to Fig. 24. If the rods are "crossed," that is, crossed when the eccentrics are placed as in Fig. 26, the lead decreases as the cut-off is shorteded and the Bilgram diagram is similar to Fig. 25. Crossed rods, however, are used but little if at all.

The amount of variation in the lead depends upon the length of the horizontal distance from the vertical center line equal to the lap, and locate *c*, *d*, *c'*, *d'*, at additional horizontal distances equal to the full gear lead, these last points being those which the eccentric centers occupy when the crank is on the centers. From these points lay down the full and dotted midgear positions of the link when the crank is on the centers, and measure the midgear travel, *ef*. Divide this in half, subtract the lap *oi* or *on* and obtain the midgeer lead *if* or *en*. Note that if the rock shaft has unequal arms, leading to inequality between the eccentric throw and the valve travel, it is most convenient to use the valve travel as the diameter of the dotted circle, the eccentric rod lengths and the link dimensions being changed from the actual in the proportion of the valve travel to the eccentric throw.

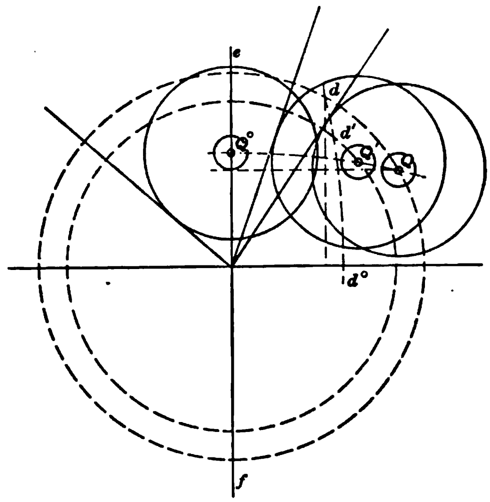


FIG. 24.

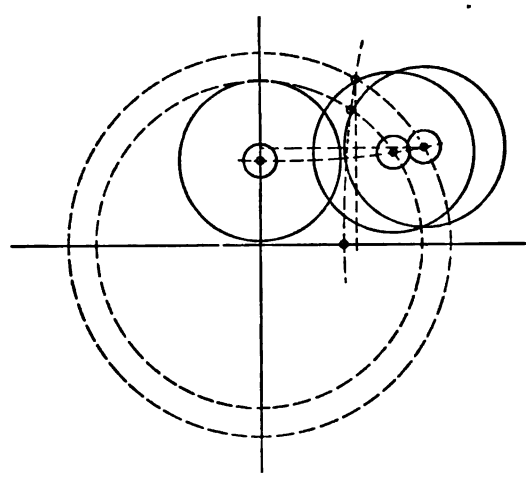


FIG. 25.

FIGS. 24 and 25.—The Bilgram diagram applied to a shifting eccentric valve gear.

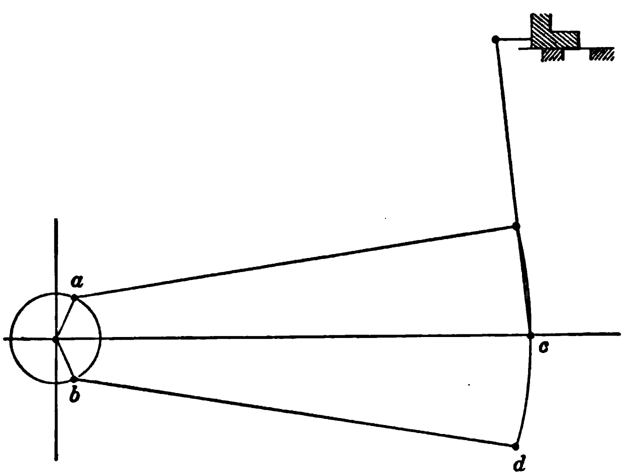


FIG. 26.—Open eccentric rods.

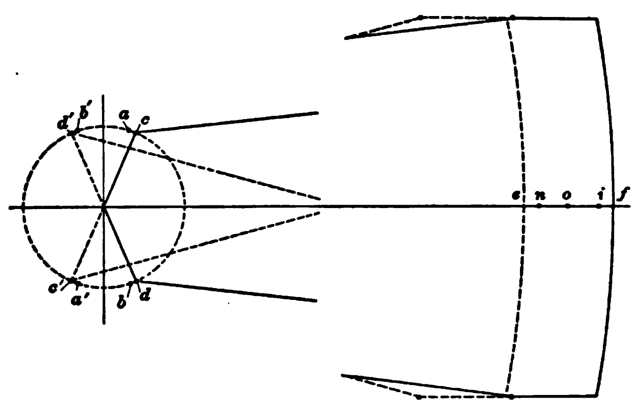


FIG. 27.—Finding the mid-gear lead of link motion.

eccentric rods—the variation increasing as the rods are shortened. The proper radius of the link is the length of the eccentric rods plus such distance as there may be between the geometrical link arc (the curved center line of the link) and the eccentric-rod pins. With any other radius the variation in the lead with varying cut-off differs for the two ends of the cylinder.

The layout of the Bilgram diagram for a link motion does not differ essentially from that for a shifting eccentric. While, however, the full gear lead is commonly given in advance, the midgear lead being dependent on the length of the rods must be found by the method shown in Fig. 27. Make the diameter of the dotted circle equal to the full gear travel of the valve and lay down points *a*, *b*, *a'*, *b'* at a

To construct the Bilgram diagram proceed as in Fig. 28. Draw the inner dotted circle equal in diameter to the full stroke valve travel, and lay down *ab* equal to the lap, make *bc* equal to the full gear lead, and *de* equal to the midgear lead, as found in Fig. 27. Now, a circle struck through *efg* will give the path on which a single shifting eccentric must travel to produce a valve movement equivalent to that given by the link of Fig. 27. Laying off *h'f'* equal to *hf*, and *Oe'* equal to *Oe*, the circular arc *e'f'* is easily drawn, on which the center of the lap circle for all points of cut-off must lie. Drawing the outer dotted circle to represent the path of the crank pin to scale, and selecting, say, the cut-off at one-third stroke for study, the point *i* is laid down such that *ij* equals one-third of the stroke, and by the perpendicular *ik* the crank line *Ok* for one-third stroke is located. Drawing a lap circle tangent to *Ok* and with its center on the line *e'f'*, we have, for the one-third cut-off: lead = *ln*, port opening = *Oo*,

valve travel $= 2Op$, exhaust opening and closure (assuming no inside lap) at crank position Og. Similarly we have for the full gear a lead rs equal to bc, a port opening Ot, a travel to twice Of', and an exhaust opening and closure at crank position Ot. To investigate the reverse motion extend the arc $e'f'$ to g'.

For particulars regarding practice with negative lead in the full gear and unequal leads in the forward and reverse gears see the author's Slide Valve Gears.

Friction of Slide Valves

The friction of slide valves formed the subject of experiments by J. A. F. Aspinall (*Proc. I. C. E.*, 1898). The experiments were upon two horizontal locomotive valves, one an ordinary unbalanced valve of phosphor bronze and the other a Richardson relieved valve of cast-iron.

As a sight-feed lubricator was used in the experiments, it was easy to watch the result of increasing the number of drops of lubricant per min., and it was found that there was a perceptible improvement in the ease of movement of the valve when the lubricant was increased.

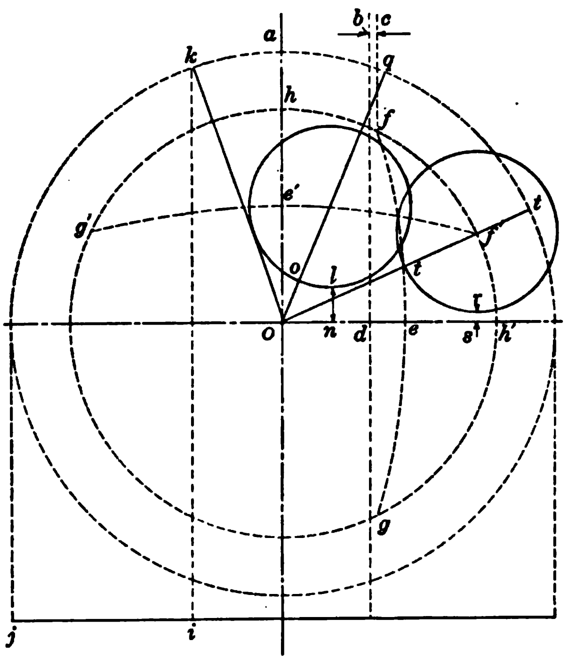


FIG. 28.—The Bilgram diagram applied to a Stephenson link motion.

The experiments show that the friction of slide valves is somewhat greater against a horizontal than against a vertical face; the coefficient of friction found in previous experiments for valves on a vertical face was .068, while in the experiments dealt with here, the average coefficient was found to be, for the unbalanced valve .0878, and for the partially balanced valve .0919.

The coefficient of friction, as given in Table 20, together with the other results of the experiments, is calculated from the whole area of the back of the plain valve, supplementary experiments by Mr. Aspinall having convinced him of the correctness of that procedure. In the author's opinion this conclusion was not warranted by the experiments, but, if the friction of other valves is calculated in the same way and from Mr. Aspinall's determinations of the coefficient of friction, the results should be sufficiently correct for all practical purposes and doubtless within the variations due to varying conditions. For the Richardson valve, the balanced area was taken as that portion which is enclosed between the strips, excluding the area of the strips themselves.

TABLE 20.—THE FRICTION OF LOCOMOTIVE SLIDE VALVES

Forward or back gear	Notch, 1 to 4	Push or pull	Lubrication	Type of valve	Boiler pressure	Steam-chest pressure	Net pressure on valve	Total force on diaphragm, corrected	Total force moving valve	Coefficient of friction
			Drops per min.		Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	
F	1	Pull	4	Phosphor-bronze "D" valve	156	155.0	24,149.0	1,549.5	2,099.0	.086
F	1	Pull	4		152	151.5	23,562.5	1,589.0	2,126.0	.090
F	2	Pull	4		158	154.0	23,405.0	1,510.5	2,056.5	.087
F	2	Pull	4		157	153.0	23,037.0	1,497.0	2,039.5	.088
F	3	Pull	4		144	137.0	19,513.0	1,090.0	1,575.5	.080
F	3	Pull	4		154	150.0	21,619.0	1,562.5	2,094.5	.092
F	4	Pull	4		146	140.0	19,455.5	1,234.0	1,730.5	.088
F	4	Pull	4		158	150.0	21,077.5	1,392.0	1,924.0	.091
B	1	Pull	4		158	155.5	23,894.5	1,497.0	2,048.5	.085
B	1	Pull	4		149	154.5	23,929.0	1,497.0	2,045.0	.085
B	2	Pull	4		152	150.0	21,924.0	1,444.5	1,976.5	.090
B	2	Pull	4		155	147.5	22,052.0	1,313.0	1,836.0	.083
B	3	Pull	4		150	145.5	20,824.0	1,300.0	1,816.0	.087
B	3	Pull	4		156	151.5	21,801.5	1,300.0	1,837.0	.084
B	4	Pull	4		155	152.0	20,819.5	1,103.0	1,642.0	.078
B	4	Pull	4		147	140.0	19,248.5	1,129.0	1,625.5	.084
F	1	Push	3		160	157.5	24,461.5	3,326.0	1,767.0	.072
F	1	Push	6		158	156.5	24,291.5	2,866.0	2,311.0	.095
F	2	Push	6		159	150.0	22,907.0	1,709.0	1,177.0	.051
F	3	Push	5		160	136.0	20,568.0	2,347.5	1,865.5	.090
F	3	Push	3		159	152.0	23,032.5	1,503.0	964.0	.041
F	4	Push	3		160	148.0	20,941.5	2,620.5	2,095.5	.100
F	4	Push	3		161	148.5	20,736.0	2,532.0	2,005.0	.096
B	1	Push	3		160	147.0	22,502.5	2,973.5	2,452.0	.108
B	1	Push	3		158	152.0	23,732.0	1,915.0	1,376.0	.057
B	2	Push	3		158	142.0	21,184.5	2,826.5	2,322.5	.109
B	2	Push	3		164	160.0	23,605.5	3,326.0	2,758.5	.112
B	3	Push	3		158	147.0	21,292.5	2,914.5	2,393.0	.112
B	3	Push	3		160	150.0	21,677.0	2,944.0	2,412.0	.111
B	4	Push	3		158	140.0	19,995.0	2,468.5	1,972.0	.098
B	4	Push	3		160	145.0	21,165.5	2,503.0	1,988.5	.093
B	1	Pull	4	Balanced	144	140.0	9,467.0	420.0	916.5	.097
B	1	Pull	4		142	140.5	9,514.5	393.5	892.0	.094
B	2	Pull	4		140	133.0	8,633.0	288.5	760.5	.088
B	2	Pull	4		158	154.5	9,976.0	393.5	941.5	.094
B	3	Pull	4		143	140.0	8,704.0	354.0	850.5	.097
B	3	Pull	4		152	149.0	9,238.0	236.0	765.0	.082

NOTE.—The throttle valve was full open in all the experiments.

Poppet Valves

Double beat poppet valves as usually made are, as is well known, difficult to keep tight. Slight differences in the coefficient of expansion of the metals composing the valve and its case, or slight differences of temperature due to the accumulation of water will cause one or other seat to lift slightly and thus leak.

Fig. 29 is a sketch showing the usual construction, from which it will be apparent that any difference of expansion between valve and case will open one or other seat. Should the valve expand the more, the seat *a* will open; while should the case expand the more, seat *b* will open. Fig. 30 shows the construction used by the Nordberg Mfg. Co. (*Amer. Mach., Aug.* 14, 1902) whereby this difficulty is overcome. Its essential feature is that the cone surfaces of the two seats have a common apex at *a*. Should the valve expand the more, its vertical expansion will tend to open the seat *b*; but its horizontal expansion, having the same increment of excess, will tend to close the seat, and the two actions will offset one another. The reverse action will take place should the case expand the more. Looked at in another way, the expansion of both valve and case is from the common center *a* and any difference of expansion is accompanied by a slight sliding of valve and seat upon one another on the line of the joints between them, but without any tendency to open either joint. This action will take place wherever the common apex *a* may be and regardless of the angle of the two seats. An actual valve by the Nordberg Mfg. Co. (a 10-in. regulating valve) is shown in Fig. 31. The lower seat is here flat but the two seats intersect at *b* and the action described in connection with Fig. 30 still holds.

Dimensions and Lift of Poppet Valves for a Given Size of Opening

D = Smaller diameter of valve seat.

d = diameter of piping to which valve opening must correspond.

$$r = \frac{D}{d}$$

Flat-seated Valves

For $r = 1$	Lift $= d \times .250 = D \times .250$
For $r = 1.25$	Lift $= d \times .200 = D \times .160$
For $r = 1.5$	Lift $= d \times .166 = D \times .111$
For $r = 2$	Lift $= d \times .125 = D \times .162$
For $r = 2.5$	Lift $= d \times .100 = D \times .040$

It will be noted that cone-seated valves require a lift from one-fifth to one-quarter greater than the corresponding flat-seated valves.

The proportions of lift given in connection with flat-seated valves are geometrically correct. It should, however, be borne in mind that flat-seated valves generally introduce a certain amount of wire drawing of the incoming charge. A slight increase over the theoretically correct lifts should consequently be provided. A definite coefficient cannot be given, as this will depend considerably upon the valve seat and valve chest design, as well as upon the proportions of the fillet between valve stem and valve head. The matter is one of personal intuition by the designer; in the best French designs the extra allowance seldom exceeds 25 per cent. of the theoretical lift. It is well to so arrange the contour of the valve and valve chamber profiles that a minimum of lift be required, as this is favorable to silence in running and to flexibility.

Fig. 29.—Incorrect construction of double beat poppet valves.

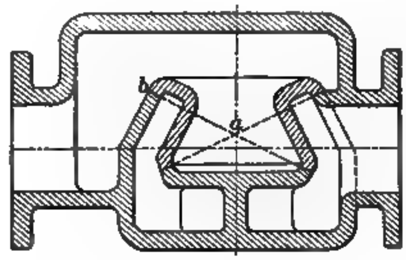


Fig. 30.—Correct construction of double beat poppet valves.

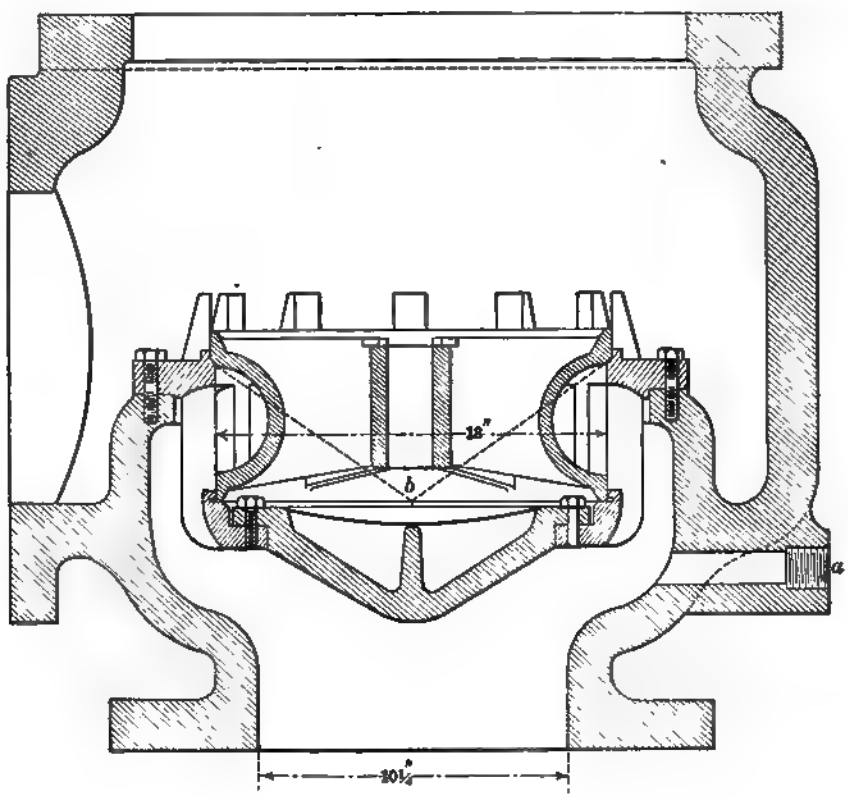


Fig. 31.—Nordberg Mfg. Co's double beat poppet valve.

Cone-seated Valves; 45 Deg. Angle of Cone

For $r = 1$	Lift $= d \times .307 = D \times .307$
For $r = 1.25$	Lift $= d \times .256 = D \times .205$
For $r = 1.5$	Lift $= d \times .219 = D \times .146$
For $r = 2$	Lift $= d \times .170 = D \times .084$
For $r = 2.5$	Lift $= d \times .138 = D \times .055$

Condensing Water

The approximate quantity of water required to condense 1 lb. of steam with a jet condenser may be obtained from the formula:

$$Q=\frac{(H+32)-T}{(T-t)}$$

in which Q = lbs. of water required to condense 1 lb. of steam,
T = temperature of discharge water, Fahr.,
t = temperature of injection water, Fahr.,
H = total heat above 32 deg. Fahr. in 1 lb. of steam to be condensed.

Table 21 by W. F. FISCHER (*Power, Sept.* 26, 1911) is based on the steam tables of Marks & Davis and is given as a guide in figuring the condensing water required per lb. of steam in condenser installations.

Example: With a vacuum of 28 ins. of mercury referred to a 30-in. barometer $(H+32)$, is found from Table 2 to be 1137 B.t.u. Substituting in the formula,

$$Q=\frac{1137-T}{T-t}$$

for a 28-in, vaccuum. In column 5 the volume in cu. ft. per lb. of steam is given, and in column 6 the weight of 1 cu. ft. of steam at the given pressure and temperature corresponding to the given vacuum.

In determining the proper value to substitute for T, care should be taken to allow a suitable drop between the steam in the condenser and the temperature of the discharge water. In practice the temperature of the discharge water is assumed to be 15 deg. lower than the steam temperature and it is customary to allow for 10 per cent. more water than the estimated quantity where actual conditions are unknown.

When estimating the quantity of water required per lb. of steam in surface condensers it is customary to take into account the temperature of the condensed steam; that is, the hotwell temperature.

Hence for surface condensers

$$Q=\frac{(H+32)-T_c}{T-t}$$

in which T_c equals the temperature of the condensed steam and Q, H, T and t repesent the same quantities as in the formula for jet condensers In the ordinary surface condenser of the single- or double-flow type, T_c may be taken from 10 to 20 deg. lower than the temperature due to the vacuum.

TABLE 21.—CONDENSING WATER PER POUND OF STEAM

Vacuum in ins. of mercury referred to a 30-in. barometer	Absolute pressure lbs. per sq. in.	Temperature of steam and water at condenser pressure	B.t.u.'s in 1 lb. of steam+32	Volume in cu. ft. per lb. of steam	Lbs. of steam per cu. ft.
29.82	.09	32	1105	3294	.0003
29.50	.25	59	1117	1249	.0008
29.00	.50	80	1127	636.8	.0016
28.50	.74	92	1132	442.2	.0023
28.00	1.00	102	1137	331.5	.0030
27.50	1.24	109	1140	272.9	.0037
27.00	1.51	116	1143	225.8	.0044
26.50	1.72	121	1145	197.9	.0050
26.00	1.99	126	1147	173.9	.0057
25.50	2.22	130	1149	157.1	.0064
25.00	2.47	134	1150	142.2	.0070
24.50	2.73	138	1152	128.9	.0077
24.00	2.96	141	1153	119.9	.0083
23.50	3.19	144	1155	111.6	.0089
23.00	3.45	147	1156	104.0	.0096
22.50	3.70	150	1157	97.0	.0103
22.00	3.96	152	1158	93.0	.0108
21.50	4.18	155	1159	86.4	.0116
21.00	4.40	157	1160	82.6	.0121
20.50	4.70	159	1161	78.0	.0125
20.00	4.90	162	1162	73.8	.0135
18.00	5.80	169	1165	63.3	.0158
16.00	6.85	176	1168	54.5	.0183
14.00	7.85	182	1171	48.12	.0207
.00	14.70	212	......	26.79	.0373

Current practice in the dimensions of gas-engine parts formed the subject of an investigation by the Department of Machine Design of Cornell University, the results being reported by SANFORD A. MOSS (*Amer. Mach.*, *Apr.* 14, 1904) and given below. The investigation included an analysis of the dimensions of engines of 76 different sizes by 20 builders.

The computed stresses are perhaps open to criticism, since the formulas may not take everything exactly into account. The numerical coefficients given are absolute, however, being taken from the actual data, and may safely be used, even though the exact stresses, bearing pressures, etc., to which they correspond may not be known.

There are also given in the last column rough formulas for average cases. For instance, in the case of the cylinder wall, the rational average formula is $t = .000204pD + \frac{1}{4}$. This gives a thickness varying with the maximum pressure p. In an average case p is 300, and if this value is substituted for p we have $t = (.000204 \times 300)pD + \frac{1}{4}$, or very nearly $t = \frac{D}{16} + \frac{1}{4}$. This formula, of course, should not be used where the pressure is much different from 300. Formulas like those in the last column are given in works on gas-engine design, without qualification, which is not correct, as these formulas have a limited range. The rational formulas given, with the mean values of the numerical coefficients substituted, are the proper formulas for general use.

The maximum explosion pressure in the engines examined varied from about 250 to 350 lbs. per sq. in., the average being 300 lbs. per sq. in. The compression pressure varied from about 50 to 100, the average being 70 lbs. per sq. in. The lower values of compression pressure and maximum pressure are for engines using gasoline, and the higher values for natural gas. This is, of course, due to the fact that pre-ignition must be avoided.

The maximum horse-power which an engine can develop is found to average very closely $1\frac{1}{8}$ times the rated horse-power for which the engine is sold.

The mechanical efficiency averages about 80 per cent. The engines examined were single-cylinder horizontal or single or multicylinder vertical engines, all single acting, varying from 5 to 100 h.p., and the formulas given apply only to such engines.

The maximum probable brake horse-power of gas engines may be obtained from Table 2 by CECIL P. POOLE (*Power*, *Mch.* 23, 1909) in connection with the formula:

Probable brake h.p. = constant from table × stroke, ins. × r.p.m.

The constants for double-acting engines include an allowance of 6 per cent. for the effect of the piston rod.

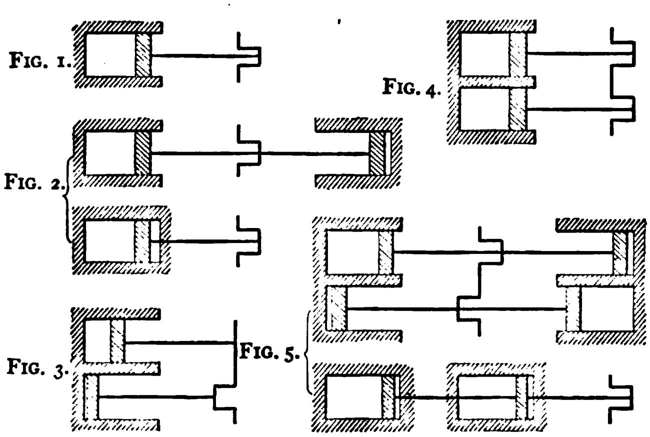
TABLE 2.—HORSE-POWER CONSTANTS FOR GAS ENGINES

Single-acting engines				Double-acting engines			
Cylin. diam.	Natural gas	Producer gas	Illum'g gas	Cylin. diam.	Natural gas	Producer gas	Illum'g gas
5	.00162	.00140	.00175	10	.0122	.0105	.0132
5¼	.00179	.00154	.00193	10½	.0135	.0116	.0145
5½	.00197	.00169	.00212	11	.0148	.0128	.0159
5¾	.00215	.00185	.00232	11½	.0161	.0139	.0173
6	.00234	.00202	.00252	12	.0175	.0151	.0189
6¼	.00254	.00219	.00274	12½	.0191	.0164	.0206
6½	.00275	.00237	.00296	13	.0207	.0178	.0222
6¾	.00297	.00255	.00319	13½	.0222	.0191	.0239
7	.00319	.00274	.00343	14	.0239	.0206	.0257
7¼	.00342	.00294	.00368	14½	.0257	.0222	.0277
7½	.00366	.00315	.00394	15	.0274	.0236	.0295
7¾	.00390	.00336	.00421	16	.0313	.0270	.0337
8	.00416	.00358	.00448	17	.0354	.0305	.0381
8¼	.00443	.00381	.00476	18	.0395	.0340	.0425
8½	.00470	.00405	.00506	19	.0441	.0380	.0474
8¾	.00498	.00429	.00536	20	.0489	.0421	.0526
9	.00526	.00454	.00567	21	.0538	.0464	.0579
9½	.00587	.00505	.00632	22	.0597	.0510	.0637
10	.00650	.00560	.00700	23	.0646	.0557	.0696
10½	.00717	.00617	.00772	24	.0703	.0606	.0759
11	.00786	.00678	.00847	25	.0763	.0657	.0827
11½	.00860	.00741	.00926	26	.0825	.0711	.0889
12	.00936	.00806	.0101	27	.0890	.0767	.0959
12½	.0101	.00875	.0109	28	.0958	.0825	.103
13	.0110	.00946	.0118	29	.103	.0885	.111
13½	.0118	.0102	.0127	30	.110	.0947	.118
14	.0127	.0110	.0137	31	.117	.101	.126
14½	.0137	.0118	.0147	32	.125	.108	.135
15	.0146	.0126	.0157	33	.133	.115	.143
16	.0166	.0143	.0179	34	.142	.122	.152
17	.0188	.0162	.0202	35	.149	.129	.161
18	.0210	.0181	.0227	36	.158	.137	.171
19	.0234	.0202	.0252	37	.168	.144	.180
20	.0260	.0224	.0280	38	.177	.152	.190
21	.0287	.0247	.0309	39	.186	.160	.200
22	.0315	.0271	.0339	40	.195	.168	.210
23	.0344	.0296	.0370	41	.205	.177	.221
24	.0374	.0323	.0403	42	.216	.186	.232
25	.0406	.0350	.0437	43	.226	.195	.243
26	.0439	.0379	.0473	44	.236	.204	.255
27	.0474	.0408	.0510	45	.247	.213	.266
28	.0510	.0439	.0549	46	.258	.223	.278
29	.0547	.0471	.0589	47	.270	.233	.291
30	.0585	.0504	.0630	48	.282	.243	304

The weight of flywheels for gas engines may be determined from the formula, by R. E. MATHOT, (*Engineering Magazine*, *June*, 1907),

$$P = K\frac{10.75\,N}{D^2an^3}$$

in which P = the weight of the rim (without arms or hub), tons,
D = diameter of the center of gravity of the rim, ft.,



FIGS. 1 to 5.—Types of gas engines in relation to weight of fly-wheels.

TABLE I.—CURRENT PRACTICE IN THE DIMENSIONS OF GAS ENGINES

Engine dimension and name of design constant upon which it depends	Notation: All dimensions in ins., all pressures and stresses in lbs. per sq. in.	Rational formula for engine dimension in terms of the design constant	Maximum, mean and minimum values of design constant	Corresponding numerical values of coefficient of formula	Assumptions made in deducing formula for average cases from rational formula; mean value of design constant always used	Reduced formula for average cases
Thickness of cyl. wall.... Stress in cyl. wall.......	t = thickness of cyl. walls. S = stress in cyl. walls. p = max. pressure. D = dia. of cylinder.	$t=\left(\frac{1}{2S}\right)pD+\frac{1}{8}$	$S=$ 1,625 2,450 3,750	$\frac{1}{2S}=$ .000308 .000204 .000133	$p=300$	$t=\frac{D}{16}+\frac{1}{8}$
Thickness of jacket walls	T = thickness of jacket wall. t = thickness of cyl. wall.	$T=ct$	$c=$ .86 .60 .43			$T=.6t$
Thickness of water jacket space.	j = thickness of jacket space. t = thickness of cyl. wall.	$j=ct$	$c=$ 1.85 1.25 1.00			$j=1\frac{1}{4}t$
Number of cyl. head studs	q = number of cyl. head studs. D = diameter of cyl.	$q=cD+2$	$c=$ 1.14 .67 .40			$q=\frac{2}{3}D+2$
Outside dia. of cyl. head studs. Stress in cyl. head studs.	o = outside diameter of cyl. head studs. q = number of cyl. head studs. p = maximum pressure. D = diameter of cyl. s = stress at root of thread.	$o=\frac{1}{\sqrt{.7s}}\sqrt{\frac{p}{q}}\,D$	$s=$ 10,900 7,800 4,500	$\frac{1}{\sqrt{.7s}}=$ .0115 .0135 .0179	$p=300$ $q=8$. This is correct for 9-in. cylinders and nearly correct for quite a range on either side.	$o=\frac{D}{12}$
Length of stroke in terms of cyl. diam.	L = length of stroke. D = cyl. diameter.	$L=cD$	$c=$ 1.8 1.5 1.0			$L=1\frac{1}{2}D$
Length of con. rod...... Ratio of con. rod to crank	C = distance from center to center of connecting rod. n = ratio of con. rod to crank. L = length of stroke.	$C=n\frac{L}{2}$	$n=$ 4.10 5.15 6.00			$C=5\frac{L}{2}$
Weight of piston.......	W = weight of piston. H = area of cyl. $=\frac{\pi}{4}D^2$					$W=1.3H$
Weight of con. rod......	V = weight of con. rod. H = area of cyl. $=\frac{\pi}{4}D^2$					$V=.8H$
Total weight of reciprocating parts. Wt. of recip. parts per sq. in. of cyl.	W = total wt. of piston. V = total wt. of con. rod. H = area of cylinder. w = weight of truly reciprocating parts per sq. in. of cyl.	$W=\frac{1}{2}V+wH$	$w=$ 1.02 1.70 2.42			$W+\frac{1}{2}V=1.7H$
Length of piston........ Bearing pressure on piston due to con. rod thrust.	B = length trunk piston. b = bearing pressure on projected area of piston (mean value during working stroke. n = ratio of con. rod to crank. p = maximum pressure D = cyl. diameter.	$B=\left(\frac{\pi}{4}\,\frac{.22}{b}\right)\frac{pD}{n}$	$b=$ 9.6 6.9 4.8	$\left(\frac{\pi}{4}\,\frac{.22}{b}\right)=$ .018 .025 .036	$p=300$ $n=5$	$B=1\frac{1}{2}D$
Bearng pressure on piston due to weight.	b' = bearing press. on proj. area of piston due to wt. of itself and portion of con. rod supported by it. w = wt. of recip. parts per sq. in. of cyl. D = diameter of cyl. B = length of piston.	$b'=\frac{\frac{\pi}{4}wD}{B}$			$w=1.7$ $B=1\frac{1}{2}D$	$b'=.89$
Thickness of rear wall of piston. Stress in rear wall of piston.	s = thickness of rear wall of piston s = stress in rear wall of piston p = max. pressure. D = cyl. diameter.	$s=\left(\frac{.41}{\sqrt{s}}\right)\sqrt{p}\,D$	$s=$ 2.860 5,320 10,200	$\left(\frac{.41}{\sqrt{s}}\right)=$ 00766 00562 00405	$p=300$	$s=\frac{D}{10}$

TABLE 1.—CURRENT PRACTICE IN THE DIMENSIONS OF GAS ENGINES—(*Continued*)

Engine dimension and name of design constant upon which it depends	Notation: All dimensions in ins., all pressures and stresses in lbs. per sq. in.	Rational formula for engine dimension in terms of the design constant	Maximum, mean and minimum values of design constant	Corresponding numerical values of coefficient of formula	Assumptions made in deducing formula for average cases from rational formula; mean value of design constant always used	Reduced formula for average cases
Length and diam. of wrist pin or piston pin. Stress and bearing pressure on wrist pin.	d'' = diam of wrist pin. l'' = length of wrist pin. p = max. pressure' D = diameter of cyl. s = stress in wrist pin. b = bearing pressure on projected area of wrist pin due to maxi: mum load.	$d''=\sqrt[4]{\frac{\pi}{4sb}}\sqrt{p}\,D$ $l''=\sqrt{\frac{\pi s}{4b}}\,d''$	$s=$ 13,300 10.500 10,000 $b=$ 3,900 2,800 2,260		$p=300$	$d''=.22D$ $l''=1\frac{3}{4}d''$
Area of mid-section of con. rod. Factor of safety in con. rod considered as a long column.	a = area of mid-section of con. rod. k = factor of safety of rod or ratio of breaking load by Ritter's formula to actual load. C = distance, center to center of rod R = diam. of mid-secton if round. Q = height of mid-secton if rectangular. r = radius of gyration of mid-section. $r^2=R^2/16$ or $Q^2/12$ D = diameter of cyl.	$a=\frac{k}{44,560}pD^2$ $\times\left(1+\frac{.00012C^2}{r^2}\right)$	$k=$ 5.44 3 90 2 23	$\frac{k}{44,560}=$ .0001220 .0000857 .0000500	$p=300$ Round rod assumed. $1+.00012\times\frac{C^2}{r^2}$ is given the average value 1.6	$R=.23D$
Length of arm of bending moment on crank pin in terms of cyl. diam.	l = length of crank-pin journal. l' = length of main bearing journal. $2m$ = distance from center to center of main bearings. $M=m-(\frac{3}{8}l+\frac{1}{4}l^1)$ = arm of effective bending moment on crank pin, for reaction on main bearing due to explosion.	$M=cD$	$c=$ .450 .609 .850			$M=.6D$
Diameter of crank pin... Stress in crank pin......	l = length of crank-pin journal. l' = length of main bearing journal. $2m$ = distance from center to center of main bearings. $M=m-(\frac{3}{8}l+\frac{1}{4}l')$. d = diam. of crank pin. s = stress in crank pin. D = diameter of cyl. p = max. pressure.	$d=\sqrt[3]{\left(\frac{4}{s}\right)MpD^2}$	$s=$ 18,800 10,600 7,500	$\left(\frac{4}{s}\right)=$ .000213 .000379 .000533	$M=.6D$ as found above $p=300$	$d=.41D$
Length of crank pin..... Bearing pressure on crank pin.	l = length of crank pin journal. d = diam of crank pin. b = bearing pressure on projected area of crank, due to the average value of load for a complete cycle.	$l=\left(\frac{.145\pi}{4b}\right)\frac{pD^2}{d}$	$b=$ 158 213 348	$\frac{.145\pi}{4b}=$ .000720 .000535 .000327	$d=.41D$ from above $p=300$	$l=.95d$
Thickness of crank throws.	x = thickness of crank throws (in direction of shaft axis). d = diam. of crank pin.	$x=cd$	$c=$ .46 .63 .80			$x=\frac{5}{8}d$
Breadth of crank throws.	y = breadth of crank throws (perpendicular to shaft axis). x = thickness of crank throws (in direction of shaft axis).	$y=cx$	$c=$ 1.50 2.12 3.00			$y=2\frac{1}{8}x$
Length of arm of equivalent bending moment on crank shaft, in terms of cyl. diam.	l' = length of main bearing journal. L = length of stroke. D = diameter of cyl. $M'=(.325l'+.090L)$ = arm of equivalent bending moment on crank shaft (at inner edge of main bearing) for reaction on main bearing due to explosion.	$M'=cD$	$c=$ .324 .400 .468			$M'=.4D$
Diameter of crank shaft. Stress in crank shaft....	s = stress in crank shaft at inner edge of main bearing journal. d' = diam. of crank shaft at main bearing. l' = length of main bearing journal. $M'=(.325l'+.090L)$. D = diameter of cyl. p = max. pressure.	$d'=\sqrt[3]{\left(\frac{4}{s}\right)pD^2M'}$	$s=$ 14,400 9,500 6,200	$\left(\frac{4}{s}\right)=$ .000278 .000422 .000644	$M'=.4D$ from above. $p=300$	$d=\frac{3}{8}D$

TABLE 1.—CURRENT PRACTICE IN THE DIMENSIONS OF GAS ENGINES—(*Continued*)

Engine dimension and name of design constant upon which it depends	Notation: All dimensions in ins., all pressures and stresses in lbs. per sq. in.	Rational formula for engine dimension in terms of the design constant	Maximum, mean and minimum values of design constant	Corresponding numerical values of coefficient of formula	Assumptions made in deducing formula for average cases from rational formula; mean value of design constant always used	Reduced formula for average cases
Length of main bearing journal. Bear'g pressure on main bearings.	l' = length of main bearing journal. d' = diam. of main bearing journal. b = bearing pressure on projected area of main bearing, due to average value of load for a complete cycle.	$l' = \left(\frac{\pi}{24b}\right)\frac{pD^2}{d'}$	$b =$ 174 123 98	$\left(\frac{\pi}{24b}\right) =$ .000752 .001068 .001334	$d' = \frac{3}{8}D$ $p = 300$	$l' = 2\frac{1}{4}d'$
Outside diameter of fly-wheel. Velocity of fly-wheel rim.	F = outside diam. of fly-wheel in ins. K = velocity of fly-wheel rim in ft. per min. n = revolutions per min.	$F = \left(\frac{12K}{\pi}\right)\frac{1}{n}$	$K =$ 4,490 3,220 2,290	$\left(\frac{12K}{\pi}\right) =$ 17,140 12,300 8,750		$F = \frac{12,300}{n}$
Weight of fly-wheel. Speed fluctuation coefficient.	U = total weight of all fly-wheels in lbs. $H.P.$ = rated horse-power. F = outside diam. of fly-wheel in ins. n = revolutions per min. f = speed fluctuation coefficient, or ratio of total variation in r.p.m. to the mean value.	$U = \frac{272,300,000,000}{f} \times \frac{H.P.}{F^2n^3}$	$f =$ .034 .054 .091 That is 3.4% 5.4% 9.1%	$\frac{272,300,000,000}{f} =$ 8,000,000,000,000 5,000,000,000,000 3,000,000,000,000	Above value of rim velocity $F = \frac{12,300}{n}$	$U = \frac{33,000 \text{ H.P.}}{n}$
Rotation speed. Inertia force at end of stroke, per sq. in. of piston.	n = rotation speed, r.p.m. I = inertia force at end of stroke per sq. in. of piston. w = weight of truly recip. parts (piston + ½ con. rod) per sq. in. of piston. L = length of stroke, ins.	$n = \frac{\sqrt{70,382I}}{\sqrt{wL}}$	$I =$ 8.14 15.40 30.80	$\sqrt{70,382I} =$ 757 1,041 1,472	$w = 1.7$ as found above	$n = \frac{800}{\sqrt{L}}$ This is equivalent, to taking the piston speed in ft. per min. as $133\sqrt{L}$
Exhaust pipe diameter. Nominal speed of gases through exhaust pipe.	E = exhaust pipe diam. v = nominal speed of gases thro' exhaust pipe, ft. per min. n = revolutions per minute. L = length of stroke. D = diameter of cyl.	$E = \left(\frac{1}{\sqrt{6v}}\right)D\sqrt{Ln}$	$v =$ 8,850 5,730 3,120	$\left(\frac{1}{\sqrt{6v}}\right) =$ .00434 .00539 .00732	$n = \frac{800}{\sqrt{L}}$ as found above. Then E depends on $\sqrt[4]{L}$ and hence varies little for different values of L. L taken as 12.	$E = .28D$
Exhaust valve diameter. Nom'l speed through exhaust valve.	e = exhaust valve diam. v = nominal speed through exhaust valve. n, L and D as above.	$e = \left(\frac{1}{\sqrt{6v}}\right)D\sqrt{Ln}$	$v =$ 6,750 5,200 3,630	$\left(\frac{1}{\sqrt{6v}}\right) =$ .00497 .00566 .00678	Same as above.	$e = .3D$
Inlet valve diameter. Nom'l speed through inlet valve.	i = inler valve dia, when there is a valve admitting whole charge. v = nominal speed thro' inlet valve. n, L and D as above.	$i = \left(\frac{1}{\sqrt{6v}}\right)D\sqrt{Ln}$	$v =$ 8,330 6,400 4,680	$\left(\frac{1}{\sqrt{6v}}\right) =$ .00447 .00510 .00598	Same as above	$i = .27D$
Gas pipe diameter. Nom'l speed through gas pipe.	G = gas pipe diam., natural gas. v = nominal speed thro' gas pipe. n, L and D as above.	$G = \left(\frac{1}{\sqrt{60v}}\right)D\sqrt{Ln}$	$v =$ 6,670 3,700 2,380	$\left(\frac{1}{\sqrt{60v}}\right) =$ .00158 .00212 .00264	Same as above	$G = .11D$
Gas valve diameter. Nom'l speed through gas valve.	g = gas valve dia., natural gas. v = nominal speed thro' gas valve. n, L and D as above.	$g = \left(\frac{1}{\sqrt{60v}}\right)D\sqrt{Ln}$	$v =$ 3,330 2,080 1,110	$\left(\frac{1}{\sqrt{60v}}\right) =$ .00224 .00283 .00387	Same as above	$g = .15D$
Air pipe diameter. Nom'l speedt hrough air pipe.	A = air pipe diameter, natural gas. v = nominal speed through air pipe. n, L and D as above.	$A = \left(\frac{1}{\sqrt{6.67v}}\right)D\sqrt{Ln}$	$v =$ 10,700 6,900 4,500	$\left(\frac{1}{\sqrt{6.67v}} = \right)$ .00374 .00466 .00577	Same as above	$A = .25D$
Maximum brake H.P. Nominal mean effective pressure.	$M.P.$ = maximum brake H.P. p' = mean effect. press. from area of indicator card. h = mechanical efficiency, or ratio of brake to indicated power. P = hp' = nom'l M.E.P. D = cylinder diameter. L = length of stroke. n = revolutions per minute.	$M.P. = \frac{D^2LnP}{1,008,500}$ (for four stroke cycle engine.)	$P =$ 50 70 85			$M.P. = \frac{D^2Ln}{14,400}$

a = the amount of allowable variation,
n = the revolutions per minute,
N = the brake horse-power,
K = coefficient varying with the type of engine,

The coefficient K, is determined as follows:

K = 44,000 for Otto-cycle engines, single-cylinder, single-acting. (Fig. 1.)

K = 28,000 for Otto-cycle engines, two opposite cylinders, single-acting, or one cylinder double-acting. (Fig. 2.)

K = 25,000 for two cylinders single-acting, with cranks set at 90 deg. (Fig. 3.)

K = 21,000 for two cylinders, single-acting. (Fig. 4.)

K = 7000 for four twin opposite cylinders, or for two tandem cylinders, double-acting. (Fig. 5.)

The factor a, the allowable amount of variation in a single revolution of the fly-wheel is as follows:

For ordinary industrial purposes .... $\frac{1}{25}$ to $\frac{1}{30}$
For electric lighting by continuous current .... $\frac{1}{50}$ to $\frac{1}{80}$
For spinning mills and similar machinery .... $\frac{1}{120}$ to $\frac{1}{130}$
For alternating-current generators in parallel .... $\frac{1}{150}$

The total weight of the fly-wheel may be considered as equal to $P \times 1.4$.

TABLE 1.—PNEUMATIC CONSTANTS

Weight and Volume of Air

1 cu. ft. = .076097 lb. = 1.217 oz.

1 lb. = 13.141 cu. ft.

Value of one Atmosphere of Pressure

Lbs. per sq. in.	Column of water, ft.	Column of mercury, ins.
14.7	33.947	30

Pressure Equivalents

1 lb. per sq. in. = 2.04 ins. of mercury = 2.309 ft. of water.
1 in. of mercury = .49 lb. per sq. in. = 1.132 ft. of water.
1 ft of water = .433 lb. per sq. in. = .883 in. of mercury.
Temperature 62 deg. Fahr.; pressure 14.7 lbs. per sq. in.

TABLE 2.—BAROMETRIC PRESSURE AT VARIOUS ALTITUDES

Altitude, ft.	Mercury column, ins.	Lbs. per sq. in.	Water column, ft.
0	30	14.7	33.95
1,000	28.88	14.15	32.68
2,000	27.80	13.62	31.46
3,000	26.76	13.11	30.28
4,000	25.76	12.62	29.15
5,000	24.79	12.15	28.05
6,000	23.86	11.69	27.00
7,000	22.97	11.26	25.99
8,000	22.11	10.83	25.02
9,000	21.28	10.43	24.08
10,000	20.48	10.04	23.18
11,000	19.72	9.66	22.32
12,000	18.98	9.30	21.48
13,000	18.27	8.95	20.67
14,000	17.59	8.62	19.90
15,000	16.93	8.30	19.16

TABLE 3.—EQUIVALENTS OF OUNCES PER SQ. IN., IN INS. OF HEIGHT OF COLUMNS OF WATER AND MERCURY

Ozs. per sq. in.	Ins. of water	Ins. of mercury
.146	.25	.018
.292	.51	.037
.438	.76	.055
.584	1.01	.074
1	1.73	.127
2	3.46	.255
3	5.20	.382
4	6.93	.510
5	8.66	.637
6	10.39	.765
7	12.12	.892
8	13.85	1.019
9	15.59	1.148
10	17.32	1.275
11	19.05	1.402
12	20.78	1.529
13	22.52	1.658
14	24.25	1.785
15	25.98	1.913
16	27.71	2.036

The word efficiency has two special meanings as applied to air compression, these being called volumetric and compression efficiency. The former refers to the volume of air taken in compared with the piston displacement. Loss of volumetric efficiency is chiefly due to re-expansion of air from the clearance spaces as the suction stroke begins. It, hence, increases with the volume of the clearance spaces and with the receiver pressure. The clearance spaces being the same, the loss is less with compound than with simple compressors because of the reduced pressure produced by the first cylinder. It is commonly measured by dividing the actual length of the suction line of the indicator card by the total length of the card, although this ignores a known but unmeasured source of loss due to the warming of the air as it enters the hot cylinder.

The compression efficiency compares the developed with the theoretical air horse-power, in which comparison two practices prevail. The first compares the actual power with that due to isothermal compression, while the second compares it with single-stage adiabatic compression. Isothermal compression being an impossible condition, the first practice has little real significance, while, adiabatic compression being the normal condition, the second practice furnishes a ready means of expressing the actual gain (often actual loss) of compound over simple compression.

Air compressors should not draw air from warm engine rooms. A suction flue connecting with the cooler out-door air gives rise to continuous economy. The gain is approximately 1 per cent. for each 5 deg. Fahr. difference of temperature, the gain appearing in increased delivery of air which costs nothing.

Compressed-air Power Calculations

The fundamental formulas for the adiabatic compression of gases are:

General	For air	For natural gas	
$\frac{p_2}{p_1}=\left(\frac{v_1}{v_2}\right)^n$	$\frac{p_2}{p_1}=\left(\frac{v_1}{v_2}\right)^{1.41}$	$\frac{p_2}{p_1}=\left(\frac{v_1}{v_2}\right)^{1.266}$	(*a*)
$\frac{t_2}{t_1}=\left(\frac{v_1}{v_2}\right)^{n-1}$	$\frac{t_2}{t_1}=\left(\frac{v_1}{v_2}\right)^{.41}$	$\frac{t_2}{t_1}=\left(\frac{v_1}{v_2}\right)^{.266}$	(*b*)
$\frac{t_2}{t_1}=\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$	$\frac{t_2}{t_1}=\left(\frac{p_2}{p_1}\right)^{.29}$	$\frac{t_2}{t_1}=\left(\frac{p_2}{p_1}\right)^{.21}$	(*c*)

in which p_1 = initial pressure, abs.,
p_2 = final pressure, abs.,
v_1 = initial volume,
v_2 = final volume,
t_1 = initial temperature, abs.,
t_2 = final temperature, abs.

$$n=\frac{\text{specific heat at constant pressure}}{\text{specific heat at constant volume}}$$

= 1.408 or, for practical purposes, 1.41 in the case of air

The work of air compression, including that due to explusion of the air from the cylinder, adiabatic compression being assumed, may be most conveniently expressed by the formula:

$$m.e.p. = 3.45p_1(r^{.29}-1) \qquad (a)$$

in which $r=\frac{p_2}{p_1}$.

Calculations of the mean effective pressure may, in most cases, be abbreviated by the use of Table 4, of which the constants of the third column multiplied by the initial pressure, lbs. per sq. in. abs., give directly the m.e.p., lbs. per sq. in. For compression at sea level the multiplication has been carried out to give both the m.e.p. and

the air h.p. per 100 cu. ft. free air compressed per min. It should, however, be noted that most compressors do not realize full atmospheric pressure in the cylinder and, in such cases, the actual pressure at the beginning of compression should be used when calculating the theoretical m.e.p. The assumption of too high an initial pressure results in a credit for cooling to which the compressor is not entitled.

Affecting the actual m.e.p. may be mentioned such cooling as there may be from the cylinder jacket, the re-expansion of the air in the clearance spaces, and the effect of the clearance spaces in reducing the actual compression ratio of volumes from the apparent ratio, all of which tend to reduce the mean pressure, while the "camel backs" due to the opening of the discharge valves tend to increase it.

The effect of jacket cooling is always small and the actual m.e.p. should not differ much from that given by formula (*d*).

TABLE 4.—CONSTANTS FOR SINGLE-STAGE COMPRESSION

$\frac{\text{Receiver pressure, abs.}}{\text{Initial pressure abs.}} = r$	$\left(\frac{\text{Receiver pressure, abs.}}{\text{Initial pressure abs.}}\right)^{.29} = r^{.29}$	$3.45(r^{.29}-1)$	Compression from 14.7 lbs., initial		
			Gage pressure, lbs.	M.e.p., lbs.	H.p. per 100 cu. ft. free air per min.
1.25	1.067	.231	3.7	3.4	1.48
1.5	1.127	.431	7.3	6.3	2.76
1.75	1.176	.607	11.0	8.9	3.89
2	1.222	.766	14.7	11.3	4.91
2.25	1.265	.914	18.4	13.4	5.86
2.5	1.304	1.049	22.0	15.4	6.73
2.75	1.341	1.176	25.7	17.3	7.54
3	1.375	1.294	29.4	19.0	8.30
3.25	1.408	1.408	33.1	20.7	9.03
3.5	1.438	1.511	36.7	22.2	9.69
3.75	1.467	1.611	40.4	23.7	10.33
4.	1.495	1.708	44.1	25.1	10.96
4.25	1.521	1.797	47.8	26.4	11.53
4.5	1.547	1.887	51.5	27.7	12.14
4.75	1.571	1.970	55.1	29.0	12.64
5.	1.595	2.053	58.8	30.2	13.17
5.25	1.617	2.129	62.5	31.3	13.66
5.5	1.640	2.208	66.2	32.5	14.16
5.75	1.661	2.280	69.8	33.5	14.63
6.	1.681	2.349	73.5	34.5	15.07
6.25	1.701	2.418	76.2	35.5	15.51
6.5	1.721	2.487	80.9	36.5	15.95
6.75	1.740	2.553	84.6	37.5	16.38
7.	1.758	2.615	88.3	38.4	16.77
7.25	1.776	2.677	91.9	39.3	17.17
7.5	1.794	2.739	95.6	40.3	17.57
7.75	1.811	2.798	99.3	41.1	17.95
8	1.828	2.857	103.0	42.0	18.33

Compound or stage compression is resorted to in order to save power and to reduce the final temperature and thereby lessen the danger of explosion of decomposed oil in the air receiver and pipes. Such explosions—whatever their explanation—have happened too many times to permit the danger to be ignored. In high-pressure work compounding becomes a mechanical necessity.

For the most economical results in compound (two stage) compression the work should be equally divided between the cylinders, and that is accomplished by making the number of compressions in each cylinder equal the square root of the total number of compressions—the number of compressions being understood as the higher divided by the lower pressure, abs.

The work of compound compression, including that due to expulsion of the air from the cylinders may be most conveniently expressed by the formula:

$$m.e.p. = 6.90p_1(r^{.145}-1) \qquad (e)$$

in which p_1 = initial pressure, lbs. per sq. in., abs.

$$r = \frac{\text{final pressure, abs.}}{\text{initial pressure, abs.}}$$

This equation gives the m.e.p. reduced to the low-pressure cylinder, under the assumption that the cylinders are proportioned as called for above, that the intercooler reduces the temperature of the air to that at which compression began and than the valves and passages offer no resistance to the flow of air.

As with the formula for simple compression, calculations of the mean effective pressure may, in most cases, be abbreviated by the use of Table 5 of which again the constants of the third column

TABLE 5.—CONSTANTS FOR TWO-STAGE COMPRESSION

$\frac{\text{Final pressure, abs.}}{\text{Initial pressure abs.}} = r$	$\left(\frac{\text{Final pressure, abs.}}{\text{Initial pressure abs.}}\right)^{.145} = r^{.145}$	$6.90(r^{.145}-1)$	Compression from 14.7 lbs. initial			
			Gage pressure, lbs.	M.e.p., lbs. reduced to low pressure piston	H.p. per 100 cu. ft. free air per min.	Possible saving by compounding, per cent.
5	1.263	1.815	58.8	26.7	11.64	11.6
5.5	1.280	1.932	66.2	28.4	12.39	12.2
6	1.297	2.049	73.5	30.1	13.14	12.8
6.5	1.312	2.153	80.9	31.7	13.81	13.4
7	1.326	2.249	88.3	33.1	14.43	14.0
7.5	1.339	2.339	95.6	34.4	15.00	14.6
8	1.352	2.429	103.0	35.7	15.58	15.2
8.5	1.364	2.512	110.2	36.9	16.11	....
9	1.375	2.587	117.6	38.0	16.59	....
9.5	1.386	2.663	125.0	39.2	17.08	....
10	1.396	2.732	132.3	40.2	17.52	....
11	1.416	2.870	147.0	42.2	18.41	....
12	1.434	2.995	161.7	44.0	19.21	....
13	1.450	3.105	176.4	45.6	19.92	....
14	1.466	3.215	191.1	47.3	20.62	....
15	1.481	3.319	205.8	48.8	21.29	....

multiplied by the initial pressure, lbs. per sq. in., abs., give directly the m.e.p., lbs. per sq. in., reduced to the low-pressure piston. For compression at sea level, the multiplication has been carried out to give both the m.e.p. and the air horse-power per 100 cu. ft. free air compressed per min., while the last column gives the theoretical saving due to compounding under the assumptions of the last paragraph.

The air horse-power for each 100 cu. ft. of free air compressed per min. for simple or compound compression.

$$= .436 \times m.e.p. \qquad (f)$$

As applied to pressures in common use for industrial purposes—say 80 to 100 lbs. per sq. in.—the margin of saving by compounding which, as Table 5 will show, is not large, may be more than offset by defective design.

This is shown in Fig. 1 from actual indicator cards. Because of deficiant capacity of the intercooler, the volume of air entering the high-pressure cylinder is not reduced to the isothermal line as it should be, while the overlapping of the high- and low-pressure cards due to inadequate valves more than offsets such gain as the intercooler of itself brings about, the final result being an actual loss of power due to compounding. That good results can be obtained by compounding, is shown in Fig. 2 (from a Nordberg compressor) in which the volume of air entering the high-pressure cylinder is carried back to the isothermal line, while the overlapping of the high- and low-pressure cards is almost negligible. If sufficiently cold water is

available, there is no reason why an efficient intercooler should not reduce the temperature of the air below that at which compression began and thus carry the volume of air entering the high-pressure cylinder within the isothermal line. As a matter of fact, this has often been done.

Air Compression at High Altitudes

The effect of altitude on air compression is to decrease both the delivery of air and the consumption of power but not in the same ratio, the net result being to increase the power consumed in producing a given volume of compressed air.

The relation of the volume of air delivered by a given compressor at sea level and at an altitude, the gage pressure of delivery being the same and ignoring clearance losses, is given by the equation:

$$\frac{v_2}{v_1}=\frac{1+\frac{P}{p_1}}{1+\frac{P}{p_2}}. \qquad (g)$$

in which v_1 = volume of delivery at sea level measured at the delivery pressure and after the heat has dissipated,

v_2 = volume of delivery at an altitude measured at the delivery pressure and after the heat has dissipated,

p_1 = barometric pressure at sea level,

p_2 = barometric pressure at an altitude,

P = gage pressure.

Table 6 has been calculated from this formula. The actual reduction of delivery due to altitude is, however, greater than the table shows. The loss due to clearance increases with the *ratio* of compression and since, for a given gage pressure, this ratio increases with the altitude, the clearance losses increase likewise. The heat due to compression is also a function of the *ratio* of compression and hence, for a given gage pressure, the temperature of the compressed air increases with the altitude. For both reasons compounding is of increasing importance with increase of altitude.

Graphic Compressed-air Power Calculations

The foregoing calculations may be made graphically by the aid of Figs. 3 and 4, by J. A. Brown (*Amer. Mach., June* 12, 1913), Fig. 3 being for single and Fig. 4 for two-stage compression, the assumption in the latter being that the intercooler reduces the air to its initial temperature.

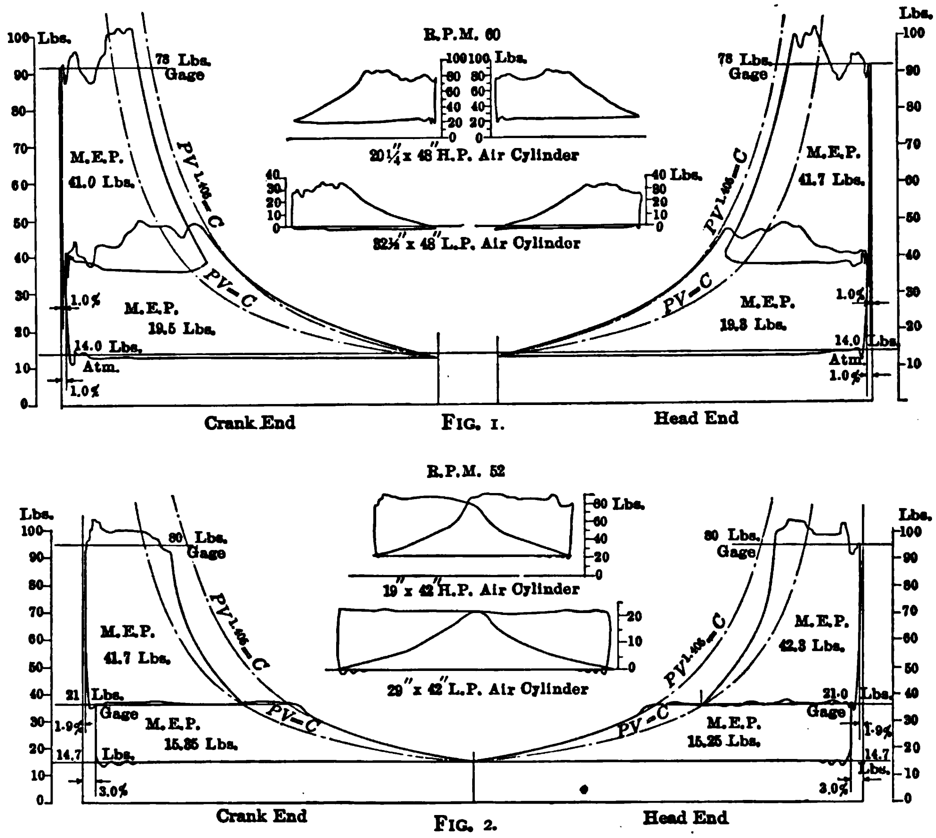
To use the chart, Fig. 3, for single-stage compression: Find the absolute final pressure on the scale at the left by adding the barometric pressure for the required altitude, Table 2, to the gage pressure. Trace horizontally to the line for the altitude, vertically to the line marked $\left(\frac{p_2}{p_1}\right)^{.29}$ and horizontally to the right where read the value of $\left(\frac{p_2}{p_1}\right)^{.29}$. Subtract, mentally, 1 from the value of $\left(\frac{p_2}{p_1}\right)^{.29}$ find the resulting value on the lower scale and from it trace vertically to the altitude line and then horizontally to the right where read the m.e.p. from the middle scale and the horse-power per 100 cu. ft. free air per min.

TABLE 6.—RELATIVE AMOUNTS OF AIR DELIVERED BY A GIVEN COMPRESSOR AT VARIOUS ALTITUDES

Altitude, ft.	Relative output at gage pressures	
	70 lbs.	100 lbs.
0	1.000	1.000
1,000	.969	.968
2,000	.938	.936
3,000	.908	.905
4,000	.880	.875
5,000	.853	.846
6,000	.826	.818
7,000	.798	.790
8,000	.772	.763
9,000	.746	.737
10,000	.720	.712
11,000	.697	.688
12,000	.675	.665
13,000	.654	.642
14,000	.632	.620
15,000	.611	.599

To find the final temperature find the value of $\frac{p_2}{p_1}$ on the scale at the top, trace vertically to the line for the suitable initial temperature and then horizontally to the scale at the center of the chart where road the final temperature, Fahr. absolute.

Fig. 4 for two-stage compression is used in precisely the same way.



FIGS. 1 and 2.—Good and poor compound air-compressor practice.

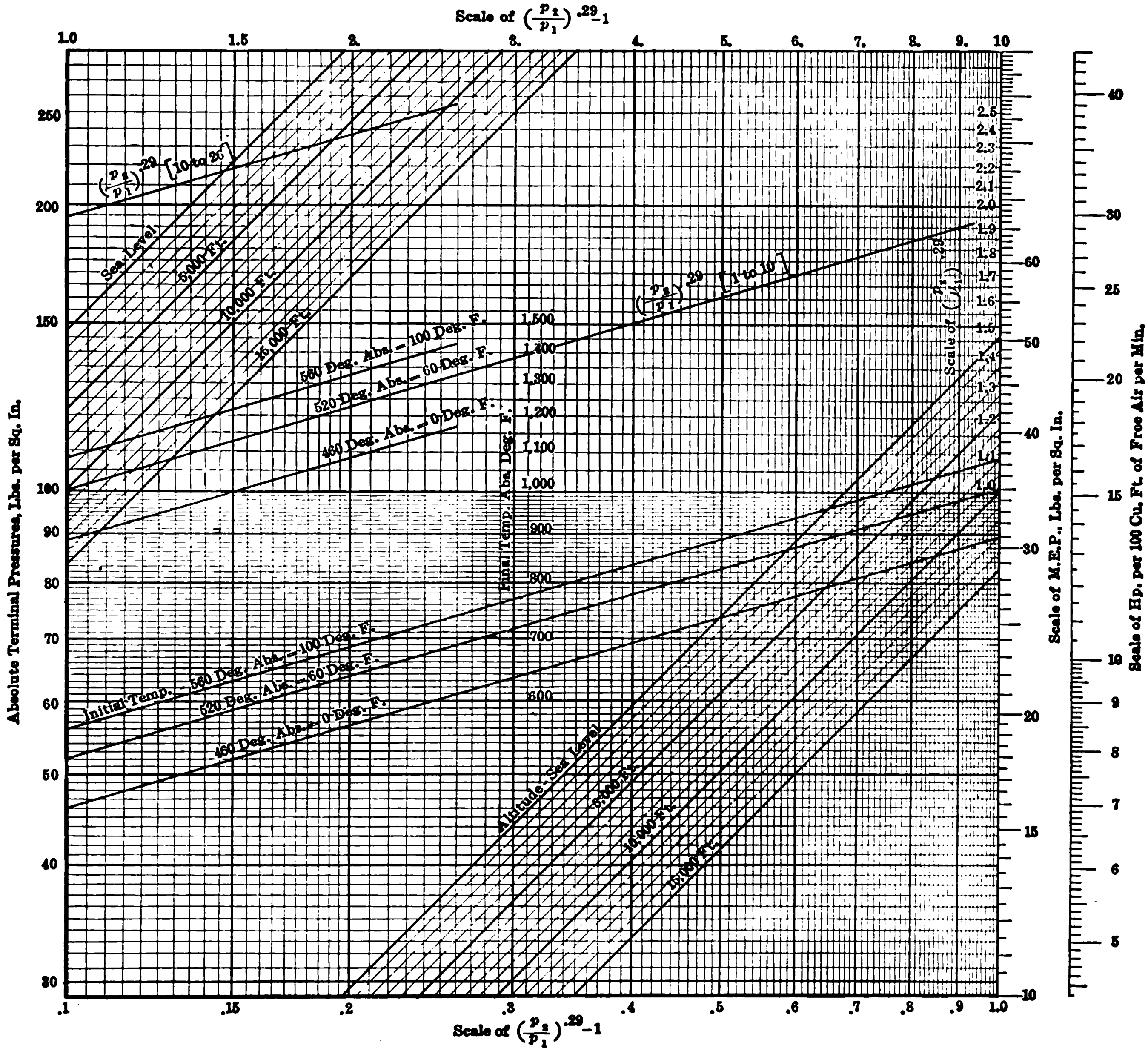


FIG. 3.—Mean effective pressure, horse power and temperature of compressed air. Single stage compression.

The Index of the Compression Curve

To find the index of an actual compression curve corresponding to the theorectical value 1.41 in equation (*a*), measure the pressure and volume at two points on the indicator card, as widely separated as possible, for which any scale may be used, inches divided into tenths being most convenient. Call the lower pressure p_1 the corresponding volume v_1, the higher pressure p_2 and the corresponding volumė v_2. The index being unknown we have then to solve the equation:

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^x$$

Let $\frac{p_2}{p_1} = R$ and $\frac{v_1}{v_2} = r$

giving $R = r^x$

That is, $\log R = \log_r r^x$

or $\log R = x \log r$

or $$x = \frac{\log R}{\log r} \qquad (h)$$

The solution of this equation will give the required index. The work may be abbreviated by the use of Table 7 as follows: Select such values of p_2 and p_1 that the former shall be an exact multiple—2, 3, 4, 5, 6, 7 or 8 times the latter.

Measure v_2 and v_1 to correspond with p_2 and p_1; divide v_1 by v_2 and find the resulting ratio in the column of $\frac{v_1}{v_2}$ standing under the ratio for $\frac{p_2}{p_1}$ selected. Opposite the ratio for $\frac{v_1}{v_2}$ will be found the value of the index.

The index of the compression curve may also be found graphically by the aid of Fig. 6, by J. A. BROWN (*Amer. Mach., June* 28, 1900). Take for example the card shown in Fig. 5. From the card measure v and p at two points and by any scale—inches and tenths being most convenient—one at the beginning of compression where p measures .44 in. and v 6.1 ins. Take any other point, say $p = 1$ in., and $v = 3.12$ ins. On the chart Fig. 6 mark the intersection of .44 on the scale of pressures with 6.1 on the scale of volumes; mark the intersection of $p = 1$ and $v = 3.12$ in the same way. Using two

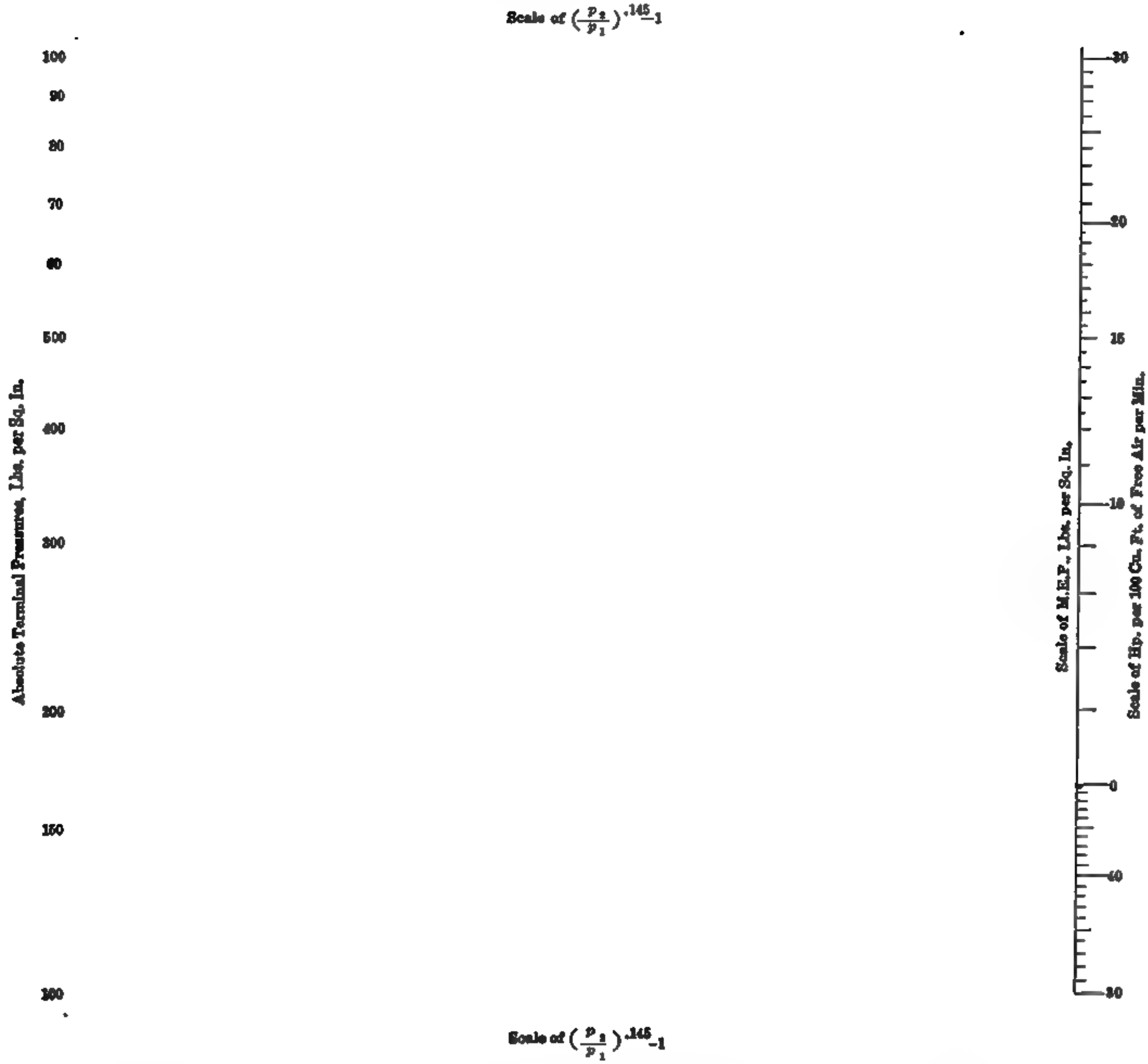


FIG. 4.—Mean effective pressure, horse power and temperature of compressed air. Two stage compression.

triangles as a parallel ruler find the index diagonal to which a line through these points is most nearly parallel and read the figures for the approximate index—in this case 1.25. If through the intersection of p and v at beginning of compression, diagonals parallel to the isothermal and adiabatic lines be drawn, intersections with these lines give corresponding values of v and p, and for every value of v, the three values of p corresponding to isothermal, adiabatic and the actual compression curve plotted on Fig. 5.

The Friction of Compressed Air in Pipe

The formula for the friction of compressed air in pipe, taking into account the increase in volume and velocity that accompanies drop in pressure, was first established by PROFESSOR UNWIN (*Transmission of Power*). Slightly transformed to make it read volume instead of weight it is as follows:

$$V=3.04\sqrt{\frac{d^5}{fl}(p_1^2-p_2^2)}$$

in which V = volume, cu. ft. free air per min. at sea level and 60 deg. Fahr.,
d = diameter of pipe, ins.,
p_1 = initial pressure, lbs. per sq. in., abs.
p_2 = terminal pressure, lbs. per sq. in., abs.,
f = coefficient of friction,
l = length of pipe, ft.

The coefficient of friction is not constant but varies with the diameter of the pipe. According to Professor Unwin it is expressed by the equation:

$$f=.0027\left(1+\frac{3}{10d}\right)$$

in which d = diameter of pipe *in ft.*

Problems involving the friction of compressed air in pipe may be solved by the use of Fig. 7 by JAS. A. BROWN (*Amer. Mach., July* 10, 1913) which incorporates both the above formulas. The use of the chart is explained below it. It will be recognized that the method of

TABLE 7.—VALUES OF THE INDEX OF COMPRESSION CURVES

Higher pressure / Lower pressure $\frac{p_2}{p_1}=R=2$		Higher pressure / Lower pressure $\frac{p_2}{p_1}=R=3$		Higher pressure / Lower pressure $\frac{p_2}{p_1}=R=4$		Higher pressure / Lower pressure $\frac{p_2}{p_1}=R=5$		Higher pressure / Lower pressure $\frac{p_2}{p_1}=R=6$		Higher pressure / Lower pressure $\frac{p_2}{p_1}=R=7$		Higher pressure / Lower pressure $\frac{p_2}{p_1}=R=8$	
Larger volume / Smaller volume $=\frac{v_1}{v_2}=r=$	Index of curve $=x=$	Larger volume / Smaller volume $=\frac{v_1}{v_2}=r=$	Index of curve $=x=$	Larger volume / Smaller volume $=\frac{v_1}{v_2}=r=$	Index of curve $=x=$	Larger volume / Smaller volume $=\frac{v_1}{v_2}=r=$	Index of curve $=x=$	Larger volume / Smaller volume $=\frac{v_1}{v_2}=r=$	Index of curve $=x=$	Larger volume / Smaller volume $=\frac{v_1}{v_2}=r=$	Index of curve $=x=$	Larger volume / Smaller volume $=\frac{v_1}{v_2}=r=$	Index of curve $=x=$
1.95	1.04	2.95	1.02	3.90	1.02	4.90	1.02	5.80	1.02	6.80	1.02	7.80	1.01
1.90	1.08	2.90	1.03	3.80	1.04	4.80	1.03	5.60	1.04	6.60	1.03	7.60	1.03
1.88	1.10	2.85	1.05	3.70	1.06	4.70	1.04	5.40	1.06	6.40	1.05	7.40	1.04
1.86	1.12	2.80	1.07	3.60	1.08	4.60	1.06	5.20	1.09	6.20	1.07	7.20	1.06
1.84	1.14	2.75	1.09	3.50	1.11	4.50	1.07	5.00	1.12	6.00	1.09	7.00	1.07
1.82	1.16	2.70	1.11	3.40	1.13	4.40	1.09	4.90	1.13	5.80	1.11	6.80	1.09
1.80	1.18	2.65	1.13	3.30	1.16	4.30	1.10	4.80	1.14	5.60	1.13	6.60	1.10
1.78	1.20	2.60	1.15	3.20	1.19	4.20	1.12	4.70	1.16	5.40	1.15	6.40	1.12
1.76	1.23	2.55	1.18	3.10	1.23	4.10	1.14	4.60	1.18	5.20	1.18	6.20	1.14
1.74	1.25	2.50	1.20	3.00	1.26	4.00	1.16	4.50	1.19	5.00	1.21	6.00	1.16
1.72	1.28	2.45	1.23	2.95	1.28	3.90	1.18	4.40	1.21	4.90	1.23	5.80	1.18
1.70	1.31	2.40	1.26	2.90	1.30	3.80	1.21	4.30	1.23	4.80	1.24	5.60	1.21
1.69	1.33	2.35	1.29	2.85	1.33	3.70	1.23	4.20	1.25	4.70	1.26	5.40	1.23
1.68	1.34	2.30	1.32	2.80	1.35	3.60	1.26	4.10	1.27	4.60	1.28	5.20	1.26
1.67	1.36	2.28	1.34	2.78	1.36	3.50	1.29	4.00	1.29	4.50	1.30	5.00	1.29
1.66	1.37	2.26	1.35	2.76	1.37	3.45	1.30	3.90	1.32	4.40	1.32	4.90	1.31
1.65	1.39	2.24	1.36	2.74	2.38	3.40	1.32	3.80	1.34	4.30	1.34	4.80	1.33
1.64	1.41	2.22	1.38	2.72	1.39	3.35	1.33	3.70	1.37	4.20	1.36	4.70	1.35
		2.20	1.40	2.70	1.40	3.30	1.35	3.60	1.40	4.10	1.38	4.60	1.36
						3.25	1.36			4.00	1.40	4.50	1.38
						3.20	1.38					4.40	1.41

introducing the length factor is due to the fact that, unlike the case of liquids, the friction loss is not in simple proportion to the length of the pipe. The factors of the table below the chart are values of $\sqrt{\frac{100}{l}}$ from which factors for other lengths may be calculated.

Plotting the Compression Curve

The plotting of the adiabatic curve involves the solution of equation (1) which, for this purpose and for air, may be more conveniently written:

$$p_2v_2^{\ 1.41} = p_1v_1^{\ 1.41}$$

Values of p_1 and v_1 are to be measured from the indicator card near the beginning of the compression when various smaller values of v_2 being taken, the corresponding values of p_2 may be calculated. The process may be greatly abbreviated by the use of the constants of Fig. 8, which gives the relative increase of pressure for the reduction of volume as the compression goes on. To use the diagram the indicator card should be divided into tenths and half-tenths, as this diagram is divided. Then, taking the absolute pressure at the beginning of compression, the product of this pressure by the multiplier at the left of each ordinate of the diagram will give the pressure

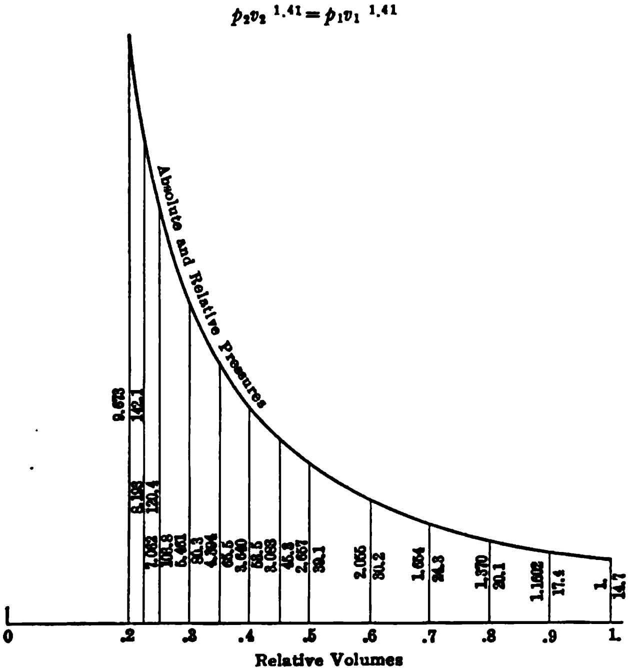


FIG. 8.—Ordinates of the adiabatic curve for air.

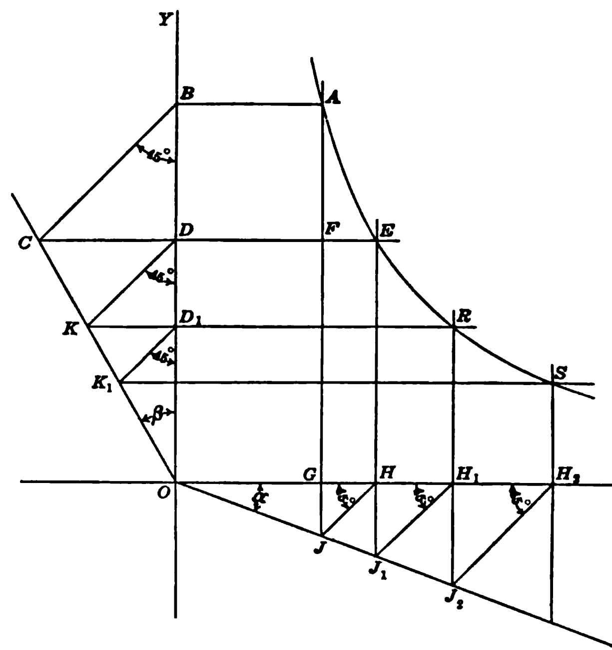


FIG. 9.—Construction of the adiabatic curve.

due to the adiabatic curve at the corresponding ordinate of the indicator card. The diagram is applicable to high and low pressure cards alike, the proper initial absolute pressure being taken, of course.

For compression from 14.7 lbs. initial the multiplications have been made and entered on the right side of the vertical lines of the diagram, from which the pressures at the various ordinates may be read directly for that initial pressure.

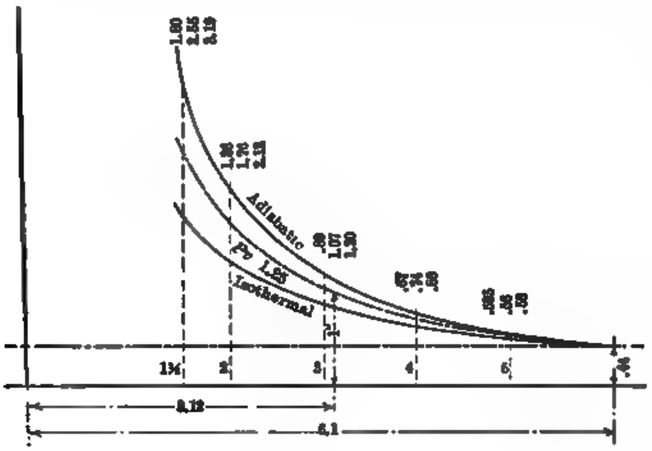


FIG. 5.—Compressed air indicator card analyzed by the chart, Fig. 6.

The use of this initial pressure is, however, seldom justified because of the suction loss which exists in most cases. In such cases the assumption of full atmospheric pressure results in a curve which is too high for the truth, and gives the compressor credit for a degree of cooling to which it is not entitled.

The plotting of the adiabatic curve for any value of the index may be done graphically as in Fig. 9 (*Amer Mach., June 21*, 1900). Draw OJ at any convenient angle α with OX. Determine the angle β from the relation

$$1+\tan\beta=(1+\tan\alpha)^n$$

in which n = the required index.

Draw OC at the angle β with OY. Through A draw AB paralle. to OX and AJ parallel to OY. Lay off BC at an angle of 45 degl with OY, and from the intersection of BC and OC draw a horizontal line CE. From the intersection of AJ and OJ draw a line at 45 deg. with OX and cutting OX at H. At H erect a perpendicular cutting CE at E and OJ produced at J_1. Then E is a point on the curve, and so proceed. The smaller angle α is taken the more closely the points of the curve will be located, but the greater the opportunity for instrumental error. Obviously the construction may be begun

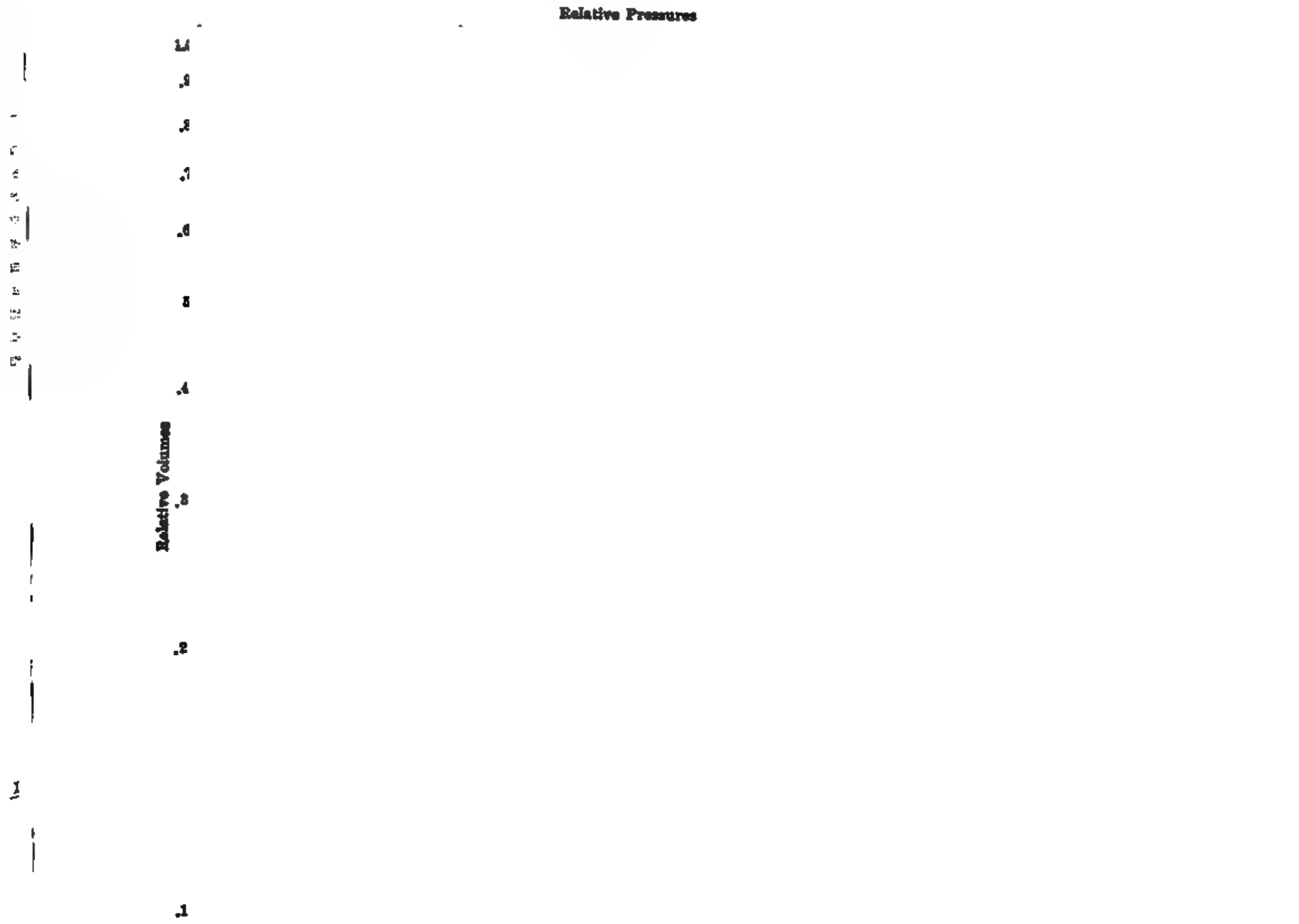


FIG. 6.—Finding the index to an actual compression curve.

at point S as readily as at point A and conversely, if we have a curve for which we wish to derive an exponent, we can, by working backward, locate the lines OC and OJ, measure the angles α and β, and solve for n.

For the index value 1.41 the method of Fig. 8 is to be preferred as, from its nature, the method of Fig. 9 involves an accumulation of error which impairs the accuracy of the result.

The isothermal curve has comparatively little application to compressed air as actual compression is always nearly adiabatic. Its equation is:

$$\frac{p_2}{p_1} = \frac{v_1}{v_2}$$

in which p_1 = initial pressure, abs.,
p_2 = find pressure, abs.,
v_1 = initial volume,
v_2 = find volume.

Scale for Value of (Initial Pressure Abs. x 2)-Drop, Lbs. per Sq. In.

Volume of Free Air, Cu. Ft. per Min.

Directions for use: Double the initial absolute pressure and from the product subtract the desired pressure loss. Find the result on the left hand vertical scale; trace horizontally to the right to the line for the desired pressure loss; thence vertically to the line for the diameter of the pipe; thence horizontally to the right hand scale where read the volume of free air in cu. ft. per min., supposing the pipe to be 100 ft. long. For other lengths multiply this volume by the proper factor from the following table:

Length, ft.	Factor	Length, ft.	Factor	Length, ft.	Factor	Length, ft.	Factor
100	1.00	750	.365	2,500	.20	7,000	.119
200	.71	1,000	.316	3,000	.183	8,000	.112
300	.578	1,250	.283	3,500	.169	9,000	.105
400	.50	1,500	.258	4,000	.158	10,000	.10
500	.448	1,750	.24	5,000	.141	15,000	.082
600	.41	2,000	.224	6,000	.129	20,000	.079

FIG. 7.—Friction of compressed air in pipes

The chief use of this equation in compressed-air work is in laying down the isothermal curve on combined indicator cards in order to determine the efficiency of the intercooler in reducing the temperature and volume of the air before entering the high-pressure cylinder. For a graphical method of constructing the curve see Isothermal Curve.

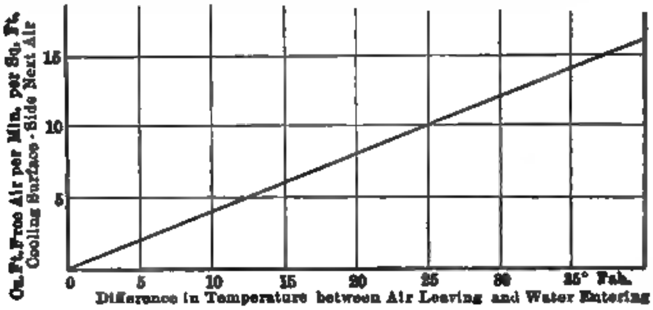


FIG. 10.—Relation of surface and capacity of intercoolers.

FIG. 11.-Plan Section

FIG. 12.-Sectional Elevation

FIGS. 11 and 12.—The Nordberg intercooler.

The Intercooler

The design of intercoolers should be such that the air and water pass through them in opposite directions in order that the incoming or coolest water may act on the air at the last stage of the cooling. The outer surface of the tubes should be the air surface as its greater area compensates, in part, for its lesser efficiency. There is no advantage in copper or brass over iron tubes, in fact, in such comparative experiments as have been made iron tubes have been found the more efficient—due probably to their greater roughness.

The cooling area required for any given final effect may be determined from Fig. 10 by H. V. HAIGHT, Chief. Engr., Canadian Ingersoll-Rand Co., (*Amer. Mach., Aug.* 30, 1906) which represents the formula (determined by experiment):

$$y = .4\,x$$

in which y = free air capacity of intercooler, cu. ft. per min. per sq. ft. of cooling surface measured on the air side,

x = difference in temperature, deg. Fahr., between the air leaving and the water entering.

The Nordberg construction of intercooler, Figs. 11 and 12, has an unusual provision to compel the water to flow equally through all the tubes in addition to the counter-current direction of air and water. The tube plates are shown at *aa* while complete tubes are shown at *bb* and others—cut off to avoid confussion—are shown at *cc*. The air enters the intercooler at *d* and is discharged at *e*, while the water enters at *f* and is discharged at *g*. Baffle plates *hhh* guide the air in the manner indicated by the arrows *iii*, while baffles in the water heads insure the flow of the water in the opposite direction as indicated by the arrows *jjj*.

Reheating Compressed Air

The gain due to reheating compressed air formed the subject of experiments by W. G. EDMONDSON and E. L. WALKER (*Amer. Mach., July* 31, 1902) a 2 h.p. shaft governor engine being used. The consumption of air per h.p. was, no doubt, greater than with

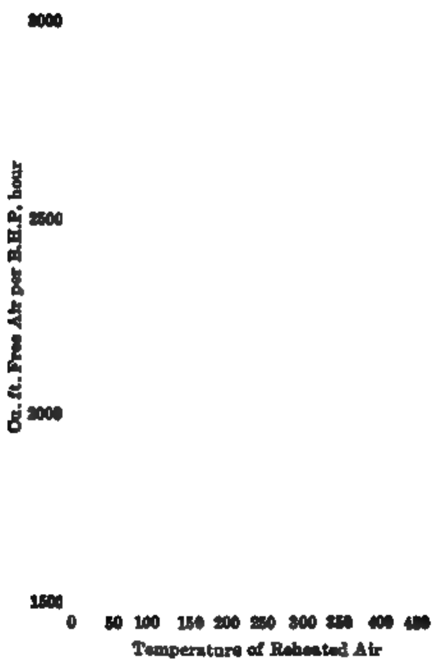


FIG. 13.—Reduced consumption of compressed air due to reheating.

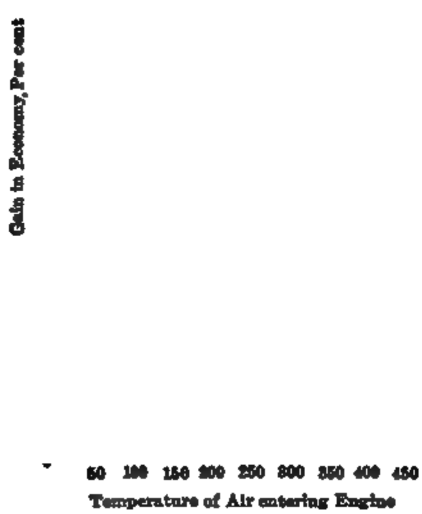


FIG. 14.—Increased economy of compressed air due to reheating.

larger engines but there is no apparent reason why the gain due to reheating should not hold. The tests were made at three pressures and various degrees of reheating. The results on the air consumption per brake h.p. are given in Fig. 13, while Fig. 14 shows the gain in economy, including the cost of reheating. In this chart, gain in economy $= 1 - \frac{\text{B.t.u.'s per h.p. reheated}}{\text{B.t.u.'s per h.p. not reheated}}$. The results found

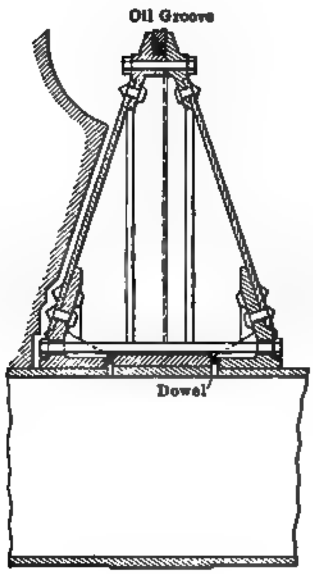


FIG. 15.

FIG. 16.

FIGS. 15 and 16.—Nordberg construction of piston and piston rod of blowing engines for very light pressures.

exceed those to be expected from the calculated expansion of the air due to the reheating. This is explained by the increased mechanical efficiency of the engine when heated air was used. Indicator and brake tests showed large friction losses due to the low temperatures when unheated air was used—losses that grew markedly less when the air was heated. The tests showed the fuel cost of the air obtained by compressing to be from 8 to 19 times that obtained by heating. Were the first cost of the compressor plant to be compared with that of the heater, the comparison would be still more striking.

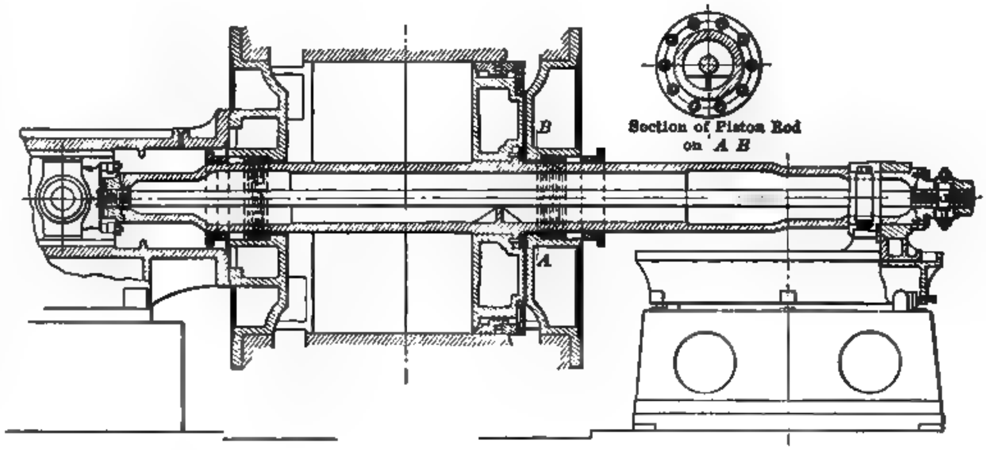


FIG. 17.—Nordberg construction of piston rod for moderate air pressures.

Details of Air Compressors

Piston and piston-rod constructions for large blowing engines under light air pressure, as used by the Nordberg Mfg. Co., (*Amer, Mach., Feb.* 7, 1907) are shown in Figs. 15 and 16. The pistons, Fig. 15, which are of 70 ins. diam. and designed for an air pressure of 40 oz. per sq. in., were made of $\frac{3}{8}$-in., boiler plate, each side in one piece and dished to the form shown. Lateral stiffness was provided by riveting the plates at the outer diameter to a ring spider in halves, the piston ring groove being between the halves. The piston was turned $2\frac{3}{8}$ ins. smaller than the bore of the cylinder and the ring did not bottom in its groove. The piston is thus a floating piston, the weight of which is carried by the piston rod. This rod, as befits its duty, and as shown in Fig. 16, is extremely light. It was made of steel pipe, its diameter providing ample stiffness. Each end carries a shoe for supporting the weight of the piston, the shoes and the guides being shown in Fig. 16.

When the slight deflection of a piston rod which suffices to transfer its weight to the cylinder is considered, the conclusion seems inevitable that the provision of slides at both ends of rods of the usual proportions is of more than doubtful value as a means of transferring the weight of the piston from the cylinder to the slide.

Another piston-rod construction used by the Nordberg Co. for air pistons under somewhat heavier pressures (in this case 7 lbs. per sq. in.) is shown in Fig. 17 (*Amer. Mach., Nov.* 23, 1905). With such low pressures, the objection to large stuffing boxes disappears and the rod is of enormous size—16 ins. diam. for an air cylinder of 62 ins. diam. It is of cast-iron, hollow and within it is a forged rod, the two being so connected that the forged rod carries the tensile and the

cast rod the compression strains. A key at *a* serves to put the forged rod under initial tension.

Air valves for a large blowing engine (62×42-in. air cylinder, pressure 7 lbs. per sq. in., speed 75 r.p.m.) as made by the Nordberg Mfg. Co. are shown in Fig. 18 (*Amer. Mach., Nov.* 23, 1905). The valves are of the Corliss type and are essentially similar to steam cylinder valves, except that their functions are reversed, the inlet air valves being similar to exhaust steam valves and the outlet air valves similar to admission steam valves. All valves are double-ported, a provision which gives to the air valves the unusually generous effective port area of 13.4 per cent. of the piston area. Fig. 4 shows the valves in relation to their seats and the method by which the inlet valves are given a double port.

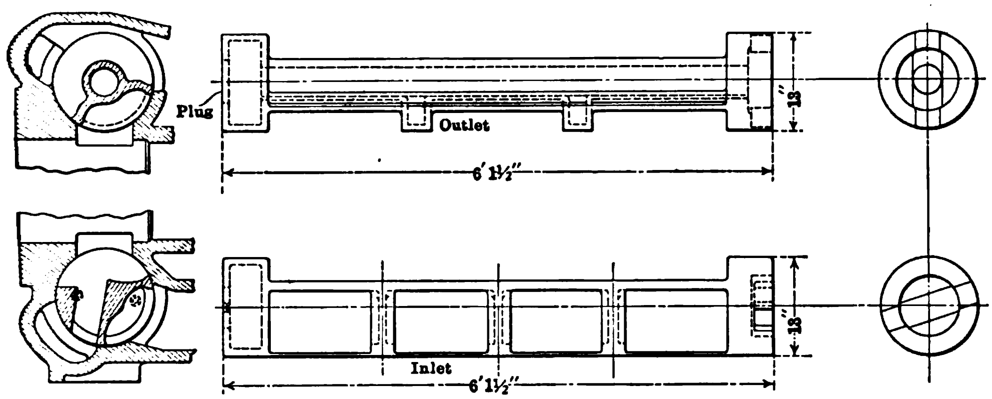


FIG. 18.—Nordberg construction of double ported Corliss valves for blowing engine cylinders.

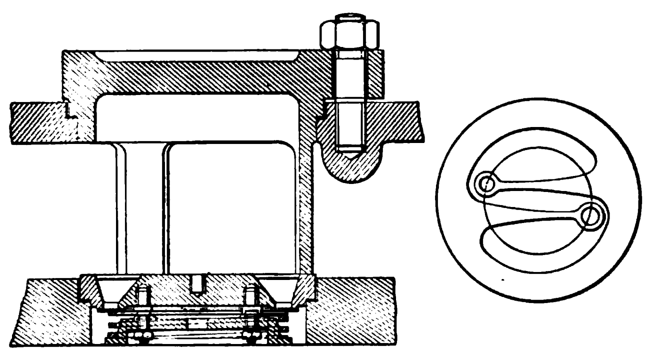


FIG. 19.—Borsig construction of air-compressor valves.

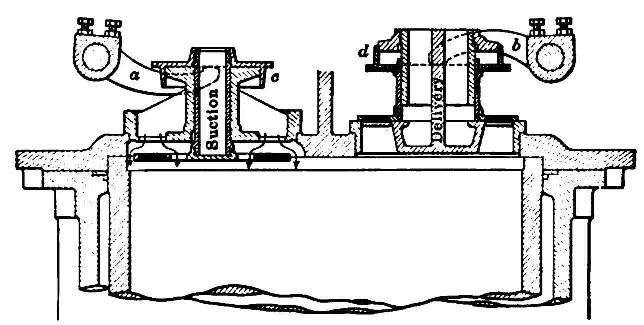


FIG. 20.—Poppet valves for air compressors—Riedler system.

Spring-closed relief or safety valves are also provided (not shown) to provide for any possible failure of the regular valves (which are positively actuated) to act.

Poppet valves for air compressor service, as made by A. Borsig, Tegel, Germany, are shown in Fig. 19 (*Amer. Mach., Nov.* 5, 1908). The valve arrangement consists of a cast-iron seat, a thin valve plate made of a steel punching, which, by means of two arms fixed by plugs, is guided without friction, the valve guard and a helical spring between valve and guard. Fig. 3 shows all these parts in detail. The only purpose that the spring has to serve is to effect the closing of the valve plate at the proper time. Suction- and discharge-valve parts are identical.

The method of inserting the valves into the casings is also shown in the illustration. The valve is inserted between the casing and guard, which former is provided with stems connecting it with the cover plate. The tightening of the valve and the cover is effected by rings made of a suitable graphitic material.

Poppet valves for air-compressor service (Riedler system) are shown in Fig. 20 (*Amer. Mach., Oct.* 16, 1902). The special feature of this system of valves is that while they are closed positively by mechanism they are opened by the air or water, as the case may be. A moving lever closes the valve at the end of the stroke, and then, before the time for the valve to open arrives again, the lever withdraws and leaves the valve free to open whenever the conditions require it. The action is the same with both suction and discharge valves, and much of the mechanism is common to both. Both sets of valves require closing at the same moment—at the extreme end of the stroke—and since that is all that the mechanism does, the same movement is obviously as appropriate to one as to the other.

Fig. 8 is a longitudinal section of the upper end of the low-pressure air cylinder and its valves, the surrounding casing being broken away. The valves will be seen to be double seated, the direction of the air currents being shown by the arrows. The manner in which the levers *ab* act to close the valves will be apparent on inspection. At *cd* are air gag or choke pots, the office of which is to prevent any rebound of the valves from the stops which limit their opening when the compressor is running at high speed. The valves of the high-pressure cylinder are similar to those of the low except that they are single seated.

Packing for a high-pressure air plunger of a 4-stage air compressor for 1000 lbs. per sq. in. pressure, by H. V. HAIGHT, Chf. Engr., Canadian Ingersoll-Rand Co. (*Amer. Mach., Apr.* 23, 1908), is shown in Fig. 21. The construction gave excellent results, although it is not easy to see the office of so many packing rings outside the oiler ring or lantern. The plunger should be ground to reduce wear of the packing rings.

The cuts in the inside rings do not lead to appreciable leakage. At most they form a long and tortuous passage with many enlargements to destroy the energy of the moving air. Moreover they fill with oil and soon close up.

For pipe fittings for high-pressure compressed air see Pipe Fittings for High-pressure Air.

Consumption of Compressed Air

The consumption of compressed air by various pneumatic tools as made by the Ingersoll-Rand Co. is given in Table 8. The figures are not mere estimates based on piston displacement but are the results of careful tests by the makers, the air being accurately measured by a water-displacement meter.

The consumption of compressed air by Curtis direct-lift air hoists is given in Table 9.

TABLE 8.—CONSUMPTION OF AIR BY PNEUMATIC TOOLS

Crown Hammers

Size no.	Cylinder Bore, ins.	Cylinder Stroke, ins.	Weight, lbs.	Cu. ft. free air per min. 60 lbs. pres.	Cu. ft. free air per min. 100 lbs. pres.	Uses
51-H	1⅛	1	9	10	17	Chipping and calking bath tub and range boilers and other light work.
52-H	1⅛	2	10	11	18	Light chipping and calking, beading flues and sealing castings.
53-H	1⅛	3	11	12	20	General chipping and calking.
54-H	1⅛	4	12	13	22	Heavy chipping and calking. Riveting light tanks and heavy sheet iron.

Imperial Hammers

Size no.	Cylinder Bore, ins.	Cylinder Stroke, ins.	Weight, lbs.	Cu. ft. free air per min. at 80 lbs. pres.	Uses
1	1$\frac{1}{16}$	1½	10	20	Chipping and calking bath tubs and range boilers and other light work.
2	1$\frac{1}{16}$	2	11½	20	Light chipping and calking beading flues and sealing castings.
3	1$\frac{1}{16}$	3	12¾	20	General chipping and calking.
4	1$\frac{1}{16}$	4	13¾	21	Heavy chipping and calking.
5	1$\frac{1}{16}$	5	15	21	Extra heavy chipping and calking.
40	1$\frac{1}{16}$	4	13¾	21	Driving rivets ¾-in. diameter and less.
50	1$\frac{1}{16}$	5-	15	21	Driving rivets ⅞-in. diameter and less.

Little David Piston Drills

Size no.	Weight lbs.	Max. diam. twist drill, ins.	Max. diam. wood bit, ins.	Max. diam. reamer, ins.	Max. diam. tap, ins.	Max. diam. flue roller, ins.	Cu. ft. free air per min. at 90 lbs. pres.
1	53	2	....	2	2	3	55
2	40	1½	....	1	1	2½	50
3	23	$\frac{11}{16}$	....	¾	¾	....	30
12	31	....	4	....	....	....	50
13	20	....	2	....	....	....	30

Imperial Motor Hoists

Size no.	Capacity, lbs.	Ft. lift per min. at 80 lbs. pres.	Max. lift, ft.	Cu. ft. free air per ft. lift of max. load
1	1,000	32	20	1.41
2	2,000	16	20	2.82
4	4,000	8	20	5.63
7	7,000	8	20	10
10	10,000	7	20	11.4

Crown Sand Rammers

	Length over all, ins.	Weight, lbs.	Cylinder Bore, ins.	Cylinder Stroke, ins.	Cu. ft. free air per min. at pres. 40 lbs.	50 lbs.	60 lbs.	70 lbs.	80 lbs.	90 lbs.	100 lbs.
Bench rammer 10-SR.	20	10	1	4	12	14½	17	19½	22	25	28
Floor rammer 20-SR.	46	22½	1½	5	12	15	18	21	24½	28	32

TABLE 9.—CONSUMPTION OF AIR BY DIRECT-LIFT AIR HOISTS

Nominal inside diameter of hoist in ins.	Capacity in lbs. at 80 lbs. air pressure. 10% allowed for friction loss	Cu. ft. of free air required to lift full load 1 ft. at 80 lbs. air pressure
4	861	.54
5	1,356	.85
6	2,050	1.22
7	2,791	1.73
8	3,616	2.24
9	4,592	2.85
10	5,636	3.29
12	8,154	5.06
14	11,270	7.13
17	16,500	10.10
19	20,900	12.50

The consumption of compressed air by direct-acting steam pumps may be determined from the formula:

$$V = a\left(.0307 \times \frac{h}{r^2} + .799\right)$$

in which V = volume of air consumed, measured at atmospheric pressure, cu. ft. per min.,

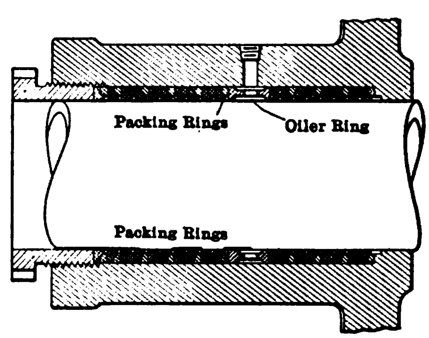
a = area of steam piston, sq. ins.,

h = head of water, ft.

$$r = \frac{\text{diameter of steam cylinder}}{\text{diameter of water cylinder}}.$$

The formula is the full rational formula reduced to its lowest terms with the added assumption that 20 per cent. of the power is consumed in friction of the pump and of the water in the pipes and that 15 per cent. of the piston displacement is lost in clearance and leakage. The compressor is assumed to operate under 14 lbs. atmospheric pressure. A useful modification of the formula is the following:

$$V = .628h + 16.9r^2$$



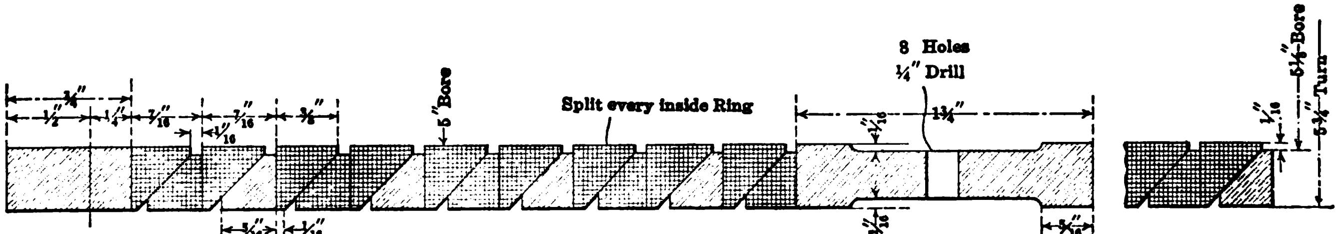


FIG. 21.—Haight's packing for high-pressure air plungers.

in which V = volume of air measured at atmospheric pressure required to pump 100 gals. of water, cu. ft.

The formulas have been verified by tests on compressors driving pumps exclusively.

The discharge of air through orifices is given in Table 10.

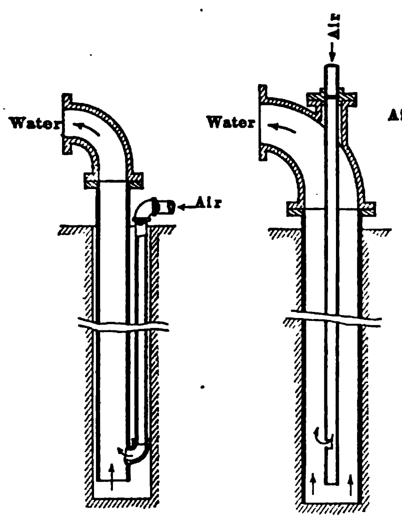


FIG. 22. FIG. 23.

FIGS. 22 and 23.—The air lift pump.

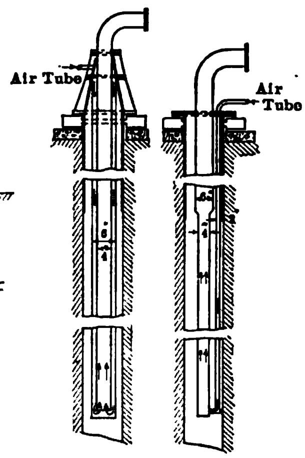


FIG. 24. FIG. 25.

FIGS. 24 and 25.—Air lift pumps tested by Mr. Kelly.

The Air-lift Pump

The air lift pump of Dr. E. S. Pohlé, a remarkable invention in point of simplicity and effectiveness, is shown in its original form in Fig. 22. Other forms are used, including that of Fig. 23, by the Ingersoll-Rand Co. Whatever the arrangement, the principle is the same. The water pipe is well submerged and, air of adequate pressure being admitted, it rises in the pipe and forms a mixture of air and water which being lighter than the surrounding water rises and, if the depth of submergence be sufficient, overflows at the outlet. Numerous installations exist in which water is raised from 600 to 1200 ft.

The theory of the pump is obscure, but the following formula by E. A. Rix and the Ingersoll-Rand Co. gives the volume of air required:

$$V = \frac{h}{C \log \frac{H+34}{34}}$$

in which V = air piston displacement cu. ft. (ordinary volumetric efficiency assumed) per gal. of water
h = vertical lift, ft.,
H = submergence of water pipe, ft.,
C = a constant from Table 11.

It must be borne in mind that the working water level, when pumping from wells, is commonly below the standing level and by an amount that cannot usually be known in advance. It is hence customary to assume certain conditions of lift and submergence based on experience and pipe the wells accordingly. After the piping is installed and working conditions arrived at, the submergence

TABLE 10.—DISCHARGE OF COMPRESSED AIR THROUGH ORIFICES

By The Ingersoll-Rand Company

Receiver gage pressure, lbs. per sq. in.	Diameter of orifice, ins.													
	1/64	1/32	1/16	1/8	1/4	3/8	1/2	5/8	3/4	7/8	1	1 1/4	1 1/2	2
	Discharge, cu. ft. of free air per min.													
2	.038	.153	.647	2.435	9.74	21.95	39	61	87.60	119.50	156	242	350	625
5	.0597	.242	.965	3.86	15.40	34.60	61.60	96.50	133	189	247	384	550	985
10	.0842	.342	.136	5.45	21.8	49	87	136	196	267	350	543	780	......
15	.103	.418	1.67	6.65	26.70	60	107	167	240	326	427	665	960	......
20	.119	.485	1.93	7.7	30.8	69	123	193	277	378	494	770	......	......
25	.133	.54	2.16	8.6	34.5	77	138	216	310	422	550	860	......	......
30	.156	.632	2.52	10	40	90	161	252	362	493	645	1000	......	......
35	.173	.71	2.80	11.2	44.7	100	179	280	400	550	715	......	......	......
40	.19	.77	3.07	12.27	49.09	110.45	196.35	306.80	441.79	601.32	785.40	......	......	......
45	.208	.843	3.36	13.4	53.8	121	215	336	482	658	860	......	......	......
50	.225	.914	3.64	14.50	58.2	130	232	364	522	710	930	......	......	......
60	.26	1.05	4.2	16.8	67	151	268	420	604	622	......	......	......	......
70	.295	1.19	4.76	19	76	171	304	476	685	930	......	......	......	......
80	.33	1.33	5.32	21.2	85	191	340	532	765	1004	......	......	......	......
90	.364	1.47	5.87	23.50	94	211	376	587	843	......	......	......	......	......
100	.40	1.61	6.45	25.8	103	231	412	645	925	......	......	......	......	......
125	.486	1.97	7.85	31.4	125	282	502	785	......	......	......	......	......	......

TABLE 11.—VALUES OF C IN FORMULA FOR AIR-LIFT PUMPS. PROPER SUBMERGENCE ASSUMED

h = Lift.	C
10 ft. to 60 ft. inclusive	245
61 ft. to 200 ft. inclusive	233
201 ft. to 500 ft.	216
501 ft. to 650 ft.	185
651 ft. to 750 ft. inclusive	156

is altered to suit by raising or lowering the pipe until the best rates are established.

The necessary percentage of submergence varies with the lift. Meaning by percentage of submergence the percentage of the total length of pipe submergeed when pumping, the range, according to the Ingersoll-Rand Co., is as follows:

For a lift of 20 ft. . . . . .66 per cent.
For a lift of 500 ft. . . . . .41 per cent.

The average best percentage in the class of work usually encountered will lie between 50 and 65 per cent.

The air pressure required does not depend upon the lift h but upon the submergence H and is greater when the pump is being started than when at work, because the submergence is greater with the water at the standing level. The starting pressure must slightly exceed the pressure due to the submergence or, say:

$$\text{starting pressure} = .44\ H$$

the pressure being in lbs. per sq. in. and the submergence in ft. The working pressure is equal to the working submergence multiplied by the same constant, but, as has been said, the working submergence is frequently unknown in advance.

It is important that the pipes be proportioned to the flow, because with too large a pipe the air rises through the water without doing all the work it should, while, with too small a pipe, the results are undue friction loss and inefficient expansion of the air bubbles. According to the Ingersoll-Rand Co., the proper dimensions and capacities of the arrangement shown in Fig. 22, which is the most economical and should be used when the well is sufficiently large, are as given in Table 12.

TABLE 12.—DIMENSIONS AND CAPACITIES OF AIR-LIFT PUMPS OF THE TYPE SHOWN IN FIG. 22

Air pipe, ins.	Water pipe, ins.	Size of well, ins.	Maximum economical capacity on moderate lift, gals. per min.
½	1	3	7
¾	1¼	4	20
1	2	4½	35
1	2½	5	60
1¼	3	6	90
1½	3½	7	120
1½	4	8	160
1½	5	9	250
2	6	10	350

In case of necessity these capacities can be increased 20 to 40 per cent. but at a decreased efficiency.

The arrangement shown in Fig. 23 is used to obtain the greatest possible output from a given size of well casing. It is not always

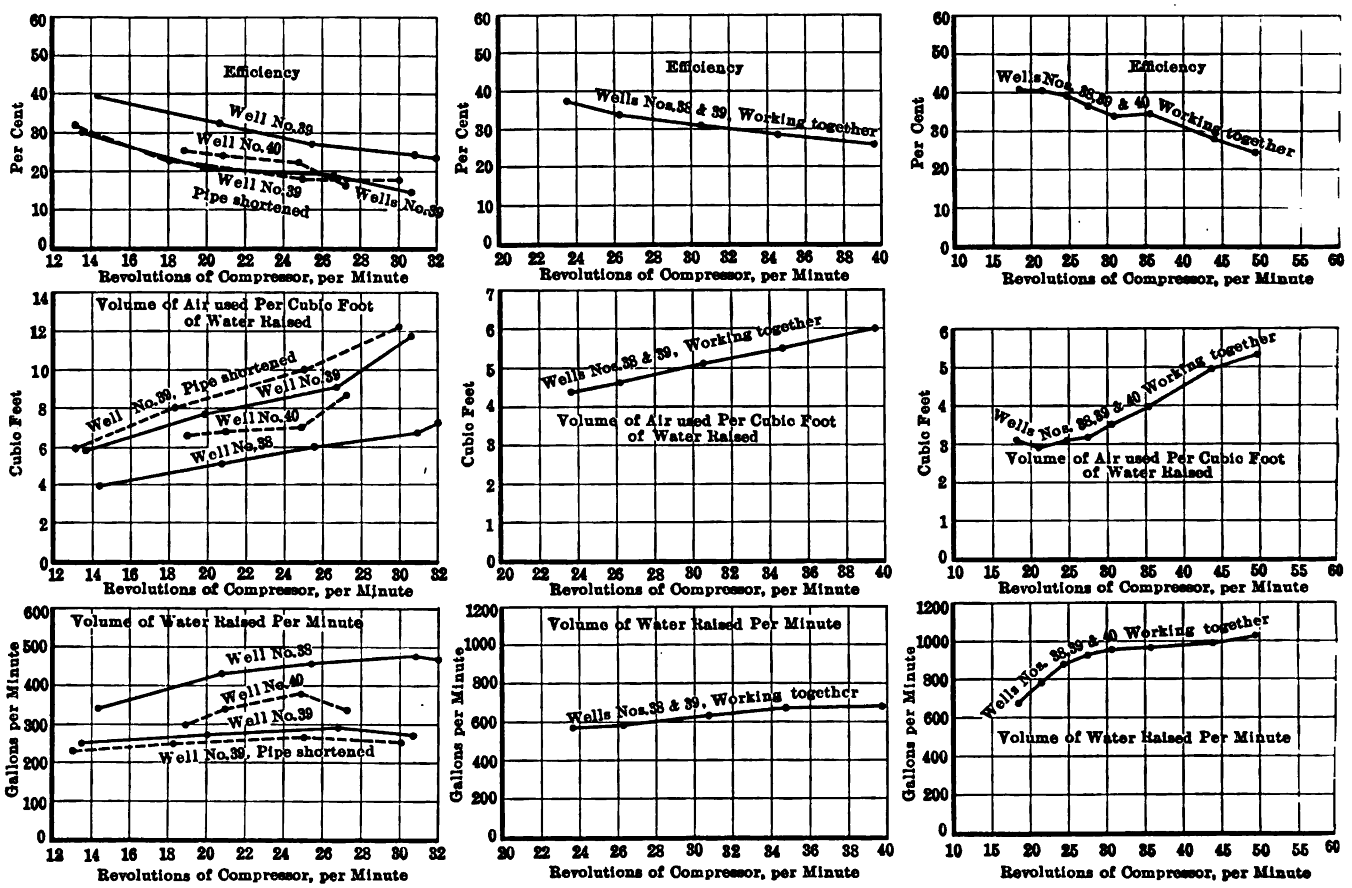


FIG. 26. FIG. 27. FIG. 28.

FIGS. 26 to 28.—Results of air lift pump tests by Mr. Kelly.

as economical as the arrangement shown in Fig. 22 but may be used when the well is very strong and a great deal of water is wanted from a few wells. According to the Ingersoll-Rand Co., the proper size of air pipe for the different sized casings and the capacities to be expected are about as given in Table 13.

TABLE 13.—DIMENSIONS AND CAPACITIES OF AIR-LIFT PUMPS OF THE TYPE SHOWN IN FIG. 23

Casing, ins.	Air pipe, ins.	Capacity, gals. per min.
3½	1¼	80 to 100
4	1½	100 to 150
5	2	150 to 250
6	2	275 to 375
8	2½	500 to 650
10	2½	775 to 1000

The efficiency of the air-lift pump is not high, but for many uses this is more than offset by its remarkable simplicity and consequent freedom from derangement and by its capacity to deliver all the water a well will supply—a capacity which is shared by no other deep-well pump.

The most complete set of test data known to the author are those of JAMES KELLY (*Proc. I. C. E.* 1906). Two arrangements, shown in Figs. 24 and 25 were tested, the dimensions of the pumps being given in Table 14, while the results are given in Figs. 26, 27 and 28. When consulting these data it should be remembered that the water was measured in British gallons. The figures for efficiency give the ratio of the work done in raising water to the work indicated in the air cylinders of the compressor. The compressor used was a compound Ingersoll-Rand having air cylinders of 28½ and 16¾ in. diameter by 24-in. stroke.

TABLE 14.—DIMENSIONS OF AIR-LIFT PUMPS TESTED BY MR. KELLY

Number of well	Depth, ft.	Diameter, ins.	Area of delivery, sq. in.	Effection area of air-tube, sq. ins.	Depth of delivery, ft.	Distance from comp. ft.
38	350	12	19.63	18.75	339.5	600
39	350	12	12.56	16.2	347.0	820
40	350	12	12.56	3.1416	326.5	5400

The mechanical advantage or increase of force due to the "mechanical powers"—lever, pulley, wheel and axle, inclined plane, wedge and screw—whether used singly or in combination, is the inverse ratio of the velocity of the applied force (power) and of the resisting force (weight). To determine the mechanical advantage it is only necessary to determine the velocities at the beginning and ending of the train of mechanism, when:

$$\frac{\text{power}}{\text{weight}}=\frac{\text{velocity of weight}}{\text{velocity of power}}$$

Such calculations assume ideal conditions, of course, that is, they ignore the losses due to friction.

Differential mechanisms are seldom successful because of a feature that is commonly overlooked. This feature was first pointed out by GEO. B. GRANT, (*Amer. Mach., Sept.* 10, 1895). Mr. Grant discussed differential gearing only, but the cause of failure of such gearing appears to be general and to operate against the success of most applications of the differential principle. The cause of failure of differential gears, as pointed out by Mr. Grant, is that the teeth operate under a combination of the heavy pressure of the driven gear and the high speed of the driving gear, this combination leading to destructive wear and to such low efficiency that the mechanical advantage for which differential mechanisms are usually designed is not realized.

If the reader will reflect a moment he will see that this combination of heavy pressure and high speed is common to all differential mechanisms, of which about the only successful example is the differential pulley block. The exception of this construction to th general law is apparently due to the fact that the bearings subject to the combined heavy loads and high speed are of a type which permits them to be made large enough for the service.

TABLE 1.—VELOCITIES DUE TO HEIGHTS OF FALL
From Clark's Manual of Rules, Tables and Data

This table gives also the spouting velocities of water—the column for height being read as head.

Height, ft.	Velocity, ft. per sec.	Height, ft.	Velocity, ft. per sec.	Height, ft.	Velocity, ft. per sec.	Height, ft.	Velocity, ft. per sec.
.01	.803	3	13.90	23	38.49	50	56.74
.02	1.14	3.5	15.01	24	39.31	100	80.25
.03	1.39	4	16.05	25	40.12	150	98.28
.04	1.61	4.5	17.03	26	40.92	200	113.5
.05	1.80	5	17.99	27	41.70	300	139
.06	1.97	5.5	18.82	28	42.47	400	160.5
.07	2.12	6	19.66	29	43.22	500	179.9
.08	2.27	6.5	20.46	30	43.95	600	196.6
.09	2.41	7	21.23	31	44.68	700	212.3
.1	2.54	7.5	21.97	32	45.39	800	226.9
.2	3.20	8	22.69	33	46.10	900	240.7
.3	4.40	8.5	23.40	34	46.79	1000	253.8
.4	5.07	9	24.07	35	47.47	1500	310.8
.5	5.68	9.5	24.73	36	48.15	2000	358.9
.6	6.22	10	25.38	37	48.81	2500	401.2
.7	6.71	11	26.62	38	49.47	3000	439.5
.8	7.18	12	27.80	39	50.11	3500	474.7
.9	7.61	13	28.93	40	50.75	4000	507.5
1.0	8.03	14	30.03	41	51.38	4500	538.3
1.2	8.79	15	31.08	42	52.01	5000	567.4
1.4	9.50	16	32.10	43	52.62	6000	621.6
1.6	10.15	17	33.09	44	53.23	7000	671.4
1.8	10.77	18	34.05	45	53.83	8000	717.8
2.0	11.35	19	34.98	46	54.43	9000	761.3
2.25	12.04	20	35.89	47	55.02	10000	802.5
2.50	12.69	21	36.77	48	55.60		
2.75	13.31	22	37.64	49	56.17		

TABLE 2.—HEIGHTS OF FALL DUE TO VELOCITIES
From Clark's Manual of Rules, Tables and Data

This table gives also the heads necessary to produce given spouting velocities of water—the column for height being read as head.

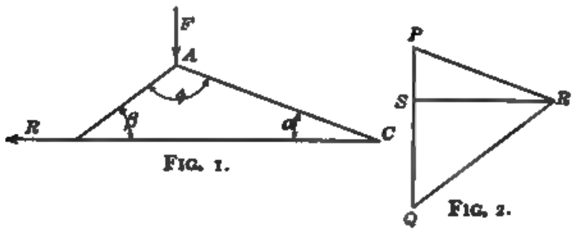
Velocity, ft. per sec.	Height, ft.	Velocity, ft. per sec.	Height, ft.	Velocity, ft. per sec.	Height, ft.	Velocity, ft. per sec.	Height, ft.
.25	.0010	19	5.61	46	32.9	73	82.7
.50	.0039	20	6.21	47	34.3	74	85
.75	.0087	21	6.85	48	35.8	75	88.4
1.00	.016	22	7.52	49	37.3	80	99.4
1.25	.024	23	8.21	50	38.8	85	112.2
1.50	.035	24	8.94	51	40.4	90	125.8
1.75	.048	25	9.71	52	42	95	140.1
2	.062	26	10.5	53	43.6	100	155.3
2.5	.097	27	11.3	54	45.3	105	171.2
3	.140	28	12.1	55	47	110	187.9
3.5	.190	29	13.1	56	48.7	115	205.4
4	.248	30	14	57	50.4	120	223.6
4.5	.314	31	14.9	58	52.2	130	262.4
5	.388	32	15.9	59	54.1	140	304.3
6	.559	33	16.9	60	55.9	150	349.4
7	.761	34	17.9	61	57.8	175	475.5
8	.994	35	19	62	59.7	200	621
9	1.26	36	20.1	63	61.6	300	1397
10	1.55	37	21.3	64	63.6	400	2484
11	1.88	38	22.4	65	65.6	500	3882
12	2.24	39	23.6	66	67.6	600	5890
13	2.62	40	24.9	67	69.7	700	7609
14	3.04	41	26.1	68	71.8	800	9938
15	3.49	42	27.4	69	73.9	900	12578
16	3.98	43	28.7	70	76.1	1000	15528
17	4.49	44	30.1	71	78.3		
18	5.03	45	31.4	72	80.5		

TABLE 3.—HEIGHTS OF FALL AND VELOCITIES DUE TO TIME
From Clark's Manual of Rules, Tables and Data

Time, sec.	Height, ft.	Velocity, ft. per sec.	Time, sec.	Height, ft.	Velocity, ft. per sec.
1	16.1	32.2	17	4,653	547.4
2	64.4	64.4	18	5,217	579.6
3	144.9	96.6	19	5,812	611.8
4	257.6	128.8	20	6,440	644
5	402.5	161	21	7,100	676.2
6	579.6	193.2	22	7,792	708.4
7	788.9	225.4	23	8,517	740.6
8	1030	257.6	24	9,273	772.8
9	1304	289.8	25	10,062	805
10	1610	322	26	10,884	837.2
11	1948	354.2	27	11,737	869.4
12	2318	386.4	28	12,622	901.6
13	2721	418.6	29	13,540	933.8
14	3156	450.8	30	14,490	966
15	3623	483	31	15,473	998.2
16	4122	515.2	32	16,487	1030

The thrust of a toggle joint may be determined graphically as in Figs. 1 and 2, the force F being applied at A, Fig. 1, and the thrust at B or C.

In the diagram, Fig. 2, make the perpendicular PQ of such length as to represent the applied force F and draw PR parallel to AC and QR parallel to AB. The lengths of PR and QR will then represent



FIGS. 1 and 2.—Forces in a toggle joint.

the stresses in the links AC and AB, respectively, while the horizontal SR represents the thrust.

Trigonometrically we have:

$$R = F\frac{\cos\alpha\cos\beta}{\sin\phi}$$

If the joint has equal arms this becomes

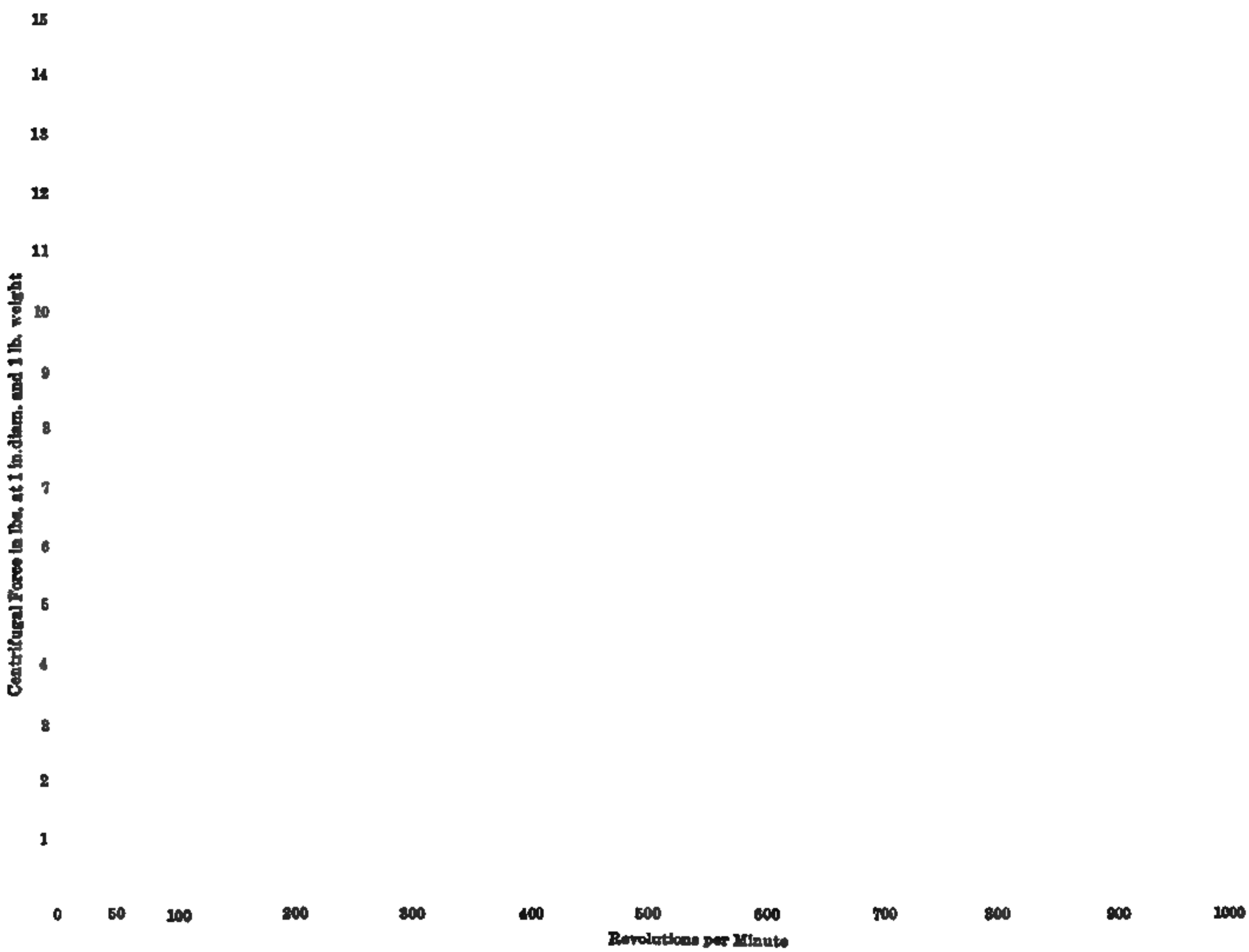
$$R = \frac{F}{2\tan\alpha}$$

The laws of falling bodies starting from a state of rest, are expressed by the equations:

$$v = 32.2\,t$$
$$v = \sqrt{64.4\,h}$$
$$= 8\sqrt{h} \text{ nearly}$$
$$h = 16.1\,t^2$$
$$t = \frac{v}{32.2}$$
$$t = \sqrt{\frac{h}{16.1}}$$
$$= \tfrac{1}{4}\sqrt{h} \text{ nearly}$$

in which v = acquired velocity, ft. per sec.,
t = time of fall, sec.,
h = height of fall, ft.

These relations of velocity, height and time are tabulated in Tables 1, 2 and 3.



Trace vertically from r.p.m. to the curve and then to the left and multiply the quantity found by the diameter, ins. and by the weight, lbs.

FIG. 4.—Centrifugal force.

The laws of motion of bodies acted upon by uniform accelerating forces are expressed by the equations:

$$v_2 = v_1 + 32.2\frac{P}{G}t$$

$$S = v_1t + 16.1\frac{P}{G}t^2$$

in which v_2 = final velocity, ft. per sec.

v_1 = initial velocity, if any, ft. per sec. (if the body starts from rest $v_1 = 0$),

P = force acting, lbs.,

G = weight of body, lbs.,

t = time during which force acts, sec.,

S = space passed through, ft.

If the force is a retarding force these become:

$$v_2 = v_1 - 32.2\frac{P}{G}t$$

$$S = v_1t - 16.1\frac{P}{G}t^2$$

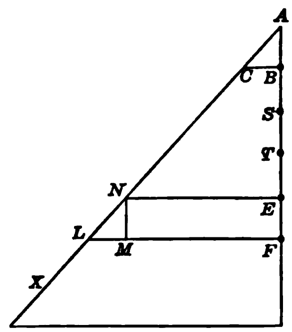


FIG. 3.—Graphical solution of problems in accelerated motion.

Problems involving the laws of uniformly accelerated motion, as falling bodies, may be solved by drawing a diagram similar to Fig. 3. On the vertical line AY lay off equal distances representing seconds, AB being unity. Make BC equal to the acceleration—32.2 ft. per sec. for gravity—and draw AX. Then, after five seconds, for example, LF = velocity, area ALF = the distance traversed and area $LFEN$ = distance traversed during the last second.

If the acceleration is not uniform and its law is known, AX may be drawn to represent it, the construction being otherwise the same.

The energy stored in moving bodies is expressed by the equation:

$$E = \frac{Gv^2}{64.4}$$

in which E = stored energy, ft. lbs.,

G = weight of body, lbs.,

v = velocity of body, ft. per sec.

The additional energy stored by an increase of velocity of a moving body is expressed by the equation:

$$E = \frac{G}{64.4}(v_2^2 - v_1^2)$$

in which E = energy stored by the increase of velocity, ft.-lbs.,

G = weight of body, lbs.,

v_1 = initial velocity of body, ft. per sec.,

v_2 = final velocity of body, ft. per sec.

The energy given out by a reduction of velocity of a moving body is expressed by the same equation with the notation of v_1 and v_2 inverted.

The centrifugal force of revolving bodies is expressed by the equation:

$$P = .0003399n^2Gr$$

in which P = centrifuagl force, lbs.,

n = revolutions per minute.,

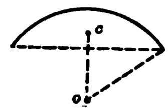
G = weight of body, lbs.,

r = radius of gyration or with sufficient accuracy for most purposes radius of the center of gravity of the body, ft.

Calculations of centrifugal force may be abridged by the use of Fig. 4, by N. J. HOPKINS (*Amer. Mach., Feb.* 10, 1898).

For the stress in a revolving ring due to centrifugal force, see Fly-wheel.

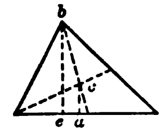
The center of gravity of many plane figures is obvious at a glance, being the same as the center of area. Following are formulas for some other figures of common occurrence, the point c being the center of gravity in all cases.



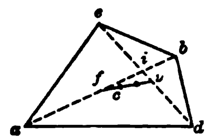
Circular arc: $oc = \dfrac{\text{chord} \times \text{radius}}{\text{arc}}$

Simicircle: $oc = .6366 \times \text{radius}.$

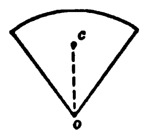
For tabulated lengths of circular arcs, see Circular Arcs.



Triangular area: Bisect two sides and connect the dvisioin points with the opposite angles. The intersection c is the center of gravity. Dropping a perpendicular ca to any side $ca = \frac{1}{3} \times \text{altitude } be.$



Any quadrilateral: Draw the diagonals ab, de; bisect ab at f; make $eg = di$; join f and g and divide fg into three equal parts; c is the center of gravity.

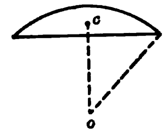


Circular sector: $oc = \dfrac{2 \times \text{chord}}{3 \times \text{arc}} \times \text{radius}.$

Semicircle: $oc = .4244 \times \text{radius}.$

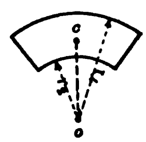
Quadrant: $oc = .6002 \times \text{radius}.$

For tabulated lengths of circular arcs see Circular Arcs.



Circular segment: $oc = \dfrac{\text{chord}^3}{12 \times \text{area}}$

For tabulated areas of segments see Areas of Circular Segments.

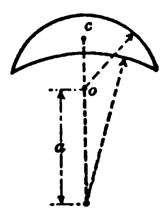


A portion of a circular ring

$$oc = \frac{2}{3}\left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right) \times \frac{\text{outer arc}}{\text{outer chord}}$$

For tabulated lengths of circular arcs, see Circular Arcs.

Circular crescent $oc = \dfrac{A_1a}{A_2}$



in which A = area of segment bounded by arc of smaller radius and common chord,

A_1 = area of segment bounded by arc of larger radius and common chord,

A_2 = area of crescent = $A - A_1$

a = distance between centers of the two arcs.

For tabulated values of areas of segments, see Areas of Circular Segments.

The center of gravity of irregular figures may be found experimentally by the method shown in Fig. 5 as follows:

Trace the figure upon heavy paper or card-board and cut it out. Suspend the figure thus made from a pin placed near the edge of the figure at A, allowing it to hang freely in a vertical plane. Suspend a plumb-line from the pin and draw upon the figure a line coincident with the position of this plumb-line. Suspend the figure from another point B and find a similar line. Where the two lines intersect is the center of gravity of the surface.

The center of gravity of irregular figures may be found graphically by the method shown in Fig. 6 by F. H. HUMMEL (*Proc. Brit. C. & M. E. S.*, 1900).

The problem usually takes the form of finding a line in a given direction passing through the center of gravity of a given section. Let the direction be OY. Draw a line OY in this direction and touching the base of the figure. If the figure is curved at the bottom the line OY must be a tangent at the lowest point of the figure. Next draw an axis OX at right angles to this. In most practical problems the section will be symmetrical, and in this case the line OX is naturally taken along the axis of symmetry, and the construction

FIG. 5.—The center of gravity of irregular figures.

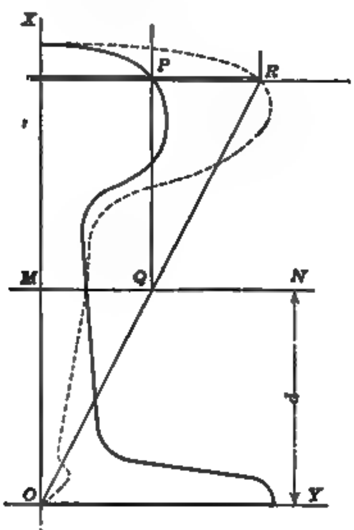


FIG. 6.—The center of gravity of irregular figures.

has then to be made for one-half of the figure only; if it is not symmetrical the axis OX can be drawn in any convenient position, and the following construction must be applied to each side of the figure. Draw a line MN parallel to OY about half-way up the figure, and at some even distance d from OY.

Next draw a series of lines PR parallel to OY. In straight parts of the section, such as the web in the figure, these can be wide apart, but where the section changes rapidly, they must be drawn closer together in order that the final curve may be quite definite. At the point P, where one of these lines cuts the section, draw a line PQ parallel to OX, intersecting MN at Q. Join O and Q and produce to R on the line PR originally drawn. R is a point on the curve we are finding. Repeat this for each of the series of lines and connect points R so found by a curve (dotted in the figure).

Then if the area of the original figure, most conveniently found by a planimeter, equals A, and the area of the new dotted figure equals G, the distance of the center of gravity from OY along OX is

$$x_0 = \frac{G}{A} d$$

If OX is not an axis of symmetry the area G must be taken as the sum of the areas of the new curves obtained for both sides, and A must be the area of the entire section.

The moments of inertia of irregular sections may be obtained from Fig. 7 by O. A. THELIN (*Amer. Mach., Aug.* 15, 1907).

The chart can be used in computing the moments of inertia of simple sections by dividing them into rectangles and computing each one separately. For more complicated sections, irregular in shape, divide into a number of equivalent rectangles and compute each one separately. The curve of the chart has been constructed to represent the formula for the moment of inertia of a rectangle about its side:

$$I = \frac{bh^3}{3}$$

in which I = moment of inertia,
b = breadth,
h = height.

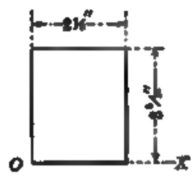


FIG. 8.

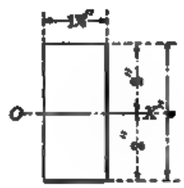


FIG. 9.

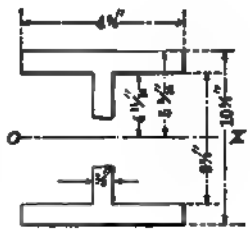


FIG. 10.

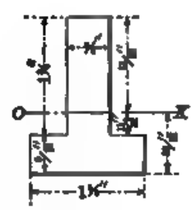


FIG. 11.

12.

FIGS. 8 to 12.—Illustrations of the use of the chart for computing moments of inertia.

In the chart the rectangle has been considered of unit width, or 1 in. The hight above the axis OX, Fig. 8, is measured on the left-hand vertical scale and varies from $\frac{1}{16}$ in. to 6 ins. The horizontal scale gives the value of I for each $\frac{1}{32}$-in. increase in the height of the unit section. The curve has been drawn in steps in order to make the horizontal or I values more definite. Should the vertical dimensions of a section run into sixty-fourths of an in., a middle point between the values for thirty-seconds can easily be read off on the curve.

As the values of I for the unit section become very small below 1 in. height, the curve is enlarged 10 times for vertical dimensions between 1 in. and $\frac{1}{2}$ in., and 100 times for such dimensions below $\frac{1}{2}$ in. in order to give accurate results. For large sections, where the distance from the neutral axis to the extreme fiber exceeds 6 ins., but is less than 12 ins., the chart may be used with following modifications: Divide all height dimensions of the section to be figured by 2. Calculate the moment of inertia from the chart, write these new dimensions and multiply the result by 8, which will give I for the original section.

A few examples will best show the method of using the chart and the advantage it has over formulas in the saving of time.

For the section, Fig. 8:

From the chart for $3\frac{5}{16}$ ins. on the vertical scale the corresponding value on the horizontal scale is 12.11.

Then $I = 2.5 \times 12.11 = 30.27$.

For the section, Fig. 9:

$$\frac{I}{2} = 1.875 \times 2.67 = 5.$$

$$I = 10.$$

For the section, Fig. 10:

$$\frac{I}{2} = 4.75\,(49.99 - 34.33) + .5 \times 34.33 = 91.55.$$

$$I = 183.1.$$

For the section, Fig. 11:

$$I = 1.625\,(.92 + .0135) + 1.625\,(.248 - .0135) = .964.$$

For the section, Fig. 12:

$$\frac{I}{2} = \frac{1}{3}(.248 - .082) + 2(.248 - .123) + (.248 - .0415) + (.248 - .0345) + (.107 - .0175) + (.082 - .0052) + .059 + .0314 + .0224.$$

$$I = .2788.$$

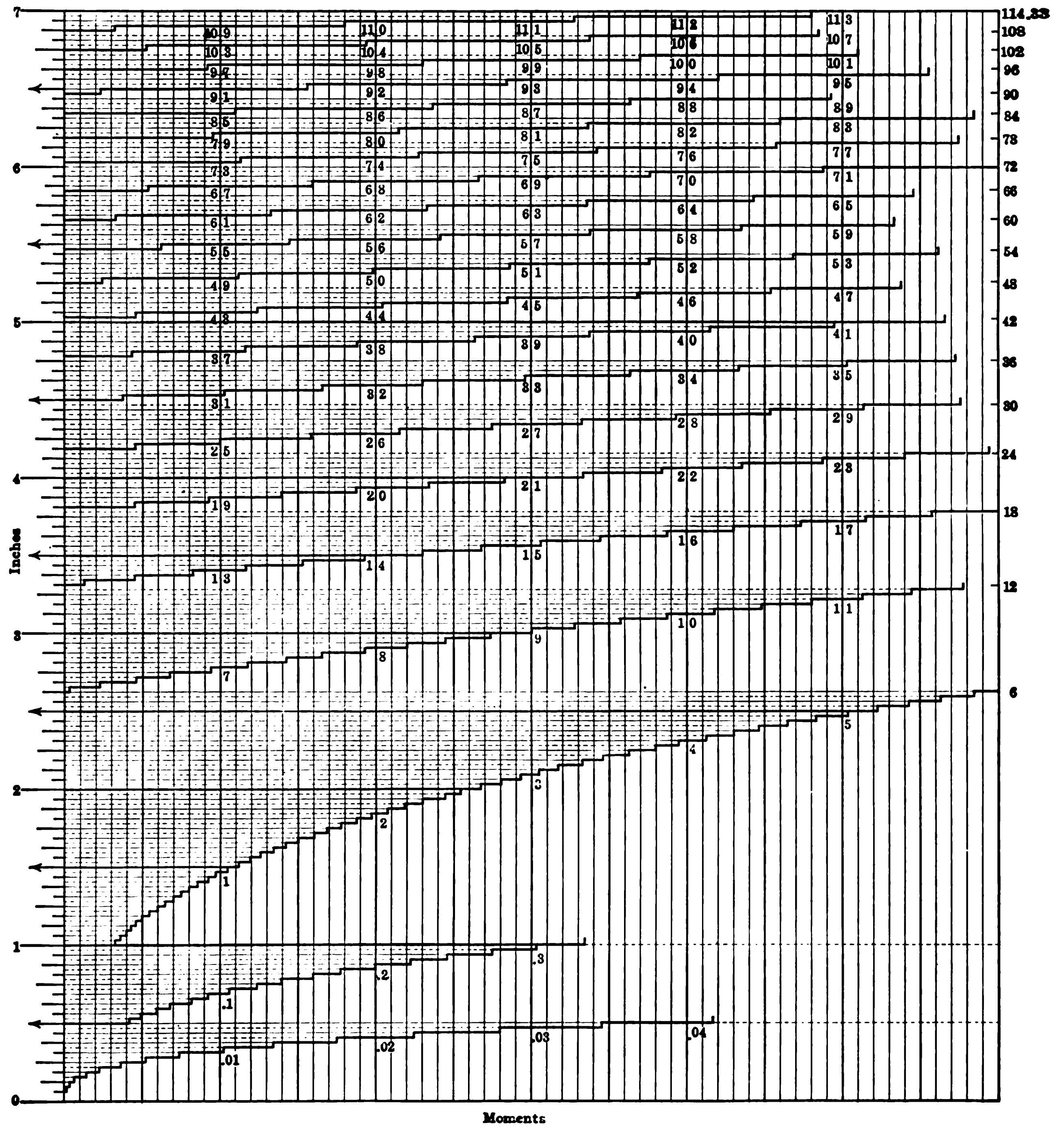


Fig. 7.—Moments of inertia of irregular figures.

Another graphical method of finding the movements of inertia of irregular figures is shown in Fig. 13, by F. H. HUMMEL (*Proc. Brit. C. & M. E. S.*, 1900).

OY is the axis about which the moment of inertia is required, and XOX_1 is a line at right angles, if possible an axis of symmetry for the same reason as in the construction of Fig. 6. Above and below OY two lines, AB and CD are drawn parallel to it and situated at some exact distance d from it. The axis OY usually divides the length of the section KL unequally, and about the best value for d is the length of the shorter of these segments OK in the figure. These lines AB and CD may or may not cut the section. Next draw a series of lines JG parallel to OY across the section, and in each case set off $CE=OJ$. Join EF, and produce if necessary to cut CD in H. Join H to O, cutting JG in G. (In this case OH is produced.) Then G is a point on the required curve.

Repeat this for each of the lines drawn across the section for lines below OY using the lower parallel AB, and join all the points so found by a curve (dotted in the figure).

Let the total area of this dotted curve $=A$; then moment of inertia of section about $OY=Ad^2$.

The areas of irregular figures may be found with sufficient accuracy for many purposes by the method shown in Figs. 14 and 15, by F. HOWKIN'S (*Amer. Mach., Nov.* 14, 1905).

To find the area of the figure shown in Fig. 14, make a tracing on thin paper and fold it along the line 2—2, adjusting it so that the areas on each side balance one another, the position when folded being shown in Fig. 15, in which, as nearly as may be, area $a=$ area b. Next open the tracing and fold it along the line 3—3, again adjusting it so that the excess and deficiency areas of the lower half balance one another, the result being that each section of the lower half represents one quarter of the original area and it only remains to find the area of one of these sections and multiply it by four to obtain the total area. This can readily be done by adopting the same principle and folding the paper on the line 4—4, making area $c=$ area $d+e$ and giving the equivalent triangle.

In the example given the two sides of the triangle measure respectively 1.85 and 1.30 ins. Instead of multiplying the area of this triangle by four we multiply the two dimensions together and then multiply by two, which will, of course, give the same result:

$$1.85\times1.30=2.4050$$
$$2.4050+2=4.81$$
$$=\text{area of original figure.}$$

The areas of irregular figures may be found by Simpson's rule which, considering its simplicity and accuracy, deserves to be more widely used than is the case. The rule is thus expressed by ANTONIO LLANO (*Amer. Mach., March.* 4, 1909):

Divide the base into an even number of equal parts, and determine the ordinates at the points of division. Form the sum of the end ordinates, four times the intermediate even ordinates, and twice the intermediate odd ordinates. Multiply this sum by one-third the distance between two consecutive ordinates. The result will be the required area.

The indicator card, Fig. 16, will serve as an illustration of the use of the rule. The length, 3.5 ins., has been divided into 14 equal parts and the intercepted or enclosed ordinates, when measured, have been found to have the lengths (ins.) marked in the figure, the value of the right-hand end ordinate being zero. The bredth of each space is therefore $\frac{3.5}{14}$ and one-third of this, as called for by the rule, is $\frac{3.5}{3\times14}$. The area of the figure is therefore:

$$\frac{3.5}{3\times14}[.9+0.+4(1.40+1.61+1.46+.97+.69+.50+.32)+2(1.59+1.52+1.20+.81+.59+.40))]=\frac{3.5}{3\times14}(.9+4\times6.95+2\times6.11)$$

$$=\frac{3.5}{3\times14}\times40.92$$
$$=3.409\text{ sq. ins.}$$

The mean ordinate is equal to this area divided by the length of the diagram, that is

$$\frac{3.409}{3.5}=.974\text{ in.}$$

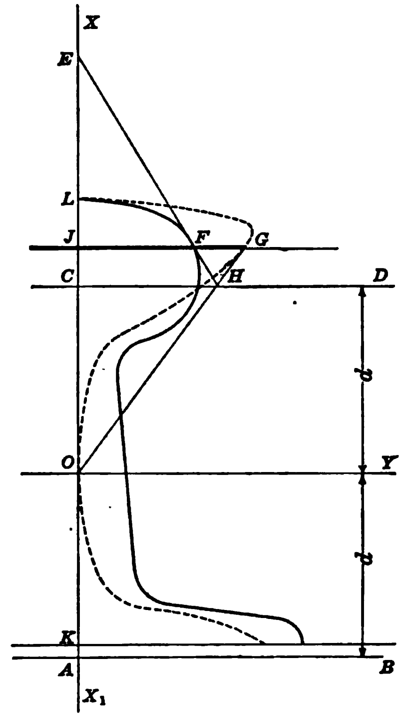


FIG. 13.—The moment of inertia of irregular figures.

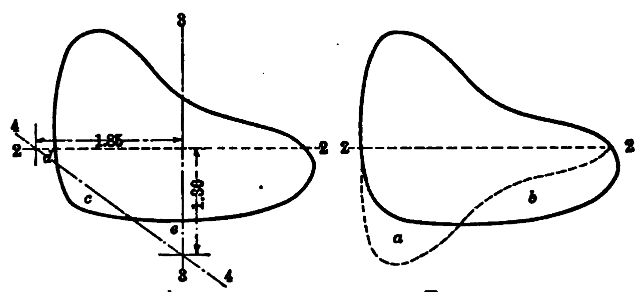


FIG. 14. FIG. 15.

FIGS. 14 and 15.—The area of irregular figures.

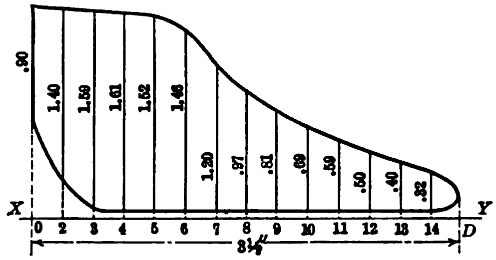


FIG. 16.—Simpson's rule applied to finding the area of indicator cards.

Since in the above we have multiplied the value of the quantity within the parenthesis, 40.92, by 3.5 when getting the area, only to

divide by it again in getting the mean ordinate, we may omit both multiplication and division and find the mean ordinate thus:

$$\frac{40.92}{3\times 14}=.974 \text{ ins., as before.}$$

If the scale of the indicator spring is 30 lbs. per in., the mean pressure is $.974\times 30=29.22$ lbs. per sq. in.

In the practical application of the rule, it is neither convenient nor accurate, when the ordinates have to be determined by actual measurement, to measure every ordinate separately. **The best way** to proceed is to mark on the edge of a long piece of paper, in succession, all the even ordinates, then measure the distance between the first and the last mark, and multiply the result by 4. Similarly for the first and last ordinate, and for the odd ordinates.

A more common method of finding the areas of indicator cards is to divide the card into strips (usually 10) as with Simpson's rule, estimate the mean ordinates by the eye, add these mean ordinates

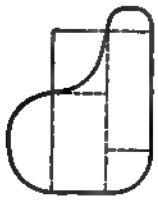


FIG. 20.

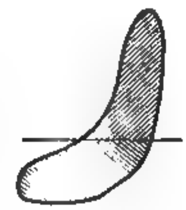


FIG. 22.

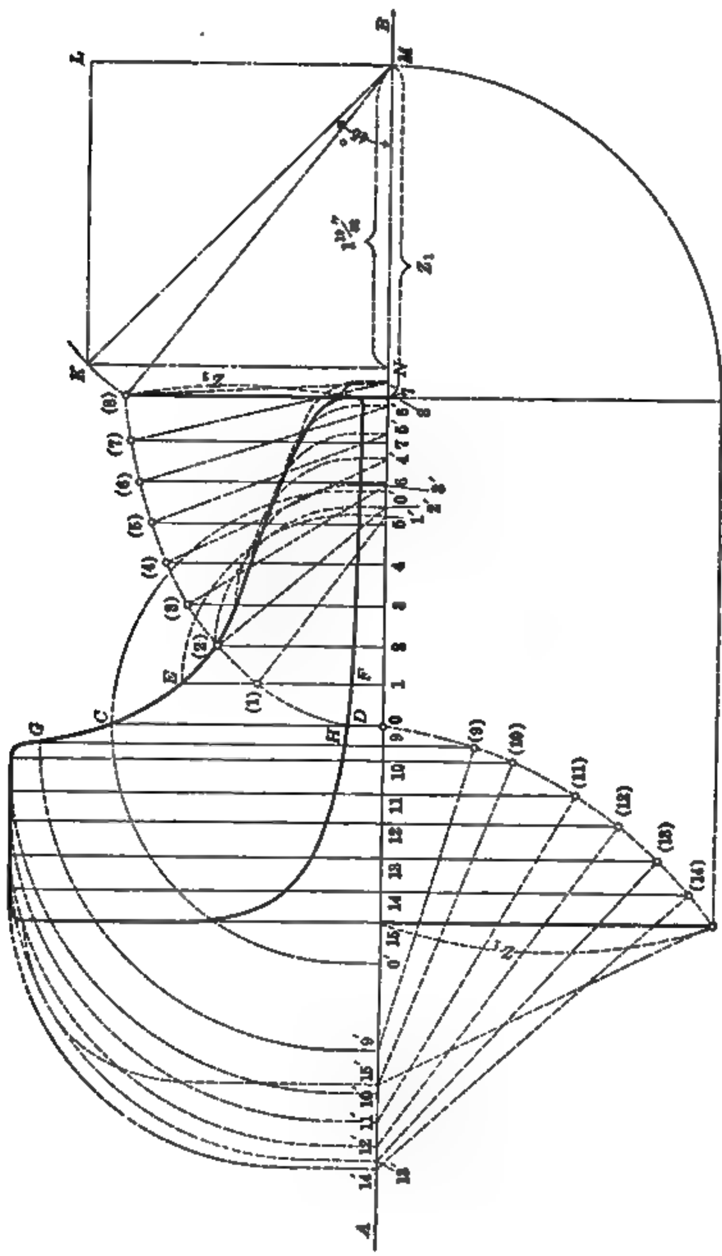


FIG. 17.

FIGS. 17 to 22.—Wiener's method of finding the area of irregular figures.

by a strip of paper as described, and divide the sum by 10. The most accurate method of locating the mean ordinate is to lay a thread horizontally across the top of each strip, equalizing the triangular areas between thread and curve and thus locating the mean ordinate.

A. E. Wiener's graphical method of finding the areas of irregular figures, which when published (*Amer. Mach., May* 19, 1898) was received with the warmest expressions of approval, is illustrated in Figs. 17–22. The complete explanation in connection with Fig. 17 leads to a multiplicity of lines and to an apparent complexity which is apparent only. The real simplicity of the method will be apparent from a glance at Figs. 18 and 19 which include all the lines drawn in actual applications.

A horizontal line AB, Fig. 17, is drawn at a convenient distance from the area to be measured, and a vertical line CD is placed at such a point as to approximately bisect that area (this is not to say that the line CD shall be about half way between the ends of the given figure, but that it shall divide it into two parts of nearly equal contents). Next a number of vertical lines are laid across the figure, close together in regions where the width of the figure undergoes rapid changes, and farther apart where it varies but gradually. At the right of CD these lines are drawn from the horizontal AB upward, while on the left hand they are extended some distance below it. The width CD is then laid off from o (the point of intersection of CD with AB) to both the right and the left, giving two points o′ upon AB, for both of which the condition $oo' = CD$ is fulfilled. For all ordinates at the right-hand side of CD, the respective widths of the irregular figure are similarly laid off on AB to the right, and for the ordinates on the left hand of CD they are swung over to the left; thus, for instance, $11' = EF$; $99' = GH$, etc. In this manner the points, 1′, 2′, 3′ . . . . . 15′ upon AB are obtained. With o′ as center, an arc is drawn from o to half-way between ordinates o and 1; this arc is continued with 1′ as center to midway between ordinates 1 and 2, at which point the center is again changed to 3′, and so on to the last ordinate 8, when the point (8) is obtained, the length 8, (8) being the end value marked Z_2. The latter is combined with the other end value 15, (15) marked Z_1 constructed in the same manner, into a right-angled triangle (8), 8, M. From M a line is drawn at an angle of 45 deg. with AB, and MK made equal to M (8). The area of the square $KLMN$ thus found, having sides of $1\frac{13}{16}$ ins. length, is 2.54 sq. ins., while the given plane, accurately measured by means of a planimeter, was found to contain 2.55 sq. ins., these figures for the planimeter being the average of 5 careful measurements. The result obtained by the graphical method, consequently, differs from the actual area by an amount of only 4–10 of 1 per cent., and could have been approached still closer if a greater number of ordinates had been taken.

While Fig. 17 has all the construction-lines dotted in for sake of explanation, in practice neither the quadrants determining the centers 1′, 2′, etc., nor the radii of the arcs forming the auxilliary curve need be shown, all that is necessary being to draw the ordinates, mark off the corresponding centers by means of a pair of dividers, and by the use of a compass find directly the lengths of the end ordinates, the geometrical mean of which (found by transforming the right-angled triangle having the end-values as sides into an isosceles right-angled triangle having the same hypothenuse) is the side of the required square.

In Figs. 18 and 19 two examples are executed in this manner, showing more strikingly the simplicity of the method. In Fig. 18 the horizontal axis cuts across the given figure, being placed through the point in which the bisecting ordinate leaves the figure; and in Fig. 19 advantage is taken of the shape of the area, and the axis placed in one of its boundaries. Figs. 18 and 19 also show the difference in effect of putting the bisecting ordinate to the right, or left, respectively, of its accurate position, in the former case the left-end value being greater than the right and the square found being some distance to the right of the given figure, while in the latter case the right-end value is greater than the left end, and the irregular figure is overlapped by its square of equal area.

Trial diagrams composed of straight lines, semi-circles, and quadrants which could be easily checked by calculation, as in Fig. 20, have been treated by this method with the result that the error seldom equals $\frac{1}{2}$ per cent. The error due to the displacement of the middle ordinate depends on the shape of the curve. If its height is uniform for some distance each side of the bisecting ordinate, as in Fig. 21, the error due to displacement of that ordinate is slight, while if a large change in this height occurs at this point, as in Fig. 22, the error is greater. The error due to the displacement of the meeting points of the arcs also varies with circumstances. If these arcs meet at a considerable angle, the error due to displacement of the meeting point is considerable, but if the arcs are more nearly tangent, this error is less.

STRENGTH OF MACHINE PARTS

For the strength of shafts, see Shafts.

For the strength of springs, see Springs.

For the strength of steam boilers, see Steam Boilers.

In a large percentage of cases the formulas for the strength of parts have but an indirect application in machine design.

In the design of a bridge, a roof or a warehouse floor, the ability of the structure to carry the load is the chief requirement, and to insure that it shall do this with safety, even under accidental strains, a factor of safety is introduced; and although the name has been often criticised, it nevertheless represents with a fair degree of accuracy the state of the designer's mind in making the calculations.

In treatises on machine design the same term is used to express the ratio between the actual working strain and the strain which would produce rupture, although there is and can be no such conception in the machine designer's mind in making the calculations. In such parts, for instance, as connecting-rod bolts, straps and keys, the stresses under the working loads will often be found to run down to 3000 lbs. per sq. in., while in engine frames the stresses seldom exceed 500 lbs., and will frequently run down to 300 lbs. per sq. in. With steel of 60,000 lbs. tensile strength, the figure for connecting-rod parts is equivalent to a factor of safety of 20, while for engine frames, cast-iron being assumed to have 20,000 lbs. tensile strength, this goes up to 40 and 70 for the two stresses named. Now, it is certain that no designer of such parts has any conception of a factor of safety, as that term is commonly understood, in his mind when he proportions these parts for such stresses, and the term "factor of safetv" in this connection is absurd.

The purpose of the designer in introducing these low stresses is not to provide a surplus of strength for accidental stresses, but to provide such a degree of stiffness that the parts will not yield unduly under the regular loads of everyday work. He has, in fact, very little thought of strength in the sense of ability to resisit rupture, his whole thought being to make the structure so rigid that the deflection under the working load shall be inappreciable, or at any rate so small as to do no harm. From this point of view the great surplus of strength is rational and understandable, while from the factor of safety standpoint it can not be defended.

A strictly scientific method of machine design would base the dimensions on the formulas for deflection rather than on those for the ultimate strength of the parts. In using the formulas for strength as he does, the designer practically converts them, in a rough and ready way, into formulas for stiffness, which is but the reciprocal for deflection, and so far as methods go, this is probably as far as we shall ever get or as it is practicable to get in most cases. That the allowable deflections under any considerable number of the infinite variety of conditions prevailing in machine construction will ever be determined is not to be expected.

Beams and Columns

The standard formulas for the strength of beams and for the usual section factors, as arranged by the Carnegie Steel Co. are given below:

Let A = area of section, sq. ins.,
l = length of span, ins.,

TABLE 1.—STRENGTH OF THE CHIEF MATERIALS OF MACHINE CONSTRUCTION

Material	Modulus of elasticity		Ultimate strength		Elastic strength	
	Tension compression	Torsion	Tension	Shear	Tension	Shear
Cast-iron (Cupola).	10,700,000 15,000,000	4,000,000 6,000,000	16,000 20,000	16,000 20,000	8,000	8000
Cast-iron (Air-furnace).	... ...	... ...	30,000 40,000	... ...	... ...	... ...
Wrought-iron..	28,000,000	11,000,000	47,000 57,000	35,000 43,000	... ...	... ...
Steel .15 carbon	30,000,000	11,800,000	60,000	45,000	40,000	...
Steel .25 carbon	30,000,000	11,800,000	70,000	52,000	45,000	...
Crucible steel (high carbon).	31,000,000	12,100,000	100,000	...	75,000	...
Steel castings..	30,600,000	11,800,000	50,000 100,000	30,000 60,000	... ...	... ...
Copper castings	12,000,000	...	22,000	...	6,000	...
Copper, rolled.	16,000,000	...	28,500 33,000	... ...	... ...	... ...
Brass cast., yel.	11,400,000	...	22,000	...	...	...
Brass cast., yel., red.	12,800,000	...	28,500	...	...	...
Gun-metal.....	15,400,000	...	42,800	...	...	...
Phosphor-bronze.	...	...	57,000	...	24,000	...

The ultimate strength of cast-iron in compression is 90,000 to 100,000 lbs. per sq. in. Its elastic strength in compression can be assumed as 25,000 lbs. per sq. in. The ultimate compressive strengths of the other materials can be taken as equal to their ultimate tensile strengths without appreciable error.

TABLE 2.—SHRINKAGE OF CASTINGS

Aluminum, pure	.2031 in. per ft.
Aluminum, nickel alloy	.1875 in. per ft.
Aluminum, special	.1718 in. per ft.
Iron, small cylinders	.0625 in. per ft.
Iron pipes	.125 in. per ft.
Iron girders and beams	.100 in. per ft.
Iron large cylinders, contraction of diameter at top	.625 in. per ft.
Iron large cylinders, contraction of diameter at bottom	.083 in. per ft.
Iron large cylinders, contraction of length	.094 in. per ft.
Brass, thin	.167 in. per ft.
Brass, thick	.150 in. per ft.
Copper	.1875 in. per ft.
Bismuth	.1563 in. per ft.
Lead	.3125 in. per ft.
Zinc	.3125 in. per ft.

W = load uniformly distributed, lbs.,
M = bending moment, lbs. ins.,
h = height of cross-section, out to out, ins.,
n = distance of center of gravity of section, from top or from bottom, ins.,
f = stress, lbs. per sq. in. in extreme fibers of beam, either top or bottom, according as n relates to distance from top or from bottom of section,
D = maximum deflection, ins.,
I = moment of inertia of section, neutral axis through center of gravity,
I' = moment of inertia of section, neutral axis parallel to above, but not through center of gravity,
d = distance between these neutral axes,
S = section modulus,
r = radius of gyration, ins.,
E = modulus of elasticity, for steel 29,000,000;

Then: $S=\frac{I}{n}$, $r=\sqrt{\frac{I}{A}}$,

$$M=\frac{fI}{n}=fS,$$

$$f=\frac{Mn}{I}=\frac{M}{S},$$

$$W=\frac{8fI}{ln}=\frac{8f}{l}S,$$

$$f=\frac{Wln}{8I}=\frac{Wl}{8S},$$

$$I'=I+Ad^2,$$

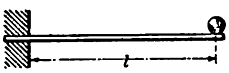
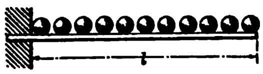
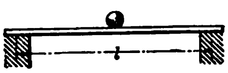
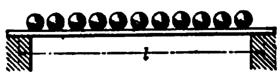
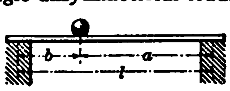
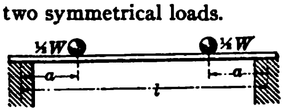
$D=\frac{5Wl^3}{384EI}$ for beam supported at both ends and uniformly loaded,

$D=\frac{Pl^3}{48EI}$ for beam supported at both ends and loaded with a single load P at middle,

$D=\frac{Wl^3}{8EI}$ for beam fixed at one end and unsupported at the other and uniformly loaded,

$D=\frac{Pl^3}{3EI}$ for beam fixed at one end and unsupported at other and loaded with a single load P at the latter end.

TABLE 3.—BENDING MOMENTS AND DEFLECTIONS OF BEAMS UNDER VARIOUS SYSTEMS OF LOADING

W = total load l = length of beam	I = moment of inertia E = modulus of elasticity
(1) Beam fixed at one end and loaded at the other.  Safe load = $\frac{1}{8}$ that given in tables. Maximum bending moment at point of support = Wl. Maximum shear at point of support = W. Deflection = $\frac{Wl^3}{3EI}$.	(2) Beam fixed at one end and uniformly loaded.  Safe load = $\frac{1}{4}$ that given in tables. Maximum bending moment at point of support = $\frac{Wl}{2}$. Maximum shear at point of support = W. Deflection = $\frac{Wl^3}{8\ EI}$.
(3) Beam supported at both ends, single load in the middle.  Safe load = $\frac{1}{2}$ that given in tables. Maximum bending moment at middle of beam = $\frac{Wl}{4}$. Maximum shear at points of support = $\frac{1}{2}W$. Deflection = $\frac{Wl^3}{48EI}$.	(4) Beam supported at both ends and uniformly loaded.  Safe load = that given in tables. Maximum bending moment at middle of beam = $\frac{Wl}{8}$. Maximum shear at points of support = $\frac{1}{2}W$. Deflection = $\frac{Wl^3}{76.8\ EI}$.
(5) Beam supported at both ends single unsymmetrical load.  Safe load = that given in tables $\times\frac{l^2}{8ab}$. Maximum bending moment under load = $\frac{Wab}{l}$ Maximum shears; at support near $a=\frac{Wb}{l}$; at other support $=\frac{Wa}{l}$. Max. deflec. $=\frac{Wab(2l-a)}{9EIl}\sqrt{\frac{1}{3}a(2l-a)}$	(6) Beam supported at both ends two symmetrical loads.  Safe load = that given in tables $\times\frac{l}{4a}$. Maximum bending moment between loads = $\frac{1}{2}Wa$. Maximum shear between load and nearer support = $\frac{1}{2}W$. Max. deflec. = $\frac{Wa}{48EI}(3l^2-4a^2)$.

Explanation of Tables of Safe Loads for I-beams

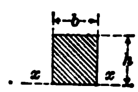
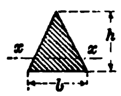
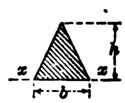
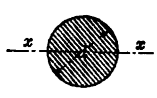
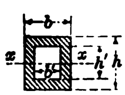
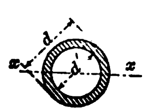
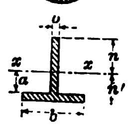
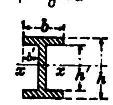
Table 7 for I-beams gives the loads which a beam will carry safely (distributed uniformly over its length) for the distances between supports indicated. These loads include the weight of the beam, which must be deducted in order to arrive at the *net load* which the beam will carry.

For beams of heavier sections than those calculated in the tables, a separate column of corrections is given for each size, stating the proper increase of safe load for every additional pound in the weight per foot of beam. The values given are based on a maximum fiber stress of 16,000 lbs. per sq. in.

It has been assumed in these tables that proper provision is made for preventing the compression flanges of the beams from deflecting sideways. They should be held in position at distances not exceed-

TABLE 4.—MOMENT OF INERTIA, I, AND SECTION MUDULUS, S, FOR USUAL SECTIONS

For methods of finding moments of inertia of irregular section see Moment of Inertia.

Sections	I	S
	$I=\frac{bh^3}{12}$	$\frac{bh^2}{6}$
	$I'=\frac{bh^3}{3}$	
	$I=\frac{bh^3}{36}$	Min. $=\frac{bh^2}{24}$
	$I'=\frac{bh^3}{12}$	
	$I=\frac{\pi d^4}{64}$ $=.0491\,d^4$	$\frac{\pi d^3}{32}$ $=.0982\,d^3$.
	$I=\frac{bh^3-b'h'^3}{12}$	$\frac{I}{.5h}$
	$I=.0491\,(d^4-d'^4)$	$.0982\left(d^3-\frac{d'^4}{d}\right)$
	$I=\frac{b'n^3+bn'^3-(b-b')a^3}{3}$	Min. $=\frac{I}{n}$
	$I=\frac{bh^3-2b'h'^3}{12}$	$\frac{I}{.5h}$.

xx denotes position of neutral axis.

TABLE 5.—MOMENTS OF INERTIA OF RECTANGLES.

Neutral Axis

Depth in ins.	Width of rectangle in ins.						
	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$
6	4.50	5.63	6.75	7.88	9.00	10.13	11.25
7	7.15	8.93	10.72	12.51	14.29	16.08	17.86
8	10.67	13.33	16.00	18.67	21.33	24.00	26.67
9	15.19	18.98	22.78	26.58	30.38	34.17	37.97
10	20.83	26.04	31.25	36.46	41.67	46.87	52.08
11	27.73	34.66	41.59	48.53	55.46	62.39	69.32
12	36.00	45.00	54.00	63.00	72.00	81.00	90.00
13	45.77	57.21	68.66	80.10	91.54	102.98	114.43
14	57.17	71.46	85.75	100.04	114.33	128.63	142.92
15	70.31	87.89	105.47	123.05	140.63	158.20	175.78
16	85.33	106.67	128.00	149.33	170.67	192.00	213.33
17	102.35	127.94	153.53	179.12	204.71	230.30	255.89
18	121.50	151.88	182.25	212.63	243.00	273.38	303.75
19	142.90	178.62	214.34	250.07	285.79	321.52	357.24
20	166.67	208.33	250.00	291.67	333.33	375.00	416.67
21	192.94	241.17	289.41	337.64	385.88	434.11	482.34
22	221.83	277.29	332.75	388.21	443.67	499.13	554.58
23	253.48	316.85	380.22	443.59	506.96	570.33	633.70
24	288.00	360.00	432.00	504.00	576.00	648.00	720.00
25	325.52	406.90	488.28	569.66	651.04	732.42	813.80
26	366.17	457.71	549.25	640.79	732.33	823 88	915.42
27	410.06	512.58	615.09	717.61	820.13	922.64	1025.16
28	457.33	571.67	686.00	800.33	914.67	1029.00	1143.33
29	508.10	635.13	762.16	889.18	1016.21	1143.23	1270.26
30	562.50	703.13	843.75	984.38	1125.00	1265.63	1406.25
31	620.65	775.81	930.97	1086.13	1241.30	1396.46	1551.62
32	682.67	853.33	1024.00	1194.67	1365.33	1536.00	1706.67
33	748.69	935.86	1123.03	1310.20	1497.38	1684.55	1871.72
34	818.83	1023.54	1228.25	1432.96	1637.67	1842.38	2047.08
35	893.23	1116.54	1339.84	1563.15	1786.46	2009.76	2233.07
36	972.00	1215.00	1458.00	1701.00	1944.00	2187.00	2430.00
37	1055.27	1319.09	1582.90	1846.72	2110.54	2374.35	2638.17
38	1143.17	1428.96	1714.75	2000.54	2286.33	2572.13	2857.92
39	1235.81	1544.77	1853.72	2162.67	2471.62	2780.58	3089.53
40	1333.33	1666.67	2000.00	2333.33	2666.67	3000.00	3333.33

ing twenty times the width of the flange, otherwise the stress allowed should be reduced as per Table 6.

TABLE 6.—BEAMS WITHOUT LATERAL SUPPORT

Length of beam	Proportion of tabular load forming greatest safe load
20 times flange width	Whole tabular load
30 times flange width	$\frac{9}{10}$ tabular load
40 times flange width	$\frac{8}{10}$ tabular load
50 times flange width	$\frac{7}{10}$ tabular load
60 times flange width	$\frac{6}{10}$ tabular load
70 times flange width	$\frac{5}{10}$ tabular load

In some instances. *deflection* rather than *absolute strength* may become the governing consideration in determining the size of beam to be used.

Table 8 gives the deflections of Carnegie beams.

The standard test specimens of the American Society for Testing Materials are shown in Fig. 1.

The strength of I-beams with reinforcing plates may be determined by the use of Table 9 or Fig. 2, by C. F. BLAKE (*Amer. Mach., May* 30, 1901). Table 9 gives the section factors of various thicknesses of cover plates when applied to different sizes of beams. The heavy figures are for plates on the compression flanges, and the light figures

TABLE 7.—SAFE LOADS UNIFORMLY DISTRIBUTED FOR STANDARD AND SPECIAL I-BEAMS IN TONS OF 2000 LBS. BY THE CARNEGIE STEEL CO.

Safe loads given include weight of beam. Maximum fiber stress 16,000 lbs. per sq. in.

Distance between supports in ft.	24"I 80 lbs.	Add for every lb. increase in weight	20"I 80 lbs.	20"I 65 lbs.	Add for every lb. increase in weight	18"I 55 lbs.	Add for every lb. increase in weight	15"I 80 lbs.	15"I 60 lbs.	15"I 42 lbs.	Add for every lb. increase in weight
12	77.33	.53	65.18	51.98	.44	39.29	.39	47.14	36.09	26.18	.33
13	71.38	.48	60.16	47.98	.40	36.27	.36	43.51	33.31	24.17	.30
14	66.28	.45	55.87	44.56	.37	33.68	.34	40.40	30.93	22.44	.28
15	61.86	.42	52.14	41.59	.35	31.43	.31	37.71	28.87	20.94	.26
16	58.00	.39	48.88	38.99	.33	29.47	.29	35.35	27.07	19.63	.24
17	54.58	.37	46.01	36.69	.31	27.74	.28	33.27	25.47	18.48	.23
18	51.56	.35	43.45	34.66	.29	26.19	.26	31.42	24.06	17.45	.22
19	48.84	.33	41.17	32.83	.28	24.82	.25	29.77	22.79	16.53	.21
20	46.40	.32	39.11	31.19	.26	23.58	.24	28.28	21.65	15.71	.20
21	44.19	.30	37.24	29.70	.25	22.45	.22	26.94	20.62	14.96	.19
22	42.18	.29	35.55	28.35	.24	21.43	.21	25.71	19.68	14.28	.18
23	40.35	.27	34.01	27.12	.23	20.50	.20	24.59	18.83	13.66	.17
24	38.67	.26	32.59	25.99	.22	19.65	.20	23.57	18.04	13.09	.16
25	37.12	.25	31.29	24.95	.21	18.86	.19	22.63	17.32	12.57	.16
26	35.69	.24	30.08	23.99	.20	18.14	.18	21.76	16.66	12.08	.15
27	34.37	.23	28.97	23.10	.19	17.46	.17	20.95	16.04	11.64	.14
28	33.14	.23	27.93	22.28	.19	16.84	.17	20.20	15.47	11.22	.14
29	32.00	.22	26.97	21.51	.18	16.26	.16	19.51	14.93	10.83	.13
30	30.93	.21	26.07	20.79	.17	15.72	.16	18.86	14.43	10.47	.13
31	29.94	.20	25.23	20.12	.17	15.21	.15	18.25	13.97	10.13	.13
32	29.00	.20	24.44	19.49	.16	14.73	.15	17.68	13.53	9.82	.12
33	28.12	.19	23.70	18.90	.16	14.29	.14	17.14	13.12	9.52	.12
34	27.29	.19	23.00	18.35	.15	13.87	.14	16.64	12.74	9.24	.11
35	26.51	.18	22.35	17.82	.15	13.47	.13	16.16	12.37	8.98	.11
36	25.78	.18	21.73	17.33	.15	13.10	.13	15.71	12.03	8.73	.11

Distance between supports in ft.	12"I 40 lbs.	12"I 31.5 lbs.	Add for every lb. increase in weight	10"I 25 lbs.	Add for every lb. increase in weight	9"I 21 lbs.	Add for every lb. increase in weight
12	19.92	15.99	.26	10.85	.22	8.39	.20
13	18.39	14.76	.24	10.02	.20	7.74	.18
14	17.08	13.70	.23	9.30	.19	7.19	.17
15	15.94	12.79	.21	8.68	.17	6.71	.16
16	14.94	11.99	.20	8.14	.16	6.29	.15
17	14.06	11.29	.19	7.66	.15	5.92	.14
18	13.28	10.66	.18	7.24	.14	5.59	.13
19	12.58	10.10	.17	6.86	.14	5.30	.12
20	11.95	9.59	.16	6.51	.13	5.03	.12
21	11.38	9.14	.15	6.20	.12	4.79	.11
22	10.87	8.72	.14	5.92	.12	4.58	.11
23	10.39	8.34	.14	5.66	.11	4.38	.10
24	9.96	7.99	.13	5.43	.11	4.19	.10
25	9.56	7.67	.13	5.21	.10	4.03	.09
26	9.19	7.38	.12	5.01	.10	3.87	.09
27	8.85	7.11	.12	4.82	.10	3.73	.09
28	8.54	6.85	.11	4.65	.09	3.59	.08
29	8.24	6.62	.11	4.49	.09	3.47	.08
30	7.97	6.40	.11	4.34	.09	3.36	.08

Distance between supports in ft.	8"I 18 lbs.	Add for every lb. increase in weight	7"I 15 lbs.	Add for every lb. increase in weight	6"I 12.25 lbs.	Add for every lb. increase in weight	5"I 9.75 lbs.	Add for every lb. increase in weight	4"I 7.5 lbs.	Add for every lb. increase in weight	3"I 55 lbs.	Add for every lb. increase in weight
5	15.17	.42	11.04	.36	7.75	.31	5.16	.26	3.18	.21	1.76	.16
6	12.64	.35	9.20	.30	6.46	.26	4.30	.22	2.65	.18	1.47	.13
7	10.84	.30	7.89	.26	5.54	.22	3.69	.19	2.27	.15	1.26	.11
8	9.48	.26	6.90	.23	4.84	.19	3.23	.16	1.99	.13	1.10	.10
9	8.43	.23	6.13	.20	4.31	.17	2.87	.14	1.77	.12	.98	.09
10	7.59	.21	5.52	.18	3.88	.16	2.58	.13	1.59	.11	.88	.08
11	6.90	.19	5.02	.16	3.52	.14	2 35	.12	1.45	.10	.80	.07
12	6.32	.18	4.60	.15	3.23	.13	2.15	.11	1.33	.09	.73	.07
13	5.83	.16	4.25	.14	2.98	.12	1.98	.10	1.22	.08	.68	.06
14	5.42	.15	3.94	.13	2.77	.11	1.84	.09	1.14	.08	.63	.06
15	5.06	.14	3.68	.12	2.58	.10	1.72	.09	1.06	.07	.59	.05
16	4.74	.13	3.45	.11	2.42	.10	1.61	.08	.99	.07	.55	.05
17	4.46	.12	3.25	.11	2.28	.09	1.52	.08	.94	.06	.52	.05
18	4.21	.12	3.07	.10	2.15	.09	1.43	.07	.88	.06	.49	.04
19	3.99	.11	2.91	.09	2.04	.08	1.36	.07	.84	.06	.46	.04
20	3.79	.11	2.76	.09	1.94	.08	1.29	.07	.80	.05	.44	.04
21	3.61	.10	2.63	.09	1.85	.07	1.23	.06	.76	.05	.42	.04

TABLE 8.—DEFLECTION COEFFICIENTS FOR I-BEAMS GIVEN IN 64THS OF AN INCH

Coefficient index	Distance between supports in ft.								
	6	8	10	12	14	16	18	20	22
C. S.	38.1	67.8	105.9	152.5	207.6	271.2	343.2	423.7	512.7
C.′ S.	29.8	53.0	82.8	119.2	162.2	211.8	268.1	331.0	400.5

Coefficient index	Distance between supports in ft.								
	24	26	28	30	32	34	36	38	40
C. S.	610.2	716.1	830.5	953.4	1085.0	1225.0	1373.0	1530	1695
C.′ S.	476.6	559.4	648.8	744.8	847.4	956.6	1073.0	1195	1324

Figures given opposite C. S. and C.′ S. are the deflection coefficients for steel shapes, subject to transverse strain for varying spans under their maximum uniformly distributed safe loads, derived fram a fiber stress of 16,000 and 12,500 respectively; the modulus of elasticity being taken at 29,000,000.

To find the deflection of any symmetrical shape used as a beam under its corresponding safe load, divide the coefficients given in the above tables by the depth of the beam. This applies to such shapes as I-beams, channels, Z-bars, etc.

Example: Required the deflection of a 12-in. I-beam, 31.5 lbs., 20 ft. span, under its maximum uniformly distributed safe load of 9.59 tons, as given in Table 7. The above tables give 423.7 as the deflection coefficient; dividing this by 12 gives 35.3 as the required deflection in 64ths of an in.

For deflections due to different systems of loading see Table 3.

TABLE 9.—SECTION FACTORS OF REINFORCING PLATES FOR I-BEAM

Width of plate in ins.	Depth of beam	Weight of beam	Section factors. Light figures for tension members. Heavy figures for compression members. Thickness of plate: ⅜ in.	½ in.	⅝ in.	¾ in.
4⅝	10	25–30	**8.67** 5.64	**11.64** 7.53	**14.54** 9.41	
5	10	35–40	**9.42** 6.32	**12.55** 8.43	**15.72** 10.57	
5	12	31½–35	**11.32** 7.57	**15.05** 10.58	**18.20** 12.12	
5¼	12	40	**11.82** 8.17	**15.85** 10.89	**18.30** 14.02	
5⅜	12	45	**12.14** 8.72	**16.15** 11.29	**20.35** 14.08	
5½	12	50	**12.37** 8.80	**16.55** 11.76	**20.71** 14.52	
5⅝	12	55	**12.68** 9.02	**16.85** 12.04	**21.35** 15.08	
5½	15	42–45	**15.03** 10.91	**20.65** 14.59	**25.91** 18.58	
5⅝	15	50	**15.12** 11.27	**21.25** 15.04	**26.60** 18.83	
5¾	15	55–60	**15.22** 11.57	**21.45** 15.49	**27.11** 19.73	
6	15	65–70	**16.92** 12.32	**22.56** 16.44	**28.37** 20.59	
6¼	15	75	**17.60** 13.17	**23.71** 17.65	**29.32** 22.09	**35.38** 26.51
6⅜	15	80–85	**18.33** 13.72	**24.47** 18.35	**30.63** 23.00	**36.82** 27.57
6½	15	90–95	**18.63** 14.52	**24.87** 18.80	**31.23** 23.80	**37.48** 28.48
6¾	15	100	**20.28** 14.57	**25.32** 19.25	**31.74** 24.10	**37.74** 29.08
6	18	55–65		**27.06** 19.74	**33.87** 24.59	**40.07** 29.65
6¼	18	70		**28.46** 20.85	**35.23** 26.09	**42.44** 31.36
6¼	20	65–70			**39.13** 29.00	**47.09** 34.86
6⅜	20	75–80			**40.83** 30.59	**48.92** 36.77
7	20	85–90			**43.90** 33.76	**52.75** 40.44
7¼	20	95–100			**45.40** 35.61	**54.76** 42.80
7	24	80–85				**63.25** 48.44
7⅛	24	90–95				**64.45** 49.70
7¼	24	100				**65.56** 51.38

for plates on the tension flanges, the area of two $\frac{13}{16}$-in. rivet holes having been deducted from the area of the latter plate. The section factors from the table are to be added to the section factor of the beam, and the sum to be multiplied by the allowable fiber stress to obtain the safe bending moment in lb-ins. for the girder.

Example: A 15-in. 42-lb. I-beam is to be reinforced with a ⅝-in. plate on each flange. What will be the safe bending moment in lb.-ins. to allow upon the girder, the fiber stress to be 12,500 lbs. per sq. in.?

From Table 10 of properties of I-beams, the section factor or modulus of a 15-in. 42-lb. beam is..............	58.9
From Table 9 the section factor of the ⅝-in. compression plate for a 15-in., 42-lb. beam is........................	25.91
and for the tension flange............................	18.58
Total section factor for girder.......................	103.39

Then $103.39 \times 12{,}500 = 1{,}292{,}375$ lb.-ins. for the allowable bending moment upon the girder.

The chart, Fig. 2, applies to beams with or without cover plates and for any bending moment and fiber stress.

The small chart in the upper corner is to be used with the short row of bending moments at the left. The letters *a*, *b*, *c* and *d* denote the position of the plates, whether on the tension or compression flange, according to the figure in the lower corner.

Example: A bending moment of 1,292,375 lb.-ins. is to be taken by a beam at a fiber stress of 12,500 lbs. per sq. in. Required the size of beam and cover plates. The nearest bending moment on the chart is 1,300,000. Follow the line from this to the diagonal line for 12,500, thence up, and read the size of beam and plates as 15-in., 42-lb. beam with two ⅝-in. cover plates.

Neither Mr. Blake's table nor chart take into account the necessity for supporting long beams against lateral deflection. For the allowances to be made in such cases, see Table 6 for beams without lateral support

Explanation of Tables

On the Properties of Carnegie Standard and Special I-beams

Table 10 on I-beams, is calculated for all weights to which each pattern is rolled.

Columns 12 and 13 give coefficients by the help of which the safe, uniformly distributed load may be readily and quickly determined. To do this, it is only necessary to divide the coefficient given by the span or distance between supports in feet.

If a section is to be selected (as will usually be the case) intended to carry a certain load for a length of span already determined on, it will only be necessary to ascertain the coefficient which this load and span will require and refer to the table for a section having a coefficient of this value. The coefficient is obtained by multiplying the load in pounds uniformly distributed by the span length in feet.

In case the load is not uniformly distributed, but is concentrated at the middle of the span, multiply the load by 2 and then consider it as uniformly distributed. The deflection will be $\frac{8}{10}$ of the deflection for the latter load.

For other cases of loading, obtain the bending moment in lb.-ft. (the most common cases are given in the table of bending moments and deflections). This multiplied by 8 will give the coefficient required.

If the loads are quiescent, the coefficients for fiber stress of 16,000 lbs. per sq. in. for steel may be used; but if moving loads are to be provided for, the coefficient for 12,500 lbs. should be taken. Inasmuch as the effects of impact may be very considerable (the stresses produced in an unyielding, inelastic material by a load suddenly

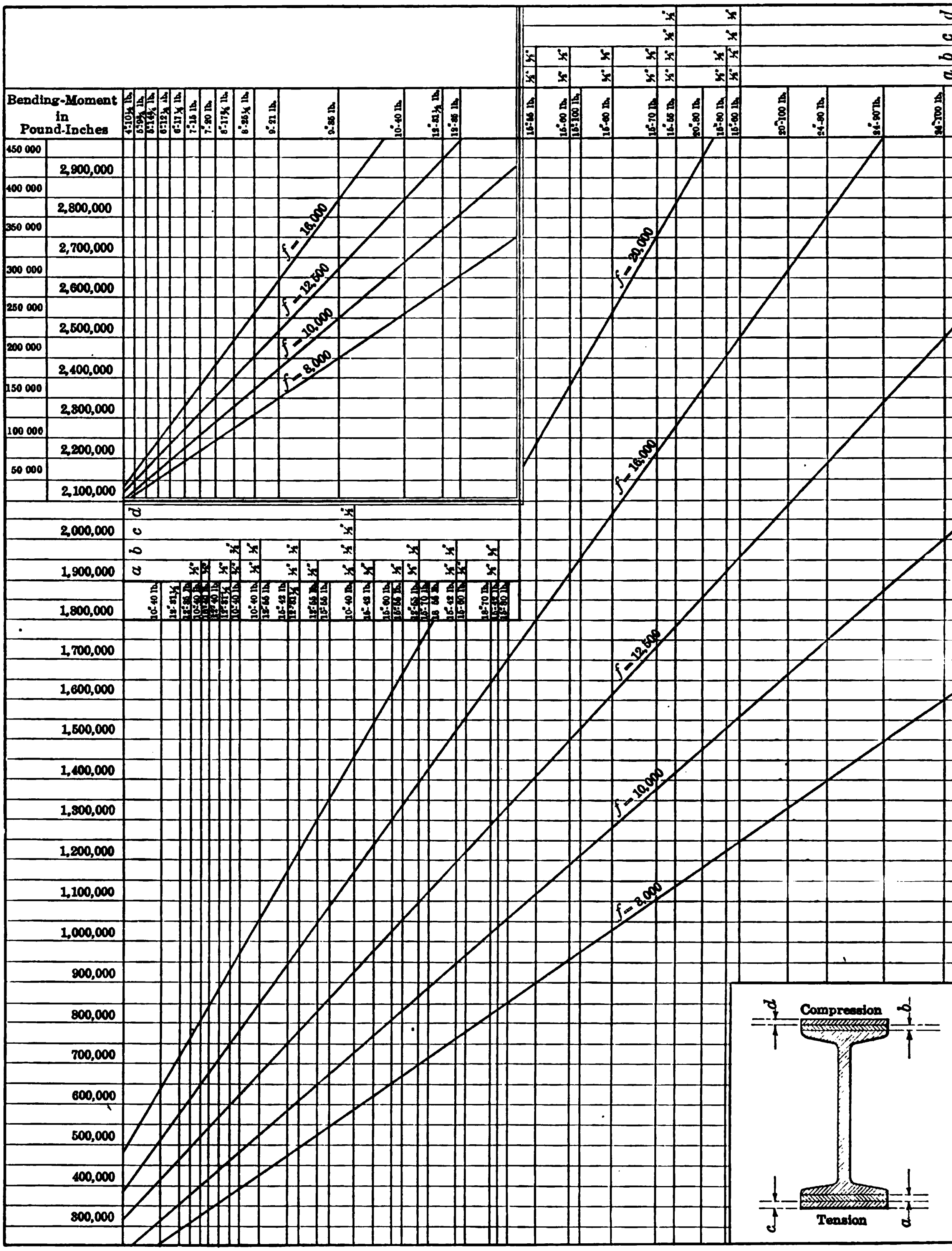


FIG. 2.—Strength of reinforced I-beams.

TABLE 10.—PROPERTIES OF I-BEAMS

By the Carnegie Steel Co.

Weights in heavy print are standard, others are special.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Section index	Depth of beam, ins.	Weight per ft., lbs.	Area of section, sq. ins.	Thickness of web, ins.	Width of flange, ins.	Mom. of inertia neutral axis perpendicular to web at center I	Mom. of inertia neutral axis coincident with center line of web I'	Radius of gyration neutral axis perpendicular to web at center r	Radius of gyration neutral axis coincident with center line of web r'	Section modulus neutral axis perpendicular to web at center S	Coefficient of strength for fiber stress of 16,000 lbs. per sq. in. Used for buildings C	Coefficient of strength for fiber stress of 12,500 lbs. per sq. in. Used for bridges C'	Distance center to center required to make radii of gyration equal	Section index
		100.00	29.41	.754	7.254	2380.3	48.56	9.00	1.28	198.4	2,115,800	1,653,000	17.82	
		95.00	27.94	.692	7.192	2309.6	47.10	9.09	1.30	192.5	2,052,900	1,603,900	17.99	
B 1	24	90.00	26.47	.631	7.131	2239.1	45.70	9.20	1.31	186.6	1,990,300	1,554,900	18.21	B 1
		85.00	25.00	.570	7.070	2168.6	44.35	9.31	1.33	180.7	1,927,600	1,505,900	18.43	
		80.00	23.32	.500	7.000	2087.9	42.86	9.46	1.36	174.0	1,855,900	1,449,900	18.72	
		100.00	29.41	.884	7.284	1655.8	52.65	7.50	1.34	165.6	1,766,100	1,379,800	14.76	
		95.00	27.94	.810	7.210	1606.8	50.78	7.58	1.35	160.7	1,713,900	1,339,000	14.92	
B 2	20	90.00	26.47	.737	7.137	1557.8	48.98	7.67	1.36	155.8	1,661,600	1,298,100	15.10	B 2
		85.00	25.00	.663	7.063	1508.7	47.25	7.77	1.37	150.9	1,609,300	1,257,200	15.30	
		80.00	23.73	.600	7.000	1466.5	45.81	7.86	1.39	146.7	1,564,300	1,222,100	15.47	
		75.00	22.06	.649	6.399	1268.9	30.25	7.58	1.17	126.9	1,353,500	1,057,400	14.98	
B 3	20	70.00	20.59	.575	6.325	1219.9	29.04	7.70	1.19	122.0	1,301,200	1,016,600	15.21	B 3
		65.00	19.08	.500	6.250	1169.6	27.86	7.83	1.21	117.0	1,247,600	974,700	15.47	
		70.00	20.59	.719	6.259	921.3	24.62	6.69	1.09	102.4	1,091,900	853,000	13.20	
B80	18	65.00	19.12	.637	6.177	881.5	23.47	6.79	1.11	97.9	1,044,800	816,200	13.40	B80
		60.00	17.65	.555	6.095	841.8	22.38	6.91	1.13	93.5	997,700	779,500	13.63	
		55.00	15.93	.460	6.000	795.6	21.19	7.07	1.15	88.4	943,000	736,700	13.95	
		100.00	29.41	1.184	6.774	900.5	50.98	5.53	1.31	120.1	1,280,700	1,000,600	10.75	
B 4	15	95.00	27.94	1.085	6.675	872.9	48.37	5.59	1.32	116.4	1,241,500	969,900	10.86	B 4
		90.00	26.47	.987	6.577	845.4	45.91	5.65	1.32	112.7	1,202,300	939,300	10.99	
		85.00	25.00	.889	6.479	817.8	43.57	5.72	1.32	109.0	1,163,000	908,600	11.13	
		80.00	23.81	.810	6.400	795.5	41.76	5.78	1.32	106.1	1,131,300	883,900	11.25	
		75.00	22.06	.882	6.292	691.2	30.68	5.60	1.18	92.2	983,000	768,000	10.95	
B 5	15	70.00	20.59	.784	6.194	663.6	29.00	5.68	1.19	88.5	943,800	737,400	11.11	B 5
		65.00	19.12	.686	6.096	636.0	27.42	5.77	1.20	84.8	904,600	706,760	11.29	
		60.00	17.67	.590	6.000	609.0	25.96	5.87	1.21	81.2	866,100	676,600	11.49	
		55.00	16.18	.656	5.746	511.0	17.06	5.62	1.02	68.1	726,800	567,800	11.05	
B 7	15	50.00	14.71	.558	5.648	483.4	16.04	5.73	1.04	64.5	687,500	537,100	11.27	B 7
		45.00	13.24	.460	5.550	455.8	15.00	5.87	1.07	60.8	648,200	506,400	11.54	
		42.00	12.48	.410	5.500	441.7	14.62	5.95	1.08	58.9	628,300	490,800	11.70	
		55.00	16.18	.822	5.612	321.0	17.46	4.45	1.04	53.5	570,600	445,800	8.65	
B 8	12	50.00	14.71	.699	5.489	303.3	16.12	4.54	1.05	50.6	539,200	421,300	8.83	B 8
		45.00	13.24	.576	5.366	285.7	14.89	4.65	1.06	47.6	507,900	396,800	9.06	
		40.00	11.84	.460	5.250	268.9	13.81	4.77	1.08	44.8	478,100	373,500	9.29	
B 9	12	35.00	10.29	.436	5.086	228.3	10.07	4.71	.99	38.0	405,800	317,000	9.21	B 9
		31.50	9.26	.350	5.000	215.8	9.50	4.83	1.01	36.0	383,700	299,700	9.45	
		40.00	11.76	.749	5.099	158.7	9.50	3.67	.90	31.7	338,500	264,500	7.12	
B11	10	35.00	10.29	.602	4.952	146.4	8.52	3.77	.91	29.3	312,400	244,100	7.32	B11
		30.00	8.82	.455	4.805	134.2	7.65	3.90	.93	26.8	286,300	223,600	7.57	
		25.00	7.37	.310	4.660	122.1	6.89	4.07	.97	24.4	260,500	203,500	7.91	
		35.00	10.29	.732	4.772	111.8	7.31	3.29	.84	24.8	265,000	207,000	6.36	
B13	9	30.00	8.82	.569	4.609	101.9	6.42	3.40	.85	22.6	241,500	188,700	7.58	B13
		25.00	7.35	.406	4.446	91.9	5.65	3.54	.88	20.4	217,900	170,300	6.86	
		21.00	6.31	.290	4.330	84.9	5.16	3.67	.90	18.9	201,300	157,300	7.12	
		25.50	7.50	.541	4.271	68.4	4.75	3.02	.80	17.1	182,500	142,600	5.82	
B15	8	23.00	6.76	.449	4.179	64.5	4.39	3.09	.81	16.1	172,000	134,400	5.96	B15
		20.50	6.03	.357	4.087	60.6	4.07	3.17	.82	15.1	161,600	126,200	6.12	
		18.00	5.33	.270	4.000	56.9	3.78	3.27	.84	14.2	151,700	118,500	6.32	
		20.00	5.88	.458	3.868	42.2	3.24	2.68	.74	12.1	128,600	100,400	5.15	
B17	7	17.50	5.15	.353	3.763	39.2	2.94	2.76	.76	11.2	119,400	93,300	5.31	B17
		15.00	4.42	.250	3.660	36.2	2.67	2.86	.78	10.4	110,400	86,300	5.50	
		17.25	5.07	.475	3.575	26.2	2.36	2.27	.68	8.7	93,100	72,800	4.33	
B19	6	14.75	4.34	.352	3.452	24.0	2.09	2.35	.69	8.0	85,300	66,660	4.49	B19
		12.25	3.61	.230	3.330	21.8	1.85	2.46	.72	7.3	77,500	60,500	4.70	
		14.75	4.34	.504	3.294	15.2	1.70	1.87	.63	6.1	64,600	50,500	....	
B21	5	12.25	3.60	.357	3.147	13.6	1.45	1.94	.63	5.4	58,100	45,400	....	B21
		9.75	2.87	.210	3.000	12.1	1.23	2.05	.65	4.8	51,600	40,300	....	
		10.50	3.09	.410	2.880	7.1	1.01	1.52	.57	3.6	38,100	29,800	....	
B23	4	9.50	2.79	.337	2.807	6.7	.93	1.55	.58	3.4	36,000	28,100	....	B23
		8.50	2.50	.263	2.733	6.4	.85	1.59	.58	3.2	33,900	26,500	....	
		7.50	2.21	.190	2.660	6.0	.77	1.64	.59	3.0	31,800	24,900	....	
		7.50	2.21	.361	2.521	2.9	.60	1.15	.52	1.9	20,700	16,200	....	
B77	3	6.50	1.91	.263	2.423	2.7	.53	1.19	.52	1.8	19,100	15,000	....	B77
		5.50	1.63	.170	2.330	2.5	.46	1.23	.53	1.7	17,600	13,800	....	

applied being double those produced by the same load in a quiescent state), it will sometimes be advisable to use still smaller fiber stresses than those given in the tables. In such cases the coefficients can readily be determined by proportion. Thus, for a fiber stress of 8,000 lbs. per sq. in. the coefficient will equal the coefficient for 16,000 lbs. fiber stress divided by 2.

The section moduli are used to determine the fiber stress per sq. in. in a beam or other shape, subjected to bending or transverse stresses, by simply dividing the same into the bending moment expressed in lb.-ins.

Column 14 gives the distance c.t.c. of beams, making the radii of gyration equal for both axes.

These tables have all been prepared with great care. No approximations have entered into any of the calculations, so that the figures given may be relied upon as accurate.

Example: What section of I-beam will be required to carry 40,000 lbs. uniformly distributed, including its own weight, over a span of 16 ft. between supports, allowing a fiber stress of 16,000 lbs. per sq. in.?

Answer: The coefficient required $= 40.000 \times 16 = 640{,}000$.

In Table 10 of Properties of I-beams, look in column 12 for the nearest number corresponding to 640,000 which is 648,200. Therefore the beam to be used is 15 in. 45 lbs.

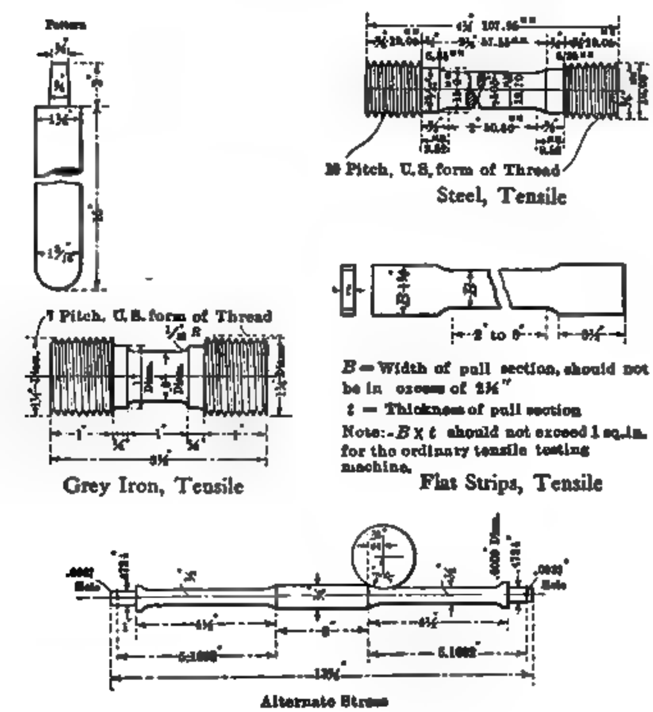


FIG. 1.—A. S. T. M. standard test specimens.

Beams of uniform strength for stakes of rivetting machines and similar structures may be laid out by the aid of Figs. 3, 4 and 5, which were originally developed at the Bement-Miles works of the Niles-Bement-Pond Co. (*Amer. Mach., Feb.* 21, 1901). The charts were designed especially for steel castings in which the compressive strength is about six-fifths of the tensile strength and, except in the case of beams of circular cross-section, the dimensions obtained from them always provide stresses in this ratio.

Instructions for use·

For full circular sections with a fiber stress of 10,000 lbs.: Read the load in tons of 2,000 lbs. on the right or left-hand scale and the length of the lever in ft. on the top or bottom scale of Fig. 3. Follow the lines from these readings to their intersection and find the required diameter of the section on the diagonals.

TABLE 11.—SAFE LOADS UNIFORMLY DISTRIBUTED FOR RECTANGULAR SPRUCE OR WHITE PINE BEAMS 1 IN. THICK
By the Carnegie Steel Co.

To obtain the safe load for any thickness: Multiply values for 1 in. by thickness of beam.

To obtain the required thickness for any load: Divide by safe load for 1 in.

This table has been calculated for extreme fiber stresses of 750 lbs. per sq. in. corresponding to the following values for Moduli of Rupture recommended by Prof. Lanza, viz.:

Spruce and white pine	3900 lbs.
Oak	4000 lbs.
Yellow pine	5000 lbs.

For oak increase values in table by $\frac{1}{3}$. For yellow pine increase values in table by $\frac{1}{4}$.

The safe load for any other values per sq. in. is found by increasing or decreasing the loads given in the table in the same proportion as the increased or decreased fiber stress.

Span in ft.	Depth of beam										
	6"	7"	8"	9"	10"	11"	12"	13"	14"	15"	16"
5	600	820	1070	1350	1670	2020	2400	2820	3270	3750	4270
6	500	680	890	1120	1390	1680	2000	2350	2730	3120	3560
7	430	580	760	960	1190	1440	1710	2010	2330	2680	3050
8	380	510	670	840	1040	1260	1500	1760	2040	2340	2670
9	330	460	590	750	930	1120	1330	1560	1810	2080	2370
10	300	410	530	670	830	1010	1200	1410	1630	1880	2130
11	270	370	490	610	760	920	1090	1280	1490	1710	1940
12	250	340	440	560	690	840	1000	1180	1360	1560	1780
13	230	310	410	520	640	780	930	1080	1260	1440	1640
14	210	290	380	480	590	720	860	1010	1170	1340	1530
15	200	270	360	450	560	670	800	940	1090	1250	1420
16	190	260	330	420	520	630	750	[illegible]	1020	1180	1330
17	180	240	310	400	400	590	710	830	960	1100	1260
18	170	230	290	370	460	560	670	780	910	1040	1100
19	160	210	[illegible]	360	440	530	630	740	860	990	1130
20	150	200	270	340	420	510	600	710	820	940	1070
21	140	190	260	320	390	480	570	670	780	890	1020
22	140	190	240	310	380	460	540	640	740	850	970
23	130	180	230	290	360	440	520	610	710	810	920
24	130	170	220	280	350	420	500	590	680	780	890
25	120	160	210	270	330	410	480	560	660	750	860
26	110	160	210	260	320	390	460	540	630	720	820
27	110	150	200	250	310	370	440	520	610	690	790
28	110	140	190	240	300	360	430	500	580	670	760
29	110	140	180	230	200	350	410	490	560	640	740

For any section of Fig. 4 with a tensile fiber stress of 10,000 and a compressive fiber stress of 12,000 lbs.: Multiply either load in tons or length of lever in ft. by the value of factor X for the section as given in Fig. 4 and proceed as before. The result given by Fig. 3 is the diameter D of the various sections of Fig. 4. The section may then be laid out by the proportional figures for the section selected. The value of D and the cross-section are to be determined for a sufficient number of points on the stake, the same proportional figures being used throughout the length of the stake, except that it should be noted that Fig. 4 will give the cross-sections at different points in the length of the stake or beam strictly according to the law of the cubic parabola. When nearing the top of the stake it is desirable to use heavier sections from Fig. 4 in order to reduce the diameters at these sections and also to avoid thin ribs which could not be cast in steel.

For any other tensile fiber stress than 10,000 lbs.: Multiply either the load in tons or the length of lever in ft. by 10,000 and divide by the desired tensile fiber stress and proceed as before. In the resulting beam the compressive fiber stress will always be equal to six-fifths of the tensile stress.

Example: Find the dimensions of section 3 of Fig. 4 for a riveter stake at a point 8 ft. below the dies. The pressure on the dies is

Feet

FIG. 3.—Beams of uniform strength.

70 tons and the tensile fiber stress is not to exceed 8,000 lbs. The value of section factor X for section 3 is $\frac{100}{75}$ and, performing the multiplication, gives:

$$70 \times \frac{100}{75} \times \frac{10000}{8000} = 117$$

Finding this load at the right and the length, 8 ft., at the top of Fig. 3, we find at the intersection of the lines through these points, $28\frac{1}{2}$ ins. as the value of D for the section.

The use of Fig. 4 for various methods of support and of loading is self-explanatory.

The Hodgkinson section of cast-iron beams with heavy tension and light compression flanges proportional in accordance with the widely differing ultimate strengths of the material in compression and tension, is now believed by many machine constructors to be fundamentally wrong when applied to machine parts. The case against it is well made out by JAS. CHRISTIE (*Proc. Engrs. Club of Phila.*, 1907) as follows:

This form of beam was largely adopted, and took precedence as long as cast-iron was used for beams in structures. We find that the same method of reasoning influenced the machine designer in disposing of cast-iron to seeming advantage in the construction of machines, massing the metal to resisit tension, and permitting high unit stress on metal in compression; and especially is this observed in machines of the open-jaw or gap type, such as presses, punching and shearing machines, etc.

I believe that usually the unit stresses should be little, if any, higher in compression than in tension, for the following reasons: In machinery rigidity or stiffness is usually the chief consideration; many machines do not fulfil the intended purpose properly, not by

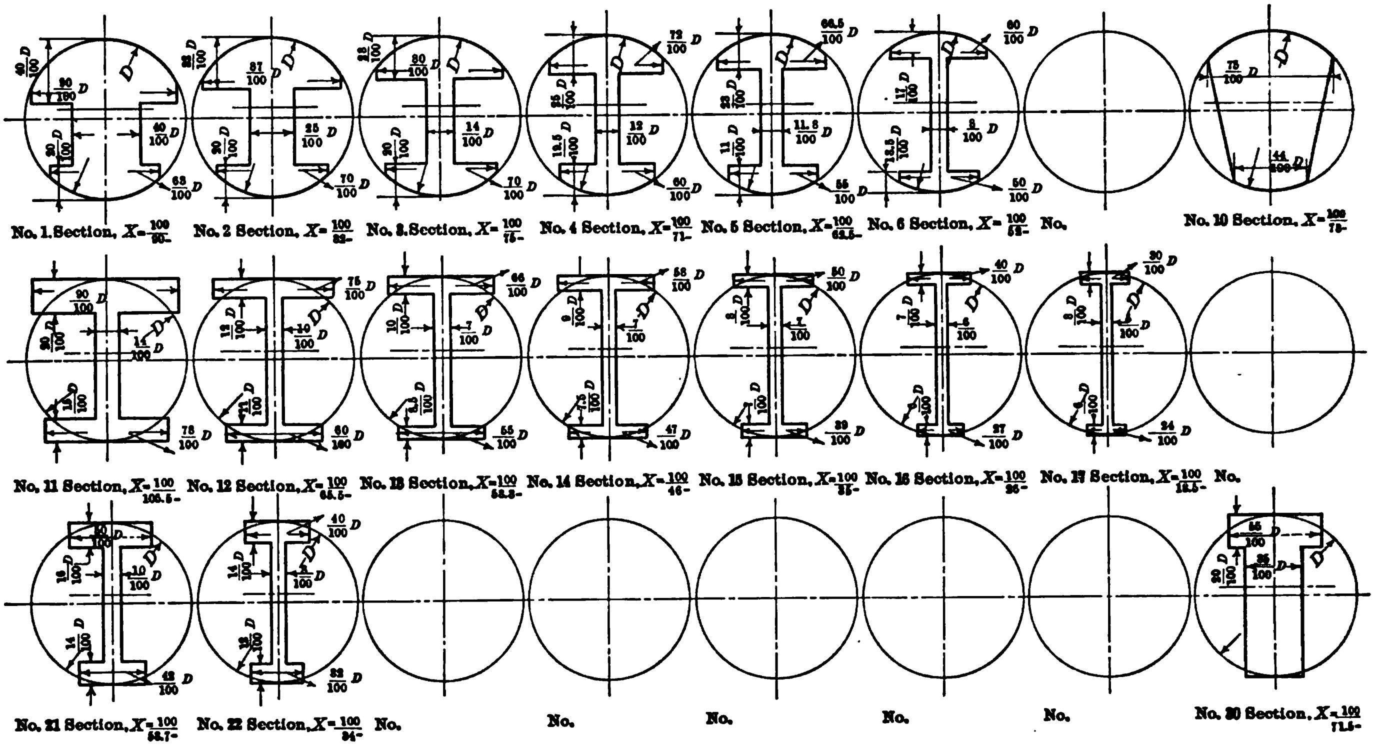


FIG. 4.—Sections of beams of uniform strength.

TABLE 12—ULTIMATE STRENGTH OF HOLLOW ROUND AND HOLLOW RECTANGULAR CAST-IRON COLUMNS

By the Carnegie Steel Co.

Ultimate Strength in Lbs. per Sq. In.:

Round Columns			Rectangular columns		
Square bearing	Pin and square	Pin bearing	Square bearing	Pin and square	Pin bearing
$\dfrac{80000}{1+\dfrac{(12l)^2}{800d^2}}$	$\dfrac{80000}{1+\dfrac{3(12l)^2}{1600d^2}}$	$\dfrac{80000}{1+\dfrac{(12l)^2}{400d^2}}$	$\dfrac{80000}{1+\dfrac{3(12l)^2}{3200d^2}}$	$\dfrac{80000}{1+\dfrac{9(12l)^2}{6400d^2}}$	$\dfrac{80000}{1+\dfrac{3(12l)^2}{1600d^2}}$

l = Length of column in ft.
d = External diameter or least side of rectangle in ins.

$\frac{l}{d}$	Round columns Ultimate strength in lbs. per sq. in.			Rectangular columns Ultimate strength in lbs. per sq. in.		
	Square bearing	Pin and square	Pin bearing	Square bearing	Pin and square	Pin bearing
1.0	67,800	62,990	58,820	70,480	66,520	62,990
1.1	65,690	60,300	55,730	68,790	64,260	60,300
1.2	63,530	57,600	52,690	67,000	61,940	57,600
1.3	61,340	54,930	49,740	65,140	59,600	54,960
1.4	59,140	52,310	46,900	63,260	57,270	52,320
1.5	56,940	49,770	44,200	61,350	54,960	49,760
1.6	54,760	47,300	41,630	59,450	52,680	47,300
1.7	52,620	44,940	39,210	57,550	50,460	44,960
1.8	50,530	42,670	36,930	55,670	48,300	42,670
1.9	48,490	40,510	34,790	53,800	46,230	40,510
2.0	46,510	38,460	32,790	51,940	44,200	38,460
2.1	44,600	36,520	30,920	50,160	42,260	36,520
2.2	42,750	34,680	29,180	48,400	40,400	34,680
2.3	40,980	32,940	27,540	46,670	38,630	32,950
2.4	39,280	31,310	26,030	44,990	36,930	31,310
2.5	37,650	29,770	24,620	43,390	35,310	29,760
2.6	36,090	28,320	23,300	41,820	33,770	28,320
2.7	34,600	26,950	22,070	40,320	32,310	26,950
2.8	33,180	25,670	20,930	38,870	30,920	25,670
2.9	31,820	24,460	19,860	37,470	29,600	24,460
3.0	30,530	23,320	18,870	36,120	28,340	23,320
3.1	29,310	22,250	17,940	34,830	27,150	22,250
3.2	28,140	21,250	17,070	33,580	26,030	21,250
3.3	27,030	20,300	16,260	32,390	24,660	20,300
3.4	25,970	19,410	15,500	31,240	23,940	19,410

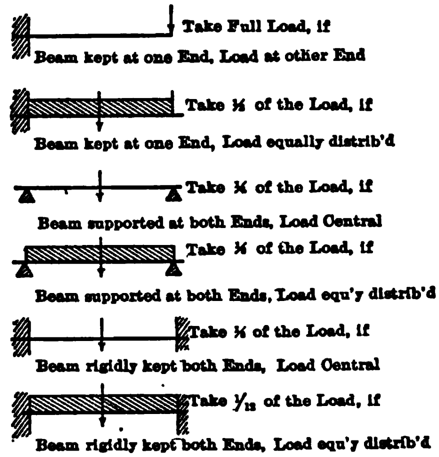


FIG. 5.—Loading and supporting beams.

failure through fracture, but by a want of sufficient stiffness. Deflection has to be limited, and when that is done, breaking from excessive tension is sufficiently guarded. Remembering that cast-iron yields to compression, as much as with the same unit stresses it yields to tension, it follows that the compressive stress should not exceed the tensile strength per unit of section if it is desired to dispose a given mass of metal with least deflection. It is believed that rupture sometimes occurs in a machine apparently through tension, where the origin of the weakness could be traced to a want of material to sufficiently resist compression, the improperly supported tension side severing by cross-bending or transverse stress.

Taking for illustration an open-gap machine with frame as illustrated in Fig. 6, tension at T and compression at C, if the section is so shaped that compressive unit stress is six times that of the tensile unit stress, then, elastic moduli being equal, the frame will yield at C six times as much by compression as it does by tension at T. This permits an oscillation of the mass at T around its center. If this oscillation becomes dangerous, by extent or frequency, the frame will break by cross-bending at the mass T, giving the impression

TABLE 13.—SAFE LOADS IN TONS OF 2000 LBS. FOR HOLLOW ROUND CAST-IRON COLUMNS
By the Carnegie Steel Co.

Outside diam., ins.	Thickness of metal	Length of columns in ft.									Sectional area ins.	Weight, lbs. of columns per ft. of length
		8	10	12	14	16	18	20	22	24		
		Tons	Tons	Tons	Tons	Tons	Tons	Tons	Tons	Tons		
6	½	26.2	23.0	20.1	17.5	15.2	13.2	11.5	....	....	8.6	26.95
6	¾	37.5	33.0	28.8	25.0	21.7	18.9	16.5	....	....	12.4	38.59
6	⅞	42.7	37.6	32.8	28.5	24.7	21.5	18.8	....	....	14.1	43.96
6	1	47.6	41.9	36.5	31.8	27.6	24.0	21.0	....	....	15.7	49.01
6	1⅛	52.2	46.0	40.1	34.8	30.2	26.3	23.0	....	....	17.2	53.76
7	¾	47.7	43.1	38.5	34.3	30.4	26.9	23.9	21.2	18.9	14.7	45.96
7	1	61.1	55.2	49.3	43.8	38.9	34.4	30.6	27.1	24.2	18.9	58.90
7	1⅛	67.2	60.8	54.3	48.3	42.8	37.9	33.7	29.9	26.7	20.8	64.77
8	¾	57.9	53.3	48.6	44.1	39.7	35.8	32.2	28.9	26.1	17.1	53.29
8	1	74.6	68.7	62.5	56.7	51.1	46.0	41.4	37.3	33.6	22.0	68.64
8	1¼	89.9	82.8	75.5	68.4	61.7	55.5	49.9	44.9	40.5	26.5	82.71
9	¾	68.1	63.6	58.9	54.2	49.6	45.2	41.2	37.5	34.1	19.4	60.65
9	1	88.0	82.3	76.2	70.0	64.1	58.4	53.2	48.4	44.1	25.1	78.40
9	1¼	106.6	99.6	92.2	84.8	77.6	70.8	64.4	58.7	53.4	30.4	94.94
9	1½	123.8	115.7	107.1	98.5	90.1	82.2	74.8	68.1	62.0	35.3	110.26
9	1¾	139.6	130.5	120.8	111.1	101.6	92.7	84.4	76.8	69.9	39.9	124.36
10	1	101.4	95.9	89.8	83.6	77.4	71.5	65.8	60.5	55.5	28.3	88.23
10	1¼	123.3	116.5	109.1	101.6	94.1	86.8	79.9	73.4	67.5	34.4	107.23
10	1½	143.7	135.8	127.3	118.5	109.7	101.2	93.2	85.6	78.7	40.1	124.99
10	1¾	162.7	153.8	144.1	134.1	124.2	114.6	105.5	97.0	89.1	45.4	141.65
11	1	114.8	109.4	103.5	97.3	91.0	84.8	80.2	73.1	67.7	31.4	98.03
11	1¼	139.9	133.3	126.1	118.6	110.9	103.3	97.8	89.4	82.5	38.3	119.46
11	1½	163.5	155.9	147.5	138.6	128.7	120.8	114.3	104.1	96.4	44.8	139.68
11	1¾	185.7	177.1	167.5	157.5	147.3	137.2	129.8	118.3	109.5	50.9	158.68
11	2	206.6	196.9	186.3	175.1	163.8	152.6	144.4	131.5	121.8	56.6	176.44
12	1	128.0	122.9	117.2	111.0	104.7	98.4	92.2	86.1	80.4	34.6	107.51
12	1¼	156.4	150.1	143.1	135.7	127.9	120.2	112.6	105.2	98.2	42.2	131.41
12	1½	183.3	175.9	167.7	159.0	149.9	140.9	132.0	123.3	115.1	49.5	154.10
12	1¾	208.7	200.4	191.0	181.1	170.7	160.4	150.3	140.5	131.1	56.4	175.53
12	2	232.7	223.4	213.0	201.9	190.4	178.9	167.6	156.6	146.1	62.8	195.75
13	1	141.2	136.3	130.7	124.7	118.5	112.1	105.8	99.5	93.5	37.7	117.53
13	1¼	172.8	166.8	160.0	152.7	145.0	137.2	129.4	121.8	114.4	46.1	143.86
13	1½	203.0	195.9	187.9	179.3	170.3	161.1	152.0	143.1	134.3	54.2	168.98
13	1¾	231.6	223.6	214.5	204.7	194.4	183.9	173.5	163.3	153.3	61.9	192.88
13	2	258.9	249.9	239.7	228.7	217.3	205.5	193.9	182.5	171.3	69.1	215.56
14	1	154.3	149.6	144.3	138.5	132.3	125.9	119.5	113.1	106.8	40.8	127.60
14	1¼	189.2	183.4	176.9	169.7	162.2	154.4	146.5	138.6	131.0	50.1	156.31
14	1½	222.6	215.8	208.1	199.7	190.8	181.7	172.3	163.1	154.1	58.9	183.67
14	1¾	254.4	246.7	237.9	228.3	218.1	207.6	197.0	186.5	176.2	67.4	210.00
14	2	284.8	276.2	266.4	255.6	244.2	232.4	220.6	208.8	197.2	75.4	235.12
15	1	167.4	162.9	157.8	152.1	146.0	139.7	133.3	126.8	120.4	44.0	137.28
15	1¼	205.5	200.0	193.7	186.7	179.3	171.5	163.6	155.7	147.9	54.0	168.48
15	1½	242.1	235.7	228.2	220.0	211.2	202.1	192.8	183.5	174.2	63.6	198.74
15	1¾	277.2	269.8	261.3	251.9	241.9	231.4	220.7	210.1	199.5	72.9	227.45
15	2	310.8	302.5	293.0	282.5	271.2	259.5	247.5	235.5	223.6	81.7	254.90

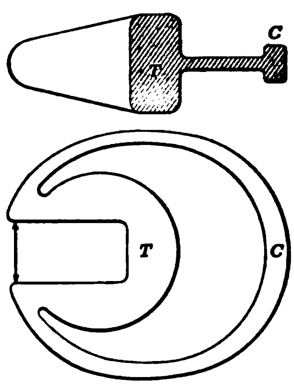


FIG. 6.—The case against the Hodgkinsen section.

that more material is needed to resist tension, whereas the fact may be that more material should be placed at C to prevent excessive yield by compression.

Flat Plates

The strength of flat plates in accordance with the formulas of Grashof and the experimental researches of Professor Bach may be determined from Table 16 by EUGENE MESSNER (*Amer. Mach., Nov.* 25, 1909). In these formulas

E = modulus of elasticity.
p = uniformly distributed load, lbs. per sq. in.,
P = total load acting at a point or over an indicated area, lbs.,
s = fiber stress due to bending, lbs. per sq. in.,
d = deflection, ins.

dimensions of plates in ins.

TABLE 14.—SAFE LOADS FOR RECTANGULAR WOODEN PILLARS (SEASONED). By the Carnegie Steel Co.

Yellow pine (southern)	White oak	White pine and spruce
$\dfrac{1125}{1+\dfrac{l^2}{1100d^2}}$	$\dfrac{925}{1+\dfrac{l^2}{1100d^2}}$	$\dfrac{800}{1+\dfrac{l^2}{1100d^2}}$

l = length of pillar ins.
d = width of smallest side ins.

These formulæ give safe loads of one-fourth the ultimate strength for short pillars decreasing to one-fifth the ultimate for long pillars.

Ratio of length to least side $\dfrac{l}{d}$	Safe load in lbs. per sq. in. of section		
	Yellow pine (southern)	White oak	White pine and spruce
12	995	818	707
14	955	785	679
16	913	750	649
18	869	715	618
20	825	678	587
22	781	642	556
24	738	607	525
26	697	575	495
28	657	541	467
30	619	509	440
32	583	479	414
34	549	451	390
36	516	425	367
38	487	400	346
40	458	377	326

Regarding the accuracy of the formulas Professor Bach's tests have demonstrated that the strength of the plates depends much on the fastening or support at the edges, the spacing of the bolts (for flanges, etc.), the forces exerted by those bolts (making a more or less elastic joint), the gaskets, the character of the tightening surfaces, etc.

The formulas assume the supports to be as shown—a rigid support being truly rigid and the plate rigidly fixed to it—conditions that seldom obtain in practice.

These formulas hold good within the limit of elasticity only. The rupturing loads cannot be found from them, this being doubly true in the case of ductile materials, such as boiler plate. With such materials the bulging under pressure leads to the formation of spherical surfaces and the destruction of the fundamental conditions on which the formulas are based. The formulas have been unjustly criticised because they do not agree with the results of tests carried to destruction, but such criticisms are based on a fundamental misunderstanding of the formulas.

Plates of much size made of ductile materials begin to bulge under moderate loads, leading to a change in the fundamental conditions even within the elastic limit, and, with such materials, the formulas have less application as the diameter increases. The formulas are most applicable to brittle materials (cast-iron) for which there is no reason to suppose that they give other than the true fiber stresses within the elastic limit. If, with such materials, they lead, as they often do, to apparently excessive thicknesses, that should be taken as an indication that crowned and not flat surfaces should be used whenever possible and that, when flat surfaces must be used, they should be ribbed.

Ribbing should be done judiciously, as otherwise it may do harm and not good. With narrow, and especially with shallow, ribs the concentration of stress on the edges of the ribs may be an added source of danger. Also, with cast-iron plates the ribs should, if possible, be on the compression side in order to take advantage of the greater strength of that material in compression than in tension.

Calculation of the strength of ribbed plates is scarcely possible, judgment and precedent being the chief guides and a free use of material the only safe course.

TABLE 15.—SAFE LOADS IN TONS OF 2000 LBS. FOR SQUARE WOODEN PILLARS BY THE CARNEGIE STEEL CO.

Unsupported length of column in ft.	Size of pillar in ins.						
	6×6	8×8	9×9	10×10	12×12	14×14	16×16
	White pine or spruce						
6	12.80	......	......	......	......	......	......
8	11.70	22.7	29.6	......	......	......	......
10	10.60	21.3	28.0	35.5	......	......	......
12	9.54	19.8	26.3	33.7	51.1	......	......
14	8.46	18.4	24.7	31.9	49.0	69.6	......
16	7.38	17.0	23.1	30.1	46.8	67.0	91.0
18	......	15.5	21.5	28.3	44.7	64.5	88.0
20	......	14.1	19.8	26.5	42.5	62.0	85.2
22	......	......	18.2	24.7	40.3	59.5	82.3
24	......	......	......	22.9	38.2	57.0	79.4
	White oak						
6	14.80	......	......	......	......	......	......
8	13.50	26.2	34.0	......	......	......	......
10	12.20	24.6	32.4	41.0	......	......	......
12	11.00	22.7	30.4	39.1	59.1	......	......
14	9.73	21.1	28.4	36.7	56.9	80.4	......
16	8.64	19.5	26.5	34.6	54.0	77.8	105.0
18	......	17.8	24.7	32.4	51.1	74.5	102.0
20	......	16.3	22.7	30.5	49.0	71.3	98.5
22	......	......	21.1	28.2	46.1	68.3	94.7
24	......	......	......	26.4	43.9	65.5	90.9
	Yellow pine (southern)						
6	18.0	......	......	......	......	......	......
8	16.4	32.0	41.6	......	......	......	......
10	14.9	29.9	39.4	50.0	......	......	......
12	13.3	27.8	36.9	47.6	72.0	......	......
14	11.9	25.8	34.7	44.7	69.1	98.0	132
16	10.4	23.7	32.3	42.3	65.5	94.6	128
18	......	21.8	30.0	39.5	62.6	90.7	124
20	......	19.8	27.8	37.0	59.8	86.9	120
22	......	......	25.7	34.6	56.2	83.6	115
24	......	......	......	32.2	53.3	80.0	111

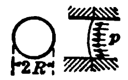
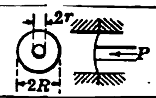
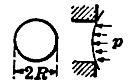
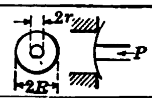
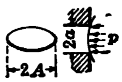
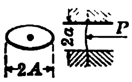
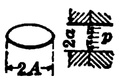
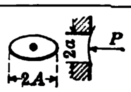
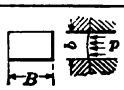
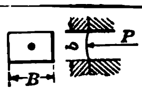
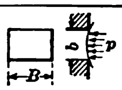
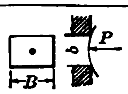
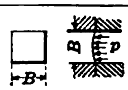
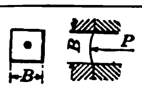
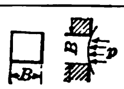
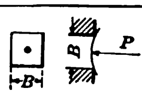
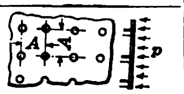
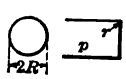
Combined Tension and Shear

The combination of direct tension or compression with shearing stresses may be made by means of Fig. 7, by E. R. DOUGLASS (*Amer. Mach.*, *July* 10, 1902).

Among the cases covered by the chart are those of shafts transmitting power and at the same time carrying heavy weights or acted on by overhung cranks and the like. The actual maximum stress at any point will be greater than that due to either the torsion or the bending alone, and will be exerted in a direction different from either.

Suppose the stresses to be combined are a tension T acting perpendicularly to the plane of a shearing stress S, these values expressing intensities, such as lbs. to the sq. in. For convenience the left-hand half of the diagram is plotted for $\frac{S}{T}$, to be used when S is less than T, and the right-hand half is plotted for $\frac{T}{S}$, to be used when S is greater than T. Then, P being the maximum resultant tensile stress and Q being the maximum resultant compressive stress, at

TABLE 16.—STRENGTH AND DEFLECTION OF FLAT PLATES

Shape and fastening of plate	Maximum fiber stress due to bending	Coefficients	Deflection in center of plate	Coefficients	Remarks
	$s=\phi\frac{R^2}{t^2}p$	Cast-iron $\phi=.8$ Steel $\phi=.45-.5$	$d=\phi\frac{R^4}{t^3}\times\frac{p}{E}$	Cast iron $\phi=0.17$	For cast-iron maximums in center, for steel at circumference.
	$s=0.438\frac{P}{t^2}\log\frac{R}{r}$		$d=.22\frac{R^2}{t^3}\frac{P}{E}$		Use Naperian logarithm for s.
	$s=\phi\frac{R^2}{t^2}p$	Cast-iron $\phi=1.2$ Steel $\phi=.67-.75$	$d=\phi\frac{R^4}{t^3}\times\frac{p}{E}$	Cast iron $\phi=.6$	Maximum s in center.
	$s=\phi\left(1-\frac{2r}{3R}\right)\frac{P}{t^2}$	Cast-iron $\phi=1.44$	$d=\phi\frac{R^2}{t^3}\frac{P}{E}$	Cast-iron $\phi=.4-.5$	
	$s=\phi\frac{a^2}{t^2}\frac{p}{1+n^2}$	Cast-iron $\phi=1.34$ Steel $\phi=.84$			$n=\frac{a}{A}$ ϕ for steel estimated.
	$s=\phi\frac{8+4n^2+3n^4}{3+2n^2+3n^4}n\frac{P}{t^2}$	Cast-iron $\phi=.76$			$n=\frac{a}{A}$
	$s=\phi\frac{a^2}{t^2}\frac{p}{1+n^2}$	Cast-iron $\phi=2.26$ Steel $\phi=1.41$			$n=\frac{a}{A}$ ϕ for steel estimated.
	$s=\phi\frac{8+4n^2+3n^4}{3+2n^2+3n^4}n\frac{P}{t^2}$	Cast-iron $\phi=.85$			$n=\frac{a}{A}$
	$s=\phi\frac{B^2b^2}{B^2+b^2}\times\frac{p}{t^2}$	Cast-iron $\phi=.38$ Steel $\phi=.24$			ϕ for steel estimated.
	$s=\phi\frac{Bb}{B^2+b^2}\times\frac{P}{t^2}$	Cast-iron $\phi=2.63$			
	$s=\phi\frac{B^2b^2}{B^2+b^2}\times\frac{p}{t^2}$	Cast-iron $\phi=.57$ Steel $\phi=.36$			ϕ for steel estimated.
	$s=\phi\frac{Bb}{B^2+b^2}\times\frac{P}{t^2}$	Cast-iron $\phi=3.0$			
	$s=\phi\frac{B^2}{t^2}p$	Cast-iron $\phi=.19$ Steel $\phi=.12$			ϕ for steel estimated.
	$s=\phi\frac{P}{t^2}$	Cast-iron $\phi=1.32$			s independent of B. Deflection only varies.
	$s=\phi\frac{B^2}{t^2}p$	Cast-iron $\phi=.28$ Steel $\phi=.18$			ϕ for steel estimated.
	$s=\phi\frac{P}{t^2}$	Cast-iron $\phi=1.5$			s independent of B. Deflection only varies.
	$s=0.228\frac{A^2}{t^2}p$		$d=.0284\frac{A^4}{t^3}\times\frac{p}{E}$		Stayed plate. The formula is for one field.
	$s=p\left[\phi\frac{r}{t}+\phi_1\left(\frac{R-.5r\left(1+\frac{r}{R}\right)}{t}\right)^2\right]$	Cast-iron $\phi=.8$, $\phi_1=.8$ Steel $\phi=.5$, $\phi_1=.33-.38$	According to stiffness of cylinder or riveted joint.		Flat boiler head with round edges.

right angles to P, the values of the ratios $\frac{P}{T}$ and $\frac{Q}{T}$ or $\frac{P}{S}$ and $\frac{Q}{S}$, and of the tangent of angle x between P and T may be read at once in terms of $\frac{S}{T}$ or $\frac{T}{S}$.

Had the original stress T been a compression instead of a tension, P would have been a compression and Q a tension, x still being the angle between P and T.

As an example of the use of this chart, suppose that in some case, as that of the shaft mentioned above, there is found to exist at a certain point a tensile stress of 8000 lbs. per sq. in. and a shearing stress of 3600 lbs. per sq. in. in a plane perpendicular to the tension. The ratio $\frac{S}{T}$ is $\frac{3600}{8000}$, or .45. Consulting the diagram we find $\frac{P}{T}=1.175$, $\frac{Q}{T}=.175$, and tan $x=.38$. Then the maximum resultant tension $P=1.175\times 8000=9400$ lbs. per sq. in., and its direction makes an angle whose tangent is .38, or 20° 50′ with the original tension, while there also exists a compression Q, normal to P, of value $.175\times 8000=1400$ lbs. per sq. in.

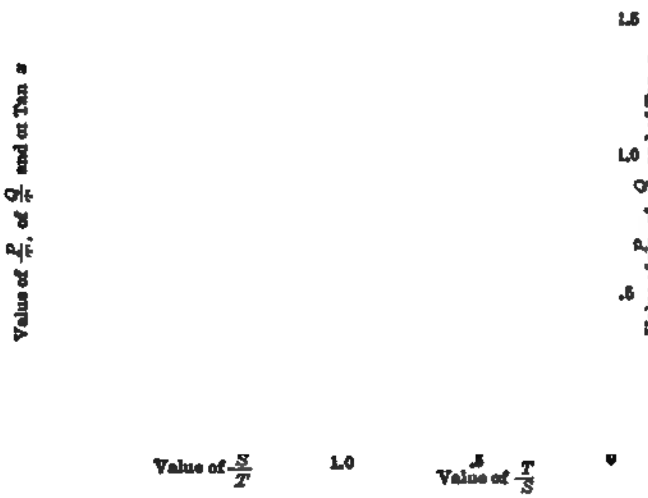


FIG. 7.—The combination of direct and shearing stresses.

The material must safely stand a stress of 9400 lbs. per sq. in. in the direction found.

Had the original tension been, for instance, 3000 lbs. per sq. in. and the shearing stress 7500 lbs. per sq. in., we would have taken $\frac{T}{S}=\frac{3000}{7500}=.4$, instead of $\frac{S}{T}$ as in the former case. Corresponding to $\frac{T}{S}=.4$ we find, in the right-hand side of the diagram, $\frac{P}{S}=1.22$, $\frac{Q}{S}=\tan x=.82$.

Whence $P=1.22\times 7500=9150$ lbs. per sq. in. maximum tension, making an angle of 39° 20′ with the original tension. $Q=7500\times .82=6150$ lbs. per sq. in. compression at right angles to P. In this case the resultant tension is more than three times the original one.

Punch and Shear Frames

The strength of cast-iron frames for punching and shearing machines formed the subject of experiments by PROF. A. L. JENKINS (*Trans. A. S. M. E., Vol.* 32). Model frames were made and tested to destruction, test bars being cast with and as part of the frame castings in order to avoid assumptions regarding the strength of the iron.

Although the experiments are not sufficiently exhaustive to justify rigid conclusions, they seem to indicate that the following statements are approximately true:

(*a*) There is no rational method for predicting the strength of curved cast-iron beams suitable for punch and shear frames.

(*b*) Of the three formulas suggested for the design of punch frames, the well-known beam formula,

$$S=\frac{Mc}{I}+\frac{W}{A}=W\left(1+\frac{Lc}{K^2}\right)$$

in which S = unit tensile stress,
M = bending moment = WL,
c = distance from the center of gravity of the section to the most extreme fiber in tension,
W = load applied,
A = area of the section considered,
L = distance from the line of application of the load to the center of gravity of the section considered,
I = moment of inertia of the section,
K = radius of gyration.

is the most accurate statement of the law of stress relations existing in such specimens.

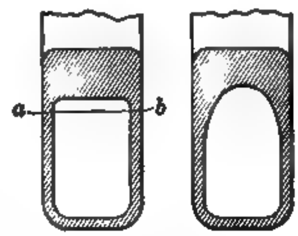


FIG. 8.—Correct and incorrect sections of punch and shear frames.

(*c*) The stress behind the inner flange at the curved portion is an important consideration that should be recognized by the designer.

(*d*) There seems to be no definite relation existing between the strength of a curved cast-iron beam and the transverse strength of a test bar cast with it.

(*e*) The Résal and Pearson-Andrews formulas are unwieldy and awkward in their application and offer many chances for error.

According to Wilfred Lewis, punch and shear frames, when made with the section shown at the left of Fig. 8, break on the line *ab*, whereas, when made with the section shown at the right, he has never known them to break.

Hoisting Hooks and Lifting Eyes

The dimensions of hoisting hooks of trapezoidal section may be determined from Fig. 10 by Axel K. Pedersen, analytical expert, of the General Electric Co. (*Amer. Mach., Dec.* 26, 1912). The charts are based on Bach's theory of curved beams. They impose one restriction, namely, that the section MN be a trapezoid. Most hook sections, especially those of large hooks, can be transformed into a trapezoid without serious error by the method shown in Fig. 9, which shows in full lines the actual shape of the important hook section, which then is transformed into the trapezoid shown in dotted lines. Only a very small reduction of the actual dimension H_1 is required, it being sufficient that the area a_1+a_2 is approximately equal to the area a_3, this being done by the eye of the observer without any refined measurement. The reduced dimension H and the increased dimension B are then used in the calculation. In designing new hooks, the theoretical dimension H is increased to H_1 and B decreased to B_1. In calculating, the selected, or actual inner radius of the hook, may be used without regard to the change of H.

The proposed capacity of the hook P in lbs, being given, we select the radius of the inside of the hook, Chart 1, Fig. 10, A, in ins. On the chart a table gives the practice of the Pawling & Harnischfeger Co. for this dimension. From these data and the allowable maximum tensile stress, s in lbs. per sq. in., we can proceed in the

following two ways in determining the dimensions of the important hook-section, MN.

(1) Calculate the dimension B in ins. from

$$B = .0225\sqrt{P} \qquad (a)$$

To facilitate this calculation, curve No. 1, Chart 2, Fig. 10, was plotted, giving the values of B for different loads P. As the dimension D_1, the shank of the hook, usually is calculated from

$$D_1 = .0225\sqrt{P}$$

which is identical with (a), we shall, after transformation, have the final dimension B_1 smaller than D_1, which is considered good practice, resulting in easy manufacture. Then select the ratio $x = H \div A$ from which then

$$H = Ax \qquad (b)$$

Suitable to most cases is $x = 2$ to 3.

Calculating the factor C from

$$C = BH\frac{s}{P} \qquad (c)$$

we use Chart 1 as follows: Locating C on its scale we trace parallel to the ratio z-scale to the curve giving the proper ratio x, thence horizontally to the left to the z-scale and read the value z. Then

$$b = Bz \qquad (d)$$

The larger the ratio x is selected the smaller we will get b, which is preferable as it tends to keep the weight of the hook reasonably low.

(2) The second method which can be employed, is the following: Select as before the ratio x which then gives

$$H = Ax$$

Then, calculate the ratio z from

$$z = \frac{1}{1+x} \qquad (e)$$

this relation between z and x usually resulting in good proportions of the hook-section. It may, however, be especially noted, that the chart can be used for any value of z; in other words, that it is not based upon any fixed relation between z and x. Now, locate this value of z on the z-scale, trace horizontally to the right to the proper curve for the value of x, thence vertically down to the C-scale and read the factor C, then

$$B = \frac{PC}{sH} \qquad (f)$$

and

$$b = Bz \qquad (g)$$

For determining the general dimensions of the hook, the following relations may serve as a guide:

$D_1 = .0225\sqrt{P}$ as already stated. $D_2 = .875D_1$, $W = 1.5A$, $h = .75H$ to $.90H$, $L_1 = 2.3A$ to $2.6A$, $L_2 = 4.3A$ to $4.5A$.

The calculation of B according to equation (a) is, of course, not necessary; however, for the reason above stated, the method gives very practical results. In calculating the dimension B or D_1 from (a) or determining it from curve 1, the nearest size of commercial available iron should be used, if the hook is to be forged from round bar iron.

The material for a new hook should preferably be a high-grade of iron rather than steel.

Most steel hooks, if overloaded, break without warning, giving no slow visible deflection as is the case with iron hooks, which open up gradually before ultimately breaking.

For hooks made from a high grade of iron and properly heat-treated, a maximum tensile stress of 17,000 lbs. per sq. in. may safely be allowed.

To check the capacity of existing hooks, measure the dimensions A, H, B and b, using the transformed section as already explained, Fig. 9. Then, calculate the ratios $x = H \div A$ and $z = b \div B$. Locate z on the z-scale of the chart, trace horizontally to the right to the curve giving the proper ratio x, thence vertically down to the C-scale and read the factor C, then

$$P = \frac{BH}{C}s \qquad (h)$$

if the capacity of the hook for a given maximum unit stress is required, and

$$s = \frac{PC}{BH} \qquad (i)$$

if the unit stress at a given load must be determined. Of course, the properties of the material from which the hook was made being unknown, the allowable maximum stress should be selected rather conservatively, an average value of 15,000 lbs. per sq. in. insuring reasonable safety.

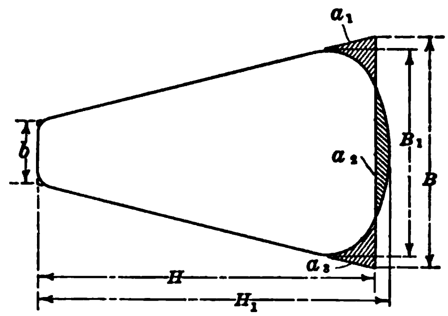


FIG. 9.—Transformation of the actual hook section into a trapezoid.

It will be observed that the chart gives solutions for all trapezoidal sections for values of $z = 0$, that is, a triangle up to $z = 1$., that is a rectangular section. The compression stress at the back of the hook at the dimension b in no case will exceed allowable limits, even for a triangular section. In other words, the hook will fail only if too high tension stresses are allowed.

Examples: Design a hoisting hook of 50-tons capacity, when the radius of the inside of the hook $A = 5$ ins., and the maximum allowable tensile stress $S = 17{,}000$ lbs. per sq. in.

According to the first method of calculatiug we would have from (a)

$$B = .0225\sqrt{100{,}000} = 7.115 \text{ ins.}$$

Say $B = 7$ ins., which also could have been determined from curve 1. Selecting $x = 2.2$ we get from (b)

$$H = 2.2 \times 5 = 11 \text{ ins.}$$

Then from (c)

$$C = 7 \times 11 \times \frac{17{,}000}{100{,}000} = 13.09 \text{ ins.}$$

Now using Chart 1, we obtain $z = .25$. Hence from (d)

$$b = .25 \times 7 = 1.75 \text{ ins.}$$

Using the second method, we would for $x = 2.2$ have $H = xA = 2.2 \times 5 = 11$ ins.; then from (e)

$$z = \frac{1}{1 + 2.2} = .31 \text{ ins.}$$

Hence from Chart 1, $C = 12.7$; and then from (f)

$$B = \frac{100{,}000}{17{,}000} \times \frac{12.7}{11} = 6.788 \text{ ins.}$$

and from (g)

$$b = .31 \times 6.788 = 2.104 \text{ ins.}$$

Thus, the two methods do not give identical proportions of the hook section. Whichever is to be preferred depends entirely upon the individual judgment of the designer, the aim being to combine strength with lightness and good appearance.

Determine the capacity of a hook of the following dimensions: $H_1 = 3.125$ ins., $B_1 = 1.75$ ins., $b = .5$ in. and $A = 1.25$ ins. After transformation into a trapezoid, we measure $H = 3$ ins. and $B = 2$ ins., hence

$$x = \frac{H}{A} = \frac{3}{1.25} = 2.4$$

and

$$z = \frac{b}{B} = \frac{.5}{2} = .25$$

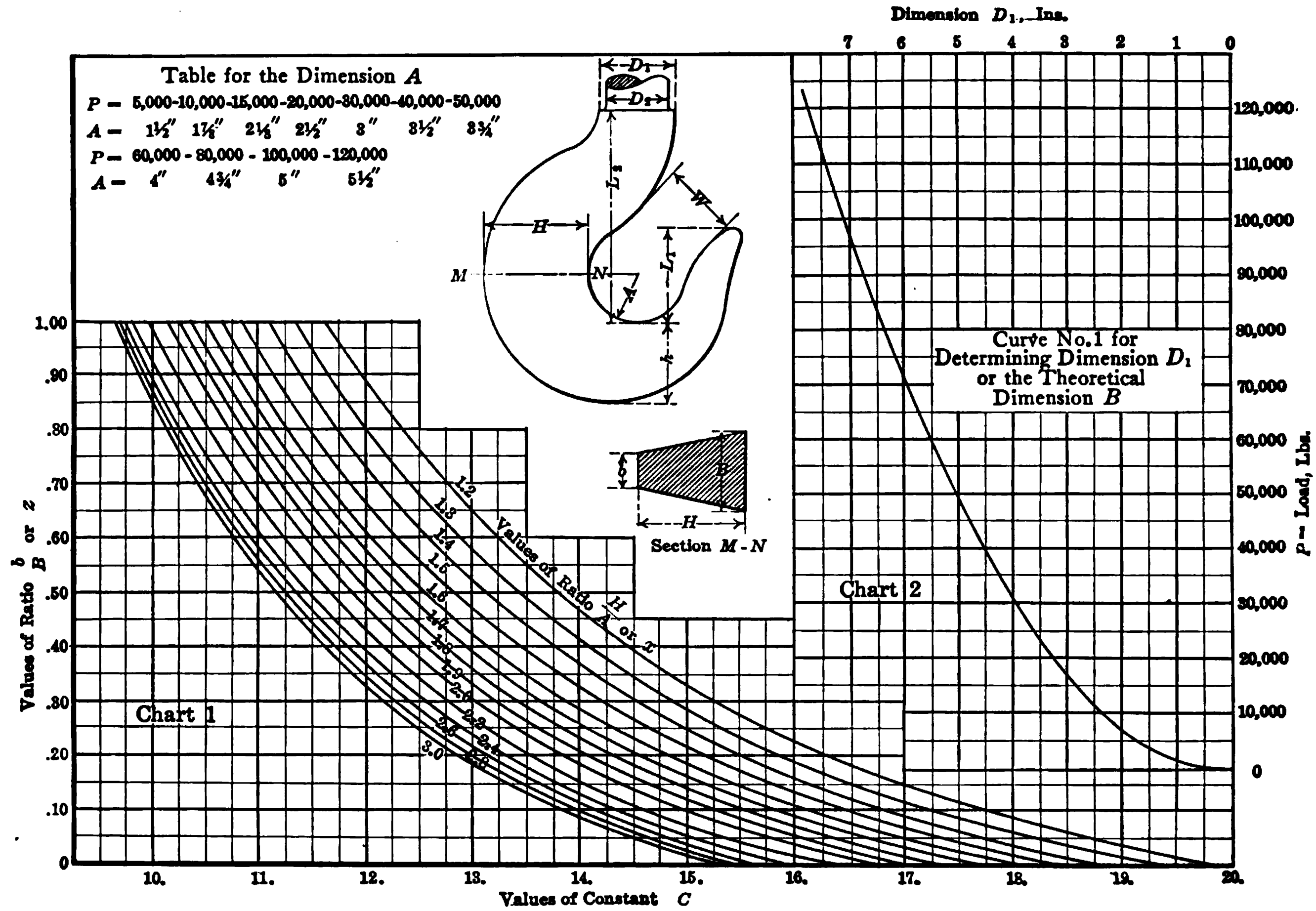


FIG. 10.—Dimensions of hoisting hooks of trapezoidal section.

Then using the chart we find $C = 12.925$, and allowing a stress of $s = 16{,}000$ lbs. per sq. in. we get from (h)

$$P = \frac{BH}{C} s = \frac{2 \times 3}{12.925} \times 16{,}000 = 7{,}427 \text{ lbs.}$$

If this same hook were to be used as a 4-ton hook or for $P = 8{,}000$ lbs., we would have the stress from (i)

$$s = \frac{PC}{BH} = \frac{8000 \times 12.925}{2 \times 3} = 17{,}233 \text{ lbs. per sq. in.}$$

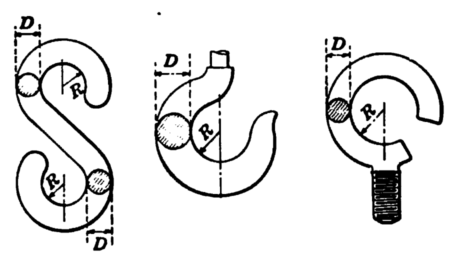


FIG. 11.—Hoisting hooks of circular cross section.

The dimensions of hoisting hooks of circular cross-section may be determined from Fig. 12, also by Mr. Pedersen and, like the preceding chart, laid out in accordance with Bach's formula. The chart is applicable to any of the hooks shown in Fig. 11. Directions for use will be found below the chart.

The dimensions of eye bolts and lifting eyes may be determined from Fig. 13, by MR. PEDERSEN (*Amer. Mach., May* 18, 1911). The chart is the outgrowth of experiments at the testing laboratory of the General Electric Co.

The chart applies to the cases shown in Figs. 14, 15 and 16 and determines the dimension D in ins., having given A and (for Fig. 14) B in ins., P, the load, in lbs. and s, the maximum allowable tensile stress, in lbs. per sq. in. occurring in the eye. T, Fig. 14, is one-half the angle which includes the unyielding part of the eye. For Fig. 16, the angle $T = 0$ and for Fig. 15, $T = 90$ deg.

Calculate the factor:

$$F = \frac{msA^2}{P}$$

using the valne $m = 2$ which was deduced from the experimental tests.

Locate F on the F-scale and trace parallel to the Z-scale to the proper curve among the curves for the sine of T and read the value of Z on the Z-scale. Then the dimension D is

$$D = Z \times A$$

To employ the proper curve for the sine of T, the following rules must be observed:

For Fig. 14 calculate the value of sine of T approximately from sine of T_1 = the ratio $B \div A$.

For Fig. 16: Use the curve, sine of $T = O$.

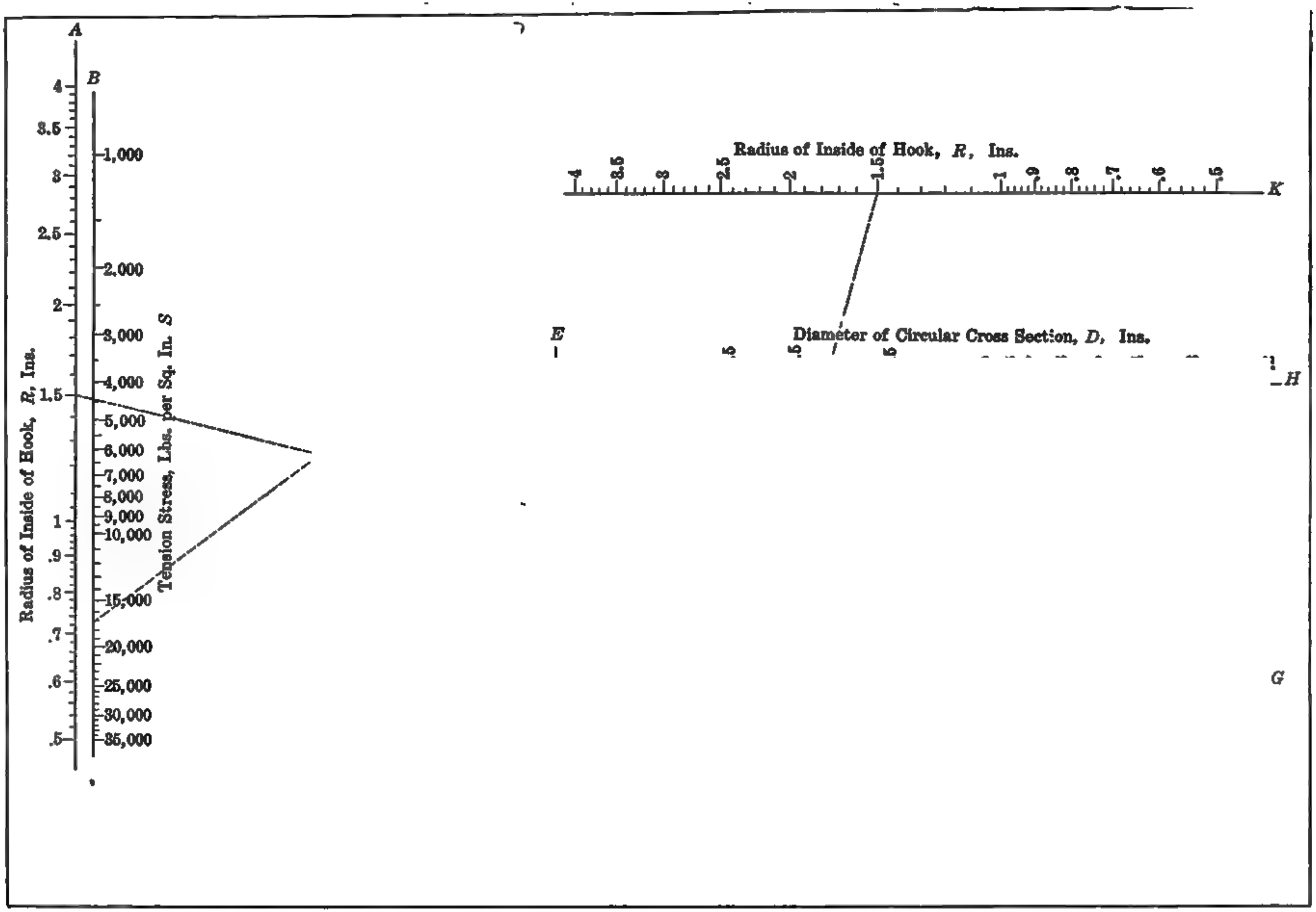
For Fig. 15: Use the curve sine of $T = 1.0$.

Allowable stresses:

For the eye: Maximum stress allowed $= \frac{2}{3}$ of the elastic limit of the iron used.

For the shank: Maximum stress $= \frac{1}{3}$ of the elastic limit of the iron used.

Example: Assume an eyebolt for 60,000 lbs. load, having an inside diameter of eye of 6 ins. Elastic limit of iron used, 30,000 lbs. per sq. in.



After selecting the radius of the inside hook R to suit the required conditions, and having decided upon a proper tensile stress—say, 17,000 lbs. per sq. in.—locate the load on the D axis of the chart, and connect this point with the point of the tensile stress s located on the B axis. This gives an intersection point with the dummy axis C. Connect this with the proper point for the radius of the inside of the hook on the A axis, and extend the connecting line to the E axis. Trace from the point thus located horizontally to the right to the curve F, thence vertically downward or upward as the case may be, to the G axis, thus fixing a point on this axis. Then connect with the proper value for the radius of the inside of the hook on the K axis. The connecting line intersects a point on the H axis, where the required diameter D of the circular cross-section is read. The general procedure is shown by the dotted lines on the chart. In the example used, we get $D = 2.10$ in. for $P = 4000$ lbs., $R = 1.5$ ins., and $s = 17{,}000$ lbs. per sq. in. In checking the capacity of a hook of given dimensions, the chart is read in the opposite direction by starting at the scales K and H.

FIG. 12.—Dimensions of hoisting hooks of circular cross-section.

Allowing a stress in the shank $= \frac{1}{3}\ 30{,}000$

$= 10{,}000$ lbs. per sq. in.,

we find the diameter at the root of the thread to be 2.77 ins., giving $B = 3\frac{1}{4}$ ins., U. S. Standard.

Allowing a stress in the eye $= \frac{2}{5}\ 30{,}000$

$= 12{,}000$ lbs. per sq. in.,

the factor F becomes:

$$F = \frac{msA^2}{P}$$

$$= \frac{2 \times 12{,}000 \times 6^2}{60{,}000}$$

$$= 14.4$$

Also $\sin T_1 = \frac{B}{A} = \frac{3.25}{6} = .54$

Now using the chart, the value of Z is found to be

$$Z = .479$$

$$\text{and } D = Z \times A = .479 \times 6 = 2.874$$

$$= 2\tfrac{7}{8} \text{ ins.}$$

Giving each of the three lifting tools, shown in Figs. 14, 15 and 16, the same dimensions, $A = 6$ ins. and $D = 2\frac{7}{8}$, as in above example, and denoting the factors F_1, F_2 and F_3 and the loads P_1, P_2 and P_3, respectively, the relative strength can be ascertained by locating $Z = .479$ on the Z-scale, and reversing the method of using the chart, determining the factors $F_1 = 14.4$ (for Fig. 14), $F_2 = 17$ (for Fig. 16) and $F_3 = 11.45$ (for Fig. 15). Then according to formula

$$F = \frac{msA^2}{P}$$

and for equal stresses, we have the proportion:

$$F_1 : F_2 : F_3 = \frac{1}{P_1} : \frac{1}{P_2} : \frac{1}{P_3} = 14.3 : 17.0 : 11.45$$

or for $P = 60{,}000$ lbs. (for Fig. 14), we get

$$P_2 = \frac{F_1}{F_2} P_1 = 51{,}000 \text{ lbs.}$$

(for Fig. 16), and

$$P_3 = \frac{F_1}{F_3} P_1 = 75{,}500 \text{ lbs.}$$

(for Fig. 15).

In calculating the shank of the eyebolt, account should be taken of any bending action of the load. Even for straight lifts, that is, lifts in the direction of the shank, it is practically impossible to avoid this bending tendency; only a low stress should therefore be allowed. For straight lifts a maximum stress in the shank equal to one-third of the elastic limit of the iron may still be considered safe.

If two or more eyebolts are used in connection with slings, the shank is subjected to heavy bending and shoulder eyebolts or a suitable spreader should be used, whenever possible. If shoulder eyebolts are employed, care should be taken to have the shoulder tight against the part to be lifted; this is often neglected. Generally, however, straight-shank eyebolts are used and, to avoid accidents, stronger eyebolts must be employed than for straight lifts.

India Rubber

The stress-strain relationship of india rubber, vulcanized for elasticity, which is unique among constructive materials, was investigated by Dr. R. H. Thurston and is presented in Fig. 17 (*Science*, 1898). It is a matter of common observation that, when this substance is subjected to a pull of steadily increasing intensity, its resistance increases, as does that of any elastic and ductile material; but that, at the end, instead of suddenly losing power of resistance, or even snapping without observable decrease of load, its resistance for a time rapidly and largely increases up to the point of rupture. This can be readily felt in even the breaking of one of the small bands of partially vulcanized rubber so universally employed for filing papers and other purposes. At the end of the period of extension the resistance rises so rapidly as to produce the sensation of bringing the hand up against a rigid obstacle, resisting further elongation.

Fig. 17 shows the property as determined in a testing machine. The substance behaves precisely like other familiar materials, up to a point which, in this case, is found at a load of 30 per cent. of the

FIG. 17.—Stress strain diagram of india rubber.

maximum, the breaking load, and at an extension one-half the maximum. At this point there exists a reversal of the line, and the curvature is thence maintained convex to the axis of X, up to the point of rupture; fracture taking place, at the end, sharply and without any indication of that method of flow of the mass which, in the case of the irons and softer steels, for example, permits a falling off of resistance after passing a point of maximum tenacity well within the breaking limit. The ratio of increase of load to increase of elongation steadily increases from the zero point, as with all substances, other than iron and steel, so far as known, up to this point of contrary flexure on the diagram, at which place the ratio is inverted and resistance increases in greater proportion than extension, finally assuming a comparatively high value.

India rubber exhibits none of the phenomena giving the characteristic form of the diagrams of the irons and steels. Even when stretched to the point of rupture it restores itself very nearly to its original dimensions, and gradually recovers a part of the loss of form at that instant observable. Its almost complete stability of form when relieved from load, and especially when in the shape of springs such as are used on railway trucks, constitutes one of its most valuable properties. Like cork, when confined laterally it is practically incapable of distortion when used as a spring. The volume of the mass remains, so far as can be seen, constant, or nearly so.

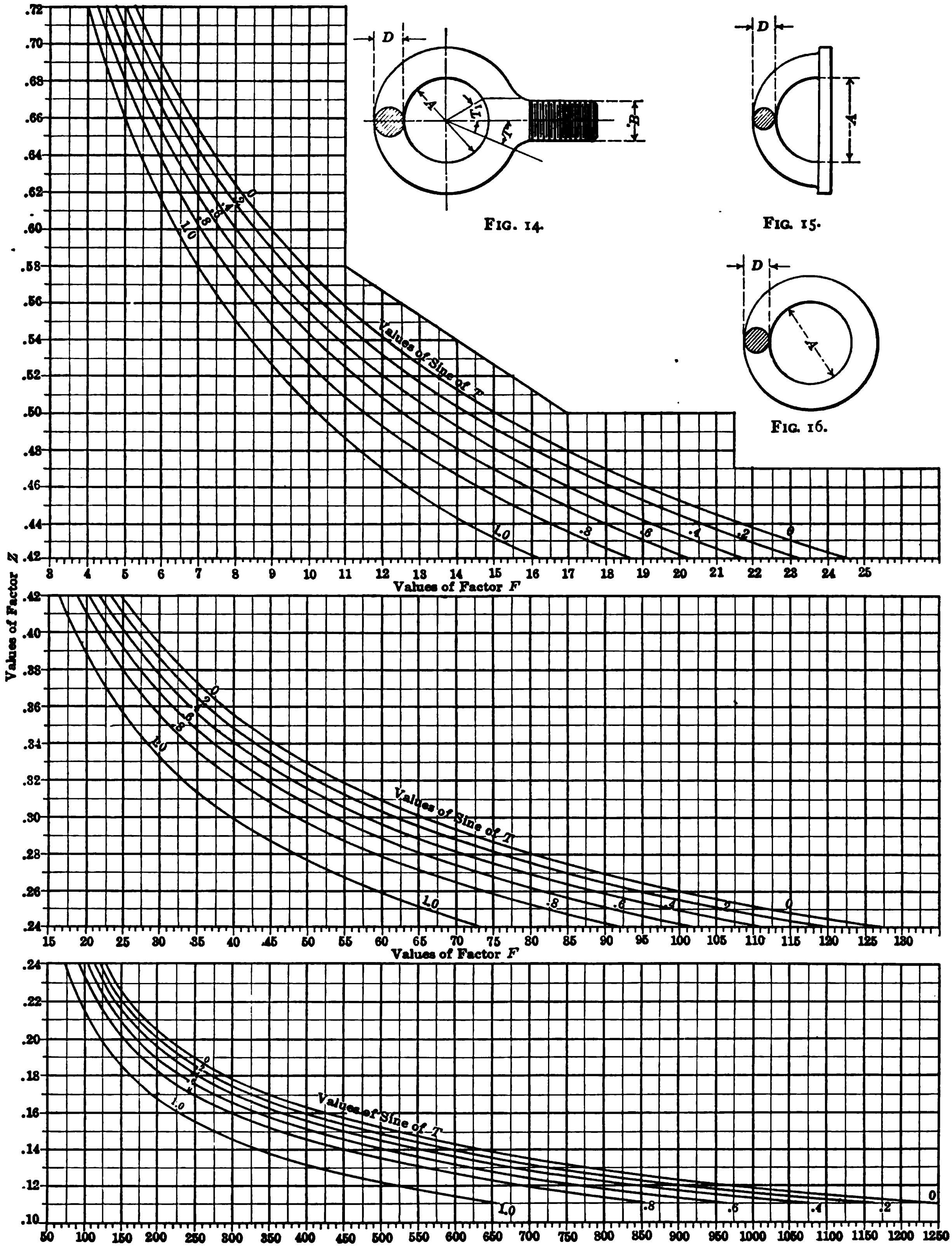


FIG. 13.—Dimensions of eye bolts and lifting eyes.

WEIGHTS AND MEASURES

While the British continue to use certain units of measurement which Americans have discarded, notably the stone and the hundredweight of 112 lbs., the fundamental units of length and weight and their chief derivatives are identical in Great Britain and the United States. Measures of capacity, unfortunately, differ.

The base of American measures of capacity for liquids is the Winchester gallon of 231 cu. ins. and the corresponding base of the British measures is the Imperial gallon of 277.274 cu. ins. The division of the two gallons into gills, pints and quarts follows the same scale. Following are the relations of the two gallons:

1 U. S. gallon	=.833 Imperial gallon.
1 Imperial gallon	=1.200 U. S. gallons.
7.48 U. S. gallons	=1 cu. ft.
6.24 Imperial gallons	=1 cu. ft.
1 U. S. gallon of water at 62 deg. Fahr.	=8.34 lbs.
1 Imperial gallon of water at 62 deg. Fahr.	=10 lbs.

The U. S. (Winchester) bushel contains 2150.42 cu. ins., while the Imperial bushel is based on the Imperial gallon, of which it contains 8 or 2218.19 cu. ins. The division of the two bushels into pints, quarts and pecks follows the same scale. Following are the relations of the two bushels:

1 U. S. bushel	= .969 Imperial bushel.
1 Imperial bushel	=1.032 U. S. bushels.
1 U. S. bushel	=1.244 cu. ft.
1 Imperial bushel	=1.284 cu. ft.

The Metric System

That monument to scientific zeal combined with ignorance of practical requirements—the metric system—is unfortunately present in the world and cannot be ignored.

The claims for the ease of adoption and the wide use of the system have been shown by S. S. Dale and the author to be grotesquely false (*Trans. A. S. M. E., Vols.* 24 and 28 and *The Metric Fallacy*). The facts are that no nation has ever made serious progress toward the adoption of the system in trade and commerce except by the force of compulsory law, and that no nation has ever discarded its old units by force of compulsory or any other law.

The case for France was officially summed up and confessed in a circular letter dated Paris, Apr. 11, 1906, from the French Minister of Commerce, Industry and Labor, to the presidents of French Chambers of Commerce, of which the following is a translation in part. The full text may be found in the Transactions of the *A. S. M. E., Vol.* 28:

"My Department at different times has been called upon to give to the Department of Weights and Measures instructions for accomplishing the total suppression of the measures and weights prohibited by the old law of July 4, 1837 by the seizure of the prohibited articles. The Department, in spite of all such efforts, has not succeeded in attaining the desired result.

"I have learned that in certain industries the advertisements, prospectuses, catalogues, etc., used by the merchants among themselves and also for sending to their customers contain the illegal expressions. . . . . They thus continue to designate in lignes and inchs all the articles they sell. . . . .

"I do not consider it worth while to enumerate here the industries and professions which have continued to employ the proscribed standards, but they are still numerous and most of them known to members of your organization."

In the metric countries of western Europe, great industries, although selling their products by the metric system, make exclusive use of the old systems in the manufacture of those products. Thus in France the leading industry is the manufacture of silk fabrics, and this industry makes exclusive use of the aune and denier as its manufacturing units of length and weight respectively. Again, in Germany, the cotton industry is based exclusively on the British yard and pound and the woolen industry is similarly based on a great variety of old German ells and pounds. Throughout the metric and non-metric world lumber is sawn to the inch.

The actual condition is diametrically the opposite of the imaginary one pictured by the metric party. For actual uniformity of measures in all industries and commerce and for actual simplicity of calculations due to that uniformity we must turn to English-speaking countries, while, for actual diversity of measures and complexity of calculations due to the necessity for repeated conversions between incommensurate units, metric countries supply an example and a warning.

Outside western Europe and contrary to oft-repeated but unfounded assertion, the system is used but little. Many countries, notably those of Spanish America, have "adopted" the system, but without compulsion, the result being that it has become an official government system used chiefly in the collection of customs duties and sometimes only partially there, while among the people it is used but little or not at all.

In other countries which are frequently classed as metric (Japan, Russia, the treaty ports of China) the law goes no further than to make the system permissive exactly as in Great Britain and the United States.

These conclusions are proven by an array of facts that is overwhelming. No serious attempt to answer them has ever been made, because such answer is impossible.

There are but two possible explanations of these facts—either the advantages of the system are not sufficient to justify its adoption or its adoption is attended with so much difficulty as to be impracticable. Either explanation is fatal to the pro-metric argument.

The feature of the system on which most stress is laid by its advocates—its convenience in the reduction or conversion of units, due to the fact that it has the same base as our unfortunate system of arithmetical notation—overlooks the fact that in the affairs of every-day life such conversions are of too infrequent occurrence to lend importance to this feature. All customary calculations of the engineer or business man are made as readily in the British as in the metric system.

Were it otherwise, the repeated conversions during the transition period of two systems of units used conjointly and bearing incommensurate ratios with one another, offset many times over even the claims made for economy of time in calculations by the metric system. Of the probable length of the transition period we may form some idea from the fact that as acknowledged by the Minister of Commerce, Industry and Labor it is still far from complete in France.

The metric system is, at best, a complete subordination of the greater to the lesser—of the function of measuring to that of calculation. Its advocates forget "that the chief function of a system of weights and measures is to weigh and measure, not to make calculations." Because of this some of its units are ill adapted to many of the purposes of life, while the decimal division of units is far inferior to binary divisions for the purposes of commerce and manufacture.

British-American-Metric Conversion Factors

(Except Capacity Measures which are American only)

From The U. S. Bureau of Standards

Lengths

Inches	Millimeters	Inches	Centimeters	Feet	Meters	U. S. yards	Meters	U. S. miles	Kilometers
.03937	1	.3937	1	1	.304801	1	.914402	.62137	1
.07874	2	.7874	2	2	.609601	1.093611	1	1	1.60935
.11811	3	1	2.54001	3	.914402	2	1.828804	1.24274	2
.15748	4	1.1811	3	3.28083	1	2.187222	2	1.86411	3
.19685	5	1.5748	4	4	1.219202	3	2.743205	2	3.21869
.23622	6	1.9685	5	5	1.524003	3.280833	3	2.48548	4
.27559	7	2	5.08001	6	1.828804	4	3.657607	3	4.82804
.31496	8	2.3622	6	6.56167	2	4.374444	4	3.10685	5
.35433	9	2.7559	7	7	2.133604	5	4.572009	3.72822	6
1	25.4001	3	7.62002	8	2.438405	5.468056	5	4	6.43739
2	50.8001	3.1496	8	9	2.743205	6	5.486411	4.34959	7
3	76.2002	3.5433	9	9.84250	3	6.561667	6	4.97096	8
4	101.6002	4	10.16002	13.12333	4	7	6.400813	5	8.04674
5	127.0003	5	12.70003	16.40417	5	7.655278	7	5.59233	9
6	152.4003	6	15.24003	19.68500	6	8	7.315215	6	9.65608
7	177.8004	7	17.78004	22.96583	7	8.748889	8	7	11.26543
8	203.2004	8	20.32004	26.24667	8	9	8.229616	8	12.87478
9	228.6005	9	22.86005	29.52750	9	9.842500	9	9	14.48412

Areas

Square inches	Square millimeters	Square inches	Square centimeters	Square feet	Square meters	Square yards	Square meters	Square miles	Square kilometers
.00155	1	.1550	1	1	.09290	1	.8361	.3861	1
.00310	2	.3100	2	2	.18581	1.1960	1	.7722	2
.00465	3	.4650	3	3	.27871	2	1.6723	1	2.5900
.00620	4	.6200	4	4	.37161	2.3920	2	1.1583	3
.00775	5	.7750	5	5	.46452	3	2.5084	1.5444	4
.00930	6	.9300	6	6	.55742	3.5880	3	1.9305	5
.01085	7	1	6.452	7	.65032	4	3.3445	2	5.1800
.01240	8	1.0850	7	8	.74323	4.7839	4	2.3166	6
.01395	9	1.2400	8	9	.83613	5	4.1807	2.7027	7
1	645.16	1.3950	9	10.764	1	5.9799	5	3	7.7700
2	1,290.33	2	12.903	21.528	2	6	5.0168	3.0888	8
3	1,935.49	3	19.355	32.292	3	7	5.8529	3.4749	9
4	2,580.65	4	25.807	43.055	4	7.1759	6	4	10.3600
5	3,225.81	5	32.258	53.819	5	8	6.6890	5	12.9500
6	3,870.98	6	38.710	64.583	6	8.3719	7	6	15.5400
7	4,516.14	7	45.161	75.347	7	9	7.5252	7	18.1300
8	5,161.30	8	51.613	86.111	8	9.5679	8	8	20.7200
9	5,806.46	9	58.065	90.875	9	10.7639	9	9	23.3100

Volumes

Cubic inches	Cubic millimeters	Cubic inches	Cubic centimeters	Cubic feet	Cubic meters	Cubic yards	Cubic meters
.000061	1	.0610	1	1	.02832	1	.7645
.000122	2	.1220	2	2	.05663	1.3079	1
.000183	3	.1831	3	3	.08495	2	1.5291
.000244	4	.2441	4	4	.11327	2.6159	2
.000305	5	.3051	5	5	.14159	3	2.2937
.000366	6	.3661	6	6	.16990	3.9238	3
.000427	7	.4272	7	7	.19822	4	3.0582
.000488	8	.4882	8	8	.22654	5	3.8228
.000549	9	.5492	9	9	.25485	5.2318	4
1	16,387.2	1	16.3872	35.314	1	6	4.5874
2	32,774.3	2	32.7743	70.629	2	6.5397	5
3	49,161.5	3	49.1615	105.943	3	7	5.3519
4	65,548.6	4	65.5486	141.258	4	7.8477	6
5	81,935.8	5	81.9358	176.572	5	8	6.1165
6	98,323.0	6	98.3230	211.887	6	9	6.8810
7	114,710.1	7	114.7101	247.201	7	9.1556	7
8	131,097.3	8	131.0973	282.516	8	10.4635	8
9	147,484.5	9	147.4845	317.830	9	11.7715	9

Areas.—*Continued*

Acres	Hectars
1	.4047
2	.8094
2.471	1
3	1.2141
4	1.6187
4.942	2
5	2.0234
6	2.4281
7	2.8328
7.413	3
8	3.2375
9	3.6422
9.884	4
12.355	5
14.826	6
17.297	7
19.768	8
22.239	9

British-American-Metric Conversion Factors—(*Continued*)

(Except Capacity Measures which are American only)

Capacities

U. S. liquid quarts	Liters	U. S. liquid gallons	Liters	U. S. dry quarts	Liters	U. S. pecks	Liters	U. S. bushels	Hectoliters
1	.94636	.26417	1	.9081	1	.11331	1	1	.35239
1.05668	1	.52834	2	1	1.1012	.22702	2	2	.70479
2	1.89272	.79251	3	1.8162	2	.34053	3	2.83774	1
2.11336	2	1	3.78543	2	2.2025	.45404	4	3	1.05718
3	2.83908	1.05668	4	2.7242	3	.56755	5	4	1 40957
3.17005	3	1.32085	5	3	3.3037	.68106	6	5	1.76196
4	3.78543	1.58502	6	3.6323	4	.79457	7	5.67548	2
4.22673	4	1.84919	7	4	4.4049	.90808	8	6	2.11436
5	4.73179	2	7.57087	4.5404	5	1	8.80982	7	2.46675
5.28341	5	2.11336	8	5	5.5061	1.02157	9	8	2.81914
6	5.67815	2.37753	9	5.4485	6	2	17.61964	8.51323	3
6.34009	6	3	11.35630	6	6.6074	3	26.42946	9	3.17154
7	6.62451	4	15.14174	6.3565	7	4	35.23928	11.35097	4
7.39677	7	5	18.92717	7	7.7086	5	44.04910	14.18871	5
8	7.57088	6	22.71261	7.2646	8	6	52.85892	17.02645	6
8.45345	8	7	26.49804	8	8.8098	7	61.66874	19.86420	7
9	8.51723	8	30.28348	8.1727	9	8	70.47856	22.70194	8
9.51014	9	9	34.06891	9	9.9110	9	79.28838	25.53968	9

Weights

Grains	Grams	Avoirdupois ounces	Grams	Troy ounces	Grams	Avoirdupois pounds	Kilograms	Troy pounds	Kilograms
1	.06480	.03527	1	.03215	1	1	.45359	1	.37324
2	.12960	.07055	2	.06430	2	2	.90718	2	.74648
3	.19440	.10582	3	.09645	3	2.20462	1	2.67923	1
4	.25920	.14110	4	.12860	4	3	1.36078	3	1.11973
5	.32399	.17637	5	.16075	5	4	1.81437	4	1.49297
6	.38879	.21164	6	.19290	6	4.40924	2	5	1.86621
7	.45359	.24692	7	.22506	7	5	2.26796	5.35846	2
8	.51839	.28219	8	.25721	8	6	2.72155	6	2.23945
9	.58319	.31747	9	.28936	9	6.61387	3	7	2.61269
15.4324	1	1	28.3495	1	31.10348	7	3.17515	8	2.98593
30.8647	2	2	56.6991	2	62.20696	8	3.62874	8.03769	3
46.2971	3	3	85.0486	3	93.31044	8.81849	4	9	3.35918
61.7294	4	4	113.3981	4	124.41392	9	4.08233	10.71691	4
77.1618	5	5	141.7476	5	155.51740	11.02311	5	13.39614	5
92.5941	6	6	170.0972	6	186.62088	13.22773	6	16.07537	6
108.0265	7	7	198.4467	7	217.72437	15.43236	7	18.75460	7
123.4589	8	8	226.7962	8	248.82785	17.63698	8	21.43383	8
138.8912	9	9	255.1457	9	279.93133	19.84160	9	24.11306	9

British-Metric and Metric-British Equivalents of Units of Length

Unit	In.	Ft.	Yd.	Rod	Furl.	Mile	Cm.	Meter	Km.	Unit
In.	1	.083	.027	.0050	…	…	2.54	.0254	…	In.
Ft.	12	1	.3	.06	.0015	.0001893	30.48	.3048	…	Ft.
Yd.	36	3	1	.18	.0045	.0005681	91.4402	.914402	.0009144	Yd.
Rod	198	16.5	5.5	1	.025	.003125	…	5.029	.005029	Rod
Furl.	7920	660	220	40	1	.125	…	201.17	.20117	Furl.
Mile	63360	5280	1760	320	8	1	…	1609.35	1.60935	Mile
Cm.	.3937	.03281	.01094	.001988	…	…	1	.01	…	Cm.
Meter	39.37	3.28083	1.09361	.19884	.00497	.0006214	100	1	.001	Meter
Km.	39370	3280.83	1093.61	198.84	4.97096	.62137	100,000	1000	1	Km.

British-Metric and Metric-British Equivalents of Units of Weight

Unit	Grain	Gram	Oz. av.	Lb. av.	Kilog.	Short cwt.	Ton			Unit
							Short	Metric	Long	
Grain	1	.0647989	.0022857	.00014286	.000064799	…	…	…	…	Grain
Gram	15.43236	1	.035274	.0022046	.001	…	…	…	…	Gram
Oz. av.	437.5	28.3495	1	.0625	.0283495	…	…	…	…	Oz. av.
Lb. av.	7000	453.592	16	1	.453592	.01	.0005	.0004536	.0004464	Lb. av.
Kilog.	15432.36	1000	35.27396	2.20462	1	.0220462	.00110231	.001	.00098421	Kilog.
Short cwt.	…	…	…	100	45.3592	1	.05	.045359	.0446429	Short cwt.
Short ton	…	…	…	2000	907.185	20	1	.907185	.8928571	Short ton
Metric ton	…	…	…	2204.62	1000	22.0462	1.10231	1	.984206	Metric ton
Long ton	…	…	…	2240	1016.05	22.4	1.12	1.01605	1	Long ton

British-American-Metric Conversion Factors for Fractional Dimensions of Length

From the U. S. Bureau of Standards

Binary Fractions of an Inch to Millimeters

½'s	¼'s	8ths	16ths	32nds	64ths	Millimeters	Decimals of an inch	Inch	½'s	¼'s	8ths	16ths	32nds	64ths	Millimeters	Decimals of an inch
					1	= .397	.015625							33	= 13.097	.515625
				1	2	= .794	.03125						17	34	= 13.494	.53125
					3	= 1.191	.046875							35	= 13.891	.546875
			1	2	4	= 1.588	.0625					9	18	36	= 14.288	.5625
					5	= 1.984	.078125							37	= 14.684	.578125
				3	6	= 2.381	.09375						19	38	= 15.081	.59375
					7	= 2.778	.109375							39	= 15.478	.609375
		1	2	4	8	= 3.175	.1250				5	10	20	40	= 15.875	.625
					9	= 3.572	.140625							41	= 16.272	.640625
				5	10	= 3.969	.15625						21	42	= 16.669	.65625
					11	= 4.366	.171875							43	= 17.066	.671875
			3	6	12	= 4.763	.1875					11	22	44	= 17.463	.6875
					13	= 5.159	.203125							45	= 17.859	.703125
				7	14	= 5.556	.21875						23	46	= 18.256	.71875
					15	= 5.953	.234375							47	= 18.653	.734375
	1	2	4	8	16	= 6.350	.2500			3	6	12	24	48	= 19.050	.75
					17	= 6.747	.265625							49	= 19.447	.765625
				9	18	= 7.144	.28125						25	50	= 19.844	.78125
					19	= 7.541	.296875							51	= 20.241	.796875
			5	10	20	= 7.938	.3125					13	26	52	= 20.638	.8125
					21	= 8.334	.328125							53	= 21.034	.828125
				11	22	= 8.731	.34375						27	54	= 21.431	.84375
					23	= 9.128	.359375							55	= 21.828	.859375
		3	6	12	24	= 9.525	.3750				7	14	28	56	= 22.225	.875
					25	= 9.922	.390625							57	= 22.622	.890625
				13	26	= 10.319	.40625						29	58	= 23.019	.90625
					27	= 10.716	.421875							59	= 23.416	.921875
			7	14	28	= 11.113	.4375					15	30	60	= 23.813	.9375
					29	= 11.509	.453125							61	= 24.209	.953125
				15	30	= 11.906	.46875						31	62	= 24.606	.96875
					31	= 12.303	.484375							63	= 25.003	.984375
1	2	4	8	16	32	= 12.700	.5	1	2	4	8	16	32	64	= 25.400	1.000

Hundredths of an Inch to Millimeters

Hundredths of an inch	0	1	2	3	4	5	6	7	8	9
	0	.254	.508	.762	1.016	1.270	1.524	1.778	2.032	2.286
10	2.540	2.794	3.048	3.302	3.556	3.810	4.064	4.318	4.572	4.826
20	5.080	5.334	5.588	5.842	6.096	6.350	6.604	6.858	7.112	7.366
30	7.620	7.874	8.128	8.382	8.636	8.890	9.144	9.398	9.652	9.906
40	10.160	10.414	10.668	10.922	11.176	11.430	11.684	11.938	12.192	12.446
50	12.700	12.954	13.208	13.462	13.716	13.970	14.224	14.478	14.732	14.986
60	15.240	15.494	15.748	16.002	16.256	16.510	16.764	17.018	17.272	17.526
70	17.780	18.034	18.288	18.542	18.796	19.050	19.304	19.558	19.812	20.066
80	20.320	20.574	20.828	21.082	21.336	21.590	21.844	22.098	22.352	22.606
90	22.860	23.114	23.368	23.622	23.876	24.130	24.384	24.638	24.892	25.146

Millimeters to Decimals of an Inch

Millimeters	0	1	2	3	4	5	6	7	8	9
	0	.03937	.07874	.11811	.15748	.19685	.23622	.27559	.31496	.35433
10	.39370	.43307	.47244	.51181	.55118	.59055	.62992	.66929	.70866	.74803
20	.78740	.82677	.86614	.90551	.94488	.98425	1.02362	1.06299	1.10236	1.14173
30	1.18110	1.22047	1.25984	1.29921	1.33858	1.37795	1.41732	1.45669	1.49606	1.53543
40	1.57480	1.61417	1.65354	1.69291	1.73228	1.77165	1.81102	1.85039	1.88976	1.92913
50	1.96850	2.00787	2.04724	2.08661	2.12598	2.16535	2.20472	2.24409	2.28346	2.32283
60	2.36220	2.40157	2.44094	2.48031	2.51968	2.55905	2.59842	2.63779	2.67716	2.71653
70	2.75590	2.79527	2.83464	2.87401	2.91338	2.95275	2.99212	3.03149	3.07086	3.11023
80	3.14960	3.18897	3.22834	3.26771	3.30708	3.34645	3.38582	3.42519	3.46456	3.50393
90	3.54330	3.58267	3.62204	3.66141	3.70078	3.74015	3.77952	3.81889	3.85826	3.89763

British-Metric Conversion Factors for Compound Units

From Clark's Manual of Rules, Tables and Data

Metric-British		
1 kg per m.	=	.672 lb. per ft..
		2.016 lbs. per yd.
1 kg. per sq. cm.	=	14.2232 lbs. per sq. ft.
1.0335 kg. per sq. cm. (1 atmosphere)	=	14.7 lbs. per sq. in.
1 kg. per sq. m.	=	.205 lbs. per sq. ft.
1 cm. of mercury	=	.394 in. of mercury
1 cm. of mercury	=	.193 lb. per sq. in.
1 kg. per cu. m.	=	.0624 lb. per cu. ft.
1 cu. m. per kg.	=	16.019 cu. ft. per lb.
1 kgm.	=	7.233 ft.-lbs.
1 metric h.p.	=	.9863 British h.p.
1 kg. per metric h.p.	=	2.235 lbs. per British h.p.
1 sq. m. per metric h.p.	=	10.913 sq. ft. per British h.p.
1 calorie	=	3.968 B.t.u.'s
1 m. per sec.	=	3.281 ft. per sec.
	=	196.860 ft. per min.
		2.236 miles per hr.
1 km. per hr.		.621 miles per hr.

British-Metric		
1 lb. per ft.	=	1.488 kg. per m.
1 lb. per yd.	=	.496 kg. per m.
1 lb. per sq. in.	=	.0703077 kg. per sq. cm
1 lb. per sq. ft.	=	4.883 kg. per sq. m.
1 in. of mercury	=	2.540 cm. of mercury
1 lb. per sq. in.	=	5.170 cm. of mercury
1 lb. per cu. ft.	=	16.020 kg. per cu. m.
1 cu. ft. per lb.	=	.0624 cu. m. per kg.
1 ft.-lb.	=	.138 kgm.
1 British h.p.	=	1.0139 metric h.p.
1 lb. per British h.p.	=	.447 kg. per metric h.p.
1 sq. ft British per h.p.	=	.0916 sq. m. per metric h.p.
1 B.t.u.	=	.252 carlorie
1 ft. per sec. or per min.	=	.305 m. per sec. or per min.
1 mile per hr.	=	.447 m. per sec.
		1.609 km. per hr.

British-Metric Conversion Factors for Units of Pressure

Lbs. per sq. in.	Kgs. per sq. centim.	Lbs. per sq. in.	Kgs. per sq. centim	Lbs. per sq. in.	Kgs. per sq. centim.	Lbs. per sq. in.	Kgs. per sq. centim.
1	.0703	26	1.828	51	3.5857	76	5.3434
2	.1406	27	1.8983	52	3.656	77	5.4138
3	.2109	28	1.9686	53	3.7263	78	5.4841
4	.2812	29	2.0389	54	3.7966	79	5.5544
5	.3515	30	2.1092	55	3.8669	80	5.6247
6	.4218	31	2.1795	56	3.9373	81	5.695
7	.4921	32	2.2498	57	4.0076	82	5.7653
8	.5624	33	2.3202	58	4.0779	83	5.8356
9	.6327	34	2.3905	59	4.1482	84	5.9059
10	.70309	35	2.4608	60	4.2185	85	5.9762
11	.7734	36	2.5311	61	4.2888	86	6.0465
12	.8437	37	2.6014	62	4.3591	87	6.1168
13	.9140	38	2.6717	63	4.4294	88	6.1872
14	.9843	39	2.7420	64	4.4997	89	6.2575
15	1.0546	40	2.8123	65	4.5700	90	6.3278
16	1.1249	41	2.8826	66	4.6404	91	6.3981
17	1.1952	42	2.9529	67	4.7107	92	6.4684
18	1.2655	43	3.0232	68	4.781	93	6.5387
19	1.3358	44	3.0936	69	4.8513	94	6.609
20	1.4062	45	3.1639	70	4.9216	95	6.6793
21	1.4765	46	3.2342	71	4.9919	96	6.7496
22	1.5468	47	3.3045	72	5.0622	97	6.8199
23	1.6171	48	3.3748	73	5.1325	98	6.8902
24	1.6874	49	3.4451	74	5.2028	99	6.9606
25	1.7577	50	3.5154	75	5.2731	100	7.0309

Metric-British Conversion Factors for Units of Pressure

Kgs. per sq. cen.	Lbs. per sq. in.	Kgs. per sq. cen.	Lbs. per sq. in.	Kgs. per sq. cen.	Lbs. per sq. in.	Kgs. per sq. cen.	Lbs. per sq. in.
1	14.223	3.6	51.203	6.2	88.183	8.8	125.162
1.1	15.645	3.7	52.625	6.3	89.605	8.9	126.585
1.2	17.068	3.8	54.047	6.4	91.027	9	128.007
1.3	18.490	3.9	55.470	6.5	92.450	9.1	129.429
1.4	19.912	4	56.892	6.6	93.872	9.2	130.852
1.5	21.335	4.1	58.314	6.7	95.294	9.3	132.274
1.6	22.757	4.2	59.737	6 8	96.716	9.4	133.696
1.7	24.179	4.3	61.159	6.9	98.139	9.5	135.119
1.8	25.601	4.4	62.581	7	99.561	9.6	136.541
1.9	27.024	4.5	64.004	7.1	100.983	9.7	137.963
2	28.446	4.6	65.426	7.2	102.406	9.8	139.385
2.1	29.868	4.7	67.848	7.3	103.828	9.9	140.808
2.2	31.291	4.8	68.270	7.4	105.250	10	142.230
2.3	32.713	4.9	69.693	7.5	106.673	10.1	143.652
2.4	34.135	5	71.115	7.6	108.095	10.2	145.074
2.5	35.558	5.1	72.537	7.7	109.517	10.3	146.497
2.6	36.980	5.2	73.960	7.8	110.939	10.4	147.919
2.7	38.402	5.3	75.382	7.9	112.362	10.5	149.341
2.8	39.824	5.4	76.804	8	113.784	10.6	150.764
2.9	41.247	5.5	78.227	8.1	115.206	10.7	152.186
3	42.669	5.6	79.649	8.2	116.629	10.8	153.608
3.1	44.091	5.7	81.071	8.3	118.051	10.9	155.030
3.2	45.514	5.8	82.493	8.4	119.473	11	156.453
3.3	46.936	5.9	83.916	8.5	120.896	11.1	157.875
3.4	48.358	6	85.338	8.6	122.318	11.2	159.297
3.5	49.781	6.1	86.760	8.7	123.740	11.3	160.720

It is for this latter reason that the millimeter is universally used as a measure of length in machinery manufacture, this little unit being *multiplied* because the decimal division of larger units has been found impracticable. It is for the former reason that units has been both dropped from and added to the original list.

Those who do not know the above facts do not know enough about the subject to make their opinions regarding the wisdom of the adoption of the system of the slightest value.

The customary tables are very misleading. It was inevitable that a schedule of units based on a rigid relationship should contain many that are redundant and fail to contain others required by considerations of convenience. The result is that the tables contain many units that are not used and they omit others which necessity or convenience has brought into use, while, of those given, they fail entirely to indicate those that are used and those that are not.

The accompanying conversion tables are but an illustration of the confusion which the system has already introduced, and every extension of it adds to this confusion, for the dream that it would supplant the old systems has proven as vain as the dream of the millenium. The whole movement for its origin and spread must be regarded as unfortunate and pernicious.

The use of the accompanying tables of equivalents is best shown by an example: Required the metric equivalent of 38.5 ins. From the proper table we find:

ins.	mm.
30	= 762.002
8	= 203.200
.5	= 12.700
38.5	987.902

American-Metric Conversion Factors for Compound Units of Value

From The U. S. Bureau of Standards

Francs per kilogram	Dollars per avoir. pouud	Francs per meter	Dollars per yard	Francs per liter	Dollars per U. S. liquid gal.	Francs per hectoliter	Dollars per U. S. bushel	Marks per kilogram	Dollars per avoir. pound	Marks per meter	Dollars per yard	Marks per liter	Dollars per U. S. liquid gal.	Marks per hectoliter	Dollars per U. S. bushel
1	= .088	1	= .176	1	= .731	1	= .068	1	= .108	1	= .218	1	= .901	1	= .084
2	= .175	2	= .353	2	=1.461	2	= .136	2	= .216	2	= .435	2	=1.802	2	= .168
3	= .263	3	= .529	3	=2.192	3	= .204	3	= .324	3	= .653	3	=2.703	3	= .252
4	= .350	4	= .705	4	=2.922	4	= .272	4	= .432	4	= .871	4	=3.604	4	= .335
5	= .438	5	= .882	5	=3.653	5	= .340	5	= .540	5	=1.088	5	=4.505	5	= .419
6	= .525	6	=1.058	6	=4.384	6	= .408	6	= .648	6	=1.306	6	=5.406	6	= .503
7	= .613	7	=1.234	7	=5.114	7	= .476	7	= .756	7	=1.523	7	=6.307	7	= .587
8	= .700	8	=1.411	8	=5.844	8	= .544	8	= .864	8	=1.741	8	=7.207	8	= .671
9	= .788	9	=1.587	9	=6.575	9	= .612	9	= .972	9	=1.959	9	=8.108	9	= .755
11.423=1		5.667=1		1.369=1		14.703=1		9.263=1		4.595=1		1.110=1		11.923=1	
22.846=2		11.334=2		2.738=2		29.407=2		18.526=2		9.190=2		2.220=2		23.847=2	
34.269=3		17.000=3		4.106=3		44.110=3		27.789=3		13.785=3		3.330=3		35.770=3	
45.691=4		22.667=4		5.475=4		58.813=4		37.052=4		18.380=4		4.440=4		47.693=4	
57.115=5		28.334=5		6.844=5		73.517=5		46.316=5		22.975=5		5.550=5		59.616=5	
68.537=6		34.001=6		8.213=6		88.220=6		55.579=6		27.570=6		6.660=6		71.540=6	
79.960=7		39.668=7		9.581=7		102.923=7		64.842=7		32.165=7		7.770=7		83.463=7	
91.383=8		45.334=8		10.950=8		117.627=8		74.105=8		36.760=8		8.880=8		95.386=8	
102.806=9		51.001=9		12.319=9		132.330=9		83.368=9		41.355=9		9.990=9		107.310=9	

Electrical Horse-power

Amperes

Volts	1	10	20	30	40	50	60	70	80	90	100	110
1	.00134	.0134	.0268	.0402	.0536	.0670	.0804	.0938	.1072	.1206	.1341	.1475
5	.00670	.0670	.1341	.2011	.2681	.3351	.4022	.4692	.5362	.6032	.6703	.7373
10	.01341	.1314	.2681	.4022	.5362	.6703	.8043	.9383	1.072	1.206	1.341	1.475
15	.02011	.2011	.4022	.6032	.8043	1.005	1.206	1.408	1.609	1.810	2.011	2.212
20	.02681	.2681	.5362	.8043	1.072	1.340	1.609	1.877	2.145	2.413	2.681	2.949
25	.03351	.3351	.6703	1.005	1.341	1.676	2.011	2.346	2.681	3.016	3.351	3.686
30	.04022	.4022	.8043	1.206	1.609	2.011	2.413	2.815	3.217	3.619	4.022	4.424
35	.04692	.4692	.9384	1.408	1.877	2.346	2.815	3.284	3.753	4.223	4.692	5.161
40	.05362	.5362	1.072	1.609	2.145	2.681	3.217	3.753	4.290	4.826	5.362	5.898
45	.06032	.6032	1.206	1.810	2.413	3.016	3.619	4.223	4.826	5.439	6.032	6.635
50	.06703	.6703	1.341	2.011	2.681	3.351	4.022	4.692	5.362	6.032	6.703	7.373
75	.10054	1.005	2.011	3.016	4.021	5.027	6.032	7.037	8.043	9.048	10.05	11.06
100	.13405	1.341	2.681	4.022	5.362	6.703	8.043	9.384	10.72	12.06	13.41	14.75
500	.67025	6.703	13.41	20.11	26.81	33.51	40.22	46.92	53.62	60.32	67.03	73.73
1,000	1.3405	13.41	26.81	40.22	53.62	67.03	80.43	93.84	107.2	120.6	134.1	147.5
5,000	6.7025	67.03	134.1	201.1	268.1	335.1	402.2	469.2	536.2	603.2	670.3	737.3
10,000	13.405	134.1	268.1	402.2	536.2	670.3	804.3	938.3	1072.	1206	1341	1475

British-Metric and Metric-British Conversion Factors for Work and Power

	Horse-power Metric to British	Horse-power British to Metric	Foot-pounds to kilogrammeters	Kilogrammeters to foot-pounds
1	.986	1.014	.1383	7.2329
2	1.973	2.028	.2765	14.4659
3	2.959	3.042	.4148	21.6988
4	3.945	4.056	.5530	28.9317
5	4.932	5.069	.6913	36.1646
6	5.918	6.083	.8295	43.3976
7	6,904	7.097	.9678	50.6305
8	7.890	8.111	1.1061	57.8634
9	8.877	9.125	1.2443	65.0963

MATHEMATICAL TABLES

The range of arithmetical tables may be greatly extended by an understanding of a few principles.

Areas of circles of fractional diameters may be obtained from tables of areas of circles whose diameters are whole numbers, by putting the diameter in the form of a decimal. For example, find the area of a circle of .97 in. diameter. The area of 97. is 7389. Point off twice as many decimal places as are in the diameter, and we have .7389 the area. Or take diameter .01 in. The area of 1 is .7854; add four decimals and we have .00007854 in. Or again, take diameter 34.7 ins. The area of 347 is 94,569, and pointing off two decimals gives 945.69 for the area belonging to diameter 34.7

It is often required to find the square or cube root of numbers larger than are given directly in the table. Suppose the square root of 12.850 is desired. Look in the column of squares for the nearest number, and it will be found that the square of 113, which is 12,769, is the nearest, but is too small, and the square root will be a fraction more than 113. To get one decimal place in the root will require two in the number; hence it would make a total of seven figures. Look down the column of squares to where there are seven figures and find the nearest to 12,850 (considering the two right-hand figures out of the seven as decimals), and the nearest number is 12,859.56, and the root is 113.4. With the usual table going up to 1,600 this method is available only for finding the square root with one decimal, of numbers between 1600 and 25,600.

TABLE 1.—FACTORS AND RELATIONS OF π

3.1416 divided by		
2 = 1.5708	68 = .0462	561 = .0056
3 = 1.0472	77 = .0408	616 = .0051
4 = .7854	84 = .0374	714 = .0044
6 = .5236	88 = .0357	748 = .0042
7 = .4488	102 = .0308	924 = .0034
8 = .3927	119 = .0264	952 = .0033
11 = .2856	132 = .0238	1,122 = .0028
12 = .2618	136 = .0231	1,309 = .0024
14 = .2244	154 = .0204	1,428 = .0022
17 = .1848	168 = .0187	1,496 = .0021
21 = .1496	187 = .0168	1,848 = .0017
22 = .1428	204 = .0154	2,244 = .0014
24 = .1309	231 = .0136	2,618 = .0012
28 = .1122	238 = .0132	2,856 = .0011
33 = .0952	264 = .0119	3,927 = .0008
34 = .0924	308 = .0102	4,488 = .0007
42 = .0748	357 = .0088	5,236 = .0006
44 = .0714	374 = .0084	7,854 = .0004
51 = .0616	408 = .0077	10,472 = .0003
56 = .0561	462 = .0068	15,708 = .0002
66 = .0476	476 = .0066	5,280 = .000595

.7854 divided by		
2 = .3927	34 = .0231	238 = .0033
3 = .2618	42 = .0187	357 = .0022
6 = .1309	51 = .0154	374 = .0021
7 = .1122	66 = .0119	462 = .0017
11 = .0714	77 = .0102	561 = .0014
14 = .0561	102 = .0077	714 = .0011
17 = .0462	119 = .0066	1,122 = .0007
21 = .0374	154 = .0051	1,309 = .0006
22 = .0357	187 = .0042	2,618 = .0003
33 = .0238	231 = .0034	3,927 = .0002

Log. π = .4971499	$\sqrt[3]{\pi} = 1.4645919$
$\frac{1}{\pi} = .3183099$	$\pi\sqrt{2} = 4.4428829$
$\frac{1}{\pi^2} = .1013212$	$\frac{\pi}{\sqrt{2}} = 2.2214415$
$\sqrt{\pi} = 1.7724538$	$\frac{\sqrt{2}}{\pi} = .4501582$
$\frac{1}{\sqrt{\pi}} = .5641896$	$\sqrt{\frac{\pi}{2}} = 1.2533141$
$\sqrt{2} = 1.4142136$	$\sqrt{\frac{2}{\pi}} = .7978846$
$\pi^2 = 9.8696044$	
$\pi^3 = 31.0062767$	

The reason for doubling the number of decimal places of the diameter comes from the fact that to find the area of a circle, the diameter is first multiplied by itself, or squared; hence there must be twice as many decimal places in the product, to conform to the rule for multiplication of decimal numbers.

Sometimes it is required to find the area of a circle larger than is in the table. The range of the table may be doubled by taking the area for half of the desired diameter and multiplying it by 4. For example: Required the area for 996 diameter: half of this is 498, the area of which is 194,782, and this multiplied by 4 = 779,128, the area required.

Referring to the table of squares, cubes and roots of numbers, which usually gives the squares and cubes of whole numbers only, it is sometimes required to know the square or cube of a fractional number. To find the square of .9 take the square of 9 and point off two decimal places, giving .81; or the cube, and point off three, giving .729, as all cubed numbers must have three times and all squared numbers two times as many decimals places as there are in the number to be cubed or squared. Finding the square or cube of a whole number and fraction is done the same way. To find the square of $7\frac{1}{4}$, take the square of 725 = 525,625, and pointing off four decimals gives 52.5625; or the cube of $7\frac{1}{4}$ = 381.078125.

By the use of the column of cubes of numbers in the same manner, the cube root with two decimal places may be found for numbers from 1,600 to 4,088; or the root with one decimal place for numbers from 4,096 up to 4,088,324. For example: Find the cube root of 3,504; the nearest number in the column of cubes is 3,375, the cube of 15. As there are to be two decimals in the cube root there must be three times this = 6 added to the number of figures which makes 10. Looking in the column of cubes we find 3,504.881359 (using the six right-hand figures as decimals), and the root is 15.19.

Always be careful to keep in mind that in finding square roots there must be twice as many decimal places in the number as in the root, and in finding cube roots there must be three times the decimal places of the root.

The value of π to eight places of decimals in 3.14159265. The ratio $\frac{355}{113}$ reduced to decimals is 3.1415929, which is far more nearly the true value than 3.1416 which is customarily used. Doubling both numerator and dominator gives $\frac{710}{226}$ which may be found without estimation on the *C*- and *D*-scales of an ordinary slide rule.

Ratios in vulgar fraction form are necessary when calculating gear trains for cutting diametral pitch worms and racks. Following are such values arranged in the order of accuracy:

$$\frac{355}{113}=3.1415929$$

$$\frac{22}{7}=3.1429$$

$$\frac{69}{22}=3.1364$$

$$\frac{47}{15}=3.1333$$

For tabulated change gears for cutting diametral pitch worms see Cutting Diametral Pitch Worms.

The value 3.1416 has many exact factors, as it is the product of $2\times3\times4\times7\times.11\times.17$. Table 1 gives various factors and other relations of π.

TABLE 2.—LOGARITHMS

The supplementary table at the right gives proportional parts without calculation. Thus to find log 2985, opposite 29 and under 8 read .4742 and in the same line of the supplementry table under 5 read 7 which added to .4742 gives .4749, the log required.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6

TABLE 2.—LOGARITHMS—(*Continued*)

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	7717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

TABLE 3.—ANTILOGARITHMS

The supplementary table at the right is used in the same manner as with the previous table. Thus to find the natural number corresponding to the logarithms 4749, opposite 47 and under 4 read 2979 and in the same line under 9 read 6 which added to 2979 gives 2985, the natural number required.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
.00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
.01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
.02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
.03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
.04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
.05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
.06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
.07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
.08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
.09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
.10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
.11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
.12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
.13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
.14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
.15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
.16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
.17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
.18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
.19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
.20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
.21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
.22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
.23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
.24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
.25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
.26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
.27	1826	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
.28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
.29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4

TABLE 3.—ANTILOGARITHMS—(*Continued*)

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
.30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
.31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
.32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
.33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
.34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
.35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
.36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
.37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
.38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
.39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
.40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
.41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
.42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
.43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
.44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
.45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
.46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
.47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
.48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
.49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6
.50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
.51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
.52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
.53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
.54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
.55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
.56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
.57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
.58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
.59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
.60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
.61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
.62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
.63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
.64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
.65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
.66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
.67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
.68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	2	4	6	7	8	9	10
.69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
.70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
.71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
.72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
.73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
.74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
.75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
.76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
.77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
.78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
.79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
.80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
.81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
.82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
.83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
.84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
.85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
.86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
.87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
.88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
.89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
.90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
.91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
.92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
.93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
.94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
.95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
.96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
.97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
.98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
.99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

TABLE 4.—HYPERBOLIC LOGARITHMS

To find the hyperbolic logarithms of larger numbers than those given in the table, divide the number by 10, find the hyp. log. of the quotient and to it add the hyp. log. of 10. Thus to find the hyp. log. of 43.6: $\frac{43.6}{10} = 4.36$; hyp. log. 4.36 = 1.4725 and hyp. log. 10 = 2.3026, the sum of which = 3.7751 = hyp. log. 43.6.

	0	1	2	3	4	5	6	7	8	9
1.0	0.0000	0.00995	0.01980	0.02956	0.03922	0.04879	0.05827	0.06766	0.07696	0.08618
1.1	0.0 531	0.1044	0.1133	0.1222	0.1310	0.1398	0.1484	0.1570	0.1655	0.1739
1.2	0.1823	0.1906	0.1988	0.2070	0.2151	0.2231	0.2311	0.2390	0.2469	0.2546
1.3	0.2624	0.2700	0.2776	0.2852	0.2927	0.3001	0.3075	0.3148	0.3221	0.3293
1.4	0.3365	0.3436	0.3507	0.3577	0.3646	0.3716	0.3784	0.3853	0.3920	0.3988
1.5	0.4055	0.4121	0.4187	0.4253	0.4318	0.4382	0.4447	0.4511	0.4574	0.4637
1.6	0.4700	0.4762	0.4824	0.4886	0.4947	0.5008	0.5068	0.5128	0.5188	0.5247
1.7	0.5306	0.5365	0.5423	0.5481	0.5539	0.5596	0.5653	0.5710	0.5766	0.5822
1.8	0.5878	0.5933	0.5988	0.6043	0.6098	0.6152	0.6206	0.6259	0.6313	0.6366
1.9	0.6418	0.6471	0.6523	0.6575	0.6627	0.6678	0.6729	p.6780	0.6831	0.6881
2.0	0.6931	0.6981	0.7031	0.7080	0.7129	0.7178	0.7227	0.7275	0.7324	0.7372
2.1	0.7419	0.7467	0.7514	0.7561	0.7608	0.7655	0.7701	0.7747	0.7793	0.7839
2.2	0.7884	0.7930	0.7975	0.8020	0.8065	0.8109	0.8154	0.8198	0.8242	0.8286
2.3	0.8329	0.8372	0.8416	0.8459	0.8502	0.8544	0.8587	0.8629	0.8671	0.8713
2.4	0.8755	0.8796	0.8838	0.8879	0.8920	0.8961	0.9002	0.9042	0.9083	0.9123
2.5	0.9163	0.9203	0.9243	0.9282	0.9322	0.9361	0.9400	0.9439	0.9478	0.9517
2.6	0.9555	0.9594	0.9632	0.9670	0.9708	0.9746	0.9783	0.9821	0.9858	0.9895
2.7	0.9933	0.9969	1.0006	1.0043	1.0080	1.0116	1.0152	1.0188	1.0225	1.0260
2.8	1.0296	1.0332	1.0367	1.0403	1.0438	1.0473	1.0508	1.0543	1.0578	1.0613
2.9	1.0647	1.0682	1.0716	1.0750	1.0784	1.0818	1.0852	1.0886	1.0919	1.0953
3.0	1.0986	1.1019	1.1053	1.1086	1.1119	1.1151	1.1184	1.1217	1.1249	1.1282
3.1	1.1314	1.1346	1.1378	1.1410	1.1442	1.1474	1.1506	1.1537	1.1569	1.1600
3.2	1.1632	1.1663	1.1694	1.1725	1.1756	1.1787	1.1817	1.1848	1.1878	1.1909
3.3	1.1939	1.1969	1.2000	1.2030	1.2060	1.2090	1.2119	1.2149	1.2179	1.2208
3.4	1.2238	1.2267	1.2296	1.2326	1.2355	1.2384	1.2413	1.2442	1.2470	1.2499
3.5	1.2528	1.2556	1.2585	1.2613	1.2641	1.2669	1.2698	1.2726	1.2754	1.2782
3.6	1.2809	1.2837	1.2865	1.2892	1.2920	1.2947	1.2975	1.3002	1.3029	1.3056
3.7	1.3083	1.3110	1.3137	1.3164	1.3191	1.3218	1.3244	1.3271	1.3297	1.3324
3.8	1.3350	1.3376	1.3403	1.3429	1.3455	1.3481	1.3507	1.3533	1.3558	1.3584
3.9	1.3610	1.3635	1.3661	1.3686	1.3712	1.3737	1.3762	1.3788	1.3813	1.3838
4.0	1.3863	1.3888	1.3913	1.3938	1.3962	1.3987	1.4012	1.4036	1.4061	1.4085
4.1	1.4110	1.4134	1.4159	1.4183	1.4207	1.4231	1.4255	1.4279	1.4303	1.4327
4.2	1.4351	1.4375	1.4398	1.4422	1.4446	1.4469	1.4403	1.4516	1.4540	1.4563
4.3	1.4586	1.4609	1.4633	1.4656	1.4679	1.4702	1.4725	1.4748	1.4770	1.4793
4.4	1.4816	1.4839	1.4861	1.4884	1.4007	1.4929	1.4051	1.4974	1.4996	1.5019
4.5	1.5041	1.5063	1.5085	1.5107	1.5129	1.5151	1.5173	1.5195	1.5217	1.5239
4.6	1.5261	1.5282	1.5304	1.5326	1.5347	1.5369	1.5390	1.5412	1.5433	1.5454
4.7	1.5476	1.5497	1.5518	1.5539	1.5560	1.5581	1.5602	1.5623	1.5644	1.5665
4.8	1.5686	1.5707	1.5728	1.5748	1.5769	1.5790	1.5810	1.5831	1.5851	1.5872
4.9	1.5892	1.5913	1.5933	1.5953	1.5974	1.5994	1.6014	1.6034	1.6054	1.6074
5.0	1.6094	1.6114	1.6134	1.6154	1.6174	1.6194	1.6214	1.6233	1.6253	1.6273
5.1	1.6292	1.6312	1.6332	1.6351	1.6371	1.6390	1.6409	1.6429	1.6448	1.6467
5.2	1.6487	1.6506	1.6525	1.6544	1.6563	1.6582	1.6601	1.6620	1.6639	1.6658
5.3	1.6677	1.6696	1.6715	1.6734	1.6752	1.6771	1.6790	1.6808	1.6827	1.6845
5.4	1.6864	1.6882	1.6901	1.6919	1.6938	1.6956	1.6974	1.6993	1.7011	1.7029
5.5	1.7047	1.7066	1.7884	1.7102	1.7120	1.7138	1.7156	1.7174	1.7192	1.7210

	0	1	2	3	4	5	6	7	8	9
5.6	1.7228	1.7246	1.7263	1.7281	1.7299	1.7317	1.7334	1.7352	1.7370	1.7387
5.7	1.7405	1.7422	1.7440	1.7457	1.7475	1.7492	1.7509	1.7527	1.7544	1.7561
5.8	1.7579	1.7596	1.7613	1.7630	1.7647	1.7664	1.7681	1.7699	1.7716	1.7733
5.9	1.7750	1.7766	1.7783	1.7800	1.7817	1.7834	1.7851	1.7867	1.7884	1.7901
6.0	1.7918	1.7934	1.7951	1.7967	1.7984	1.8001	1.8017	1.8034	1.8050	1.8066
6.1	1.8083	1.8099	1.8116	1.8132	1.8148	1.8165	1.8181	1.8197	1.8213	1.8229
6.2	1.8245	1.8262	1.8278	1.8294	1.8310	1.8326	1.8342	1.8358	1.8374	1.8390
6.3	1.8405	1.8421	1.8437	1.8453	1.8469	1.8485	1.8500	1.8516	1.8532	1.8547
6.4	1.8563	1.8579	1.8594	1.8610	1.8625	1.8641	1.8656	1.8672	1.8687	1.8703
6.5	1.8718	1.8733	1.8749	1.8764	1.8779	1.8795	1.8810	1.8825	1.8840	1.8856
6.6	1.8871	1.8886	1.8901	1.8916	1.8931	1.8946	1.8961	1.8976	1.8991	1.9006
6.7	1.9021	1.9036	1.9051	1.9066	1.9081	1.9095	1.9110	1.9125	1.9140	1.9155
6.8	1.9169	1.9184	1.9199	1.9213	1.9228	1.9242	1.9257	1.9272	1.9286	1.9301
6.9	1.9315	1.9330	1.9344	1.9359	1.9373	1.9387	1.9402	1.9416	1.9430	1.9445
7.0	1.9459	1.9473	1.9488	1.9502	1.9516	1.9530	1.9544	1.9559	1.9573	1.9587
7.1	1.9601	1.9615	1.9629	1.9643	1.9657	1.9671	1.9685	1.9699	1.9713	1.9727
7.2	1.9741	1.9755	1.9769	1.9782	1.9796	1.9810	1.9824	1.9838	1.9851	1.9865
7.3	1.9879	1.9892	1.9906	1.9920	1.9933	1.9947	1.9961	1.9974	1.9988	2.0001
7.4	2.0015	2.0028	2.0042	2.0055	2.0069	2.0082	2.0096	2.0109	2.0122	2.0136
7.5	2.0149	2.0162	2.0176	2.0189	2.0202	2.0215	2.0229	2.0242	2.0255	2.0268
7.6	2.0281	2.0295	2.0308	2.0321	2.0334	2.0347	2.0360	2.0373	2.0386	2.0399
7.7	2.0412	2.0425	2.0438	2.0451	2.0464	2.0477	2.0490	2.0503	2.0516	2.0528
7.8	2.0541	2.0554	2.0567	2.0580	2.0592	2.0605	2.0618	2.0631	2.0643	2.0656
7.9	2.0668	2.0681	2.0694	2.0707	2.0719	2.0732	2.0744	2.0757	2.0769	2.0782
8.0	2.0794	2.0807	2.0819	2.0832	2.0844	2.0857	2.0860	2.0881	2.0894	2.0906
8.1	2.0919	2.0931	2.0943	2.0956	2.0968	2.0980	2.0992	2.1005	2.1017	2.1029
8.2	2.1041	2.1054	2.1066	2.1078	2.1090	2.1102	2.1114	2.1126	2.1138	2.1150
8.3	2.1163	2.1175	2.1187	2.1199	2.1211	2.1223	2.1235	2.1247	2.1258	2.1270
8.4	2.1282	2.1294	2.1306	2.1318	2.1330	2.1342	2.1353	2.1365	2.1377	2.1389
8.5	2.1401	2.1412	2.1424	2.1436	2.1448	2.1459	2.1471	2.1483	2.1494	2.1506
8.6	2.1518	2.1529	2.1541	2.1552	2.1564	2.1576	2.1587	2.1599	2.1610	2.1622
8.7	2.1633	2.1645	2.1656	2.1668	2.1679	2.1691	2.1702	2.1713	2.1725	2.1736
8.8	2.1748	2.1759	2.1770	2.1782	2.1793	2.1804	2.1815	2.1827	2.1838	2.1849
8.9	2.1861	2.1872	2.1883	2.1894	2.1905	2.1917	2.1928	2.1939	2.1950	2.1961
9.0	2.1972	2.1983	2.1994	2.2006	2.2017	2.2028	2.2039	2.2050	2.2061	2.2072
9.1	2.2083	2.2094	2.2105	2.2116	2.2127	2.2138	2.2148	2.2159	2.2170	2.2181
9.2	2.2192	2.2203	2.2214	2.2225	2.2235	2.2246	2.2257	2.2268	2.2279	2.2289
9.3	2.2300	2.2311	2.2322	2.2332	2.2343	2.2354	2.2364	2.2375	2.2386	2.2396
9.4	2.2407	2.2418	2.2428	2.2439	2.2450	2.2460	2.2471	2.2481	2.2492	2.2502
9.5	2.2513	2.2523	2.2534	2.2544	2.2555	2.2565	2.2576	2.2586	2.2597	2.2607
9.6	2.2618	2.2628	2.2638	2.2649	2.2659	2.2670	2.2680	2.2690	2.2701	2.2711
9.7	2.2721	2.2732	2.2742	2.2752	2.2762	2.2773	2.2783	2.2793	2.2803	2.2814
9.8	2.2824	2.2834	2.2844	2.2854	2.2865	2.2875	2.2885	2.2895	2.2905	2.2915
9.9	2.2925	2.2935	2.2946	2.2956	2.2966	2.2976	2.2986	2.2996	2.3006	2.3016
10.0	2.3026									

TABLE 5.—NATURAL TRIGONOMETRIC FUNCTIONS

′	0° Tan.	0° Co-tan.	1° Tan.	1° Co-tan.	2° Tan.	2° Co-tan.	3° Tan.	3° Co-tan.	′
0	.00000	Infinite.	.01746	57.2900	.03492	28.6363	.05241	19.0811	60
1	.00029	3437.750	.01775	56.3506	.03521	28.3994	.05270	18.9755	59
2	.00058	1718.870	.01804	55.4415	.03550	28.1664	.05299	18.8711	58
3	.00087	1145.920	.01833	54.5613	.03579	27.9372	.05328	18.7678	57
4	.00116	859.436	.01862	53.7086	.03609	27.7117	.05357	18.6656	56
5	.00145	687.549	.01891	52.8821	.03638	27.4899	.05387	18.5645	55
6	.00175	572.957	.01920	52.0807	.03667	27.2715	.05416	18.4645	54
7	.00204	491.106	.01949	51.3032	.03696	27.0566	.05445	18.3655	53
8	.00233	429.718	.01978	50.5485	.03725	26.8450	.05474	18.2677	52
9	.00262	381.971	.02007	49.8157	.03754	26.6367	.05503	18.1708	51
10	.00291	343.774	.02036	49.1039	.03783	26.4316	.05533	18.0750	50
11	.00320	312.521	.02066	48.4121	.03812	26.2296	.05562	17.9802	49
12	.00349	286.478	.02095	47.7395	.03842	26.0307	.05591	17.8863	48
13	.00378	264.441	.02124	47.0853	.03871	25.8348	.05620	17.7934	47
14	.00407	245.552	.02153	46.4489	.03900	25.6418	.05649	17.7015	46
15	.00436	229.182	.02182	45.8294	.03929	25.4517	.05678	17.6106	45
16	.00465	214.858	.02211	45.2261	.03958	25.2644	.05708	17.5205	44
17	.00495	202.219	.02240	44.6386	.03987	25.0798	.05737	17.4314	43
18	.00524	190.984	.02269	44.0661	.04016	24.8978	.05766	17.3432	42
19	.00553	180.932	.02298	43.5081	.04046	24.7185	.05795	17.2558	41
20	.00582	171.885	.02328	42.9641	.04075	24.5418	.05824	17.1693	40
21	.00611	163.700	.02357	42.4335	.04104	24.3675	.05854	17.0837	39
22	.00640	156.259	. 386	41.9158	.04133	24.1957	.05883	16.9990	38
23	.00669	149.465	.02415	41.4106	.04162	24.0263	.05912	16.9150	37
24	.00698	143.237	.02444	40.9174	.04191	23.8593	.05941	16.8319	36
25	.00727	137.507	.02473	40.4358	.04220	23.6945	.05970	16.7496	35
26	.00756	132.219	.02502	39.9655	.04250	23.5321	.05999	16.6681	34
27	.00785	127.321	.02531	39.5059	.04279	23.3718	.06029	16.5874	33
28	.00814	122.774	.02560	39.0568	.04308	23.2137	.06058	16.5075	32
29	.00844	118.540	.02589	38.6177	.04337	23.0577	.06087	16.4283	31
30	.00873	114.589	.02619	38.1885	.04366	22.9038	.06116	16.3499	30
31	.00902	110.892	.02648	37.7686	.04395	22.7519	.06145	16.2722	29
32	.00931	107.426	.02677	37.3579	.04424	22.6020	.06175	16.1952	28
33	.00960	104.171	.02706	36.9560	.04454	22.4541	.06204	16.1190	27
34	.00989	101.107	.02735	36.5627	.04483	22.3081	.06233	16.0435	26
35	.01018	98.2179	.02764	36.1776	.04512	22.1640	.06262	15.9687	25
36	.01047	95.4895	.02793	35.8006	.04541	22.0217	.06291	15.8945	24
37	.01076	92.9085	.02822	35.4313	.04570	21.8813	.06321	15.8211	23
38	.01105	90.4633	.02851	35.0695	.04599	21.7426	.06350	15.7483	22
39	.01135	88.1436	.02881	34.7151	.04628	21.6056	.06379	15.6762	21
40	.01164	85.9398	.02910	34.3678	.04658	21.4704	.06408	15.6048	20
41	.01193	83.8435	.02939	34.0273	.04687	21.3369	.06437	15.5340	19
42	.01222	81.8470	.02968	33.6935	.04716	21.2049	.06467	15.4638	18
43	.01251	79.9434	.02997	33.3662	.04745	21.0747	.06496	15.3943	17
44	.01280	78.1263	.03026	33.0452	.04774	20.9460	.06525	15.3254	16
45	.01309	76.3900	.03055	32.7303	.04803	20.8188	.06554	15.2571	15
46	.01338	74.7292	.03084	32.4213	.04832	20.6932	.06584	15.1893	14
47	.01367	73.1390	.03114	32.1181	.04862	20.5691	.06613	15.1222	13
48	.01396	71.6151	.03143	31.8205	.04891	20.4465	.06642	15.0557	12
49	.01425	70.1533	.03172	31.5284	.04920	20.3253	.06671	14.9898	11
50	.01455	68.7501	.03201	31.2416	.04949	20.2056	.06700	14.9244	10
51	.01484	67.4019	.03230	30.9599	.04978	20.0872	.06730	14.8596	9
52	.01513	66.1055	.03259	30.6833	.05007	19.9702	.06759	14.7954	8
53	.01542	64.8580	.03288	30.4116	.05037	19.8546	.06788	14.7317	7
54	.01571	63.6567	.03317	30.1446	.05066	19.7403	.06817	14.6685	6
55	.01600	62.4992	.03346	29.8823	.05095	19.6273	.06847	14.6059	5
56	.01629	61.3829	.03376	29.6245	.05124	19.5156	.06876	14.5438	4
57	.01658	60.3058	.03405	29.3711	.05153	19.4051	.06905	14.4823	3
58	.01687	59.2659	.03434	29.1220	.05182	19.2959	.06934	14.4212	2
59	.01716	58.2612	.03463	28.8771	.05212	19.1879	.06963	14.3607	1
60	.01746	57.2900	.03492	28.6363	.05241	19.0811	.06993	14.3007	0
′	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	′
	89°		88°		87°		86°		

′	4° Tan.	4° Co-tan.	5° Tan.	5° Co-tan.	6° Tan.	6° Co-tan.	7° Tan.	7° Co-tan.	′
0	.06993	14.3007	.08749	11.4301	.10510	9.51436	.12278	8.14435	60
1	.07022	14.2411	.08778	11.3919	.10540	9.48781	.12308	8.12481	59
2	.07051	14.1821	.08807	11.3540	.10569	9.46141	.12338	8.10536	58
3	.07080	14.1235	.08837	11.3163	.10599	9.43515	.12367	8.08600	57
4	.07110	14.0655	.08866	11.2789	.10628	9.40904	.12397	8.06674	56
5	.07139	14.0079	.08895	11.2417	.10657	9.38307	.12426	8.04756	55
0	.07168	13.9507	.08925	11.2048	.10687	9.35724	.12456	8.02848	54
7	.07197	13.8940	.08954	11.1681	.10716	9.33154	.12485	8.00948	53
8	.07227	13.8378	.08983	11.1316	.10746	9.30599	.12515	7.99058	52
9	.07256	13.7821	.09013	11.0954	.10775	9.28058	.12544	7.97176	51
10	.07285	13.7267	.09042	11.0594	.10805	9.25530	.12574	7.95302	50
11	.07314	13.6719	.09071	11.0237	.10834	9.23016	.12603	7.93438	49
12	.07344	13.6174	.09101	10.9882	.10863	9.20516	.12633	7.91582	48
13	.07373	13.5634	.09130	10.9529	.10893	9.18028	.12662	7.89734	47
14	.07402	13.5098	.09159	10.9178	.10922	9.15554	.12692	7.87895	46
15	.07431	13.4566	.09189	10.8829	.10952	9.13093	.12722	7.86064	45
16	.07461	13.4039	.09218	10.8483	.10981	9.10646	.12751	7.84242	44
17	.07490	13.3515	.09247	10.8139	.11011	9.08211	.12781	7.82428	43
18	.07519	13.2996	.09277	10.7797	.11040	9.05789	.12810	7.80622	42
19	.07548	13.2480	.09306	10.7457	.11070	9.03379	.12840	7.78825	41
20	.07578	13.1969	.09335	10.7119	. 11099	9.00983	.12869	7.77035	40
21	.07607	13.1461	.09365	10.6783	.11128	8.98598	.12899	7.75254	39
22	.07636	13.0958	.09394	10.6450	.11158	8.96227	.12929	7.73480	38
23	.07665	13.0458	.09423	10.6118	.11187	8.93867	.12958	7.71715	37
24	.07695	12.9962	.09453	10.5789	.11217	8.91520	.12988	7.69957	36
25	.07724	12.9469	.09482	10.5462	.11246	8.89185	.13017	7.68208	35
26	.07753	12.8981	.09511	10.5136	.11276	8.86862	.13047	7.66466	34
27	.07782	12.8496	.09541	10.4813	.11305	8.84551	.13076	7.64732	33
28	.07812	12.8014	.09570	10.4491	.11335	8.82252	.13106	7.63005	32
29	.07841	12.7536	.09600	10.4172	.11364	8.79964	.13136	7.61287	31
30	.07870	12.7062	.09629	10.3854	.11394	8.77689	.13165	7.59575	30
31	.07899	12.6591	.09658	10.3538	.11423	8.75425	.13195	7.57872	29
32	.07929	12.6124	.09688	10.3224	.11452	8.73172	.13224	7.56176	28
33	.07958	12.5660	.09717	10.2913	.11482	8.70931	.13254	7.54487	27
34	.07987	12.5199	.09746	10.2602	.11511	8.68701	.13284	7.52806	26
35	.08017	12.4742	.09776	10.2294	.11541	8.66482	.13313	7.51132	25
36	.08046	12.4288	.09805	10.1988	.11570	8.64275	.13343	7.49465	24
37	.08075	12.3838	.09834	10.1683	.11600	8.62078	.13372	7.47806	23
38	.08104	12.3390	.09864	10.1381	.11629	8.59893	.13402	7.46154	22
39	.08134	12.2946	.09893	10.1080	.11659	8.57718	.13432	7.44509	21
40	.08163	12.2505	.09923	10.0780	.11688	8.55555	.13461	7.42871	20
41	.08192	12.2067	.09952	10.0483	.11718	8.53402	.13491	7.41240	19
42	.08221	12.1632	.09981	10.0187	.11747	8.51259	.13521	7.39616	18
43	.08251	12.1201	.10011	9.98931	.11777	8.49128	.13550	7.37999	17
44	.08280	12.0772	.10040	9.96007	.11806	8.47007	.13580	7.36389	16
45	.08309	12.0346	.10069	9.93101	.11836	8.44806	.13609	7.34786	15
46	.08339	11.9923	.10099	9.90211	.11865	8.42795	.13639	7.33190	14
47	.08368	11.9504	.10128	9.87338	.11895	8.40705	.13669	7.31600	13
48	.08397	11.9087	.10158	9.84482	.11924	8.38625	.13698	7.30018	12
49	.08427	11.8673	.10187	9.81641	.11954	8.36555	.13728	7.28442	11
50	.08456	11.8262	.10216	9.78817	.11983	8.34496	.13758	7.26873	10
51	.08485	11.7853	.10246	9.76009	.12013	8.32446	.13787	7.25310	9
52	.08514	11.7448	.10275	9.73217	.12042	8.30406	.13817	7.23754	8
53	.08544	11.7045	.10305	9.70441	.12072	8.28376	.13846	7.22204	7
54	.08573	11.6645	.10334	9.67680	.12101	8.26355	.13876	7.20661	6
55	.08602	11.6248	.10363	9.64935	.12131	8.24345	.13906	7.19125	5
56	.08632	11.5853	.10393	9.62205	.12160	8.22344	.13935	7.17594	4
57	.08661	11.5461	.10422	9.59490	.12190	8.20352	.13965	7.16071	3
58	.08690	11.5072	.10452	9.56791	.12219	8.18370	.13995	7.14553	2
59	.08720	11.4685	.10481	9.54106	.12249	8.16398	.14024	7.13042	1
60	.08749	11.4301	.10510	9.51436	.12278	8.14435	.14054	7.11537	0
′	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	′
	85°		84°		83°		82°		

TABLE 5.—NATURAL TRIGONOMETRIC FUNCTIONS—(*Continued*)

′	8°		9°		10°		11°	
	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.
0	.14054	7.11537	.15838	6.31375	.17633	5.67128	.19438	5.14455
1	.14084	7.10038	.15868	6.30189	.17663	5.66165	.19468	5.13658
2	.14113	7.08546	.15898	6.29007	.17693	5.65205	.19498	5.12862
3	.14143	7.07059	.15928	6.27829	.17723	5.64248	.19529	5.12069
4	.14173	7.05579	.15958	6.26655	.17753	5.63295	.19559	5.11279
5	.14202	7.04105	.15988	6.25486	.17783	5.62344	.19589	5.10490
6	.14232	7.02637	.16017	6.24321	.17813	5.61397	.19619	5.09704
7	.14262	7.01174	.16047	6.23160	.17843	5.60452	.19649	5.08921
8	.14291	6.99718	.16077	6.22003	.17873	5.59511	.19680	5.08139
9	.14321	6.98268	.16107	6.20851	.17903	5.58573	.19710	5.07360
10	.14351	6.96823	.16137	6.19703	.17933	5.57638	.19740	5.06584
11	.14381	6.95385	.16167	6.18559	.17963	5.56706	.19770	5.05809
12	.14410	6.93952	.16196	6.17419	.17993	5.55777	.19801	5.05037
13	.14440	6.92525	.16226	6.16283	.18023	5.54851	.19831	5.04267
14	.14470	6.91104	.16256	6.15151	.18053	5.53927	.19861	5.03499
15	.14499	6.89688	.16286	6.14023	.18083	5.53007	.19891	5.02734
16	.14529	6.88278	.16316	6.12899	.18113	5.52090	.19921	5.01971
17	.14559	6.86874	.16346	6.11779	.18143	5.51176	.19952	5.01210
18	.14588	6.85475	.16376	6.10664	.18173	5.50264	.19982	5.00451
19	.14618	6.84082	.16405	6.09552	.18203	5.49356	.20012	4.99695
20	.14648	6.82694	.16435	6.08444	.18233	5.48451	.20042	4.98940
21	.14678	6.81312	.16465	6.07340	.18263	5.47548	.20073	4.98188
22	.14707	6.79936	.16495	6.06240	.18293	5.46648	.20103	4.97438
23	.14737	6.78564	.16525	6.05143	.18323	5.45751	.20133	4.96690
24	.14767	6.77199	.16555	6.04051	.18353	5.44857	.20164	4.95945
25	.14796	6.75838	.16585	6.02962	.18384	5.43966	.20194	4.95201
26	.14826	6.74483	.16615	6.01878	.18414	5.43077	.20224	4.94460
27	.14856	6.73133	.16645	6.00797	.18444	5.42192	.20254	4.93721
28	.14886	6.71789	.16674	5.99720	.18474	5.41309	.20285	4.92984
29	.14915	6.70450	.16704	5.98646	.18504	5.40429	.20315	4.92249
30	.14945	6.69116	.16734	5.97576	.18534	5.39552	.20345	4.91516
31	.14975	6.67787	.16764	5.96510	.18564	5.38677	.20376	4.90785
32	.15005	6.66463	.16794	5.95448	.18594	5.37805	.20406	4.90056
33	.15034	6.65144	.16824	5.94390	.18624	5.36936	.20436	4.89330
34	.15064	6.63831	.16854	5.93335	.18654	5.36070	.20466	4.88605
35	.15094	6.62523	.16884	5.92283	.18684	5.35206	.20497	4.87882
36	.15124	6.61219	.16914	5.91236	.18714	5.34345	.20527	4.87162
37	.15153	6.59921	.16944	5.90191	.18745	5.33487	.20557	4.86444
38	.15183	6.58627	.16974	5.89151	.18775	5.32631	.20588	4.85727
39	.15213	6.57339	.17004	5.88114	.18805	5.31778	.20618	4.85013
40	.15243	6.56055	.17033	5.87080	.18835	5.30928	.20648	4.84300
41	.15272	6.54777	.17063	5.86051	.18865	5.30080	.20679	4.83590
42	.15302	6.53503	.17093	5.85024	.18895	5.29235	.20709	4.82882
43	.15332	6.52234	.17123	5.84001	.18925	5.28393	.20739	4.82175
44	.15362	6.50970	.17153	5.82982	.18955	5.27553	.20770	4.81471
45	.15391	6.49710	.17183	5.81966	.18986	5.26715	.20800	4.80769
46	.15421	6.48456	.17213	5.80953	.19016	5.25880	.20830	4.80068
47	.15451	6.47206	.17243	5.79944	.19046	5.25048	.20861	4.79370
48	.15481	6.45961	.17273	5.78938	.19076	5.24218	.20891	4.78673
49	.15511	6.44720	.17303	5.77936	.19106	5.23391	.20921	4.77978
50	.15540	6.43484	.17333	5.76937	.19136	5.22566	.20952	4.77286
51	.15570	6.42253	.17363	5.75941	.19166	5.21744	.20982	4.76595
52	.15600	6.41026	.17393	5.74949	.19197	5.20925	.21013	4.75906
53	.15630	6.39804	.17423	5.73960	.19227	5.20107	.21043	4.75219
54	.15660	6.38587	.17453	5.72974	.19257	5.19293	.21073	4.74534
55	.15689	6.37374	.17483	5.71992	.19287	5.18480	.21104	4.73851
56	.15719	6.36165	.17513	5.71013	.19317	5.17671	.21134	4.73170
57	.15749	6.34961	.17543	5.70037	.19347	5.16863	.21164	4.72490
58	.15779	6.33761	.17573	5.69064	.19378	5.16058	.21195	4.71813
59	.15809	6.32566	.17603	5.68094	.19408	5.15256	.21225	4.71137
60	.15838	6.31375	.17633	5.67128	.19438	5.14455	.21256	4.70463
′	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.
	81°		80°		79°		78°	

′	12°		13°		14°		15°		′
	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	
0	.21256	4.70463	.23087	4.33148	.24933	4.01078	.26795	3.73205	60
1	.21286	4.69791	.23117	4.32573	.24964	4.00582	.26826	3.72771	59
2	.21316	4.69121	.23148	4.32001	.24995	4.00086	.26857	3.72338	58
3	.21347	4.68452	.23179	4.31430	.25026	3.99592	.26888	3.71907	57
4	.21377	4.67786	.23209	4.30860	.25056	3.99099	.26920	3.71476	56
5	.21408	4.67121	.23240	4.30291	.25087	3.98607	.26951	3.71046	55
6	.21438	4.66458	.23271	4.29724	.25118	3.98117	.26982	3.70616	54
7	.21469	4.65797	.23301	4.29159	.25149	3.97627	.27013	3.70188	53
8	.21499	4.65138	.23332	4.28595	.25180	3.97139	.27044	3.69761	52
9	.21529	4.64480	.23363	4.28032	.25211	3.96651	.27076	3.69335	51
10	.21560	4.63825	.23393	4.27471	.25242	3.96165	.27107	3.68909	50
11	.21590	4.63171	.23424	4.26911	.25273	3.95680	.27138	3.68485	49
12	.21621	4.62518	.23455	4.26352	.25304	3.95196	.27169	3.68061	48
13	.21651	4.61868	.23485	4.25795	.25335	3.94713	.27201	3.67638	47
14	.21682	4.61219	.23516	4.25239	.25366	3.94232	.27232	3.67217	46
15	.21712	4.60572	.23547	4.24685	.25397	3.93751	.27263	3.66796	45
16	.21743	4.59927	.23578	4.24132	.25428	3.93271	.27294	3.66376	44
17	.21773	4.59283	.23608	4.23580	.25459	3.92793	.27326	3.65957	43
18	.21804	4.58641	.23639	4.23030	.25490	3.92316	.27357	3.65538	42
19	.21834	4.58001	.23670	4.22481	.25521	3.91839	.27388	3.65121	41
20	.21864	4.57363	.23700	4.21933	.25552	3.91364	.27419	3.64705	40
21	.21895	4.56726	.23731	4.21387	.25583	3.90890	.27451	3.64289	39
22	.21925	4.56091	.23762	4.20842	.25614	3.90417	.27482	3.63874	38
23	.21956	4.55458	.23793	4.20298	.25645	3.89945	.27513	3.63461	37
24	.21986	4.54826	.23823	4.19756	.25676	3.89474	.27545	3.63048	36
25	.22017	4.54196	.23854	4.19215	.25707	3.89004	.27576	3.62636	35
26	.22047	4.53568	.23885	4.18675	.25738	3.88536	.27607	3.62224	34
27	.22078	4.52941	.23916	4.18137	.25769	3.88068	.27638	3.61814	33
28	.22108	4.52316	.23946	4.17600	.25800	3.87601	.27670	3.61405	32
29	.22139	4.51693	.23977	4.17064	.25831	3.87136	.27701	3.60996	31
30	.22169	4.51071	.24008	4.16530	.25862	3.86671	.27732	3.60588	30
31	.22200	4.50451	.24039	4.15997	.25893	3.86208	.27764	3.60181	29
32	.22231	4.49832	.24069	4.15465	.25924	3.85745	.27795	3.59775	28
33	.22261	4.49215	.24100	4.14934	.25955	3.85284	.27826	3.59370	27
34	.22292	4.48600	.24131	4.14405	.25986	3.84824	.27858	3.58966	26
35	.22322	4.47986	.24162	4.13877	.26017	3.84364	.27889	3.58562	25
36	.22353	4.47374	.24193	4.13350	.26048	3.83906	.27921	3.58160	24
37	.22383	4.46764	.24223	4.12825	.26079	3.83449	.27952	3.57758	23
38	.22414	4.46155	.24254	4.12301	.26110	3.82992	.27983	3.57357	22
39	.22444	4.45548	.24285	4.11778	.26141	3.82537	.28015	3.56957	21
40	.22475	4.44942	.24316	4.11256	.26172	3.82083	.28046	3.56557	20
41	.22505	4.44338	.24347	4.10736	.26203	3.81630	.28077	3.56159	19
42	.22536	4.43735	.24377	4.10216	.26235	3.81177	.28109	3.55761	18
43	.22567	4.43134	.24408	4.09699	.26266	3.80726	.28140	3.55364	17
44	.22597	4.42534	.24439	4.09182	.26297	3.80276	.28172	3.54968	16
45	.22628	4.41936	.24470	4.08666	.26328	3.79827	.28203	3.54573	15
46	.22658	4.41340	.24501	4.08152	.26359	3.79378	.28234	3.54179	14
47	.22689	4.40745	.24532	4.07639	.26390	3.78931	.28266	3.53785	13
48	.22719	4.40152	.24562	4.07127	.26421	3.78485	.28297	3.53393	12
49	.22750	4.39560	.24593	4.06616	.26452	3.78040	.28329	3.53001	11
50	.22781	4.38969	.24624	4.06107	.26483	3.77595	.28360	3.52609	10
51	.22811	4.38381	.24655	4.05599	.26515	3.77152	.28391	3.52219	9
52	.22842	4.37793	.24686	4.05092	.26546	3.76709	.28423	3.51829	8
53	.22872	4.37207	.24717	4.04586	.26577	3.76268	.28454	3.51441	7
54	.22903	4.36623	.24747	4.04081	.26608	3.75828	.28486	3.51053	6
55	.22934	4.36040	.24778	4.03578	.26639	3.75388	.28517	3.50666	5
56	.22964	4.35459	.24809	4.03075	.26670	3.74950	.28549	3.50279	4
57	.22995	4.34879	.24840	4.02574	.26701	3.74512	.28580	3.49894	3
58	.23026	4.34300	.24871	4.02074	.26733	3.74075	.28612	3.49509	2
59	.23056	4.33723	.24902	4.01576	.26764	3.73640	.28643	3.49125	1
60	.23087	4.33148	.24933	4.01078	.26795	3.73205	.28675	3.48741	0
′	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	′
	77°		76°		75°		74°		

TABLE 5.—NATURAL TRIGONOMETRIC FUNCTIONS—(*Continued*)

′	16°		17°		18°		19°		′
	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	
0	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	60
1	.28706	3.48359	.30605	3.26745	.32524	3.07464	.34465	2.90147	59
2	.28738	3.47977	.30637	3.26406	.32556	3.07160	.34498	2.89873	58
3	.28769	3.47596	.30669	3.26067	.32588	3.06857	.34530	2.89600	57
4	.28800	3.47216	.30700	3.25729	.32621	3.06554	.34563	2.89327	56
5	.28832	3.46837	.30732	3.25392	.32653	3.06252	.34596	2.89055	55
6	.28864	3.46458	.30764	3.25055	.32685	3.05950	.34628	2.88783	54
7	.28895	3.46080	.30796	3.24719	.32717	3.05649	.34661	2.88511	53
8	.28927	3.45703	.30828	3.24383	.32749	3.05349	.34693	2.88240	52
9	.28958	3.45327	.30860	3.24049	.32782	3.05049	.34726	2.87970	51
10	.28990	3.44951	.30891	3.23714	.32814	3.04749	.34758	2.87700	50
11	.29021	3.44576	.30923	3.23381	.32846	3.04450	.34791	2.87430	49
12	.29053	3.44202	.30955	3.23048	.32878	3.04152	.34824	2.87161	48
13	.29084	3.43829	.30987	3.22715	.32911	3.03854	.34856	2.86892	47
14	.29116	3.43456	.31019	3.22384	.32943	3.03556	.34889	2.86624	46
15	.29147	3.43084	.31051	3.22053	.32975	3.03260	.34922	2.86356	45
16	.29179	3.42713	.31083	3.21722	.33007	3.02963	.34954	2.86089	44
17	.29210	3.42343	.31115	3.21392	.33040	3.02667	.34987	2.85822	43
18	.29242	3.41973	.31147	3.21063	.33072	3.02372	.35019	2.85555	42
19	.29274	3.41604	.31178	3.20734	.33104	3.02077	.35052	2.85289	41
20	.29305	3.41236	.31210	3.20406	.33136	3.01783	.35085	2.85023	40
21	.29337	3.40869	.31242	3.20079	.33160	3.01489	.35117	2.84758	39
22	.29368	3.40502	.31274	3.19752	.33201	3.01196	.35150	2.84494	38
23	.29400	3.40136	.31306	3.19426	.33233	3.00903	.35183	2.84229	37
24	.29432	3.39771	.31338	3.19100	.33266	3.00611	.35216	2.83965	36
25	.29463	3.39406	.31370	3.18775	.33298	3.00319	.35248	2.83702	35
26	.29495	3.39042	.31402	3.18451	.33330	3.00028	.35281	2.83439	34
27	.29526	3.38679	.31434	3.18127	.33363	2.99738	.35314	2.83176	33
28	.29558	3.38317	.31466	3.17804	.33395	2.99447	.35346	2.82914	32
29	.29590	3.37955	.31498	3.17481	.33427	2.99158	.35379	2.82653	31
30	.29621	3.37594	.31530	3.17159	.33460	2.98868	.35412	2.82391	30
31	.29653	3.37234	.31562	3.16838	.33492	2.98580	.35445	2.82130	29
32	.29685	3.36875	.31594	3.16517	.33524	2.98292	.35477	2.81870	28
33	.29716	3.36516	.31626	3.16197	.33557	2.98004	.35510	2.81610	27
34	.29748	3.36158	.31658	3.15877	.33589	2.97717	.35543	2.81350	26
35	.29780	3.35800	.31690	3.15558	.33621	2.97430	.35576	2.81091	25
36	.29811	3.35443	.31722	3.15240	.33654	2.97144	.35608	2.80833	24
37	.29843	3.35087	.31754	3.14922	.33686	2.96858	.35641	2.80574	23
38	.29875	3.34732	.31786	3.14605	.33718	2.96573	.35674	2.80316	22
39	.29906	3.34377	.31818	3.14288	.33751	2.96288	.35707	2.80059	21
40	.29938	3.34023	.31850	3.13972	.33783	2.96004	.35740	2.79802	20
41	.29970	3.33670	.31882	3.13656	.33816	2.95721	.35772	2.79545	19
42	.30001	3.33317	.31914	3.13341	.33848	2.95437	.35805	2.79289	18
43	.30033	3.32965	.31946	3.13027	.33881	2.95155	.35838	2.79033	17
44	.30065	3.32614	.31978	3.12713	.33913	2.94872	.35871	2.78778	16
45	.30097	3.32264	.32010	3.12400	.33945	2.94590	.35904	2.78523	15
46	.30128	3.31914	.32042	3.12087	.33978	2.94309	.35937	2.78269	14
47	.30160	3.31565	.32074	3.11775	.34010	2.94028	.35969	2.78014	13
48	.30192	3.31216	.32106	3.11464	.34043	2.93748	.36002	2.77761	12
49	.30224	3.30868	.32139	3.11153	.34075	2.93468	.36035	2.77507	11
50	.30255	3.30521	.32171	3.10842	.34108	2.93189	.36068	2.77254	10
51	.30287	3.30174	.32203	3.10532	.34140	2.92910	.36101	2.77002	9
52	.30319	3.29829	.32235	3.10223	.34173	2.92632	.36134	2.76750	8
53	.30351	3.29483	.32267	3.09914	.34205	2.92354	.36167	2.76498	7
54	.30382	3.29139	.32299	3.09606	.34238	2.92076	.36199	2.76247	6
55	.30414	3.28795	.32331	3.09298	.34270	2.91799	.36232	2.75996	5
56	.30446	3.28452	.32363	3.08991	.34303	2.91523	.36265	2.75746	4
57	.30478	3.28109	.32396	3.08685	.34335	2.91246	.36298	2.75496	3
58	.30509	3.27767	.32428	3.08379	.34368	2.90971	.36331	2.75246	2
59	.30541	3.27426	.32460	3.08073	.34400	2.90696	.36364	2.74997	1
60	.30573	3.27085	.32492	3.07768	.34433	2.90421	.36397	2.74748	0
′	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	′
	73°		72°		71°		70°		

′	20°		21°		22°		23°		′
	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	
0	.36397	2.74748	.38386	2.60509	.40403	2.47509	.42447	2.35585	60
1	.36430	2.74499	.38420	2.60283	.40436	2.47302	.42482	2.35395	59
2	.36463	2.74251	.38453	2.60057	.40470	2.47095	.42516	2.35205	58
3	.36496	2.74004	.38487	2.59831	.40504	2.46888	.42551	2.35015	57
4	.36529	2.73756	.38520	2.59606	.40538	2.46682	.42585	2.34825	56
5	.36562	2.73509	.38553	2.59381	.40572	2.46476	.42619	2.34636	55
6	.36595	2.73263	.38587	2.59156	.40606	2.46270	.42654	2.34447	54
7	.36628	2.73017	.38620	2.58932	.40640	2.46065	.42688	2.34258	53
8	.36661	2.72771	.38654	2.58708	.40674	2.45860	.42722	2.34069	52
9	.36694	2.72526	.38687	2.58484	.40707	2.45655	.42757	2.33881	51
10	.36727	2.72281	.38721	2.58261	.40741	2.45451	.42791	2.33693	50
11	.36760	2.72036	.38754	2.58038	.40775	2.45246	.42826	2.33505	49
12	.36793	2.71792	.38787	2.57815	.40809	2.45043	.42860	2.33317	48
13	.36826	2.71548	.38821	2.57593	.40843	2.44839	.42894	2.33130	47
14	.36859	2.71305	.38854	2.57371	.40877	2.44636	.42929	2.32943	46
15	.36892	2.71062	.38888	2.57150	.40911	2.44433	.42963	2.32756	45
16	.36925	2.70819	.38921	2.56928	.40945	2.44230	.42998	2.32570	44
17	.36958	2.70577	.38955	2.56707	.40979	2.44027	.43032	2.32383	43
18	.36991	2.70335	.38988	2.56487	.41013	2.43825	.43067	2.32197	42
19	.37024	2.70094	.39022	2.56266	.41047	2.43623	.43101	2.32012	41
20	.37057	2.69853	.39055	2.56046	.41081	2.43422	.43136	2.31826	40
21	.37090	2.69612	.39089	2.55827	.41115	2.43220	.43170	2.31641	39
22	.37124	2.69371	.39122	2.55608	.41149	2.43019	.43205	2.31456	38
23	.37157	2.69131	.39156	2.55389	.41183	2.42819	.43239	2.31271	37
24	.37190	2.68892	.39190	2.55170	.41217	2.42618	.43274	2.31086	36
25	.37223	2.68653	.39223	2.54952	.41251	2.42418	.43308	2.30902	35
26	.37256	2.68414	.39257	2.54734	.41285	2.42218	.43343	2.30718	34
27	.37289	2.68175	.39290	2.54516	.41319	2.42019	.43378	2.30534	33
28	.37322	2.67937	.39324	2.54299	.41353	2.41819	.43412	2.30351	32
29	.37355	2.67700	.39357	2.54082	.41387	2.41620	.43447	2.30167	31
30	.37388	2.67462	.39391	2.53865	.41421	2.41421	.43481	2.29984	30
31	.37422	2.67225	.39425	2.53648	.41455	2.41223	.43516	2.29801	29
32	.37455	2.66989	.39458	2.53432	.41490	2.41025	.43550	2.29619	28
33	.37488	2.66752	.39492	2.53217	.41524	2.40827	.43585	2.29437	27
34	.37521	2.66516	.39526	2.53001	.41558	2.40629	.43620	2.29254	26
35	.37554	2.66281	.39559	2.52786	.41592	2.40432	.43654	2.29073	25
36	.37588	2.66046	.39593	2.52571	.41626	2.40235	.43689	2.28891	24
37	.37621	2.65811	.39626	2.52357	.41660	2.40038	.43724	2.28710	23
38	.37654	2.65576	.39660	2.52142	.41694	2.39841	.43758	2.28528	22
39	.37687	2.65342	.39694	2.51929	.41728	2.39645	.43793	2.28348	21
40	.37720	2.65109	.39727	2.51715	.41763	2.39449	.43828	2.28167	20
41	.37754	2.64875	.39761	2.51502	.41797	2.39253	.43862	2.27987	19
42	.37787	2.64642	.39795	2.51289	.41831	2.39058	.43897	2.27806	18
43	.37820	2.64410	.39829	2.51076	.41865	2.38862	.43932	2.27626	17
44	.37853	2.64177	.39862	2.50864	.41899	2.38668	.43966	2.27447	16
45	.37887	2.63945	.39896	2.50652	.41933	2.38473	.44001	2.27267	15
46	.37920	2.63714	.39930	2.50440	.41968	2.38279	.44036	2.27088	14
47	.37953	2.63483	.39963	2.50229	.42002	2.38084	.44071	2.26909	13
48	.37986	2.63252	.39997	2.50018	.42036	2.37891	.44105	2.26730	12
49	.38020	2.63021	.40031	2.49807	.42070	2.37697	.44140	2.26552	11
50	.38053	2.62791	.40065	2.49597	.42105	2.37504	.44175	2.26374	10
51	.38086	2.62561	.40098	2.49386	.42139	2.37311	.44210	2.26196	9
52	.38120	2.62332	.40132	2.49177	.42173	2.37118	.44244	2.26018	8
53	.38153	2.62103	.40166	2.48967	.42207	2.36925	.44279	2.25840	7
54	.38186	2.61874	.40200	2.48758	.42242	2.36733	.44314	2.25663	6
55	.38220	2.61646	.40234	2.48549	.42276	2.36541	.44349	2.25486	5
56	.38253	2.61418	.40267	2.48340	.42310	2.36349	.44384	2.25309	4
57	.38286	2.61190	.40301	2.48132	.42345	2.36158	.44418	2.25132	3
58	.38320	2.60963	.40335	2.47924	.42379	2.35967	.44453	2.24956	2
59	.38353	2.60736	.40369	2.47716	.42413	2.35776	.44488	2.24780	1
60	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	0
′	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	′
	69°		68°		67°		66°		

TABLE 5.—NATURAL TRIGONOMETRIC FUNCTIONS—(*Continued*)

′	24°		25°		26°		27°				28°		29°		30°		31°		
	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.			Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	
0	.44523	2.24604	.46631	2.14451	.48773	2.05030	.50953	1.96261	60	0	.53171	1.88073	.55431	1.80405	.57735	1.73205	.60086	1.66428	60
1	.44558	2.24428	.46666	2.14288	.48809	2.04879	.50989	1.96120	59	1	.53208	1.87941	.55469	1.80281	.57774	1.73089	.60126	1.66318	59
2	.44593	2.24252	.46702	2.14125	.48845	2.04728	.51026	1.95979	58	2	.53246	1.87809	.55507	1.80158	.57813	1.72973	.60165	1.66209	58
3	.44627	2.24077	.46737	2.13963	.48881	2.04577	.51063	1.95838	57	3	.53283	1.87677	.55545	1.80034	.57851	1.72857	.60205	1.66099	57
4	.44662	2.23902	.46772	2.13801	.48917	2.04426	.51099	1.95698	56	4	.53320	1.87546	.55583	1.79911	.57890	1.72741	.60245	1.65990	56
5	.44697	2.23727	.46808	2.13639	.48953	2.04276	.51136	1.95557	55	5	.53358	1.87415	.55621	1.79788	.57929	1.72625	.60234	1.65881	55
6	.44732	2.23553	.46843	2.13477	.48989	2.04125	.51173	1.95417	54	6	.53395	1.87283	.55659	1.79665	.57968	1.72509	.60324	1.65772	54
7	.44767	2.23378	.46879	2.13316	.49026	2.03975	.51209	1.95277	53	7	.53432	1.87152	.55697	1.79542	.58007	1.72393	.60364	1.65663	53
8	.44802	2.23204	.46914	2.13154	.49062	2.03825	.51246	1.95137	52	8	.53470	1.87021	.55736	1.79419	.58046	1.72278	.60403	1.65534	52
9	.44837	2.23030	.46950	2.12993	.49098	2.03675	.51283	1.94997	51	9	.53507	1.86891	.55774	1.79296	.58085	1.72163	.60443	1.65445	51
10	.44872	2.22857	.46985	2.12832	.49134	2.03526	.51319	1.94858	50	10	.53545	1.86760	.55812	1.79174	.58124	1.72047	.60483	1.65337	50
11	.44907	2.22683	.47021	2.12671	.49170	2.03376	.51356	1.94718	49	11	.53582	1.86630	.55850	1.79051	.58162	1.71932	.60522	1.65228	49
12	.44942	2.22510	.47056	2.12511	.49206	2.03227	.51393	1.94579	48	12	.53620	1.86499	.55888	1.78929	.58201	1.71817	.60562	1.65120	48
13	.44977	2.22337	.47092	2.12350	.49242	2.03078	.51430	1.94440	47	13	.53657	1.86369	.55926	1.78807	.58240	1.71702	.60602	1.65011	47
14	.45012	2.22164	.47128	2.12190	.49278	2.02929	.51467	1.94301	46	14	.53694	1.86239	.55964	1.78685	.58279	1.71588	.60642	1.64903	46
15	.45047	2.21992	.47163	2.12030	.49315	2.02780	.51503	1.94162	45	15	.53732	1.86109	.56003	1.78563	.58318	1.71473	.60681	1.64795	45
16	.45082	2.21819	.47199	2.11871	.49351	2.02631	.51540	1.94023	44	16	.53769	1.85979	.56041	1.78441	.58357	1.71358	.60721	1.64687	44
17	.45117	2.21647	.47234	2.11711	.49387	2.02483	.51577	1.93885	43	17	.53807	1.85850	.56079	1.78319	.58396	1.71244	.60761	1.64579	43
18	.45152	2.21475	.47270	2.11552	.49423	2.02335	.51614	1.93746	42	18	.53844	1.85720	.56117	1.78198	.58435	1.71129	.60801	1.64471	42
19	.45187	2.21304	.47305	2.11392	.49459	2.02187	.51651	1.93608	41	19	.53882	1.85591	.56156	1.78077	.58474	1.71015	.60841	1.64363	41
20	.45222	2.21132	.47341	2.11233	.49495	2.02039	.51688	1.93470	40	20	.53920	1.85462	.56194	1.77955	.58513	1.70901	.60881	1.64256	40
21	.45257	2.20961	.47377	2.11075	.49532	2.01891	.51724	1.93332	39	21	.53957	1.85333	.56232	1.77834	.58552	1.70787	.60921	1.64148	39
22	.45292	2.20790	.47412	2.10916	.49568	2.01743	.51761	1.93195	38	22	.53995	1.85204	.56270	1.77713	.58591	1.70673	.60960	1.64041	38
23	.45327	2.20619	.47448	2.10758	.49604	2.01596	.51798	1.93057	37	23	.54032	1.85075	.56309	1.77592	.58631	1.70560	.61000	1.63934	37
24	.45362	2.20449	.47483	2.10600	.49640	2.01449	.51835	1.92920	36	24	.54070	1.84946	.56347	1.77471	.58670	1.70446	.61040	1.63826	36
25	.45397	2.20278	.47519	2.10442	.49677	2.01302	.51872	1.92782	35	25	.54107	1.84818	.56385	1.77351	.58709	1.70332	.61080	1.63719	35
26	.45432	2.20108	.47555	2.10284	.49713	2.01155	.51909	1.92645	34	26	.54145	1.84689	.56424	1.77230	.58748	1.70219	.61120	1.63612	34
27	.45467	2.19938	.47590	2.10126	.49749	2.01008	.51946	1.92508	33	27	.54183	1.84561	.56462	1.77110	.58787	1.70106	.61160	1.63505	33
28	.45502	2.19769	.47626	2.09969	.49786	2.00862	.51983	1.92371	32	28	.54220	1.84433	.56500	1.76990	.58826	1.69992	.61200	1.63398	32
29	.45537	2.19599	.47662	2.09811	.49822	2.00715	.52020	1.92235	31	29	.54258	1.84305	.56539	1.76869	.58865	1.69879	.61240	1.63292	31
30	.45573	2.19430	.47698	2.09654	.49858	2.00569	.52057	1.92098	30	30	.54296	1.84177	.56577	1.76749	.58904	1.69766	.61280	1.63185	30
31	.45608	2.19261	.47733	2.09498	.49894	2.00423	.52094	1.91962	29	31	.54333	1.84049	.56616	1.76630	.58944	1.69653	.61320	1.63079	29
32	.45643	2.19092	.47769	2.09341	.49931	2.00277	.52131	1.91826	28	32	.54371	1.83922	.56654	1.76510	.58983	1.69541	.61360	1.62972	28
33	.45678	2.18923	.47805	2.09184	.49967	2.00131	.52168	1.91690	27	33	.54409	1.83794	.56693	1.76390	.59022	1.69428	.61400	1.62866	27
34	.45713	2.18755	.47840	2.09028	.50004	1.99986	.52205	1.91554	26	34	.54446	1.83667	.56731	1.76271	.59061	1.69316	.61440	1.62760	26
35	.45748	2.18587	.47876	2.08872	.50040	1.99841	.52242	1.91418	25	35	.54484	1.83540	.56769	1.76151	.59101	1.69203	.61480	1.62654	25
36	.45784	2.18419	.47912	2.08716	.50076	1.99695	.52279	1.91282	24	36	.54522	1.83413	.56808	1.76032	.59140	1.69091	.61520	1.62548	24
37	.45819	2.18251	.47948	2.08560	.50113	1.99550	.52316	1.91147	23	37	.54560	1.83286	.56846	1.75913	.59179	1.68979	.61561	1.62442	23
38	.45854	2.18084	.47984	2.08405	.50149	1.99406	.52353	1.91012	22	38	.54597	1.83159	.56885	1.75794	.59218	1.68866	.61601	1.62336	22
39	.45889	2.17916	.48019	2.08250	.50185	1.99261	.52390	1.90876	21	39	.54635	1.83033	.56923	1.75675	.59258	1.68754	.61641	1.62230	21
40	.45924	2.17740	.48055	2.08094	.50222	1.99116	.52427	1.90741	20	40	.54673	1.82906	.56962	1.75556	.59297	1.68643	.61681	1.62125	20
41	.45960	2.17582	.48091	2.07939	.50258	1.98972	.52464	1.90607	19	41	.54711	1.82780	.57000	1.75437	.59336	1.68531	.61721	1.62019	19
42	.45995	2.17416	.48127	2.07785	.50295	1.98828	.52501	1.90472	18	42	.54748	1.82654	.57030	1.75319	.59376	1.68419	.61761	1.61914	18
43	.46030	2.17249	.48163	2.07630	.50331	1.98684	.52538	1.90337	17	43	.54786	1.82528	.57078	1.75200	.59415	1.68308	.61801	1.61808	17
44	.46065	2.17083	.48198	2.07476	.50368	1.98540	.52575	1.90203	16	44	.54824	1.82402	.57116	1.75082	.59454	1.68196	.61842	1.61703	16
45	.46101	2.16917	.48234	2.07321	.50404	1.98396	.52613	1.90069	15	45	.54862	1.82276	.57155	1.74964	.59494	1.68085	.61882	1.61598	15
46	.46136	2.16751	.48270	2.07167	.50441	1.98253	.52650	1.89935	14	46	.54900	1.82150	.57193	1.74846	.59533	1.67974	.61922	1.61403	14
47	.46171	2.16585	.48306	2.07014	.50477	1.98110	.52687	1.89801	13	47	.54938	1.82025	.57232	1.74728	.59573	1.67863	.61962	1.61388	13
48	.46206	2.16420	.48342	2.06860	.50514	1.97966	.52724	1.89667	12	48	.54975	1.81899	.57271	1.74610	.59612	1.67752	.62003	1.61283	12
49	.46242	2.16255	.48378	2.06706	.50550	1.97823	.52761	1.89533	11	49	.55013	1.81774	.57309	1.74492	.59651	1.67641	.62043	1.61179	11
50	.46277	2.16090	.48414	2.06553	.50587	1.97680	.52798	1.89400	10	50	.55051	1.81649	.57348	1.74375	.59691	1.67530	.62083	1.61074	10
51	.46312	2.15925	.48450	2.06400	.50623	1.97538	.52836	1.89266	9	51	.55089	1.81524	.57386	1.74257	.59730	1.67419	.62124	1.60970	9
52	.46348	2.15760	.48486	2.06247	.50660	1.97395	.52873	1.89133	8	52	.55127	1.81399	.57425	1.74140	.59770	1.67309	.62164	1.60865	8
53	.46383	2.15596	.48521	2.06094	.50696	1.97253	.52910	1.89000	7	53	.55165	1.81274	.57464	1.74022	.59809	1.67198	.62204	1.60761	7
54	.46418	2.15432	.48557	2.05942	.50733	1.97111	.52947	1.88867	6	54	.55203	1.81150	.57503	1.73905	.59849	1.67088	.62245	1.60657	6
55	.46454	2.15268	.48593	2.05790	.50769	1.96969	.52984	1.88734	5	55	.55241	1.81025	.57541	1.73788	.59888	1.66978	.62285	1.60553	5
56	.46489	2.15104	.48629	2.05637	.50806	1.96827	.53022	1.88602	4	56	.55279	1.80901	.57580	1.73671	.59928	1.66867	.62325	1.60449	4
57	.46525	2.14940	.48665	2.05485	.50843	1.96685	.53059	1.88469	3	57	.55317	1.80777	.57619	1.73555	.59967	1.66757	.62366	1.60345	3
58	.46560	2.14777	.48701	2.05333	.50879	1.96544	.53096	1.88337	2	58	.55355	1.80653	.57657	1.73438	.60007	1.66647	.62406	1.60241	2
59	.46595	2.14614	.48737	2.05182	.50916	1.96402	.53134	1.88205	1	59	.55393	1.80529	.57696	1.73321	.60046	1.66538	.62446	1.60137	1
60	.46631	2.14451	.48773	2.05030	.50953	1.96261	.53171	1.88073	0	60	.55431	1.80405	.57735	1.73205	.60086	1.66428	62487	1.60033	0
′	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	′		Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	
	65°		64°		63°		62°				61°		60°		59°		58°		

TABLE 5.—NATURAL TRIGONOMETRIC FUNCTIONS—(*Continued*)

′	32° Tan.	32° Co-tan.	33° Tan.	33° Co-tan.	34° Tan.	34° Co-tan.	35° Tan.	35° Co-tan.	′
0	.62487	1.60033	.64941	1.53986	.67451	1.48256	.70021	1.42815	60
1	.62527	1.59930	.64982	1.53888	.67493	1.48163	.70064	1.42726	59
2	.62568	1.59826	.65023	1.53791	.67536	1.48070	.70107	1.42638	58
3	.62608	1.59723	.65065	1.53693	.67578	1.47977	.70151	1.42550	57
4	.62649	1.59620	.65106	1.53595	.67620	1.47885	.70194	1.42462	56
5	.62689	1.59517	.65148	1.53497	.67663	1.47792	.70238	1.42374	55
6	.62730	1.59414	.65189	1.53400	.67705	1.47699	.70281	1.42286	54
7	.62770	1.59311	.65231	1.53302	.67748	1.47607	.70325	1.42198	53
8	.62811	1.59208	.65272	1.53205	.67790	1.47514	.70368	1.42110	52
9	.62852	1.59105	.65314	1.53107	.67832	1.47422	.70412	1.42022	51
10	.62892	1.59002	.65355	1.53010	.67875	1.47330	.70455	1.41934	50
11	.62933	1.58900	.65397	1.52913	.67917	1.47238	.70499	1.41847	49
12	.62973	1.58797	.65438	1.52816	.67960	1.47146	.70542	1.41759	48
13	.63014	1.58695	.65480	1.52719	.68002	1.47053	.70586	1.41672	47
14	.63055	1.58593	.65521	1.52622	.68045	1.46962	.70629	1.41584	46
15	.63095	1.58490	.65563	1.52525	.68088	1.46870	.70673	1.41497	45
16	.63136	1.58388	.65604	1.52429	.68130	1.46778	.70717	1.41409	44
17	.63177	1.58286	.65646	1.52332	.68173	1.46686	.70760	1.41322	43
18	.63217	1.58184	.65688	1.52235	.68215	1.46595	.70804	1.41235	42
19	.63258	1.58083	.65729	1.52139	.68258	1.46503	.70848	1.41148	41
20	.63299	1.57981	.65771	1.52043	.68301	1.46411	.70891	1.41061	40
21	.63340	1.57879	.65813	1.51946	.68343	1.46320	.70935	1.40974	39
22	.63380	1.57778	.65854	1.51850	.68386	1.46229	.70979	1.40887	38
23	.63421	1.57676	.65896	1.51754	.68429	1.46137	.71023	1.40800	37
24	.63462	1.57575	.65938	1.51658	.68471	1.46046	.71066	1.40714	36
25	.63503	1.57474	.65980	1.51562	.68514	1.45955	.71110	1.40627	35
26	.63544	1.57372	.66021	1.51466	.68557	1.45864	.71154	1.40540	34
27	.63584	1.57271	.66063	1.51370	.68600	1.45773	.71198	1.40454	33
28	.63625	1.57170	.66105	1.51275	.68642	1.45682	.71242	1.40367	32
29	.63666	1.57069	.66147	1.51179	.68685	1.45592	.71285	1.40281	31
30	.63707	1.56969	.66189	1.51084	.68728	1.45501	.71329	1.40195	30
31	.63748	1.56868	.66230	1.50988	.68771	1.45410	.71373	1.40109	29
32	.63789	1.56767	.66272	1.50893	.68814	1.45320	.71417	1.40022	28
33	.63830	1.56667	.66314	1.50797	.68857	1.45229	.71461	1.39936	27
34	.63371	1.56566	.66356	1.50702	.68900	1.45139	.71505	1.39850	26
35	.63912	1.56466	.66398	1.50607	.68942	1.45049	.71549	1.39764	25
36	.63953	1.56366	.66440	1.50512	.68985	1.44958	.71593	1.39679	24
37	.63994	1.56265	.66482	1.50417	.69028	1.44868	.71637	1.39593	23
38	.64035	1.56165	.66524	1.50322	.69071	1.44778	.71681	1.39507	22
39	.64076	1.56065	.66566	1.50228	.69114	1.44688	.71725	1.39421	21
40	.64117	1.55966	.66608	1.50133	.69157	1.44598	.71769	1.39336	20
41	.64158	1.55866	.66650	1.50038	.69200	1.44508	.71813	1.39250	19
42	.64199	1.55766	.66692	1.49944	.69243	1.44418	.71857	1.39165	18
43	.64240	1.55666	.66734	1.49849	.69286	1.44329	.71901	1.39079	17
44	.64281	1.55567	.66776	1.49755	.69329	1.44239	.71946	1.38994	16
45	.64322	1.55467	.66818	1.49661	.69372	1.44149	.71990	1.38909	15
46	.64363	1.55368	.66860	1.49566	.69416	1.44060	.72034	1.38824	14
47	.64404	1.55269	.66902	1.49472	.69459	1.43970	.72078	1.38738	13
48	.64446	1.55170	.66944	1.49378	.69502	1.43881	.72122	1.38653	12
49	.64487	1.55071	.66986	1.49284	.69545	1.43792	.72166	1.38568	11
50	.64528	1.54972	.67028	1.49190	.69588	1.43703	.72211	1.38484	10
51	.64569	1.54873	.67071	1.49097	.69631	1.43614	.72255	1.38399	9
52	.64610	1.54774	.67113	1.49003	.69675	1.43525	.72299	1.38314	8
53	.64652	1.54675	.67155	1.48909	.69718	1.43436	.72344	1.38229	7
54	.64693	1.54576	.67197	1.48816	.69761	1.43347	.72388	1.38145	6
55	.64734	1.54478	.67239	1.48722	.69804	1.43258	.72432	1.38060	5
56	.64775	1.54379	.67282	1.48629	.69847	1.43169	.72477	1.37976	4
57	.64817	1.54281	.67324	1.48536	.69891	1.43080	.72521	1.37891	3
58	.64858	1.54183	.67366	1.48442	.69934	1.42992	.72565	1.37807	2
59	.64899	1.54085	.67409	1.48349	.69977	1.42903	.72610	1.37722	1
60	.64941	1.53986	.67451	1.48256	.70021	1.42815	.72654	1.37638	0
′	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	′
	57°		56°		55°		54°		

′	36° Tan.	36° Co-tan.	37° Tan.	37° Co-tan.	38° Tan.	38° Co-tan.	39° Tan.	39° Co-tan.	′
0	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	60
1	.72699	1.37554	.75401	1.32624	.78175	1.27917	.81027	1.23416	59
2	.72743	1.37470	.75447	1.32544	.78222	1.27841	.81075	1.23343	58
3	.72788	1.37386	.75492	1.32464	.78269	1.27764	.81123	1.23270	57
4	.72832	1.37302	.75538	1.32384	.78316	1.27688	.81171	1.23196	56
5	.72877	1.37218	.75584	1.32304	.78363	1.27611	.81220	1.23123	55
6	.72921	1.37134	.75629	1.32224	.78410	1.27535	.81268	1.23050	54
7	.72966	1.37050	.75675	1.32144	.78457	1.27458	.81316	1.22977	53
8	.73010	1.36967	.75721	1.32064	.78504	1.27382	.81364	1.22904	52
9	.73055	1.36883	.75767	1.31984	.78551	1.27306	.81413	1.22831	51
10	.73100	1.36800	.75812	1.31904	.78598	1.27230	.81461	1.22758	50
11	.73144	1.36716	.75858	1.31825	.78645	1.27153	.81510	1.22685	49
12	.73189	1.36633	.75904	1.31745	.78692	1.27077	.81558	1.22612	48
13	.73234	1.36549	.75950	1.31666	.78739	1.27001	.81606	1.22539	47
14	.73278	1.36466	.75996	1.31586	.78786	1.26925	.81655	1.22467	46
15	.73323	1.36383	.76042	1.31507	.78834	1.26849	.81703	1.22394	45
16	.73368	1.36300	.76088	1.31427	.78881	1.26774	.81752	1.22321	44
17	.73413	1.36217	.76134	1.31348	.78928	1.26698	.81800	1.22249	43
18	.73457	1.36133	.76180	1.31269	.78975	1.26622	.81849	1.22176	42
19	.73502	1.36051	.76226	1.31190	.79022	1.26546	.81898	1.22104	41
20	.73547	1.35968	.76272	1.31110	.79070	1.26471	.81946	1.22031	40
21	.73592	1.35885	.76318	1.31031	.79117	1.26395	.81995	1.21959	39
22	.73637	1.35802	.76364	1.30952	.79164	1.26319	.82044	1.21886	38
23	.73681	1.35719	.76410	1.30873	.79212	1.26244	.82092	1.21814	37
24	.73726	1.35637	.76456	1.30795	.79259	1.26169	.82141	1.21742	36
25	.73771	1.35554	.76502	1.30716	.79306	1.26093	.82190	1.21670	35
26	.73816	1.35472	.76548	1.30637	.79354	1.26018	.82238	1.21598	34
27	.73861	1.35389	.76594	1.30558	.79401	1.25943	.82287	1.21526	33
28	.73906	1.35307	.76640	1.30480	.79449	1.25867	.82336	1.21454	32
29	.73951	1.35224	.76686	1.30401	.79496	1.25792	.82385	1.21382	31
30	.73996	1.35142	.76733	1.30323	.79544	1.25717	.82434	1.21310	30
31	.74041	1.35060	.76779	1.30244	.79591	1.25642	.82483	1.21238	29
32	.74086	1.34978	.76825	1.30166	.79639	1.25567	.82531	1.21166	28
33	.74131	1.34896	.76871	1.30087	.79686	1.25492	.82580	1.21094	27
34	.74176	1.34814	.76918	1.30009	.79734	1.25417	.82629	1.21023	26
35	.74221	1.34732	.76964	1.29931	.79781	1.25343	.82678	1.20951	25
36	.74267	1.34650	.77010	1.29853	.79829	1.25268	.82727	1.20879	24
37	.74312	1.34568	.77057	1.29775	.79877	1.25193	.82776	1.20808	23
38	.74357	1.34487	.77103	1.29696	.79924	1.25118	.82825	1.20736	22
39	.74402	1.34405	.77149	1.29618	.79972	1.25044	.82874	1.20665	21
40	.74447	1.34323	.77196	1.29541	.80020	1.24969	.82923	1.20593	20
41	.74492	1.34242	.77242	1.29463	.80067	1.24895	.82972	1.20522	19
42	.74538	1.34160	.77289	1.29385	.80115	1.24820	.83022	1.20451	18
43	.74583	1.34079	.77335	1.29307	.80163	1.24746	.83071	1.20379	17
44	.74628	1.33998	.77382	1.29229	.80211	1.24672	.83120	1.20308	16
45	.74674	1.33916	.77428	1.29152	.80258	1.24597	.83169	1.20237	15
46	.74719	1.33835	.77475	1.29074	.80306	1.24523	.83218	1.20166	14
47	.74764	1.33754	.77521	1.28997	.80354	1.24449	.83268	1.20095	13
48	.74810	1.33673	.77568	1.28919	.80402	1.24375	.83317	1.20024	12
49	.74855	1.33592	.77615	1.28842	.80450	1.24301	.83366	1.19953	11
50	.74900	1.33511	.77661	1.28764	.80498	1.24227	.83415	1.19882	10
51	.74946	1.33430	.77708	1.28687	.80546	1.24153	.83465	1.19811	9
52	.74991	1.33349	.77754	1.28610	.80594	1.24079	.83514	1.19740	8
53	.75037	1.33268	.77801	1.28533	.80642	1.24005	.83564	1.19669	7
54	.75082	1.33187	.77848	1.28456	.80690	1.23931	.83613	1.19599	6
55	.75128	1.33107	.77895	1.28379	.80738	1.23858	.83662	1.19528	5
56	.75173	1.33026	.77941	1.28302	.80786	1.23784	.83712	1.19457	4
57	.75219	1.32946	.77988	1.28225	.80834	1.23710	.83761	1.19387	3
58	.75264	1.32865	.78035	1.28148	.80882	1.23637	.83811	1.19316	2
59	.75310	1.32785	.78082	1.28071	.80930	1.23563	.83860	1.19246	1
60	.75355	1.32704	.78129	1.27994	.80978	1.23490	.83910	1.19175	0
′	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan	′
	53°		52°		51°		50°		

TABLE 5.—NATURAL TRIGONOMETRIC FUNCTIONS—(*Continued*)

′	40° Tan.	40° Co-tan.	41° Tan.	41° Co-tan.	42° Tan.	42° Co-tan.	43° Tan.	43° Co-tan.	′
0	.83910	1.19175	.86929	1.15037	.90040	1.11061	.93252	1.07237	60
1	.83960	1.19105	.86980	1.14969	.90093	1.10996	.93306	1.07174	59
2	.84009	1.19035	.87031	1.14902	.90146	1.10931	.93360	1.07112	58
3	.84059	1.18964	.87082	1.14834	.90199	1.10867	.93415	1.07049	57
4	.84108	1.18894	.87133	1.14767	.90251	1.10802	.93469	1.06987	56
5	.84158	1.18824	.87184	1.14699	.90304	1.10737	.93524	1.06925	55
6	.84208	1.18754	.87236	1.14632	.90357	1.10672	.93578	1.06862	54
7	.84258	1.18684	.87287	1.14565	.90410	1.10607	.93633	1.06800	53
8	.84307	1.18614	.87338	1.14498	.90463	1.10543	.93688	1.06738	52
9	.84357	1.18544	.87389	1.14430	.90516	1.10478	.93742	1.06676	51
10	.84407	1.18474	.87441	1.14363	.90569	1.10414	.93797	1.06613	50
11	.84457	1.18404	.87492	1.14296	.90621	1.10349	.93852	1.06551	49
12	.84507	1.18334	.87543	1.14229	.90674	1.10285	.93906	1.06489	48
13	.84556	1.18264	.87595	1.14162	.90727	1.10220	.93961	1.06427	47
14	.84606	1.18194	.87646	1.14095	.90781	1.10156	.94016	1.06365	46
15	.84656	1.18125	.87698	1.14028	.90834	1.10091	.94071	1.06303	45
16	.84706	1.18055	.87749	1.13961	.90887	1.10027	.94125	1.06241	44
17	.84756	1.17986	.87801	1.13894	.90940	1.09963	.94180	1.06179	43
18	.84806	1.17916	.87852	1.13828	.90993	1.09899	.94235	1.06117	42
19	.84856	1.17846	.87904	1.13761	.91046	1.09834	.94290	1.06056	41
20	.84906	1.17777	.87955	1.13694	.91099	1.09770	.94345	1.05994	40
21	.84956	1.17708	.88007	1.13627	.91153	1.09706	.94400	1.05932	39
22	.85006	1.17638	.88059	1.13561	.91206	1.09642	.94455	1.05870	38
23	.85057	1.17569	.88110	1.13494	.91259	1.09578	.94510	1.05809	37
24	.85107	1.17500	.88162	1.13428	.91313	1.09514	.94565	1.05747	36
25	.85157	1.17430	.88214	1.13361	.91366	1.09450	.94620	1.05685	35
26	.85207	1.17361	.88265	1.13295	.91419	1.09386	.94676	1.05624	34
27	.85257	1.17292	.88317	1.13228	.91473	1.09322	.94731	1.05562	33
28	.85307	1.17223	.88369	1.13162	.91526	1.09258	.94786	1.05501	32
29	.85358	1.17154	.88421	1.13096	.91580	1.09195	.94841	1.05439	31
30	.85408	1.17085	.88473	1.13029	.91633	1.09131	.94896	1.05378	30
31	.85458	1.17016	.88524	1.12963	.91687	1.09067	.94952	1.05317	29
32	.85509	1.16947	.88576	1.12897	.91740	1.09003	.95007	1.05255	28
33	.85559	1.16878	.88628	1.12831	.91794	1.08940	.95062	1.05194	27
34	.85609	1.16809	.88680	1.12765	.91847	1.08876	.95118	1.05133	26
35	.85660	1.16741	.88732	1.12699	.91901	1.08813	.95173	1.05072	25
36	.85710	1.16672	.88784	1.12633	.91955	1.08749	.95229	1.05010	24
37	.85761	1.16603	.88836	1.12567	.92008	1.08686	.95284	1.04949	23
38	.85811	1.16535	.88888	1.12501	.92062	1.08622	.95340	1.04888	22
39	.85862	1.16466	.88940	1.12435	.92116	1.08559	.95395	1.04827	21
40	.85912	1.16398	.88992	1.12369	.92170	1.08496	.95451	1.04766	20
41	.85963	1.16329	.89045	1.12303	.92224	1.08432	.95506	1.04705	19
42	.86014	1.16261	.89097	1.12238	.92277	1.08369	.95562	1.04644	18
43	.86064	1.16192	.89149	1.12172	.92331	1.08306	.95618	1.04583	17
44	.86115	1.16124	.89201	1.12106	.92385	1.08243	.95673	1.04522	16
45	.86166	1.16056	.89253	1.12041	.92439	1.08179	.95729	1.04461	15
46	.86216	1.15987	.89306	1.11975	.92493	1.08116	.95785	1.04401	14
47	.86267	1.15919	.89358	1.11909	.92547	1.08053	.95841	1.04340	13
48	.86318	1.15851	.89410	1.11844	.92601	1.07990	.95897	1.04279	12
49	.86368	1.15783	.89463	1.11778	.92655	1.07927	.95952	1.04218	11
50	.86419	1.15715	.89515	1.11713	.92709	1.07864	.96008	1.04158	10
51	.86470	1.15647	.89567	1.11648	.92763	1.07801	.96064	1.04097	9
52	.86521	1.15579	.89620	1.11582	.92817	1.07738	.96120	1.04036	8
53	.86572	1.15511	.89672	1.11517	.92872	1.07676	.96176	1.03976	7
54	.86623	1.15443	.89725	1.11452	.92926	1.07613	.96232	1.03915	6
55	.86674	1.15375	.89777	1.11387	.92980	1.07550	.96288	1.03855	5
56	.86725	1.15308	.89830	1.11321	.93034	1.07487	.96344	1.03794	4
57	.86776	1.15240	.89883	1.11256	.93088	1.07425	.96400	1.03734	3
58	.86827	1.15172	.89935	1.11191	.93143	1.07362	.96457	1.03674	2
59	.86878	1.15104	.89988	1.11126	.93197	1.07299	.96513	1.03613	1
60	.86929	1.15037	.90040	1.11061	.93252	1.07237	.96569	1.03553	0
′	Co-tan. 49°	Tan. 49°	Co-tan. 48°	Tan. 48°	Co-tan. 47°	Tan. 47°	Co-tan. 46°	Tan. 46°	′

′	44° Tan.	44° Co-tan.	′	′	44° Tan.	44° Co-tan.	′	′	44° Tan.	44° Co-tan.	′
0	.96569	1.03553	60	21	.97756	1.02295	39	41	.98901	1.01112	19
1	.96625	1.03493	59	22	.97813	1.02236	38	42	.98958	1.01053	18
2	.96681	1.03433	58	23	.97870	1.02176	37	43	.99016	1.00994	17
3	.96738	1.03372	57	24	.97927	1.02117	36	44	.99073	1.00935	16
4	.96794	1.03312	56	25	.97984	1.02057	35	45	.99131	1.00876	15
5	.96850	1.03252	55	26	.98041	1.01998	34	46	.99189	1.00818	14
6	.96907	1.03192	54	27	.98098	1.01939	33	47	.99247	1.00759	13
7	.96963	1.03132	53	28	.98155	1.01879	32	48	.99304	1.00701	12
8	.97020	1.03072	52	29	.98213	1.01820	31	49	.99362	1.00642	11
9	.97076	1.03012	51	30	.98270	1.01761	30	50	.99420	1.00583	10
10	.97133	1.02952	50	31	.98327	1.01702	29	51	.99478	1.00525	9
11	.97189	1.02892	49	32	.98384	1.01642	28	52	.99536	1.00467	8
12	.97246	1.02832	48	33	.98441	1.01583	27	53	.99594	1.00408	7
13	.97302	1.02772	47	34	.98499	1.01524	26	54	.99652	1.00350	6
14	.97359	1.02713	46	35	.98556	1.01465	25	55	.99710	1.00291	5
15	.97416	1.02653	45	36	.98613	1.01406	24	56	.99768	1.00233	4
16	.97472	1.02593	44	37	.98671	1.01347	23	57	.99826	1.00175	3
17	.97529	1.02533	43	38	.98728	1.01288	22	58	.99884	1.00116	2
18	.97586	1.02474	42	39	.98786	1.01229	21	59	.99942	1.00058	1
19	.97643	1.02414	41	40	.98843	1.01170	20	60	1	1	0
20	.97700	1.02355	40								
′	Co-tan. 45°	Tan. 45°	′	′	Co-tan. 45°	Tan. 45°	′	′	Co-tan. 45°	Tan. 45°	′

NATURAL SINES AND COSINES

′	0° Sine	0° Cosine	′	′	0° Sine	0° Cosine	′	′	0° Sine	0° Cosine	′
0	.00000	1	60	21	.00611	.99998	39	41	.01193	.99993	19
1	.00029	1	59	22	.00640	.99998	38	42	.01222	.99993	18
2	.00058	1	58	23	.00669	.99998	37	43	.01251	.99992	17
3	.00087	1	57	24	.00698	.99998	36	44	.01280	.99992	16
4	.00116	1	56	25	.00727	.99997	35	45	.01309	.99991	15
5	.00145	1	55	26	.00756	.99997	34	46	.01338	.99991	14
6	.00175	1	54	27	.00785	.99997	33	47	.01367	.99991	13
7	.00204	1	53	28	.00814	.99997	32	48	.01396	.99990	12
8	.00233	1	52	29	.00844	.99996	31	49	.01425	.99990	11
9	.00262	1	51	30	.00873	.99996	30	50	.01454	.99989	10
10	.00291	1	50	31	.00902	.99996	29	51	.01483	.99989	9
11	.00320	.99999	49	32	.00931	.99996	28	52	.01513	.99989	8
12	.00349	.99999	48	33	.00960	.99995	27	53	.01542	.99988	7
13	.00378	.99999	47	34	.00989	.99995	26	54	.01571	.99988	6
14	.00407	.99999	46	35	.01018	.99995	25	55	.01600	.99987	5
15	.00436	.99999	45	36	.01047	.99995	24	56	.01629	.99987	4
16	.00465	.99999	44	37	.01076	.99994	23	57	.01658	.99986	3
17	.00495	.99999	43	38	.01105	.99994	22	58	.01687	.99986	2
18	.00524	.99999	42	39	.01134	.99994	21	59	.01716	.99985	1
19	.00553	.99998	41	40	.01164	.99993	20	60	.01745	.99985	0
20	.00582	.99998	40								
′	Cosine 89°	Sine 89°	′	′	Cosine 89°	Sine 89°	′	′	Cosine 89°	Sine 89°	′

TABLE 5.—NATURAL TRIGONOMETRIC FUNCTIONS—(*Continued*)

′	1° Sine	1° Cosine	2° Sine	2° Cosine	3° Sine	3° Cosine	4° Sine	4° Cosine	′
0	.01745	.99985	.03490	.99939	.05234	.99863		.99756	60
1	.01774	.99984	.03519	.99938	.05263	.99861		.99754	59
2	.01803	.99984	.03548	.99937	.05292	.99860		.99752	58
3	.01832	.99983	.03577	.99936	.05321	.99858		.99750	57
4	.01862	.99983	.03606	.99935	.05350	.99857		.99748	56
5	.01891	.99982	.03635	.99934	.05379	.99855		.99746	55
6	.01920	.99982	.03664	.99933	.05408	.99854		.99744	54
7	.01949	.99981	.03693	.99932	.05437	.99852		.99742	53
8	.01978	.99980	.03723	.99931	.05466	.99851		.99740	52
9	.02007	.99980	.03752	.99930	.05495	.99849		.99738	51
10	.02036	.99979	.03781	.99929	.05524	.99847		.99736	50
11	.02065	.99979	.03810	.99927	.05553	.99846		.99734	49
12	.02094	.99978	.03839	.99926	.05582	.99844		.99731	48
13	.02123	.99977	.03868	.99925	.05611	.99842		.99729	47
14	.02152	.99977	.03897	.99924	.05640	.99841		.99727	46
15	.02181	.99976	.03926	.99923	.05669	.99839		.99725	45
16	.02211	.99976	.03955	.99922	.05698	.99838		.99723	44
17	.02240	.99975	.03984	.99921	.05727	.99836		.99721	43
18	.02269	.99974	.04013	.99919	.05756	.99834		.99719	42
19	.02298	.99974	.04042	.99918	.05785	.99833		.99716	41
20	.02327	.99973	.04071	.99917	.05814	.99831		.99714	40
21	.02356	.99972	.04100	.99916	.05844	.99829		.99712	39
22	.02385	.99972	.04129	.99915	.05873	.99827		.99710	38
23	.02414	.99971	.04159	.99913	.05902	.99826		.99708	37
24	.02443	.99970	.04188	.99912	.05931	.99824		.99705	36
25	.02472	.99969	.04217	.99911	.05960	.99822		.99703	35
26	.02501	.99969	.04246	.99910	.05989	.99821		.99701	34
27	.02530	.99968	.04275	.99909	.06018	.99819		.99699	33
28	.02560	.99967	.04304	.99907	.06047	.99817		.99696	32
29	.02589	.99966	.04333	.99906	.06076	.99815		.99694	31
30	.02618	.99966	.04362	.99905	.06105	.99813		.99692	30
31	.02647	.99965	.04391	.99904	.06134	.99812		.99689	29
32	.02676	.99964	.04420	.99902	.06163	.99810		.99687	28
33	.02705	.99963	.04449	.99901	.06192	.99808		.99685	27
34	.02734	.99963	.04478	.99900	.06221	.99806		.99683	26
35	.02763	.99962	.04507	.99898	.06250	.99804		.99680	25
36	.02792	.99961	.04536	.99897	.06279	.99803		.99678	24
37	.02821	.99960	.04565	.99896	.06308	.99801		.99676	23
38	.02850	.99959	.04594	.99894	.06337	.99799		.99673	22
39	.02879	.99959	.04623	.99893	.06366	.99797		.99671	21
40	.02908	.99958	.04653	.99892	.06395	.99795		.99668	20
41	.02938	.99957	.04682	.99890	.06424	.99793		.99666	19
42	.02967	.99956	.04711	.99889	.06453	.99792		.99664	18
43	.02996	.99955	.04740	.99888	.06482	.99790		.99661	17
44	.03025	.99954	.04769	.99886	.06511	.99788		.99659	16
45	.03054	.99953	.04798	.99885	.06540	.99786		.99657	15
46	.03083	.99952	.04827	.99883	.06569	.99784		.99654	14
47	.03112	.99952	.04856	.99882	.06598	.99782		.99652	13
48	.03141	.99951	.04885	.99881	.06627	.99780		.99649	12
49	.03170	.99950	.04914	.99879	.06656	.99778		.99647	11
50	.03199	.99949	.04943	.99878	.06685	.99776		.99644	10
51	.03228	.99948	.04972	.99876	.06714	.99774		.99642	9
52	.03257	.99947	.05001	.99875	.06743	.99772		.99639	8
53	.03286	.99946	.05030	.99873	.06773	.99770		.99637	7
54	.03316	.99945	.05059	.99872	.06802	.99768		.99635	6
55	.03345	.99944	.05088	.99870	.06831	.99766		.99632	5
56	.03374	.99943	.05117	.99869	.06860	.99764		.99630	4
57	.03403	.99942	.05146	.99867	.06889	.99762		.99627	3
58	.03432	.99941	.05175	.99866	.06918	.99760		.99625	2
59	.03461	.99940	.05205	.99864	.06947	.99758		.99622	1
60	.03490	.99939	.05234	.99863	.06976	.99756		.99619	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	88°		87°		86°		85°		

′	5° Sine	5° Cosine	6° Sine	6° Cosine	7° Sine	7° Cosine	8° Sine	8° Cosine	′
0	.08716	.99619	.10453	.99452	.12187	.99255	.13917	.99027	60
1	.08745	.99617	.10482	.99449	.12216	.99251	13046	.99023	59
2	.08774	.99614	.10511	.99446	.12245	.99248	.13075	.99019	58
3	.08803	.99612	.10540	[illegible]	.12274	[illegible]	.14004	.99015	57
4	.08831	.99609	.10569	.99440	.12302	.99240	.14033	.99011	56
5	.08860	.99607	.10507	.99437	.12331	.99237	.14061	.99006	55
6	.08889	.99604	.10626	.99434	.12360	.99233	.14090	.99002	54
7	.08918	.99602	.10655	.99431	.12389	.99230	.14119	.98998	53
8	.08947	[illegible]	.10684	.99428	.12418	.99226	.14148	.98994	52
9	.08976	.99596	.10713	.99424	.12447	.99222	.14177	.98990	51
10	.09005	.99594	.10742	.99421	.12476	.99219	.14205	.98986	50
11	.09034	.99591	.10771	.99418	.12504	.99215	.14234	.98982	49
12	.09063	.99588	.10800	.99415	.12533	.99211	.14263	.98978	48
13	.09092	.99586	.10829	.99412	.12562	.99208	.14292	.98973	47
14	.09121	.99583	.10858	.99409	.12591	.99204	.14320	.98969	46
15	.09150	.99580	.10887	.99406	.12620	.99200	.14349	.98965	45
16	.09179	.99578	.10916	.99402	.12649	.99197	.14378	.98961	[illegible]
17	.09208	.99575	.10945	[illegible]	.12678	.99193	[illegible]	.98957	43
18	.09237	.99572	.10973	.99396	.12706	.99189	.14436	.98953	[illegible]
19	.09266	.99570	.11002	.99393	.12735	.99186	.14464	.98948	41
20	.09295	.99567	.11031	.99390	.12764	.99182	[illegible]	.98944	40
21	.09324	.99564	.11060	.99386	.12793	.99178	.14522	.98940	39
22	.09353	.99562	.11089	.99383	.12822	.99175	.14551	.98936	38
23	.09382	.99559	.11118	.99380	.12851	.99171	.14580	.98931	37
24	.09411	.99556	.11147	.99377	12880	.99167	.14608	.98927	36
25	.09440	.99553	.11176	.99374	12908	.99163	.14637	.98923	35
26	.09469	.99551	.11205	.99370	.12937	.99160	.14666	.98919	34
27	.09498	.99548	.11234	.99367	12966	.99156	.14695	.98914	33
28	.09527	.99545	.11263	.99364	.12995	.99152	.14723	.98910	32
29	.09556	.99542	.11291	.99360	13024	.99148	.14752	.98906	31
30	.09585	.99540	.11320	.99357	.13053	.99144	.14781	.98902	30
31	.09614	.99537	.11349	.99354	.13081	.99141	.14810	.98897	29
32	.09642	.99534	.11378	.99351	.13110	.99137	.14838	.98893	28
33	.09671	.99531	.11407	.99347	.13139	.99133	.14867	.98889	27
34	.09700	.99528	.11436	.99344	.13168	.99129	.14896	.98884	26
35	.09729	.99526	.11465	.99341	.13197	.99125	.14025	.98880	25
36	.09758	.99523	.11494	.99337	.13226	.99122	[illegible]	.98876	24
37	.09787	.99520	.11523	.99334	.13254	.99118	.14982	.98871	23
38	.09816	.99517	.11552	.99331	.13283	.99114	.15011	.98867	22
39	.09845	.99514	.11580	.99327	.13312	.99110	.15040	.98863	21
40	.09874	.99511	.11609	.99324	.13341	.99106	.15069	.98858	20
41	.09903	.99508	.11638	.99320	.13370	.99102	.15097	.98854	19
[illegible]	.09932	.99506	.11667	.99317	.13399	.99098	.15126	.98849	18
43	.09961	.99503	.11696	.99314	.13427	.99094	.15155	.98845	17
44	.09990	.99500	.11725	.99310	.13456	.99091	.15184	.98841	16
45	.10019	.99497	.11754	.99307	.13485	.99087	.15212	.98836	15
46	.10048	.99494	.11783	.99303	.13514	.99083	.15241	.98832	14
47	.10077	.99491	.11812	.99300	.13543	.99079	.15270	.98827	13
48	.10106	.99488	.11840	.99297	.13572	.99075	.15299	.98823	12
49	.10135	.99485	.11869	.99293	.13600	.99071	.15327	.98818	11
50	.10164	.99482	.11898	.99290	.13629	.99067	.15356	.98814	10
51	.10192	.99479	.11927	.99286	.13658	.99063	.15385	.98809	9
52	.10221	.99476	.11956	.99283	.13687	.99059	.15414	.98805	8
53	.10250	.99473	.11985	.99279	.13716	.99055	.15442	.98800	7
54	.10279	.99470	.12014	.99276	.13744	.99051	.15471	.98796	6
55	.10308	.99467	.12043	.99272	.13773	.99047	.15500	.98791	5
56	.10337	.99464	.12071	.99269	.13802	.99043	.15529	.98787	4
57	.10366	.99461	.12100	.99265	.13831	.99039	.15557	.98782	3
58	.10395	.99458	.12129	.99262	.13860	.99035	.15586	.98778	2
59	.10424	.99455	.12158	.99258	.13889	.99031	.15615	.98773	1
60	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	84°		83°		82°		81°		

TABLE 5.—NATURAL TRIGONOMETRIC FUNCTIONS—(*Continued*)

′	9° Sine	9° Cosine	10° Sine	10° Cosine	11° Sine	11° Cosine	12° Sine	12° Cosine	′
0	.15643	.98769	.17365	.98481	.19081	.98163	.20791	.97815	60
1	.15672	.98764	.17393	.98476	.19109	.98157	.20820	.97809	59
2	.15701	.98760	.17422	.98471	.19138	.98152	.20848	.97803	58
3	.15730	.98755	.17451	.98466	.19167	.98146	.20877	.97797	57
4	.15758	.98751	.17479	.98461	.19195	.98140	.20905	.97791	56
5	.15787	.98746	.17508	.98455	.19224	.98135	.20933	.97784	55
6	.15816	.98741	.17537	.98450	.19252	.98129	.20962	.97778	54
7	.15845	.98737	.17565	.98445	.19281	.98124	.20990	.97772	53
8	.15873	.98732	.17594	.98440	.19309	.98118	.21019	.97766	52
9	.15902	.98728	.17623	.98435	.19338	.98112	.21047	.97760	51
10	.15931	.98723	.17651	.98430	.19366	.98107	.21076	.97754	50
11	.15959	.98718	.17680	.98425	.19395	.98101	.21104	.97748	49
12	.15988	.98714	.17708	.98420	.19423	.98096	.21132	.97742	48
13	.16017	.98709	.17737	.98414	.19452	.98090	.21161	.97735	47
14	.16046	.98704	.17766	.98409	.19481	.98084	.21189	.97729	46
15	.16074	.98700	.17794	.98404	.19509	.98079	.21218	.97723	45
16	.16103	.98695	.17823	.98399	.19538	.98073	.21246	.97717	44
17	.16132	.98690	.17852	.98394	.19566	.98067	.21275	.97711	43
18	.16160	.98689	.17880	.98389	.19595	.98061	.21303	.97705	42
19	.16189	.98681	.17909	.98383	.19623	.98056	.21331	.97698	41
20	.16218	.98676	.17937	.98378	.19652	.98050	.21360	.97692	40
21	.16246	.98671	.17966	.98373	.19680	.98044	.21388	.97686	39
22	.16275	.98667	.17995	.98368	.19709	.98039	.21417	.97680	38
23	.16304	.98662	.18023	.98362	.19737	.98033	.21445	.97673	37
24	.16333	.98657	.18052	.98357	.19766	.98027	.21474	.97667	36
25	.16361	.98652	.18081	.98352	.19794	.98021	.21502	.97661	35
26	.16390	.98648	.18109	.98347	.19823	.98016	.21530	.97655	34
27	.16419	.98643	.18138	.98341	.19851	.98010	.21559	.97648	33
28	.16447	.98638	.18166	.98336	.19880	.98004	.21587	.97642	32
29	.16476	.98633	.18195	.98331	.19908	.97987	.21616	.97636	31
30	.16505	.98629	.18224	.98325	.19937	.97992	.21644	.97630	30
31	.16533	.98624	.18252	.98320	.19965	.97987	.21672	.97623	29
32	.16562	.98619	.18281	.98315	.19994	.97981	.21701	.97617	28
33	.16591	.98614	.18309	.98310	.20022	.97975	.21729	.97611	27
34	.16620	.98609	.18338	.98304	.20031	.97969	.21758	.97604	26
35	.16648	.98604	.18367	.98299	.20079	.97963	.21786	.97598	25
36	.16677	.98600	.18395	.98294	.20108	.97958	.21814	.97592	24
37	.16706	.98595	.18424	.98288	.20136	.97952	.21843	.97585	23
38	.16734	.98590	.18452	.98283	.20165	.97946	.21871	.97579	22
39	.16763	.98585	.18481	.98277	.20193	.97940	.21899	.97573	21
40	.16792	.98580	.18509	.98272	.20222	.97934	.21928	.97566	20
41	.16820	.98575	.18538	.98267	.20250	.97928	.21956	.97560	19
42	.16849	.98570	.18567	.98261	.20279	.97922	.21985	.97553	18
43	.16878	.98565	.18595	.98256	.20307	.97916	.22013	.97547	17
44	.16906	.98561	.18624	.98250	.20336	.97910	.22041	.97541	16
45	.16935	.98556	.18652	.98245	.20364	.97905	.22070	.97534	15
46	.16964	.98551	.18681	.98240	.20393	.97899	.22098	.97528	14
47	.16992	.98546	.18710	.98234	.20421	.97893	.22126	.97521	13
48	.17021	.98541	.18738	.98229	.20450	.97887	.22155	.97515	12
49	.17050	.98536	.18767	.98223	.20478	.97881	.22183	.97508	11
50	.17078	.98531	.18795	.98218	.20507	.97875	.22212	.97502	10
51	.17107	.98526	.18824	.98212	.20535	.97869	.22240	.97496	9
52	.17136	.98521	.18852	.98207	.20563	.97863	.22268	.97489	8
53	.17164	.98516	.18881	.98201	.20592	.97857	.22297	.97483	7
54	.17193	.98511	.18910	.98196	.20620	.97851	.22325	.97476	6
55	.17222	.98506	.18938	.98190	.20649	.97845	.22353	.97470	5
56	.17250	.98501	.18967	.98185	.20677	.97839	.22382	.97463	4
57	.17279	.98496	.18995	.98179	.20706	.97833	.22410	.97457	3
58	.17308	.98491	.19024	.98174	.20734	.97827	.22438	.97450	2
59	.17336	.98486	.19052	.98168	.20763	.97821	.22467	.97444	1
60	.17365	.98481	.19081	.98163	.20791	.97815	.22495	.97437	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	80°		79°		78°		77°		

′	13° Sine	13° Cosine	14° Sine	14° Cosine	15° Sine	15° Cosine	16° Sine	16° Cosine	′
0	.22495	.97437	.24192	.97030	.25882	.96593	.27564	.96126	60
1	.22523	.97430	.24220	.97023	.25910	.96585	.27592	.96118	59
2	.22552	.97424	.24249	.97015	.25938	.96578	.27620	.96110	58
3	.22580	.97417	.24277	.97008	.25966	.96570	.27648	.96102	57
4	.22608	.97411	.24305	.97001	.25994	.96562	.27676	.96094	56
5	.22637	.97404	.24333	.96994	.26022	.96555	.27704	.96086	55
6	.22665	.97398	.24362	.96987	.26050	.96547	.27731	.96078	54
7	.22693	.97391	.24390	.96980	.26079	.96540	.27759	.96070	53
8	.22722	.97384	.24418	.96973	.26107	.96532	.27787	.96062	52
9	.22750	.97378	.24446	.96966	.26135	.96524	.27815	.96054	51
10	.22778	.97371	.24474	.96959	.26163	.96517	.27843	.96046	50
11	.22807	.97365	.24503	.96952	.26191	.96509	.27871	.96037	49
12	.22835	.97358	.24531	.96945	.26219	.96502	.27899	.96029	48
13	.22863	.97351	.24559	.96937	.26247	.96494	.27927	.96021	47
14	.22892	.97345	.24587	.96930	.26275	.96486	.27955	.96013	46
15	.22920	.97338	.24615	.96923	.26303	.96479	.27983	.96005	45
16	.22948	.97331	.24644	.96916	.26331	.96471	.28011	.95997	44
17	.22977	.97325	.24672	.96909	.26359	.96463	.28039	.95989	43
18	.23005	.97318	.24700	.96902	.26387	.96456	.28067	.95981	42
19	.23033	.97311	.24728	.96894	.26415	.96448	.28095	.95972	41
20	.23062	.97304	.24756	.96887	.26443	.96440	.28123	.95964	40
21	.23090	.97298	.24784	.96880	.26471	.96433	.28150	.95956	39
22	.23118	.97291	.24813	.96873	.26500	.96425	.28178	.95948	38
23	.23146	.97284	.24841	.96866	.26528	.96417	.28206	.95940	37
24	.23175	.97278	.24869	.96858	.26556	.96410	.28234	.95931	36
25	.23203	.97271	.24897	.96851	.26584	.96402	.28262	.95923	35
26	.23231	.97264	.24925	.96844	.26612	.96394	.28290	.95915	34
27	.23260	.97257	.24954	.96837	.26640	.96386	.28318	.95907	33
28	.23288	.97251	.24982	.96829	.26668	.96379	.28346	.95898	32
29	.23316	.97244	.25010	.96822	.26696	.96371	.28374	.95890	31
30	.23345	.97237	.25038	.96815	.26724	.96363	.28402	.95882	30
31	.23373	.97230	.25066	.96807	.26752	.96355	.28429	.95874	29
32	.23401	.97223	.25094	.96800	.26780	.96347	.28457	.95865	28
33	.23429	.97217	.25122	.96793	.26808	.96340	.28485	.95857	27
34	.23458	.97210	.25151	.96786	.26836	.96332	.28513	.95849	26
35	.23486	.97203	.25179	.96778	.26864	.96324	.28541	.95841	25
36	.23514	.97196	.25207	.96771	.26892	.96316	.28569	.95832	24
37	.23542	.97189	.25235	.96764	.26920	.96308	.28597	.95824	23
38	.23571	.97182	.25263	.96756	.26948	.96301	.28625	.95816	22
39	.23599	.97176	.25291	.96749	.26976	.96293	.28652	.95807	21
40	.23627	.97169	.25320	.96742	.27004	.96285	.28680	.95799	20
41	.23656	.97162	.25348	.96734	.27032	.96277	.28708	.95791	19
42	.23684	.97155	.25376	.96727	.27060	.96269	.28736	.95782	18
43	.23712	.97148	.25404	.96719	.27088	.96261	.28764	.95774	17
44	.23740	.97141	.25432	.96712	.27116	.96253	.28792	.95766	16
45	.23769	.97134	.25460	.96705	.27144	.96246	.28820	.95757	15
46	.23797	.97127	.25488	.96697	.27172	.96238	.28847	.95740	14
47	.23825	.97120	.25516	.96690	.27200	.96230	.28875	.95740	13
48	.23853	.97113	.25545	.96682	.27228	.96222	.28903	.95732	12
49	.23882	.97106	.25573	.96675	.27256	.96214	.28931	.95724	11
50	.23910	.97100	.25601	.96667	.27284	.96206	.28959	.95715	10
51	.23938	.97093	.25629	.96660	.27312	.96198	.28987	.95707	9
52	.23966	.97086	.25657	.96653	.27340	.96190	.29015	.95698	8
53	.23995	.97079	.25685	.96645	.27368	.96182	.29042	.95690	7
54	.24023	.97072	.25713	.96638	.27396	.96174	.29070	.95681	6
55	.24051	.97065	.25741	.96630	.27424	.96166	.29098	.95673	5
56	.24079	.97058	.25769	.96623	.27452	.96158	.29126	.95664	4
57	.24108	.97051	.25798	.96615	.27480	.96150	.29154	.95656	3
58	.24136	.97044	.25826	.96608	.27508	.96142	.29182	.95647	2
59	.24164	.97037	.25854	.96600	.27536	.96134	.29209	.95639	1
60	.24192	.97030	.25882	.96593	.27564	.96126	.29237	.95630	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	76°		75°		74°		73°		

TABLE 5.—NATURAL TRIGONOMETRIC FUNCTIONS—(*Continued*)

′	17° Sine	17° Cosine	18° Sine	18° Cosine	19° Sine	19° Cosine	20° Sine	20° Cosine	′
0	.29237	.95630	.30902	.95106	.32557	.94552	.34202	.93969	60
1	.29265	.95622	.30929	.95097	.32584	.94542	.34229	.93959	59
2	.29293	.95613	.30957	.95088	.32612	.94533	.34257	.93949	58
3	.29321	.95605	.30985	.95079	.32639	.94523	.34284	.93939	57
4	.29348	.95596	.31012	.95070	.32667	.94514	.34311	.93929	56
5	.29376	.95588	.31040	.95061	.32694	.94504	.34339	.93919	55
6	.29404	.95579	.31068	.95052	.32722	.94495	.34366	.93909	54
7	.29432	.95571	.31095	.95043	.32749	.94485	.34393	.93899	53
8	.29460	.95562	.31123	.95033	.32777	.94476	.34421	.93889	52
9	.29487	.95554	.31151	.95024	.32804	.94466	.34448	.93879	51
10	.29515	.95545	.31178	.95015	.32832	.94457	.34475	.93869	50
11	.29543	.95536	.31206	.95006	.32859	.94447	.34503	.93859	49
12	.29571	.95528	.31233	.94997	.32887	.94438	.34530	.93849	48
13	.29599	.95519	.31261	.94988	.32914	.94428	.34557	.93839	47
14	.29626	.95511	.31289	.94979	.32942	.94418	.34584	.93829	46
15	.29654	.95502	.31316	.94970	.32969	.94409	.34612	.93819	45
16	.29682	.95493	.31344	.94961	.32997	.94399	.34639	.93809	44
17	.29710	.95485	.31372	.94952	.33024	.94390	.34666	.93799	43
18	.29737	.95476	.31399	.94943	.33051	.94380	.34694	.93789	42
19	.29765	.95467	.31427	.94933	.33079	.94370	.34721	.93779	41
20	.29793	.95459	.31454	.94924	.33106	.94361	.34748	.93769	40
21	.29821	.95450	.31482	.94915	.33134	.94351	.34775	.93759	39
22	.29849	.95441	.31510	.94906	.33161	.94342	.34803	.93748	38
23	.29876	.95433	.31537	.94897	.33189	.94332	.34830	.93738	37
24	.29904	.95424	.31565	.94888	.33216	.94322	.34857	.93728	36
25	.29932	.95415	.31593	.94878	.33244	.94313	.34884	.93718	35
26	.29960	.95407	.31620	.94869	.33271	.94303	.34912	.93708	34
27	.29987	.95398	.31648	.94860	.33298	.94293	.34939	.93698	33
28	.30015	.95389	.31675	.94851	.33326	.94284	.34966	.93688	32
29	.30043	.95380	.31703	.94842	.33353	.94274	.34993	.93677	31
30	.30071	.95372	.31730	.94832	.33381	.94264	.35021	.93667	30
31	.30098	.95363	.31758	.94823	.33408	.94254	.35048	.93657	29
32	.30126	.95354	.31786	.94814	.33436	.94245	.35075	.93647	28
33	.30154	.95345	.31813	.94805	.33463	.94235	.35102	.93637	27
34	.30182	.95337	.31841	.94795	.33490	.94225	.35130	.93626	26
35	.30209	.95328	.31868	.94786	.33518	.94215	.35157	.93616	25
36	.30237	.95319	.31896	.94777	.33545	.94206	.35184	.93606	24
37	.30265	.95310	.31923	.94768	.33573	.94196	.35211	.93596	23
38	.30292	.95301	.31951	.94758	.33600	.94186	.35239	.93585	22
39	.30320	.95293	.31979	.94749	.33627	.94176	.35266	.93575	21
40	.30348	.95284	.32006	.94740	.33655	.94167	.35293	.93565	20
41	.30376	.95275	.32034	.94730	.33682	.94157	.35320	.93555	19
42	.30403	.95266	.32061	.94721	.33710	.94147	.35347	.93544	18
43	.30431	.95257	.32089	.94712	.33737	.94137	.35375	.93534	17
44	.30459	.95248	.32116	.94702	.33764	.94127	.35402	.93524	16
45	.30486	.95240	.32144	.94693	.33792	.94118	.35429	.93514	15
46	.30514	.95231	.32171	.94684	.33819	.94108	.35456	.93503	14
47	.30542	.95222	.32199	.94674	.33846	.94098	.35484	.93493	13
48	.30570	.95213	.32227	.94665	.33874	.94088	.35511	.93483	12
49	.30597	.95204	.32254	.94656	.33901	.94078	.35538	.93472	11
50	.30625	.95195	.32282	.94646	.33929	.94068	.35565	.93462	10
51	.30653	.95186	.32309	.94637	.33956	.94058	.35592	.93452	9
52	.30680	.95177	.32337	.94627	.33983	.94049	.35619	.93441	8
53	.30708	.95168	.32364	.94618	.34011	.94039	.35647	.93431	7
54	.30736	.95159	.32392	.94609	.34038	.94029	.35674	.93420	6
55	.30763	.95150	.32419	.94599	.34065	.94019	.35701	.93410	5
56	.30791	.95142	.32447	.94590	.34093	.94009	.35728	.93400	4
57	.30819	.95133	.32474	.94580	.34120	.93999	.35755	.93389	3
58	.30846	.95124	.32502	.94571	.34147	.93989	.35782	.93379	2
59	.30874	.95115	.32529	.94561	.34175	.93979	.35810	.93368	1
60	.30902	.95106	.32557	.94552	.34202	.93969	.35837	.93358	0
′	Cosine 72°	Sine 72°	Cosine 71°	Sine 71°	Cosine 70°	Sine 70°	Cosine 69°	Sine 69°	′

′	21° Sine	21° Cosine	22° Sine	22° Cosine	23° Sine	23° Cosine	24° Sine	24° Cosine	′
0	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	60
1	.35864	.93348	.37488	.92707	.39100	.92039	.40700	.91343	59
2	.35891	.93337	.37515	.92697	.39127	.92028	.40727	.91331	58
3	.35918	.93327	.37542	.92686	.39153	.92016	.40753	.91319	57
4	.35945	.93316	.37569	.92675	.39180	.92005	.40780	.91307	56
5	.35973	.93306	.37595	.92664	.39207	.91994	.40806	.91295	55
6	.36000	.93295	.37622	.92653	.39234	.91982	.40833	.91283	54
7	.36027	.93285	.37649	.92642	.39260	.91971	.40860	.91272	53
8	.36054	.93274	.37676	.92631	.39287	.91959	.40886	.91260	52
9	.36081	.93264	.37703	.92620	.39314	.91948	.40913	.91248	51
10	.36108	.93253	.37730	.92609	.39341	.91936	.40939	.91236	50
11	.36135	.93243	.37757	.92598	.39367	.91925	.40966	.91224	49
12	.36162	.93232	.37784	.92587	.39394	.91914	.40992	.91212	48
13	.36190	.93222	.37811	.92576	.39421	.91902	.41019	.91200	47
14	.36217	.93211	.37838	.92565	.39448	.91891	.41045	.91188	46
15	.36244	.93201	.37865	.92554	.39474	.91879	.41072	.91176	45
16	.36271	.93190	.37892	.92543	.39501	.91868	.41098	.91164	44
17	.36298	.93180	.37919	.92532	.39528	.91856	.41125	.91152	43
18	.36325	.93169	.37946	.92521	.39555	.91845	.41151	.91140	42
19	.36352	.93159	.37973	.92510	.39581	.91833	.41178	.91128	41
20	.36379	.93148	.37999	.92499	.39608	.91822	.41204	.91116	40
21	.36406	.93137	.38026	.92488	.39635	.91810	.41231	.91104	39
22	.36434	.93127	.38053	.92477	.39661	.91799	.41257	.91092	38
23	.36461	.93116	.38080	.92466	.39688	.91787	.41284	.91080	37
24	.36488	.93106	.38107	.92455	.39715	.91775	.41310	.91068	36
25	.36515	.93095	.38134	.92444	.39741	.91764	.41337	.91056	35
26	.36542	.93084	.38161	.92432	.39768	.91752	.41363	.91044	34
27	.36569	.93074	.38188	.92421	.39795	.91741	.41390	.91032	33
28	.36596	.93063	.38215	.92410	.39822	.91729	.41416	.91020	32
29	.36623	.93052	.38241	.92399	.39848	.91718	.41443	.91008	31
30	.36650	.93042	.38268	.92388	.39875	.91706	.41469	.90996	30
31	.36677	.93031	.38295	.92377	.39902	.91694	.41496	.90984	29
32	.36704	.93020	.38322	.92366	.39928	.91683	.41522	.90972	28
33	.36731	.93010	.38349	.92355	.39955	.91671	.41549	.90960	27
34	.36758	.92999	.38376	.92343	.39982	.91660	.41575	.90948	26
35	.36785	.92988	.38403	.92332	.40008	.91648	.41602	.90936	25
36	.36812	.92978	.38430	.92321	.40035	.91636	.41628	.90924	24
37	.36839	.92967	.38456	.92310	.40062	.91625	.41655	.90911	23
38	.36867	.92956	.38483	.92299	.40088	.91613	.41681	.90899	22
39	.36894	.92945	.38510	.92287	.40115	.91601	.41707	.90887	21
40	.36921	.92935	.38537	.92276	.40141	.91590	.41734	.90875	20
41	.36948	.92924	.38564	.92265	.40168	.91578	.41760	.90863	19
42	.36975	.92913	.38591	.92254	.40195	.91566	.41787	.90851	18
43	.37002	.92902	.38617	.92243	.40221	.91555	.41813	.90839	17
44	.37029	.92892	.38644	.92231	.40248	.91543	.41840	.90826	16
45	.37056	.92881	.38671	.92220	.40275	.91531	.41866	.90814	15
46	.37083	.92870	.38698	.92209	.40301	.91519	.41892	.90802	14
47	.37110	.92859	.38725	.92198	.40328	.91508	.41919	.90790	13
48	.37137	.92849	.38752	.92186	.40355	.91496	.41945	.90778	12
49	.37164	.92838	.38778	.92175	.40381	.91484	.41972	.90766	11
50	.37191	.92827	.38805	.92164	.40408	.91472	.41998	.90753	10
51	.37218	.92816	.38832	.92152	.40434	.91461	.42024	.90741	9
52	.37245	.92805	.38859	.92141	.40461	.91449	.42051	.90729	8
53	.37272	.92794	.38886	.92130	.40488	.91437	.42077	.90717	7
54	.37299	.92784	.38912	.92119	.40514	.91425	.42104	.90704	6
55	.37326	.92773	.38939	.92107	.40541	.91414	.42130	.90692	5
56	.37353	.92762	.38966	.92096	.40567	.91402	.42156	.90680	4
57	.37380	.92751	.38993	.92085	.40594	.91390	.42183	.90668	3
58	.37407	.92740	.39020	.92073	.40621	.91378	.42209	.90655	2
59	.37434	.92729	.39046	.92062	.40647	.91366	.42235	.90643	1
60	.37461	.92718	.39073	.92050	.40674	.91355	.42262	.90631	0
′	Cosine 68°	Sine 68°	Cosine 67°	Sine 67°	Cosine 66°	Sine 66°	Cosine 65°	Sine 65°	′

TABLE 5.—NATURAL TRIGONOMETRIC FUNCTIONS—(*Continued*)

	25°		26°		27°		28°		
′	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	′
0	.42262	.90631	.43837	.89879	.45399	.89101	.46947	.88295	60
1	.42288	.90618	.43863	.89867	.45425	.89087	.46973	.88281	59
2	.42315	.90606	.43889	.89854	.45451	.89074	.46999	.88267	58
3	.42341	.90594	.43916	.89841	.45477	.89061	.47024	.88254	57
4	.42367	.90582	.43942	.89828	.45503	.89048	.47050	.88240	56
5	.42394	.90569	.43968	.89816	.45529	.89035	.47076	.88226	55
6	.42420	.90557	.43994	.89803	.45554	.89021	.47101	.88213	54
7	.42446	.90545	.44020	.89790	.45580	.89008	.47127	.88199	53
8	.42473	.90532	.44046	.89777	.45606	.88995	.47153	.88185	52
9	.42499	.90520	.44072	.89764	.45632	.88981	.47178	.88172	51
10	.42525	.90507	.44098	.89752	.45658	.88968	.47204	.88158	50
11	.42552	.90495	.44124	.89739	.45684	.88955	.47229	.88144	49
12	.42578	.90483	.44151	.89726	.45710	.88942	.47255	.88130	48
13	.42604	.90470	.44177	.89713	.45736	.88928	.47281	.88117	47
14	.42631	.90458	.44203	.89700	.45762	.88915	.47306	.88103	46
15	.42657	.90446	.44229	.89687	.45787	.88902	.47332	.88089	45
16	.42683	.90433	.44255	.89674	.45813	.88888	.47358	.88075	44
17	.42709	.90421	.44281	.89662	.45839	.88875	.47383	.88062	43
18	.42736	.90408	.44307	.89649	.45865	.88862	.47409	.88048	42
19	.42762	.90396	.44333	.89636	.45891	.88848	.47434	.88034	41
20	.42788	.90383	.44359	.89623	.45917	.88835	.47460	.88020	40
21	.42815	.90371	.44385	.89610	.45942	.88822	.47486	.88006	39
22	.42841	.90358	.44411	.89597	.45968	.88808	.47511	.87993	38
23	.42867	.90346	.44437	.89584	.45994	.88795	.47537	.87979	37
24	.42894	.90334	.44464	.89571	.46020	.88782	.47562	.87965	36
25	.42920	.90321	.44490	.89558	.46046	.88768	.47588	.87951	35
26	.42946	.90309	.44516	.89545	.46072	.88755	.47614	.87937	34
27	.42972	.90296	.44542	.89532	.46097	.88741	.47639	.87923	33
28	.42999	.90284	.44568	.89519	.46123	.88728	.47665	.87909	32
29	.43025	.90271	.44594	.89506	.46149	.88715	.47690	.87896	31
30	.43051	.90259	.44620	.89493	.46175	.88701	.47716	.87882	30
31	.43077	.90246	.44646	.89480	.46201	.88688	.47741	.87868	29
32	.43104	.90233	.44672	.89467	.46226	.88674	.47767	.87854	28
33	.43130	.90221	.44698	.89454	.46252	.88661	.47793	.87840	27
34	.43156	.90208	.44724	.89441	.46278	.88647	.47818	.87826	26
35	.43182	.90196	.44750	.89428	.46304	.88634	.47844	.87812	25
36	.43209	.90183	.44776	.89415	.46330	.88620	.47869	.87798	24
37	.43235	.90171	.44802	.89402	.46355	.88607	.47895	.87784	23
38	.43261	.90158	.44828	.89389	.46381	.88593	.47920	.87770	22
39	.43287	.90146	.44854	.89376	.46407	.88580	.47946	.87756	21
40	.43313	.90133	.44880	.89363	.46433	.88566	.47971	.87743	20
41	.43340	.90120	.44906	.89350	.46458	.88553	.47997	.87729	19
42	.43366	.90108	.44932	.89337	.46484	.88539	.48022	.87715	18
43	.43392	.90095	.44958	.89324	.46510	.88526	.48048	.87701	17
44	.43418	.90082	.44984	.89311	.46536	.88512	.48073	.87687	16
45	.43445	.90070	.45010	.89298	.46561	.88499	.48099	.87673	15
46	.43471	.90057	.45036	.89285	.46587	.88485	.48124	.87659	14
47	.43497	.90045	.45062	.89272	.46613	.88472	.48150	.87645	13
48	.43523	.90032	.45088	.89259	.46639	.88458	.48175	.87631	12
49	.43549	.90019	.45114	.89245	.46664	.88445	.48201	.87617	11
50	.43575	.90007	.45140	.89232	.46690	.88431	.48226	.87603	10
51	.43602	.80094	.45166	.89219	.46716	.88417	.48252	.87589	9
52	.43628	.89081	45192	.89206	.46742	.88404	.48277	.87575	8
53	.43654	.89068	.45218	.89193	.46767	.88390	.48303	.87561	7
54	.43680	.89956	.45243	.89180	.46793	.88377	.48328	.87546	6
55	.43706	.89043	.45269	.89167	.46819	.88363	.48354	.87532	5
56	.43733	.89030	.45295	.89153	.46844	.88349	.48379	.87518	4
57	.43759	.89918	.45321	.89140	.46870	.88336	.48405	.87504	3
58	.43785	.89905	.45347	.89127	.46896	.88322	.48430	.87490	2
59	.43811	.89892	.45373	.89114	.46921	.88308	.48456	.87476	1
60	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	64°		63°		62°		61°		

	29°		30°		31°		32°		
′	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	′
0	.48481	.87462	.50000	.86603	.51504	.85717	.52992	.84805	60
1	.48506	.87448	.50025	.86588	.51529	.85702	.53017	.84789	59
2	.48532	.87434	.50050	.86573	.51554	.85687	.53041	.84774	58
3	.48557	.87420	.50076	.86559	.51579	.85672	.53066	.84759	57
4	.48583	.87406	.50101	.86544	.51604	.85657	.53091	.84743	56
5	.48608	.87391	.50126	.86530	.51628	.85642	.53115	.84728	55
6	.48634	.87377	.50151	.86515	.51653	.85627	.53140	.84712	54
7	.48659	.87363	.50176	.86501	.51678	.85612	.53164	.84697	53
8	.48684	.87349	.50201	.86486	.51703	.85597	.53189	.84681	52
9	.48710	.87335	.50227	.86471	.51728	.85582	.53214	.84666	51
10	.48735	.87321	.50252	.86457	.51753	.85567	.53238	.84650	50
11	.48761	.87306	.50277	.86442	.51778	.85551	.53263	.84635	49
12	.48786	.87292	.50302	.86427	.51803	.85536	.53288	.84619	48
13	.48811	.87278	.50327	.86413	.51828	.85521	.53312	.84604	47
14	.48837	.87264	.50352	.86398	.51852	.85506	.53337	.84588	46
15	.48862	.87250	.50377	.86384	.51877	.85491	.53361	.84573	45
16	.48888	.87235	.50403	.86369	.51902	.85476	.53386	.84557	44
17	.48913	.87221	.50428	.86354	.51927	.85461	.53411	.84542	43
18	.48938	.87207	.50453	.86340	.51952	.85446	.53435	.84526	42
19	.48964	.87193	.50478	.86325	.51977	.85431	.53460	.84511	41
20	.48989	.87178	.50503	.86310	.52002	.85416	.53484	.84495	40
21	.49014	.87164	.50528	.86295	.52026	.85401	.53509	.84480	39
22	.49040	.87150	.50553	.86281	.52051	.85385	.53534	.84464	38
23	.49065	.87136	.50578	.86266	.52076	.85370	.53558	.84448	37
24	.49090	.87121	.50603	.86251	.52101	.85355	.53583	.84433	36
25	.49116	.87107	.50628	.86237	.52126	.85340	.53607	.84417	35
26	.49141	.87093	.50654	.86222	.52151	.85325	.53632	.84402	34
27	.49166	.87079	.50679	.86207	.52175	.85310	.53656	.84386	33
28	.49192	.87064	.50704	.86192	.52200	.85294	.53681	.84370	32
29	.49217	.87050	.50729	.86178	.52225	.85279	.53705	.84355	31
30	.49242	.87036	.50754	.86163	.52250	.85264	.53730	.84339	30
31	.49268	.87021	.50779	.86148	.52275	.85249	.53754	.84324	29
32	.49293	.87007	.50804	.86133	.52299	.85234	.53779	.84308	28
33	.49318	.86993	.50829	.86119	.52324	.85218	.53804	.84292	27
34	.49344	.86978	.50854	.86104	.52349	.85203	.53828	.84277	26
35	.49369	.86964	.50879	.86089	.52374	.85188	.53853	.84261	25
36	.49394	.86949	.50904	.86074	.52399	.85173	.53877	.84245	24
37	.49419	.86935	.50929	.86059	.52423	.85157	.53902	.84230	23
38	.49445	.86921	.50954	.86045	.52448	.85142	.53926	.84214	22
39	.49470	.86906	.50979	.86030	.52473	.85127	.53951	.84198	21
40	.49495	.86892	.51004	.86015	.52498	.85112	.53975	.84182	20
41	.49521	.86878	.51029	.86000	.52522	.85096	.54000	.84167	19
42	.49546	.86863	.51054	.85985	.52547	.85081	.54024	.84151	18
43	.49571	.86849	.51079	.85970	.52572	.85066	.54049	.84135	17
44	.49596	.86834	.51104	.85956	.52597	.85051	.54073	.84120	16
45	.49622	.86820	.51129	.85941	.52621	.85035	.54097	.84104	15
46	.49647	.86805	.51154	.85926	.52646	.85020	.54122	.84088	14
47	.49672	.86791	.51179	.85911	.52671	.85005	.54146	.84072	13
48	.49697	.86777	.51204	.85896	.52696	.84989	.54171	.84057	12
49	.49723	.86762	.51229	.85881	.52720	.84974	.54195	.84041	11
50	.49748	.86748	.51254	.85866	.52745	.84959	.54220	.84025	10
51	.49773	.86733	.51279	.85851	.52770	.84943	.54244	.84009	9
52	.49798	.86719	.51304	.85836	.52794	.84928	.54269	.83994	8
53	.40824	.86704	.51329	.85821	.52819	.84913	.54293	.83978	7
54	.49849	.86690	.51354	.85806	.52844	.84897	.54317	.83962	6
55	.49874	.86675	.51379	.85792	.52869	.84882	.54342	.83946	5
56	.49899	.86661	.51404	.85777	.52893	.84866	.54366	.83930	4
57	.49924	.86646	.51429	.85762	.52918	.84851	.54391	.83915	3
58	.49950	.86632	.51454	.85747	.52943	.84836	.54415	.83899	2
59	.49975	.86617	.51479	.85732	.52967	.84820	.54440	.83883	1
60	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	60°		59°		58°		57°		

TABLE 5.—NATURAL TRIGONOMETRIC FUNCTIONS—(*Continued*)

′	33° Sine	33° Cosine	34° Sine	34° Cosine	35° Sine	35° Cosine	36° Sine	36° Cosine	′
0	.54464	.83867	.55919	.82904	.57358	.81915	.58779	.80902	60
1	.54488	.83851	.55943	.82887	.57381	.81899	.58802	.80885	59
2	.54513	.83835	.55968	.82871	.57405	.81882	.58826	.80867	58
3	.54537	.83819	.55992	.82855	.57429	.81865	.58849	.80850	57
4	.54561	.83804	.56016	.82839	.57453	.81848	.58873	.80833	56
5	.54586	.83788	.56040	.82822	.57477	.81832	.58896	.80816	55
6	.54610	.83772	.56064	.82806	.57501	.81815	.58920	.80799	54
7	.54635	.83756	.56088	.82790	.57524	.81798	.58943	.80782	53
8	.54659	.83740	.56112	.82773	.57548	.81782	.58967	.80765	52
9	.54683	.83724	.56136	.82757	.57572	.81765	.58990	.80748	51
10	.54708	.83708	.56160	.82741	.57596	.81748	.59014	.80730	50
11	.54732	.83692	.56184	.82724	.57619	.81731	.59037	.80713	49
12	.54756	.83676	.56208	.82708	.57643	.81714	.59061	.80696	48
13	.54781	.83660	.56232	.82692	.57667	.81698	.59084	.80679	47
14	.54805	.83645	.56256	.82675	.57691	.81681	.59108	.80662	46
15	.54829	.83629	.56280	.82659	.57715	.81664	.59131	.80644	45
16	.54854	.83613	.56305	.82643	.57738	.81647	.59154	.80627	44
17	.54878	.83597	.56329	.82626	.57762	.81631	.59178	.80610	43
18	.54902	.83581	.56353	.82610	.57786	.81614	.59201	.80593	42
19	.54927	.83565	.56377	.82593	.57810	.81597	.59225	.80576	41
20	.54951	.83549	.56401	.82577	.57833	.81580	.59248	.80558	40
21	.54975	.83533	.56425	.82561	.57857	.81563	.59272	.80541	39
22	.54999	.83517	.56449	.82544	.57881	.81546	.59295	.80524	38
23	.55024	.83501	.56473	.82528	.57904	.81530	.59318	.80507	37
24	.55048	.83485	.56497	.82511	.57928	.81513	.59342	.80489	36
25	.55072	.83469	.56521	.82495	.57952	.81496	.59365	.80472	35
26	.55097	.83453	.56545	.82478	.57976	.81479	.59389	.80455	34
27	.55121	.83437	.56569	.82462	.57999	.81462	.59412	.80438	33
28	.55145	.83421	.56593	.82446	.58023	.81445	.59436	.80420	32
29	.55169	.83405	.56617	.82429	.58047	.81428	.59459	.80403	31
30	.55194	.83389	.56641	.82413	.58070	.81412	.59482	.80386	30
31	.55218	.83373	.56665	.82396	.58094	.81395	.59506	.80368	29
32	.55242	.83356	.56689	.823 0	.58118	.81378	.59529	.80351	28
33	.55266	.83340	.56713	.82363	.58141	.81361	.59552	.80334	27
34	.55291	.83324	.56736	.82347	.58165	.81344	.59576	.80316	26
35	.55315	.83308	.56760	.82330	.58189	.81327	.59599	.80299	25
36	.55339	.83292	.56784	.82314	.58212	.81310	.59622	.80282	24
37	.55363	.83276	.56808	.82297	.58236	.81293	.59646	.80264	23
38	.55388	.83260	.56832	.82281	.58260	.81276	.59669	.80247	22
39	.55412	.83244	.56856	.82264	.58283	.81259	.59693	.80230	21
40	.55436	.83228	.56880	.82248	.58307	.81242	.59716	.80212	20
41	.55460	.83212	.56904	.82231	.58330	.81225	.59739	.80195	19
42	.55484	.83195	.56928	.82214	.58354	.81208	.59763	.80178	18
43	.55509	.83179	.56952	.82198	.58378	.81191	.59786	.80160	17
44	.55533	.83163	.56976	.82181	.58401	.81174	.59809	.80143	16
45	.55557	.83147	.57000	.82165	.58425	.81157	.59832	.80125	15
46	.55581	.83131	.57024	.82148	.58449	.81140	.59856	.80108	14
47	.55605	.83115	.57047	.82132	.58472	.81123	.59879	.80091	13
48	.55630	.83098	.57071	.82115	.58496	.81106	.59902	.80073	12
49	.55654	.83082	.57095	.82098	.58519	.81089	.59926	.80056	11
50	.55678	.83066	.57119	.82082	.58543	.81072	.59949	.80038	10
51	.55702	.83050	.57143	.82065	.58567	.81055	.59972	.80021	9
52	.55726	.83034	.57167	.82048	.58590	.81038	.59995	.80003	8
53	.55750	.83017	.57191	.82032	.58614	.81021	.60019	.79986	7
54	.55775	.83001	.57215	.82015	.58637	.81004	.60042	.79968	6
55	.55799	.82985	.57238	.81999	.58661	.80987	.60065	.79951	5
56	.55823	.82969	.57262	.81982	.58684	.80970	.60089	.79934	4
57	.55847	.82953	.57286	.81965	.58708	.80953	.60112	.79916	3
58	.55871	.82936	.57310	.81949	.58731	.80936	.60135	.79899	2
59	.55895	.82920	.57334	.81932	.58755	.80919	.60158	.79881	1
60	.55919	.82904	.57358	.81915	.58779	.80902	.60182	.79864	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	56°		55°		54°		53°		

′	37° Sine	37° Cosine	38° Sine	38° Cosine	39° Sine	39° Cosine	40° Sine	40° Cosine	′
0	.60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	60
1	.60205	.79846	.61589	.78783	.62955	.77696	.64301	.76586	59
2	.60228	.79829	.61612	.78765	.62977	.77678	.64323	.76567	58
3	.60251	.79811	.61635	.78747	.63000	.77660	.64346	.76548	57
4	.60274	.79793	.61658	.78729	.63022	.77641	.64368	.76530	56
5	.60298	.79776	.61681	.78711	.63045	.77623	.64390	.76511	55
6	.60321	.79758	.61704	.78694	.63068	.77605	.64412	.76492	54
7	.60344	.79741	.61726	.78676	.63090	.77586	.64435	.76473	53
8	.60367	.79723	.61749	.78658	.63113	.77568	.64457	.76455	52
9	.60390	.79706	.61772	.78640	.63135	.77550	.64479	.76436	51
10	.60414	.79688	.61795	.78622	.63158	.77531	.64501	.76417	50
11	.60437	.79671	.61818	.78604	.63180	.77513	.64524	.76398	49
12	.60460	.79653	.61841	.78586	.63203	.77494	.64546	.76380	48
13	.60483	.79635	.61864	.78568	.63225	.77476	.64568	.76361	47
14	.60506	.79618	.61887	.78550	.63248	.77458	.64590	.76342	46
15	.60529	.79600	.61909	.78532	.63271	.77439	.64612	.76323	45
16	.60553	.79583	.61932	.78514	.63293	.77421	.64635	.76304	44
17	.60576	.79565	.61955	.78496	.63316	.77402	.64657	.76286	43
18	.60599	.79547	.61978	.78478	.63338	.77384	.64679	.76267	42
19	.60622	.79530	.62001	.78460	.63361	.77366	.64701	.76248	41
20	.60645	.79512	.62024	.78442	.63383	.77347	.64723	.76229	40
21	.60668	.79494	.62046	.78424	.63406	.77329	.64746	.76210	39
22	.60691	.79477	.62069	.78405	.63428	.77310	.64768	.76192	38
23	.60714	.79459	.62092	.78387	.63451	.77292	.64790	.76173	37
24	.60738	.79441	.62115	.78369	.63473	.77273	.64812	.76154	36
25	.60761	.79424	.62138	.78351	.63496	.77255	.64834	.76135	35
26	.60784	.79406	.62160	.78333	.63518	.77236	.64856	.76116	34
27	.60807	.79388	.62183	.78315	.63540	.77218	.64878	.76097	33
28	.60830	.79371	.62206	.78297	.63563	.77199	.64901	.76078	32
29	.60853	.79353	.62229	.78279	.63585	.77181	.64923	.76059	31
30	.60876	.79335	.62251	.78261	.63608	.77162	.64945	.76041	30
31	.60899	.79318	.62274	.78243	.63630	.77144	.64967	.76022	29
32	.60922	.79300	.62297	.78225	.63653	.77125	.64989	.76003	28
33	.60945	.79282	.62320	.78206	.63675	.77107	.65011	.75984	27
34	.60968	.79264	.62342	.78188	.63698	.77088	.65033	.75965	26
35	.60991	.79247	.62365	.78170	.63720	.77070	.65055	.75946	25
36	.61015	.79229	.62388	.78152	.63742	.77051	.65077	.75927	24
37	.61038	.79211	.62411	.78134	.63765	.77033	.65100	.75908	23
38	.61061	.79193	.62433	.78116	.63787	.77014	.65122	.75889	22
39	.61084	.79176	.62456	.78098	.63810	.76996	.65144	.75870	21
40	.61107	.79158	.62479	.78079	.63832	.76977	.65166	.75851	20
41	.61130	.79140	.62502	.78061	.63854	.76959	.65188	.75832	19
42	.61153	.79122	.62524	.78043	.63877	.76940	.65210	.75813	18
43	.61176	.79105	.62547	.78025	.63899	.76921	.65232	.75794	17
44	.61199	.79087	.62570	.78007	.63922	.76903	.65254	.75775	16
45	.61222	.79069	.62592	.77988	.63944	.76884	.65276	.75756	15
46	.61245	.79051	.62615	.77970	.63966	.76866	.65298	.75738	14
47	.61268	.79033	.62638	.77952	.63989	.76847	.65320	.75719	13
48	.61291	.79016	.62660	.77934	.64011	.76828	.65342	.75700	12
49	.61314	.78998	.62683	.77916	.64033	.76810	.65364	.75680	11
50	.61337	.78980	.62706	.77897	.64056	.76791	.65386	.75661	10
51	.61360	.78962	.62728	.77879	.64078	.76772	.65408	.75642	9
52	.61383	.78944	.62751	.77861	.64100	.76754	.65430	.75623	8
53	.61406	.78926	.62774	.77843	.64123	.76735	.65452	.75604	7
54	.61429	.78908	.62796	.77824	.64145	.76717	.65474	.75585	6
55	.61451	.78891	.62819	.77806	.64167	.76698	.65496	.75566	5
56	.61474	.78873	.62842	.77788	.64190	.76679	.65518	.75547	4
57	.61497	.78855	.62864	.77769	.64212	.76661	.65540	.75528	3
58	.61520	.78837	.62887	.77751	.64234	.76642	.65562	.75509	2
59	.61543	.78819	.62909	.77733	.64256	.76623	.65584	.75490	1
60	.61566	.78801	.62932	.77715	.64279	.76604	.65606	.75471	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	52°		51°		50°		49°		

TABLE 5.—NATURAL TRIGONOMETRIC FUNCTIONS—(*Continued*)

′	41° Sine	41° Cosine	42° Sine	42° Cosine	43° Sine	43° Cosine	44° Sine	44° Cosine	′
0	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	60
1	.65628	.75452	.66935	.74295	.68221	.73116	.69487	.71914	59
2	.65650	.75433	.66956	.74276	.68242	.73096	.69508	.71894	58
3	.65672	.75414	.66978	.74256	.68264	.73076	.69529	.71873	57
4	.65694	.75395	.66999	.74237	.68285	.73056	.69549	.71853	56
5	.65716	.75375	.67021	.74217	.68306	.73036	.69570	.71833	55
6	.65738	.75356	.67043	.74198	.68327	.73016	.69591	.71813	54
7	.65759	.75337	.67064	.74178	.68349	.72996	.69612	.71792	53
8	.65781	.75318	.67086	.74159	.68370	.72976	.69633	.71772	52
9	.65803	.75299	.67107	.74139	.68391	.72957	.69654	.71752	51
10	.65825	.75280	.67129	.74120	.68412	.72937	.69675	.71732	50
11	.65847	.75261	.67151	.74100	.68434	.72917	.69696	.71711	49
12	.65869	.75241	.67172	.74080	.68455	.72897	.69717	.71691	48
13	.65891	.75222	.67194	.74061	.68476	.72877	.69737	.71671	47
14	.65913	.75203	.67215	.74041	.68497	.72857	.69758	.71650	46
15	.65935	.75184	.67237	.74022	.68518	.72837	.69779	.71630	45
16	.65956	.75165	.67258	.74002	.68539	.72817	.69800	.71610	44
17	.65978	.75146	.67280	.73983	.68561	.72797	.69821	.71590	43
18	.66000	.75126	.67301	.73963	.68582	.72777	.69842	.71569	42
19	.66022	.75107	.67323	.73944	.68603	.72757	.69862	.71549	41
20	.66044	.75088	.67344	.73924	.68624	.72737	.69883	.71529	40
21	.66066	.75069	.67366	.73904	.68645	.72717	.69904	.71508	39
22	.66088	.75050	.67387	.73885	.68666	.72697	.69925	.71488	38
23	.66109	.75030	.67409	.73865	.68688	.72677	.69946	.71468	37
24	.66131	.75011	.67430	.73846	.68709	.72657	.69966	.71447	36
25	.66153	.74992	.67452	.73826	.68730	.72637	.69987	.71427	35
26	.66175	.74973	.67473	.73806	.68751	.72617	.70008	.71407	34
27	.66197	.74953	.67495	.73787	.68772	.72597	.70029	.71386	33
28	.66218	.74934	.67516	.73767	.68793	.72577	.70049	.71366	32
29	.66240	.74915	.67538	.73747	.68814	.72557	.70070	.71345	31
30	.66262	.74896	.67559	.73728	.68835	.72537	.70091	.71325	30
31	.66284	.74876	.67580	.73708	.68857	.72517	.70112	.71305	29
32	.66306	.74857	.67602	.73688	.68878	.72497	.70132	.71284	28
33	.66327	.74838	.67623	.73669	.68899	.72477	.70153	.71264	27
34	.66349	.74818	.67645	.73649	.68920	.72457	.70174	.71243	26
35	.66371	.74799	.67666	.73629	.68941	.72437	.70195	.71223	25
36	.66393	.74780	.67688	.73610	.68962	.72417	.70215	.71203	24
37	.66414	.74760	.67709	.73590	.68983	.72397	.70236	.71182	23
38	.66436	.74741	.67730	.73570	.69004	.72377	.70257	.71162	22
39	.66458	.74722	.67752	.73551	.69025	.72357	.70277	.71141	21
40	.66480	.74703	.67773	.73531	.69046	.72337	.70298	.71121	20
41	.66501	.74683	.67795	.73511	.69067	.72317	.70319	.71100	19
42	.66523	.74664	.67816	.73491	.69088	.72297	.70339	.71080	18
43	.66545	.74644	.67837	.73472	.69109	.72277	.70360	.71059	17
44	.66566	.74625	.67859	.73452	.69130	.72257	.70381	.71039	16
45	.66588	.74606	.67880	.73432	.69151	.72236	.70401	.71019	15
46	.66610	.74586	.67901	.73413	.69172	.72216	.70422	.70998	14
47	.66632	.74567	.67923	.73393	.69193	.72196	.70443	.70978	13
48	.66653	.74548	.67944	.73373	.69214	.72176	.70463	.70957	12
49	.66675	.74528	.67965	.73353	.69235	.72156	.70484	.70937	11
50	.66697	.74509	.67987	.73333	.69256	.72136	.70505	.70916	10
51	.66718	.74489	.68008	.73314	.69277	.72116	.70525	.70896	9
52	.66740	.74470	.68029	.73294	.69298	.72095	.70546	.70875	8
53	.66762	.74451	.68051	.73274	.69319	.72075	.70567	.70855	7
54	.66783	.74431	.68072	.73254	.69340	.72055	.70587	.70834	6
55	.66805	.74412	.68093	.73234	.69361	.72035	.70608	.70813	5
56	.66827	.74392	.68115	.73215	.69382	.72015	.70628	.70793	4
57	.66848	.74373	.68136	.73195	.69403	.71995	.70649	.70772	3
58	.66870	.74353	.68157	.73175	.69424	.71974	.70670	.70752	2
59	.66891	.74334	.68179	.73155	.69445	.71954	.70690	.70731	1
60	.66913	.74314	.68200	.73135	.69466	.71934	.70711	.70711	0
′	Cosine 48°	Sine 48°	Cosine 47°	Sine 47°	Cosine 46°	Sine 46°	Cosine 45°	Sine 45°	′

′	0° Sec.	0° Co-sec.	1° Sec.	1° Co-sec.	2° Sec.	2° Co-sec.	3° Sec.	3° Co-sec.	′
0	1	Infinite.	1.0001	57.299	1.0006	28.654	1.0014	19.107	60
1	1	3437.70	1.0001	56.359	1.0006	28.417	1.0014	19.002	59
2	1	1718.90	1.0002	55.450	1.0006	28.184	1.0014	18.897	58
3	1	1145.90	1.0002	54.570	1.0006	27.955	1.0014	18.704	57
4	1	859.44	1.0002	53.718	1.0006	27.730	1.0014	18.692	56
5	1	687.55	1.0002	52.891	1.0007	27.508	1.0014	18.591	55
6	1	572.96	1.0002	52.090	1.0007	27.290	1.0015	18.491	54
7	1	491.11	1.0002	51.313	1.0007	27.075	1.0015	18.393	53
8	1	429.72	1.0002	50.558	1.0007	26.864	1.0015	18.295	52
9	1	381.97	1.0002	49.826	1.0007	26.655	1.0015	18.198	51
10	1	343.77	1.0002	49.114	1.0007	26.450	1.0015	18.103	50
11	1	312.52	1.0002	48.422	1.0007	26.249	1.0015	18.008	49
12	1	286.48	1.0002	47.750	1.0007	26.050	1.0016	17.914	48
13	1	264.44	1.0002	47.096	1.0007	25.854	1.0016	17.821	47
14	1	245.55	1.0002	46.460	1.0008	25.661	1.0016	17.730	46
15	1	229.18	1.0002	45.840	1.0008	25.471	1.0016	17.639	45
16	1	214.86	1.0002	45.237	1.0008	25.284	1.0016	17.549	44
17	1	202.22	1.0002	44.650	1.0008	25.100	1.0016	17.460	43
18	1	190.99	1.0002	44.077	1.0008	24.918	1.0017	17.372	42
19	1	180.73	1.0003	43.520	1.0008	24.739	1.0017	17.285	41
20	1	171.89	1.0003	42.976	1.0008	24.562	1.0017	17.198	40
21	1	163.70	1.0003	42.445	1.0008	24.358	1.0017	17.113	39
22	1	156.26	1.0003	41.928	1.0008	24.216	1.0017	17.028	38
23	1	149.47	1.0003	41.423	1 0009	24.047	1.0017	16.944	37
24	1	143.24	1.0003	40.930	1.0009	23.880	1.0018	16.861	36
25	1	137.51	1.0003	40.448	1.0009	23.716	1.0018	16.779	35
26	1	132.22	1.0003	39.978	1.0009	23.553	1.0018	16.698	34
27	1	127.32	1.0003	39.518	1.0009	23.393	1.0018	16.617	33
28	1	122.78	1.0003	39.069	1.0009	23.235	1.0018	16.538	32
29	1	118.54	1.0003	38.631	1.0009	23.079	1.0018	16.459	31
30	1	114.59	1.0003	38.201	1.0009	22.925	1.0019	16.380	30
31	1	110.90	1.0003	37.782	1.0010	22.774	1.0019	16.303	29
32	1	107.43	1.0003	37.371	1.0010	22.624	1.0019	16.226	28
33	1	104.17	1.0004	36.969	1.0010	22.476	1.0019	16.150	27
34	1	101.11	1.0004	36.576	1.0010	22.330	1.0019	16.075	26
35	1	95.223	1.0004	36.191	1.0010	22.186	1.0019	16.000	25
36	1	95.405	1.0004	35.814	1.0010	22.044	1.0020	15.926	24
37	1	92.914	1.0004	35.445	1.0010	21.904	1.0020	15.853	23
38	1.0001	92.469	1.0004	35.084	1.0010	21.765	1.0020	15.780	22
39	1.0001	88.149	1.0004	34.729	1.0011	21.629	1.0020	15.708	21
40	1.0001	85.946	1.0004	34.382	1.0011	21.494	1.0020	15.637	20
41	1.0001	83.849	1.0004	34.042	1.0011	21.360	1.0021	15.566	19
42	1.0001	81.853	1.0004	33.708	1.0011	21.228	1.0021	15.496	18
43	1.0001	79.950	1.0004	33.381	1.0011	21.098	1.0021	15.427	17
44	1.0001	78.133	1.0004	33.060	1.0011	20.970	1.0021	15.358	16
45	1.0001	76.396	1.0005	32.745	1.0011	20.843	1.0021	15.290	15
46	1.0001	74.736	1.0005	32.437	1.0012	20.717	1.0022	15.222	14
47	1.0001	73.146	1.0005	32.134	1.0012	20.593	1.0022	15.155	13
48	1.0001	71.622	1.0005	31.836	1.0012	20.471	1.0022	15.089	12
49	1.0001	71.160	1.0005	31.544	1.0012	20.350	1.0022	15.023	11
50	1.0001	68.757	1.0005	31.257	1.0012	20.230	1.0022	14.958	10
51	1.0001	67.409	1.0005	30.976	1.0012	20.112	1.0023	14.803	9
52	1.0001	66.113	1.0005	30.699	1.0012	19.995	1.0023	14.829	8
53	1.0001	64.866	1.0005	30.428	1.0013	19.880	1.0023	14.765	7
54	1.0001	63.664	1.0005	30.161	1.0013	19.766	1.0023	14.702	6
55	1.0001	62.507	1.0005	29.899	1.0013	19.653	1.0023	14.640	5
56	1.0001	61.391	1.0006	29.641	1.0013	19.541	1.0024	14.578	4
57	1.0001	61.314	1.0006	29.388	1.0013	19.431	1.0024	14.517	3
58	1.0001	59.274	1.0006	29.130	1.0013	19.322	1.0024	14.456	2
59	1.0001	58.270	1.0006	28.894	1.0013	19.214	1.0024	14.395	1
60	1.0001	57.299	1.0006	28.654	1.0014	19.107	1.0024	14.335	0
′	Co-sec. 89°	Sec. 89°	Co-sec. 88°	Sec. 88°	Co-sec. 87°	Sec. 87°	Co-sec. 86°	Sec. 86°	

TABLE 5.—NATURAL TRIGONOMETRIC FUNCTIONS—(*Continued*)

	4°		5°		6°		7°		
′	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec	Sec.	Co-sec.	′
0	1.0024	14.335	1.0038	11.474	1.0055	9.5668	1.0075	8.2055	60
1	1.0025	14.276	1.0038	11.436	1.0055	9.5404	1.0075	8.1861	59
2	1.0025	14.217	1.0039	11.398	1.0056	9.5141	1.0076	8.1668	58
3	1.0025	14.159	1.0039	11.360	1.0056	9.4880	1.0076	8.1476	57
4	1.0025	14.101	1.0039	11.323	1.0056	9.4620	1.0076	8.1285	56
5	1.0025	14.043	1.0039	11.286	1.0057	9.4362	1.0077	8.1094	55
6	1.0026	13.986	1.0040	11.249	1.0057	9.4105	1.0077	8.0905	54
7	1.0026	13.930	1.0040	11.213	1.0057	9.3850	1.0078	8.0717	53
8	1.0026	13.874	1.0040	11.176	1.0057	9.3596	1.0078	8.0529	52
9	1.0026	13.818	1.0040	11.140	1.0058	9.3343	1.0078	8.0342	51
10	1.0026	13.763	1.0041	11.104	1.0058	9.3092	1.0079	8.0156	50
11	1.0027	13.708	1.0041	11.069	1.0058	9.2842	1.0079	7.9971	49
12	1.0027	13.654	1.0041	11.033	1.0059	9.2593	1.0079	7.9787	48
13	1.0027	13.600	1.0041	10.988	1.0059	9.2346	1.0080	7.9604	47
14	1.0027	13.547	1.0042	10.963	1.0059	9.2100	1.0080	7.9421	46
15	1.0027	13.494	1.0042	10.929	1.0060	9.1855	1.0080	7.9240	45
16	1.0028	13.441	1.0042	10.894	1.0060	9.1612	1.0081	7.9059	44
17	1.0028	13.389	1.0043	10.860	1.0060	9.1370	1.0081	7.8879	43
18	1.0028	13.337	1.0043	10.826	1.0061	9.1129	1.0082	7.8700	42
19	1.0028	13.286	1.0043	10.792	1.0061	9.0890	1.0082	7.8522	41
20	1.0029	13.235	1.0043	10.758	1.0061	9.0651	1.0082	7.8344	40
21	1.0029	13.184	1.0044	10.725	1.0062	9.0414	1.0083	7.8168	39
22	1.0029	13.134	1.0044	10.692	1 0062	9.0179	1.0083	7.7992	38
23	1.0029	13.084	1.0044	10.659	1.0062	8.9944	1.0084	7.7817	37
24	1.0029	13.034	1.0044	10.626	1.0063	8.9711	1.0084	7.7642	36
25	1.0030	12.985	1.0045	10.593	1.0063	8.9479	1.0084	7.7469	35
26	1.0030	12.937	1.0045	10.561	1.0063	8.9248	1.0085	7.7296	34
27	1.0030	12.888	1.0045	10.529	1.0064	8.9018	1.0085	7.7124	33
28	1.0030	12.840	1.0046	10.497	1.0064	8.8790	1.0085	7.6953	32
29	1.0031	12.793	1.0046	10.465	1.0064	8.8563	1.0086	7.6783	31
30	1.0031	12.745	1.0046	10.433	1.0065	8.8337	1.0086	7.6613	30
31	1.0031	12.698	1.0046	10.402	1.0065	8.8112	1.0087	7.6444	29
32	1.0031	12.652	1.0047	10.371	1.0065	8.7888	1.0087	7.6276	28
33	1.0032	12.606	1.0047	10.340	1.0066	8.7665	1.0087	7.6108	27
34	1.0032	12.560	1.0047	10.309	1.0066	8.7444	1.0088	7.5942	26
35	1.0032	12.514	1.0048	10.278	1.0066	8.7223	1.0088	7.5776	25
36	1.0032	12.469	1.0048	10.248	1.0067	8.7004	1.0089	7.5611	24
37	1.0032	12.424	1.0048	10.217	1.0067	8.6786	1.0089	7.5446	23
38	1.0033	12.379	1.0048	10.187	1.0067	8.6569	1.0089	7.5282	22
39	1.0033	12.335	1.0049	10.157	1.0068	8.6353	1.0090	7.5119	21
40	1.0033	12.291	1.0049	10.127	1.0068	8.6138	1'0090	7.4957	20
41	1.0033	12.248	1.0049	10.098	1.0068	8.5924	1.0090	7.4795	19
42	1.0034	12.204	1.0050	10.068	1.0069	8.5711	1.0091	7.4634	18
43	1.0034	12.161	1.0050	10.039	1.0069	8.5499	1.0091	7.4474	17
44	1.0034	12.118	1.0050	10.010	1.0069	8.5289	1.0092	7.4315	16
45	1.0034	12.076	1.0050	9.9812	1.0070	8.5079	1.0092	7.4156	15
46	1.0035	12.034	1.0051	9.9525	1.0070	8.4871	1.0092	7.3998	14
47	1.0035	11.992	1.0051	9.0239	1.0070	8.4663	1.0093	7.3840	13
48	1.0035	11.950	1.0051	9.8955	1.0071	8.4457	1.0093	7.3683	12
49	1.0035	11.909	1.0052	9.8672	1.0071	8.4251	1.0094	7.3527	11
50	1.0036	11.868	1.0052	9.8391	1.0071	8.4046	1.0094	7.3372	10
51	1.0036	11.828	1.0052	9.8112	1.0072	8.3843	1.0094	7.3217	9
52	1.0036	11.787	1.0053	9.7834	1.0072	8.3640	1.0095	7.3063	8
53	1.0036	11.747	1.0053	9.7558	1.0073	8.3439	1.0095	7.2909	7
54	1.0037	11.707	1.0053	9.7283	1.0073	8.3238	1.0096	7.2757	6
55	1.0037	11.668	1.0053	9.7010	1.0073	8.3039	1.0096	7.2604	5
56	1.0037	11.628	1.0054	9.6739	1.0074	8.2840	1.0097	7.2453	4
57	1.0037	11.589	1.0054	9.6469	1.0074	8.2642	1.0097	7.2302	3
58	1.0038	11.550	1.0054	9.6200	1.0074	8.2446	1.0097	7.2152	2
59	1.0038	11.512	1.0055	9.5933	1.0075	8.2250	1.0098	7.2002	1
60	1.0038	11.474	1.0055	9.5668	1.0075	8.2055	1.0098	7.1853	0
′	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	′
	85°		84°		83°		82°		

	8°		9°		10°		11°		
′	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	Co.sec.	Sec.	Co-sec.	′
0	1.0098	7.1853	1.0125	6.3924	1.0154	5.7588	1.0187	5.2408	60
1	1.0099	7.1704	1.0125	6.3807	1.0155	5.7493	1.0188	5.2330	59
2	1.0099	7.1557	1.0125	6.3690	1.0155	5.7398	1.0188	5.2252	58
3	1.0099	7.1409	1.0126	6.3574	1.0156	5.7304	1.0189	5.2174	57
4	1.0100	7.1263	1.0126	6.3458	1.0156	5.7210	1.0189	5.2097	56
5	1.0100	7.1117	1.0127	6.3343	1.0157	5.7117	1.0190	5.2019	55
6	1.0101	7.0972	1.0127	6.3228	1.0157	5.7023	1.0191	5.1942	54
7	1.0101	7.0827	1.0128	6.3113	1.0158	5.6930	1.0191	5.1865	53
8	1.0102	7.0683	1.0128	6.2999	1.0158	5.6838	1.0192	5.1788	52
9	1.0102	7.0539	1.0129	6.2885	1.0159	5.6745	1.0192	5.1712	51
10	1.0102	7.0396	1.0129	6.2772	1.0159	5.6653	1.0193	5.1636	50
11	1.0103	7.0254	1.0130	6.2659	1.0160	5.6561	1.0193	5.1560	49
12	1.0103	7.0112	1.0130	6.2546	1.0160	5.6470	1.0194	5.1484	48
13	1.0104	6.9971	1.0131	6.2434	1.0161	5.6379	1.0195	5.1409	47
14	1.0104	6.9830	1.0131	6.2322	1.0162	5.6288	1.0195	5.1333	46
15	1.0104	6.9690	1.0132	6.2211	1.0162	5.6197	1.0196	5.1258	45
16	1.0105	6.9550	1.0132	6.2100	1.0163	5.6107	1.0196	5.1183	44
17	1.0105	6.9411	1.0133	6.1990	1.0163	5.6017	1.0197	5.1109	43
18	1.0106	6.9273	1.0133	6.1880	1.0164	5.5928	1.0198	5.1034	42
19	1.0106	6.9135	1.0134	6.1770	1.0164	5.5838	1.0198	5.0960	41
20	1.0107	6.8998	1.0134	6.1661	1.0165	5.5749	1.0199	5.0886	40
21	1.0107	6.8861	1.0135	6.1552	1.0165	5.5660	1.0199	5.0812	39
22	1.0107	6.8725	1.0135	6.1443	1.0166	5.5572	1.0200	5.0739	38
23	1.0108	6.8589	1.0136	6.1335	1.0166	5.5484	1.0201	5.0666	37
24	1.0108	6.8454	1.0136	6.1227	1.0167	5.5396	1.0201	5.0593	36
25	1.0109	6.8320	1.0136	6.1120	1.0167	5.5308	1.0202	5.0520	35
26	1.0109	6.8185	1.0137	6.1013	1.0168	5.5221	1.0202	5.0447	34
27	1.0110	6.8052	1.0137	6.0906	1.0169	5.5134	1.0203	5 0375	33
28	1.0110	6.7919	1.0138	6.0800	1.0169	5.5047	1.0204	5.0302	32
29	1.0111	6.7787	1.0138	6.0694	1.0170	5.4960	1.0204	5.0230	31
30	1.0111	6.7655	1.0139	6.0588	1.0170	5.4874	1.0205	5.0158	30
31	1.0111	6.7523	1.0139	6.0483	1.0171	5.4788	1.0205	5.0087	29
32	1.0112	6.7392	1.0140	6.0379	1.0171	5.4702	1.0206	5.0015	28
33	1.0112	6.7262	1.0140	6.0274	1.0172	5.4617	1.0207	4.9944	27
34	1.0113	6.7132	1.0141	6.0170	1.0172	5.4532	1.0207	4.9873	26
35	1.0113	6.7003	1.0141	6.0066	1.0173	5.4447	1.0208	4.9802	25
36	1.0114	6.6874	1.0142	5.9963	1.0174	5.4362	1.0208	4.9732	24
37	1.0114	6.6745	1.0142	5.9860	1.0174	5.4278	1.0209	4.9661	23
38	1.0115	6.6617	1.0143	5.9758	1.0175	5.4194	1.0210	4.9591	22
39	1.0115	6.6490	1.0143	5.9655	1.0175	5.4110	1.0210	4.9521	21
40	1.0115	6.6363	1.0144	5.9554	1.0176	5.4026	1.0211	4.9452	20
41	1.0116	6.6237	1.0144	5.9452	1.0176	5.3943	1.0211	4.9382	19
42	1.0116	6.6111	1.0145	5.9351	1.0177	5.3860	1.0212	4.9313	18
43	1.0117	6.5985	1.0145	5.9250	1.0177	5.3777	1.0213	4.9243	17
44	1.0117	6.5860	1.0146	5.9150	1.0178	5.3695	1.0213	4.9175	16
45	1.0118	6.5736	1.0146	5.9049	1.0179	5.3612	1.0214	4.9106	15
46	1.0118	6.5612	1.0147	5.8950	1.0179	5.3530	1.0215	4.9037	14
47	1.0119	6.5488	1.0147	5.8850	1.0180	5 3449	1.0215	4.8969	13
48	1.0119	6.5365	1.0148	5.8751	1.0180	5.3367	1.0216	4.8901	12
49	1.0119	6.5243	1.0148	5.8652	1.0181	5.3286	1.0216	4.8833	11
50	1.0120	6.5121	1.0149	5.8554	1.0181	5.3205	1.0217	4.8765	10
51	1.0120	6.4999	1.0150	5.8456	1.0182	5.3124	1.0218	4.8697	9
52	1.0121	6.4878	1.0150	5.8358	1.0182	5.3044	1.0218	4.8630	8
53	1.0121	6.4757	1.0151	5.8261	1.0183	5.2963	1.0219	4.8563	7
54	1.0122	6.4637	1.0151	5.8163	1.0184	5.2883	1.0220	4.8496	6
55	1.0122	6.4517	1.0152	5.8067	1.0184	5.2803	1.0220	4.8429	5
56	1.0123	6.4398	1.0152	5.7970	1.0185	5.2724	1.0221	4.8362	4
57	1.0123	6.4279	1.0153	5.7874	1.0185	5.2645	1.0221	4.8296	3
58	1.0124	6.4160	1.0153	5.7778	1.0186	5.2566	1.0222	4.8229	2
59	1.0124	6.4042	1.0154	5.7683	1.0186	5.2487	1.0223	4.8163	1
60	1.0125	6.3924	1.0154	5.7588	1.0187	5.2408	1.0223	4.8097	0
′	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	′
	81°		80°		79°		78°		

TABLE 5.—NATURAL TRIGONOMETRIC FUNCTIONS—(*Continued*)

′	12° Sec.	12° Co-sec.	13° Sec.	13° Co-sec.	14° Sec.	14° Co-sec.	15° Sec.	15° Co-sec.	′
0	1.0223	4.8097	1.0263	4.4454	1.0306	4.1336	1.0353	3.8637	60
1	1.0224	4.8032	1.0264	4.4398	1.0307	4.1287	1.0353	3.8595	59
2	1.0225	4.7966	1.0264	4.4342	1.0308	4.1239	1.0354	3.8553	58
3	1.0225	4.7901	1.0265	4.4287	1.0308	4.1191	1.0355	3.8512	57
4	1.0226	4.7835	1.0266	4.4231	1.0309	4.1144	1.0356	3.8470	56
5	1.0226	4.7770	1.0266	4.4176	1.0310	4.1096	1.0357	3.8428	55
6	1.0227	4.7706	1.0267	4.4121	1.0311	4.1048	1.0358	3.8387	54
7	1.0228	4.7641	1.0268	4.4065	1.0311	4.1001	1.0358	3.8346	53
8	1.0228	4.7576	1.0268	4.4011	1.0312	4.0953	1.0359	3.8304	52
9	1.0229	4.7512	1.0269	4.3956	1.0313	4.0906	1.0360	3.8263	51
10	1.0230	4.7448	1.0270	4.3910	1.0314	4.0859	1.0361	3.8222	50
11	1.0230	4.7384	1.0271	4.3847	1.0314	4.0812	1.0362	3.8181	49
12	1.0231	4.7320	1.0271	4.3792	1.0315	4.0765	1.0362	3.8140	48
13	1.0232	4.7257	1.0272	4.3738	1.0316	4.0718	1.0363	3.8100	47
14	1.0232	4.7193	1.0273	4.3684	1.0317	4.0672	1.0364	3.8059	46
15	1.0233	4.7130	1.0273	4.3630	1.0317	4.0625	1.0365	3.8018	45
16	1.0234	4.7067	1.0274	4.3576	1.0318	4.0579	1.0366	3.7978	44
17	1.0234	4.7004	1.0275	4.3522	1.0319	4.0532	1.0367	3.7937	43
18	1.0235	4.6942	1.0276	4.3469	1.0320	4.0486	1.0367	3.7897	42
19	1.0235	4.6879	1.0276	4.3415	1.0320	4.0440	1.0368	3.7857	41
20	1.0236	4.6817	1.0277	4.3362	1.0321	4.0394	1.0369	3.7816	40
21	1.0237	4.6754	1.0278	4.3309	1.0322	4.0348	1.0370	3.7776	39
22	1.0237	4.6692	1.0278	4.3256	1.0323	4.0302	1.0371	3.7736	38
23	1.0238	4.6631	1.0279	4.3203	1.0323	4.0256	1.0371	3.7697	37
24	1.0239	4.6569	1.0280	4.3150	1.0324	4.0211	1.0372	3.7657	36
25	1.0239	4.6507	1.0280	4.3098	1.0325	4.0165	1.0373	3.7617	35
26	1.0240	4.6446	1.0281	4.3045	1.0326	4.0120	1.0374	3.7577	34
27	1.0241	4.6385	1.0282	4.2993	1.0327	4.0074	1.0375	3.7538	33
28	1.0241	4.6324	1.0283	4.2941	1.0327	4.0029	1.0376	3.7498	32
29	1.0242	4.6263	1.0283	4.2888	1.0328	3.9984	1.0376	3.7459	31
30	1.0243	4.6202	1.0284	4.2836	1.0329	3.9939	1.0377	3.7420	30
31	1.0243	4.6142	1.0285	4.2785	1.0330	3.9894	1.0378	3.7380	29
32	1.0244	4.6081	1.0285	4.2733	1.0330	3.9850	1.0379	3.7341	28
33	1.0245	4.6021	1.0286	4.2681	1.0331	3.9805	1.0380	3.7302	27
34	1.0245	4.5961	1.0287	4.2630	1.0332	3.9760	1.0381	3.7263	26
35	1.0246	4.5901	1.0288	4.2579	1.0333	3.9716	1.0382	3.7224	25
36	1.0247	4.5841	1.0288	4.2527	1.0334	3.9672	1.0382	3.7186	24
37	1.0247	4.5782	1.0289	4.2476	1.0334	3.9627	1.0383	3.7147	23
38	1.0248	4.5722	1.0290	4.2425	1.0335	3.9583	1.0384	3.7108	22
39	1.0249	4.5663	1.0291	4.2375	1.0336	3.9539	1.0385	3.7070	21
40	1.0249	4.5604	1.0291	4.2324	1.0337	3.9495	1.0386	3.7031	20
41	1.0250	4.5545	1.0292	4.2273	1.0338	3.9451	1.0387	3.6993	19
42	1.0251	4.5486	1.0293	4.2223	1.0338	3.9408	1.0387	3.6955	18
43	1.0251	4.5428	1.0293	4.2173	1.0339	3.9364	1.0388	3.6917	17
44	1.0252	4.5369	1.0294	4.2122	1.0340	3.9320	1.0389	3.6878	16
45	1.0253	4.5311	1.0295	4.2072	1.0341	3.9277	1.0390	3.6840	15
46	1.0253	4.5253	1.0296	4.2022	1.0341	3.9234	1.0391	3.6802	14
47	1.0254	4.5195	1.0296	4.1972	1.0342	3.9199	1.0392	3.6765	13
48	1.0255	4.5137	1.0297	4.1923	1.0343	3.9147	1.0393	3.6727	12
49	1.0255	4.5079	1.0298	4.1873	1.0344	3.9104	1.0393	3.6689	11
50	1.0256	4.5021	1.0299	4.1824	1.0345	3.9061	1.0394	3.6651	10
51	1.0257	4.4964	1.0299	4.1774	1.0345	3.9018	1.0395	3.6614	9
52	1.0257	4.4907	1.0300	4.1725	1.0346	3.8976	1.0396	3.6576	8
53	1.0258	4.4850	1.0301	4.1676	1.0347	3.8933	1.0397	3.6539	7
54	1.0259	4.4793	1.0302	4.1627	1.0348	3.8990	1.0398	3.6502	6
55	1.0260	4.4736	1.0302	4.1578	1.0349	3.8848	1.0399	3.6464	5
56	1.0260	4.4679	1.0303	4.1529	1.0349	3.8805	1.0399	3.6427	4
57	1.0261	4.4623	1.0304	4.1481	1.0350	3.8763	1.0400	3.6390	3
58	1.0262	4.4566	1.0305	4.1432	1.0351	3.8721	1.0401	3.6353	2
59	1.0262	4.4510	1.0305	4.1384	1.0352	3.8679	1.0402	3.6316	1
60	1.0263	4.4454	1.0306	4.1336	1.0353	3.8637	1.0403	3.6279	0
′	77° Co-sec.	77° Sec.	76° Co-sec.	76° Sec.	75° Co-sec.	75° Sec.	74° Co-sec.	74° Sec.	′

′	16° Sec.	16° Co-sec.	17° Sec.	17° Co-sec.	18° Sec.	18° Co-sec.	19° Sec.	19° Co-sec.	′
0	1.0403	3.6279	1.0457	3.4203	1.0515	3.2361	1.0576	3.0715	60
1	1.0404	3.6243	1.0458	3.4170	1.0516	3.2332	1.0577	3.0690	59
2	1.0405	3.6206	1.0459	3.4138	1.0517	3.2303	1.0578	3.0664	58
3	1.0406	3.6169	1.0460	3.4106	1.0518	3.2274	1.0579	3.0638	57
4	1.0406	3.6133	1.0461	3.4073	1.0519	3.2245	1.0580	3.0612	56
5	1.0407	5.6096	1.0461	3.4041	1.0520	3.2216	1.0581	3.0586	55
6	1.0408	3.6060	1.0462	3.4009	1.0521	3.2188	1.0582	3.0561	54
7	1.0409	3.6024	1.0463	3.3977	1.0522	3.2159	1.0584	3.0535	53
8	1.0410	3.5987	1.0464	3.3945	1.0523	3.2131	1.0585	3.0509	52
9	1.0411	3.5951	1.0465	3.3913	1.0524	3.2102	1.0586	3.0484	51
10	1.0412	3.5915	1.0466	3.3881	1.0525	3.2074	1.0587	3.0458	50
11	1.0413	3.5879	1.0467	3.3849	1.0526	3.2045	1.0588	3.0433	49
12	1.0413	3.5843	1.0468	3.3817	1.0527	3.2017	1.0589	3.0407	48
13	1.0414	3.5807	1.0469	3.3785	1.0528	3.1989	1.0590	3.0382	47
14	1.0415	3.5772	1.0470	3.3754	1.0529	3.1960	1.0591	3.0357	46
15	1.0416	3.5736	1.0471	3.3722	1.0530	3.1932	1.0592	3.0331	45
16	1.0417	3.5700	1.0472	3.3690	1.0531	3.1904	1.0593	3.0306	44
17	1.0418	3.5665	1.0473	3.3659	1.0532	3.187	1.0594	3.0281	43
18	1.0419	3.5629	1.0474	3.3627	1.0533	3.1848	1.0595	3.0256	42
19	1.0420	3.5594	1.0475	3.3596	1.0534	3.1820	1.0596	3.0231	41
20	1.0420	3.5559	1.0476	3.3565	1.0535	3.1792	1.0598	3.0206	40
21	1.0421	3.5523	1.0477	3.3534	1.0536	3.1764	1.0599	3.0181	39
22	1.0422	3.5488	1.0478	3.3502	1.0537	3.1736	1.0600	3.0156	38
23	1.0423	3.5453	1.0478	3.3471	1.0538	3.1708	1.0601	3.0131	37
24	1.0424	3.5418	1.0479	3.3440	1.0539	3.1681	1.0602	3.0106	36
25	1.0425	3.5383	1.0480	3.3409	1.0540	3.1653	1.0603	3.0081	35
26	1.0426	3.5348	1.0481	3.3378	1.0541	3.1625	1.0604	3.0056	34
27	1.0427	3.5313	1.0482	3.3347	1.0542	3.1598	1.0605	3.0031	33
28	1.0428	3.5279	1.0483	3.3316	1.0543	3.1570	1.0606	3.0007	32
29	1.0428	3.5244	1.0484	3.3286	1.0544	3.1543	1.0607	2.9982	31
30	1.0429	3.5209	1.0485	3.3255	1.0545	3.1515	1.0608	2.9957	30
31	1.0430	3.5175	1.0486	3.3224	1.0546	3.1488	1.0609	2.9933	29
32	1.0431	3.5140	1.0487	3.3194	1.0547	3.1461	1.0611	2.9908	28
33	1.0432	3.5106	1.0488	3.3163	1.0548	3.1433	1.0612	2.9884	27
34	1.0433	3.5072	1.0489	3.3133	1.0549	3.1406	1.0613	2.9859	26
35	1.0434	3.5037	1.0490	3.3102	1.0550	3.1379	1.0614	2.9835	25
36	1.0435	3.5003	1.0491	3.3072	1.0551	3.1352	1.0615	2.9810	24
37	1.0436	3.4969	1.0492	3.3042	1.0552	3.1325	1.0616	2.9786	23
38	1.0437	3.4935	1.0493	3.3011	1.0553	3.1298	1.0617	2.9762	22
39	1.0438	3.4901	1.0494	3.2981	1.0554	3.1271	1.0618	2.9738	21
40	1.0438	3.4867	1.0495	3.2951	1.0555	3.1244	1.0619	2.9713	20
41	1.0439	3.4833	1.0496	3.2921	1.0556	3.1217	1.0620	2.9689	19
42	1.0440	3.4799	1.0497	3.2891	1.0557	3.1190	1.0622	2.9665	18
43	1.0441	3.4766	1.0498	3.2861	1.0558	3.1163	1.0623	2.9641	17
44	1.0442	3.4732	1.0499	3.2831	1.0559	3.1137	1.0624	2.9617	16
45	1.0443	3.4698	1.0500	3.2801	1.0560	3.1110	1.0625	2.9593	15
46	1.0444	3.4665	1.0501	3.2772	1.0561	3.1083	1.0626	2.9569	14
47	1.0445	3.4632	1.0502	3.2742	1.0562	3.1057	1.0627	2.9545	13
48	1.0446	3.4598	1.0503	3.2712	1.0563	3.1030	1.0628	2.9521	12
40	1.0447	3.4565	1.0504	3.2683	1.0565	3.1004	1.0629	2.9497	11
50	1.0448	3.4532	1.0505	3.2653	1.0566	3.0977	1.0630	2.9474	10
51	1.0448	3.4498	1.0506	3.2624	1.0567	3.0951	1.0632	2.9450	9
52	1.0449	3.4465	1.0507	3.2594	1.0568	3.0925	1.0633	2.9426	8
53	1.0450	3.4432	1.0508	3.2565	1.0569	3.0898	1.0634	2.9402	7
54	1.0451	3.4399	1.0509	3.2535	1.0570	3.0872	1.0635	2.9379	6
55	1.0452	3.4366	1.0510	3.2506	1.0571	3.0846	1.0636	2.9355	5
56	1.0453	3.4334	1.0511	3.2477	1.0572	3.0820	1.0637	2.9332	4
57	1.0454	3.4301	1.0512	3.2448	1.0573	3.0793	1.0638	2.9308	3
58	1.0455	3.4268	1.0513	3.2419	1.0574	3.0767	1.0639	2.9285	2
59	1.0456	3.4236	1.0514	3.2390	1.0575	3.0741	1.0641	2.9261	1
60	1.0457	3.4203	1.0515	3.2361	1.0576	3.0715	1.0642	2.9238	0
′	73° Co-sec.	73° Sec.	72° Co-sec.	72° Sec.	71° Co-sec.	71° Sec.	70° Co-sec.	70° Sec.	′

TALBE 5.—NATURAL TRIGONOMETRIC FUNCTIONS—(*Continued*)

′	20°		21°		22°		23°		′
	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	
0	1.0642	2.9238	1.0711	2.7904	1.0785	2.6695	1.0864	2.5593	60
1	1.0643	2.9215	1.0713	2.7883	1.0787	2.6675	1.0865	2.5575	59
2	1.0644	2.9191	1.0714	2.7862	1.0788	2.6656	1.0866	2.5558	58
3	1.0645	2.9168	1.0715	2.7841	1.0789	2.6637	1.0868	2.5540	57
4	1.0646	2.9145	1.0716	2.7820	1.0790	2.6618	1.0869	2.5523	56
5	1.0647	2.9122	1.0717	2.7799	1.0792	2.6599	1.0870	2.5506	55
6	1.0648	2.9098	1.0719	2.7778	1.0793	2.6580	1.0872	2.5488	54
7	1.0650	2.9075	1.0720	2.7757	1.0794	2.6561	1.0873	2.5471	53
8	1.0651	2.9052	1.0721	2.7736	1.0795	2.6542	1.0874	2.5453	52
9	1.0652	2.9029	1.0722	2.7715	1.0797	2.6523	1.0876	2.5436	51
10	1.0653	2.9006	1.0723	2.7694	1.0798	2.6504	1.0877	2.5419	50
11	1.0654	2.8983	1.0725	2.7674	1.0799	2.6485	1.0878	2.5402	49
12	1.0655	2.8960	1.0726	2.7653	1.0801	2.6466	1.0880	2.5384	48
13	1.0656	2.8937	1.0727	2.7632	1.0802	2.6447	1.0881	2.5367	47
14	1.0658	2.8915	1.0728	2.7611	1.0803	2.6428	1.0882	2.5350	46
15	1.0659	2.8892	1.0729	2.7591	1.0804	2.6410	1.0884	2.5333	45
16	1.0660	2.8869	1.0731	2.7570	1.0806	2.6391	1.0885	2.5316	44
17	1.0661	2.8846	1.0732	2.7550	1.0807	2.6372	1.0886	2.5299	43
18	1.0662	2.8824	1.0733	2.7529	1.0808	2.6353	1.0888	2.5281	42
19	1.0663	2.8801	1.0734	2.7509	1.0810	2.6335	1.0889	2.5264	41
20	1.0664	2.8778	1.0736	2.7488	1.0811	2.6316	1.0891	2.5247	40
21	1.0666	2.8756	1.0737	2.7468	1.0812	2.6297	1.0892	2.5230	39
22	1.0667	2.8733	1.0738	2.7447	1.0813	2.6279	1.0893	2.5213	38
23	1.0668	2.8711	1.0739	2.7427	1.0815	2.6260	1.0895	2.5196	37
24	1.0669	2.8688	1.0740	2.7406	1.0816	2.6242	1.0896	2.5179	36
25	1.0670	2.8666	1.0742	2.7386	1.0817	2.6223	1.0897	2.5163	35
26	1.0671	2.8644	1.0743	2.7366	1.0819	2.6205	1.0899	2.5146	34
27	1.0673	2.8621	1.0744	2.7346	1.0820	2.6186	1.0900	2.5129	33
28	1.0674	2.8599	1.0745	2.7325	1.0821	2.6168	1.0902	2.5112	32
29	1.0675	2.8577	1.0747	2.7305	1.0823	2.6150	1.0903	2.5095	31
30	1.0676	2.8554	1.0748	2.7285	1.0824	2.6131	1.0904	2.5078	30
31	1.0677	2.8532	1.0749	2.7265	1.0825	2.6113	1.0906	2.5062	29
32	1.0678	2.8510	1.0750	2.7245	1.0826	2.6095	1.0907	2.5045	28
33	1.0679	2.8488	1.0751	2.7225	1.0828	2.6076	1.0908	2.5028	27
34	1.0681	2.8466	1.0753	2.7205	1.0829	2.6058	1.0910	2.5011	26
35	1.0682	2.8444	1.0754	2.7185	1.0830	2.6040	1.0911	2.4995	25
36	1.0683	2.8422	1.0755	2.7165	1.0832	2.6022	1.0913	2.4978	24
37	1.0684	2.8400	1.0756	2.7145	1.0833	2.6003	1.0914	2.4961	23
38	1.0685	2.8378	1.0758	2.7125	1.0834	2.5985	1.0915	2.4945	22
39	1.0686	2.8356	1.0759	2.7105	1.0836	2.5967	1.0917	2.4928	21
40	1.0688	2.8334	1.0760	2.7085	1.0837	2.5949	1.0918	2.4912	20
41	1.0689	2.8312	1.0761	2.7065	1.0838	2.5931	1.0920	2.4895	19
42	1.0690	2.8290	1.0763	2.7045	1.0840	2.5913	1.0921	2.4879	18
43	1.0691	2.8269	1.0764	2.7026	1.0841	2.5895	1.0922	2.4862	17
44	1.0692	2.8247	1.0765	2.7006	1.0842	2.5877	1.0924	2.4846	16
45	1.0694	2.8225	1.0766	2.6986	1.0844	2.5859	1.0925	2.4829	15
46	1.0695	2.8204	1.0768	2.6967	1.0845	2.5841	1.0927	2.4813	14
47	1.0696	2.8182	1.0769	2.6947	1.0846	2.5823	1.0928	2.4797	13
48	1.0697	2.8160	1.0770	2.6927	1.0847	2.5805	1.0929	2.4780	12
49	1.0698	2.8139	1.0771	2.6908	1.0849	2.5787	1.0931	2.4764	11
50	1.0699	2.8117	1.0773	2.6888	1.0850	2.5770	1.0932	2.4748	10
51	1.0701	2.8096	1.0774	2.6869	1.0851	2.5752	1.0934	2.4731	9
52	1.0702	2.8074	1.0775	2.6849	1.0853	2.5734	1.0935	2.4715	8
53	1.0703	2.8053	1.0776	2.6830	1.0854	2.5716	1.0936	2.4699	7
54	1.0704	2.8032	1.0778	2.6810	1.0855	2.5699	1.0938	2.4683	6
55	1.0705	2.8010	1.0779	2.6791	1.0857	2.5681	1.0939	2.4666	5
56	1.0707	2.7989	1.0780	2.6772	1.0858	2.5663	1.0941	2.4650	4
57	1.0708	2.7968	1.0781	2.6752	1.0859	2.5646	1.0942	2.4634	3
58	1.0709	2.7947	1.0783	2.6733	1.0861	2.5628	1.0943	2.4618	2
59	1.0710	2.7925	1.0784	2.6714	1.0862	2.5610	1.0945	2.4602	1
60	1.0711	2.7904	1.0785	2.6695	1.0864	2.5593	1.0946	2.4586	0
′	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	′
	69°		68°		67°		66°		

′	24°		25°		26°		27°		′
	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	
0	1.0946	2.4586	1.1034	2.3662	1.1126	2.2812	1.1223	2.2027	60
1	1.0948	2.4570	1.1035	2.3647	1.1127	2.2798	1.1225	2.2014	59
2	1.0949	2.4554	1.1037	2.3632	1.1129	2.2784	1.1226	2.2002	58
3	1.0951	2.4538	1.1038	2.3618	1.1131	2.2771	1.1228	2.1989	57
4	1.0952	2.4522	1.1040	2.3603	1.1132	2.2757	1.1230	2.1977	56
5	1.0953	2.4506	1.1041	2.3588	1.1134	2.2744	1.1231	2.1964	55
6	1.0955	2.4490	1.1043	2.3574	1.1135	2.2730	1.1233	2.1952	54
7	1.0956	2.4474	1.1044	2.3559	1.1137	2.2717	1.1235	2.1939	53
8	1.0958	2.4458	1.1046	2.3544	1.1139	2.2703	1.1237	2.1927	52
9	1.0959	2.4442	1.1047	2.3530	1.1140	2.2690	1.1238	2.1914	51
10	1.0961	2.4426	1.1049	2.3515	1.1142	2.2676	1.1240	2.1902	50
11	1.0962	2.4411	1.1050	2.3501	1.1143	2.2663	1.1242	2.1889	49
12	1.0963	2.4395	1.1052	2.3486	1.1145	2.2650	1.1243	2.1877	48
13	1.0965	2.4379	1.1053	2.3472	1.1147	2.2636	1.1245	2.1865	47
14	1.0966	2.4363	1.1055	2.3457	1.1148	2.2623	1.1247	2.1852	46
15	1.0968	2.4347	1.1056	2.3443	1.1150	2.2610	1.1248	2.1840	45
16	1.0969	2.4332	1.1058	2.3428	1.1151	2.2596	1.1250	2.1828	44
17	1.0971	2.4316	1.1059	2.3414	1.1153	2.2583	1.1252	2.1815	43
18	1.0972	2.4300	1.1061	2.3399	1.1155	2.2570	1.1253	2.1803	42
19	1.0973	2.4285	1.1062	2.3385	1.1156	2.2556	1.1255	2.1791	41
20	1.0975	2.4269	1.1064	2.3371	1.1158	2.2543	1.1257	2.1778	40
21	1.0976	2.4254	1.1065	2.3356	1.1159	2.2530	1.1258	2.1766	39
22	1.0978	2.4238	1.1067	2.3342	1.1161	2.2517	1.1260	2.1754	38
23	1.0979	2.4222	1.1068	2.3328	1.1163	2.2503	1.1262	2.1742	37
24	1.0981	2.4207	1.1070	2.3313	1.1164	2.2490	1.1264	2.1730	36
25	1.0982	2.4191	1.1072	2.3299	1.1166	2.2477	1.1265	2.1717	35
26	1.0984	2.4176	1.1073	2.3285	1.1167	2.2464	1.1267	2.1705	34
27	1.0985	2.4160	1.1075	2.3271	1.1169	2.2451	1.1269	2.1693	33
28	1.0986	2.4145	1.1076	2.3256	1.1171	2.2438	1.1270	2.1681	32
29	1.0988	2.4130	1.1078	2.3242	1.1172	2.2425	1.1272	2.1669	51
30	1.0989	2.4114	1.1079	2.3228	1.1174	2.2411	1.1274	2.1657	30
31	1.0991	2.4099	1.1081	2.3214	1.1176	2.2398	1.1275	2.1645	29
32	1.0992	2.4083	1.1082	2.3200	1.1177	2.2385	1.1277	2.1633	28
33	1.0994	2.4068	1.1084	2.3186	1.1179	2.2372	1.1279	2.1620	27
34	1.0995	2.4053	1.1085	2.3172	1.1180	2.2359	1.1281	2.1608	26
35	1.0997	2.4037	1.1087	2.3158	1.1182	2.2346	1.1282	2.1596	25
36	1.0998	2.4022	1.1088	2.3143	1.1184	2.2333	1.1284	2.1584	24
37	1.1000	2.4007	1.1090	2.3129	1.1185	2.2320	1.1286	2.1572	23
38	1.1001	2.3992	1.1092	2.3115	1.1187	2.2307	1.1287	2.1560	22
39	1.1003	2.3976	1.1093	2.3101	1.1189	2.2294	1.1289	2.1548	21
40	1.1004	2.3961	1.1095	2.3087	1.1190	2.2282	1.1291	2.1536	20
41	1.1005	2.3946	1.1096	2.3073	1.1192	2.2269	1.1293	2.1525	19
42	1.1007	2.3931	1.1098	2.3059	1.1193	2.2256	1.1294	2.1513	18
43	1.1008	2.3916	1.1099	2.3046	1.1195	2.2243	1.1296	2.1501	17
44	1.1010	2.3901	1.1101	2.3032	1.1197	2.2230	1.1298	2.1489	16
45	1.1011	2.3886	1.1102	2.3018	1.1198	2.2217	1.1299	2.1477	15
46	1.1013	2.3871	1.1104	2.3004	1.1200	2.2204	1.1301	2.1465	14
47	1.1014	2.3856	1.1106	2.2990	1.1202	2.2192	1.1303	2.1453	13
48	1.1016	2.3841	1.1107	2.2976	1.1203	2.2179	1.1305	2.1441	12
40	1.1017	2.3826	1.1109	2.2962	1.1205	2.2166	1.1306	2.1430	11
50	1.1019	2.3811	1.1110	2.2949	1.1207	2.2153	1.1308	2.1418	10
51	1.1020	2.3796	1.1112	2.2935	1.1208	2.2141	1.1310	2.1406	9
52	1.1022	2.3781	1.1113	2.2921	1.1210	2.2128	1.1312	2.1394	8
53	1.1023	2.3766	1.1115	2.2907	1.1212	2.2115	1.1313	2.1382	7
54	1.1025	2.3751	1.1116	2.2894	1.1213	2.2103	1.1315	2.1371	6
55	1.1026	2.3736	1.1118	2.2880	1.1215	2.2090	1.1317	2.1359	5
56	1.1028	2.3721	1.1120	2.2866	1.1217	2.2077	1.1319	2.1347	4
57	1.1029	2.3706	1.1121	2.2853	1.1218	2.2065	1.1320	2.1335	3
58	1.1031	2.3691	1.1123	2.2839	1.1220	2.2052	1.1322	2.1324	2
50	1.1032	2.3677	1.1124	2.2825	1.1222	2.2039	1.1324	2.1312	1
60	1.1034	2.3662	1.1126	2.2812	1.1223	2.2027	1.1326	2.1300	0
′	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	′
	65°		64°		63°		62°		

TALBE 5.—NATURAL TRIGONOMETRIC FUNCTIONS—(*Continued*)

′	28°		29°		30°		31°		′
	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	
0	1.1326	2.1300	1.1433	2.0627	1.1547	2.0000	1.1666	1.9416	60
1	1.1327	2.1289	1.1435	2.0616	1.1549	1.9990	1.1668	1.9407	59
2	1.1329	2.1277	1.1437	2.0605	1.1551	1.9980	1.1670	1.9397	58
3	1.1331	2.1266	1.1439	2.0594	1.1553	1.9970	1.1672	1.9388	57
4	1.1333	2.1254	1.1441	2.0583	1.1555	1.9960	1.1674	1.9378	56
5	1.1334	2.1242	1.1443	2.0573	1.1557	1.9950	1.1676	1.9369	55
6	1.1336	2.1231	1.1445	2.0562	1.1559	1.9940	1.1678	1.9360	54
7	1.1338	2.1219	1.1446	2.0551	1.1561	1.9930	1.1681	1.9350	53
8	1.1340	2.1208	1.1448	2.0540	1.1562	1.9920	1.1683	1.9341	52
9	1.1341	2.1196	1.1450	2.0530	1.1564	1.9910	1.1685	1.9332	51
10	1.1343	2.1185	1.1452	2.0519	1.1566	1.9900	1.1687	1.9322	50
11	1.1345	2.1173	1.1454	2.0508	1.1568	1.9890	1.1689	1.9313	49
12	1.1347	2.1162	1.1456	2.0498	1.1570	1.9880	1.1691	1.9304	48
13	1.1349	2.1150	1.1458	2.0487	1.1572	1.9870	1.1693	1.9295	47
14	1.1350	2.1139	1.1459	2.0476	1.1574	1.9860	1.1695	1.9285	46
15	1.1352	2.1127	1.1461	2.0466	1.1576	1.9850	1.1697	1.9276	45
16	1.1354	2.1116	1.1463	2.0455	1.1578	1.9840	1.1699	1.9267	44
17	1.1356	2.1104	1.1465	2.0444	1.1580	1.9830	1.1701	1.9258	43
18	1.1357	2.1093	1.1467	2.0434	1.1582	1.9820	1.1703	1.9248	42
19	1.1359	2.1082	1.1469	2.0423	1.1584	1.9811	1.1705	1.9239	41
20	1.1361	2.1070	1.1471	2.0413	1.1586	1.9801	1.1707	1.9230	40
21	1.1363	2.1059	1.1473	2.0402	1.1588	1.9791	1.1709	1.9221	39
22	1.1365	2.1048	1.1474	2.0392	1.1590	1.9781	1.1712	1.9212	38
23	1.1366	2.1036	1.1476	2.0381	1.1592	1.9771	1.1714	1.9203	37
24	1.1368	2.1025	1.1478	2.0370	1.1594	1.9761	1.1716	1.9193	36
25	1.1370	2.1014	1.1480	2.0360	1.1596	1.9752	1.1718	1.9184	35
26	1.1372	2.1002	1.1482	2.0349	1.1598	1.9742	1.1720	1.9175	34
27	1.1373	2.0991	1.1484	2.0339	1.1600	1.9732	1.1722	1.9166	33
28	1.1375	2.0980	1.1486	2.0329	1.1602	1.9722	1.1724	1.9157	32
29	1.1377	2.0969	1.1488	2.0318	1.1604	1.9713	1.1726	1.9148	31
30	1.1379	2.0957	1.1489	2.0308	1.1606	1.9703	1.1728	1.9139	30
31	1.1381	2.0946	1.1491	2.0297	1.1608	1.9693	1.1730	1.9130	29
32	1.1382	2.0935	1.1493	2.0287	1.1610	1.9683	1.1732	1.9121	28
33	1.1384	2.0924	1.1495	2.0276	1.1612	1.9674	1.1734	1.9112	27
34	1.1386	2.0912	1.1497	2.0266	1.1614	1.9664	1.1737	1.9102	26
35	1.1388	2.0901	1.1499	2.0256	1.1616	1.9654	1.1739	1.9093	25
36	1.1390	2.0890	1.1501	2.0245	1.1618	1.9645	1.1741	1.9084	24
37	1.1391	2.0879	1.1503	2.0235	1.1620	1.9635	1.1743	1.9075	23
38	1.1393	2.0868	1.1505	2.0224	1.1622	1.9625	1.1745	1.9066	22
39	1.1395	2.0857	1.1507	2.0214	1.1624	1.9616	1.1747	1.9057	21
40	1.1397	2.0846	1.1508	2.0204	1.1626	1.9606	1.1749	1.9048	20
41	1.1399	2.0835	1.1510	2.0194	1.1628	1.9596	1.1751	1.9039	19
42	1.1401	2.0824	1.1512	2.0183	1.1630	1.9587	1.1753	1.9030	18
43	1.1402	2.0812	1.1514	2.0173	1.1632	1.9577	1.1756	1.9021	17
44	1.1404	2.0801	1.1516	2.0163	1.1634	1.9568	1.1758	1.9013	16
45	1.1406	2.0790	1.1518	2.0152	1.1636	1.9558	1.1760	1.9004	15
46	1.1408	2.0779	1.1520	2.0142	1.1638	1.9549	1.1762	1.8995	14
47	1.1410	2.0768	1.1522	2.0132	1.1640	1.9539	1.1764	1.8986	13
48	1.1411	2.0757	1.1524	2.0122	1.1642	1.9530	1.1766	1.8977	12
49	1.1413	2.0746	1.1526	2.0111	1.1644	1.9520	1.1768	1.8968	11
50	1.1415	2.0735	1.1528	2.0101	1.1646	1.9510	1.1770	1.8959	10
51	1.1417	2.0725	1.1530	2.0091	1.1648	1.9501	1.1772	1.8950	9
52	1.1419	2.0714	1.1531	2.0081	1.1650	1.9491	1.1775	1.8941	8
53	1.1421	2.0703	1.1533	2.0071	1.1652	1.9482	1.1777	1.8932	7
54	1.1422	2.0692	1.1535	2.0061	1.1654	1.9473	1.1779	1.8924	6
55	1.1424	2.0681	1.1537	2.0050	1.1656	1.9463	1.1781	1.8915	5
56	1.1426	2.0670	1.1539	2.0040	1.1658	1.9454	1.1783	1.8906	4
57	1.1428	2.0659	1.1541	2.0030	1.1660	1.9444	1.1785	1.8897	3
58	1.1430	2.0648	1.1543	2.0020	1.1662	1.9435	1.1787	1.8888	2
59	1.1432	2.0637	1.1545	2.0010	1.1664	1.9425	1.1790	1.8879	1
60	1.1433	2.0627	1.1547	2.0000	1.1666	1.9416	1.1792	1.8871	0
′	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	′
	61°		60°		59°		58°		

′	32°		33°		34°		35°		′
	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	
0	1.1792	1.8871	1.1924	1.8361	1.2062	1.7883	1.2208	1.7434	60
1	1.1794	1.8862	1.1926	1.8352	1.2064	1.7875	1.2210	1.7427	59
2	1.1796	1.8853	1.1928	1.8344	1.2067	1.7867	1.2213	1.7420	58
3	1.1798	1.8844	1.1930	1.8336	1.2069	1.7860	1.2215	1.7413	57
4	1.1800	1.8836	1.1933	1.8328	1.2072	1.7852	1.2218	1.7405	56
5	1.1802	1.8827	1.1935	1.8320	1.2074	1.7844	1.2220	1.7398	55
6	1.1805	1.8818	1.1937	1.8311	1.2076	1.7837	1.2223	1.7391	54
7	1.1807	1.8809	1.1939	1.8303	1.2079	1.7829	1.2225	1.7384	53
8	1.1809	1.8801	1.1942	1.8295	1.2081	1.7821	1.2228	1.7377	52
9	1.1811	1.8792	1.1944	1.8287	1.2083	1.7814	1.2230	1.7369	51
10	1.1813	1.8783	1.1946	1.8279	1.2086	1.7806	1.2233	1.7362	50
11	1.1815	1.8785	1.1948	1.8271	1.2088	1.7798	1.2235	1.7355	49
12	1.1818	1.8766	1.1951	1.8263	1.2091	1.7791	1.2238	1.7348	48
13	1.1820	1.8757	1.1953	1.8255	1.2093	1.7783	1.2240	1.7341	47
14	1.1822	1.8749	1.1955	1.8246	1.2095	1.7776	1.2243	1.7334	46
15	1.1824	1.8740	1.1958	1.8238	1.2098	1.7768	1.2245	1.7327	45
16	1.1826	1.8731	1.1960	1.8230	1.2100	1.7760	1.2248	1.7319	44
17	1.1828	1.8723	1.1962	1.8222	1.2103	1.7753	1.2250	1.7312	43
18	1.1831	1.8714	1.1964	1.8214	1.2105	1.7745	1.2253	1.7305	42
19	1.1833	1.8706	1.1967	1.8206	1.2107	1.7738	1.2255	1.7298	41
20	1.1835	1.8697	1.1969	1.8198	1.2110	1.7730	1.2258	1.7291	40
21	1.1837	1.8688	1.1971	1.8190	1.2112	1.7723	1.2260	1.7284	39
22	1.1839	1.8680	1.1974	1.8182	1.2115	1.7715	1.2263	1.7277	38
23	1.1841	1.8671	1.1976	1.8174	1.2117	1.7708	1.2265	1.7270	37
24	1.1844	1.8663	1.1978	1.8166	1.2119	1.7700	1.2268	1.7263	36
25	1.1846	1.8654	1.1980	1.8158	1.2122	1.7693	1.2270	1.7256	35
26	1.1848	1.8646	1.1983	1.8150	1.2124	1.7685	1.2273	1.7249	34
27	1.1850	1.8637	1.1985	1.8142	1.2127	1.7678	1.2276	1.7242	33
28	1.1852	1.8629	1.1987	1.8134	1.2129	1.7670	1.2278	1.7234	32
29	1.1855	1.8620	1.1990	1.8126	1.2132	1.7663	1.2281	1.7227	31
30	1.1857	1.8611	1.1992	1.8118	1.2134	1.7655	1.2283	1.7220	30
31	1.1859	1.8603	1.1994	1.8110	1.2136	1.7648	1.2286	1.7213	29
32	1.1861	1.8595	1.1997	1.8102	1.2139	1.7640	1.2288	1.7206	28
33	1.1863	1.8586	1.1999	1.8094	1.2141	1.7633	1.2291	1.7199	27
34	1.1866	1.8578	1.2001	1.8086	1.2144	1.7625	1.2293	1.7192	26
35	1.1868	1.8569	1.2004	1.8078	1.2146	1.7618	1.2296	1.7185	25
36	1.1870	1.8561	1.2006	1.8070	1.2149	1.7610	1.2298	1.7178	24
37	1.1872	1.8552	1.2008	1.8062	1.2151	1.7603	1.2301	1.7171	23
38	1.1874	1.8544	1.2010	1.8054	1.2153	1.7596	1.2304	1.7164	22
39	1.1877	1.8535	1.2013	1.8047	1.2156	1.7588	1.2306	1.7157	21
40	1.1879	1.8527	1.2015	1.8039	1.2158	1.7581	1.2309	1.7151	20
41	1.1881	1.8519	1.2017	1.8031	1.2161	1.7573	1.2311	1.7144	19
42	1.1883	1.8510	1.2020	1.8023	1.2163	1.7566	1.2314	1.7137	18
43	1.1886	1.8502	1.2022	1.8015	1.2166	1.7559	1.2316	1.7130	17
44	1.1888	1.8493	1.2024	1.8007	1.2168	1.7551	1.2319	1.7123	16
45	1.1890	1.8485	1.2027	1.7999	1.2171	1.7544	1.2322	1.7116	15
46	1.1892	1.8477	1.2029	1.7992	1.2173	1.7537	1.2324	1.7109	14
47	1.1894	1.8468	1.2031	1.7984	1.2175	1.7529	1.2327	1.7102	13
48	1.1897	1.8460	1.2034	1.7976	1.2178	1.7522	1.2329	1.7095	12
49	1.1899	1.8452	1.2036	1.7968	1.2180	1.7514	1.2332	1.7088	11
50	1.1901	1.8443	1.2039	1.7960	1.2183	1.7507	1.2335	1.7081	10
51	1.1903	1.8435	1.2041	1.7953	1.2185	1.7500	1.2337	1.7075	9
52	1.1906	1.8427	1.2043	1.7945	1.2188	1.7493	1.2340	1.7068	8
53	1.1908	1.8418	1.2046	1.7937	1.2190	1.7485	1.2342	1.7061	7
54	1.1910	1.8410	1.2048	1.7929	1.2193	1.7478	1.2345	1.7054	6
55	1.1912	1.8402	1.2050	1.7921	1.2195	1.7471	1.2348	1.7047	5
56	1.1915	1.8394	1.2053	1.7914	1.2198	1.7463	1.2350	1.7040	4
57	1.1917	1.8385	1.2055	1.7906	1.2200	1.7456	1.2353	1.7033	3
58	1.1919	1.8377	1.2057	1.7898	1.2203	1.7449	1.2355	1.7027	2
59	1.1921	1.8369	1.2060	1.7891	1.2205	1.7442	1.2358	1.7020	1
60	1.1922	1.8361	1.2062	1.7883	1.2208	1.7434	1.2361	1.7013	0
′	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	′
	57°		56°		55°		54°		

TABLE 5.—NATURAL TRIGONOMETRIC FUNCTIONS—(*Continued*)

′	36° Sec.	36° Co-sec.	37° Sec.	37° Co-sec.	38° Sec.	38° Co-sec.	39° Sec.	39° Co-sec.	′
0	1.2361	1.7013	1.2521	1.6616	1.2690	1.6243	1.2867	1.5890	60
1	1.2363	1.7006	1.2524	1.6610	1.2693	1.6237	1.2871	1.5884	59
2	1.2366	1.6999	1.2527	1.6603	1.2696	1.6231	1.2874	1.5879	58
3	1.2368	1.6993	1.2530	1.6597	1.2699	1.6224	1.2877	1.5873	57
4	1.2371	1.6986	1.2532	1.6591	1.2702	1.6218	1.2880	1.5867	56
5	1.2374	1.6979	1.2535	1.6584	1.2705	1.6212	1.2883	1.5862	55
6	1.2376	1.6972	1.2538	1.6578	1.2707	1.6206	1.2886	1.5856	54
7	1.2379	1.6965	1.2541	1.6572	1.2710	1.6200	1.2889	1.5850	53
8	1.2382	1.6959	1.2543	1.6565	1.2713	1.6194	1.2892	1.5845	52
9	1.2384	1.6952	1.2546	1.6559	1.2716	1.6188	1.2895	1.5839	51
10	1.2387	1.6945	1.2549	1.6552	1.2719	1.6182	1.2898	1.5833	50
11	1.2389	1.6938	1.2552	1.6546	1.2722	1.6176	1.2901	1.5828	49
12	1.2392	1.6932	1.2554	1.6540	1.2725	1.6170	1.2904	1.5822	48
13	1.2395	1.6925	1.2557	1.6533	1.2728	1.6164	1.2907	1.5816	47
14	1.2397	1.6918	1.2560	1.6527	1.2731	1.6159	1.2910	1.5811	46
15	1.2400	1.6912	1.2563	1.6521	1.2734	1.6153	1.2913	1.5805	45
16	1.2403	1.6905	1.2565	1.6514	1.2737	1.6147	1.2916	1.5799	44
17	1.2405	1.6898	1.2568	1.6508	1.2739	1.6141	1.2919	1.5794	43
18	1.2408	1.6891	1.2571	1.6502	1.2742	1.6135	1.2922	1.5788	42
19	1.2411	1.6885	1.2574	1.6496	1.2745	1.6129	1.2926	1.5783	41
20	1.2413	1.6878	1.2577	1.6489	1.2748	1.6123	1.2929	1.5777	40
21	1.2416	1.6871	1.2579	1.6483	1.2751	1.6117	1.2932	1.5771	39
22	1.2419	1.6865	1.2582	1.6477	1.2754	1.6111	1.2935	1.5766	38
23	1.2421	1.6858	1.2585	1.6470	1.2757	1.6105	1.2938	1.5760	37
24	1.2424	1.6851	1.2588	1.6464	1.2760	1.6099	1.2941	1.5755	36
25	1.2427	1.6845	1.2591	1.6458	1.2763	1.6093	1.2944	1.5749	35
26	1.2429	1.6838	1.2593	1.6452	1.2766	1.6087	1.2947	1.5743	34
27	1.2432	1.6831	1.2596	1.6445	1.2769	1.6081	1.2950	1.5738	33
28	1.2435	1.6825	1.2599	1.6439	1.2772	1.6077	1.2953	1.5732	32
29	1.2437	1.6818	1.2602	1.6433	1.2775	1.6070	1.2956	1.5727	31
30	1.2440	1.6812	1.2605	1.6427	1.2778	1.6064	1.2960	1.5721	30
31	1.2443	1.6805	1.2607	1.6420	1.2781	1.6058	1.2963	1.5716	29
32	1.2445	1.6798	1.2610	1.6414	1.2784	1.6052	1.2966	1.5710	28
33	1.2448	1.6792	1.2613	1.6408	1.2787	1.6046	1.2969	1.5705	27
34	1.2451	1.6785	1.2616	1.6402	1.2790	1.6040	1.2972	1.5699	26
35	1.2453	1.6779	1.2619	1.6396	1.2793	1.6034	1.2975	1.5694	25
36	1.2456	1.6772	1.2622	1.6389	1.2795	1.6029	1.2978	1.5688	24
37	1.2459	1.6766	1.2624	1.6383	1.2798	1.6023	1.2981	1.5683	23
38	1.2461	1.6759	1.2627	1.6377	1.2801	1.6017	1.2985	1.5677	22
39	1.2464	1.6752	1.2630	1.6371	1.2804	1.6011	1.2988	1.5672	21
40	1.2467	1.6746	1.2633	1.6365	1.2807	1.6005	1.2991	1.5666	20
41	1.2470	1.6739	1.2636	1.6359	1.2810	1.6000	1.2994	1.5661	19
42	1.2472	1.6733	1.2639	1.6352	1.2813	1.5994	1.2997	1.5655	18
43	1.2475	1.6726	1.2641	1.6346	1.2816	1.5988	1.3000	1.5650	17
44	1.2478	1.6720	1.2644	1.6340	1.2819	1.5982	1.3003	1.5644	16
45	1.2480	1.6713	1.2647	1.6334	1.2822	1.5976	1.3006	1.5639	15
46	1.2483	1.6707	1.2650	1.6328	1.2825	1.5971	1.3010	1.5633	14
47	1.2486	1.6700	1.2653	1.6322	1.2828	1.5965	1.3013	1.5628	13
48	1.2488	1.6694	1.2656	1.6316	1.2831	1.5959	1.3016	1.5622	12
49	1.2490	1.6687	1.2659	1.6309	1.2834	1.5953	1.3019	1.5617	11
50	1.2494	1.6681	1.2661	1.6303	1.2837	1.5947	1.3022	1.5611	10
51	1.2497	1.6674	1.2664	1.6297	1.2840	1.5942	1.3025	1.5606	9
52	1.2499	1.6668	1.2667	1.6291	1.2843	1.5936	1.3029	1.5600	8
53	1.2502	1.6661	1.2670	1.6285	1.2846	1.5930	1.3032	1.5595	7
54	1.2505	1.6655	1.2673	1.6279	1.2849	1.5924	1.3035	1.5590	6
55	1.2508	1.6648	1.2676	1.6273	1.2852	1.5919	1.3038	1.5584	5
56	1.2510	1.6642	1.2679	1.6267	1.2855	1.5913	1.3041	1.5579	4
57	1.2513	1.6636	1.2681	1.6261	1.2858	1.5907	1.3044	1.5573	3
58	1.2516	1.6629	1.2684	1.6255	1.2861	1.5901	1.3048	1.5568	2
59	1.2519	1.6623	1.2687	1.6249	1.2864	1.5896	1.3051	1.5563	1
60	1.2521	1.6616	1.2690	1.6243	1.2867	1.5890	1.3054	1.5557	0
′	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	′
	53°		52°		51°		50°		

′	40° Sec.	40° Co-sec.	41° Sec.	41° Co-sec.	42° Sec.	42° Co-sec.	43° Sec.	43° Co-sec.	′
0	1.3054	1.5557	1.3250	1.5242	1.3456	1.4945	1.3673	1.4663	60
1	1.3057	1.5552	1.3253	1.5237	1.3460	1.4940	1.3677	1.4658	59
2	1.3060	1.5546	1.3257	1.5232	1.3463	1.4935	1.3681	1.4654	58
3	1.3064	1.5541	1.3260	1.5227	1.3467	1.4930	1.3684	1.4649	57
4	1.3067	1.5536	1.3263	1.5222	1.3470	1.4925	1.3688	1.4644	56
5	1.3070	1.5530	1.3267	1.5217	1.3474	1.4921	1.3692	1.4640	55
6	1.3073	1.5525	1.3270	1.5212	1.3477	1.4916	1.3695	1.4635	54
7	1.3076	1.5520	1.3274	1.5207	1.3481	1.4911	1.3699	1.4631	53
8	1.3080	1.5514	1.3277	1.5202	1.3485	1.4906	1.3703	1.4626	52
9	1.3083	1.5509	1.3280	1.5197	1.3488	1.4901	1.3707	1.4622	51
10	1.3086	1.5503	1.3284	1.5192	1.3492	1.4897	1.3710	1.4617	50
11	1.3089	1.5498	1.3287	1.5187	1.3495	1.4892	1.3714	1.4613	49
12	1.3092	1.5493	1.3290	1.5182	1.3499	1.4887	1.3718	1.4608	48
13	1.3096	1.5487	1.3294	1.5177	1.3502	1.4882	1.3722	1.4604	47
14	1.3099	1.5482	1.3297	1.5171	1.3506	1.4877	1.3725	1.4599	46
15	1.3102	1.5477	1.3301	1.5166	1.3509	1.4873	1.3729	1.4595	45
16	1.3105	1.5471	1.3304	1.5161	1.3513	1.4868	1.3733	1.4590	44
17	1.3109	1.5466	1.3307	1.5156	1.3517	1.4863	1.3737	1.4586	43
18	1.3112	1.5461	1.3311	1.5151	1.3520	1.4858	1.3740	1.4581	42
19	1.3115	1.5456	1.3314	1.5146	1.3524	1.4854	1.3744	1.4577	41
20	1.3118	1.5450	1.3318	1.5141	1.3527	1.4849	1.3748	1.4572	40
21	1.3121	1.5445	1.3321	1.5136	1.3531	1.4844	1.3752	1.4568	39
22	1.3125	1.5440	1.3324	1.5131	1.3534	1.4839	1.3756	1.4563	38
23	1.3128	1.5434	1.3328	1.5126	1.3538	1.4835	1.3759	1.4559	37
24	1.3131	1.5429	1.3331	1.5121	1.3542	1.4830	1.3763	1.4554	36
25	1.3134	1.5424	1.3335	1.5116	1.3545	1.4825	1.3767	1.4550	35
26	1.3138	1.5419	1.3338	1.5111	1.3549	1.4821	1.3771	1.4545	34
27	1.3141	1.5413	1.3342	1.5106	1.3552	1.4816	1.3774	1.4541	33
28	1.3144	1.5408	1.3345	1.5101	1.3556	1.4811	1.3778	1.4536	32
29	1.3148	1.5403	1.3348	1.5096	1.3560	1.4806	1.3782	1.4532	31
30	1.3151	1.5398	1.3352	1.5092	1.3563	1.4802	1.3786	1.4527	30
31	1.3154	1.5392	1.3355	1.5087	1.3567	1.4797	1.3790	1.4523	29
32	1.3157	1.5387	1.3359	1.5082	1.3571	1.4792	1.3794	1.4518	28
33	1.3161	1.5382	1.3362	1.5077	1.3574	1.4788	1.3797	1.4514	27
34	1.3164	1.5377	1.3366	1.5072	1.3578	1.4783	1.3801	1.4510	26
35	1.3167	1.5371	1.3369	1.5067	1.3581	1.4778	1.3805	1.4505	25
36	1.3170	1.5366	1.3372	1.5062	1.3585	1.4774	1.3809	1.4501	24
37	1.3174	1.5361	1.3376	1.5057	1.3589	1.4769	1.3813	1.4496	23
38	1.3177	1.5356	1.3379	1.5052	1.3592	1.4764	1.3816	1.4492	22
39	1.3180	1.5351	1.3383	1.5047	1.3596	1.4760	1.3820	1.4487	21
40	1.3184	1.5345	1.3386	1.5042	1.3600	1.4755	1.3824	1.4483	20
41	1.3187	1.5340	1.3390	1.5037	1.3603	1.4750	1.3828	1.4479	19
42	1.3190	1.5335	1.3393	1.5032	1.3607	1.4746	1.3832	1.4474	18
43	1.3193	1.5330	1.3397	1.5027	1.3611	1.4741	1.3836	1.4470	17
44	1.3197	1.5325	1.3400	1.5022	1.3614	1.4736	1.3839	1.4465	16
45	1.3200	1.5319	1.3404	1.5018	1.3618	1.4732	1.3843	1.4461	15
46	1.3203	1.5314	1.3407	1.5013	1.3622	1.4727	1.3847	1.4457	14
47	1.3207	1.5309	1.3411	1.5008	1.3625	1.4723	1.3851	1.4452	13
48	1.3210	1.5304	1.3414	1.5003	1.3629	1.4718	1.3855	1.4448	12
49	1.3213	1.5299	1.3418	1.4998	1.3633	1.4713	1.3859	1.4443	11
50	1.3217	1.5294	1.3421	1.4993	1.3636	1.4709	1.3863	1.4439	10
51	1.3220	1.5289	1.3425	1.4988	1.3640	1.4704	1.3867	1.4435	9
52	1.3223	1.5283	1.3428	1.4983	1.3644	1.4699	1.3870	1.4430	8
53	1.3227	1.5278	1.3432	1.4979	1.3647	1.4695	1.3874	1.4426	7
54	1.3230	1.5273	1.3435	1.4974	1.3651	1.4690	1.3878	1.4422	6
55	1.3233	1.5268	1.3439	1.4969	1.3655	1.4686	1.3882	1.4417	5
56	1.3237	1.5263	1.3442	1.4964	1.3658	1.4681	1.3886	1.4413	4
57	1.3240	1.5258	1.3446	1.4959	1.3662	1.4676	1.3890	1.4408	3
58	1.3243	1.5253	1.3449	1.4954	1.3666	1.4672	1.3894	1.4404	2
59	1.3247	1.5248	1.3453	1.4949	1.3669	1.4667	1.3898	1.4400	1
60	1.3250	1.5242	1.3456	1.4945	1.3673	1.4663	1.3902	1.4395	0
′	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	′
	49°		48°		47°		46°		

TABLE 5.—NATURAL TRIGONOMETRIC FUNCTIONS—(*Continued*)

′	44° Sec.	44° Co-sec.	′	′	44° Sec.	44° Co-sec.	′	′	44° Sec	44° Co-sec.	′
0	1.3902	1.4395	60	21	1.3984	1.4305	39	41	1.4065	1.4221	19
1	1.3905	1.4391	59	22	1.3988	1.4301	38	42	1.4069	1.4217	18
2	1.3909	1.4387	58	23	1.3992	1.4297	37	43	1.4073	1.4212	17
3	1.3913	1.4382	57	24	1.3996	1.4292	36	44	1.4077	1.4208	16
4	1.3917	1.4378	56	25	1.4000	1.4288	35	45	1.4081	1.4204	15
5	1.3921	1.4374	55	26	1.4004	1.4284	34	46	1.4085	1.4200	14
6	1.3925	1.4370	54	27	1.4008	1.4280	33	47	1.4089	1.4196	13
7	1.3929	1.4365	53	28	1.4012	1.4276	32	48	1.4093	1.4192	12
8	1.3933	1.4361	52	29	1.4016	1.4271	31	49	1.4097	1.4188	11
9	1.3937	1.4357	51	30	1.4020	1.4267	30	50	1.4101	1.4183	10
10	1.3941	1.4352	50	31	1.4024	1.4263	29	51	1.4105	1.4179	9
11	1.3945	1.4348	49	32	1.4028	1.4259	28	52	1.4109	1.4175	8
12	1.3949	1.4344	48	33	1.4032	1.4254	27	53	1.4113	1.4171	7
13	1.3953	1.4339	47	34	1.4036	1.4250	26	54	1.4117	1.4167	6
14	1.3957	1.4335	46	35	1.4040	1.4246	25	55	1.4122	1.4163	5
15	1.3960	1.4331	45	36	1.4044	1.4242	24	56	1.4126	1.4159	4
16	1.3964	1.4327	44	37	1.4048	1.4238	23	57	1.4130	1.4154	3
17	1.3968	1.4322	43	38	1.4052	1.4233	22	58	1.4134	1.4150	2
18	1.3972	1.4318	42	39	1.4056	1.4229	21	59	1.4138	1.4146	1
19	1.3976	1.4314	41	40	1.4060	1.4225	20	60	1.4142	1.4142	0
20	1.3980	1.4310	40								
′	Co-sec. 45°	Sec. 45°	′	′	Co-sec. 45°	Sec. 45°	′	′	Co-sec 45°	Sec. 45°	′

The factoring method of extracting roots (also called the successive approximation method) is applicable to any root and, except for the square root, is less laborious than other arithmetical methods. It is greatly facilitated by the use of Table 6 of factors.

To find the square root proceed as follows: Look in the table of second powers for the number nearest the given number of which the root is desired, and take the factor of that power. Divide the given number by that factor. Then divide the number by the half sum of the factor and quotient for a second approximation. Divide the number again by the half sum of the second approximation and the second quotient.

This process can be continued until the result is obtained to any required degree of exactness. Usually two divisions give the root to as great a number of places as is necessary.

To find the cube root proceed in a similar way, as follows: Look in the column of third powers for the number nearest the given number. Take out the factor. Divide the given number by that factor. Divide the quotient also by the factor. Take one-third the sum of the two divisors and the last quotient for a second divisor. Divide the given number by this second divisor and divide the quotient also by it. Take one-third the sum of twice the second divisor and the final quotient for a third divisor. It may not be necessary to divide the number a third time. This third divisor may be the cube root as close as needed. For the fourth root, the new divisor will be one-fourth the sum of the last quotient and three times the preceding divisor. For the fifth root, one-fifth the sum of the last quotient and four times the preceding divisor, and so for any root required.

Whatever the root, to the number of decimal places that the divisor and quotient agree, they are correct—comparison of the two showing at once the degree of approximation to which the process has been carried.

The sides and angles of polygons and the distance between holes in circles may be obtained from Table 8, by F. W. SEIDENSTICKER (*Amer. Mach., Dec.* 17, 1908). The table gives the number of sides (3 to

TABLE 6.—FACTORS FOR USE IN EXTRACTING ROOTS BY THE FACTORING METHOD

Factors	2d power The factor is the sq. root	3d power The factor is the third root	4th power The factor is the fourth root	5th power The factor is the fifth root
1	1	1	1	1
2	4	8	16	32
3	9	27	81	243
4	16	64	256	1,024
5	25	125	625	3,125
6	36	216	1,296	7,776
7	49	343	2,401	16,807
8	64	512	4,096	32,768
9	81	729	6,561	59,049
10	100	1,000	10,000	100,000
11	121	1,331	14,641	161,051
12	144	1,728	20,736	248,832
13	169	2,197	28,561	371,293
14	196	2,744	38,416	537,824
15	225	3,375	50,625	759,375
16	256	4,096	65,536	1,048,576
17	289	4,913	83,521	1,419,857
18	324	5,832	104,976	1,889,568
19	361	6,859	130,321	2,476,099
20	400	8,000	160,000	3,200,000
21	441	9,261	194,481	4,084,101
22	484	10,648	234,256	5,153,632
23	529	12,167	279,841	6,436,343
24	576	13,824	331,776	7,962,624
25	625	15,625	390,625	9,765,625

TABLE 7.—SHOWING SIZES OF LARGEST SQUARES (CORNERS BEING SHARP) WHICH CAN BE OBTAINED FROM ROUND STOCK

Diameter of stock	Decimal equivalent	Diameter	Diameter of stock	Decimal equivalent	Diameter	Diameter of stock	Decimal equivalent	Diameter
1/8	.125	.0883+	1 1/16	1.0625	1.7551+	2	2.000	1.414
3/16	.1875	.1325+	1 1/8	1.125	.7953+	2 1/16	2.0625	1.4581+
1/4	.250	.1767+	1 3/16	1.1875	.8395+	2 1/8	2.125	1.5023+
5/16	.3125	.2209+	1 1/4	1.250	.8837+	2 3/16	2.1875	1.5465+
3/8	.375	.2651+	1 5/16	1.3125	.9279+	2 1/4	2.250	1.5907+
7/16	.4375	.3093+	1 3/8	1.375	.9721+	2 5/16	2.3125	1.6349+
1/2	.500	.3535	1 7/16	1.4375	1.0163+	2 3/8	2.375	1.6791+
9/16	.5625	.3976+	1 1/2	1.500	1.0605	2 7/16	2.4375	1.7233+
5/8	.625	.4418+	1 9/16	1.5625	1.1046+	2 1/2	2.500	1.7675
11/16	.6875	.4860+	1 5/8	1.625	1.1488+	2 9/16	2.5625	1.8116+
3/4	.750	.5302+	1 11/16	1.6875	1.1930+	2 5/8	2.625	1.8558+
13/16	.8125	.5744+	1 3/4	1.750	1.2372+	2 11/16	2.6875	1.9000+
7/8	.875	.6186+	1 13/16	1.8125	1.2814+	2 3/4	2.750	1.9442+
15/16	.9375	.6628+	1 7/8	1.875	1.3256+	2 13/16	2.8125	1.9884+
1	1.000	.707	1 15/16	1.9375	1.3698+	2 7/8	2.875	2.0326+
						2 15/16	2.9375	2.0768+
						3	3.000	2.121

Rule

Multiply diameter of stock by the constant .707

Example 3/4 in. = .750 × .707 = .53025

Table 8.—Sides and Angles of Polygons

No. sides	Angle	Sine
3	60-	.8660254
4	45-	.7071068
5	36-	.5877853
6	30-	.5000000
7	25-42-51.42	.4338837
8	22-30	.3826834
9	20-	.3420201
10	18-	.3090170
11	16-21-49.09	.2817325
12	15-	.2588190
13	13-50-46.15	.2393157
14	12-51-25.71	.2225208
15	12-	.2079116
16	11-15	.1950903
17	10-35-17.64	.1837495
18	10-	.1736481
19	9-28-25.26	.1645945
20	9-	.1564344
21	8-34-17.14	.1490422
22	8-10-54.54	.1423148
23	7-49-33.91	.1361666
24	7-30	.1305262
25	7-12	.1253332
26	6-55-23.07	.1205366
27	6-40	.1160929
28	6-25-42.85	.1119644
29	6-12-24.82	.1081189
30	6-	.1045284
31	5-48-23.22	.1011683
32	5-37-30	.0980171
33	5-27-16.36	.0950560
34	5-17-38.82	.0922683
35	5- 8-34.28	.0896392
36	5-	.0871557
37	4-51-53.51	.0848058
38	4-44-12.63	.0825793
39	4-36-55.38	.0804665
40	4-30	.0784591
41	4-23-24.87	.0765492
42	4-17- 8.57	.0747301
43	4-11- 9.76	.0729952
44	4- 5-27.27	.0713391
45	4-	.0697565
46	3-54-46.95	.0682423
47	3-49-47.23	.0667926
48	3-45	.0654031
49	3-40-24.49	.0640702
50	3-36	.0627905
51	3-31-45.88	.0615609
52	3-27-41.53	.0603784
53	3-23-46.41	.0592405
54	3-20	.0581448
55	3-16-21.81	.0570887
56	3-12-51.42	.0560704
57	3- 9-28.42	.0550877
58	3- 6-12.41	.0541388
59	3- 3- 3.05	.0532221
60	3-	.0523360
61	2-57- 2.95	.0514787
62	2-54-11.61	.0506491
63	2-51-25.71	.0498458
64	2-48-45	.0490676
65	2-46- 9.23	.0483133
66	2-43-38.18	.0475819
67	2-41-11.64	.0468722
68	2-38-49.41	.0461834
69	2-36-31.30	.0455145
70	2-34-17.14	.0448648
71	2-32- 6.76	.0442333
72	2-30	.0436194
73	2-27-56.71	.0430222

No. sides	Angle	Sine
74	2-25-56.75	.0422411
75	2-24	.0418757
76	2-22- 6.31	.0413249
77	2-20-15.58	.0407885
78	2-18-27.69	.0402659
79	2-16-42.53	.0397565
80	2-15	.0392598
81	2-13-20	.0387753
82	2-11-42.45	.0383027
83	2-10- 7.22	.0378414
84	2- 8-34.28	.0373911
85	2- 7- 3.54	.0369515
86	2- 5-34.88	.0365220
87	2- 4- 8.27	.0361023
88	2- 2-43.63	.0356923
89	2- 1-20.89	.0352914
90	2-	.0348995
91	1-58-40.87	.0345160
92	1-57-23.47	.0341410
93	1-56- 7.74	.0337741
94	1-54-53.61	.0334149
95	1-53-41.05	.0330633
96	1-52-30	.0327190
97	1-51-20.41	.0323818
98	1-50-12.24	.0320515
99	1-49- 5.45	.0317279
100	1-48	.0314107
101	1-46-55.84	.0310998
102	1-45-52.94	.0307950
103	1-44-51.26	.0304961
104	1-43-50.76	.0302029
105	1-42-51.42	.0299154
106	1-41-53.20	.0296332
107	1-40-56.07	.0293564
108	1-40-	.0290847
109	1-39- 4.95	.0288179
110	1-38-10.90	.0285560
111	1-37-17.83	.0282488
112	1-36-25.71	.0280462
113	1-35-34.51	.0277981
114	1-34-44.21	.0275543
115	1-33-54.78	.0273147
116	1-33- 6.20	.0270793
117	1-32-18.46	.0268479
118	1-31-31.52	.0266204
119	1-30-45.38	.0263968
120	1-30	.0261769
121	1-29-15.37	.0259606
122	1-28-31.47	.0257478
123	1-27-48.29	.0255386
124	1-27- 5.80	.0253326
125	1-26-24	.0251300
126	1-25-42.85	.0249306
127	1-25- 2.36	.0247344
128	1-24-22.50	.0245412
129	1-23-43.25	.0243509
130	1-23 -4.61	.0241637
131	1-22-26.56	.0239793
132	1-21-49.09	.0237976
133	1-21-12.18	.0236188
134	1-20-35.82	.0234425
135	1-20	.0232689
136	1-19-24.70	.0230978
137	1-18-49.92	.0229292
138	1-18-15.65	.0227631
139	1-17-41.87	.0225994
140	1-17- 8.57	.0224380
141	1-16-35.74	.0222789
142	1-16- 3.38	.0221220
143	1-15-31.46	.0219673
144	1-15	.0218148

No. sides	Angle	Sine
145	1-14-28.96	.0216644
146	1-13-58.35	.0215160
147	1-13-28.16	.0213697
148	1-12-58.37	.0212253
149	1-12-28.99	.0210829
150	1-12	.0209424
151	1-11-31.39	.0208037
152	1-11- 3.15	.0206668
153	1-10-35.29	.0205318
154	1-10- 7.79	.0203985
155	1- 9-40.64	.0202669
156	1- 9-13.84	.0201370
157	1- 8-47.38	.0200087
158	1- 8-21.26	.0198821
159	1- 7-55.47	.0197571
160	1- 7-30	.0196336
161	1- 7- 4.84	.0195117
162	1- 6-40	.0193913
163	1- 6-15.46	.0192723
164	1- 5-51.21	.0191548
165	1- 5-27.27	.0190387
166	1- 5- 3.61	.0189241
167	1- 4-40.23	.0188107
168	1- 4-17.14	.0186988
169	1- 3-54.31	.0185881
170	1- 3-31.76	.0184788
171	1- 3- 9.47	.0183708
172	1- 2-47.44	.0182640
173	1- 2-25.66	.0181584
174	1- 2- 4.13	.0180541
175	1- 1-42.85	.0179509
176	1- 1-21.81	.0178489
177	1- 1- 1.01	.0177481
178	1- 0-40.44	.0176484
179	1-0-20.11	.0175498
180	1-	.0174524
181	59-40.11	.0173559
182	59-20.43	.0172605
183	59- 0.98	.0171663
184	58-41.73	.0170730
185	58-22.70	.0169807
186	58- 3.87	.0168894
187	57-45.24	.0167991
188	57-26.30	.0167097
189	57- 8.57	.0166214
190	56-50.52	.0165339
191	56-32.67	.0164473
192	56-15	.0163617
193	55-57.51	.0162769
194	55-40.20	.0161930
195	55-23.07	.0161100
196	55- 6.12	.0160278
197	54-49.34	.0159464
198	52-32.72	.0158659
199	54-16.28	.0157862
200	54-	.0157073
201	53-43.88	.0156294
202	53-27.92	.0155518
203	53-12.12	.0154752
204	52-56.47	.0153993
205	52-40.97	.0153242
206	52-25.63	.0152498
207	52-10.44	.0151764
208	51-55.38	.0151033
209	51-40.48	.0150310
210	51-25.71	.0149595
211	51-11.09	.0148886
212	50-56.60	.0148183
213	50-42.25	.0147487
214	50-28.04	.0146798
215	50-13.96	.0146115

TABLE 8.—SIDES AND ANGLES OF POLYGONS—(*Continued*)

No. sides	Angle	Sine
216	50–	.0145439
217	49–46.17	.0144769
218	49–32.48	.0144104
219	49–18.91	.0143446
220	49– 5.46	.0142794
221	48–52.13	.0142148
222	48–38.92	.0141508
223	48–25.83	.0140874
224	48–12.86	.0140245
225	48–	.0139622
226	47–47.26	.0139004
227	47–34.63	.0138392
228	47–22.11	.0137785
229	47– 9.69	.0137183
230	46–57.39	.0136587
231	46–45.19	.0135995
232	46–33.10	.0135409
233	46–21.11	.0134828
234	46– 9.23	.0134252
235	45–57.45	.0133681
235	45–45.76	.0133115
237	45–34.18	.0132553
238	45–22.69	.0131996
239	45–11.29	.0131444
240	45–	.0130896
241	44–48.80	.0130353
242	44–37.68	.0129814
243	44–26.67	.0129280
244	44–15.74	.0128750
245	44– 4.90	.0128225
246	43–54.15	.0127704
247	43–43.48	.0127187
248	43–32.90	.0126674
249	43–22.41	.0126165
250	43–12	.0125661
251	43– 1.67	.0125160
252	42–51.43	.0124663
253	42–41.26	.0124171
254	42–31.18	.0123682
255	42–21.18	.0123197
256	42–11.25	.0122715
257	42– 1.40	.0122238
258	41–51.63	.0121764
259	41–41.93	.0121294
260	41–32.31	.0120827
261	41–22.76	.0120364
262	41–13.28	.0119905
263	41– 3.88	.0119449
264	40–54.54	.0118997
265	40–45.28	.0118548
266	40–36.09	.0118102
267	40–26.96	.0117660
268	40–17.91	.0117221
269	40– 8.93	.0116786
270	40–	.0116353
271	39–51.14	.0115923
272	39–42.35	.0115497
273	39–33.63	.0115074
274	39–24.96	.0114654
275	39–16.36	.0114237
276	39– 7.83	.0113823
277	38–59.35	.0113412
278	38–50.94	.0113004
279	38–42.58	.0112599
280	38–34.28	.0112197
281	38–26.05	.0111798
282	38–17.87	.0111401
283	38– 9.75	.0111008
284	38– 1.69	.0110617
285	37–53.68	.0110229
286	37–45.73	.0109844

No. sides	Angle	Sine
287	37–37.84	.0109461
288	37–30	.0109081
289	37–22.21	.0108704
290	37–14.48	.0108329
291	37– 6.80	.0107957
292	36–59.18	.0107587
293	36–51.60	.0107220
294	36–44.08	.0106855
295	36–36.61	.0106493
296	36–29.19	.0106133
297	36–21.82	.0105776
298	36–14.50	.0105421
299	36– 7.22	.0105068
300	36–	.0104718
301	35–52.82	.0104370
302	35–45.69	.0104024
303	35–38.61	.0103681
304	35–31.58	.0103340
305	35–24.59	.0103001
306	35–17.65	.0102665
307	35–10.75	.0102330
308	35– 3.90	.0101998
309	34–57.09	.0101668
310	34–50.32	.0101340
311	34–43.60	.0101014
312	34–36.92	.0100690
313	34–30.29	.0100368
314	34–23.69	.0100049
315	34–17.14	.0099731
316	34–10.63	.0099415
317	34– 4.16	.0099102
318	33–57.74	.0098791
319	33–51.35	.0098482
320	33–45	.0098174
321	33–38.69	.0097868
322	33–32.42	.0097564
323	33–26.19	.0097261
324	33–20	.0096961
325	33–13.85	.0096663
326	33– 7.73	.0096367
327	33– 1.65	.0096072
328	32–55.61	.0095779
329	32–49.60	.0095488
330	32–43.64	.0095198
331	32–37.70	.0094911
332	32–31.81	.0094625
333	32–25.95	.0094341
334	32–20.12	.0094059
335	32–14.33	.0093778
336	32– 8.57	.0093499
337	32– 2.85	.0093221
338	31–57.16	.0092945
339	31–51.50	.0092671
340	31–45.88	.0092398
341	31–40.29	.0092127
342	31–34.74	.0091858
343	31–29.21	.0091590
344	31–23.72	.0091324
345	31–18.26	.0091059
346	31–12.83	.0090796
347	31– 7.44	.0090534
348	31– 2.07	.0090274
349	30–56.73	.0090016
350	30–51.43	.0089758
351	30–46.15	.0089502
352	30–40.91	.0089248
353	30–35.69	.0088996
354	30–30.51	.0088744
355	30–25.35	.0088494
356	30–20.22	.0088245
357	30–15.12	.0087998

No. sides	Angle	Sine
358	30–10.05	.0087753
359	30– 5.01	.0087508
360	30–	.0087265
361	29–55.01	.0087023
362	29–50.05	.0086783
363	29–45.12	.0086544
364	29–40.22	.0086306
365	29–35.34	.0086070
366	29–30.49	.0085835
367	29–25.67	.0085601
368	29–20.87	.0085368
369	29–16.10	.0085137
370	29–11.35	.0084907
371	29– 6.63	.0084678
372	29– 1.94	.0084451
373	28–57.27	.0084224
374	28–52.62	.0083999
375	28–48	.0083775
376	28–43.40	.0083552
377	28–38.83	.0083331
378	28–34.28	.0083110
379	28–29.76	.0082891
380	28–25.26	.0082673
381	28–20.78	.0082456
382	28–16.33	.0082240
383	28–11.91	.0082025
384	28– 7.50	.0081812
385	28– 3.12	.0081599
386	27–58.76	.0081387
387	27–54.42	.0081177
388	27–50.10	.0080968
389	27–45.81	.0080760
390	27–41.54	.0080553
391	27–37.29	.0080347
392	27–33.06	.0080142
393	27–28.85	.0079938
394	27–24.67	.0079735
395	27–20.51	.0079533
396	27–16.36	.0079332
397	27–12.24	.0079132
398	27– 8.14	.0078934
399	27– 4.06	.0078736
400	27–	.0078539
401	26–55.96	.0078343
402	26–51.94	.0078148
403	26–47.94	.0077954
404	26–43.96	.0077761
405	26–40	.0077569
406	26–36.06	.0077378
407	26–32.14	.0077188
408	26–28.23	.0076999
409	26–24.35	.0076811
410	26–20.49	.0076623
411	26–16.64	.0076437
412	26–12.82	.0076251
413	26– 9.01	.0076067
414	26– 5.22	.0075883
415	26– 1.45	.0075700
416	25–57.70	.0075518
417	25–53.96	.0075337
418	25–50.24	.0075157
419	25–46.54	.0074977
420	25–42.86	.0074799
421	25–39.19	.0074621
422	25–35.54	.0074444
423	25–31.91	.0074268
424	25–28.30	.0074093
425	25–24.70	.0073919
426	25–21.12	.0073745
427	25–17.56	.0073573
428	25–14.02	.0073401

TABLE 8.—SIDES AND ANGLES OF POLYGONS—(*Continued*)

Sides	Angle	Sine	Sides	Angle	Sine	Sides	Angle	Sine
429	25-10.49	.0073230	453	23-50.46	.0069351	478	22-35.65	.0065723
			454	23-47.31	.0069198	479	22-32.82	.0065585
430	25- 6.98	.0073059	455	23-44.17	.0069046			
431	25- 3.48	.0072890	456	23-41.05	.0068894	480	22-30	.0065449
432	25-	.0072721	457	23-37.94	.0068744	481	22-27.20	.0065313
433	24-56.54	.0072553	458	23-34.84	.0068594	482	22-24.40	.0065178
434	24-53.09	.0072386	459	23-31.76	.0068444	483	22-21.61	.0065043
435	24-49.66	.0072220				484	22-18.84	.0064909
436	24-46.24	.0072054	460	23-28.69	.0068295	485	22-16.08	.0064775
437	24-42.84	.0071889	461	23-25.64	.0068147	486	22-13.33	.0064641
438	24-39.45	.0071725	462	23-22.60	.0067999	487	22-10.59	.0064509
439	24-36.08	.0071562	463	23-19.57	.0067852	488	22- 7.87	.0064377
			464	23-16.55	.0067706	489	22- 5.16	.0064245
440	24-32.73	.0071399	465	23-13.55	.0067561			
441	24-29.39	.0071237	466	23-10.56	.0067416	490	22- 2.45	.0064114
442	24-26.06	.0071076	467	23- 7.58	.0067272	491	21-59.75	.0063983
443	24-22.75	.0070916	468	23- 4.61	.0067128	492	21-57.07	.0063853
444	24-19.46	.0070756	469	23- 1.66	.0066985	493	21-54.40	.0063723
445	24-16.18	.0070597				494	21-51.74	.0063594
446	24-12.91	.0070439	470	22-58.72	.0066842	495	21-49.09	.0063466
447	24- 9.66	.0070281	471	22-55.79	.0066700	496	21-46.45	.0063338
448	24- 6.43	.0070124	472	22-52.88	.0066559	497	21-43.82	.0063211
449	24- 3.21	.0069968	473	22-49.98	.0066418	498	21-41.20	.0063084
			474	22-47.09	.0066278	499	21-38.59	.0062957
450	24-	.0069813	475	22-44.21	.0066138			
451	23-56.81	.0069658	476	22-41.34	.0065999	500	21-36	.0062831
452	23-53.63	.0069504	477	22-38.49	.0065861			

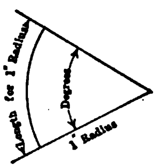


TABLE 9.—LENGTHS OF CIRCULAR ARCS TO RADIUS OF 1 IN.

Deg.	Length	Deg.	Length	Deg.	Length	Min.	Length
1	.0175	61	1.0647	121	2.1118	1	.0003
2	.0349	62	1.0821	122	2.1293	2	.0006
3	.0524	63	1.0996	123	2.1468	3	.0009
4	.0698	64	1.1170	124	2.1642	4	.0012
5	.0873	65	1.1345	125	2.1817	5	.0015
6	.1047	66	1.1519	126	2.1991	6	.0017
7	.1222	67	1.1694	127	2.2166	7	.0020
8	.1396	68	1.1868	128	2.2340	8	.0023
9	.1571	69	1.2043	129	2.2515	9	.0026
10	.1745	70	1.2217	130	2.2690	10	.0029
11	.1920	71	1.2392	131	2.2864	11	.0032
12	.2094	72	1.2566	132	2.3038	12	.0035
13	.2269	73	1.2741	133	2.3132	13	.0038
14	.2443	74	1.2915	134	2.3387	14	.0041
15	.2618	75	1.3090	135	2.3562	15	.0044
16	.2793	76	1.3265	136	2.3736	16	.0047
17	.2967	77	1.3439	137	2.3911	17	.0050
18	.3142	78	1.3614	138	2.4086	18	.0052
19	.3316	79	1.3788	139	2.4260	19	.0055
20	.3491	80	1.3963	140	2.4435	20	.0058
21	.3665	81	1.4137	141	2.4609	21	.0061
22	.3840	82	1.4312	142	2.4784	22	.0064
23	.4014	83	1.4486	143	2.4958	23	.0067
24	.4189	84	1.4661	144	2.5133	24	.0070
25	.4363	85	1.4835	145	2.5307	25	.0073
26	.4538	86	1.5010	146	2.5482	26	.0076
27	.4712	87	1.5184	147	2.5656	27	.0079
28	.4887	88	1.5359	148	2.5831	28	.0081
29	.5061	89	1.5533	149	2.6005	29	.0084
30	.5236	90	1.5708	150	2.6180	30	.0087
31	.5411	91	1.5882	151	2.6354	31	.0090
32	.5585	92	1.6057	152	2.6529	32	.0093
33	.5760	93	1.6232	153	2.6704	33	.0096
34	.5934	94	1.6406	154	2.6878	34	.0099
35	.6109	95	1.6581	155	2.7052	35	.0102
36	.6283	96	1.6755	156	2.7227	36	.0105
37	.6458	97	1.6930	157	2.7402	37	.0108
38	.6632	98	1.7104	158	2.7576	38	.0111
39	.6807	99	1.7279	159	2.7751	39	.1130
40	.6981	100	1.7453	160	2.7925	40	.1160
41	.7156	101	1.7628	161	2.8100	41	.1190
42	.7330	102	1.7802	162	2.8274	42	.0122
43	.7505	103	1.7977	163	2.8449	43	.0125
44	.7679	104	1.8151	164	2.8623	44	.0128
45	.7854	105	1.8326	165	2.8798	45	.0131
46	.8029	106	1.8500	166	2.8972	46	.0134
47	.8203	107	1.8675	167	2.9147	47	.0137
48	.8378	108	1.8850	168	2.9322	48	.0140
49	.8552	109	1.9024	169	2.9496	49	.0143
50	.8728	110	1.9199	170	2.9671	50	.0145
51	.8901	111	1.9373	171	2.9845	51	.0148
52	.9076	112	1.9548	172	3.0020	52	.0151
53	.9250	113	1.9722	173	3.0194	53	.0154
54	.9425	114	1.9897	174	3.0369	54	.0157
55	.9599	115	2.0071	175	3.0543	55	.0160
56	.9774	116	2.0246	176	3.0718	56	.0163
57	.9948	117	2.0420	177	3.0892	57	.0166
58	1.0123	118	2.0595	178	3.1067	58	.0169
59	1.0297	119	2.0769	179	3.1241	59	.0172
60	1.0472	120	2.0944	180	3.1416	60	.0175

500 inclusive) of polygons, one-half the angle subtended by a side and the sine of that half angle. To get the length of a side of any polygon, of a number of sides included in the limits of the table, multiply the corresponding sine in the table by the diameter of the circumscribed circle.

The squares of mixed numbers not found in Table 10 are most conveniently computed by remembering that $(a+b)^2=a^2+2ab+b^2$, that is to say, add the square of the whole number, the square of the fraction and twice the product of the whole number and fraction. Squares of binary fractions will be found in Table 22 and decimal equivalents in Table 16 which will greatly facilitate the process.

Example: Required the square of $27\frac{1}{16}$

square of 27	729.0000
square of $\frac{1}{16}$	.0039
twice the product of 27 and $\frac{1}{16}$	
$=2\times27\times.0625$	3.375
square of $27\frac{1}{16}$	732.3789

Table 10.—Squares of Mixed Numbers

(*W. L. and R. E. Tyron, Amer. Mach.*, Dec. 23, 1909)

	No.	0	1	2	3	4	5	6	7
Added sixty-fourths	0	.000000	1.00000	4.00000	9.00000	16.00000	25.00000	36.00000	49.00000
	1/64	.000244	1.03149	4.06274	9.09399	16.12524	25.15049	36.18774	49.21899
	1/32	.000977	1.06348	4.12598	9.18848	16.25098	25.31348	36.37598	49.43848
	3/64	.002197	1.09595	4.18970	9.28345	16.37720	25.47095	36.56470	49.65845
	1/16	.003906	1.12891	4.25391	9.37891	16.50391	25.62891	36.75391	49.87891
	5/64	.006104	1.16235	4.31860	9.47485	16.63110	25.78735	36.94360	50.09985
	3/32	.008789	1.19629	4.38379	9.57129	16.75879	25.94629	37.13379	50.32129
	7/64	.011963	1.23071	4.44946	9.66821	16.88696	26.10571	37.32446	50.54321
	1/8	.015625	1.26563	4.51563	9.76563	17.01563	26.26563	37.51563	50.76563
	9/64	.019775	1.30103	4.58228	9.86353	17.14478	26.42603	37.70728	50.98853
	5/32	.024414	1.33691	4.64941	9.96191	17.27441	26.58691	37.89941	51.21191
	11/64	.029541	1.37329	4.71704	10.06079	17.40454	26.74829	38.09204	51.43579
	3/16	.035156	1.41016	4.78516	10.16016	17.53516	26.91016	38.28516	51.66016
	13/64	.041260	1.44751	4.85376	10.26001	17.66626	27.07251	38.47876	51.88501
	7/32	.047852	1.48535	4.92285	10.36035	17.79785	27.23535	38.67285	52.11035
	15/64	.054932	1.52368	4.99243	10.46118	17.92993	27.39868	38.86743	52.33618
	1/4	.062500	1.56250	5.06250	10.56250	18.06250	27.56250	39.06250	52.56250
	17/64	.070557	1.60181	5.13306	10.66431	18.19556	27.72681	39.25806	52.78931
	9/32	.079102	1.64160	5.20410	10.76660	18.32910	27.89160	39.45410	53.01660
	19/64	.088135	1.68188	5.27563	10.85938	18.46313	28.05688	39.65063	53.24438
	5/16	.097656	1.72266	5.34766	10.97266	18.59766	28.22266	39.84766	53.47266
	21/64	.107666	1.76392	5.42017	11.07642	18.73267	28.38892	40.04517	53.70142
	11/32	.118164	1.80566	5.49316	11.18066	18.86816	28.55566	40.24316	53.93066
	23/64	.129151	1.84790	5.56665	11.28540	19.00415	28.72290	40.44165	54.16040
	3/8	.140625	1.89063	5.64063	11.39063	19.14063	28.89063	40.64063	54.39063
	25/64	.152588	1.93384	5.71509	11.49634	19.27759	29.05884	40.84009	54.62134
	13/32	.165039	1.97754	5.79004	11.60254	19.41504	29.22754	41.04004	54.85254
	27/64	.177979	2.02173	5.86548	11.70923	19.55298	29.39673	41.24048	55.08423
	7/16	.191406	2.06641	5.94141	11.81641	19.69141	29.56641	41.44141	55.31641
	29/64	.205322	2.11157	6.01782	11.92407	19.83032	29.73657	41.64282	55.54907
	15/32	.219726	2.15723	6.09473	12.03223	19.96973	29.90723	41.84473	55.78223
	31/64	.234619	2.20337	6.17212	12.14087	20.10962	30.07837	42.04712	56.01587
	1/2	.250000	2.25000	6.25000	12.25000	20.25000	30.25000	42.25000	56.20500
	33/64	.265869	2.29712	6.32837	12.35962	20.39087	30.42212	42.45337	56.48462
	17/32	.282227	2.34473	6.40723	12.46973	20.53223	30.59473	42.65723	56.71973
	35/64	.299072	2.39282	6.48657	12.58032	20.67407	30.76782	42.86157	56.95532
	9/16	.316406	2.44141	6.56641	12.69141	12.81641	30.94141	43.06641	57.19141
	37/64	.334229	2.49048	6.64673	12.80298	20.95923	31.11548	43.27173	57.42798
	19/32	.352539	2.54004	6.72754	12.91504	21.10254	31.29004	43.47754	57.66504
	39/64	.371338	2.59009	6.80884	13.02759	21.24634	31.46509	43.68384	57.90259
	5/8	.390625	2.64063	6.89063	13.14063	21.39063	31.64063	43.89063	58.14063
	41/64	.410400	2.69165	6.97290	13.25415	21.53540	31.81665	44.09790	58.37915
	21/32	.430664	2.74316	7.05566	13.36816	21.68066	31.99316	44.30566	58.61816
	43/64	.451416	2.79517	7.13892	13.48267	21.82642	32.17017	44.51392	58.85767
	11/16	.472656	2.84766	7.22266	13.59766	21.97266	32.34766	44.72266	59.09766
	45/64	.494385	2.90063	7.30688	13.71313	22.11938	32.52563	44.93188	59.33813
	23/32	.516602	2.95410	7.39160	13.82910	22.26660	32.70410	45.14160	59.57910
	47/64	.539307	3.00806	7.47681	13.94556	22.41431	32.88306	45.35181	59.82056
	3/4	.562500	3.06250	7.56250	14.06250	22.56250	33.06250	45.56250	60.06250
	49/64	.586182	3.11743	7.64868	14.17993	22.71118	33.24243	45.77368	60.30493
	25/32	.610352	3.17285	7.73535	14.29785	22.86035	33.42285	45.98535	60.54785
	51/64	.635010	3.22876	7.82251	14.41626	23.01001	33.60376	46.19751	60.79126
	13/16	.660156	3.28516	7.91016	14.53516	23.16016	33.78516	46.41016	61.03516
	53/64	.685791	3.34204	7.99829	14.65454	23.31079	33.96704	46.62329	61.27954
	27/32	.711914	3.39941	8.08691	14.77441	23.46191	34.14941	46.83691	61.52441
	55/64	.738526	3.45728	8.17603	14.89478	23.61353	34.33228	47.05103	61.76978
	7/8	.765625	3.51563	8.26563	15.01563	23.76563	34.51563	47.26563	62.01563
	57/64	.793213	3.57446	8.35571	15.13696	23.91821	34.69946	47.48071	62.26196
	29/32	.821289	3.63779	8.44629	15.25879	24.07129	34.88379	47.69629	62.50879
	59/64	.849854	3.69360	8.53735	15.38110	24.22485	35.06860	47.91235	62.75610
	15/16	.878906	3.75391	8.62891	15.50391	24.37891	35.25391	48.12891	63.00391
	61/64	.908447	3.81470	8.72095	15.62720	24.53345	35.43970	48.34595	63.25220
	31/32	.938477	3.87598	8.81348	15.75098	24.68848	35.62598	48.56348	63.50098
	63/64	.968994	3.93774	8.90649	15.87524	24.84399	35.81274	48.78149	63.75024

TABLE 10.—SQUARES OF MIXED NUMBERS—(*Continued*)
(*W. L. and R. E. Tyron, Amer. Mach., Dec.* 23, 1909.)

No. (Added thirty-seconds)	8	9	10	11	12	13	14	15
0	64.0000	81.0000	100.0000	121.0000	144.0000	169.0000	196.0000	225.0000
1/32	64.5010	81.5635	100.6260	121.6885	144.7510	169.8135	196.8760	225.9385
1/16	65.0039	82.1289	101.2539	122.3789	145.5039	170.6289	197.7539	226.8789
3/32	65.5088	82.6963	101.8838	123.0713	146.2588	171.4463	198.6338	227.8213
1/8	66.0156	83.2656	102.5156	123.7656	147.0156	172.2656	199.5156	228.7656
5/32	66.5244	83.8369	103.1494	124.4619	147.7744	173.0869	200.3994	229.7119
3/16	67.0352	84.4102	103.7852	125.1602	148.5352	173.9102	201.2852	230.6602
7/32	67.5479	84.9854	104.4229	125.8604	149.2979	174.7354	202.1725	231.6104
1/4	68.0625	85.5625	105.0625	126.5625	150.0625	175.5625	203.0625	232.5625
9/32	68.5791	86.1416	105.7041	127.2666	150.8291	176.3916	203.9541	233.5166
5/16	69.0977	86.7227	106.3477	127.9727	151.5977	177.2227	204.8477	234.4727
11/32	69.6182	87.3057	106.9932	128.6807	152.3682	178.0557	205.7432	235.4307
3/8	70.1406	87.8906	107.6406	129.3906	153.1406	178.8906	206.6406	236.3906
13/32	70.6650	88.4775	108.2900	130.1025	153.9150	179.7278	207.5404	237.3535
7/16	71.1914	89.0664	108.9414	130.8164	154.6914	180.5664	208.4414	238.3164
15/32	71.7197	89.6572	109.5947	131.5322	155.4697	181.4072	209.3447	239.2822
1/2	72.2500	90.2500	110.2500	132.2500	156.2500	182.2500	210.2500	240.2500
17/32	72.7822	90.8447	110.9072	132.9697	157.0322	183.0947	211.1572	241.2197
9/16	73.3164	91.4414	111.5664	133.6914	157.8164	183.9414	212.0664	242.1914
19/32	73.8525	92.0400	112.2275	134.4150	158.6025	184.7900	212.9775	243.1650
5/8	74.3906	92.6406	112.8906	135.1406	159.3906	185.6406	213.8906	244.1406
21/32	74.9307	93.2432	113.5557	135.8682	160.1807	186.4932	214.8057	245.1182
11/16	75.4727	93.8477	114.2227	136.5977	160.9727	187.3477	215.7227	246.0977
23/32	76.0166	94.4541	114.8916	137.3291	161.7666	188.2041	216.6416	247.0791
3/4	76.5625	95.0625	115.5625	138.0625	162.5625	189.0625	217.5625	248.0625
25/32	77.1104	95.6729	116.2354	138.7979	163.3604	189.9229	218.4854	249.0479
13/16	77.6602	96.2852	116.9102	139.5352	164.1602	190.7852	219.4102	250.0352
27/32	78.2119	96.8994	117.5869	140.2744	164.9619	191.6494	220.3369	251.0244
7/8	78.7656	97.5156	118.2656	141.0156	165.7656	192.5156	221.2656	252.0156
29/32	79.3213	98.1338	118.9463	141.7588	166.5713	193.3838	222.1963	253.0088
15/16	79.8789	98.7539	119.6289	142.5039	167.3789	194.2539	223.1289	254.0039
31/32	80.4385	99.3760	120.3135	143.2510	168.1885	195.1260	224.0635	255.0010

No. (Added thirty-seconds)	16	17	18	19	20	21	22	23
0	256.000	289.000	324.000	361.000	400.000	441.000	484.000	529.000
1/32	257.001	290.063	325.126	362.188	401.251	442.313	485.376	530.438
1/16	258.004	291.129	326.254	363.379	402.504	443.629	486.754	531.879
3/32	259.009	292.196	327.384	364.571	403.759	444.946	488.134	533.321
1/8	260.016	293.266	328.516	365.766	405.016	446.266	489.516	534.766
5/32	261.024	294.337	329.649	366.962	406.274	447.587	490.899	536.212
3/16	262.035	295.410	330.785	368.160	407.535	448.910	492.286	537.660
7/32	263.048	296.485	331.923	369.360	408.798	450.235	493.673	539.110
1/4	264.063	297.563	333.063	370.563	410.063	451.563	495.063	540.563
9/32	265.079	298.642	334.204	371.767	411.329	452.892	496.454	542.017
5/16	266.098	299.723	335.348	372.973	412.598	454.223	497.848	543.473
11/32	267.118	300.806	336.493	374.181	413.868	455.556	499.243	544.931
3/8	268.141	301.891	337.641	375.391	415.141	456.891	500.641	546.391
13/32	269.165	302.978	338.790	376.603	416.415	458.228	502.040	547.853
7/16	270.191	304.066	339.941	377.816	417.691	459.566	503.441	549.316
15/32	271.220	305.157	341.095	379.032	418.970	460.907	504.845	550.782
1/2	272.250	306.250	342.250	380.250	420.250	462.250	506.250	552.250
17/32	273.282	307.345	343.407	381.470	421.532	463.595	507.657	553.720
9/16	274.316	308.441	344.566	382.691	422.816	464.941	509.066	555.191
19/32	275.353	309.540	345.728	383.915	424.103	466.290	510.478	556.665
5/8	276.391	310.641	346.891	385.141	425.391	467.641	511.891	558.141
21/32	277.431	311.743	348.056	386.368	426.681	468.993	513.306	559.618
11/16	278.473	312.848	349.223	387.598	427.973	470.348	514.723	561.098
23/32	279.517	313.954	350.392	388.829	429.267	471.704	516.142	562.579
3/4	280.563	315.063	351.563	390.063	430.563	473.063	517.563	564.063
25/32	281.610	316.173	352.735	391.299	431.860	474.423	518.985	565.548
13/16	282.660	317.285	353.910	392.535	433.160	475.785	520.410	567.035
27/32	283.712	318.399	355.087	393.774	434.462	477.149	521.837	568.524
7/8	284.766	319.516	356.266	395.016	435.766	478.516	523.266	570.016
29/32	285.821	320.634	357.446	396.259	437.071	479.884	524.696	571.509
15/16	286.879	321.754	358.629	397.504	438.379	481.254	526.129	573.004
31/32	287.938	322.876	359.813	398.751	439.688	482.626	527.563	574.501

TABLE 10.—SQUARES OF MIXED NUMBERS—(*Continued*)

(*W. L. and R. E. Tryon, Amer. Mach., Dec.* 23, 1909)

Added thirty-seconds

No.	24	25	26	27	28	29	30	31
0	576.000	625.000	676.000	729.000	784.000	841.000	900.000	961.000
1/32	577.501	626.564	677.626	730.689	785.751	842.814	901.876	962.939
1/16	579.004	628.129	679.254	732.379	787.504	844.629	903.754	964.879
3/32	580.509	629.696	680.884	734.071	789.259	846.446	905.634	966.821
1/8	582.016	631.266	682.516	735.766	791.016	848.266	907.516	968.766
5/32	583.524	632.837	683.149	737.462	792.774	850.087	909.399	970.712
3/16	585.035	634.410	685.785	739.160	794.535	851.910	911.286	972.660
7/32	586.548	635.985	687.423	740.860	796.298	853.735	913.173	974.610
1/4	588.063	637.563	689.063	742.563	798.063	855.563	915.063	976.563
9/32	589.579	639.142	690.704	744.267	799.829	857.392	916.954	978.517
5/16	591.098	640.723	692.348	745.973	801.598	859.223	918.848	980.473
11/32	592.618	642.306	693.993	747.681	803.368	861.056	920.743	982.431
3/8	594.141	643.891	695.641	749.391	805.141	862.891	922.641	984.391
13/32	595.665	645.478	697.290	751.103	806.915	864.728	924.540	986.353
7/16	597.191	647.066	698.941	752.816	808.691	866.566	926.441	988.316
15/32	598.720	648.657	700.595	754.532	810.470	868.407	928.345	990.282
1/2	600.250	650.250	702.250	756.250	812.250	870.250	930.250	992.250
17/32	601.782	651.845	703.907	757.970	814.032	872.095	932.157	994.220
9/16	603.316	653.441	705.566	759.691	815.816	873.941	934.066	996.191
19/32	604.853	655.040	707.228	761.415	817.603	875.790	935.978	998.165
5/8	606.391	656.641	708.891	763.141	819.391	877.641	937.891	1000.141
21/32	607.931	658.243	710.556	764.868	821.181	879.493	939.806	1002.118
11/16	609.473	659.848	712.223	766.598	822.973	881.348	941.723	1004.098
23/32	611.017	661.454	713.892	768.329	824.767	883.204	943.642	1006.079
3/4	612.063	663.063	715.563	770.063	826.563	885.063	945.563	1008.063
25/32	614.110	664.673	717.235	771.798	828.360	886.923	947.486	1010.048
13/16	615.660	666.285	718.910	773.535	830.160	888.785	949.410	1012.035
27/32	617.212	667.899	720.587	775.274	831.962	890.650	951.337	1014.024
7/8	618.766	669.516	722.266	777.016	833.766	892.516	953.266	1016.016
29/32	620.321	671.134	723.946	778.759	835.571	894.384	955.196	1018.009
15/16	621.879	672.754	725.629	780.504	837.379	896.254	957.129	1020.004
31/32	623.439	674.376	727.314	782.251	839.188	898.126	959.064	1022.001

Added thirty-seconds

No.	32	33	34	35	36	37	38	39
0	1024.00	1089.00	1156.00	1225.00	1296.00	1369.00	1444.00	1521.00
1/32	1026.00	1091.06	1158.13	1227.19	1298.25	1371.31	1446.38	1523.44
1/16	1028.00	1093.13	1160.25	1229.38	1300.50	1373.63	1448.75	1525.88
3/32	1030.01	1095.20	1162.38	1231.57	1302.76	1375.95	1451.13	1528.32
1/8	1032.02	1097.27	1164.52	1233.77	1305.02	1378.27	1453.52	1530.77
5/32	1034.02	1099.34	1166.65	1235.96	1307.27	1380.59	1455.90	1533.21
3/16	1036.04	1101.41	1168.79	1238.16	1309.54	1382.91	1458.29	1535.66
7/32	1038.05	1103.49	1170.92	1240.36	1311.80	1385.24	1460.67	1538.11
1/4	1040.06	1105.56	1173.06	1242.56	1314.06	1387.56	1463.06	1540.56
9/32	1042.08	1107.64	1175.20	1244.77	1316.33	1389.89	1465.45	1543.02
5/16	1044.10	1109.72	1177.35	1246.97	1318.60	1392.22	1467.85	1545.47
11/32	1046.12	1111.81	1179.49	1249.18	1320.87	1394.56	1470.24	1547.93
3/8	1048.14	1113.89	1181.64	1251.39	1323.14	1396.89	1472.64	1550.39
13/32	1050.17	1115.98	1183.79	1253.60	1325.42	1399.23	1475.04	1552.85
7/16	1052.19	1118.07	1185.94	1255.82	1327.69	1401.57	1477.44	1555.32
15/32	1054.22	1120.16	1188.09	1258.03	1329.97	1403.91	1479.84	1557.78
1/2	1056.25	1122.25	1190.25	1260.25	1332.25	1406.25	1482.25	1560.25
17/32	1058.28	1124.34	1192.41	1262.47	1334.53	1408.59	1484.66	1562.72
9/16	1060.32	1126.44	1194.57	1264.69	1336.82	1410.94	1487.07	1565.19
19/32	1062.35	1128.54	1196.73	1266.91	1339.10	1413.29	1489.48	1567.67
5/8	1064.39	1130.64	1198.89	1269.14	1341.39	1415.64	1491.89	1570.14
21/32	1066.43	1132.74	1201.06	1271.37	1343.68	1417.99	1494.31	1572.62
11/16	1068.47	1134.85	1203.22	1273.60	1345.97	1420.35	1496.72	1575.10
23/32	1070.52	1135.95	1205.39	1275.83	1348.27	1422.70	1499.14	1577.58
3/4	1072.56	1139.06	1207.56	1278.06	1350.56	1425.06	1501.56	1580.06
25/32	1074.61	1141.17	1209.74	1280.30	1352.86	1427.42	1503.99	1582.55
13/16	1076.66	1143.29	1211.91	1282.54	1355.16	1429.79	1506.41	1585.04
27/32	1078.71	1145.40	1214.09	1284.77	1357.46	1432.15	1508.84	1587.52
7/8	1080.77	1147.52	1216.26	1287.02	1359.77	1434.52	1511.26	1590.02
29/32	1082.82	1149.63	1218.45	1289.26	1362.07	1436.88	1513.70	1592.51
15/16	1084.88	1151.75	1220.63	1291.50	1364.38	1439.25	1516.13	1595.00
31/32	1086.94	1153.88	1222.81	1293.75	1366.69	1441.63	1518.56	1597.50

Table 10.—Squares of Mixed Numbers—(*Continued*)

(*W. L. and R. E. Tryon, Amer. Mach., Dec.* 23, 1909)

Added Eighths

No.	40	41	42	43	44	45	46	47
0	1600.00	1681.00	1764.00	1849.00	1936.00	2025.00	2116.00	2209.00
⅛	1610.02	1691.27	1774.52	1859.77	1947.02	2036.27	2127.52	2220.77
¼	1620.06	1701.56	1785.06	1870.56	1958.06	2047.56	2139.06	2232.56
⅜	1630.14	1711.89	1795.64	1881.39	1969.14	2058.89	2150.64	2244.39
½	1640.25	1722.25	1806.25	1892.25	1980.25	2070.25	2162.25	2256.25
⅝	1650.39	1732.64	1816.89	1903.14	1991.39	2081.64	2173.89	2268.14
¾	1660.56	1743.06	1827.56	1914.06	2002.56	2093.06	2185.56	2280.06
⅞	1670.77	1753.52	1838.27	1925.02	2013.77	2104.52	2197.27	2292.02

No.	48	49	50	51	52	53	54	55
0	2304.00	2401.00	2500.00	2601.00	2704.00	2809.00	2916.00	3025.00
⅛	2316.02	2413.27	2512.52	2613.77	2717.02	2822.27	2929.52	3038.77
¼	2328.06	2425.56	2525.06	2626.56	2730.06	2835.56	2943.06	3052.56
⅜	2340.14	2437.89	2537.64	2639.39	2743.14	2848.89	2956.64	3066.39
½	2352.25	2450.25	2550.25	2652.25	2756.25	2862.25	2970.25	3080.25
⅝	2364.39	2462.64	2562.89	2665.14	2769.39	2875.64	2983.89	3094.14
¾	2376.56	2475.06	2575.56	2678.06	2782.56	2889.06	2997.56	3108.06
⅞	2388.77	2487.52	2588.27	2691.02	2775.77	2902.52	3011.27	3122.02

No.	56	57	58	59	60	61	62	63
0	3136.00	3249.00	3364.00	3481.00	3600.00	3721.00	3844.00	3969.00
⅛	3150.02	3263.27	3378.52	3495.77	3615.02	3736.27	3859.52	3984.77
¼	3164.06	3277.56	3393.06	3510.56	3630.06	3751.56	3875.06	4000.56
⅜	3178.14	3291.89	3407.64	3525.39	3645.14	3766.89	3890.64	4016.39
½	3192.25	3306.25	3422.25	3540.25	3660.25	3782.25	3906.25	4032.25
⅝	3206.39	3320.64	3436.89	3555.14	3675.39	3797.64	3921.89	4048.14
¾	3220.56	3335.06	3451.56	3570.06	3690.56	3813.06	3937.56	4064.06
⅞	3234.77	3349.52	3466.27	3585.02	3705.77	3828.52	3953.27	4080.02

No.	64	65	66	67	68	69	70	71
0	4096.00	4225.00	4356.00	4489.00	4624.00	4761.00	4900.00	5041.00
⅛	4112.02	4241.27	4372.52	4505.77	4641.02	4778.27	4917.52	5058.77
¼	4128.06	4257.56	4389.06	4522.56	4658.06	4795.56	4935.06	5076.56
⅜	4144.14	4273.89	4405.64	4539.39	4675.14	4812.89	4952.64	5094.39
½	4160.25	4290.25	4422.25	4556.25	4692.25	4830.25	4970.25	5112.25
⅝	4176.39	4306.64	4438.89	4573.14	4709.39	4847.64	4987.89	5130.14
¾	4192.56	4323.06	4455.56	4590.06	4726.56	4865.06	5005.56	5148.06
⅞	4208.77	4339.52	4472.27	4607.02	4743.77	4882.52	5023.27	5166.02

No.	72	73	74	75	76	77	78	79
0	5184.00	5329.00	5476.00	5625.00	5776.00	5929.00	6084.00	6241.00
⅛	5202.02	5347.27	5494.52	5643.77	5795.02	5948.27	6103.52	6260.77
¼	5220.06	5365.56	5513.06	5662.56	5814.06	5967.56	6123.06	6280.56
⅜	5238.14	5383.89	5531.64	5681.39	5833.14	5986.89	6142.64	6300.39
½	5256.25	5402.25	5550.25	5700.25	5852.25	6006.25	6162.25	6320.25
⅝	5274.39	5420.64	5568.89	5719.14	5871.39	6025.64	6181.89	6340.14
¾	5292.56	5439.06	5587.56	5738.06	5890.56	6045.06	6201.56	6360.06
⅞	5310.77	5457.52	5606.27	5757.02	5909.77	6064.52	6221.27	6380.02

No.	80	81	82	83	84	85	86	87
0	6400.00	6561.00	6724.00	6889.00	7056.00	7225.00	7396.00	7569.00
⅛	6420.02	6581.27	6744.52	6909.77	7077.02	7246.27	7417.52	7590.77
¼	6440.06	6601.56	6765.06	6930.56	7098.06	7267.56	7439.06	7612.56
⅜	6460.14	6621.89	6785.64	6951.39	7119.14	7288.89	7460.64	7634.39
½	6480.25	6642.25	6806.25	6972.25	7140.25	7310.25	7482.25	7656.25
⅝	6500.39	6662.64	6826.89	6993.14	7161.39	7331.64	7503.89	7678.14
¾	6520.56	6683.06	6847.56	7014.06	7182.56	7353.06	7525.56	7700.06
⅞	6540.77	6703.52	6868.27	7035.02	7203.77	7374.52	7547.27	7722.02

No.	88	89	90	91	92	93	94	95
0	7744.00	7921.00	8100.00	8281.00	8464.00	8649.00	8836.00	9025.00
⅛	7766.02	7943.27	8122.52	8303.77	8487.02	8672.27	8859.52	9048.77
¼	7788.06	7965.56	8145.06	8326.56	8510.06	8695.56	8883.06	9072.56
⅜	7810.14	7987.89	8167.64	8349.39	8533.14	8718.89	8906.64	9096.39
½	7832.25	8010.25	8190.25	8372.25	8556.25	8742.25	8930.25	9120.25
⅝	7854.39	8032.64	8212.89	8395.14	8579.39	8765.64	8953.89	9144.14
¾	7876.56	8055.06	8235.56	8418.06	8602.56	8789.06	8977.56	9168.06
⅞	7898.77	8077.52	8258.27	8441.02	8625.77	8812.52	9001.27	9192.02

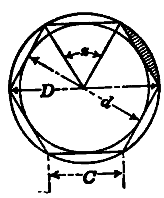
No.	96	97	98	99	100	101	102	103
0	9216.00	9409.00	9604.00	9801.00	10000.00	10201.00	10404.00	10609.00
⅛	9240.02	9433.27	9628.52	9825.77	10025.02	10226.27	10429.52	10634.77
¼	9264.06	9457.56	9653.06	9850.56	10050.06	10251.56	10455.06	10660.56
⅜	9288.14	9481.89	9677.64	9875.39	10075.14	10276.89	10480.64	10686.39
½	9312.25	9506.25	9702.25	9900.25	10100.25	10302.25	10506.25	10712.25
⅝	9336.39	9530.64	9726.89	9925.14	10125.39	10327.64	10531.89	10738.14
¾	9360.56	9555.06	9751.56	9950.06	10150.56	10353.06	10557.56	10764.06
⅞	9384.77	9579.52	9776.27	9975.02	10175.77	10378.52	10583.27	10790.02

Table 11.—Formulas and Constants for the Computation of Regular Polygons, by W. L. Benitz (*Amer. Mach., May* 23, 1907)
Factors and their Logarithms, and Central Angles, for Polygons of from 3 to 25 Sides

N	F	Log F	M	Log M	H	Log H	K	Log K	B	Log B	s
3	5.19615	.715682	5.19615	.715682	.324760	$\bar{1}$.511562	.433013	$\bar{1}$.636501	.384900	$\bar{1}$.585348	120°
4	4.00000	.602060	5.65685	.752575	.500000	$\bar{1}$.698970	1.00000	.000000	.353553	$\bar{1}$.548455	90°
5	3.63271	.560231	5.87785	.769219	.594410	$\bar{1}$.774086	1.72048	.235649	.340260	$\bar{1}$.531811	72°
6	3.46410	.539591	6.00000	.778151	.649519	$\bar{1}$.812592	2.59808	.414652	.333333	$\bar{1}$.522879	60°
7	3.37100	.527759	6.07435	.783500	.684103	$\bar{1}$.835122	3.63393	.560377	.329254	$\bar{1}$.517531	51° 25′ 43″
8	3.31371	.520314	6.12294	.786960	.707107	$\bar{1}$.849485	4.82843	.683806	.326641	$\bar{1}$.514070	45°
9	3.27573	.515308	6.15636	.789324	.723136	$\bar{1}$.859220	6.18182	.791117	.324867	$\bar{1}$.511706	40°
10	3.24920	.511776	6.18034	.791012	.734732	$\bar{1}$.866129	7.69421	.886164	.323607	$\bar{1}$.510018	36°
11	3.22989	.509187	6.19811	.792259	.743380	$\bar{1}$.871211	9.36566	.971538	.322679	$\bar{1}$.508771	32° 43′ 38″
12	3.21539	.507234	6.21166	.793207	.750000	$\bar{1}$.875061	11.1962	1.049069	.321975	$\bar{1}$.507823	30°
13	3.20420	.505720	6.22219	.793943	.755173	$\bar{1}$.878047	13.1858	1.120107	.321430	$\bar{1}$.507087	27° 41′ 32″
14	3.19543	.504529	6.23062	.794532	.759293	$\bar{1}$.880410	15.3344	1.185667	.320995	$\bar{1}$.506499	25° 42′ 51″
15	3.18835	.503566	6.23735	.795000	.762631	$\bar{1}$.882315	17.6424	1.246557	.320649	$\bar{1}$.506030	24°
16	3.18260	.502782	6.24289	.795386	.765367	$\bar{1}$.883870	20.1094	1.303398	.320364	$\bar{1}$.505644	22° 30′
17	3.17788	.502138	6.24754	.795709	.767636	$\bar{1}$.885155	22.7353	1.356700	.320126	$\bar{1}$.505321	21° 10′ 35″
18	3.17389	.501591	6.25133	.795973	.769545	$\bar{1}$.886234	25.5208	1.406894	.319932	$\bar{1}$.505057	20°
19	3.17051	.501130	6.25455	.796196	.771166	$\bar{1}$.887148	28.4654	1.454318	.319767	$\bar{1}$.504834	18° 56′ 51″
20	3.16769	.500743	6.25738	.796392	.772543	$\bar{1}$.887922	31.5688	1.499258	.319623	$\bar{1}$.504638	18°
21	3.16523	.500405	6.25975	.796557	.773729	$\bar{1}$.888589	34.8316	1.541974	.319502	$\bar{1}$.504473	17° 8′ 34″
22	3.16317	.500123	6.26195	.796710	.774763	$\bar{1}$.889169	38.2527	1.582663	.319389	$\bar{1}$.504320	16° 21′ 49″
23	3.16129	.499864	6.26369	.796830	.775668	$\bar{1}$.889676	41.8342	1.621532	.319301	$\bar{1}$.504200	15° 39′ 8″
24	3.15966	.499640	6.26526	.796939	.776457	$\bar{1}$.890118	45.5745	1.658722	.319221	$\bar{1}$.504091	15°
25	3.15824	.499444	6.26666	.797036	.777156	$\bar{1}$.890508	49.4738	1.694376	.319149	$\bar{1}$.503994	14° 24′

Symbols and Equations

s = Angle subtended at center by side.
P = Perimeter of polygon.
C = Length of one side.
A = Area of polygon.
N = Number of sides.
d = Diameter of inscribed circle.
D = Diameter of circumscribed circle.



To Find	Knowing: P	A	C	D	d
P		$P = 2\sqrt{FA}$	$P = CN$	$P = \frac{MD}{2}$	$P = Fd$
A	$A = \frac{KP^2}{N^2}$		$A = KC^2$	$A = HD^2$	$A = \frac{Fd^2}{4}$
C	$C = \frac{P}{N}$	$C = \frac{2\sqrt{FA}}{N}$		$C = \frac{MD}{2N}$	$C = \frac{Fd}{N}$
D	$D = BP$	$D = 2B\sqrt{FA}$	$D = NBC$		$D = BFd$
d	$d = \frac{4KP}{N^2}$	$d = \frac{4\sqrt{AK}}{N}$	$d = \frac{4KC}{N}$	$d = \frac{2MKD}{N^2}$	

The following factors are used in the calculations, their values being found in Table 11:

$$F = N \tan \frac{180°}{N}; \quad M = 2N \sin \frac{180°}{N}; \quad H = \frac{N}{8} \sin \frac{360°}{N};$$

$$K = \frac{N}{4} \cot \frac{180°}{N}; \quad B = \frac{1}{N} \operatorname{cosec} \frac{180°}{N}.$$

Table 12.—Diameters and Spacings of Circles With Nearest Whole Number of Divisions
(*Jas. Fraser, Amer. Mach., May* 14, 1908)

Diameter	Distance on circumference: 1/32″	1/16″	1/8″	3/16″	1/4″	5/16″	3/8″	7/16″	1/2″	9/16″	5/8″	11/16″	3/4″	7/8″	1″
1/4	25	12	6												
5/16	31	16	8												
3/8	38	19	9	6											
7/16	44	22	11	7	5										
1/2	50	25	13	8	6										
5/8	63	31	16	10	8	6									
3/4	75	38	19	13	9	8	6								
7/8	88	44	22	15	11	9	7	6							
1	100	50	25	17	13	10	8	7	6						
1/4	126	63	31	21	16	13	10	9	8	7	6				
1/2	150	75	38	25	19	15	13	11	10	8	7				
3/4	176	88	44	29	22	18	15	13	11	10	9	8	7	6	
2	200	100	50	34	25	20	17	14	12	11	10	9	8	7	6
1/4	226	113	56	38	28	23	19	16	14	13	11	10	9	8	7
1/2	251	125	63	42	31	25	21	18	16	14	12	11	10	9	8
3/4	277	138	69	46	35	28	23	20	17	15	14	13	12	10	9
3	302	151	75	50	38	30	25	22	19	17	15	14	13	11	9
1/4	327	163	82	54	41	33	27	23	20	18	16	15	14	12	10
1/2	352	176	88	59	44	35	30	25	22	20	18	16	15	13	11
3/4	378	189	94	63	47	38	31	27	24	21	19	17	16	14	12
4	402	201	100	67	50	40	34	29	25	22	20	18	17	15	13
1/4	428	214	107	71	53	43	36	31	27	24	21	19	18	15	13
1/2	454	227	114	76	57	45	38	32	28	25	23	21	19	16	14
3/4	478	239	119	79	60	48	40	34	30	27	24	22	20	17	15
5	503	252	126	84	63	50	42	36	31	28	25	23	21	18	16
1/4	528	264	132	88	66	53	44	38	33	29	26	24	22	19	16
1/2	554	277	138	92	69	55	46	40	35	31	27	25	23	20	17
3/4	579	289	145	96	73	58	48	41	36	32	28	26	24	21	18
6	604	302	151	101	76	61	50	44	38	34	30	27	25	22	19

Table 13.—Areas of Circular Segments

From Trautwine's Civil Engineer's Pocket Book

To find the area of a segment: Divide the height by the diameter. Opposite the result in this table find the area constant and multiply it by the square of the diameter.

Height	Area	Height	Area	Height	Area	Height	Area	Height	Area	Height	Area	Height	Area
.001	.000042	.073	.025714	.145	.070329	.217	.125634	.289	.188141	.361	.255511	.433	.325900
.002	.000119	.074	.026236	.146	.071034	.218	.126459	.290	.189048	.362	.256472	.434	.326891
.003	.000219	.075	.026761	.147	.071741	.219	.127286	.291	.189956	.363	.257433	.435	.327883
.004	.000337	.076	.027290	.148	.072450	.220	.128114	.292	.190865	.364	.258395	.436	.328874
.005	.000471	.077	.027821	.149	.073162	.221	.128943	.293	.191774	.365	.259358	.437	.329866
.006	.000619	.078	.028356	.150	.073875	.222	.129773	.294	.192685	.366	.260321	.438	.330858
.007	.000779	.079	.028894	.151	.074590	.223	.130605	.295	.193597	.367	.261285	.439	.331851
.008	.000952	.080	.029435	.152	.075307	.224	.131438	.296	.194509	.368	.262249	.440	.332843
.009	.001135	.081	.029979	.153	.076026	.225	.132273	.297	.195423	.369	.263214	.441	.333836
.010	.001329	.082	.030526	.154	.076747	.226	.133109	.298	.196337	.370	.264179	.442	.334829
.011	.001533	.083	.031077	.155	.077470	.227	.133946	.299	.197252	.371	.265145	.443	.335823
.012	.001746	.084	.031630	.156	.078194	.228	.134784	.300	.198168	.372	.266111	.444	.336816
.013	.001969	.085	.032186	.157	.078921	.229	.135624	.301	.199085	.373	.267078	.445	.337810
.014	.002199	.086	.032746	.158	.079650	.230	.136465	.302	.200003	.374	.268046	.446	.338804
.015	.002438	.087	.033308	.159	.080380	.231	.137307	.303	.200922	.375	.269014	.447	.339799
.016	.002685	.088	.033873	.160	.081112	.232	.138151	.304	.201841	.376	.269982	.448	.340793
.017	.002940	.089	.034441	.161	.081847	.233	.138996	.305	.202762	.377	.270951	.449	.341788
.018	.003202	.090	.035012	.162	.082582	.234	.139842	.306	.203683	.378	.271921	.450	.342783
.019	.003472	.091	.035586	.163	.083320	.235	.140689	.307	.204605	.379	.272891	.451	.343778
.020	.003749	.092	.036162	.164	.084060	.236	.141538	.308	.205528	.380	.273861	.452	.344773
.021	.004032	.093	.036742	.165	.084801	.237	.142388	.309	.206452	.381	.274832	.453	.345768
.022	.004322	.094	.037324	.166	.085545	.238	.143239	.310	.207376	.382	.275804	.454	.346764
.023	.004619	.095	.037909	.167	.086290	.239	.144091	.311	.208302	.383	.276776	.455	.347760
.024	.004922	.096	.038497	.168	.087037	.240	.144945	.312	.209228	.384	.277748	.456	.348756
.025	.005231	.097	.039087	.169	.087785	.241	.145800	.313	.210155	.385	.278721	.457	.349752
.026	.005546	.098	.039681	.170	.088536	.242	.146656	.314	.211083	.386	.279695	.458	.350749
.027	.005867	.099	.040277	.171	.089288	.243	.147513	.315	.212011	.387	.280669	.459	.351745
.028	.006194	.100	.040875	.172	.090042	.244	.148371	.316	.212941	.388	.281643	.460	.352742
.029	.006527	.101	.041477	.173	.090797	.245	.149231	.317	.213871	.389	.282618	.461	.353739
.030	.006866	.102	.042081	.174	.091555	.246	.150091	.318	.214802	.390	.283593	.462	.354736
.031	.007209	.103	.042687	.175	.092314	.247	.150953	.319	.215734	.391	.284569	.463	.355733
.032	.007559	.104	.043296	.176	.093074	.248	.151816	.320	.216666	.392	.285545	.464	.356730
.033	.007913	.105	.043908	.177	.093837	.249	.152681	.321	.217600	.393	.286521	.465	.357728
.034	.008273	.106	.044523	.178	.094601	.250	.153546	.322	.218534	.394	.287499	.466	.358725
.035	.008638	.107	.045140	.179	.095367	.251	.154413	.323	.219469	.395	.288476	.467	.359723
.036	.009008	.108	.045759	.180	.096135	.252	.155281	.324	.220404	.396	.289454	.468	.360721
.037	.009383	.109	.046381	.181	.096904	.253	.156149	.325	.221341	.397	.290432	.469	.361719
.038	.009764	.110	.047006	.182	.097675	.254	.157019	.326	.222278	.398	.291411	.470	.362717
.039	.010148	.111	.047633	.183	.098447	.255	.157891	.327	.223216	.399	.292390	.471	.363715
.040	.010538	.112	.048262	.184	.099221	.256	.158763	.328	.224154	.400	.293370	.472	.364714
.041	.010932	.113	.048894	.185	.099997	.257	.159636	.329	.225094	.401	.294350	.473	.365712
.042	.011331	.114	.049529	.186	.100774	.258	.160511	.330	.226034	.402	.295330	.474	.366711
.043	.011734	.115	.050165	.187	.101553	.259	.161386	.331	.226974	.403	.296311	.475	.367710
.044	.012142	.116	.050805	.188	.102334	.260	.162263	.332	.227916	.404	.297292	.476	.368708
.045	.012555	.117	.051446	.189	.103116	.261	.163141	.333	.228858	.405	.298274	.477	.369707
.046	.012971	.118	.052090	.190	.103900	.262	.164020	.334	.229801	.406	.299256	.478	.370706
.047	.013393	.119	.052737	.191	.104686	.263	.164900	.335	.230745	.407	.300238	.479	.371705
.048	.013818	.120	.053385	.192	.105472	.264	.165781	.336	.231689	.408	.301221	.480	.372704
.049	.014248	.121	.054037	.193	.106261	.265	.166663	.337	.232634	.409	.302204	.481	.373704
.050	.014681	.122	.054690	.194	.107051	.266	.167546	.338	.233580	.410	.303187	.482	.374703
.051	.015119	.123	.055346	.195	.107843	.267	.168431	.339	.234526	.411	.304171	.483	.375702
.052	.015561	.124	.056004	.196	.108636	.268	.169316	.340	.235473	.412	.305156	.484	.376702
.053	.016008	.125	.056664	.197	.109431	.269	.170202	.341	.236421	.413	.306140	.485	.377701
.054	.016458	.126	.057327	.198	.110227	.270	.171090	.342	.237369	.414	.307125	.486	.378701
.055	.016912	.127	.057991	.199	.111025	.271	.171978	.343	.238319	.415	.308110	.487	.379701
.056	.017369	.128	.058658	.200	.111824	.272	.172868	.344	.239268	.416	.309096	.488	.380700
.057	.017831	.129	.059328	.201	.112625	.273	.173758	.345	.240219	.417	.310082	.489	.381700
.058	.018297	.130	.059999	.202	.113427	.274	.174650	.346	.241170	.418	.311068	.490	.382700
.059	.018766	.131	.060673	.203	.114231	.275	.175542	.347	.242122	.419	.312055	.491	.383700
.060	.019239	.132	.061349	.204	.115036	.276	.176436	.348	.243074	.420	.313042	.492	.384699
.061	.019716	.133	.062027	.205	.115842	.277	.177330	.349	.244027	.421	.314029	.493	.385699
.062	.020197	.134	.062707	.206	.116651	.278	.178226	.350	.244980	.422	.315017	.494	.386699
.063	.020681	.135	.063389	.207	.117460	.279	.179122	.351	.245935	.423	.316005	.495	.387699
.064	.021168	.136	.064074	.208	.118271	.280	.180020	.352	.246890	.424	.316993	.496	.388699
.065	.021660	.137	.064761	.209	.119084	.281	.180918	.353	.247845	.425	.317981	.497	.389699
.066	.022155	.138	.065449	.210	.119898	.282	.181818	.354	.248801	.426	.318970	.498	.390699
.067	.022653	.139	.066140	.211	.120713	.283	.182718	.355	.249758	.427	.319959	.499	.391699
.068	.023155	.140	.066833	.212	.121530	.284	.183619	.356	.250715	.428	.320949	.500	.392699
.069	.023660	.141	.067528	.213	.122348	.285	.184522	.357	.251673	.429	.321938		
.070	.024168	.142	.068225	.214	.123167	.286	.185425	.358	.252632	.430	.322928		
.071	.024680	.143	.068924	.215	.123988	.287	.186329	.359	.253591	.431	.323919		
.072	.025196	.144	.069626	.216	.124811	.288	.187235	.360	.254551	.432	.324909		

TABLE 14.—ANGLES OF REGULAR POLYGONS

No. of sides	Included angle	Angles at center of circles	Angles for sides of figures
3	120°	30°	30°
4	90°	45°	45°
5	72°	18°—54°	36°—72°
6	60°	30°	30°
8	45°	45°	22° 30′
10	36°	54°—18°	18°—54°
12	30°	60°	15°—45°
14	25° 43′	64° 17′—38° 34′ 12° 51′	12° 51′—38° 34′ 64° 17′
16	22° 30′	67° 30′—45°	11° 15′—33° 45′
18	20°	70°—50°—30° 10°	10°—30°—50° 70°
20	18°	72°—54°	9°—27°—45°
24	15°	75°—60°—45°	7° 30′—22° 30′ 37° 30′

TABLE 15.—CIRCUMFERENTIAL SPEEDS IN FT. PER MIN.
By W. L. TRYON (*Amer. Mach., Dec.* 19, 1912)

Diam., ins.	Revolutions per minute										
	50	100	150	200	250	300	350	400	450	500	550
1	13	26	39	52	65	79	92	105	118	131	144
2	26	52	79	105	131	157	183	209	236	262	288
3	39	79	118	157	196	236	275	314	353	393	432
4	52	105	157	209	262	314	367	419	471	523	576
5	65	131	196	262	328	393	458	524	589	654	720
6	79	157	236	314	393	471	550	628	707	785	863
7	92	183	275	367	458	550	641	733	825	916	1,008
8	105	209	314	419	524	628	733	838	942	1,047	1,152
9	118	236	353	471	589	707	825	942	1,060	1,178	1,296
10	131	262	393	524	655	785	916	1,047	1,178	1,309	1,440
11	144	288	432	576	720	864	1008	1,152	1,296	1,440	1,584
12	157	314	471	628	785	943	1100	1,257	1,414	1,571	1,728
13	170	340	511	681	851	1021	1191	1,361	1,532	1,701	1,872
14	183	367	550	733	916	1100	1283	1,466	1,649	1,832	2,016
15	196	393	589	785	982	1178	1375	1,571	1,767	1,963	2,160
16	209	419	628	838	1047	1257	1466	1,675	1,885	2,094	2,304
17	223	445	668	890	1113	1335	1558	1,780	2,003	2,225	2,244
18	236	471	707	943	1178	1414	1649	1,885	2,121	2,356	2,592
19	249	497	746	995	1244	1492	1741	1,990	2,238	2,487	2,736
20	262	524	785	1047	1309	1571	1833	2,094	2,356	2,618	2,880
21	275	550	825	1100	1374	1649	1924	2,199	2,474	2,749	3,024
22	288	576	864	1152	1440	1728	2016	2,304	2,592	2,880	3,168
23	301	602	903	1204	1505	1806	2107	2,409	2,710	3,011	3,312
24	314	628	943	1257	1571	1885	2199	2,513	2,827	3,142	3,456
25	327	655	982	1309	1636	1963	2291	2,618	2,945	3,273	3,600
26	340	681	1021	1361	1702	2042	2382	2,723	3,063	3,403	3,744
27	353	707	1060	1414	1767	2121	2474	2,827	3,181	3,534	3,888
28	367	733	1100	1466	1837	2199	2566	2,932	3,299	3,665	4,032
29	380	759	1139	1518	1898	2278	2657	3,037	3,417	3,796	4,176
30	393	785	1178	1571	1964	2356	2749	3,142	3,534	3,927	4,320
31	406	812	1217	1623	2029	2435	2840	3,246	3,652	4,058	4,464
32	419	838	1257	1676	2094	2513	2932	3,351	3,770	4,189	4,608
33	432	864	1296	1728	2160	2592	3024	3,456	3,888	4,320	4,752
34	445	890	1335	1780	2225	2670	3115	3,560	4,006	4,451	4,896
35	458	916	1375	1833	2291	2749	3206	3,665	4,123	4,581	5,040
36	471	943	1414	1885	2356	2827	3299	3,770	4,241	4,712	5,184
37	484	969	1453	1937	2422	2906	3390	3,875	4,359	4,843	5,328
38	497	995	1492	1990	2487	2985	3482	3,979	4,477	4,974	5,472
39	511	1021	1532	2042	2553	3063	3573	4,084	4,595	5,105	5,616
40	524	1047	1571	2094	2618	3142	3665	4,189	4,712	5,236	5,760
41	537	1073	1610	2147	2683	3220	3757	4,294	4,831	5,367	5,904
42	550	1100	1649	2199	2749	3299	3848	4,398	4,948	5,498	6,048
43	563	1126	1689	2251	2814	3377	3940	4,503	5,066	5,629	6,192
44	576	1152	1728	2304	2880	3456	4032	4,608	5,184	5,760	6,336
45	589	1178	1767	2356	2945	3534	4123	4,712	5,301	5,891	6,480
46	602	1204	1806	2408	3011	3613	4215	4,817	5,419	6,021	6,623
47	615	1231	1846	2461	3076	3692	4307	4,922	5,537	6,152	6,768
48	628	1257	1885	2513	3142	3770	4398	5,027	5,655	6,283	6,912
49	641	1283	1924	2566	3207	3849	4490	5,131	5,773	6,414	7,056
50	655	1309	1963	2618	3273	3927	4581	5,236	5,891	6,545	7,200
51	668	1335	2003	2670	3338	4006	4673	5,341	6,008	6,676	7,343
52	681	1361	2042	2723	3403	4084	4764	5,445	6,126	6,807	7,487
53	694	1388	2081	2775	3469	4163	4856	5,550	6,244	6,938	7,631
54	707	1414	2121	2827	3534	4241	4948	5,655	6,362	7,069	7,775
55	720	1440	2160	2880	3600	4320	5040	5,760	6,480	7,199	7,919
56	733	1466	2199	2932	3665	4398	5131	5,864	6,597	7,330	8,063
57	746	1492	2238	2985	3731	4477	5223	5,969	6,715	7,461	8,207
58	759	1518	2278	3037	3796	4555	5314	6,074	6,833	7,592	8,351
59	772	1545	2317	3089	3862	4634	5406	6,178	6,951	7,723	8,495
60	785	1571	2356	3142	3927	4712	5498	6,283	7,069	7,854	8,639
61	799	1597	2395	3194	3992	4791	5589	6,388	7,186	7,985	8,783
62	812	1623	2435	3246	4058	4870	5681	6,493	7,304	8,116	8,927
63	825	1649	2474	3299	4123	4948	5773	6,597	7,422	8,247	9,071
64	838	1676	2513	3351	4189	5027	5864	6,702	7,540	8,378	9,215
65	851	1702	2552	3403	4254	5105	5956	6,807	7,658	8,508	9,359
66	864	1728	2592	3456	4320	5184	6040	6,912	7,775	8,640	9,503
67	877	1754	2631	3508	4385	5262	6139	7,016	7,893	8,770	9,647
68	890	1780	2670	3650	4451	5341	6231	7,121	8,011	8,901	9,791
69	903	1806	2710	3613	4516	5419	6322	7,226	8,129	9,032	9,935
70	916	1833	2749	3665	4581	5498	6414	7,330	8,247	9,163	10,079
71	929	1859	2788	3718	4647	5576	6506	7,435	8,365	9,294	10,223
72	943	1885	2827	3770	4712	5655	6597	7,540	8,482	9,425	10,367
73	956	1911	2867	3822	4778	5733	6689	7,644	8,600	9,556	10,511
74	969	1937	2906	3875	4843	5812	6781	7,749	8,718	9,687	10,655
75	982	1964	2945	3927	4909	5890	6872	7,854	8,836	9,818	10,799
76	995	1990	2985	3979	4974	5969	6964	7,959	8,954	9,948	10,943
77	1008	2016	3024	4032	5040	6048	7056	8,063	9,072	10,079	11,087
78	1021	2042	3063	4084	5105	6126	7147	8,168	9,189	10,210	11,231
79	1034	2068	3102	4136	5171	6205	7239	8,273	9,307	10,341	11,375
80	1047	2094	3142	4189	5236	6283	7330	8,378	9,425	10,472	11,519
81	1060	2121	3181	4241	5301	6362	7422	8,482	9,543	10,603	11,663
82	1073	2147	3220	4294	5367	6440	7514	8,587	9,660	10,734	11,807
83	1087	2173	3259	4346	5432	6519	7605	8,692	9,778	10,865	11,951
84	1100	2199	3299	4398	5498	6597	7697	8,797	9,896	10,996	12,095
85	1113	2225	3338	4451	5563	6676	7769	8,901	10,014	11,127	12,239
86	1126	2251	3377	4503	5629	6754	7880	9,006	10,132	11,257	12,383
87	1139	2278	3417	4555	5694	6833	7972	9,111	10,249	11,388	12,527
88	1152	2304	3456	4607	5760	6912	8063	9,215	10,367	11,519	
89	1161	2330	3495	4660	5825	6990	8155	9,320	10,485	11,650	
90	1178	2356	3534	4712	5891	7069	8247	9,425	10,603	11,780	
91	1191	2382	3574	4765	5956	7147	8338	9,530	10,721	11,912	
92	1204	2408	3613	4817	6021	7226	8430	9,634	10,839	12,043	
93	1217	2435	3652	4870	6087	7304	8522	9,739	10,956	12,174	
94	1231	2461	3692	4922	6152	7383	8613	9,844	11,074	12,305	
95	1244	2487	3731	4974	6217	7461	8704	9,948	11,192	12,436	
96	1257	2513	3770	5027	6283	7540	8796	10,053	11,310	12,566	
97	1270	2539	3809	5079	6349	7618	8888	10,158	11,428		
98	1283	2566	3849	5131	6414	7697	8980	10,263	11,545		
99	1296	2592	3888	5183	6480	7775	9071	10,367	11,663		
100	1309	2618	3927	5236	6545	7854	9163	10,472	11,781		

Circumferential speeds for diameters greater than those given in Table 15, can be obtained by adding together the speeds for two diameters whose sum equals that of the diameter for which we require the speed. For example, to find the speed at a 120-in. diameter and 200 r.p.m. the following calculation is readily made:

speed for 100-in. diameter—5236 ft.
speed for 20-in. diameter—1047 ft.

speed for 120-in. diameter—6283 ft.

To interpolate, we can use the values given for speed for 1- to 10 in. diameters, dividing them by 10, 100, 1000, etc., to obtain speeds for tenths, hundredths, thousandths, etc. For instance, if the speed for 550 r.p.m. and 46.186-in. diameter is required, we proceed as follows:

For 46 in. dia. . . . . speed = 6623 ft.
For .1 in. dia. = $\frac{1}{10}$ of 1-in. dia. speed = 14.4 ft.
For .08 in. dia. = $\frac{1}{100}$ of 8-in. dia. speed = 11.5 ft.
For .006 in. dia. = $\frac{1}{1000}$ of 6-in. dia. speed = .9 ft.
For 46.186 in. dia. . . . . speed = 6650 ft.

It is noted that in the above addition the digit in the tenths place, being 8, has been replaced by adding 1 to the units digit, or in other words 6649.8 has been called 6650. This is following the method used in making up the tables, for when the decimal part of the speed was .5, or more, 1 was added, but when it was less than .5, it was dropped entirely.

The same method can be used for interpolating for r.p.m. between those given in the table.

Table 15.—Circumferential Speeds in Ft. per Min.—(*Continued*)
By W. L. Tyron (*Amer. Mach. Dec.* 19, 1912)

Revolutions per minute

Diameter in ins.			2400	2600	2800	3000	3200	3400	3600	3800	4000	4400
			1200	1300	1400	1500	1600	1700	1800.	1900	2000	2200
			600	650	700	750	800	850	900	950	1000	1100
1/4	1/2	1	157	170	183	196	209	223	236	249	262	288
1/2	1	2	314	340	307	393	419	445	471	497	524	576
3/4	1 1/2	3	471	510	550	589	628	668	707	746	785	863
1	2	4	628	681	733	785	838	890	942	995	1,047	1,152
1 1/4	2 1/2	5	785	851	916	982	1,047	1,113	1,178	1,244	1,309	1,440
1 1/2	3	6	942	1,021	1,100	1,178	1,257	1,335	1,414	1,492	1,571	1,728
1 3/4	3 1/2	7	1100	1,191	1,283	1,375	1,466	1,558	1,649	1,741	1,832	2,016
2	4	8	1257	1,361	1,466	1,571	1,675	1,780	1,885	1,990	2,094	2,304
2 1/4	4 1/2	9	1414	1 531	1,649	1,767	1,885	2,003	2,121	2,238	2,356	2,592
2 1/2	5	10	1571	1,702	1,833	1,964	2,094	2,225	2,356	2,487	2,618	2,880
2 3/4	5 1/2	11	1728	1,872	2,016	2,160	2,304	2,448	2,592	2,736	2,880	3,168
3	6	12	1885	2,042	2,199	2,356	2,513	2,670	2,827	2,984	3,143	3,456
3 1/4	6 1/2	13	2042	2,212	2,382	2,552	2,723	2,893	3,063	3,233	3,403	3,744
3 1/2	7	14	2199	2,382	2,566	2,749	2,932	3,115	3,299	3,482	3,665	4,032
3 3/4	7 1/2	15	2356	2,552	2,749	2,945	3,142	3,338	3,534	3,731	3,927	4,320
4	8	16	2513	2,723	2,932	3,142	3,351	3,560	3,770	3,979	4,189	4,608
4 1/4	8 1/2	17	2670	2,893	3,115	3,338	3,560	3,783	4,006	4,228	4,451	4,896
4 1/2	9	18	2827	3,063	3,299	3,534	3,770	4,006	4,241	4,477	4,712	5,184
4 3/4	9 1/2	19	2985	3,233	3,482	3,731	3,979	4,228	4,477	4,725	4,974	5,472
5	10	20	3142	3,403	3,665	3,927	4,189	4,451	4,712	4,974	5,236	5,760
5 1/4	10 1/2	21	3299	3,573	3,848	4,123	4,398	4,673	4,948	5,223	5,498	6,048
5 1/2	11	22	3456	3,744	4,032	4,320	4,608	4,896	5,184	5,472	5,760	6,336
5 3/4	11 1/2	23	3613	3,914	4,215	4,516	4,817	5,118	5,419	5,720	6,021	6,623
6	12	24	3770	4,084	4,398	4,712	5,027	5,341	5,655	5,969	6,283	6,912
6 1/4	12 1/2	25	3927	4,254	4,581	4,909	5,236	5,563	5,891	6,218	6,545	7,200
6 1/2	13	26	4084	4,424	4,764	5,105	5,445	5,786	6,126	6,466	6,807	7,487
6 3/4	13 1/2	27	4241	4,594	4,948	5,301	5,655	6,008	6,362	6,715	7,069	7,775
7	14	28	4398	4,764	5,131	5,498	5,864	6,231	6,597	6,063	7,330	8,063
7 1/4	14 1/2	29	4555	4,935	5,314	5,694	6,074	6,453	6,833	7,213	7,592	8,351
7 1/2	15	30	4712	5,105	5,498	5,890	6,283	6,676	7,069	7,461	7,854	8,639
7 3/4	15 1/2	31	4870	5,275	5,681	6,086	6,493	6,898	7,304	7,710	8,116	8,927
8	16	32	5027	5,445	5,864	6,283	6,702	7,121	7,540	7,959	8,378	9,215
8 1/4	16 1/2	33	5184	5,615	6,048	6,479	6,912	7,343	7,775	8,207	8,640	9,503
8 1/2	17	34	5341	5,785	6,231	6,676	7,121	7,566	8,011	8,456	8,901	9,791
8 3/4	17 1/2	35	5498	5,956	6,414	6,872	7,330	7,789	8,247	8,705	9,163	10,079
9	18	36	5655	6,126	6,597	7,069	7,540	8,011	8,482	8,954	9,425	10,367
9 1/4	18 1/2	37	5812	6,296	6,781	7,265	7,749	8,234	8,718	9,202	9,687	10,655
9 1/2	19	38	5969	6,466	6,964	7,461	7,959	8,456	8,954	9,451	9,948	10,943
9 3/4	19 1/2	39	6126	6,637	7,147	7,658	8,168	8,679	9,189	9,700	10,210	
10	20	40	6283	6,807	7,330	7,854	8,378	8,901	9,425	9,948	10,472	
10 1/4	20 1/2	41	6440	6,977	7,514	8,050	8,587	9,124	9,660	10,197	10,734	
10 1/2	21	42	6597	7,147	7,697	8,247	8,797	9,346	9,896	10,446	10,996	
10 3/4	21 1/2	34	6754	7,317	7,880	8,443	9,006	9,569	10,131	10,695		
11	22	44	6912	7,487	8,063	8,639	9,215	9,791	10,367	10,943		
11 1/4	22 1/2	45	7069	7,658	8,247	8,836	9,425	10,014	10,603			
11 1/2	23	46	7226	7,828	8,430	9,032	9,634	10,236	10,839			
11 3/4	23 1/2	47	7383	7,998	8,613	9,228	9,844	10,459				
12	24	48	7540	8,168	8,797	9,425	10,053	10,681				
12 1/4	24 1/2	49	7697	8,338	8,980	9,621	10,263	10,903				
12 1/2	25	50	7854	8,508	9,163	9,818	10,472					
12 3/4	25 1/2	51	8011	8,679	9,346	10,014	10,681					
13	26	52	8168	8,849	9,529	10,210	10,891					
13 1/4	26 1/2	53	8325	9,019	9,712	10,407						
13 1/2	27	54	8482	9,189	9,896	10,603						
13 3/4	27 1/2	55	8639	9,359	10,079	10,799						
14	28	56	8797	9,530	10,263	10,996						
14 1/4	28 1/2	57	8954	9,700	10,446							
14 1/2	29	58	9111	9,870	10,629							
14 3/4	29 1/2	59	9268	10,040	10,812							
15	30	60	9425	10,210	10,996							

Revolutions per minute

Diameter			2400	2600
			1200	1300
			600	650
15 1/4	30 1/2	61	9,582	10,380
15 1/2	31	62	9,739	10,551
15 3/4	31 1/2	63	9,896	10,721
16	32	64	10,053	10,891
16 1/4	32 1/2	65	10,210	
16 1/2	33	66	10,367	
16 3/4	33 1/2	67	10,524	
17	34	68	10,681	
17 1/4	34 1/2	69	10,839	

Table 16.—Decimal Equivalents of Binary Fractions

1/64	.015625
1/32	.03125
3/64	.046875
1/16	.0625
5/64	.078125
3/32	.09375
7/64	.109375
1/8	.125
9/64	.140625
5/32	.15625
11/64	.171875
3/16	.1875
13/64	.203125
7/32	.21875
15/64	.234375
1/4	.250
17/64	.265625
9/32	.28125
19/64	.296875
5/16	.3125
21/64	.328125
11/32	.34375
23/64	.359375
3/8	.375
25/64	.390625
13/32	.40625
27/64	.421875
7/16	.4375
29/64	.453125
15/32	.46875
31/64	.484375
1/2	.500
33/64	.515625
17/32	.53125
35/64	.546875
9/16	.5625
37/64	.578125
19/32	.59375
39/64	.609375
5/8	.625
41/64	.640625
21/32	.65625
43/64	.671875
11/16	.6875
45/64	.703125
23/32	.71875
47/64	.734375
3/4	.750
49/64	.765625
25/32	.78125
51/64	.796875
13/16	.8125
53/64	.828125
27/32	.84375
55/64	.859375
7/8	.875
57/64	.890625
29/32	.90625
59/64	.921875
15/16	.9375
61/64	.953125
31/32	.96875
63/64	.984375

Table 17.—Decimal Equivalents of Other than Binary Fractions

Fr.	Decimal.	Nearest 64th	Fr.	Decimal.	Nearest 64th	Fr.	Decimal.	Nearest 64th	Fr.	Decimal.	Nearest 64th
1/32	.0313	1/32	3/32	.0938	3/32	4/23	.1739		7/27	.2593	
1/31	.0323		2/21	.0952		3/17	.1765		6/23	.2609	
1/30	.0333		3/31	.0968		5/28	.1785		5/19	.2632	
1/29	.0345		1/10	.1000		2/11	.1818		4/15	.2667	17/64
1/28	.0357		3/29	.1034		5/27	.1852		7/26	.2692	
1/27	.0370		2/19	.1053		3/16	.1875	3/16	3/11	.2727	
1/26	.0385		3/28	.1071		4/21	.1905		8/29	.2759	
1/25	.0400		1/9	.1111	7/64	5/26	.1923		5/18	.2778	
1/24	.0417		3/26	.1154		6/31	.1935		7/25	.2800	
1/23	.0435		2/17	.1176		1/5	.2000	13/64	9/32	.2813	9/32
1/22	.0455		3/25	.1200		6/29	.2069		2/7	.2857	
1/21	.0476	3/64	1/8	.1250	1/8	5/24	.2083		9/31	.2903	
1/20	.0500		4/31	.1290		4/19	.2105		7/24	.2917	
1/19	.0526		3/23	.1304		3/14	.2143		5/17	.2941	
1/18	.0556		2/15	.1333		5/23	.2174		8/27	.2963	19/64
1/17	.0589		3/22	.1364		7/32	.2188	7/32	3/10	.3000	
1/16	.0625	1/16	4/29	.1379		2/9	.2222		7/23	.3043	
2/31	.0645		1/7	.1429	9/64	7/31	.2258		4/13	.3077	
1/15	.0667		4/27	.1481		5/22	.2273		9/29	.3103	
2/29	.0690		3/20	.1500		3/13	.2308		5/16	.3125	5/16
1/14	.0714		2/13	.1538		7/30	.2333	15/64	6/19	.3158	
2/27	.0740		5/32	.1563	5/32	4/17	.2353		7/22	.3182	
1/13	.0769	5/64	3/19	.1579		5/21	.2381		8/25	.3200	
2/25	0800		4/25	.1600		6/25	.2400		9/28	.3214	
1/12	.0833		5/31	.1613		7/29	.2414		10/31	.3226	21/64
2/23	.0870		1/6	.1667		1/4	.2500	1/4	1/3	.3333	
1/11	.0909		5/29	.1724	11/64	8/31	.2580		11/32	.3438	11/32
10/29	.3448		13/31	.4194		16/31	.5161	33/64	17/29	.5862	
9/26	.3462		8/19	.4211	27/64	15/29	.5172		10/17	.5882	
8/23	.3478		11/26	.4231		14/27	.5185		13/22	.5909	
7/20	.3500		3/7	.4286		13/25	.5200		16/27	.5926	
6/17	.3529		13/30	.4333		12/23	.5217		19/32	.5938	19/32
11/31	.3548		10/23	.4348		11/21	.5238		3/5	.6000	
5/14	.3571		7/16	.4375	7/16	10/19	.5263		17/28	.6071	
9/25	.3600	23/64	11/25	.4400		9/17	.5294		14/23	.6087	39/64
4/11	.3636		4/9	.4444		17/32	.5313	17/32	11/18	.6111	
11/30	.3667		13/29	.4483		8/15	.5333		19/31	.6129	
7/19	.3684		9/20	.4500		15/28	.5357		8/13	.6154	
10/27	.3704		14/31	.4516	29/64	7/13	.5385		13/21	.6190	
3/8	.3750	3/8	5/11	.4545		13/24	.5417		18/29	.6207	
11/29	.3793		11/24	.4583		6/11	.5455	35/64	5/8	.6250	5/8
8/21	.3810		6/13	.4615		17/31	.5484		17/27	.6296	
5/13	.3846		13/28	.4642		11/20	.5500		12/19	.6316	
12/31	.3871		7/15	.4667		16/29	.5517		19/30	.6333	
7/18	.3889		15/32	.4688	15/32	5/9	.5556		7/11	.6364	
9/23	.3913	25/64	8/17	.4706		14/25	.5600		16/25	.6400	41/64
11/28	.3929		9/19	.4737		9/16	.5625	9/16	9/14	.6429	
2/5	.4000		10/21	.4762		13/23	.5652		20/31	.6452	
13/32	.4063	13/32	11/23	.4783		17/30	.5667		11/17	.6471	
11/27	.4074		12/25	.4800		4/7	.5714		13/20	.6500	
9/22	.4091		13/27	.4815		15/26	.5769		15/23	.6522	
7/17	.4118		14/29	.4828	31/64	11/19	.5789	37/64	17/26	.6538	
12/29	.4138		15/31	.4839		18/31	.5806		19/29	.6552	
5/12	.4167		1/2	.5000	1/2	7/12	.5833		21/32	.6563	21/32
2/3	.6667	43/64	3/4	.7500	3/4	5/6	.8334		21/23	.9130	
21/31	.6774		22/29	.7586		26/31	.8387		11/12	.9167	
19/28	.6786		19/25	.7600		21/25	.8400		23/25	.9200	
17/25	.6800		16/21	.7619		16/19	.8421		12/13	.9231	59/64
15/22	.6818		13/17	.7647	49/64	27/32	.8438	27/32	25/27	.9259	
13/19	.6842		23/30	.7667		11/13	.8462		13/14	.9286	
11/16	.6875	11/16	10/13	.7692		17/20	.8500		27/29	.9310	
20/29	.6897		17/22	.7727		23/27	.8519		14/15	.9333	
9/13	.6923		24/31	.7742		6/7	.8571	55/64	29/31	.9355	
16/23	.6957		7/9	.7778		25/29	.8621		15/16	.9375	15/16
7/10	.7000		25/32	.7813	25/32	19/22	.8636		16/17	.9412	
19/27	.7037	45/64	19/23	.7826		13/15	.8667		17/18	.9444	
12/17	.7059		11/14	.7857		20/23	.8696		18/19	.9474	
17/24	.7083		15/19	.7895		27/31	.8710	7/8	19/20	.9500	
22/31	.7097		19/24	.7917		7/8	.875c		20/21	.9524	61/64
5/7	.7143		23/29	.7931	51/64	22/25	.8800		21/22	.9545	
23/32	.7188	23/32	4/5	.8000		15/17	.8824		22/23	.9565	
18/25	.7200		25/31	.8065		23/26	.8846		23/24	.9583	
13/18	.7222		21/26	.8077		8/9	.8889		24/25	.9600	
21/29	.7241		17/21	.8095		25/28	.8929	57/64	25/26	.9615	
8/11	.7273		13/16	.8125	13/16	17/19	.8947		26/27	.9630	
19/26	.7308		22/27	.8148		26/29	.8966		27/28	.9643	
11/15	.7333	47/64	9/11	.8182		9/10	.9000		28/29	9655	
14/19	.7368		23/28	.8214		28/31	.9032		29/30	.9667	
17/23	.7391		14/17	.8235		19/21	.9048		30/31	.9678	
20/27	.7407		19/23	.8261		29/32	.9063	29/32	31/32	.9688	31/32
23/31	.7419		24/29	.8276	53/64	10/11	.9091		1	1.0000	

TABLE 18.—DECIMAL EQUIVALENTS OF OTHER THAN BINARY FRACTIONS.

Thirds, sixths, twelfths and twenty-fourths		Sevenths, fourteenths and twenty-eighths	
1/24	.041666	1/28	.035714
1/12	.083333	1/14	.071429
3/24	.125	3/28	.107143
1/6	.166666	1/7	.142857
5/24	.208333	5/28	.178571
3/12	.250	3/14	.214286
7/24	.291666	7/28	.250
1/3	.333333	2/7	.285714
9/24	.375	9/28	.321429
5/12	.416666	5/14	.357143
11/24	.458333	11/28	.392857
3/6	.500	3/7	.428571
13/24	.541666	13/28	.464286
7/12	.583333	7/14	.500
15/24	.625	15/28	.535714
2/3	.666666	4/7	.571429
17/24	.708333	17/28	.607143
9/12	.750	9/14	.642857
19/24	.791666	19/28	.678571
5/6	.833333	5/7	.714286
21/24	.875	21/28	.750
11/12	.916666	11/14	.785714
23/24	.958333	23/28	.821429
1	1.000	6/7	.857143
		25/28	.892857
		13/14	.928571
		27/28	.964286
		1	1.000

TABLE 19.—SURFACES AND VOLUMES OF SPHERES

(From Trautwine's Civil Engineers Pocket-Book)

Diam. ins.	Surface, sq. ins.	Volume, cu. ins.	Diam. ins.	Surface, sq. ins.	Volume, cu. ins.	Diam. ins.	Surface, sq. ins.	Volume, cu. ins.
1/64	.00077		5/32	4.2000	.80939	11/32	17.258	6.7412
1/32	.00307	.00002	3/16	4.4301	.87681	3/8	17.721	7.0144
3/64	.00690	.00005	7/32	4.6664	.94786	13/32	18.190	7.2949
1/16	.01227	.00013	1/4	4.9088	1.0227	7/16	18.666	7.5829
3/32	.02761	.00043	9/32	5.1573	1.1013	15/32	19.147	7.8783
1/8	.04909	.00102	5/16	5.4119	1.1839	1/2	19.635	8.1813
5/32	.07670	.00200	11/32	5.6728	1.2704	17/32	20.129	8.4919
3/16	.11045	.00345	3/8	5.9396	1.3611	9/16	20.629	8.8103
7/32	.15033	.00548	13/32	6.2126	1.4561	19/32	21.135	9.1366
1/4	.19635	.00818	7/16	6.4919	1.5553	5/8	21.648	9.4708
9/32	.24851	.01165	15/32	6.7771	1.6590	21/32	22.166	9.8131
5/16	.30680	.01598	1/2	7.0686	1.7671	11/16	22.691	10.164
11/32	.37123	.02127	17/32	7.3663	1.8799	23/32	23.222	10.522
3/8	.44179	.02761	9/16	7.6699	1.9974	3/4	23.758	10.889
13/32	.51848	.03511	19/32	7.9798	2.1196	25/32	24.302	11.265
7/16	.60132	.04385	5/8	8.2957	2.2468	13/16	24.850	11.649
15/32	.69028	.05393	21/32	8.6180	2.3789	27/32	25.405	12.041
1/2	.78540	.06545	11/16	8.9461	2.5161	7/8	25.967	12.443
17/32	.88664	.07850	23/32	9.2805	2.6586	29/32	26.535	12.853
9/16	.99403	.09319	3/4	9.6211	2.8062	15/16	27.109	13.272
19/32	1.1075	.10960	25/32	9.9678	2.9592	31/32	27.688	13.700
5/8	1.2272	.12783	13/16	10.321	3.1177	3.	28.274	14.137
21/32	1.3530	.14798	27/32	10.680	3.2818	1/16	29.465	15.039
11/16	1.4849	.17014	7/8	11.044	3.4514	1/8	30.680	15.979
23/32	1.6230	.19442	29/32	11.416	3.6270	3/16	31.919	16.957
3/4	1.7671	.22089	15/16	11.793	3.8083	1/4	33.183	17.974
25/32	1.9175	.24967	31/32	12.177	3.9956	5/16	34.472	19.031
13/16	2.0739	.28084	2.	12.566	4.1888	3/8	35.784	20.129
27/32	2.2365	.31451	1/32	12.962	4.3882	7/16	37.122	21.268
7/8	2.4053	.35077	1/16	13.364	4.5939	1/2	38.484	22.449
29/32	2.5802	.38971	3/32	13.772	4.8060	9/16	39.872	23.674
15/16	2.7611	.43143	1/8	14.186	5.0243	5/8	41.283	24.942
31/32	2.9483	.47603	5/32	14.607	5.2493	11/16	42.719	26.254
1.	3.1416	.52360	3/16	15.033	5.4809	3/4	44.179	27.611
1/32	3.3410	.57424	7/32	15.466	5.7190	13/16	45.664	29.016
1/16	3.5466	.62804	1/4	15.904	5.9641	7/8	47.173	30.466
3/32	3.7583	.68511	9/32	16.349	6.2161	15/16	48.708	31.965
1/8	3.9761	.74551	5/16	16.800	6.4751	4.	50.265	33.510

TABLE 19.—SURFACES AND VOLUMES OF SPHERES—(*Continued*)

(From Trautwine's Civil Engineers Pocket-Book)

Diam. ins.	Surface, sq. ins.	Volume, cu. ins.	Diam. ins.	Surface, sq. ins.	Volume, cu. ins.	Diam. ins.	Surface, sq. ins.	Volume, cu. ins.
1/16	51.848	35.106	1/2	415.48	796.33	7/8	1369.0	4763.0
1/8	53.456	36.751	5/8	424.56	822.58	21.	1385.5	4849.1
3/16	55.089	38.448	3/4	433.73	849.40	1/8	1402.0	4936.2
1/4	56.745	40.195	7/8	443.01	876.79	1/4	1418.6	5024.3
5/16	58.427	41.994	12.	452.39	904.78	3/8	1435.4	5113.5
3/8	60.133	43.847	1/8	461.87	933.34	1/2	1452.2	5203.7
7/16	61.863	45.752	1/4	471.44	962.52	5/8	1469.2	5295.1
1/2	63.617	47.713	3/8	481.11	992.28	3/4	1486.2	5387.4
9/16	65.397	49.729	1/2	490.87	1022.7	7/8	1503.3	5480.8
5/8	67.201	51.801	5/8	500.73	1053.6	22.	1520.5	5575.3
11/16	69.030	53.929	3/4	510.71	1085.3	1/8	1537.9	5670.8
3/4	70.883	56.116	7/8	520.77	1117.5	1/4	1555.3	5767.6
13/16	72.759	58.359	13.	530.93	1150.3	3/8	1572.8	5865.2
7/8	74.663	60.663	1/8	541.19	1183.8	1/2	1590.4	5964.1
15/16	76.589	63.026	1/4	551.55	1218.0	5/8	1608.2	6064.1
5.	78.540	65.450	3/8	562.00	1252.7	3/4	1626.0	6165.2
1/16	80.516	67.935	1/2	572.55	1288.3	7/8	1643.9	6267.3
1/8	82.516	70.482	5/8	583.20	1324.4	23.	1661.9	6370.6
3/16	84.541	73.092	3/4	593.95	1361.2	1/8	1680.0	6475.0
1/4	86.591	75.767	7/8	604.80	1398.6	1/4	1698.2	6580.6
5/16	88.664	78.505	14.	615.75	1436.8	3/8	1716.5	6687.3
3/8	90.763	81.308	1/8	626.80	1475.6	1/2	1735.0	6795.2
7/16	92.887	84.178	1/4	637.95	1515.1	5/8	1753.5	6904.2
1/2	95.033	87.113	3/8	649.17	1555.3	3/4	1772.1	7014.3
9/16	97.205	90.118	1/2	660.52	1596.3	7/8	1790.8	7125.6
5/8	99.401	93.189	5/8	671.95	1637.9	24.	1809.6	7238.2
11/16	101.62	96.331	3/4	683.49	1680.3	1/8	1828.5	7351.9
3/4	103.87	99.541	7/8	695.13	1723.3	1/4	1847.5	7466.7
13/16	106.14	102.82	15.	706.85	1767.2	3/8	1866.6	7583.0
7/8	108.44	106.18	1/8	718.69	1811.7	1/2	1885.8	7700.1
15/16	110.75	109.60	1/4	730.63	1857.0	5/8	1905.1	7818.6
6.	113.10	113.10	3/8	742.65	1903.0	3/4	1924.4	7938.3
1/8	117.87	120.31	1/2	754.77	1949.8	7/8	1943.9	8059.2
1/4	122.72	127.83	5/8	767.00	1997.4	25.	1963.5	8181.3
3/8	127.68	135.66	3/4	779.32	2045.7	1/8	1983.2	8304.7
1/2	132.73	143.79	7/8	791.73	2094.8	1/4	2002.9	8429.2
5/8	137.89	152.25	16.	804.25	2144.7	3/8	2022.9	8554.9
3/4	143.14	161.03	1/8	816.85	2195.3	1/2	2042.8	8682.0
7/8	148.49	170.14	1/4	829.57	2246.8	5/8	2062.9	8810.3
7.	153.94	179.59	3/8	842.40	2299.1	3/4	2083.0	8939.9
1/8	159.49	189.39	1/2	855.29	2352.1	7/8	2103.4	9070.6
1/4	165.13	199.53	5/8	868.31	2406.0	26.	2123.7	9202.8
3/8	170.87	210.03	3/4	881.42	2460.6	1/8	2144.2	9336.2
1/2	176.71	220.89	7/8	894.63	2516.1	1/4	2164.7	9470.8
5/8	182.66	232.13	17.	907.93	2572.4	3/8	2185.5	9606.7
3/4	188.69	243.73	1/8	921.33	2629.6	1/2	2206.2	9744.0
7/8	194.83	255.72	1/4	934.83	2687.6	5/8	2227.1	9882.5
8.	201.06	268.08	3/8	948.43	2746.5	3/4	2248.0	10022
1/8	207.39	280.85	1/2	962.12	2806.2	7/8	2269.1	10164
1/4	213.82	294.01	5/8	975.91	2866.8	27.	2290.2	10306
3/8	220.36	307.58	3/4	989.80	2928.2	1/8	2311.5	10450
1/2	226.98	321.56	7/8	1003.8	2990.5	1/4	2332.8	10595
5/8	233.71	335.95	18.	1017.9	3053.6	3/8	2354.3	10741
3/4	240.53	350.77	1/8	1032.1	3117.7	1/2	2375.8	10889
7/8	247.45	366.02	1/4	1046.4	3182.6	5/8	2397.5	11038
9.	254.47	381.70	3/8	1060.8	3248.5	3/4	2419.2	11189
1/8	261.59	397.83	1/2	1075.2	3315.3	7/8	2441.1	11341
1/4	268.81	414.41	5/8	1089.8	3382.9	28.	2463.0	11494
3/8	276.12	431.44	3/4	1104.5	3451.5	1/8	2485.1	11649
1/2	283.53	448.92	7/8	1119.3	3521.0	1/4	2507.2	11805
5/8	291.04	466.87	19.	1134.1	3591.4	3/8	2529.5	11962
3/4	298.65	485.31	1/8	1149.1	3662.8	1/2	2551.8	12121
7/8	306.36	504.21	1/4	1164.2	3735.0	5/8	2574.3	12281
10.	314.16	523.60	3/8	1179.3	3808.2	3/4	2596.7	12443
1/8	322.06	543.48	1/2	1194.6	3882.5	7/8	2619.4	12606
1/4	330.06	563.86	5/8	1210.0	3957.6	29.	2642.1	12770
3/8	338.16	584.74	3/4	1225.4	4033.7	1/8	2665.0	12936
1/2	346.36	606.13	7/8	1241.0	4110.8	1/4	2687.8	13103
5/8	354.66	628.04	20.	1256.7	4188.8	3/8	2710.9	13272
3/4	363.05	650.46	1/8	1272.4	4267.8	1/2	2734.0	13442
7/8	371.54	673.42	1/4	1288.3	4347.8	5/8	2757.3	13614
11.	380.13	696.91	3/8	1304.2	4428.8	3/4	2780.5	13787
1/8	388.83	720.95	1/2	1320.3	4510.9	7/8	2804.0	13961
1/4	397.61	745.51	5/8	1336.4	4593.9	30.	2827.4	14137
3/8	406.49	770.64	3/4	1352.7	4677.9			

TABLE 20.—SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, RECIPROCALS, CIRCUMFERENCES AND CIRCULAR AREAS

No.	Square	Cube	Sq. root	Cu. root	1000× recip.	No. = Dia. Circum.	No. = Dia. Area	No.	Square	Cube	Sq. root	Cu. root	1000× recip.	No. = Dia. Circum.	No. = Dia. Area
1	1	1	1.0000	1.0000	1000.000	3.142	.7854	70	4900	343,000	8.3666	4.1213	14.2857	219.91	3848.45
2	4	8	1.4142	1.2599	500.000	6.283	3.1416	71	5041	357,911	8.4261	4.1408	14.0845	223.05	3959.19
3	9	27	1.7321	1.4422	333.333	9.524	7.0686	72	5184	373,248	8.4853	4.1602	13.8889	226.19	4071.50
4	16	64	2.0000	1.5874	250.000	12.566	12.5664	73	5329	389,017	8.5440	4.1793	13.6986	229.34	4185.39
5	25	125	2.2361	1.7100	200.000	15.708	19.6350	74	5476	405,224	8.6023	4.1983	13.5135	232.48	4300.84
6	36	216	2.4495	1.8171	166.667	18.850	28.2743	75	5625	421,875	8.6603	4.2172	13.3333	235.62	4417.86
7	49	343	2.6458	1.9129	142.857	21.991	38.4845	76	5776	438,976	8.7178	4.2358	13.1579	238.76	4536.46
8	64	512	2.8284	2.0000	125.000	25.133	50.2655	77	5929	456,533	8.7750	4.2543	12.9870	241.90	4656.63
9	81	729	3.0000	2.0801	111.111	28.274	63.6173	78	6084	474,552	8.8318	4.2727	12.8205	245.04	4778.36
10	100	1,000	3.1623	2.1544	100.000	31.416	78.5398	79	6241	493,039	8.8882	4.2908	12.6582	248.19	4901.67
11	121	1,331	3.3166	2.2240	90.9091	34.558	95.0332	80	6400	512,000	8.9443	4.3089	12.5000	251.33	5026.55
12	144	1,728	3.4641	2.2894	83.3333	37.699	113.097	81	6561	531,441	9.0000	4.3267	12.3457	254.47	5153.00
13	169	2,197	3.6056	2.3513	76.9231	40.841	132.732	82	6724	551,368	9.0554	4.3445	12.1951	257.61	5281.02
14	196	2,744	3.7417	2.4101	71.4286	43.982	153.938	83	6889	571,787	9.1104	4.3621	12.0482	260.75	5410.61
15	225	3,375	3.8730	2.4662	66.6667	47.124	176.715	84	7056	592,704	9.1652	4.3795	11.9048	263.89	5541.77
16	256	4,096	4.0000	2.5198	62.5000	50.265	201.062	85	7225	614,125	9.2195	4.3968	11.7647	267.04	5674.50
17	289	4,913	4.1231	2.5713	58.8235	53.407	226.980	86	7396	636,056	9.2736	4.4140	11.6279	270.18	5808.80
18	324	5,832	4.2426	2.6207	55.5556	56.549	254.469	87	7569	658,503	9.3274	4.4310	11.4943	273.32	5944.68
19	361	6,859	4.3589	2.6684	52.6316	59.690	283.529	88	7744	681,472	9.3808	4.4480	11.3636	276.46	6082.12
20	400	8,000	4.4721	2.7144	50.0000	62.832	314.159	89	7921	704,969	9.4340	4.4647	11.2360	279.60	6221.14
21	441	9,261	4.5826	2.7589	47.6190	65.973	346.361	90	8100	729,000	9.4868	4.4814	11.1111	282.74	6361.73
22	484	10,648	4.6904	2.8020	45.4545	69.115	380.133	91	8281	753,571	9.5394	4.4979	10.9890	285.88	6503.88
23	529	12,167	4.7958	2.8439	43.4783	72.257	415.476	92	8464	778,688	9.5917	4.5144	10.8696	289.03	6647.61
24	576	13,824	4.8990	2.8845	41.6667	75.398	452.389	93	8649	804,357	9.6437	4.5307	10.7527	292.17	6792.91
25	625	15,625	5.0000	2.9240	40.0000	78.540	490.874	94	8836	830,584	9.6954	4.5468	10.6383	295.31	6939.78
26	676	17,576	5.0990	2.9625	38.4615	81.681	530.929	95	9025	857,375	9.7468	4.5629	10.5263	298.45	7088.22
27	729	19,683	5.1962	3.0000	37.0370	84.823	572.555	96	9216	884,736	9.7980	4.5789	10.4167	301.59	7238.23
28	784	21,952	5.2915	3.0366	35.7143	87.965	615.752	97	9409	912,673	9.8489	4.5947	10.3093	304.73	7389.81
29	841	24,389	5.3852	3.0723	34.4828	91.106	660.520	98	9604	941,192	9.8995	4.6104	10.2041	307.88	7542.96
30	900	27,000	5.4772	3.1072	33.3333	94.248	706.858	99	9801	970,299	9.9499	4.6261	10.1010	311.02	7697.69
31	961	29,791	5.5678	3.1414	32.2581	97.389	754.768	100	10,000	1,000,000	10.0000	4.6416	10.0000	314.16	7,853.98
32	1024	32,768	5.6569	3.1748	31.2500	100.531	804.248	101	10,201	1,030,301	10.0499	4.6570	9.90099	317.30	8,011.85
33	1089	35,937	5.7446	3.2075	30.3030	103.673	855.299	102	10,404	1,061,208	10.0995	4.6723	9.80392	320.44	8,171.28
34	1156	39,304	5.8310	3.2396	29.4118	106.814	907.920	103	10,609	1,092,727	10.1489	4.6875	9.70874	323.58	8,332.29
35	1225	42,875	5.9161	3.2711	28.5714	109.956	962.113	104	10,816	1,124,864	10.1980	4.7027	9.61538	326.73	8,494.87
36	1296	46,656	6.0000	3.3019	27.7778	113.097	1017.88	105	11,025	1,157,625	10.2470	4.7177	9.52381	329.87	8,659.01
37	1369	50,653	6.0828	3.3322	27.0270	116.239	1075.21	106	11,236	1,191,016	10.2956	4.7326	9.43396	333.01	8,824.73
38	1444	54,872	6.1644	3.3620	26.3158	119.381	1134.11	107	11,449	1,225,043	10.3441	4.7475	9.34579	336.15	8,992.02
39	1521	59,319	6.2450	3.3912	25.6410	122.522	1194.59	108	11,664	1,259,712	10.3923	4.7622	9.25926	339.29	9,160.88
40	1600	64,000	6.3246	3.4200	25.0000	125.66	1256.64	109	11,881	1,295,029	10.4403	4.7769	9.17431	342.43	9,331.32
41	1681	68,921	6.4031	3.4482	24.3902	128.81	1320.25	110	12,100	1,331,000	10.4881	4.7914	9.09091	345.58	9,503.32
42	1764	74,088	6.4807	3.4760	23.8095	131.95	1385.44	111	12,321	1,367,631	10.5357	4.8059	9.00901	348.72	9,676.89
43	1849	79,507	6.5574	3.5034	23.2558	135.09	1452.20	112	12,544	1,404,928	10.5830	4.8203	8.92857	351.86	9,852.03
44	1936	85,184	6.6332	3.5303	22.7273	138.23	1520.53	113	12,769	1,442,897	10.6301	4.8346	8.84956	355.00	10,028.7
45	2025	91,125	6.7082	3.5569	22.2222	141.37	1590.43	114	12,996	1,481,544	10.6771	4.8488	8.77193	358.14	10,207.0
46	2116	97,336	6.7823	3.5830	21.7391	144.51	1661.90	115	13,225	1,520,875	10.7238	4.8629	8.69565	361.28	10,386.9
47	2209	103,823	6.8557	3.6088	21.2766	147.65	1734.94	116	13,456	1,560,896	10.7703	4.8770	8.62069	364.42	10,568.3
48	2304	110,592	6.9282	3.6342	20.8333	150.80	1809.56	117	13,689	1,601,613	10.8167	4.8910	8.54701	367.57	10,751.3
49	2401	117,649	7.0000	3.6593	20.4082	153.94	1885.74	118	13,924	1,643,032	10.8628	4.9049	8.47458	370.71	10,935.9
50	2500	125,000	7.0711	3.6840	20.0000	157.08	1963.50	119	14,161	1,685,159	10.9087	4.9187	8.40336	373.85	11,122.0
51	2601	132,651	7.1414	3.7084	19.6078	160.22	2042.82	120	14,400	1,728,000	10.9545	4.9324	8.33333	376.99	11,309.7
52	2704	140,608	7.2111	3.7325	19.2308	163.36	2123.72	121	14,641	1,771,561	11.0000	4.9461	8.26446	380.13	11,499.0
53	2809	148,877	7.2801	3.7563	18.8679	166.50	2206.18	122	14,884	1,815,848	11.0454	4.9597	8.19672	383.27	11,689.9
54	2916	157,464	7.3485	3.7798	18.5185	169.65	2290.22	123	15,129	1,860,867	11.0905	4.9732	8.13008	386.42	11,882.3
55	3025	166,375	7.4162	3.8030	18.1818	172.79	2375.83	124	15,376	1,906,624	11.1355	4.9866	8.06452	389.56	12,076.3
56	3136	175,616	7.4833	3.8259	17.8571	175.93	2463.01	125	15,625	1,953,125	11.1803	5.0000	8.00000	392.70	12,271.8
57	3249	185,193	7.5498	3.8485	17.5439	179.07	2551.76	126	15,876	2,000,376	11.2250	5.0133	7.93651	395.84	12,469.0
58	3364	195,112	7.6158	3.8709	17.2414	182.21	2642.08	127	16,129	2,048,383	11.2694	5.0265	7.87402	398.98	12,667.7
59	3481	205,379	7.6811	3.8930	16.9492	185.35	2733.97	128	16,384	2,097,152	11.3137	5.0397	7.81250	402.12	12,868.0
60	3600	216,000	7.7460	3.9149	16.6667	188.50	2827.43	129	16,641	2,146,689	11.3578	5.0528	7.75194	405.27	13,069.8
61	3721	226,981	7.8102	3.9365	16.3934	191.64	2922.47	130	16,900	2,197,000	11.4018	5.0658	7.69231	408.41	13,273.2
62	3844	238,328	7.8740	3.9579	16.1290	194.78	3019.07	131	17,161	2,248,091	11.4455	5.0788	7.63359	411.55	13,478.2
63	3969	250,047	7.9373	3.9791	15.8730	197.92	3117.25	132	17,424	2,299,968	11.4891	5.0916	7.57576	414.69	13,684.8
64	4096	262,144	8.0000	4.0000	15.6250	201.06	3216.99	133	17,689	2,352,637	11.5326	5.1045	7.51880	417.83	13,892.9
65	4225	274,625	8.0623	4.0207	15.3846	204.20	3318.31	134	17,956	2,406,104	11.5758	5.1172	7.46269	420.97	14,102.6
66	4356	287,496	8.1240	4.0412	15.1515	207.35	3421.19	135	18,225	2,460,375	11.6190	5.1299	7.40741	424.12	14,313.9
67	4489	300,763	8.1854	4.0615	14.9254	210.49	3525.65	136	18,496	2,515,456	11.6619	5.1426	7.35294	427.26	14,526.7
68	4624	314,432	8.2462	4.0817	14.7059	213.63	3631.68	137	18,769	2,571,353	11.7047	5.1551	7.29927	430.40	14,741.1
69	4761	328,509	8.3066	4.1016	14.4928	216.77	3739.28	138	19,044	2,628,072	11.7473	5.1676	7.24638	433.54	14,957.1

TABLE 20.—SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, RECIPROCALS, CIRCUMFERENCES AND CIRCULAR AREAS—(*Continued*)

No.	Square	Cube	Sq. root	Cu. root	1000× recip.	No. = Dia. Circum.	No. = Dia. Area
139	19,321	2,685,619	11.7898	5.1801	7.19424	436.68	15,174.7
140	19,600	2,744,000	11.8322	5.1925	7.14286	439.82	15,393.8
141	19,881	2,803,221	11.8743	5.2048	7.09220	442.96	15,614.5
142	20,164	2,863,288	11.9164	5.2171	7.04255	446.11	15,836.8
143	20,449	2,924,207	11.9583	5.2293	6.99301	449.25	16,060.6
144	20,736	2,985,984	12.0000	5.2415	6.94444	452.39	16,286.0
145	21,025	3,048,625	12.0416	5.2536	6.89655	455.53	16,513.0
146	21,316	3,112,136	12.0830	5.2656	6.84932	458.67	16,741.5
147	21,609	3,176,523	12.1244	5.2776	6.80272	461.81	16,971.7
148	21,904	3,241,792	12.1655	5.2896	6.75676	464.96	17,203.4
149	22,201	3,307,949	12.2066	5.3015	6.71141	468.10	17,436.6
150	22,500	3,375,000	12.2474	5.3133	6.66667	471.24	17,671.5
151	22,801	3,442,951	12.2882	5.3251	6.62252	474.38	17,907.9
152	23,104	3,511,808	12.3288	5.3368	6.57895	477.52	18,145.8
153	23,409	3,581,577	12.3693	5.3485	6.53595	480.66	18,385.4
154	23,716	3,652,264	12.4097	5.3601	6.49351	483.81	18,626.5
155	24,025	3,723,875	12.4499	5.3717	6.45161	486.95	18,869.2
156	24,336	3,796,416	12.4900	5.3832	6.41026	490.09	19,113.4
157	24,649	3,869,893	12.5300	5.3947	6.36943	493.23	19,359.3
158	24,964	3,944,312	12.5698	5.4061	6.32911	496.37	19,606.7
159	25,281	4,019,679	12.6095	5.4175	6.28931	499.51	19,855.7
160	25,600	4,096,000	12.6491	5.4288	6.25000	502.65	20,106.2
161	25,921	4,173,281	12.6886	5.4401	6.21118	505.80	20,358.3
162	26,244	4,251,528	12.7279	5.4514	6.17284	508.94	20,612.0
163	26,569	4,330,747	12.7671	5.4626	6.13497	512.08	20,867.2
164	26,896	4,410,944	12.8062	5.4737	6.09756	515.22	21,124.1
165	27,225	4,492,125	12.8452	5.4848	6.06061	518.36	21,382.5
166	27,556	4,574,296	12.8841	5.4959	6.02410	521.50	21,642.4
167	27,889	4,657,463	12.9228	5.5069	5.98802	524.65	21,904.0
168	28,224	4,741,632	12.9615	5.5178	5.95238	527.79	22,167.1
169	28,561	4,826,809	13.0000	5.5288	5.91716	530.93	22,431.8
170	28,900	4,913,000	13.0384	5.5397	5.88235	534.07	22,698.0
171	29,241	5,000,211	13.0767	5.5505	5.84795	537.21	22,965.8
172	29,584	5,088,448	13.1149	5.5613	5.81395	540.35	23,235.2
173	29,929	5,177,717	13.1529	5.5721	5.78035	543.50	23,506.2
174	30,276	5,268,024	13.1909	5.5828	5.74713	546.64	23,778.7
175	30,625	5,359,375	13.2288	5.5934	5.71429	549.78	24,052.8
176	30,976	5,451,776	13.2665	5.6041	5.68182	552.92	24,328.5
177	31,329	5,545,233	13.3041	5.6147	5.64972	556.06	24,605.7
178	31,684	5,639,752	13.3417	5.6252	5.61798	559.20	24,884.6
179	32,041	5,735,339	13.3791	5.6357	5.58659	562.35	25,164.9
180	32,400	5,832,000	13.4164	5.6462	5.55556	565.49	25,446.9
181	32,761	5,929,741	13.4536	5.6567	5.52486	568.63	25,730.4
182	33,124	6,028,568	13.4907	5.6671	5.49451	571.77	26,015.5
183	33,489	6,128,487	13.5277	5.6774	5.46448	574.91	26,302.2
184	33,856	6,229,504	13.5647	5.6877	5.43478	578.05	26,590.4
185	34,225	6,331,625	13.6015	5.6980	5.40541	581.19	26,880.3
186	34,596	6,434,856	13.6382	5.7083	5.37634	584.34	27,171.6
187	34,969	6,539,203	13.6748	5.7185	5.34759	587.48	27,464.6
188	35,344	6,644,672	13.7113	5.7287	5.31915	590.62	27,759.1
189	35,721	6,751,269	13.7477	5.7388	5.29101	593.76	28,055.2
190	36,100	6,859,000	13.7840	5.7489	5.26316	596.90	28,352.9
191	36,481	6,967,871	13.8203	5.7590	5.23560	600.04	28,652.1
192	36,864	7,077,888	13.8564	5.7690	5.20833	603.19	28,952.9
193	37,249	7,189,057	13.8924	5.7790	5.18135	606.33	29,255.3
194	37,636	7,301,384	13.9284	5.7890	5.15464	609.47	29,559.2
195	38,025	7,414,875	13.9642	5.7989	5.12821	612.61	29,864.8
196	38,416	7,529,536	14.0000	5.8088	5.10204	615.75	30,171.9
197	38,809	7,645,373	14.0357	5.8186	5.07614	618.89	30,480.5
198	39,204	7,762,392	14.0712	5.8285	5.05051	622.04	30,790.7
199	39,601	7,880,599	14.1067	5.8383	5.02513	625.18	31,102.6
200	40,000	8,000,000	14.1421	5.8480	5.00000	628.32	31,415.9
201	40,401	8,120,601	14.1774	5.8578	4.97512	631.46	31,730.9
202	40,804	8,242,408	14.2127	5.8675	4.95050	634.60	32,047.4
203	41,209	8,365,427	14.2478	5.8771	4.92611	637.74	32,365.5
204	41,616	8,489,664	14.2829	5.8868	4.90196	640.89	32,685.1
205	42,025	8,615,125	14.3178	5.8964	4.87805	644.03	33,006.4
206	42,436	8,741,816	14.3527	5.9059	4.85437	647.17	33,329.2
207	42,849	8,869,743	14.3875	5.9155	4.83092	650.31	33,653.5

No.	Square	Cube	Sq. root	Cu. root	1000× recip.	No. = Dia. Circum.	No. = Dia. Area
208	43,264	8,998,912	14.4222	5.9250	4.80769	653.45	33,979.5
209	43,681	9,129,329	14.4568	5.9345	4.78469	656.59	34,307.0
210	44,100	9,261,000	14.4914	5.9439	4.76190	659.73	34,636.1
211	44,521	9,393,931	14.5258	5.9533	4.73934	662.88	34,966.7
212	44,944	9,528,128	14.5602	5.9627	4.71698	666.02	35,298.9
213	45,369	9,663,597	14.5945	5.9721	4.69484	669.16	35,632.7
214	45,796	9,800,344	14.6287	5.9814	4.67290	672.30	35,968.1
215	46,225	9,938,375	14.6629	5.9907	4.65116	675.44	36,305.0
216	46,656	10,077,696	14.6969	6.0000	4.62963	678.58	36,643.5
217	47,089	10,218,313	14.7309	6.0092	4.60829	681.73	36,983.6
218	47,524	10,360,232	14.7648	6.0185	4.58716	684.87	37,325.3
219	47,961	10,503,459	14.7986	6.0277	4.56621	688.01	37,668.5
220	48,400	10,648,000	14.8324	6.0368	4.54545	691.15	38,013.3
221	48,841	10,793,861	14.8661	6.0459	4.52489	694.29	38,359.6
222	49,284	10,941,048	14.8997	6.0550	4.50450	697.43	38,707.6
223	49,729	11,089,567	14.9332	6.0641	4.48431	700.58	39,057.1
224	50,176	11,239,424	14.9666	6.0732	4.46429	703.72	39,408.1
225	50,625	11,390,625	15.0000	6.0822	4.44444	706.86	39,760.8
226	51,076	11,543,176	15.0333	6.0912	4.42478	710.00	40,115.0
227	51,529	11,697,083	15.0665	6.1002	4.40529	713.14	40,470.8
228	51,984	11,852,352	15.0997	6.1091	4.38596	716.28	40,828.1
229	52,441	12,008,989	15.1327	6.1180	4.36681	719.42	41,187.1
230	52,900	12,167,000	15.1658	6.1269	4.34783	722.57	41,547.6
231	53,361	12,326,391	15.1987	6.1358	4.32900	725.71	41,909.6
232	53,824	12,487,168	15.2315	6.1446	4.31034	728.85	42,273.3
233	54,289	12,649,337	15.2643	6.1534	4.29185	731.99	42,638.5
234	54,756	12,812,904	15.2971	6.1622	4.27350	735.13	43,005.3
235	55,225	12,977,875	15.3297	6.1710	4.25532	738.27	43,373.6
236	55,696	13,144,256	15.3623	6.1797	4.23729	741.42	43,743.5
237	56,169	13,312,053	15.3948	6.1885	4.21941	744.56	44,115.0
238	56,644	13,481,272	15.4272	6.1972	4.20168	747.70	44,488.1
239	57,121	13,651,919	15.4596	6.2058	4.18410	750.84	44,862.7
240	57,600	13,824,000	15.4919	6.2145	4.16667	753.98	45,238.9
241	58,081	13,997,521	15.5242	6.2231	4.14938	757.12	45,616.7
242	58,564	14,172,488	15.5563	6.2317	4.13223	760.27	45,996.1
243	59,049	14,348,907	15.5885	6.2403	4.11523	763.41	46,377.0
244	59,536	14,526,784	15.6205	6.2488	4.09836	766.55	46,759.5
245	60,025	14,706,125	15.6525	6.2573	4.08163	769.69	47,143.5
246	60,516	14,886,936	15.6844	6.2658	4.06504	772.83	47,529.2
247	61,009	15,069,223	15.7162	6.2743	4.04858	775.97	47,916.4
248	61,504	15,252,992	15.7480	6.2828	4.03226	779.12	48,305.1
249	62,001	15,438,249	15.7797	6.2912	4.01606	782.26	48,695.5
250	62,500	15,625,000	15.8114	6.2996	4.00000	785.40	49,087.4
251	63,001	15,813,251	15.8430	6.3080	3.98406	788.54	49,480.9
252	63,504	16,003,008	15.8745	6.3164	3.96825	791.68	49,875.9
253	64,009	16,194,277	15.9060	6.3247	3.95257	794.82	50,272.6
254	64,516	16,387,064	15.9374	6.3330	3.93701	797.96	50,670.7
255	65,025	16,581,375	15.9687	6.3413	3.92157	801.11	51,070.5
256	65,536	16,777,216	16.0000	6.3496	3.90625	804.25	51,471.9
257	66,049	16,974,593	16.0312	6.3579	3.89105	807.39	51,874.8
258	66,564	17,173,512	16.0624	6.3661	3.87597	810.53	52,279.2
259	67,081	17,373,979	16.0935	6.3743	3.86100	813.67	52,685.3
260	67,600	17,576,000	16.1245	6.3825	3.84615	816.81	53,092.9
261	68,121	17,779,581	16.1555	6.3907	3.83142	819.96	53,502.1
262	68,644	17,984,728	16.1864	6.3988	3.81679	823.10	53,912.9
263	69,169	18,191,447	16.2173	6.4070	3.80228	826.24	54,325.2
264	69,696	18,399,744	16.2481	6.4151	3.78788	829.38	54,739.1
265	70,225	18,609,625	16.2788	6.4232	3.77358	832.52	55,154.6
266	70,756	18,821,096	16.3095	6.4312	3.75940	835.66	55,571.6
267	71,289	19,034,163	16.3401	6.4393	3.74532	838.81	55,990.3
268	71,824	19,248,832	16.3707	6.4473	3.73134	841.95	56,410.4
269	72,361	19,465,109	16.4012	6.4553	3.71747	845.09	56,832.2
270	72,900	19,683,000	16.4317	6.4633	3.70370	848.23	57,255.5
271	73,441	19,902,511	16.4621	6.4713	3.69004	851.37	57,680.4
272	73,984	20,123,648	16.4924	6.4792	3.67647	854.51	58,106.9
273	74,529	20,346,417	16.5227	6.4872	3.66300	857.66	58,534.9
274	75,076	20,570,824	16.5529	6.4951	3.64964	860.80	58,964.6
275	75,625	20,796,875	16.5831	6.5030	3.63636	863.94	59,395.7
276	76,176	21,024,576	16.6132	6.5108	3.62319	867.08	59,828.5

TABLE 20.—SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, RECIPROCALS, CIRCUMFERENCES AND CIRCULAR AREAS—(*Continued*)

No.	Square	Cube	Sq. root	Cu. root	1000× recip.	No. = Dia.		No.	Square	Cube	Sq. root	Cu. root	1000× recip.	No. = Dia.	
						Circum.	Area							Circum.	Area
277	76,729	21,253,933	16.6433	6.5187	3.61011	870.22	60,262.8	346	119,716	41,421,736	18.6011	7.0203	2.89017	1087.0	94,024.7
278	77,284	21,484,952	16.6733	6.5265	3.59712	873.36	60,698.7	347	120,409	41,781,923	18.6279	7.0271	2.88184	1090.1	94,569.0
279	77,841	21,717,639	16.7033	6.5343	3.58423	876.50	61,136.2	348	121,104	42,144,192	18.6548	7.0338	2.87356	1093.3	95,114.9
280	74,400	21,952,000	16.7332	6.5421	3.57143	879.65	61,575.2	349	121,801	42,508,549	18.6815	7.0406	2.86533	1096.4	95,662.3
281	78,961	22,188,041	16.7631	6.5499	3.55872	882.79	62,015.8	350	122,500	42,875,000	18.7083	7.0473	2.85714	1099.6	96,211.3
282	79,524	22,425,768	16.7929	6.5577	3.54610	885.93	62,458.0	351	123,201	43,243,551	18.7350	7.0540	2.84900	1102.7	96,761.8
283	80,089	22,665,187	16.8226	6.5654	3.53357	889.07	62,901.8	352	123,904	43,614,208	18.7617	7.0607	2.84091	1105.8	97,314.0
284	80,656	22,906,304	16.8523	6.5731	3.52113	892.21	63,347.1	353	124,609	43,986,977	18.7883	7.0674	2.83286	1109.0	97,867.7
285	81,255	23,149,125	16.8819	6.5808	3.50877	895.35	63,794.0	354	125,316	44,361,864	18.8149	7.0740	2.82486	1112.1	98,423.0
286	81,796	23,393,656	16.9115	6.5885	3.49650	898.50	64,242.4	355	126,025	44,738,875	18.8414	7.0807	2.81690	1115.3	98,979.8
287	82,369	23,639,903	16.9411	6.5962	3.48432	901.64	64,692.5	356	126,736	45,118,016	18.8680	7.0873	2.80899	1118.4	99,538.2
288	82,944	23,887,872	16.9706	6.6039	3.47222	904.78	65,144.1	357	127,449	45,499,293	18.8944	7.0940	2.80112	1121.5	100,098
289	83,521	24,137,569	17.0000	6.6115	3.46021	907.92	65,597.2	358	128,164	45,882,712	18.9209	7.1006	2.79330	1124.7	100,660
290	84,100	24,389,000	17.0294	6.6191	3.44828	911.06	66,052.0	359	128,881	46,268,279	18.9473	7.1072	2.78552	1127.8	101,223
291	84,681	24,642,171	17.0587	6.6267	3.43643	914.20	66,508.3	360	129,600	46,656,000	18.9737	7.1138	2.77778	1131.0	101,788
292	85,264	24,897,088	17.0880	6.6343	3.42466	917.35	66,966.2	361	130,321	47,045,881	19.0000	7.1204	2.77008	1134.1	102,354
293	85,849	25,153,757	17.1172	6.6419	3.41297	920.49	67,425.6	362	131,044	47,437,928	19.0263	7.1269	2.76243	1137.3	102,922
294	86,436	25,412,184	17.1464	6.6494	3.40136	923.63	67,886.7	363	131,769	47,832,147	19.0526	7.1335	2.75482	1140.4	103,491
295	87,025	25,672,375	17.1756	6.6569	3.38983	926.77	68,349.3	364	132,496	48,228,544	19.0788	7.1400	2.74725	1143.5	104,062
296	87,616	25,934,336	17.2047	6.6644	3.37838	929.91	68,813.5	365	133,225	48,627,125	19.1050	7.1466	2.73973	1146.7	104,635
297	88,209	26,198,073	17.2337	6.6719	3.36700	933.05	69,279.2	366	133,956	49,027,896	19.1311	7.1531	2.73224	1149.8	105,209
298	88,804	26,463,592	17.2627	6.6794	3.35570	936.19	69,746.5	367	134,689	49,430,863	19.1572	7.1596	2.72480	1153.0	105,785
299	89,401	26,730,899	17.2916	6.6869	3.34448	939.34	70,215.4	368	135,424	49,836,032	19.1833	7.1661	2.71739	1156.1	106,362
300	90,000	27,000,000	17.3205	6.6943	3.33333	942.48	70,685.8	369	136,161	50,243,409	19.2094	7.1726	2.71003	1159.2	106,941
301	90,601	27,270,901	17.3494	6.7018	3.32226	945.62	71,157.9	370	136,900	50,653,000	19.2354	7.1791	2.70270	1162.4	107,521
302	91,204	27,543,608	17.3781	6.7092	3.31126	948.76	71,631.5	371	137,641	51,064,811	19.2614	7.1855	2.69542	1165.5	108,103
303	91,809	27,818,127	17.4069	6.7166	3.30033	951.90	72,106.6	372	138,384	51,478,848	19.2873	7.1920	2.68817	1168.7	108,687
304	92,416	28,094,464	17.4356	6.7240	3.28947	955.04	72,583.4	373	139,129	51,895,117	19.3132	7.1984	2.68097	1171.8	109,272
305	93,025	28,372,625	17.4642	6.7313	3.27869	958.19	73,061.7	374	139,876	52,313,624	19.3391	7.2048	2.67380	1175.0	109,858
306	93,636	28,652,616	17.4929	6.7387	3.26797	961.33	73,541.5	375	140,625	52,734,375	19.3649	7.2112	2.66667	1178.1	110,447
307	94,249	28,934,443	17.5214	6.7460	3.25733	964.47	74,023.0	376	141,376	53,157,376	19.3907	7.2177	2.65957	1181.2	111,036
308	94,864	29,218,112	17.5499	6.7533	3.24675	967.61	74,506.0	377	142,129	53,582,633	19.4165	7.2240	2.65252	1184.4	111,628
309	95,481	29,503,629	17.5784	6.7606	3.23625	970.75	74,990.6	378	142,884	54,010,152	19.4422	7.2304	2.64550	1187.5	112,221
310	96,100	29,791,000	17.6068	6.7679	3.22581	973.89	75,476.8	379	143,641	54,439,939	19.4679	7.2368	2.63852	1190.7	112,815
311	96,721	30,080,231	17.6352	6.7752	3.21543	977.04	75,964.5	380	144,400	54,872,000	19.4936	7.2432	2.63158	1193.8	113,411
312	97,344	30,371,328	17.6635	6.7824	3.20513	980.18	76,453.8	381	145,161	55,306,341	19.5192	7.2495	2.62467	1196.9	114,009
313	97,969	30,664,297	17.6918	6.7897	3.19489	983.32	76,944.7	382	145,924	55,742,968	19.5448	7.2558	2.61780	1200.1	114,608
314	98,596	30,959,144	17.7200	6.7969	3.18471	986.46	77,437.1	383	146,689	56,181,887	19.5704	7.2622	2.61097	1203.2	115,209
315	99,225	31,255,875	17.7482	6.8041	3.17460	989.60	77,931.1	384	147,456	56,623,104	19.5959	7.2685	2.60417	1206.4	115,812
316	99,856	31,554,496	17.7764	6.8113	3.16456	992.74	78,426.7	385	148,225	57,066,625	19.6214	7.2748	2.59740	1209.5	116,416
317	100,489	31,855,013	17.8045	6.8185	3.15457	995.88	78,923.9	386	148,996	57,512,456	19.6469	7.2811	2.59067	1212.7	117,021
318	101,124	32,157,432	17.8326	6.8256	3.14465	999.03	79,422.6	387	149,769	57,960,603	19.6723	7.2874	2.58398	1215.8	117,628
319	101,761	32,461,759	17.8606	6.8328	3.13480	1002.2	79,922.9	388	150,544	58,411,072	19.6977	7.2936	2.57732	1218.9	118,237
320	102,400	32,768,000	17.8885	6.8399	3.12500	1005.3	80,424.8	389	151,321	58,863,869	19.7231	7.2999	2.57069	1221.1	118,847
321	103,041	33,076,161	17.9165	6.8470	3.11527	1008.5	80,928.2	390	152,100	59,319,000	19.7484	7.3061	2.56410	1225.2	119,459
322	103,684	33,386,248	17.9444	6.8541	3.10559	1011.6	81,433.2	391	152,881	59,776,471	19.7737	7.3124	2.55755	1228.4	120,072
323	104,329	33,698,267	17.9722	6.8612	3.09598	1014.7	81,939.8	392	153,664	60,236,288	19.7990	7.3186	2.55102	1231.5	120,687
324	104,976	34,012,224	18.0000	6.8683	3.08642	1017.9	82,448.0	393	154,449	60,698,457	19.8242	7.3248	2.54453	1234.6	121,304
325	105,625	34,328,125	18.0278	6.8753	3.07692	1021.0	82,957.7	394	155,236	61,162,984	19.8494	7.3310	2.53807	1237.8	121,922
326	106,276	34,645,976	18.0555	6.8824	3.06749	1024.2	83,469.0	395	156,025	61,629,875	19.8746	7.3372	2.53165	1240.9	122,542
327	106,929	34,965,783	18.0831	6.8894	3.05810	1027.3	83,981.8	396	156,816	62,099,136	19.8997	7.3434	2.52525	1244.1	123,163
328	107,584	35,287,552	18.1108	6.8964	3.04878	1030.4	84,496.3	397	157,609	62,570,773	19.9249	7.3496	2.51889	1247.2	123,786
329	108,241	35,611,289	18.1384	6.9034	3.03951	1033.6	85,012.3	398	158,404	63,044,792	19.9499	7.3558	2.51256	1250.4	124,410
330	108,900	35,937,000	18.1659	6.9104	3.03030	1036.7	85,529.9	399	159,201	63,521,199	19.9750	7.3619	2.50627	1253.5	125,036
331	109,561	36,264,691	18.1934	6.9174	3.02115	1039.9	86,049.0	400	160,000	64,000,000	20.0000	7.3681	2.50000	1256.6	125,664
332	110,224	36,594,368	18.2209	6.9244	3.01205	1043.0	86,569.7	401	160,801	64,481,201	20.0250	7.3742	2.49377	1259.8	126,293
333	110,889	36,926,037	18.2483	6.9313	3.00300	1046.2	87,092.0	402	161,604	64,964,808	20.0499	7.3803	2.48756	1262.9	126,923
334	111,556	37,259,704	18.2757	6.9382	2.99401	1049.3	87,615.9	403	162,409	65,450,827	20.0749	7.3864	2.48139	1266.1	127,556
335	112,225	37,595,375	18.3030	6.9451	2.98507	1052.4	88,141.3	404	163,216	65,939,264	20.0998	7.3925	2.47525	1269.2	128,190
336	112,896	37,933,056	18.3303	6.9521	2.97619	1055.6	88,668.3	405	164,025	66,430,125	20.1246	7.3986	2.46914	1272.3	128,825
337	113,569	38,272,753	18.3576	6.9589	2.96736	1058.7	89,196.9	406	164,836	66,923,416	20.1494	7.4047	2.46305	1275.5	129,462
338	114,244	38,614,472	18.3848	6.9658	2.95858	1061.9	89,727.0	407	165,649	67,419,143	20.1742	7.4108	2.45700	1278.6	130,100
339	114,921	38,958,219	18.4120	6.9727	2.94985	1065.0	90,258.7	408	166,464	67,917,312	20.1990	7.4169	2.45098	1281.8	130,741
340	115,600	39,304,000	18.4391	6.9795	2.94118	1068.1	90,792.0	409	167,281	68,417,929	20.2237	7.4229	2.44499	1284.9	131,382
341	116,281	39,651,821	18.1662	6.9864	2.93255	1071.3	91,326.9	410	168,100	68,921,000	20.2485	7.4290	2.43902	1288.1	132,025
342	116,964	40,001,688	18.4932	6.9932	2.92398	1074.4	91,863.3	411	168,921	69,426,531	20.2731	7.4350	2.43309	1291.2	132,670
343	117,649	40,353,607	18.5203	7.0000	2.91545	1077.6	92,401.3	412	169,744	69,934,528	20.2978	7.4410	2.42718	1294.3	133,317
344	118,336	40,707,584	18.5472	7.0068	2.90698	1080.7	92,940.9	413	170,569	70,444,997	20.3224	7.4470	2.42131	1297.5	133,965
345	119,025	41,063,625	18.5742	7.0136	2.89855	1083.8	93,482.0	414	171,396	70,957,944	20.3470	7.4530	2.41546	1300.6	134,614

TABLE 20.—SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, RECIPROCALS, CIRCUMFERENCES AND CIRCULAR AREAS—(*Continued*)

No.	Square	Cube	Sq. root	Cu. root	1000× recip.	No. = Dia. Circum.	No. = Dia. Area
415	172,225	71,473,375	20.3715	7.4590	2.40964	1303.8	135,265
416	173,056	71,991,296	20.3961	7.4650	2.40385	1306.9	135,918
417	173,889	72,511,713	20.4206	7.4710	2.39808	1310.0	136,572
418	174,724	73,034,632	20.4450	7.4770	2.39234	1313.2	137,228
419	175,561	73,560,059	20.4695	7.4829	2.38664	1316.3	137,885
420	176,400	74,088,000	20.4939	7.4889	2.38095	1319.5	138,544
421	177,241	74,618,461	20.5183	7.4948	2.37530	1322.6	139,205
422	178,084	75,151,448	20.5426	7.5007	2.36967	1325.8	139,867
423	178,929	75,686,967	20.5670	7.5067	2.36407	1328.0	140,531
424	179,776	76,225,024	20.5913	7.5126	2.35849	1332.0	141,196
425	180,625	76,765,625	20.6155	7.5185	2.35294	1335.2	141,863
426	181,476	77,308,776	20.6398	7.5244	2.34742	1338.3	142,531
427	182,329	77,854,483	20.6640	7.5302	2.34192	1341.5	143,201
428	183,184	78,402,752	20.6882	7.5361	2.33645	1344.6	143,872
429	184,041	78,953,589	20.7123	7.5420	2.33100	1347.7	144,545
430	184,900	79,507,000	20.7364	7.5478	2.32558	1350.9	145,220
431	185,761	80,062,991	20.7605	7.5537	2.32019	1354.0	145,896
432	186,624	80,621,568	20.7846	7.5595	2.31482	1357.2	146,574
433	187,489	81,182,737	20.8087	7.5654	2.30947	1360.3	147,254
434	188,356	81,746,504	20.8327	7.5712	2.30415	1363.5	147,934
435	189,225	82,312,875	20.8567	7.5770	2.29885	1366.6	148,617
436	190,096	82,881,856	20.8806	7.5828	2.29358	1369.7	149,301
437	190,969	83,453,453	20.9045	7.5886	2.28833	1372.9	149,987
438	191,844	84,027,672	20.9284	7.5944	2.28311	1376.0	150,674
439	192,721	84,604,519	20.9523	7.6001	2.27790	1379.2	151,363
440	193,600	85,184,000	20.9762	7.6059	2.27273	1382.3	152,053
441	194,481	85,766,121	21.0000	7.6117	2.26757	1385.4	152,745
442	195,364	86,350,888	21.0238	7.6174	2.26244	1388.6	153,439
443	196,249	86,938,307	21.0476	7.6232	2.25734	1391.7	154,134
444	197,136	87,528,384	21.0713	7.6289	2.25225	1394.9	154,830
445	198,025	88,121,125	21.0950	7.6346	2.24719	1398.0	155,528
446	198,916	88,716,536	21.1187	7.6403	2.24215	1401.2	156,228
447	199,809	89,314,623	21.1424	7.6460	2.23714	1404.3	156,930
448	200,704	89,915,392	21.1660	7.6517	2.23214	1407.4	157,633
449	201,601	90,518,849	21.1896	7.6574	2.22717	1410.6	158,337
450	202,500	91,125,000	21.2132	7.6631	2.22222	1413.7	159,043
451	203,401	91,733,851	21.2368	7.6688	2.21730	1416.9	159,751
452	204,304	92,345,408	21.2603	7.6744	2.21239	1420.0	160,460
453	205,209	92,959,677	21.2838	7.6801	2.20751	1423.1	161,171
454	206,116	93,576,664	21.3073	7.6857	2.20264	1426.3	161,883
455	207,025	94,196,375	21.3307	7.6914	2.19780	1429.4	162,597
456	207,936	94,818,816	21.3542	7.6970	2.19298	1432.6	163,313
457	208,849	95,443,993	21.3776	7.7026	2.18818	1435.7	164,030
458	209,764	96,071,912	21.4009	7.7082	2.18341	1438.9	164,748
459	210,681	96,702,579	21.4243	7.7138	2.17865	1442.0	165,468
460	211,600	97,336,000	21.4476	7.7194	2.17391	1445.1	166,190
461	212,521	97,972,181	21.4709	7.7250	2.16920	1448.3	166,914
462	213,444	98,611,128	21.4942	7.7306	2.16450	1451.4	167,639
463	214,369	99,252,847	21.5174	7.7362	2.15983	1454.6	168,365
464	215,296	99,897,344	21.5407	7.7418	2.15517	1457.7	169,093
465	216,225	100,544,625	21.5639	7.7473	2.15054	1460.8	169,823
466	217,156	101,194,696	21.5870	7.7529	2.14592	1464.0	170,554
467	218,089	101,847,563	21.6102	7.7584	2.14133	1467.1	171,287
468	219,024	102,503,232	21.6333	7.7639	2.13675	1470.3	172,021
469	219,961	103,161,709	21.6564	7.7695	2.13220	1473.4	172,757
470	220,900	103,823,000	21.6795	7.7750	2.12766	1476.5	173,494
471	221,841	104,487,111	21.7025	7.7805	2.12314	1479.7	174,234
472	222,784	105,154,048	21.7256	7.7860	2.11864	1482.8	174,974
473	223,729	105,823,817	21.7486	7.7915	2.11417	1486.0	175,716
474	224,676	106,496,424	21.7715	7.7970	2.10971	1489.1	176,460
475	225,625	107,171,875	21.7945	7.8025	2.10526	1492.3	177,205
476	226,576	107,850,176	21.8174	7.8079	2.10084	1495.4	177,952
477	227,529	108,531,333	21.8403	7.8134	2.09644	1498.5	178,701
478	228,484	109,215,352	21.8632	7.8188	2.09205	1501.7	179,451
479	229,441	109,902,239	21.8861	7.8243	2.08768	1504.8	180,203
480	230,400	110,592,000	21.9089	7.8297	2.08333	1508.0	180,956
481	231,361	111,284,641	21.9317	7.8352	2.07900	1511.1	181,711
482	232,324	111,980,168	21.9545	7.8406	2.07469	1514.3	182,467
483	233,289	112,678,587	21.9773	7.8460	2.07039	1517.4	183,225

No.	Square	Cube	Sq. root	Cu. root	1000× recip.	No. = Dia. Circum.	No. = Dia. Area
484	234,256	113,379,904	22.0000	7.8514	2.06612	1520.5	183,984
485	235,225	114,084,125	22.0227	7.8568	2.06186	1523.7	184,745
486	236,196	114,791,256	22.0454	7.8622	2.05761	1526.8	185,508
487	237,169	115,501,303	22.0681	7.8676	2.05339	1530.0	186,272
488	238,144	116,214,272	22.0907	7.8730	2.04918	1533.1	187,038
489	239,121	116,930,169	22.1133	7.8784	2.04499	1536.2	187,805
490	240,100	117,649,000	22.1359	7.8837	2.04082	1539.4	188,574
491	241,081	118,370,771	22.1585	7.8891	2.03666	1542.5	189,345
492	242,064	119,095,488	22.1811	7.8944	2.03252	1545.7	190,117
493	243,049	119,823,157	22.2036	7.8998	2.02840	1548.8	190,890
494	244,036	120,553,784	22.2261	7.9051	2.02429	1551.9	191,665
495	245,025	121,287,375	22.2486	7.9105	2.02020	1555.1	192,442
496	246,016	122,023,936	22.2711	7.9158	2.01613	1558.2	193,221
497	247,009	122,763,473	22.2935	7.9211	2.01207	1561.4	194,000
498	248,004	123,505,992	22.3159	7.9264	2.00803	1564.5	194,782
499	249,001	124,251,499	22.3383	7.9317	2.00401	1567.7	195,565
500	250,000	125,000,000	22.3607	7.9370	2.00000	1570.8	196,350
501	251,001	125,751,501	22.3830	7.9423	1.99601	1573.9	197,136
502	252,004	126,506,008	22.4054	7.9476	1.99203	1577.1	197,923
503	253,009	127,263,527	22.4277	7.9528	1.98807	1580.2	198,713
504	254,016	128,024,064	22.4499	7.9581	1.98413	1583.4	199,504
505	255,025	128,787,625	22.4722	7.9634	1.98020	1586.5	200,296
506	256,036	129,554,216	22.4944	7.9686	1.97629	1589.7	201,090
507	257,049	130,323,843	22.5167	7.9739	1.97239	1592.8	201,886
508	258,064	131,096,512	22.5389	7.9791	1.96850	1595.9	202,683
509	259,081	131,872,229	22.5610	7.9843	1.96464	1599.1	202,482
510	260,100	132,651,000	22.5832	7.9896	1.96078	1602.2	204,282
511	261,121	133,432,831	22.6053	7.9948	1.95695	1605.4	205,084
512	262,144	134,217,728	22.6274	8.0000	1.95312	1608.5	205,887
513	263,169	135,005,697	22.6495	8.0052	1.94932	1611.6	206,692
514	264,196	135,796,744	22.6716	8.0104	1.94553	1614.8	207,499
515	265,225	136,590,875	22.6936	8.0156	1.94175	1617.9	208,307
516	266,256	137,388,096	22.7156	8.0208	1.93798	1621.1	209,117
517	267,289	138,188,413	22.7376	8.0260	1.93424	1624.2	209,928
518	268,324	138,991,832	22.7596	8.0311	1.93050	1627.3	210,741
519	269,361	139,798,359	22.7816	8.0363	1.92678	1630.5	211,556
520	270,400	140,608,000	22.8035	8.0415	1.92308	1633.6	212,372
521	271,441	141,420,761	22.8254	8.0466	1.91939	1636.8	213,189
522	272,484	142,236,648	22.8473	8.0517	1.91571	1639.9	214,008
523	273,529	143,055,667	22.8692	8.0569	1.91205	1643.1	214,829
524	274,576	143,877,824	22.8910	8.0620	1.90840	1646.2	215,651
525	275,625	144,703,125	22.9129	8.0671	1.90476	1649.3	216,475
526	276,676	145,531,576	22.9347	8.0723	1.90114	1652.5	217,301
527	277,729	146,363,183	22.9565	8.0774	1.89753	1655.6	218,128
528	278,784	147,197,952	22.9783	8.0825	1.89394	1658.8	218,956
529	279,841	148,035,889	23.0000	8.0876	1.89036	1661.9	219,787
530	280,900	148,877,000	23.0217	8.0927	1.88679	1665.0	220,618
431	281,961	149,721,291	23.0434	8.0978	1.88324	1668.2	221,452
532	283,024	150,568,768	23.0651	8.1028	1.87970	1671.3	222,287
533	284,089	151,419,437	23.0868	8.1079	1.87617	1674.5	223,123
534	285,156	152,273,304	23.1084	8.1130	1.87266	1677.6	223,961
535	286,225	153,130,375	23.1301	8.1180	1.86916	1680.8	224,801
536	287,296	153,990,656	23.1517	8.1231	1.86567	1683.9	225,642
537	288,369	154,854,153	23.1733	8.1281	1.86220	1687.0	226,484
538	289,444	155,720,872	23.1948	8.1332	1.85874	1690.2	227,329
539	290,521	156,590,819	23.2164	8.1382	1.85529	1693.3	228,175
540	291,600	157,464,000	23.2379	8.1433	1.85185	1696.5	229,022
541	292,681	158,340,421	23.2594	8.1483	1.84843	1699.6	229,871
542	293,764	159,220,088	23.2809	8.1533	1.84502	1702.7	230,722
543	294,849	160,103,007	23.3024	8.1583	1.84162	1705.9	231,574
544	295,936	160,989,184	23.3238	8.1633	1.83824	1709.0	232,428
545	297,025	161,878,625	23.3452	8.1683	1.83486	1712.2	233,283
546	298,116	162,771,336	23.3666	8.1733	1.83150	1715.3	234,140
547	299,209	163,667,323	23.3880	8.1783	1.82815	1718.5	234,998
548	300,304	164,566,592	23.4094	8.1833	1.82482	1721.6	235,858
549	301,401	165,469,149	23.4307	8.1882	1.82149	1724.7	236,720
550	302,500	166,375,000	23.4521	8.1932	1.81818	1727.9	237,583
551	303,601	167,284,151	23.4734	8.1982	1.81488	1731.0	238,448
552	304,704	168,196,608	23.4947	8.2031	1.81159	1734.2	239,314

TABLE 20.—SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, RECIPROCALS, CIRCUMFERENCES AND CIRCULAR AREAS—(*Continued*)

No.	Square	Cube	Sq. root	Cu. root	1000× recip.	No. = Dia. Circum.	No. = Dia. Area
553	305,809	169,112,377	23.5160	8.2081	1.80832	1737.3	240,182
554	306,916	170,031,464	23.5372	8.2130	1.80505	1740.4	241,051
555	308,025	170,953,875	23.5584	8.2180	1.80180	1743.6	241,922
556	309,136	171,879,616	23.5797	8.2229	1.79856	1746.7	242,795
557	310,249	172,808,693	23.6008	8.2278	1.79533	1743.9	243,669
558	311,364	173,741,112	23.6220	8.2327	1.79211	1759.0	244,545
559	312,481	174,676,879	23.6432	8.2377	1.78891	1756.2	245,422
560	313,600	175,616,000	23.6643	8.2426	1.78571	1759.3	246,301
561	314,721	176,558,481	23.6854	8.2475	1.78253	1762.4	247,181
562	315,844	177,504,328	23.7065	8.2524	1.77936	1765.6	248,063
563	316,969	178,453,547	23.7276	8.2573	1.77620	1768.7	248,947
564	318,096	179,406,144	23.7487	8.2621	1.77305	1771.9	249,832
565	319,225	180,362,125	23.7697	8.2670	1.76991	1775.0	250,719
566	320,356	181,321,496	23.7908	8.2719	1.76678	1778.1	251,607
567	321,489	182,284,263	23.8118	8.2768	1.76367	1781.3	252,497
568	322,624	183,250,432	23.8328	8.2816	1.76056	1784.4	253,388
569	323,761	184,220,009	23.8537	8.2865	1.75747	1787.6	254,281
570	324,900	185,193,000	23.8747	8.2913	1.75439	1790.7	255,176
571	326,041	186,169,411	23.8956	8.2962	1.75131	1793.9	256,072
572	327,184	187,149,248	23.9165	8.3010	1.74825	1797.0	256,970
573	328,329	188,132,517	23.9374	8.3059	1.74520	1800.1	257,869
574	329,476	189,119,224	23.9583	8.3107	1.74216	1803.3	258,770
575	330,625	190,109,375	23.9792	8.3155	1.73913	1806.4	259,672
576	331,776	191,102,976	24.0000	8.3203	1.73611	1809.6	260,576
577	332,929	192,100,033	24.0208	8.3251	1.73310	1812.7	261,482
578	334,084	193,100,552	24.0416	8.3300	1.73010	1815.8	262,389
579	335,241	194,104,539	24.0624	8.3348	1.72712	1819.0	263,298
580	336,400	195,112,000	24.0832	8.3396	1.72414	1822.1	264,208
581	337,561	196,122,941	24.1039	8.3443	1.72117	1825.3	265,120
582	338,724	197,137,368	24.1247	8.3491	1.71821	1828.4	266,033
583	339,889	198,155,287	24.1454	8.3539	1.71527	1831.6	266,948
584	341,056	199,176,704	24.1661	8.3587	1.71233	1834.7	267,865
585	342,225	200,201,625	24.1868	8.3634	1.70940	1837.8	268,783
586	343,396	201,230,056	24.2074	8.3682	1.70649	1841.0	269,701
587	344,569	202,262,003	24.2281	8.3730	1.70358	1844.1	270,624
588	345,744	203,297,472	24.2487	8.3777	1.70068	1847.3	271,547
589	346,921	204,336,469	24.2693	8.3825	1.69779	1850.4	272,471
590	348,100	205,379,000	24.2899	8.3872	1.69492	1853.5	273,397
591	349,281	206,425,071	24.3105	8.3919	1.69205	1856.7	274,325
592	350,464	207,474,688	24.3311	8.3967	1.68919	1859.8	275,254
593	351,649	208,527,857	24.3516	8.4014	1.68634	1863.0	276,184
594	352,836	209,584,584	24.3721	8.4061	1.68350	1866.1	277,117
595	354,025	210,644,875	24.3926	8.4108	1.68067	1869.3	278,051
596	355,216	211,708,736	24.4131	8.4155	1.67785	1872.4	278,986
597	356,409	212,776,173	24.4336	8.4202	1.67504	1875.5	279,923
598	357,604	213,847,192	24.4540	8.4249	1.67224	1878.7	280,862
599	358,801	214,921,799	24.4745	8.4296	1.66945	1881.8	281,802
600	360,000	216,000,000	24.4949	8.4343	1.66667	1885.0	282,743
601	361,201	217,081,801	24.5153	8.4390	1.66389	1888.1	283,687
602	362,404	218,167,208	24.5357	8.4437	1.66113	1891.2	284,631
603	363,609	219,256,227	24.5561	8.4484	1.65837	1894.4	285,578
604	364,816	220,348,864	24.5764	8.4530	1.65563	1897.5	286,526
605	366,025	221,445,125	24.5967	8.4577	1.65289	1900.7	287,475
606	367,236	222,545,016	24.6171	8.4623	1.65017	1903.8	288,426
607	368,449	223,648,543	24.6374	8.4670	1.64745	1907.0	289,379
608	369,664	224,755,712	24.6577	8.4716	1.64474	1910.1	290,333
609	370,881	225,866,529	24.6779	8.4763	1.64204	1913.2	291,289
610	372,100	226,981,000	24.6982	8.4809	1.63934	1916.4	292,247
611	373,321	228,099,131	24.7184	8.4856	1.63666	1919.5	293,206
612	374,544	229,220,928	24.7386	8.4902	1.63399	1922.7	294,166
613	375,769	230,346,397	24.7588	8.4948	1.63132	1925.8	295,128
614	376,996	231,475,544	24.7790	8.4994	1.62866	1928.9	296,092
615	378,225	232,608,375	24.7992	8.5040	1.62602	1932.1	297,057
616	379,456	233,744,896	24.8193	8.5086	1.62338	1935.2	298,024
617	380,689	234,885,113	24.8395	8.5132	1.62075	1938.4	298,992
618	381,924	236,029,032	24.8596	8.5178	1.61812	1941.5	299,962
619	383,161	237,176,659	24.8797	8.5224	1.61551	1944.7	300,934
620	384,400	238,328,000	24.8998	8.5270	1.61290	1947.8	301,907
621	385,641	239,483,061	24.9199	8.5316	1.61031	1950.9	302,882
622	386,884	240,641,848	24.9399	8.5362	1.60772	1954.1	303,858
623	388,129	241,804,367	24.9600	8.5408	1.60514	1957.2	304,836
624	389,376	242,970,624	24.9800	8.5453	1.60256	1960.4	305,815
625	390,625	244,140,625	25.0000	8.5499	1.60000	1963.5	306,796
626	391,876	245,314,376	25.0200	8.5544	1.59744	1966.6	307,779
627	393,129	246,491,883	25.0400	8.5590	1.59490	1969.8	308,763
628	394,384	247,673,152	25.0599	8.5635	1.59236	1972.9	309,748
629	395,641	248,858,189	25.0799	8.5681	1.58983	1976.1	310,736
630	396,900	250,047,000	25.0998	8.5726	1.58730	1979.2	311,725
631	398,161	251,239,591	25.1197	8.5772	1.58479	1982.4	312,715
632	399,424	252,435,968	25.1396	8.5817	1.58228	1985.5	313,707
633	400,689	253,636,137	25.1595	8.5862	1.57978	1988.6	314,700
634	401,956	254,840,104	25.1794	8.5907	1.57729	1991.8	315,696
635	403,225	256,047,875	25.1992	8.5952	1.57480	1994.9	316,692
636	404,496	257,259,456	25.2190	8.5997	1.57233	1998.1	317,690
637	405,769	258,474,853	25.2389	8.6043	1.56986	2001.2	318,690
638	407,044	259,694,072	25.2587	8.6088	1.56740	2004.3	319,692
639	408,321	260,917,119	25.2784	8.6132	1.56495	2007.5	320,965
640	409,600	262,144,000	25.2982	8.6177	1.56250	2010.6	321,699
641	410,881	263,374,721	25.3180	8.6222	1.56006	2013.8	322,705
642	412,164	264,609,288	25.3377	8.6267	1.55763	2016.9	323,713
643	413,449	265,847,707	25.3574	8.6312	1.55521	2020.0	324,722
644	414,736	267,089,984	25.3772	8.6357	1.55280	2023.2	325,733
645	416,025	268,336,125	25.3969	8.6401	1.55039	2026.3	326,745
646	417,316	269,586,136	25.4165	8.6446	1.54799	2029.5	327,759
647	418,609	270,840,023	25.4362	8.6490	1.54560	2032.6	328,775
648	419,904	272,097,792	25.4558	8.6535	1.54321	2035.8	329,792
649	421,201	273,359,449	25.4755	8.6579	1.54083	2038.9	330,810
650	422,500	274,625,000	25.4951	8.6624	1.53846	2042.0	331,831
651	423,801	275,894,451	25.5147	8.6668	1.53610	2045.2	332,853
652	425,104	277,167,808	25.5343	8.6713	1.53374	2048.3	333,876
653	426,409	278,445,077	25.5539	8.6757	1.53139	2051.5	334,901
654	427,716	279,726,264	25.5734	8.6801	1.52905	2054.6	335,927
655	429,025	281,011,375	25.5930	8.6845	1.52672	2057.7	336,955
656	430,336	282,300,416	25.6125	8.6890	1.52439	2060.9	337,985
657	431,649	283,593,393	25.6320	8.6934	1.52207	2064.0	339,016
658	432,964	284,890,312	25.6515	8.6978	1.51976	2067.2	340,049
659	434,281	286,191,179	25.6710	8.7022	1.51745	2070.3	341,084
660	435,600	287,496,000	25.6905	8.7066	1.51515	2073.5	324,119
661	436,921	288,804,781	25.7099	8.7110	1.51286	2076.6	343,157
662	438,244	290,117,528	25.7294	8.7154	1.51057	2079.7	344,196
663	439,569	291,434,247	25.7488	8.7198	1.50830	2082.9	345,237
664	440,896	292,754,944	25.7682	8.7241	1.50602	2086.0	346,279
665	442,225	294,079,625	25.7876	8.7285	1.50376	2089.2	347,323
666	443,556	295,408,296	25.8070	8.7329	1.50150	2092.3	348,368
667	444,889	296,740,963	25.8263	8.7373	1.49925	2095.4	349,415
668	446,224	298,077,632	25.8457	8.7416	1.49701	2098.6	350,464
669	447,561	299,418,309	25.8650	8.7460	1.49477	2101.7	351,514
670	448,900	300,763,000	25.8844	8.7503	1.49254	2104.9	352,565
671	450,241	302,111,711	25.9037	8.7547	1.49031	2108.0	353,618
672	451,584	303,464,448	25.9230	8.7590	1.48810	2111.2	354,673
673	452,929	304,821,217	25.9422	8.7634	1.48588	2114.3	355,730
674	454,276	306,182,024	25.9615	8.7677	1.48368	2117.4	356,788
675	455,625	307,546,875	25.9808	8.7721	1.48148	2120.6	357,847
676	456,976	308,915,776	26.0000	8.7764	1.47929	2123.7	358,908
677	458,329	310,288,733	26.0192	8.7807	1.47711	2126.9	359,971
678	459,684	311,665,752	26.0384	8.7850	1.47493	2130.0	361,035
679	461,041	313,046,839	26.0576	8.7893	1.47275	2133.1	362,101
680	462,400	314,432,000	26.0768	8.7937	1.47059	2136.3	363,168
681	463,761	315,821,241	26.0960	8.7980	1.46843	2139.4	364,237
682	465,124	317,214,568	26.1151	8.8023	1.46628	2142.6	365,308
683	466,489	318,611,987	26.1343	8.8066	1.46413	2145.7	366,380
684	467,856	320,013,504	26.1534	8.8109	1.46199	2148.9	367,453
685	469,225	321,419,125	26.1725	8.8152	1.45985	2152.0	368,528
686	470,596	322,828,856	26.1916	8.8194	1.45773	2155.1	369,605
687	471,969	324,242,703	26.2107	8.8237	1.45560	2158.3	370,684
688	473,344	325,660,672	26.2298	8.8280	1.45349	2161.4	371,764
689	474,721	327,082,769	26.2488	8.8323	1.45138	2164.6	372,845
690	476,100	328,509,000	26.2679	8.8366	1.44928	2167.7	373,928

TABLE 20.—SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, RECIPROCALS, CIRCUMFERENCES AND CIRCULAR AREAS—(*Continued*)

No.	Square	Cube	Sq. root	Cu. root	1000× recip.	No. = Dia		No.	Square	Cube	Sq. root	Cu. root	1000× recip.	No. = Dia.	
						Circum.	Area							Circum.	Area
691	477,481	329,939,371	26.2869	8.8408	1.44718	2170.8	375,013	760	577,600	438,976,000	27.5681	9.1258	1.31579	2387.6	453,646
692	478,864	331,373,888	26.3059	8.8451	1.44509	2174.0	376,099	761	579,121	440,711,081	27.5862	9.1298	1.31406	2390.8	454,841
693	480,249	332,812,557	26.3249	8.8493	1.44300	2177.1	377,187	762	580,644	442,450,728	27.6043	9.1338	1.31234	2393.9	456,037
694	481,636	334,255,384	26.3439	8.8536	1.44092	2180.3	378,276	763	582,169	444,194,947	27.6225	9.1378	1.31062	2397.0	457,234
695	483,025	335,702,375	26.3629	8.8578	1.43885	2183.4	379,367	764	583,696	445,943,744	27.6405	9.1418	1.30890	2400.2	458,434
696	484,416	337,153,536	26.3818	8.8621	1.43678	2186.6	380,459	765	585,225	447,697,125	27.6586	9.1458	1.30719	2403.3	459,635
697	485,809	338,608,873	26.4008	8.8663	1.43472	2189.7	381,554	766	586,756	449,455,096	27.6767	9.1498	1.30548	2406.5	460,837
698	487,204	340,068,392	26.4197	8.8706	1.43267	2192.8	382,649	767	588,289	451,217,663	27.6948	9.1537	1.30378	2409.6	462,042
699	488,601	341,532,099	26.4386	8.8748	1.43062	2196.0	383,746	768	589,824	452,984,832	27.7128	9.1577	1.30208	2412.7	463,247
700	490,000	343,000,000	26.4575	8.8790	1.42857	2199.1	384,845	769	591,361	454,756,609	27.7308	9.1617	1.30039	2415.9	464,454
701	491,401	344,472,101	26.4764	8.8833	1.42653	2202.3	385,945	770	592,900	456,533,000	27.7489	9.1657	1.29870	2419.0	465,663
702	492,804	345,948,408	26.4953	8.8875	1.42450	2205.4	387,047	771	594,441	458,314,011	27.7669	9.1696	1.29702	2422.2	466,873
703	494,209	347,428,927	26.5141	8.8917	1.42248	2208.5	388,151	772	595,984	460,099,648	27.7849	9.1736	1.29534	2425.3	468,085
704	495,616	348,913,664	26.5330	8.8959	1.42046	2211.7	389,256	773	597,529	461,889,917	27.8029	9.1775	1.29366	2428.5	469,298
705	497,025	350,402,625	26.5518	8.9001	1.41844	2214.8	390,363	774	599,076	463,684,824	27.8209	9.1815	1.29199	2431.6	470,513
706	498,436	351,895,816	26.5707	8.9043	1.41643	2218.0	391,471	775	600,625	465,484,375	27.8388	9.1855	1.29032	2434.7	471,730
707	499,849	353,393,243	26.5895	8.9085	1.41443	2221.1	392,580	776	602,176	467,288,576	27.8568	9.1894	1.28866	2437.9	472,948
708	501,264	354,894,912	26.6083	8.9127	1.41243	2224.3	393,692	777	603,729	469,097,433	27.8747	9.1933	1.28700	2441.0	474,168
709	502,681	356,400,829	26.6271	8.9169	1.41044	2227.4	394,805	778	605,284	470,910,952	27.8927	9.1973	1.28535	2444.2	475,389
710	504,100	357,911,000	26.6458	8.9211	1.40845	2230.5	395,919	779	606,841	472,729,139	27.9106	9.2012	1.28370	2447.3	476,612
711	505,521	359,425,431	26.6646	8.9253	1.40647	2233.7	397,035	780	608,400	474,552,000	27.9285	9.2052	1.28205	2450.4	477,836
712	506,944	360,944,128	26.6833	8.9295	1.40449	2236.8	398,153	781	609,961	476,379,541	27.9464	9.2091	1.28041	2453.6	479,062
713	508,369	362,467,097	26.7021	8.9337	1.40253	2240.0	399,272	782	611,524	478,211,768	27.9643	9.2130	1.27877	2456.7	480,290
714	509,796	363,994,344	26.7208	8.9378	1.40056	2243.1	400,393	783	613,089	480,048,687	27.9821	9.2170	1.27714	2459.9	481,519
715	511,225	365,525,875	26.7395	8.9420	1.39860	2246.2	401,515	784	614,656	481,890,304	28.0000	9.2209	1.27551	2463.0	482,750
716	512,656	367,061,696	26.7582	8.9462	1.39665	2249.4	402,639	785	616,225	483,736,625	28.0179	9.2248	1.27389	2466.2	483,982
717	514,089	368,601,813	26.7769	8.9503	1.39470	2252.5	403,765	786	617,796	485,587,656	28.0357	9.2287	1.27226	2469.3	485,216
718	515 524	370,146,232	26.7955	8.9545	1.39276	2255.7	404,892	787	619,369	487,443,403	28.0535	9.2326	1.27065	2472.4	486,451
719	516,961	371,694,959	28.8142	8.9587	1.39082	2258.8	406,020	788	620,944	489,303,872	28.0713	9.2365	1.26904	2475.6	487,688
720	518,400	373,248,000	26.8328	8.9628	1.38889	2261.9	407,150	789	622,521	491,169,069	28.0891	9.2404	1.26743	2478.7	488,927
721	519,841	374,805,361	26.8514	8.9670	1.38696	2265.1	408,282	790	624,100	493,039,000	28.1069	9.2443	1.26582	2481.9	490,167
722	521,284	376,367,048	26.8701	8.9711	1.38504	2268.2	409,416	791	625,681	494,913,671	28.1247	9.2482	1.26422	2485.0	491,409
723	522,729	377,933,067	26.8887	8.9752	1.38313	2271.4	410,550	792	627,264	496,793,088	28.1425	9.2521	1.26263	2488.1	492,652
724	524,176	379,503,424	26.9072	8.9794	1.38122	2274.5	411,687	793	628,849	498,677,257	28.1603	9.2560	1.26103	2491.3	493,897
725	525,625	381,078,125	26.9258	8.9835	1.37931	2277.7	412,825	794	630,436	500,566,184	28.1780	9.2599	1.25945	2494.4	495,143
726	527,076	382,657,176	26.9444	8.9876	1.37741	2280.8	413,965	795	632,025	502,459,875	28.1957	9.2638	1.25786	2497.6	496,391
727	528,529	384,240,583	26.9629	8.9918	1.37552	2283.9	415,106	796	633,616	504,358,336	28.2135	9.2677	1.25628	2500.7	497,641
728	529,984	385,828,352	26.9815	8.9959	1.37363	2287.1	416,248	797	635,209	506,261,573	28.2312	9.2716	1.25471	2503.8	498,892
729	531,441	387,420,489	27.0000	9.0000	1.37174	2290.2	417,393	798	636,804	508,169,592	28.2489	9.2754	1.25313	2507.0	500,145
730	532,900	389,017,000	27.0185	9.0041	1.36986	2293.4	418,539	799	638,401	510,082,399	28.2666	9.2793	1.25156	2510.1	501,399
731	534,361	390,617,891	27.0370	9.0082	1.36799	2296.5	419,686	800	640,000	512,000,000	28.2843	9.2832	1.25000	2513.3	502,655
732	535,824	392,223,168	27.0555	9.0123	1.36612	2299.7	420,835	801	641,601	513,922,401	28.3019	9.2870	1.24844	2516.4	503,912
733	537,289	393,832,837	27.0740	9.0164	1.36426	2302.8	421,986	802	643,204	515,849,608	28.3196	9.2909	1.24688	2519.6	505,171
734	538,756	395,446,904	27.0924	9.0205	1.36240	2305.9	423,138	803	644,809	517,781,627	28.3373	9.2948	1.24533	2522.7	506,432
735	540,225	397,065,375	27.1109	9.0246	1.36054	2309.1	424,293	804	646,416	519,718,464	28.3549	9.2986	1.24378	2525.8	507,694
736	541,696	398,688,256	27.1293	9.0287	1.35870	2312.2	425,448	805	648,025	521,660,125	28.3725	9.3025	1.24224	2529.0	508,958
737	543,169	400,315,553	27.1477	9.0328	1.35685	2315.4	426,604	806	649,636	523,606,616	28.3901	9.3063	1.24069	2532.1	510,223
738	544,644	401,947,272	27.1662	9.0369	1.35501	2318.5	427,762	807	651,249	525,557,943	28.4077	9.3102	1.23916	2535.3	511,490
739	546,121	403,583,419	27.1846	9.0410	1.35318	2321.6	428,922	808	652,864	527,514,112	28.4253	9.3149	1.23762	2538.4	512,758
740	547,600	405,224,000	27.2029	9.0450	1.35135	2324.8	430,084	809	654,481	529,475,129	28.4429	9.3179	1.23609	2541.5	514,028
741	549,081	406,869,021	27.2213	9.0491	1.34953	2327.9	431,247	810	656,100	531,441,000	28.4605	9.3217	1.23457	2544.7	515,300
742	550,564	408,518,488	27.2397	9.0532	1.34771	2331.1	432,412	811	657,721	533,411,731	28.4781	9.3255	1.23305	2547.8	516,573
743	552,049	410,172,407	27.2580	9.0572	1.34590	2334.2	433,578	812	659,344	535,387,328	28.4956	9.3294	1.23153	2551.0	517,848
744	553,536	411,830,784	27.2764	9.0613	1.34409	2337.3	434,746	813	660,969	537,367,797	28.5132	9.3332	1.23001	2554.1	519,124
745	555,025	413,493,625	27.2947	9.0654	1.34228	2340.5	435,916	814	662,596	539,353,144	28.5307	9.3370	1.22850	2557.3	520,402
746	556,516	415,160,936	27.3130	9.0694	1.34048	2343.6	437,087	815	664,225	541,343,375	28.5482	9.3408	1.22699	2560.4	521,681
747	558,009	416,832,723	27.3313	9.0735	1.33869	2346.8	438,259	816	665,856	543,338,496	28.5657	9.3447	1.22549	2563.5	522,962
748	559,504	418,508,992	27.3496	9.0775	1.33690	2349.9	439,423	817	667,489	545,338,513	28.5832	9.3485	1.22399	2566.7	524,245
749	561,001	420,189,749	27.3679	9.0816	1.33511	2353.1	440,609	818	669,124	547,343,432	28.6007	9.3523	1.22249	2569.8	525,529
750	562,500	421,875,000	27.3861	9.0856	1.33333	2356.2	441,786	819	670,761	549,353,259	28.6182	9.3561	1.22100	2573.0	526,814
751	564,001	423,564,751	27.4044	9.0896	1.33156	2359.3	442,965	820	672,400	551,368,000	28.6356	9.3599	1.21951	2576.1	528,102
752	565,504	425,259,008	27.4226	9.0937	1.32979	2362.5	444,146	821	674,041	553,387,661	28.6531	9.3637	1.21803	2579.2	529,391
753	567,009	426,957,777	27.4408	9.0977	1.32802	2365.6	445,328	822	675,684	555,412,248	28.6705	9.3675	1.21655	2582.4	530,681
754	568,516	428,661,064	27.4591	9.1017	1.32626	2368.8	446,511	823	677,329	557,441,767	28.6880	9.3713	1.21507	2585.5	531,973
755	570,025	430,368,875	27.4773	9.1057	1.32450	2371.9	447,697	824	678,976	559,476,224	28.7054	9.3751	1.21359	2588.7	533,267
756	571,536	432,081,216	27.4955	9.1098	1.32275	2375.0	448,883	825	680,625	561,515,625	28.7228	9.3789	1.21212	2591.8	534,562
757	573,049	433,798,093	27.5136	9.1138	1.32100	2378.2	450,072	826	682,276	563,559,976	28.7402	9.3827	1.21065	2595.0	535,858
758	574,564	435,519,512	27.5318	9.1178	1.31926	2381.3	451,262	827	683,929	565,609,283	28.7576	9.3865	1.20919	2598.1	537,157
759	576,081	437,245,479	27.5500	9.1218	1.31752	2384.5	452,453	828	685,584	567,663,552	28.7750	9.3902	1.20773	2601.2	538,456

TABLE 20.—SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, RECIPROCALS CIRCUMFERENCES AND CIRCULAR AREAS—(*Continued*)

No.	Square	Cube	Sq. root	Cu. root	1000× recip.	No. = Dia. Circum.	No. = Dia. Area
829	687,241	569,722,789	28.7924	9.3940	1.20627	2604.4	539,758
830	688,900	571,787,000	28.8097	9.3978	1.20482	2607.5	541,061
831	690,561	573,856,191	28.8271	9.4016	1.20337	2610.7	542,365
832	692,224	575,930,368	28.8444	9.4053	1.20192	2613.8	543,671
833	693,889	578,009,537	28.8617	9.4091	1.20048	2616.9	544,979
834	695,556	580,093,704	28.8791	9.4129	1.19904	2620.1	546,288
835	697,225	582,182,875	28.8964	9.4166	1.19760	2623.2	547,599
836	698,896	584,277,056	28.9137	9.4204	1.19617	2626.4	548,912
837	700,569	586,376,253	28.9310	9.4241	1.19474	2629.5	550,226
838	702,244	588,480,472	28.9482	9.4279	1.19332	2632.7	551,541
839	703,921	590,589,719	28.9655	9.4316	1.19189	2635.8	552,858
840	705,600	592,704,000	28.9828	9.4354	1.19048	2638.9	554,177
841	707,281	594,823,321	29.0000	9.4391	1.18906	2642.1	555,497
842	708,964	596,947,688	29.0172	9.4429	1.18765	2645.2	556,819
843	710,649	599,077,107	29.0345	9.4466	1.18624	2648.4	558,142
844	712,336	601,211,584	29.0517	9.4503	1.18483	2651.5	559,467
845	714,025	603,351,125	29.0689	9.4541	1.18343	2654.6	560,794
846	715,716	605,495,736	29.0861	9.4578	1.18203	2657.8	562,122
847	717,409	607,645,423	29.1033	9.4615	1.18064	2660.9	563,452
848	719,104	609,800,192	29.1204	9.4652	1.17925	2664.1	564,783
849	720,801	611,960,049	29.1376	9.4690	1.17786	2667.2	566,116
850	722,500	614,125,000	29.1548	9.4727	1.17647	2670.4	567,450
851	724,201	616,295,051	29.1719	9.4764	1.17509	2673.5	568,786
852	725,904	618,470,208	29.1890	9.4801	1.17371	2676.6	570,124
853	727,609	620,650,477	29.2062	9.4838	1.17233	2679.8	571,463
854	729,316	622,835,864	29.2233	9.4875	1.17096	2682.9	572,803
855	731,025	625,026,375	29.2404	9.4912	1.16959	2686.1	574,146
856	732,736	627,222,016	29.2575	9.4949	1.16822	2689.2	575,490
857	734,449	629,422,793	29.2746	9.4986	1.16686	2692.3	576,835
858	736,164	631,628,712	29.2916	9.5023	1.16550	2695.5	578,182
859	737,881	633,839,779	29.3087	9.5060	1.16414	2698.6	579,530
860	739,600	636,056,000	29.3258	9.5097	1.16279	2701.8	580,880
861	741,321	638,277,381	29.3428	9.5134	1.16144	2704.9	582,232
862	743,044	640,503,928	29.3598	9.5171	1.16009	2708.1	583,585
863	744,769	642,735,647	29.3769	9.5207	1.15875	2711.2	584,940
864	746,496	644,972,544	29.3939	9.5244	1.15741	2714.3	586,297
865	748,225	647,214,625	29.4109	9.5281	1.15607	2717.5	587,655
866	749,956	649,461,896	29.4279	9.5317	1.15473	2720.6	589,014
867	751,689	651,714,363	29.4449	9.5354	1.15340	2723.8	590,375
868	753,424	653,972,032	29.4618	9.5391	1.15207	2726.9	591,738
869	755,161	656,234,909	29.4788	9.5427	1.15075	2730.0	593,102
870	756,900	658,503,000	29.4958	9.5464	1.14943	2733.2	594,468
871	758,641	660,776,311	29.5127	9.5501	1.14811	2736.3	595,835
872	760,384	663,054,848	29.5296	9.5537	1.14679	2739.5	597,204
873	762,129	665,338,617	29.5466	9.5574	1.14548	2742.6	598,575
874	763,876	667,627,624	29.5635	9.5610	1.14416	2745.8	599,947
875	765,625	669,921,875	29.5804	9.5647	1.14286	2748.9	601,320
876	767,376	672,221,376	29.5973	9.5683	1.14155	2752.0	602,696
877	769,129	674,526,133	29.6142	9.5719	1.14025	2755.2	604,073
878	770,884	676,836,152	29.6311	9.5756	1.13895	2758.3	605,451
879	772,641	679,151,439	29.6479	9.5792	1.13766	2761.5	606,831
880	774,400	681,472,000	29.6648	9.5828	1.13636	2764.6	608,212
881	776,161	683,797,841	29.6816	9.5865	1.13507	2767.7	609,595
882	777,924	686,128,968	29.6985	9.5901	1.13379	2770.9	610,980
883	779,689	688,465,387	29.7153	9.5937	1.13250	2774.0	612,366
884	781,456	690,807,104	29.7321	9.5973	1.13122	2777.2	613,754
885	783,225	693,154,125	29.7489	9.6010	1.12994	2780.3	615,143
886	784,996	695,506,456	29.7658	9.6046	1.12867	2783.5	616,534
887	786,769	697,864,103	29.7825	9.6082	1.12740	2786.6	617,927
888	788,544	700,227,072	29.7993	9.6118	1.12613	2789.7	619,321
889	790,321	702,595,369	29.8161	9.6154	1.12486	2792.9	620,717
890	792,100	704,969,000	29.8329	9.6190	1.12360	2796.0	622,114
891	793,881	707,347,971	29.8496	9.6226	1.12233	2799.2	623,513
892	795,664	709,732,288	29.8664	9.6262	1.12108	2802.3	624,913
893	797,449	712,121,957	29.8831	9.6298	1.11982	2805.4	626,315
894	799,236	714,516,984	29.8998	9.6334	1.11857	2808.6	627,718
895	801,025	716,917,375	29.9166	9.6370	1.11732	2811.7	629,124
896	802,816	719,323,136	29.9333	9.6406	1.11607	2814.9	630,530
897	804,609	721,734,273	29.9500	9.6442	1.11483	2818.0	631,938

No.	Square	Cube	Sq. root	Cu. root	1000× recip.	No. = Dia. Circum.	No. = Dia. Area
898	806,404	724,150,792	29.9666	9.6477	1.11359	2821.2	633,348
899	808,201	726,572,699	29.9833	9.6513	1.11235	2824.3	634,760
900	810,000	729,000,000	30.0000	9.6549	1.11111	2827.4	636,173
901	811,801	731,432,701	30.0167	9.6585	1.10988	2830.6	637,587
902	813,604	733,870,808	30.0333	9.6620	1.10865	2833.7	639,003
903	815,409	736,314,327	30.0500	9.6656	1.10742	2836.9	640,421
904	817,216	738,763,264	30.0666	9.6692	1.10619	2840.0	641,840
905	819,025	741,217,625	30.0832	9.6727	1.10497	2843.1	643,261
906	820,836	743,677,416	30.0998	9.6763	1.10375	2846.3	644,683
907	822,649	746,142,643	30.1164	9.6799	1.10254	2849.4	646,107
908	824,464	748,613,312	30.1330	9.6834	1.10132	2852.6	647,533
909	826,281	751,089,429	30.1496	9.6870	1.10011	2855.7	648,960
910	828,100	753,571,000	30.1662	9.6905	1.09890	2858.8	650,388
911	829,921	756,058,031	30.1828	9.6941	1.09769	2862.0	651,818
912	831,744	758,550,528	30.1993	9.6976	1.09649	2865.1	653,250
913	833,569	761,048,497	30.2159	9.7012	1.09529	2868.3	654,684
914	835,396	763,551,944	30.2324	9.7047	1.09409	2871.4	656,118
915	837,225	766,060,875	30.2490	9.7082	1.09290	2874.6	657,555
916	839,056	768,575,296	30.2655	9.7118	1.09170	2877.7	658,993
917	840,889	771,095,213	30.2820	9.7153	1.09051	2880.8	660,433
918	842,724	773,620,632	30.2985	9.7188	1.08932	2884.0	661,874
919	844,561	776,151,559	30.3150	9.7224	1.08814	2887.1	663,317
920	846,400	778,688,000	30.3315	9.7259	1.08696	2890.3	664,761
921	848,241	781,229,961	30.3480	9.7294	1.08578	2893.4	666,207
922	850,084	783,777,448	30.3645	9.7329	1.08460	2896.5	667,654
923	851,929	786,330,467	30.3809	9.7364	1.08342	2899.7	669,103
924	853,776	788,889,024	30.3974	9.7400	1.08225	2902.8	670,554
925	855,625	791,453,125	30.4138	9.7435	1.08108	2906.0	672,006
926	857,476	794,022,776	30.4302	9.7470	1.07991	2909.1	673,460
927	859,329	796,597,983	30.4467	9.7505	1.07875	2912.3	674,915
928	861,184	799,178,752	30.4631	9.7540	1.07759	2915.4	676,372
929	863,041	801,765,089	30.4795	9.7575	1.07643	2918.5	677,831
930	864,900	804,357,000	30.4959	9.7610	1.07527	2921.7	679,291
931	866,761	806,954,491	30.5123	9.7645	1.07411	2924.9	680,752
932	868,624	809,557,568	30.5287	9.7680	1.07296	2928.0	682,216
933	870,489	812,166,237	30.5450	9.7715	1.07181	2931.1	683,680
934	872,356	814,780,504	30.5614	9.7750	1.07066	2934.2	685,147
935	874,225	817,400,375	30.5778	9.7785	1.06952	2937.4	686,615
936	876,096	820,025,856	30.5941	9.7819	1.06838	2940.5	688,084
937	877,969	822,656,953	30.6105	9.7854	1.06724	2943.7	689,555
938	879,844	825,293,672	30.6268	9.7889	1.06610	2946.8	691,028
939	881,721	827,936,019	30.6431	9.7924	1.06496	2950.0	692,502
940	883,600	830,584,000	30.6594	9.7959	1.06383	2953.1	693,978
941	885,481	833,237,621	30.6757	9.7993	1.06270	2956.2	695,455
942	887,364	835,896,888	30.6920	9.8028	1.06157	2959.4	696,934
943	889,249	838,561,807	30.7083	9.8063	1.06045	2962.5	698,415
944	891,136	841,232,384	30.7246	9.8097	1.05932	2965.7	699,897
945	893,025	843,908,625	30.7409	9.8132	1.05820	2968.8	701,380
946	894,916	846,590,536	30.7571	9.8167	1.05708	2971.9	702,865
947	896,809	849,278,123	30.7734	9.8201	1.05597	2975.1	704,352
948	898,704	851,971,392	30.7896	9.8236	1.05485	2978.2	705,840
949	900,601	854,670,349	30.8058	9.8270	1.05374	2981.4	707,330
950	902,500	857,375,000	30.8221	9.8305	1.05263	2984.5	708,822
951	904,401	860,085,351	30.8383	9.8339	1.05152	2987.7	710,315
952	906,304	862,801,408	30.8545	9.8374	1.05042	2990.8	711,809
953	908,209	865,523,177	30.8707	9.8408	1.04932	2993.9	713,306
954	910,116	868,250,664	30.8869	9.8443	1.04822	2997.1	714,803
955	912,025	870,983,875	30.9031	9.8477	1.04712	3000.2	716,303
956	913,936	873,722,816	30.9192	9.8511	1.04603	3003.4	717,804
957	915,849	876,467,493	30.9354	9.8546	1.04493	3006.5	719,306
958	917,764	879,217,912	30.9516	9.8580	1.04384	3009.6	720,810
959	919,681	881,974,079	30.9677	9.8614	1.04275	3012.8	722,316
960	921,600	884,736,000	30.9839	9.8648	1.04167	3015.9	723,823
961	923,521	887,503,681	31.0000	9.8683	1.04058	3019.1	725,332
962	925,444	890,277,128	31.0161	9.8717	1.03950	3022.2	726,842
963	927,369	893,056,347	31.0322	9.8751	1.03842	3025.4	728,354
964	929,296	895,841,344	31.0483	9.8785	1.03734	3028.5	729,867
965	931,225	898,632,125	31.0644	9.8819	1.03627	3031.6	731,382
966	933,156	901,428,696	31.0805	9.8854	1.03520	3034.8	732,899

TABLE 20.—SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, RECIPROCALS, CIRCUMFERENCES AND CIRCULAR AREAS—(*Continued*)

No.	Square	Cube	Sq. root	Cu. root	1000× recip.	No. = Dia. Circum.	No. = Dia. Area
967	935,089	904,231,063	31.0966	9.8888	1.03413	3037.9	734,417
968	937,024	907,039,232	31.1127	9.8922	1.03306	3041.1	735,937
969	938,961	909,853,209	31.1288	9.8956	1.03199	3044.2	737,458
970	940,900	912,673,000	31.1448	9.8990	1.03093	3047.3	738,981
971	942,841	915,498,611	31.1609	9.9024	1.02987	3050.5	740,506
972	944,784	918,330,048	31.1769	9.9058	1.02881	3053.6	742,032
973	946,729	921,167,317	31.1929	9.9092	1.02775	3056.8	743,559
974	948,676	924,010,424	31.2090	9.9126	1.02669	3059.9	745,088
975	950,625	926,859,375	31.2250	9.9160	1.02564	3063.1	746,619
976	952,576	929,714,176	31.2410	9.9194	1.02459	3066.2	748,151
977	954,529	932,574,833	31.2570	9.9227	1.02354	3069.3	749,685
978	956,484	935,441,352	31.2730	9.9261	1.02249	3072.5	751,221
979	958,441	938,313,739	31.2890	9.9295	1.02145	3075.6	752,758
980	960,400	941,192,000	31.3050	9.9329	1.02041	3078.8	754,296
981	962,361	944,076,141	31.3209	9.9363	1.01937	3081.9	755,837
982	964,324	946,966,168	31.3369	9.9396	1.01833	3085.0	757,378
983	966,289	949,862,087	31.3528	9.9430	1.01729	3088.2	758,922
984	968,256	952,763,904	31.3688	9.9464	1.01626	3091.3	760,466
985	970,225	955,671,625	31.3847	9.9497	1.01523	3094.5	762,013
986	972,196	958,585,256	31.4006	9.9531	1.01420	3097.6	763,561
987	974,169	961,504,803	31.4166	9.9565	1.01317	3100.8	765,111
988	976,144	964,430,272	31.4325	9.9598	1.01215	3103.9	766,662
989	978,121	967,361,669	31.4484	9.9632	1.01112	3107.0	768,214
990	980,100	970,299,000	31.4643	9.9666	1.01010	3110.2	769,769
991	982,081	973,242,271	31.4802	9.9699	1.00908	3113.3	771,325
992	984,064	976,191,488	31.4960	9.9733	1.00806	3116.5	772,882
993	986,049	979,146,657	31.5119	9.9766	1.00705	3119.6	774,441
994	988,036	982,107,784	31.5278	9.9800	1.00604	3122.7	776,002
995	990,025	985,074,875	31.5436	9.9833	1.00503	3125.9	777,564
996	992,016	988,047,936	31.5595	9.9866	1.00402	3129.0	779,128
997	994,009	991,026,973	31.5753	9.9900	1.00301	3132.2	780,693
998	996,004	994,011,992	31.5911	9.9933	1.00200	3135.3	782,260
999	998,001	997,002,999	31.6070	9.9967	1.00100	3138.5	783,828

TABLE 21.—AREAS AND CIRCUMFERENCES OF CIRCLES—DECIMAL DIVISIONS

Diameter	Area	Circumference	Diameter	Area	Circumference	Diameter	Area	Circumference	Diameter	Area	Circumference
0.0			.5	15.9043	14.1372	9.0	63.6173	28.2743	.5	143.1388	42.4115
.1	.007854	.31416	.6	16.6190	14.4513	.1	65.0388	28.5885	.6	145.2672	42.7257
.2	.031416	.62832	.7	17.3494	14.7655	.2	66.4761	28.9027	.7	147.4114	43.0398
.3	.070686	.94248	.8	18.0956	15.0796	.3	67.9291	29.2168	.8	149.5712	43.3540
.4	.12566	1.2566	.9	18.8574	15.3938	.4	69.3978	29.5310	.9	151.7468	43.6681
.5	.19635	1.5708	5.0	19.6350	15.7080	.5	70.8822	29.8451	14.0	153.9380	43.9823
.6	.28274	1.8850	.1	20.4282	16.0221	.6	72.3823	30.1593	.1	156.1450	44.2965
.7	.38485	2.1991	.2	21.2372	16.3363	.7	73.8981	30.4734	.2	158.3677	44.6106
.8	.50265	2.5133	.3	22.0618	16.6504	.8	75.4296	30.7876	.3	160.6061	44.9248
.9	.63617	2.8274	.4	22.9022	16.9646	.9	76.9769	31.1018	.4	162.8602	45.2389
1.0	.7854	3.1416	.5	23.7583	17.2788	10.0	78.5398	31.4159	.5	165.1300	45.5531
.1	.9503	3.4558	.6	24.6301	17.5929	.1	80.1185	31.7301	.6	167.4155	45.8673
.2	1.1310	3.7699	.7	25.5176	17.9071	.2	81.7128	32.0442	.7	169.7167	46.1814
.3	1.3273	4.0841	.8	26.4208	18.2212	.3	83.3229	32.3584	.8	172.0336	46.4956
.4	1.5394	4.3982	.9	27.3397	18.5354	.4	84.9487	32.6726	.9	174.3662	46.8097
.5	1.7671	4.7124	6.0	28.2743	18.8496	.5	86.5901	32.9867	15.0	176.7146	47.1239
.6	2.0106	5.0265	.1	29.2247	19.1637	.6	88.2473	33.3009	.1	179.0786	47.4380
.7	2.2698	5.3407	.2	30.1907	19.4779	.7	89.9202	33.6150	.2	181.4584	47.7522
.8	2.5447	5.6549	.3	31.1725	19.7920	.8	91.6088	33.9292	.3	183.8539	48.0664
.9	2.8353	5.9690	.4	32.1699	20.1062	.9	93.3132	34.2434	.4	186.2650	48.3805
2.0	3.1416	6.2832	.5	33.1831	20.4204	11.0	95.0332	34.5575	.5	188.6919	48.6947
.1	3.4636	6.5973	.6	34.2119	20.7345	.1	96.7689	34.8717	.6	191.1345	49.0088
.2	3.8013	6.9115	.7	35.2565	21.0487	.2	98.5203	35.1858	.7	193.5928	49.3230
.3	4.1548	7.2257	.8	36.3168	21.3628	.3	100.2875	35.5000	.8	196.0668	49.6372
.4	4.5239	7.5398	.9	37.3928	21.6770	.4	102.0703	35.8142	.9	198.5565	49.9513
.5	4.9087	7.8540	7.0	38.4845	21.9911	.5	103.8689	36.1283	16.0	201.0619	50.2655
.6	5.3093	8.1681	.1	39.5919	22.3053	.6	105.6832	36.4425	.1	203.5831	50.5796
.7	5.7256	8.4823	.2	40.7150	22.6195	.7	107.5132	36.7566	.2	206.1199	50.8938
.8	6.1575	8.7965	.3	41.8539	22.9336	.8	109.3588	37.0708	.3	208.6724	51.2080
.9	6.6052	9.1106	.4	43.0084	23.2478	.9	111.2202	37.3850	.4	211.2407	51.5221
3.0	7.0686	9.4248	.5	44.1786	23.5619	12.0	113.0973	37.6991	.5	213.8246	51.8386
.1	7.5477	9.7389	.6	45.3646	23.8761	.1	114.9901	38.0133	.6	216.4243	52.1504
.2	8.0425	10.0531	.7	46.5663	24.1903	.2	116.8987	38.3274	.7	219.0397	52.4646
.3	8.5530	10.3673	.8	47.7836	24.5044	.3	118.8229	38.6416	.8	221.6708	52.7788
.4	9.0792	10.6814	.9	49.1067	24.8186	.4	120.7628	38.9557	.9	224.3176	53.0929
.5	9.6211	10.9956	8.0	50.2655	25.1327	.5	122.7185	39.2699	17.0	226.9801	53.4071
.6	10.1788	11.3097	.1	51.5300	25.4469	.6	124.6898	39.5841	.1	229.6583	53.7212
.7	10.7521	11.6239	.2	52.8102	25.7611	.7	126.6769	39.8982	.2	232.3522	54.0354
.8	11.3411	11.9381	.3	54.1061	26.0752	.8	128.6796	40.2124	.3	235.0618	54.3496
.9	11.9459	12.2522	.4	55.4177	26.3894	.9	130.6981	40.5265	.4	237.7871	54.6637
4.0	12.5664	12.5664	.5	56.7450	26.7035	13.0	132.7323	40.8407	.5	240.5282	54.9779
.1	13.2025	12.8805	.6	58.0880	27.0177	.1	134.7822	41.1549	.6	243.2849	55.2920
.2	13.8544	13.1947	.7	59.4468	27.3319	.2	136.8478	41.4690	.7	246.0574	55.6062
.3	14.5220	13.5088	.8	60.8212	27.6460	.3	138.9291	41.7832	.8	248.8456	55.9203
.4	15.2053	13.8230	.9	62.2114	27.9602	.4	141.0261	42.0973	.9	251.6494	56.2345

TABLE 21.—AREAS AND CIRCUMFERENCES OF CIRCLES—DECIMAL DIVISIONS—(*Continued*)

Diameter	Area	Circumference	Diameter	Area	Circumference	Diameter	Area	Circumference	Diameter	Area	Circumference
18.0	254.4690	56.5487	.5	471.4352	76.9690	31.0	754.7676	97.3894	.5	1104.4662	117.8097
.1	257.3043	56.8628	.6	475.2916	77.2832	.1	759.6450	97.7035	.6	1110.3645	118.1239
.2	260.1553	57.1770	.7	479.1636	77.5973	.2	764.5380	98.0177	.7	1116.2786	118.4380
.3	263.0220	57.4911	.8	483.0513	77.9115	.3	769.4467	98.3319	.8	1122.2083	118.7522
.4	265.9044	57.8053	.9	486.9547	78.2257	.4	774.3712	98.6460	.9	1128.1538	119.0664
.5	268.8025	58.1195	25.0	490.8739	78.5398	.5	779.3113	98.9602	38.0	1134.1149	119.3805
.6	271.7163	58.4336	.1	494.8087	78.8540	.6	784.2672	99.2743	.1	1140.0918	119.6947
.7	274.6459	58.7478	.2	498.7592	79.1681	.7	789.2388	99.5885	.2	1146.0844	120.0088
.8	277.5911	59.0619	.3	502.7255	79.4823	.8	794.2260	99.9026	.3	1152.0927	120.3230
.9	280.5521	59.3761	.4	506.7075	79.7965	.9	799.2290	100.2168	.4	1158.1167	120.6372
19.0	283.5287	59.6903	.5	510.7052	80.1106	32.0	804.2477	100.5310	.5	1164.1564	120.9513
.1	286.5211	60.0044	.6	514.7185	80.4248	.1	809.2821	100.8451	.6	1170.2118	121.2655
.2	289.5292	60.3186	.7	518.7476	80.7389	.2	814.3322	101.1593	.7	1176.2830	121.5796
.3	292.5530	60.6327	.8	522.7924	81.0531	.3	819.3980	101.4734	.8	1182.3698	121.8938
.4	295.5925	60.9469	.9	526.8529	81.3672	.4	824.4796	101.7876	.9	1188.4723	122.2080
.5	298.6477	61.2611	26.0	530.9292	81.6814	.5	829.5768	102.1018	39.0	1194.5906	122.5221
.6	301.7186	61.5752	.1	535.0211	81.9956	.6	834.6897	102.4159	.1	1200.7246	122.8363
.7	304.8052	61.8894	.2	539.1287	82.3097	.7	839.8184	102.7301	.2	1206.8742	123.1504
.8	307.9075	62.2035	.3	543.2521	82.6239	.8	844.9628	103.0442	.3	1213.0396	123.4646
.9	311.0255	62.5177	.4	547.3911	82.9380	.9	850.1229	103.3584	.4	1219.2207	123.7788
20.0	314.1593	62.8319	.5	551.5459	83.2522	33.0	855.2986	103.6726	.5	1225.4175	124.0929
.1	317.3087	63.1460	.6	555.7163	83.5664	.1	860.4902	103.9867	.6	1231.6300	124.4071
.2	320.4739	63.4602	.7	559.9025	83.8805	.2	865.6973	104.3009	.7	1237.8582	124.7212
.3	323.6547	63.7743	.8	564.1044	84.1947	.3	870.9202	104.6150	.8	1244.1021	125.0354
.4	326.8513	64.0885	.9	568.3220	84.5088	.4	876.1588	104.9292	.9	1250.3617	125.3495
.5	330.0636	64.4026	27.0	572.5553	84.8230	.5	881.4131	105.2434	40.0	1256.6371	125.6637
.6	333.2916	64.7168	.1	576.8043	85.1372	.6	886.6831	105.5575	.1	1262.9281	125.9779
.7	336.5353	65.0310	.2	581.0690	85.4513	.7	891.9688	105.8717	.2	1269.2348	126.2920
.8	339.7947	65.3451	.3	585.3494	85.7655	.8	897.2703	106.1858	.3	1275.5573	126.6062
.9	343.0698	65.6593	.4	589.6455	86.0796	.9	902.5874	106.5000	.4	1281.8955	126.9203
21.0	346.3606	65.9734	.5	593.9574	86.3938	34.0	907.9203	106.8142	.5	1288.2493	127.2345
.1	349.6671	66.2876	.6	598.2849	86.7080	.1	913.2688	107.1283	.6	1294.6189	127.5487
.2	352.9893	66.6018	.7	602.6282	87.0221	.2	918.6331	107.4425	.7	1301.0042	127.8628
.3	356.3273	66.9159	.8	606.9871	87.3363	.3	924.0131	107.7566	.8	1307.4052	128.1770
.4	359.6809	67.2301	.9	611.3618	87.6504	.4	929.4088	108.0708	.9	1313.8219	128.4911
.5	363.0503	67.5442	28.0	615.7522	87.9646	.5	934.8202	108.3846	41.0	1320.2543	128.8053
.6	366.4354	67.8584	.1	620.1582	88.2788	.6	940.2473	108.6991	.1	1326.7024	129.1195
.7	369.8361	68.1726	.2	624.5800	88.5929	.7	945.6901	109.0133	.2	1333.1663	129.4336
.8	373.2526	68.4867	.3	629.0175	88.9071	.8	951.1486	109.3274	.3	1339.6458	129.7478
.9	376.6848	68.8009	.4	633.4707	89.2212	.9	956.6228	109.6416	.4	1346.1410	130.0619
22.0	380.1327	69.1150	.5	637.9397	89.5354	35.0	962.1127	109.9557	.5	1352.6520	130.3761
.1	383.5963	69.4292	.6	642.4243	89.8495	.1	967.6184	110.2699	.6	1359.1786	130.6903
.2	387.0756	69.7434	.7	646.9246	90.1637	.2	973.1397	110.5841	.7	1365.7210	131.0044
.3	390.5707	70.0575	.8	651.4406	90.4779	.3	978.6768	110.8982	.8	1372.2791	131.3186
.4	394.0814	70.3717	.9	655.9724	90.7920	.4	984.2296	111.2124	.9	1378.8529	131.6327
.5	397.6078	70.6858	29.0	660.5199	91.1062	.5	989.7980	111.5265	42.0	1385.4424	131.9469
.6	401.1500	71.0000	.1	665.0830	91.4203	.6	995.3822	111.8407	.1	1392.0476	132.2611
.7	404.7078	71.3142	.2	669.6619	91.7345	.7	1000.9821	112.1549	.2	1398.6685	132.5752
.8	408.2814	71.6283	.3	674.2565	92.0487	.8	1006.5977	112.4690	.3	1405.3051	132.8894
.9	411.8706	71.9425	.4	678.8668	92.3628	.9	1012.2290	112.7832	.4	1411.9574	133.2035
23.0	415.4756	72.2566	.5	683.4927	92.6770	36.0	1017.8760	113.0973	.5	1418.6254	133.5177
.1	419.0963	72.5708	.6	688.1345	92.9911	.1	1023.5387	113.4115	.6	1425.3092	133.8318
.2	422.7327	72.8849	.7	692.7919	93.3053	.2	1029.2172	113.7257	.7	1432.0086	134.1460
.3	426.3848	73.1991	.8	697.4650	93.6195	.3	1034.9113	114.0398	.8	1438.7238	134.4602
.4	430.0526	73.5133	.9	702.1538	93.9336	.4	1040.6211	114.3540	.9	1445.4546	134.7743
.5	433.7361	73.8274	30.0	706.8583	94.2478	.5	1046.3467	114.6681	43.0	1452.2012	135.0885
.6	437.4354	74.1416	.1	711.5786	94.5619	.6	1052.0880	114.9823	.1	1458.9635	135.4026
.7	441.1503	74.4557	.2	716.3145	94.8761	.7	1057.8449	115.2965	.2	1465.7415	135.7168
.8	444.8809	74.7699	.3	721.0662	95.1903	.8	1063.6176	115.6106	.3	1472.5352	136.0310
.9	448.6273	75.0841	.4	725.8336	95.5044	.9	1069.4060	115.9248	.4	1479.3446	136.3451
24.0	452.3893	75.3982	.5	730.6167	95.8186	37.0	1075.2101	116.2389	.5	1486.1697	136.6593
.1	456.1671	75.7124	.6	735.4154	96.1327	.1	1081.0299	116.5531	.6	1493.0105	136.9734
.2	459.9606	76.0265	.7	740.2299	96.4469	.2	1086.8654	116.8672	.7	1499.8670	137.2876
.3	463.7698	76.3407	.8	745.0601	96.7611	.3	1092.7166	117.1814	.8	1506.7393	137.6018
.4	467.5946	76.6549	.9	749.9060	97.0752	.4	1098.5835	117.4956	.9	1513.6272	137.9159

TABLE 21.—AREAS AND CIRCUMFERENCES OF CIRCLES—DECIMAL DIVISI.

Diameter	Area	Circumference	Diameter	Area	Circumference	Diameter	Area	Circumference			
44.0	1520.5308	138.2301	.5	2002.9617	158.6504	57.0	2551.7586	179.0708			
.1	1527.4502	138.5442	.6	2010.9020	158.9646	.1	2560.7200	179.3849	.6		
.2	1534.3853	138.8584	.7	2018.8581	159.2787	.2	2569.6971	179.6991	.7		
.3	1541.3360	139.1726	.8	2026.8299	159.5929	.3	2578.6899	180.0133	.8	31.	
.4	1548.3025	139.4867	.9	2034.8174	159.9071	.4	2587.6984	180.3274	.9	3206.	
.5	1555.2847	139.8009	51.0	2042.8206	160.2212	.5	2596.7227	180.6416	64.0	3216.9909	
.6	1562.2826	140.1150	.1	2050.8395	160.5354	.6	2605.7626	180.9557	.1	3227.0518	
.7	1569.2962	140.4292	.2	2058.8742	160.8495	.7	2614.8182	181.2699	.2	3237.1285	201.
.8	1576.3255	140.7434	.3	2066.9245	161.1637	.8	2623.8896	181.5841	.3	3247.2209	202.00
.9	1583.3705	141.0575	.4	2074.9905	161.4779	.9	2632.9766	181.8982	.4	3257.3289	202.3186
45.0	1590.4313	141.3717	.5	2083.0723	161.7920	58.0	2642.0794	182.2124	.5	3267.4527	202.6327
.1	1597.5077	141.6858	.6	2091.1697	162.1062	.1	2651.1979	182.5265	.6	3277.5922	202.9469
.2	1604.5999	142.0000	.7	2099.2829	162.4203	.2	2660.3321	182.8407	.7	3287.7474	203.2610
.3	1611.7077	142.3141	.8	2107.4118	162.7345	.3	2669.4820	183.1549	.8	3297.9183	203.5752
.4	1618.8313	142.6283	.9	2115.5563	163.0487	.4	2678.6475	183.4690	.9	3308.1049	203.8894
.5	1625.9705	142.9425	52.0	2123.7166	163.3628	.5	2687.8289	183.7832	65.0	3318.3072	204.2035
.6	1633.1255	143.2566	.1	2131.8926	163.6770	.6	2697.0259	184.0973	.1	3328.5253	204.5177
.7	1640.2962	143.5708	.2	2140.0843	163.9911	.7	2706.2386	184.4115	.2	3338.7590	204.8318
.8	1647.4826	143.8849	.3	2148.2917	164.3053	.8	2715.4670	184.7256	.3	3349.0085	205.1460
.9	1654.6847	144.1991	.4	2156.5149	164.6195	.9	2724.7112	185.0398	.4	3359.2736	205.4602
46.0	1661.9025	144.5133	.5	2164.7537	164.9336	59.0	2733.9710	185.3540	.5	3369.5545	205.7743
.1	1669.1360	144.8274	.6	2173.0082	165.2478	.1	2743.2465	185.6681	.6	3379.8510	206.0885
.2	1676.3852	145.1416	.7	2181.2785	165.5619	.2	2752.5378	185.9823	.7	3390.1633	206.4026
.3	1683.6502	145.4557	.8	2189.5644	165.8761	.3	2761.8448	186.2964	.8	3400.4913	206.7168
.4	1690.9308	145.7699	.9	2197.8661	166.1903	.4	2771.1675	186.6106	.9	3410.8350	207.0310
.5	1698.2272	146.0841	53.0	2206.1834	166.5044	.5	2780.5058	186.9248	66.0	3421.1944	207.3451
.6	1705.5392	146.3982	.1	2214.5165	166.8186	.6	2789.8599	187.2389	.1	3431.5695	207.6593
.7	1712.8670	146.7124	.2	2222.8653	167.1327	.7	2799.2297	187.5531	.2	3441.9603	207.9734
.8	1720.2105	147.0265	.3	2231.2298	167.4469	.8	2808.6152	187.8672	.3	3452.3669	208.2876
.9	1727.5696	147.3407	.4	2239.6100	167.7610	.9	2818.0165	188.1814	.4	3462.7891	208.6017
47.0	1734.9445	147.6549	.5	2248.0059	168.1752	60.0	2827.4334	188.4956	.5	3473.2270	208.9159
.1	1742.3351	147.9690	.6	2256.4175	168.3894	.1	2836.8660	188.8097	.6	3483.6807	209.2301
.2	1749.7414	148.2832	.7	2264.8448	168.7035	.2	2846.3143	189.1239	.7	3494.1500	209.5442
.3	1757.1634	148.5973	.8	2273.2879	169.0177	.3	2855.7784	189.4380	.8	3504.6351	209.8584
.4	1764.6012	148.9115	.9	2281.7466	169.3318	.4	2865.2582	189.7522	.9	3515.1359	210.1725
.5	1772.0546	149.2257	54.0	2290.2210	169.6460	.5	2874.7536	190.0664	67.0	3525.6524	210.4867
.6	1779.5237	149.5398	.1	2298.7112	169.9602	.6	2884.2648	190.3805	.1	3536.1845	210.8009
.7	1787.0086	149.8540	.2	2307.2171	170.2743	.7	2893.7917	190.6947	.2	3546.7324	211.1150
.8	1794.5091	150.1681	.3	2315.7386	170.5885	.8	2903.3343	191.0088	.3	3557.2960	211.4292
.9	1802.0254	150.4823	.4	2324.2759	170.9026	.9	2912.8925	191.3230	.4	3567.8754	211.7433
48.0	1809.5574	150.7964	.5	2332.8289	171.2168	61.0	2922.4666	191.6372	.5	3578.4704	212.0575
.1	1817.1050	151.1106	.6	2341.3976	171.5310	.1	2932.0563	191.9513	.6	3589.0811	212.3717
.2	1824.6684	151.4248	.7	2349.9820	171.8451	.2	2941.6617	192.2655	.7	3599.7075	212.6858
.3	1832.2475	151.7389	.8	2358.5821	172.1593	.3	2951.2828	192.5796	.8	3610.3497	213.0000
.4	1839.8423	152.0531	.9	2367.1979	172.4734	.4	2960.9196	192.8938	.9	3621.0075	213.3141
.5	1847.4528	152.3672	55.0	2375.8294	172.7876	.5	2970.5722	193.2079	68.0	3631.6811	213.6283
.6	1855.0790	152.6814	.1	2384.4767	173.1018	.6	2980.2404	193.5221	.1	3642.3704	213.9425
.7	1862.7210	152.9956	.2	2393.1396	173.4159	.7	2989.9244	193.8363	.2	3653.0754	214.2566
.8	1870.3786	153.3097	.3	2401.8183	173.7301	.8	2999.6241	194.1504	.3	3663.7960	214.5708
.9	1878.0519	153.6239	.4	2410.5126	174.0442	.9	3009.3394	194.4646	.4	3674.5324	214.8849
49.0	1885.7410	153.9380	.5	2419.2227	174.3584	62.0	3019.0705	194.7787	.5	3685.2845	215.1991
.1	1893.4457	154.2522	.6	2427.9485	174.6726	.1	3028.8173	195.0929	.6	3696.0523	215.5133
2	1901.1662	154.5664	.7	2436.6899	174.9867	.2	3038.5798	195.4071	.7	3706.8359	215.8274
.3	1908.9024	154.8805	.8	2445.4417	175.3009	.3	3048.3580	195.7212	.8	3717.6351	216.1416
.4	1916.6543	155.1947	.9	2454.2200	175.6150	.4	3058.1519	196.0354	.9	3728.4500	216.4556
.5	1924.4218	155.5088	56.0	2463.0086	175.9292	.5	3067.9616	196.3495	69.0	3739.2807	216.7699
.6	1932.2051	155.8230	.1	2471.8129	176.2433	.6	3077.7869	196.6637	.1	3750.1270	217.0841
.7	1940.0041	156.1372	.2	2480.6330	176.5575	.7	3087.6279	196.9779	.2	3760.9891	217.3982
.8	1947.8189	156.4513	.3	2489.4687	176.8717	.8	3097.4847	197.2920	.3	3771.8668	217.7124
.9	1955.6493	156.7655	.4	2498.3201	177.1858	.9	3107.3571	197.6062	.4	3782.7603	218.0265
50.0	1963.4954	157.0796	.5	2507.1873	177.5000	63.0	3117.2453	197.9203	.5	3793.6695	218.3407
.1	1971.3572	157.3938	.6	2516.0701	177.8141	.1	3127.1492	198.2345	.6	3804.5944	218.6548
.2	1979.2348	157.7080	.7	2524.9687	178.1283	.2	3137.0687	198.5487	.7	3815.5350	218.9690
.3	1987.1280	158.0221	.8	2533.8830	178.4425	.3	3147.0040	198.8628	.8	3826.4913	219.2832
.4	1995.0370	158.3363	.9	2542.8129	178.7566	.4	3156.9550	199.1770	.9	3837.4633	219.5973

TABLE 21.—AREAS AND CIRCUMFERENCES OF CIRCLES—DECIMAL DIVISIONS—(*Continued*)

Diameter	Area	Circumference	Diameter	Area	Circumference	Diameter	Area	Circumference	Diameter	Area	Circumference
70.0	3848.4510	219.9115	.5	4596.3464	240.3318	83.0	5410.6079	260.7522	.5	6291.2356	281.1725
.1	3859.4544	220.2256	.6	4608.3708	240.6460	.1	5423.6534	261.0663	.6	6305.3021	281.4867
.2	3870.4735	220.5398	.7	4620.4110	240.9602	.2	5436.7146	261.3805	.7	6319.3843	281.8009
.3	3881.5084	220.8540	.8	4632.4668	241.2743	.3	5449.7914	261.6947	.8	6333.4822	282.1150
.4	3892.5589	221.1681	.9	4644.5384	241.5885	.4	5462.8840	262.0088	.9	6347.5958	282.4292
.5	3903.6252	221.4823	77.0	4656.6257	241.9026	.5	5475.9923	262.3230	90.0	6361.7251	282.7433
.6	3914.7072	221.7964	.1	4668.7287	242.2168	.6	5489.1163	262.6371	.1	6375.8701	283.0575
.7	3925.8048	222.1106	.2	4680.8474	242.5310	.7	5502.2560	262.9513	.2	6390.0308	283.3717
.8	3936.9182	222.4248	.3	4692.9818	242.8451	.8	5515.4115	263.2655	.3	6404.2073	283.6858
.9	3948.0473	222.7389	.4	4705.1319	243.1592	.9	5528.5826	263.5796	.4	6418.3994	284.0000
71.0	3959.1921	223.0531	.5	4717.2977	243.4734	84.0	5541.7694	263.8938	.5	6432.6073	284.3141
.1	3970.3526	223.3672	.6	4729.4792	243.7876	.1	5554.9720	264.2079	.6	6446.8308	284.6283
.2	3981.5288	223.6814	.7	4741.6765	244.1017	.2	5568.1902	264.5221	.7	6461.0701	284.9425
.3	3992.7208	223.9956	.8	4753.8894	244.4159	.3	5581.4242	264.8363	.8	6475.3251	285.2566
.4	4003.9284	224.3097	.9	4766.1180	244.7301	.4	5594.6738	265.1504	.9	6489.5958	285.5708
.5	4015.1517	224.6239	78.0	4778.3624	245.0442	.5	5607.9392	265.4646	91.0	6503.8822	285.8849
.6	4026.3908	224.9380	.1	4790.6225	245.3584	.6	5621.2203	265.7787	.1	6518.1843	286.1991
.7	4037.6455	225.2522	.2	4802.8982	245.6725	.7	5634.5171	266.0929	.2	6532.5021	286.5132
.8	4048.9160	225.5664	.3	4815.1897	245.9867	.8	5647.8296	266.4071	.3	6546.8356	286.8274
.9	4060.2022	225.8805	.4	4827.4969	246.3009	.9	5661.1578	266.7212	.4	6561.1848	287.1416
72.0	4071.5041	226.1947	.5	4839.8198	246.6150	85.0	5674.5017	267.0354	.5	6575.5497	287.4557
.1	4082.8216	226.5088	.6	4852.1584	246.9292	.1	5687.8613	267.3495	.6	6589.9304	287.7699
.2	4094.1549	226.8230	.7	4864.5127	247.2433	.2	5701.2367	267.6637	.7	6604.3267	288.0840
.3	4105.5039	227.1371	.8	4876.8828	247.5575	.3	5714.6277	267.9779	.8	6618.7388	288.3982
.4	4116.8687	227.4513	.9	4889.2685	247.8717	.4	5728.0344	268.2920	.9	6633.1666	288.7124
.5	4128.2491	227.7655	79.0	4901.6699	248.1858	.5	5741.4569	268.6062	92.0	6647.6100	289.0265
.6	4139.6452	228.0796	.1	4914.0871	248.5000	.6	5754.8951	268.9203	.1	6662.0692	289.3407
.7	4151.0570	228.3938	.2	4926.5199	248.8141	.7	5768.3489	269.2345	.2	6676.5441	289.6548
.8	4162.4846	228.7079	.3	4938.9685	249.1283	.8	5781.8185	269.5486	.3	6691.0347	289.9690
.9	4173.9278	229.0221	.4	4951.4328	249.4425	.9	5795.3038	269.8628	.4	6705.5410	290.2832
73.0	4185.3868	229.3363	.5	4963.9127	249.7566	86.0	5808.8048	270.1770	.5	6720.0630	290.5973
.1	4196.8615	229.6504	.6	4976.4084	250.0708	.1	5822.3215	270.4911	.6	6734.6007	290.9115
.2	4208.3518	229.9646	.7	4988.9198	250.3849	.2	5835.8539	270.8053	.7	6749.1542	291.2256
.3	4219.8579	230.2787	.8	5001.4469	250.6991	.3	5849.4020	271.1194	.8	6763.7233	291.5398
.4	4231.3797	230.5929	.9	5013.9897	251.0133	.4	5862.9659	271.4336	.9	6778.3081	291.8540
.5	4242.9172	230.9071	80.0	5026.5482	251.3274	.5	5876.5454	271.7478	93.0	6792.9087	292.1681
.6	4254.4704	231.2212	.1	5039.1224	251.6416	.6	5890.1406	272.0619	.1	6807.5249	292.4823
.7	4266.0393	231.5354	.2	5051.7124	251.9557	.7	5903.7516	272.3761	.2	6822.1569	292.7964
.8	4277.6240	231.8495	.3	5064.3180	252.2699	.8	5917.3782	272.6902	.3	6836.8046	293.1106
.9	4289.2243	232.1637	.4	5076.9394	252.5840	.9	5931.0206	273.0044	.4	6851.4680	293.4248
74.0	4300.8403	232.4779	.5	5089.5764	252.8982	87.0	5944.6787	273.3186	.5	6866.1471	293.7389
.1	4312.4721	232.7920	.6	5102.2292	253.2124	.1	5958.3525	273.6327	.6	6880.8419	294.0531
.2	4324.1195	233.1062	.7	5114.8977	253.5265	.2	5972.0419	273.9469	.7	6895.5524	294.3672
.3	4335.7827	233.4203	.8	5127.5818	253.8407	.3	5985.7471	274.2610	.8	6910.2786	294.6814
.4	4347.4616	233.7345	.9	5140.2817	254.1548	.4	5999.4680	274.5752	.9	6925.0205	294.9956
.5	4359.1562	234.0487	81.0	5152.9973	254.4690	.5	6013.2047	274.8894	94.0	6939.7781	295.3097
.6	4370.8664	234.3628	.1	5165.7286	254.7832	.6	6026.9570	275.2035	.1	6954.5515	295.6239
.7	4382.5924	234.6770	.2	5178.4756	255.0973	.7	6040.7250	275.5177	.2	6969.3405	295.9380
.8	4394.3341	234.9911	.3	5191.2384	255.4115	.8	6054.5088	275.8318	.3	6984.1453	296.2522
.9	4406.0915	235.3053	.4	5204.0168	255.7256	.9	6068.3082	276.1460	.4	6998.9657	296.5663
75.0	4417.8647	235.6194	.5	5216.8109	256.0398	88.0	6082.1234	276.4602	.5	7013.8019	296.8805
.1	4429.6535	235.9336	.6	5229.6208	256.3540	.1	6095.9542	276.7743	.6	7028.6538	297.1947
.2	4441.4580	236.2478	.7	5242.4463	256.6681	.2	6109.8008	277.0885	.7	7043.5214	297.5088
.3	4453.2783	236.5619	.8	5255.2876	256.9823	.3	6123.6631	277.4026	.8	7058.4047	297.8230
.4	4465.1142	236.8761	.9	5268.1446	257.2964	.4	6137.5410	277.7168	.9	7073.3037	298.1371
.5	4476.9659	237.1902	82.0	5281.0172	257.6106	.5	6151.4347	278.0309	95.0	7088.2184	298.4513
.6	4488.8332	237.5044	.1	5293.9056	257.9248	.6	6165.3441	278.3451	.1	7103.1488	298.7655
.7	4500.7163	237.8186	.2	5306.8097	258.2389	.7	6179.2692	278.6593	.2	7118.0949	299.0796
.8	4512.6151	238.1327	.3	5319.7295	258.5531	.8	6193.2101	278.9734	.3	7133.0568	299.3938
.9	4524.5296	238.4469	.4	5332.6650	258.8672	.9	6207.1666	279.2876	.4	7148.0343	299.7079
76.0	4536.4598	238.7610	.5	5345.6162	259.1814	89.0	6221.1388	279.6017	.5	7163.0276	300.0221
.1	4548.4057	239.0752	.6	5358.5832	259.4956	.1	6235.1268	279.9159	.6	7178.0365	300.3363
.2	4560.3673	239.3894	.7	5371.5658	259.8097	.2	6249.1304	280.2301	.7	7193.0612	300.6504
.3	4572.3446	239.7035	.8	5384.5641	260.1239	.3	6263.1498	280.5442	.8	7208.1016	300.9646
.4	4584.3376	240.0177	.9	5397.5782	260.4380	.4	6277.1848	280.8584	.9	7223.1577	301.2787

TABLE 21.—AREAS AND CIRCUMFERENCES OF CIRCLES—DECIMAL DIVISIONS—(*Continued*)

Diameter	Area	Circumference	Diameter	Area	Circumference	Diameter	Area	Circumference	Diameter	Area	Circumference
96.0	7238.2294	301.5929	97.0	7389.8113	304.7345	98.0	7542.9639	307.8761	99.0	7697.6874	311.0177
.1	7253.3169	301.9071	.1	7405.0559	305.0486	.1	7558.3656	308.1902	.1	7713.2461	311.3318
.2	7268.4201	302.2212	.2	7420.3162	305.3628	.2	7573.7830	308.5044	.2	7728.8205	311.6460
.3	7283.5391	302.5354	.3	7435.5921	305.6770	.3	7589.2161	308.8186	.3	7744.4107	311.9602
.4	7298.6737	302.8495	.4	7450.8838	305.9911	.4	7604.6648	309.1327	.4	7760.0166	312.2743
.5	7313.8240	303.1637	.5	7466.1913	306.3053	.5	7620.1293	309.4469	.5	7775.6381	312.5885
.6	7328.9901	303.4779	.6	7481.5144	306.6194	.6	7635.6095	309.7610	.6	7791.2754	312.9026
.7	7344.1718	303.7920	.7	7496.8532	306.9336	.7	7651.1054	310.0752	.7	7806.9284	313.2168
.8	7359.3693	304.1062	.8	7512.2077	307.2478	.8	7666.6170	310.3894	.8	7822.5971	313.5309
.9	7374.5824	304.4203	.9	7527.5780	307.5619	.9	7682.1443	310.7035	.9	7838.2815	313.8415
									100.0	7853.9816	314.1593

For larger integral circles see Table 20.

TABLE 22.—AREAS, CIRCUMFERENCES, SQUARES, CUBES AND FOURTH POWERS OF BINARY FRACTIONAL QUANTITIES
(*F. E. Kelley, Amer. Mach., July* 25, 1901)

	Area	Cir.	Square	Cube	4th power		Area	Cir.	Square	Cube	4th power
1/64	.00019	.04909	.000244	.0000038	.00000006	1	.78540	3.14159	1.0	1.0	1.0
1/32	.00077	.0982	.000977	.0000305	.0000009	1/32	.83525	3.2398	1.0635	1.0967	1.13
3/64	.00173	.1473	.002197	.0001030	.0000048	1/16	.88664	3.3379	1.1289	1.1995	1.27
1/16	.0031	.1963	.003906	.0002441	.0000152	3/32	.93956	3.4361	1.1963	1.3084	1.43
5/64	.0048	.2454	.006104	.0004768	.0000372	1/8	.99402	3.5343	1.2656	1.4238	1.60
3/32	.0069	.2945	.008789	.0008240	.0000771	5/32	1.05	3.6325	1.3369	1.5458	1.78
7/64	.0094	.3436	.011963	.0013084	.0001430	3/16	1.1075	3.7306	1.4102	1.6746	1.98
1/8	.0123	.3927	.015625	.0019531	.000244	7/32	1.1666	3.8288	1.4854	1.8103	2.20
9/64	.0155	.4418	.019775	.002781	.0003908	1/4	1.2272	3.9270	1.5625	1.9531	2.45
5/32	.0192	.4909	.024414	.003815	.0005954	9/32	1.2893	4.0252	1.6416	2.1033	2.69
11/64	.0232	.5400	.029541	.005077	.0008702	5/16	1.3530	4.1233	1.7227	2.2610	2.97
3/16	.0276	.5890	.035156	.006592	.001232	11/32	1.4182	4.2215	1.8057	2.4264	3.26
13/64	.0324	.6381	.041260	.008381	.001697	3/8	1.4849	4.3197	1.8906	2.5996	3.57
7/32	.0376	.6872	.047852	.010468	.002284	13/32	1.5532	4.4179	1.9775	2.7809	3.91
15/64	.0431	.7363	.054932	.012875	.003014	7/16	1.6230	4.5160	2.0664	2.9705	4.27
1/4	.0491	.7854	.0625	.015625	.003906	15/32	1.6943	4.6142	2.1572	3.1684	4.65
17/64	.0554	.8345	.070557	.018742	.004970	1/2	1.7671	4.7124	2.25	3.375	5.06
9/32	.0621	.8836	.079102	.022247	.006256	17/32	1.8415	4.8106	2.3447	3.5904	5.47
19/64	.0692	.9327	.088135	.026165	.007762	9/16	1.9175	4.9087	2.4414	3.8147	5.95
5/16	.0767	.9817	.097656	.030518	.009530	19/32	1.9949	5.0069	2.5400	4.0482	6.45
21/64	.0846	1.0308	.107666	.035328	.01158	5/8	2.0739	5.1051	2.6406	4.2910	6.97
11/32	.0928	1.0799	.118164	.040619	.01395	21/32	2.1545	5.2033	2.7432	4.5434	7.50
23/64	.1014	1.1290	.129150	.046413	.01668	11/16	2.2365	5.3014	2.8477	4.8054	8.07
3/8	.1104	1.1781	.140625	.052734	.01977	23/32	2.3201	5.3996	2.9541	5.0774	8.70
25/64	.1198	1.2272	.152588	.059605	.02328	3/4	2.4053	5.4978	3.0625	5.3594	9.38
13/32	.1296	1.2763	.165039	.067047	.02723	25/32	2.4920	5.5960	3.1729	5.6516	10.05
27/64	.1398	1.3254	.177979	.075085	.03165	13/16	2.5802	5.6941	3.2852	5.9543	10.76
7/16	.1503	1.3744	.191406	.083740	.03663	27/32	2.6699	5.7923	3.3994	6.2677	11.56
29/64	.1613	1.4235	.205322	.093037	.04215	7/8	2.7612	5.8905	3.5156	6.5918	12.4
15/32	.1726	1.4726	.219727	.102997	.04837	29/32	2.8540	5.9887	3.6338	6.9269	13.2
31/64	.1843	1.5217	.234619	.113644	.05504	15/16	2.9483	6.0868	3.7539	7.2732	14.1
1/2	.1963	1.5708	.25	.125	.0625	31/32	3.0442	6.1850	3.8760	7.6308	14.9
33/64	.2088	1.6199	.265869	.137089	.07071	2	3.1416	6.2832	4.0	8.0	16.0
17/32	.2217	1.6690	.282227	.149933	.07963	1/16	3.3410	6.4795	4.2539	8.7737	18.1
35/64	.2349	1.7181	.299072	.163555	.08946	1/8	3.5466	6.6759	4.5156	9.5957	20.4
9/16	.2485	1.7671	.316406	.177979	.10011	3/16	3.7583	6.8722	4.7852	10.4675	22.9
37/64	.2625	1.8162	.334229	.193226	.11169	1/4	3.9761	7.0686	5.0625	11.3906	25.6
19/32	.2769	1.8653	.352559	.209320	.124191	5/16	4.2000	7.2649	5.3477	12.3665	28.6
39/64	.2916	1.9144	.371338	.226284	.137901	3/8	4.4301	7.4613	5.6406	13.3965	31.8
5/8	.3068	1.9635	.390625	.244141	.152568	7/16	4.6664	7.6576	5.9414	14.4822	35.3
41/64	.3252	2.0126	.410400	.262913	.168428	1/2	4.9087	7.8540	6.25	15.625	39.1
21/32	.3382	2.0617	.430664	.282623	.18546	9/16	5.1572	8.0503	6.5664	16.8264	43.0
43/64	.3545	2.1108	.451416	.303295	.20376	5/8	5.4119	8.2467	6.8906	18.0879	47.5
11/16	.3712	2.1598	.472656	.324951	.22340	11/16	5.6727	8.4430	7.2227	19.4109	52.1
45/64	.3883	2.2089	.494385	.347614	.24438	3/4	5.9396	8.6394	7.5625	20.7969	57.2
23/32	.4057	2.2580	.516602	.371307	.26688	13/16	6.2126	8.8357	7.9102	22.2473	62.6
47/64	.4236	2.3071	.539307	.396053	.29084	7/8	6.4918	9.0321	8.2656	23.7637	68.3
3/4	.4418	2.3562	.5625	.421875	.31641	15/16	6.7771	9.2284	8.6289	25.3474	74.5
49/64	.4602	2.4048	.586182	.448795	.34357	3	7.0686	9.4248	9.0	27.0	81.0
25/32	.4794	2.4544	.610352	.476837	.3721	1/8	7.6699	9.8175	9.7656	30.5176	95.4
51/64	.4987	2.5030	.635010	.506023	.4032	1/4	8.2958	10.210	10.5626	34.3281	112.0
13/16	.5185	2.5525	.660156	.536377	.4356	3/8	8.9462	10.603	11.391	38.443	130
53/64	.5387	2.6011	.685791	.567921	.4704	1/2	9.6211	10.996	12.25	42.875	150
27/32	.5591	2.6507	.711914	.600677	.5069	5/8	10.321	11.388	13.141	47.635	173
55/64	.5796	2.7017	.738525	.634670	.5456	3/4	11.045	11.781	14.062	52.734	198
7/8	.6013	2.7499	.765625	.669922	.5861	7/8	11.793	12.174	15.016	58.186	225
57/64	.6228	2.7960	.793213	.706455	.6292	4	12.566	12.566	16.0	64.0	256
29/32	.6450	2.8471	.821289	.744293	.6750	1/8	13.364	12.959	17.016	70.189	290
59/64	.6675	2.8965	.849854	.783459	.7225	1/4	14.186	13.352	18.062	76.766	326
15/16	.6903	2.9452	.878906	.823975	.7725	3/8	15.033	13.744	19.141	83.740	366
61/64	.7131	2.9939	.908447	.865864	.8248	1/2	15.904	14.137	20.25	91.125	410
31/32	.7371	3.0434	.938477	.909149	.8798	5/8	16.800	14.530	21.391	98.932	458
63/64	.7610	3.0913	.968994	.953854	.9385	3/4	17.721	14.923	22.562	107.172	509
1	.7854	3.1416	1.0	1.0	1.0000	5	19.635	15.708	25.0	125.0	625

Table 23.—Areas and Circumferences of Circles—Binary Divisions

For more Minute Divisions of Small Circles see Table 22

Diam.	Cir.	Area
1/16 in.	.1963	.00307
1/8	.3927	.0123
1/4	.7854	.0491
3/8	1.1781	.1104
1/2	1.5708	.1963
5/8	1.9635	.3068
3/4	2.3562	.4418
7/8	2.7489	.6013
1 in.	3.1416	.7854
1/8	3.5343	.9940
1/4	3.9270	1.2272
3/8	4.3197	1.4849
1/2	4.7124	1.7671
5/8	5.1051	2.0739
3/4	5.4978	2.4053
7/8	5.8905	2.7611
2 ins.	6.2832	3.1416
1/8	6.6759	3.5466
1/4	7.0686	3.9761
3/8	7.4613	4.4302
1/2	7.8540	4.9087
5/8	8.2467	5.4119
3/4	8.6394	5.9396
7/8	9.0321	6.4918
3 ins.	9.4248	7.0686
1/8	9.8175	7.6699
1/4	10.210	8.2958
3/8	10.603	8.9462
1/2	10.996	9.6211
5/8	11.388	10.321
3/4	11.781	11.045
7/8	12.174	11.793
4 ins.	12.566	12.566
1/8	12.959	13.364
1/4	13.352	14.186
3/8	13.744	15.033
1/2	14.137	15.904
5/8	14.530	16.800
3/4	14.923	17.721
7/8	15.315	18.665
5 ins.	15.708	19.635
1/8	16.101	20.629
1/4	16.493	21.648
3/8	16.886	22.691
1/2	17.279	23.758
5/8	17.671	24.850
3/4	18.064	25.967
7/8	18.457	27.109
6 ins.	18.850	28.274
1/8	19.242	29.465
1/4	19.635	30.680
3/8	20.028	31.919
1/2	20.420	33.183
5/8	20.813	34.472
3/4	21.205	35.784
7/8	21.598	37.122
7 ins.	21.991	38.485
1/8	22.384	39.871
1/4	22.776	41.282
3/8	23.169	42.718
1/2	23.562	44.179
5/8	23.955	45.664
3/4	24.347	47.173
7/8	24.740	48.707
8 ins.	25.133	50.265
1/8	25.525	51.849
1/4	25.918	53.456
3/8	26.311	55.088
1/2	26.704	56.745
5/8	27.096	58.426
3/4	27.489	60.132
7/8	27.882	61.862

Diam.	Cir.	Area
9 ins.	28.274	63.617
1/8	28.667	65.397
1/4	29.060	67.201
3/8	29.452	69.029
1/2	29.845	70.882
5/8	30.238	72.760
3/4	30.631	74.662
7/8	31.023	76.589
10 ins.	31.416	78.540
1/8	31.809	80.516
1/4	32.201	82.516
3/8	32.594	84.541
1/2	32.987	86.590
5/8	33.379	88.664
3/4	33.772	90.763
7/8	34.165	92.886
11 ins.	34.558	95.033
1/8	34.950	97.205
1/4	35.343	99.402
3/8	35.736	101.623
1/2	36.128	103.869
5/8	36.521	106.139
3/4	36.914	108.434
7/8	37.306	110.753
12 ins.	37.699	113.097
1/8	38.092	115.466
1/4	38.485	117.859
3/8	38.877	120.276
1/2	39.270	122.718
5/8	39.663	125.184
3/4	40.055	127.676
7/8	40.448	130.192
13 ins.	40.841	132.732
1/8	41.233	135.297
1/4	41.626	137.886
3/8	42.019	140.500
1/2	42.412	143.139
5/8	42.804	145.802
3/4	43.197	148.489
7/8	43.590	151.201
14 ins.	43.982	153.938
1/8	44.375	156.699
1/4	44.768	159.485
3/8	45.160	162.295
1/2	45.553	165.130
5/8	45.946	167.989
3/4	46.338	170.873
7/8	46.731	173.782
15 ins.	47.124	176.715
1/8	47.517	179.672
1/4	47.909	182.654
3/8	48.302	185.661
1/2	48.694	188.692
5/8	49.087	191.748
3/4	49.480	194.828
7/8	49.873	197.933
16 ins.	50.265	201.062
1/8	50.658	204.216
1/4	51.051	207.394
3/8	51.444	210.597
1/2	51.836	213.825
5/8	52.229	217.077
3/4	52.622	220.353
7/8	53.014	223.654
17 ins.	53.407	226.980
1/8	53.780	230.330
1/4	54.192	233.705
3/8	54.585	237.104
1/2	54.978	240.528
5/8	55.371	243.977
3/4	55.763	247.450
7/8	56.156	250.947

Diam.	Cir.	Area
18 ins.	56.549	254.469
1/8	56.941	258.016
1/4	57.334	261.587
3/8	57.727	265.182
1/2	58.119	268.803
5/8	58.512	272.447
3/4	58.905	276.117
7/8	59.298	279.811
19 ins.	59.690	283.529
1/8	60.083	287.272
1/4	60.476	291.039
3/8	60.868	294.831
1/2	61.261	298.648
5/8	61.654	302.489
3/4	62.046	306.355
7/8	62.439	310.245
20 ins.	62.832	314.160
1/8	63.225	318.099
1/4	63.617	322.063
3/8	64.010	326.051
1/2	64.403	330.064
5/8	64.795	334.101
3/4	65.188	338.163
7/8	65.581	342.250
21 ins.	65.973	346.361
1/8	66.366	350.497
1/4	66.759	354.657
3/8	67.152	358.841
1/2	67.544	363.051
5/8	67.937	367.284
3/4	68.330	371.543
7/8	68.722	375.826
22 ins.	69.115	380.133
1/8	69.508	384.465
1/4	69.900	388.822
3/8	70.293	393.203
1/2	70.686	397.608
5/8	71.079	402.038
3/4	71.471	406.493
7/8	71.864	410.972
23 ins.	72.257	415.476
1/8	72.649	420.004
1/4	73.042	424.557
3/8	73.435	429.135
1/2	73.827	433.731
5/8	74.220	438.363
3/4	74.613	443.014
7/8	75.006	447.699
24 ins.	75.398	452.390
1/8	75.791	457.115
1/4	76.184	461.864
3/8	76.576	466.638
1/2	76.969	471.436
5/8	77.362	476.259
3/4	77.754	481.106
7/8	78.147	485.978
25 ins.	78.540	490.875
1/8	78.933	495.796
1/4	79.325	500.741
3/8	79.718	505.711
1/2	80.111	510.706
5/8	80.503	515.725
3/4	80.896	520.769
7/8	81.289	525.837
26 ins.	81.681	530.930
1/8	82.074	536.047
1/4	82.467	541.189
3/8	82.860	546.356
1/2	83.252	551.547
5/8	83.645	556.762
3/4	84.038	562.002
7/8	84.430	567.267

Diam.	Cir.	Area
27 ins.	84.823	572.556
1/8	85.216	577.870
1/4	85.608	583.208
3/8	86.001	588.571
1/2	86.394	593.958
5/8	86.786	599.370
3/4	87.179	604.807
7/8	87.572	610.268
28 ins.	87.965	615.753
1/8	88.357	621.263
1/4	88.750	626.798
3/8	89.143	632.357
1/2	89.535	637.941
5/8	89.928	643.554
3/4	90.321	649.182
7/8	90.713	654.839
29 ins.	91.106	660.521
1/8	91.499	666.237
1/4	91.892	671.958
3/8	92.284	677.714
1/2	92.677	683.494
5/8	93.070	689.298
3/4	93.462	695.128
7/8	93.855	700.981
30 ins.	94.248	706.860
1/8	94.640	712.762
1/4	95.033	718.690
3/8	95.426	724.641
1/2	95.819	730.618
5/8	96.211	736.619
3/4	96.604	742.644
7/8	96.997	748.694
31 ins.	97.389	754.769
1/8	97.782	760.868
1/4	98.175	766.992
3/8	98.567	773.140
1/2	98.960	779.313
5/8	99.353	785.510
3/4	99.746	791.732
7/8	100.138	797.978
32 ins.	100.531	804.249
1/8	100.924	810.545
1/4	101.316	816.865
3/8	101.709	823.209
1/2	102.102	829.578
5/8	102.494	835.972
3/4	102.887	842.390
7/8	103.280	848.833
33 ins.	103.673	855.30
1/8	104.065	861.79
1/4	104.458	868.30
3/8	104.851	874.85
1/2	105.243	881.41
5/8	105.636	888.00
3/4	106.029	894.62
7/8	106.421	901.26
34 ins.	106.814	907.92
1/8	107.207	914.61
1/4	107.600	921.32
3/8	107.992	928.06
1/2	108.385	934.82
5/8	108.778	941.61
3/4	109.170	948.42
7/8	109.563	955.25
35 ins.	109.956	962.11
1/8	110.348	969.00
1/4	110.741	975.91
3/8	111.134	982.84
1/2	111.527	989.80
5/8	111.919	996.78
3/4	112.312	1003.78
7/8	112.705	1010.82

Diam.	Cir.	Area
36 ins.	113.097	1017.87
1/8	113.490	1024.95
1/4	113.883	1032.06
3/8	114.275	1039.19
1/2	114.668	1046.35
5/8	115.061	1053.52
3/4	115.453	1060.73
7/8	115.846	1067.95
37 ins.	116.239	1075.21
1/8	116.631	1082.48
1/4	117.024	1089.79
3/8	117.417	1097.11
1/2	117.810	1104.46
5/8	118.202	1111.84
3/4	118.596	1119.24
7/8	118.988	1126.66
38 ins.	119.381	1134.11
1/8	119.773	1141.59
1/4	120.166	1149.08
3/8	120.559	1156.61
1/2	120.951	1164.15
5/8	121.344	1171.73
3/4	121.737	1179.32
7/8	122.129	1186.94
39 ins.	122.522	1194.59
1/8	122.915	1202.26
1/4	123.308	1209.95
3/8	123.700	1217.67
1/2	124.093	1225.42
5/8	124.486	1233.18
3/4	124.878	1240.98
7/8	125.271	1248.79
40 ins.	125.664	1256.64
1/8	126.056	1264.50
1/4	126.449	1272.39
3/8	126.842	1280.31
1/2	127.235	1288.25
5/8	127.627	1296.21
3/4	128.020	1304.20
7/8	128.413	1312.21
41 ins.	128.805	1320.25
1/8	129.198	1328.32
1/4	129.591	1336.40
3/8	129.983	1344.51
1/2	130.376	1352.65
5/8	130.769	1360.81
3/4	131.161	1369.00
7/8	131.554	1377.21
42 ins.	131.947	1385.44
1/8	132.340	1393.70
1/4	132.732	1401.98
3/8	133.125	1410.29
1/2	133.518	1418.62
5/8	133.910	1426.98
3/4	134.303	1435.36
7/8	134.696	1443.77
43 ins.	135.088	1452.20
1/8	135.481	1460.65
1/4	135.874	1469.13
3/8	136.267	1477.63
1/2	136.659	1486.17
5/8	137.052	1494.72
3/4	137.445	1503.30
7/8	137.837	1511.90
44 ins.	138.230	1520.53
1/8	138.623	1529.18
1/4	139.015	1537.86
3/8	139.408	1546.55
1/2	139.801	1555.28
5/8	140.194	1564.03
3/4	140.586	1572.81
7/8	140.979	1581.61

Diam.	Cir.	Area
45 ins.	141.372	1590.43
1/8	141.764	1599.28
1/4	142.157	1608.15
3/8	142.550	1617.04
1/2	142.942	1625.97
5/8	143.335	1634.92
3/4	143.728	1643.89
7/8	144.121	1652.88
46 ins.	144.513	1661.90
1/8	144.906	1670.95
1/4	145.299	1680.01
3/8	145.691	1689.10
1/2	146.084	1698.23
5/8	146.477	1707.37
3/4	146.869	1716.54
7/8	147.262	1725.73
47 ins.	147.655	1734.94
1/8	148.048	1744.18
1/4	148.440	1753.45
3/8	148.833	1762.73
1/2	149.226	1772.05
5/8	149.618	1781.39
3/4	150.011	1790.76
7/8	150.404	1800.14
48 ins.	150.796	1809.56
1/8	151.189	1818.99
1/4	151.582	1828.46
3/8	151.975	1837.93
1/2	152.367	1847.45
5/8	152.760	1856.99
3/4	153.153	1866.55
7/8	153.545	1876.13
49 ins.	153.938	1885.74
1/8	154.331	1895.37
1/4	154.723	1905.03
3/8	155.116	1914.70
1/2	155.509	1924.42
5/8	155.902	1934.15
3/4	156.294	1943.91
7/8	156.687	1953.69
50 ins.	157.080	1963.50
1/4	157.865	1983.18
1/2	158.650	2002.96
3/4	159.436	2022.84
51 ins.	160.221	2042.82
1/4	161.007	2062.90
1/2	161.792	2083.07
3/4	162.577	2103.35
52 ins.	163.363	2123.72
1/4	164.148	2144.19
1/2	164.934	2164.75
3/4	165.719	2185.42
53 ins.	166.504	2206.18
1/4	167.290	2227.05
1/2	168.075	2248.01
3/4	168.861	2269.06
54 ins.	169.646	2290.22
1/4	170.431	2311.48
1/2	171.217	2332.83
3/4	172.002	2354.28
55 ins.	172.788	2375.83
1/4	173.573	2397.48
1/2	174.358	2419.22
3/4	175.144	2441.07
56 ins.	175.929	2463.01
1/4	176.715	2485.05
1/2	177.500	2507.19
3/4	178.285	2529.42

TABLE 23.—AREAS AND CIRCUMFERENCES OF CIRCLES—BINARY DIVISIONS—(Continued)

Diam.	Cir.	Area
57 ins.	179.071	2551.76
¼	179.856	2574.19
½	180.642	2596.72
¾	181.427	2619.35
58 ins.	182.212	2642.08
¼	182.998	2664.91
½	183.783	2687.83
¾	184.569	2710.85
59 ins.	185.354	2733.97
¼	186.139	2757.19
½	186.925	2780.51
¾	187.710	2803.92
60 ins.	188.496	2827.44
¼	189.281	2851.05
½	190.066	2874.76
¾	190.852	2898.56
61 ins.	191.637	2922.47
¼	192.423	2946.47
½	193.208	2970.57
¾	193.993	2994.77
62 ins.	194.779	3019.07
¼	195.564	3043.47
½	196.350	3067.96
¾	197.135	3092.56
63 ins.	197.920	3117.25
¼	198.706	3142.04
½	199.491	3166.92
¾	200.277	3191.91
64 ins.	201.062	3216.99
¼	201.847	3242.17
½	202.633	3267.46
¾	203.418	3292.83
65 ins.	204.204	3318.31
¼	204.989	3343.88
½	205.774	3369.56
¾	206.560	3395.33
66 ins.	207.345	3421.20
¼	208.131	3447.16
½	208.916	3473.23
¾	209.701	3499.39
67 ins.	210.487	3525.66
¼	211.272	3552.01
½	212.058	3578.47
¾	212.843	3605.03
68 ins.	213.628	3631.68
¼	214.414	3658.44
½	215.199	3685.29
¾	215.984	3712.24
69 ins.	216.770	3739.28
¼	217.555	3766.43
½	218.341	3793.67
¾	219.126	3821.02
70 ins.	219.911	3848.46
¼	220.697	3875.99
½	221.482	3903.63
¾	222.268	3931.36
71 ins.	223.053	3959.20
¼	223.838	3987.13
½	224.624	4015.16
¾	225.409	4043.28
72 ins.	226.195	4071.53
¼	226.980	4099.85
½	227.765	4128.21
¾	228.551	4156.77
73 ins.	229.336	4185.39
¼	230.122	4214.11
½	230.907	4242.92
¾	231.692	4271.83
74 ins.	232.478	4300.85
¼	233.263	4329.95
½	234.049	4359.16
¾	234.834	4388.47
75 ins.	235.619	4417.87
¼	236.405	4447.37
½	237.190	4476.97
¾	237.976	4506.67
76 ins.	238.761	4536.47
¼	239.546	4566.36
½	240.332	4596.35
¾	241.117	4626.44
77 ins.	241.903	4656.63
¼	242.688	4686.92
½	243.473	4717.30
¾	244.259	4747.79
78 ins.	245.044	4778.37
¼	245.830	4809.05
½	246.615	4839.83
¾	247.400	4870.70
79 ins.	248.186	4901.68
¼	248.971	4932.75
½	249.757	4963.92
¾	250.542	4995.19
80 ins.	251.327	5026.56
½	252.898	5089.58
81 ins.	254.469	5153.00
½	256.040	5216.82
82 ins.	257.611	5281.02
½	259.181	5345.62
83 ins.	260.752	5410.62
½	262.323	5476.00
84 ins.	263.894	5541.78
½	265.465	5607.95
85 ins.	267.035	5674.51
½	268.606	5741.47
86 ins.	270.177	5808.81
½	271.748	5876.55
87 ins.	273.319	5944.66
½	274.889	6013.21
88 ins.	276.460	6082.13
½	278.031	6151.44
89 ins.	279.602	6221.15
½	281.173	6291.25
90 ins.	282.743	6361.74
½	284.314	6432.62
91 ins.	285.885	6503.89
½	287.456	6575.56
92 ins.	289.027	6647.62
½	290.597	6720.07
93 ins.	292.168	6792.92
½	293.739	6866.16
94 ins.	295.310	6939.79
½	296.881	7013.81
95 ins.	298.451	7088.23
½	300.022	7163.04
96	301.593	7238.25
97	304.734	7389.83
98	307.876	7542.98
99	311.018	7697.70
100	314.159	7854.00
101	317.301	8011.86
102	320.442	8171.28
103	323.584	8332.29
104	326.725	8494.87
105	329.867	8659.01
106	333.009	8824.73
107	336.150	8992.02
108	339.292	9160.88
109	342.433	9331.32
110	345.575	9503.32

For larger integral circles see Table 20.

TABLE 24.—AREAS OF CIRCLES OF WIRE GAGE DIAMETERS

No. of gage	Birmingham or Stubb's iron wire	Brown and Sharpe	British Imperial or new British	United States	Stubb's steel wire	American Steel & Wire Co.
0000	.162	.167	.126	.130	....	.122
000	.142	.132	.108	.111	....	.104
00	.113	.104	.095	.093	....	.086
0	.091	.083	.082	.077	....	.074
1	.071	.066	.071	.062	.041	.063
2	.064	.053	.060	.056	.038	.054
3	.053	.041	.050	.049	.035	.047
4	.045	.033	.042	.043	.034	.040
5	.038	.026	.035	.038	.033	.034
6	.032	.020	.029	.032	.031	.029
7	.025	.016	.024	.027	.031	.024
8	.021	.013	.020	.024	.031	.020
9	.017	.010	.016	.019	.030	.017
10	.014	.008	.013	.016	.028	.014
11	.011	.0063	.010	.013	.027	.012
12	.009	.0055	.009	.009	.027	.009
13	.0071	.0039	.0063	.0071	.026	.0063
14	.0055	.0031	.0047	.0047	.025	.0047
15	.0039	.0024	.0039	.0039	.025	.0039
16	.0031	.0024	.0031	.0031	.024	.0031
17	.0024	.0016	.0024	.0024	.024	.0024
18	.0016	.0016	.0016	.0024	.022	.0016
19	.0016	.0008	.0016	.0016	.021	.0016
20	.0008	.0008	.0008	.0008	.020	.0008
21	.0008	.0006	.0008	.0008	.020	.0008
22	.0006	.0005	.0006	.0008	.019	.0006
23	.0005	.0004	.0005	.0006	.018	.0005
24	.0004	.0003	.0004	.0005	.018	.0004
25	.0003	.0002	.0003	.0004	.017	.0003
26	.0002	.0002	.0002	.0003	.016	.0002

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